ECFA WORKSHOP ON LEP 200

Aachen, Federal Republic of Germany
29 September – 1 October 1986

PROCEEDINGS
Editors: A. Böhm
W. Hoogland

Vol. I
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ABSTRACT

On the initiative of the European Committee for Future Accelerators, a workshop was organized to study experimental and theoretical aspects of the physics feasible at LEP with the beam energy increased to its design value of 100 GeV per beam. These two volumes provide the written versions of the reports presented by nine working groups to the workshop, as well as the presentations on the machine upgrade itself and the potential of hadron colliders and of HERA for LEP200 physics. Each working group studied a specific topic, evaluating in some detail the requirements both for the detectors and for the machine of W mass measurements, W decay properties, W production dynamics, electroweak radiative corrections, two-photon physics, new heavy quarks and leptons, Higgs particles, supersymmetric particles, and composite models. The Proceedings also include some of the more detailed work done in the framework of the working groups.
PROGRAM OF THE ECFA-WORKSHOP LEP200

Monday 29 September 1986

Morning session
9.30 - 10.00  Opening of the ECFA-Workshop LEP200
             H.-D. Ohlenbusch : Rektor of the RWTH Aachen
             W. Hoogland : Chairman of the Workshop
              H. Schopper : Director-General of CERN

10.00 - 10.30 E. Picasso  Status of LEP
11.00 - 12.00 E. Keil  LEP at 200 GeV c.m. energies
12.00 - 13.00 H. Lengeler  Upgrading of LEP-energies by superconducting cavities

Afternoon session
15.00 - 16.00 P. Roudeau  W-mass measurement
16.30 - 17.30 E. Longo  W-decay properties
17.30 - 18.30 H. Davier  W-production dynamics

Tuesday 30 September 1986

Morning session
9.00 - 10.00  F. Dydak  Electroweak radiative correction
10.30 - 11.30 D.J. Miller  Two-photon physics
11.30 - 12.30 P. Igo-Kemenes  New heavy quarks and leptons

Afternoon session
14.30 - 15.30 Sau Lan Wu  Higgs particles
16.00 - 17.00 C. Dionisi  Supersymmetric particles
17.00 - 18.00 D. Treille  Composite models

Wednesday 1 October 1986

Morning session
9.00 - 10.00  R. Rückl  Physics at HERA
10.30 - 11.30 P. Jenni  Future physics at hadron colliders
11.30 - 12.30 D. Perkins  Workshop summary
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CONVENORS OF WORKING GROUP

W-mass measurement
W-decay properties
W-production dynamics
Electroweak radiative corrections
Two-photon physics
New heavy quarks and leptons
Search for Higgs particles
Supersymmetric particles
Composite models
P. Roudeau
E. Longo
M. Davier
F. Dydak
D.J. Miller
P. Igo-Kemenes
Sau Lan Wu
C. Dionisi
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FOREWORD

In March 1985 the LEPC organized a jamboree to update the physics with LEP in view of both the experimental and theoretical developments since the LEP program was formulated. Some work was also done on the physics that could be done when the beam energy would be increased to the design value of 100 GeV.

These studies clearly confirmed the unique physics potential of LEP when crossing the W-pair threshold. However, it was felt that a more elaborate study was needed to better assess the experimental requirements of LEP200 physics both for the machine and the detectors.

In the beginning of 1986 ECFA therefore took the initiative to organize a workshop specializing on LEP200. It was fortunate to find the Rheinisches Westfälisches Technische Hochschule Aachen willing to act as a local organizer and host a three-day meeting, in which the work of a series of working groups would be summarized. The meeting was held in September 1986, in order to have the results available to the CERN management in time for discussions on the long range program of CERN and for the planning of the construction program after 1990. The timescale was also dictated by the heavy commitments of the physicists as a consequence of the installation schedule of the LEP experiments.

Nine working groups were defined covering the following topics:

- W-mass measurements
- W-decay properties
- W-production dynamics
- Electroweak radiative corrections
- Two-photon physics
- New heavy quarks and leptons
- Higgs particles
- Supersymmetric particles
- Composite models

Moreover, at the Aachen meeting there were presentations on the machine upgrade itself as well as on the potential of hadron colliders and of HERA for LEP200 physics.

The working groups had an experimental convener, who is actively participating in one of the four LEP experiments, supported by a theoretical subconvener. The link with the LEPC jamboree was provided by having the organizers of the latter, Ellis and Peccei, act as supervisors. Care was taken that the four experiments were all represented in the individual working groups. There was an excellent and fruitful cooperation between the experimentalists and the theorists in the study groups, which has greatly contributed to the success of the workshop.

The working groups were asked to assess the requirements with respect to the machine performance i.e. beam energies, energy spread, minimum luminosity and possibly polarisation, as well as to study the capabilities of the presently constructed detectors for the experimental program foreseen with LEP200.

A standard list of parameters was given to the working groups so that results would be based on common assumptions. Three beam energies were suggested: 67, 88 and 95 GeV, with beam energy spread 60, 125 and 160 MeV respectively. The total yearly integrated luminosity was fixed to 500 pb$^{-1}$. In addition the groups were asked to evaluate the need for beam energies above 95 GeV. The $Z^0$ and W masses were fixed to 93 GeV and 82 GeV respectively, leading to a value of $\sin^2\theta_W=0.223$.

The meeting in Aachen was attended by more than 200 participants. All member countries were represented. In addition there were participants from the USA, Canada, the GDR, Finland and Israel.

The write-ups of the talks in Aachen address in detail the questions asked by the organizers on the physics accessible to LEP200, on the requirements concerning beam energy, luminosity etcetera and
on the performance of the presently constructed detectors. Here we give a very brief summary of the major conclusions:

- The studies confirm the outstanding physics potential of LEP200. In many ways LEP200 will offer unique possibilities in checking the standard electroweak theory, searching for Higgs particles and exploring new physics, the simplicity of the e⁺e⁻ system being a decisive asset.
- The upgrade of LEP to a machine having a maximum beam energy of 95 GeV appears to be feasible without major problems. In particular the technology for superconducting RF cavities is so far advanced that cavities can be produced by industry. With reasonable assumptions on budget profiles and production rates for SC cavities, LEP could reach its design goal of about 100 GeV/beam around 1995.
- A beam energy of 95-100 GeV seems to be sufficient for most of the physics goals. However, the effects of new physics on the cross section of e⁺e⁻ → W⁺W⁻ would show up clearer if we were running at energies well above the W⁺W⁻ threshold.
- Some running below the threshold for W-pair production was wanted by several study groups.
- The luminosity of the machine seems adequate for most purposes, but an increase would help in improving certain limits.
- Polarisation, either longitudinal or transverse, was not a big issue for LEP200 physics. It was, however, stressed, that for Z⁰ physics it would be extremely important to have longitudinally polarised beams.
- The presently constructed detectors seem to be adequate for the physics anticipated with LEP200. However, some improvements are recommended: the hermiticity of the detectors should possibly be improved. High resolution vertex detectors should be installed to identify heavy quarks. A small diameter of the beam pipe would help here.
- More work on radiative corrections is needed. In particular a standard approach for handling the effect of radiative corrections on the data is required.

Thanks to the large amount of work of the many people participating in the study groups, and in particular the tremendous efforts and the enthusiasm of the conveners, the LEP200 workshop has become a great success. We thank all the speakers and participants for their contributions, which not only made the meeting in Aachen a very stimulating three days of discussion on very interesting physics issues, but also led to a better assessment of the experimental requirements for both machine and detectors. The excellent organisation by the local organizers and the hospitality extended to the participants by the RWTH Aachen and the city of Aachen largely contributed to the excellent atmosphere at the meeting and thus to its success.

Finally we wish to express our gratitude to the Bundesministerium für Forschung und Technologie, the Deutsche Forschungsgemeinschaft and the Wissenschaftsministerium des Landes Nordrhein-Westfalen, for the financial support of the workshop. We also thank the CERN management for its support, in particular for the printing of these proceedings.

A. Böhm and W. Hoogland
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LEP200 WORKSHOP SUMMARY

D.H. Perkins,
University of Oxford, Nuclear Physics Laboratory, Oxford.

1. INTRODUCTION

The LEP200 ECFA Workshop took place in the historic city of Aachen, which besides being the 9th Century capital of Charlemagne, was also the home of the very first event of the type $\bar{\nu}_\mu e \rightarrow \bar{\nu}_e e$, found in December 1972 in film from the Gargamelle chamber. When I saw this event—Helmut Faisstner brought it over to Oxford—I was convinced that neutral currents really existed, because I had spent some time calculating the background and knew it must be less than 0.01 events. Recall that, to within a few months, December 1972 was historic for another reason—the 100th anniversary of the publication by Maxwell of his treatise on the unification of electricity and magnetism. Apparently Maxwell’s powers of prediction were even greater than we thought!

I want to say that, as an outsider with no personal involvement in LEP, I was enormously impressed by the tremendous efforts made by so many physicists in assessing the physics possibilities at LEP200 and evaluating how this physics would be dug out from the various LEP detectors. The results were presented by some 14 speakers over 2½ days. I am not able or competent to discuss all these contributions, and will select what I thought were the most interesting and crucial points, and offer my deep apologies to those whose work I left out because of shortage of time.

2. THE MACHINE SCENARIOS

The upgrade of LEP from 55 GeV per beam (to be realised in Spring 1989) to the full design energy (100 GeV per beam) was discussed by Picasso, Keil and Lengeler. Basically, there are 2 possibilities (Table 1). Scenario 1 consists of adding 64 superconducting cavities (reaching fields of 5–10 MV/m) to the 128 copper cavities (1.5 MV/m) to reach $E_b = 73–77$ GeV as a first stage; and later replacing the Cu cavities with 128 SC cavities, reaching 84–92 GeV. This scheme replaces copper cavities at 2 RF stations, 2 and 6. Scenario 2 envisages as a first stage, 126 Cu cavities plus 64 SC cavities in RF stations 2 and 6 as before, then 64 SC cavities at new RF stations in intersection regions 4 and 8, (giving $E_b = 82–88$ GeV), and finally 128 SC cavities in 4 and 8 and 128 SC cavities (no copper) in 2 and 6 (giving $E_b = 90–98$ GeV). The second scenario involves excavation of new klystron galleries in 4 and 8 and is thus considerably more expensive. The timescale for manufacture and assembly for scenario 1 is 4–5 years and scenario 2, 5–6 years (see Fig.1).

<table>
<thead>
<tr>
<th>Scenario</th>
<th># cavities</th>
<th>$E_b$ (GeV)</th>
<th>Cost (Msf)</th>
<th>Timescale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>128 Cu, 64 SC</td>
<td>73–77</td>
<td>91</td>
<td>4–5 yrs</td>
</tr>
<tr>
<td>192 SC</td>
<td>84–92</td>
<td>152</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>IR 2 &amp; 6</td>
<td>IR 4 &amp; 8</td>
<td>$E_b$ (GeV)</td>
<td>Cost (Msf)</td>
</tr>
<tr>
<td></td>
<td>128 Cu, 64 SC</td>
<td>73–77</td>
<td>91</td>
<td>5–6 yrs</td>
</tr>
<tr>
<td>128 Cu, 64 SC</td>
<td>64 SC</td>
<td>82–88</td>
<td>172</td>
<td></td>
</tr>
<tr>
<td>128 SC</td>
<td>128 SC</td>
<td>90–98</td>
<td>234</td>
<td></td>
</tr>
</tbody>
</table>

The staged upgrade of LEP should proceed to as high an energy as possible, consistent with finance and the political will of the Member States to get the job completed. Incidentally, the limit set by the
FIG. 1

LEP 200 - possible schedule

- Production: 8 cavity system/month
- Installation: 8 cavity system/week
dipole magnets is $E_b \sim 110$ GeV.

While, for $M_W = 82$ GeV, $E_b = 84$ GeV is adequate to achieve $W^+W^-$ pair production (with half the peak cross-section at $E_b = 95$ GeV), it is too low to be useful for detailed checks of the gauge couplings and measurement of transverse and longitudinal $W$ cross-sections, for which a greater range of beam energies, up to $E_b \sim 90 - 95$ GeV is needed. Obviously, the highest possible energy must be achieved also in the search for new phenomena.

The situation in LEP200 luminosity is unclear; different assumptions could have $\mathcal{L}$ increasing or decreasing as the energy increases. However the general expectation is that $\mathcal{L}$ will increase with $E_b$, perhaps even compensating for the fall-off of the pointlike cross-section ($\sigma_{\text{point}} \propto 1/E_b^2$). The results presented assumed a luminosity $\mathcal{L} \sim 2.10^{31} \text{cm}^{-2}\text{sec}^{-1}$, or 500 pb$^{-1}$ per crossing point for experimental runs extending over 1 or 2 years. This assumes a beam current of 3mA, although 6mA may be possible. The beam energy spread was assumed to vary from 100 MeV at $E_b = 84$ GeV to 160 MeV at $E_b = 95$ GeV.

A major issue is whether, as at LEP100, longitudinally-polarized $e^\pm$ beams will be necessary. The general consensus was that, unless large departures from the standard model behaviour are found using unpolarized beams, it is probably not worth the effort and reduction in luminosity.

3. MAIN PHYSICS OBJECTIVES

It seems to me that the physics objective of LEP200 fall into 3 main categories, which I shall discuss later. They are:

(i) "Surefire" Physics

These include the processes which we know have to exist, and will constitute a unique confirmation (or refutation) of the standard model. I regard these things as the "bread and butter" of the LEP200 project. They include:
- $\sigma(e^+e^- \rightarrow W^+W^-); \gamma WW, ZWW$ couplings
- Precise measurement of $M_W$, check on radiative corrections
- Ratio of longitudinal to transverse $\sigma_{W_\ell}/\sigma_{W_T}$ and production angular distributions $d\sigma_W/d\cos \theta$
- $d\sigma_W/d\cos \theta$, limits on anomalous magnetic/quadrupole moments of $W$.
- $W \rightarrow Q\bar{Q}, \ell\nu$. Study of leptonic and hadronic decays of $W^\pm$, identification of heavy flavours of quark, measurement of elements of $KM$ matrix.

(ii) "Probable" Physics

These include the processes which ought to be seen, and which we know should exist, provided $E_b$ is high enough. The outstanding example in this class is the search for the Higgs boson, the so-far missing link of the electroweak theory. If the Higgs does not exist or is a composite, other indications on the nature of spontaneous-symmetry breaking ought to be detectable.

(iii) "Possible" Physics

Many new phenomena have been predicted by theorists on the energy range $\sqrt{s} = 0.1 - 10$ TeV, that is, roughly on the Fermi scale of energy, $G^{-1}$. These include composite quarks and leptons, excited leptons, new heavy leptons and bosons, supersymmetric particles, anomalous electroweak couplings etc: and, of course, completely new and unexpected processes. There is neither any certainty that any of these things will exist, or that the energy of LEP200 will be sufficient to reveal them if they do. Nevertheless, possible new physics is surely a very strong reason for the upgrade of LEP100 to LEP200.
4. W PAIR PRODUCTION AND THE W MASS

The pair production of W bosons

\[ e^+e^- \rightarrow W^+W^- \]  \hspace{1cm} (1)

has a cross-section shown in Fig.2. It rises steeply from the threshold \( E_b = M_W = 82 \) GeV, with \( \sigma_{\text{max}} \approx 18 \text{pb} \) at \( E_b \approx 100 \) GeV; thereafter it falls as \( (\ln s)/s \). For an integrated luminosity of 500pb\(^{-1}\) we expect, for \( \sigma = \sigma_{\text{max}} \), 5–10,000 events for each of the 4 LEP experiments. (The corresponding peak cross-section for \( e^+e^- \rightarrow Z^0Z^0 \) is about one order of magnitude less).

![Fig. 2.](image)

The reaction (1) can be detected in the 4-jet cross-section, with both W's decaying to \( QQ \) pairs, and accounting for about half of all the sample. The QCD 4-jet background has a cross-section \( \sigma_{\text{QCD}} \sim a_3^2 R_a \) (point), where \( R \approx 5 \), i.e. \( \sigma_{\text{QCD}} \sim 0.4 \text{pb} \). By requiring that the 4-jet energy be consistent with \( 2E_b \), and that the minimum value of the jet-pair energy satisfy \( (E_b - E_{jj})_{\text{min}} < 10 \) GeV, the background can be reduced below 1% and the genuine WW pair events be retained with 70% efficiency.

The leptonic decays \( W_1 \rightarrow Qq, W_2 \rightarrow t\nu \) and \( W_1, W_2 \rightarrow \ell, \nu \) are equally clear. They are identified on the basis of 2 jets + isolated charged lepton (or 2 isolated leptons) with longitudinal momentum imbalance \( \Delta p_L > 20 \) GeV/c. It is estimated that genuine events can be detected with 55% efficiency and the background is again < 1%.

Using the expected sample of events, Roudeau discussed precision determination of the W mass, by 4 methods:

(i) Measurement of the excitation function \( \sigma_{WW}(E) \) near threshold.

(ii) Determination of the end point of the electron spectrum in \( W \rightarrow e\nu_e \).

(iii) Jet-jet invariant mass from decays \( W \rightarrow 2 \) jets.

(iv) Reconstruction of leptonic decay events, and determination of \( e\nu \) invariant mass.

Regarding Method (i), for a total exposure of 500pb\(^{-1}\) optimally distributed at 5 beam energies near threshold, the total standard error—systematic and statistical—is estimated to be \( \sigma_{M_W} < 100 \) MeV.

Fig.3 shows the form of the electron spectrum for Method (ii) near the end-point. Even with excellent lepton energy resolution, \( \sigma_E/E \leq 0.12/\sqrt{E} \) (\( E \) in GeV) and with < 1% calibration errors, it seems unlikely that the error on \( M_W \) by this method could be reduced below 300 MeV.

The proposed measurement of the 2-jet invariant mass, using Method (iii), uses re-scaling procedures to correct systematic errors in calibration of the calorimeters (beam energy constraint). Monte Carlo calculations suggest one could reach \( \sigma_{M_W} \sim 100 \) MeV.
The errors in Method (iv)—reconstruction of $W$ mass from leptonic decay $W \to e\nu$, taking account of beam energy and longitudinal and transverse momentum imbalance to determine the neutrino momentum vector—also depend crucially on absolute electron energy measurements. A number of studies by different groups suggest that a precision of order 100 MeV in $M_W$ might be attainable.

In summary, it should be possible to attain a precision on $M_W$ of 100 MeV or better, the most promising method being the observed variation of $WW$ cross-section with beam energy near threshold. Unfortunately, this does not provide the maximum number of $W$ events or $W$ pair production in the region $E_b \sim 90$ GeV, which is considered interesting from the viewpoint of production dynamics, discussed below.

The significance of the precise measurement of $M_W$ is in testing radiative corrections to the electroweak theory. The mass of the $W$ and $Z$ are related by the formula

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2G(1 - M_W^2/M_Z^2)} \left(1 - \Delta r\right)} \frac{1}{\Delta r}$$

where $\Delta r$ is the radiative correction to the boson mass, arising from loop diagrams due to virtual boson and fermion exchanges as in Fig.4. For a top quark mass $M_t = 35$ GeV and Higgs mass $M_H = 100$ GeV, $\Delta r \approx 0.07$. Most of the radiative correction arises from the photon exchange loop, and corresponds to the QED renormalization of $\alpha$ in (2), and is not interesting. The interesting contributions to $\Delta r$ are at the level of $\sim 0.01$ only. For example, an increase of the Higgs mass $M_H$ from 100 GeV to 1 TeV decreases $\Delta r$
\[ \delta(1 - \Delta r) \approx \sqrt{20 \left( \frac{\delta M_W}{M_W} \right)^2 + 40 \left( \frac{\delta M_Z}{M_Z} \right)^2} \]  

so that \( \delta M_W/M_W = \delta M_Z/M_Z = 0.001 \) gives \( \delta(1 - \Delta r) \approx .008 \). Therefore, one must aim for a precision in the measurements of \( M_Z \) and \( M_W \) to 1 part per mil or better. This precision is on \( M_Z \) is certainly attainable at LEP100, and it seems that a similar precision on \( M_W \) should be obtained at LEP200. There are therefore good grounds to expect that with LEP200, a “final” and sensitive test of the electroweak theory will be possible.

Precise tests of radiative corrections of course depend critically on the performance of the LEP detectors, their energy resolution, absolute energy calibration and sensitivity, for example, to low energy bremsstrahlung from electrons produced in leptonic \( W \)-decays. Each detector will be different in these respects, and Dydak gave a comprehensive review of the problems of comparing results from the different detectors, and made a plea for each experiment to provide a subset of data with identical “canonical cuts”. One advantage of 4 LEP detectors is that one will quadruple the total number of events, and each experiment will cross-check the others, but is is indeed most important to have a clear and agreed basis for comparison.

5. \( W \) PRODUCTION DYNAMICS

The process \( e^+e^- \rightarrow W^+W^- \) proceeds in leading order via 3 diagrams, as shown in Fig.5. Each of these diagrams on its own is divergent, leading to \( \sigma \propto s \). Recall that introduction of the \( Z^0 \) exchange in Fig.5(b) in the Weinberg-Salam model was expressly to cancel the divergences in the other diagrams. The finiteness of the cross-section as a function of energy (Fig.6) indeed depends on gauge cancellations and on those having the exact values of the couplings prescribed by the electroweak theory (e.g. \( g_{ZWW} = e \cot \theta_W \)). So the study for \( \sigma(e\bar{e} \rightarrow W\bar{W}) \) as a function of energy will provide, for the first time, detailed tests of the 3 gauge vertex.

![Diagrams](image)

Fig. 5.

It might be useful to emphasize here that the process \( e^+e^- \rightarrow Z^0\bar{Z}^0 \) is not so interesting from this viewpoint; the relevant diagram is analogous to Fig.5(a), but the analogue of Fig.5(b) should not exist, since the \( ZZZ \) coupling vanishes (conservation of weak isospin; \( I = 1, I_3 = 0 \) \( \lor \) \( I = 1, I_3 = 0 \) \( \lor \) \( I = 1, I_3 = 0 \)).

The second important point about the dynamics of the process \( e^+e^- \rightarrow W^+W^- \) is that it also provides, for the first time, a look at the longitudinal components of the \( W \) boson. Let us recall that, in exact \( SU(2) \times U(1) \) symmetry, the weak isospin triplet \( w^+, w^-, w^0 \) and singlet \( b^0 \) are massless vector bosons, which are therefore purely transverse and occur in only 2 helicity states, \( \lambda = +1 \) and \( \lambda = -1 \).
Spontaneous symmetry breaking is provided by the Higgs mechanism. The Higgs (in the Weinberg-Salam model) is a weak isospin doublet of complex fields, with 4 components denoted \( \phi^+, \phi^-, (\phi - \bar{\phi})/\sqrt{2} \) and \((\phi + \bar{\phi})/\sqrt{2}\). The Higgs generates mass by self-interaction. One composite of the \(w^\circ\) and \(B\) remains massless as the photon, while the orthogonal combination absorbs \((\phi - \bar{\phi})/\sqrt{2}\), to become the massive \(Z^0\), while \(w^\pm\) “eat” the fields \(\phi^+\) and \(\phi^-\) to become the massive \(W^+\) and \(W^-\). These Higgs fields therefore appear as the longitudinal degrees of freedom (helicity \(\lambda = 0\) states) of \(Z^0, W^\pm\). Hence these latter provide a crucial test of the theory. In contrast, in the production of \(W, Z\) in \(p\bar{p}\) collisions or in \(e^+e^- \rightarrow Z^0\), only the transverse components are involved.

![Graph](image)

**Fig. 6.**

Fig. 7 indicates the helicity states involved in (a) electron scattering via a vector or axial vector field. Helicity is conserved (in the relativistic limit), a LH (RH) electron being scattered as a LH (RH) electron. Replacing an outgoing electron by an incoming positron, in the process \(e^+e^- \rightarrow \text{anything}\), only LR and RL combinations are involved. Because the \(\nu\) exchange graph (Fig.5c) dominates the process \(e^+e^- \rightarrow W^+W^-\), the \(e_L^e\) and \(e_R^e\) combination is the more important, and for this reason also, the \(W^+\) and \(W^-\) produced are peaked in the directions \(\vec{p}_e^+\) and \(\vec{p}_e^-\) respectively. For exact forward-backward production, as indicated in Fig.7(b), the initial state of \(J_z = -1\) requires \(\lambda = 1\) for \(W^+\) and \(\lambda = 0\) for \(W^-\), or \(\lambda = 0\) for \(W^+\) and \(\lambda = -1\) for \(W^-\). In any case, the longitudinal component \(\lambda = 0\) should occur with frequency comparable with (and equal to, in the forward-backward limit) the transverse state \(\lambda = +1\) or \(-1\).

![Diagram](image)

**Fig. 7.**
Fig. 8 shows (a) the angular distribution of $W^+$ production at two energies; (b) the contributions for one $W$ being transverse (and the other, either $L$ or $T$), and for one $W$ being $L$ (and the other, $L$ or $T$), for both $W^+$ and $W^-$ combined. (This for the case where $W \rightarrow 2$ jets and the sign of charge is not determined); (c) and (d), contributions for the various $LL$, $TT$, $TL$ combinations at two beam energies. As explained before, for $\theta = 0$ or $\pi$, the $LL$ and $TT$ contributions vanish.

These plots show that the production angular distribution depends sensitively on beam energy and provides a good test of the standard model.

The measurement of the $W$ angular distribution is more sensitive to QCD background, in the case where both $W$'s $\rightarrow 2$ jets, than is the total cross-section. So, the events in which one $W \rightarrow e\nu$ are safer, and one can even measure the double differential distribution with respect to the production angle $\theta$ of the $W$ relative to the beam axis, and the decay angle $\theta^*$ of the lepton in the $W$ rest-frame. The latter have
distributions of the form $\sin^2 \theta^*$ for $W_L$ and $(1 + \cos^2 \theta^*)$ for $W_T$ (adding $W^+$ and $W^-$)

$$\frac{d^2 \sigma}{d \cos \theta d \cos \theta^*} = \frac{3}{4} \sin^2 \theta^* \left( \frac{d\sigma_L}{d \cos \theta} \right) + \frac{3}{8} (1 + \cos^2 \theta^*) \left( \frac{d\sigma_T}{d \cos \theta} \right)$$

allowing the extraction of the polarized cross-sections.

6. ANOMALOUS W COUPLINGS

The plots given in Fig.8 were for the standard model couplings. However, Davier also mentioned the possibility of anomalous $\gamma WW$ and $ZWW$ couplings. An anomalous $\gamma WW$ coupling, for example, will result in magnetic dipole and electric quadrupole moments of the $W$ described by coefficients $k$ and $\lambda$:

$$\mu_W = \frac{e}{2M_W} (1 + k - \lambda)$$
$$Q_W = -\frac{e}{M_W^2} (k - \lambda)$$

where, in the standard model, $k = 1$ and $\lambda = 0$. Analogous parameters $k'$ and $\lambda'$ correspond to anomalous $ZWW$ couplings. These parameters determine deviations in the production angular distribution from the standard model. The best that can be done is to hold 3 of the 4 parameters $k$, $\lambda$, $k'$, $\lambda'$ fixed and ask what effect a variation in the fourth will have. In a 500pb$^{-1}$ run, roughly 20% deviations in $k$, $\lambda$ should be detectable. In any case, large anomalous couplings should stand out clearly (although in the total cross-section, they are harder to detect).

The advantages of polarized beams was a strong point of discussion at the LEP200 workshop. With unpolarized beams, the process $e^+e^- \rightarrow W^+W^-$ strongly selects the $\epsilon_L^+\epsilon_R^-$ combination. The unfavoured combination $\epsilon_R^+\epsilon_L^-$ with standard couplings gives rates 1–2 orders of magnitude less—and therefore hardly observable. So, one would not learn very much in this case. However, if measurable deviations from the $SM$ predictions are observed with unpolarized beams, it would obviously be necessary to use polarized beams in order to investigate the anomaly. So the policy would be to wait and see. Since polarized beams will be available, eventually, at LEP100, it would not be a very major operation to provide them at LEP200, if the scientific case is demonstrated in the early runs at the higher beam energy.

7. W DECAY AND WEAK QUARK AND LEPTON COUPLINGS

The existence at LEP200 of a large number of $W$-decays makes possible unique tests of the universality of the weak couplings of leptons and quarks, as discussed at the Workshop by Longo.

First, for an integrated luminosity of 500pb$^{-1}$, it should be possible to compare the leptonic branching ratios of the $W$’s to an accuracy of order 2%:

$$\Delta(g_\mu/g_e) \simeq \pm.018 \quad \Delta(g_\tau/g_\mu) \simeq \pm.020$$

The second result represents an improvement by a factor 4 on the accuracy of present measurements (at UA1).

For the hadronic decays, attempts would be made to isolate the decays $W \rightarrow t\bar{b}$ and $c\bar{s}$, using cuts in aplanarity, thrust and sphericity to select heavy quark jets. In particular, it might be possible to “tag” $b$ and $c$ decays by detecting the heavy quark decay vertex (i.e. by measuring the impact parameter for non-pointing tracks). If this is possible—and it will need very good vertex detectors—the branching ratio $W \rightarrow t\bar{b}$ provides a sensitive measure of the $t$ quark mass, with $\Delta M_t \sim 1–2$ GeV for 500pb$^{-1}$ luminosity.

If LEP200 experiments are able to identify decays of $W$’s to different quark flavours, they should provide dramatic improvements in the accuracy of measurement of the elements of the $KM$ matrix. An
important feature is that some of the systematic errors cancel since one is measuring branching ratios and since 5-10,000 $W \to 2$ jet decays would be involved in each of the 4 experiments, statistical errors would be very small.

8. THE HIGGS BOSON SEARCH

If the conventional neutral Higgs is not massive ($M_H \leq 40$ GeV) there are good prospects for detecting it at LEP100. If $M_H < M_t$ where $\theta = t\bar{t}$ is the toponium bound state, then the branching ratio for

$$\theta(= t\bar{t}) \to H^0\gamma$$  \hspace{1cm} \text{(6)}

will be at the percent level and easily detectable. If $M_H < M_Z$ then the decay

$$e^+e^- \to Z^0 \to H^0l^+l^-$$  \hspace{1cm} \text{(7)}

will have a small but measurable branching ratio of order $10^{-5}$ or more, if $M_H \leq 40$ GeV.

For larger Higgs masses, $40 < M_H < 80$ GeV, LEP200 presents unique possibilities, reviewed by S.L. Wu at the workshop. The most suitable reaction is

$$e^+e^- \to H^0Z$$  \hspace{1cm} \text{(8)}

via the diagram

and with a cross section shown in Fig.9. For $M_H < E_{\text{beam}}$, the cross-section ranges from 0.3pb ($M_H = 100$ GeV) to 2pb ($M_H = 40$ GeV). Thus, production rates per crossing-point are in the region of 100-1000 events for 500pb$^{-1}$ integrated luminosity. This is a dismally low rate (1 event per day $\to$ 1 event per week)—a typically high risk, high return situation which a physicist should face as a challenge.

The principal Higgs decay modes with branching ratios and topologies are as follows:

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>BR</th>
<th>Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \to H^0Z^0$</td>
<td>18%</td>
<td>(2j + M\mu)</td>
</tr>
<tr>
<td>$\downarrow \to \nu\bar{\nu}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\downarrow b\bar{b}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\to H^0Z^0$</td>
<td>8%</td>
<td>(2j + 2\ell)</td>
</tr>
<tr>
<td>$\downarrow \to e\bar{e}, \mu\bar{\mu}, \tau\bar{\tau}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\downarrow b\bar{b}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\to H^0Z^0$</td>
<td>74%</td>
<td>(4j)</td>
</tr>
<tr>
<td>$\downarrow Q\bar{Q}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Clearly, since the Higgs coupling is proportional to fermion mass, decay to heavy quark pairs is the only detectable process.
The main backgrounds to reaction 9(a) are $e^+e^- \rightarrow (Z^0 \rightarrow 2j) + (Z^0 \rightarrow \nu\bar{\nu}), e^+e^- \rightarrow (W \rightarrow 2j) + (W \rightarrow \nu\tau, \tau \rightarrow \nu\bar{\nu} + \text{soft hadron})$, and $e^+e^- \rightarrow Q\bar{Q}Q\bar{Q} + \cdots$ (QCD). These have cross-sections respectively of 4, 26 and 120 times the Higgs cross-section for $M_H = 80$ GeV. To reduce background, cuts are proposed on the missing-mass to 2 jets ($MM > 92$ GeV); on $p_T$, $p_T$ imbalance ($\Sigma p_T > 30$ GeV, $\Sigma p_Z > 60$ GeV); and on charged multiplicity. Furthermore, constraining $MM = M_Z$ and requiring $\Sigma E = E_{CMS}, \Sigma p' = 0$ for all particles also reduces the background. Table 2(a) shows the results expected from a Monte Carlo simulation.

For reaction 9(b), the main backgrounds are $e^+e^- \rightarrow (Z^0 \rightarrow 2j) + (Z^0 \rightarrow \ell\bar{\ell})$ and $e^+e^- \rightarrow (t \rightarrow e+\text{jet})+(\bar{t} \rightarrow \bar{e}+\text{jet})$. Again, requiring $M(e^+e^-) \equiv M_{Z^0} (93 \pm 5$ GeV), $n_{ch} > 4$, and with $M(2-jet) \equiv M_{Z^0}$ and the overall energy/momentum constraint from the beams, the signal and backgrounds expected are shown in Table 2(b).

Finally, the most prolific rate is for reaction 9(c), which also presents the greatest background problems. The main backgrounds are $e^+e^- \rightarrow (W, Z \rightarrow 2j) + (W, Z \rightarrow 2j)$ with $\sigma \sim 10$ pb, and $e^+e^- \rightarrow Q\bar{Q}Q\bar{Q}$ (QCD 4-jet events), with $\sigma \sim 70$ pb. It is proposed, firstly, to reconstruct the 6 possible $2j$ mass pairs in each 4j event, and then use the overall energy/momentum constraint to exclude non-physical solutions (where extra energy/momentum is missing); thereafter to include events with 2 pairs of jets of equal $M_{jj} > 75$ GeV and thus due to $WW$ or $ZZ$ pairs; to retain events not converging in the $WW, ZZ$ fits but fitting $H^0Z^0$ (for any Higgs mass) with $> 1\%$ probability. Thereafter, cuts in sphereicity are proposed in order to reject QCD background and preferentially select massive quark jets (e.g. $H^0 \rightarrow b\bar{b}$). The results are shown in Table 2(c).
TABLE 2

Higgs event rates ($\int L dt = 500 \text{pb}^{-1}$); $E_{\text{cm, s}} = 200 \text{ GeV}$

<table>
<thead>
<tr>
<th>$M_H$</th>
<th>No. of generated events</th>
<th>No. of events after cuts</th>
<th>No. of background</th>
<th>S/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>107</td>
<td>50</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>60</td>
<td>83</td>
<td>34</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>80</td>
<td>55</td>
<td>12</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M_H$</th>
<th>No. of generated events</th>
<th>No. of events after cuts</th>
<th>No. of background</th>
<th>S/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>36</td>
<td>24</td>
<td>0.2</td>
<td>120</td>
</tr>
<tr>
<td>60</td>
<td>28</td>
<td>17</td>
<td>0.6</td>
<td>26</td>
</tr>
<tr>
<td>80</td>
<td>18</td>
<td>11</td>
<td>3</td>
<td>3-4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M_H$</th>
<th>No. of generated events</th>
<th>No. of events after cuts</th>
<th>No. of background</th>
<th>S/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>430</td>
<td>78</td>
<td>23</td>
<td>3.4</td>
</tr>
<tr>
<td>60</td>
<td>340</td>
<td>60</td>
<td>31</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Fig. 10 shows examples of the mass spectra of signal and background from MC simulations for $M_H = 60$ GeV for reactions 9(a) and 9(c).

Clearly the 4 jet events 9(c) are not useful if $M_H = 80 - 90$ GeV, since there could be complete confusion with $WW$ and $ZZ$ production, and for $M_H > 90$ GeV, the event rate is too low. It is clear that more detailed studies need to be undertaken, but already one can conclude that

(i) LEP200 should detect the standard Higgs if $M_H < 80$ GeV.
(ii) The fact that the energy and momentum of the $HZ^*$ system in the initial state is known, serves as a crucial constraint in extracting the signal; this is not true for $ep$ and $pp, pp$ colliders.

9. NEW PHYSICS AT LEP200

Three contributions were made on new phenomena outside the standard model; a paper on new quarks and leptons by Igo-Kemenes, one by Dionisi on SUSY particles, and one by Treille on composite
models.

a) **New Leptons and Quarks**

The detection of a possible 4th generation of charged lepton $L^\pm$, or of a new heavy neutral lepton, $L^0$, should be possible if they have standard couplings and can be pair-produced. Cross-sections are only of order 1pb but the signatures are very distinctive. For example if $M_{L^\pm} > M_W$, the process would be

$$e^+e^- \rightarrow L^+ L^- \downarrow \downarrow \nu W^+ \nu W^-$$

with substantial missing energy/momentum, and confusion only if $M_{L^\pm} \approx M_W$. As another example, for $M_{L^0} < M_W$, the reaction

$$e^+e^- \rightarrow L^0 \bar{L}^0 \rightarrow e^+ \bar{e}^- \nu \rightarrow e^- \bar{e}^+ \nu$$

results in 4 charged leptons and again, missing energy/momentum.

For heavy quarks—the $t$-quark or a new 4th generation $b'$ quark, the threshold can be determined from measurement of $R$ and the events identified by leptonic decay with a high $p_T$ lepton.

Broadly speaking, the accessible mass range for detection of new leptons and quarks extends up to the kinematic limits of the machine i.e. $M \sim \frac{1}{2} E_{beam} \sim 90$ GeV.

b) **New Bosons**

Massive bosons $Z'$ could result if the $SU(2) \times U(1)$ symmetry of the SM is extended. For $M_{Z'} < 200$ GeV, there would be spectacular effects observable in $\mu^+\mu^-$ asymmetries. For higher values of $M_{Z'}$, the deviations would be small and hard to measure.

c) **Supersymmetric Particles**

The signatures of SUSY particles e.g. $\tilde{\chi} \rightarrow \tilde{\gamma} + \tilde{e}$ are well known: acoplanar pairs of high energy, high $p_T$ electrons with large missing $p_T$ and energy. Again, the mass limits for detection of $\tilde{\chi}, \tilde{\mu}, \tilde{\tau}, \tilde{\nu}, \tilde{W}, \tilde{Z}$ are all set by the pair-production kinematics, that is, $M < 80 - 90$ GeV. The LEP200 experiments should give clear and clean signals. Despite the low cross-sections (typically in the pb region), an integrated luminosity of 500pb$^{-1}$ is more than adequate and it seems clear that the background (e.g. $WW$ pairs) should not be too severe.

d) **Composite Models**

The proliferation of quark and lepton flavours have led to the suggestion that these particles are composites of more fundamental objects or preons. Presumably the preons are bound into quarks and/or leptons by a strong interaction characterized by a compositeness scale $\Lambda$ (essentially, the binding energy). Manifestations of this model could be, for example, leptons with colour, excited leptons, and, at energies small compared with $\Lambda$, residual contact interactions in addition to the known interactions. For example, in Bhabha scattering, this interaction would add a term of the type

$$L_{eff} = \frac{g^2}{2\Lambda^2} \sum_{i,j=L,R} n_{ij}(\bar{\epsilon}_i\gamma_\mu\epsilon_i)(\bar{\epsilon}_j\gamma_\mu\epsilon_j)$$

where $\epsilon_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\epsilon$, $\eta = \pm 1$ and $g^2/4\pi = 1$. The residual interaction would then manifest itself as peculiar combinations of $V$, $A$ or $L$, $R$ couplings between the electrons. Similar phenomena could be observed in $e^+e^- \rightarrow \mu^+\mu^-$, and Fig.11 shows results on the total cross-section and angular asymmetry for $\Lambda_{\mu} = 4$ TeV.
Fig. 11: Top: Departures of total cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ from standard model, due to compositeness with scale $\Lambda_{\mu\mu} = 4$ TeV.
Bottom: Departures in $e^+e^- \rightarrow \mu^+\mu^-$ asymmetry due to contact $VV$ interaction on scale $\Lambda_{\mu\mu} = 4$ TeV.

Typical limits on $\Lambda$—depending somewhat on the $V$, $A$ structures of the contact interaction—for LEP200 are of order $\Lambda \leq 10$ TeV, that is a factor 4 larger than in today's $e^+e^-$ experiments.

10. SUMMARY
In summary, my conclusion is that the case for completing LEP to the full design energy of 200 GeV in the CMS is just as strong today as it was when the project was proposed 5 years ago. In particular LEP 200 offers:

(1) A solid programme for 1995–2000+ of outstanding physics not accessible in any other way. This constitutes the "final check" of the EWI theory via:
   a) $e^+e^- \rightarrow W^+W^-$; checking the gauge vertices.
   b) precise $M_{W^\pm}$; radiative corrections sensitive to $M_H$ etc.
   c) measurement of $\sigma_L$ and $\sigma_T$ for the $W$ from production angular distribution, detection of anomalies in electric and magnetic moments.
   d) study of $W \rightarrow \ell\nu$, $QQ'$; precise measurements of $KM$ matrix element, and of lepton universality.

(2) Big challenges on fundamental questions, of which the most important is the existence of the Higgs boson, and the nature of spontaneous symmetry—breaking. LEP200 should detect the Higgs if
$40 < M_H < 80$ GeV, but it will be difficult. It is not at all clear that hadron colliders (of any energy) would be able to detect the Higgs at all.

(3) A new regime of physics, provided by the factor 4 increase in phase-space in the transition LEP100—LEP200. Many new particles—composites, supersymmetric particles, leptoquarks, new leptons, quarks, $W$’s, $Z$’s have all been predicted and some may even exist. In this respect, the peculiar advantages of $e^+e^-$ collider should be emphasized. The ability to extract rare and unexpected signals from background depends critically on the exploitation of kinematic constraints, which are not available in $p\bar{p}$, $pp$ or $ep$ colliders, where the total energy and momentum of the fundamental participants in the collision are unknown.

(4) Finally it is clear that this programme of physics will place severe demands on accelerator and detectors. Resolution and hermeticity of detectors are clearly important when dealing with event rates of order 1 per day. On the machine energy, it is clear that $E_{\text{beam}} > 90$ GeV is necessary to cover most of the demands of the programme. Polarized beams, which will be provided for LEP100, are not so obviously necessary at the higher energy.
LEP AT 200 GeV c.m. ENERGY

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ABSTRACT

After a brief review of the basic facts of e⁺e⁻ storage ring physics, the scaling laws are given which apply to a doubling of LEP Phase 1 energy. The lattice and equipment modifications needed are presented. The expected performance, energy uncertainty, and beam lifetimes at up to about 100 GeV beam energy are estimated. The problems of background and polarization are briefly reviewed.

1. INTRODUCTION

In this paper, the implications of raising the LEP c.m. energy to about 200 GeV are discussed, as far as beam dynamics and performance are concerned. This approximate doubling of the LEP Phase 1 energy will be achieved by adding a large superconducting RF system which is described in a companion paper [1].

This paper is organized as follows: some basic e⁺e⁻ storage ring physics is discussed in Chapter 2. It is used in Chapter 3 to present the scaling laws for the most important LEP parameters, the requirements on the LEP lattice, and the necessary equipment changes. Chapter 4 contains the LEP performance estimates at a few beam energies up to 95 GeV, and several relevant machine parameters, i.e. the energy spread and energy uncertainty, and the beam lifetime. Background to the experiments and polarization are briefly reviewed in Chapter 5 and 6, respectively. Chapter 7 contains the conclusions.

Table I shows a comparison of LEP parameters most relevant to the experiments at two different beam energies. The parameters at 55 GeV are derived from the LEP Design Report [2], with some later modifications[3].

2. BASIC e⁺e⁻ STORAGE RING PHYSICS

2.1 Collision physics

For beams with total intensity $N$ in each beam, $k$ bunches each, rms beam radii $\sigma_x$ and $\sigma_y$ at the interaction point, revolution frequency $f$, and Gaussian density distribution, the luminosity $L$ is given by:

$$ L = \frac{N^2 f}{4\pi k \sigma_x \sigma_y}.$$  \hspace{1cm} (1)
<table>
<thead>
<tr>
<th>Energy</th>
<th>55</th>
<th>95 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole field</td>
<td>0.0645</td>
<td>0.1114 T</td>
</tr>
<tr>
<td>Injection energy</td>
<td>20</td>
<td>20 GeV</td>
</tr>
<tr>
<td>Tunes</td>
<td>70.35</td>
<td>78.2</td>
</tr>
<tr>
<td>Cell length</td>
<td>79</td>
<td>79 m</td>
</tr>
<tr>
<td>Max. and min. $\beta$-functions</td>
<td>135</td>
<td>46</td>
</tr>
</tbody>
</table>

Experimental Interaction Region Parameters

| Space between quadrupoles | ± 3.5 | ± 3.5 m |
| β-functions               | 1.75  | 0.07  |
| RMS beam radii            | 255   | 15.3  | 209 | 10.8 μm |
| RMS bunch length          | 17.2  | 13.9 mm |
| RMS energy spread         | 0.92 x 10^{-3} | 2.06 x 10^{-3} |

Parameters of the Radiofrequency System

| Frequency          | 352 | 352 MHz |
| Peak voltage       | 400 | 2730 MV |
| Active length of 128 Cu cavities | 272 | - m |
| Active length of 256 s.c. cavities | - | 435 m |
| Peak power of 16 klystrons | 16 | 16 MW |

Performance Parameters

| Current in 4 bunches | 3 | 3 mA |
| Beam-beam tune shift | 0.03 | 0.03 |
| Nominal luminosity   | $1.6 \times 10^{31}$ | $2.7 \times 10^{31}$ cm^{-2}s^{-1} |

Table I - LEP parameters at 55 and 95 GeV

One limit on $L$ is given by the vertical beam-beam tune shift $\xi_y$

$$\xi_y = N r_e e \beta y / 2 \pi k \gamma \sigma_x \sigma_y ,$$

with classical electron radius $r_e$, relativistic factor $\gamma$. The beam-beam tune shift $\xi_y$ describes the change of the tune, i.e. the number of betatron oscillations around the machine, which particles with betatron oscillation amplitudes small compared to $\sigma_x$ and $\sigma_y$ undergo due to the electromagnetic forces between the bunches. Experience with e+e- storage rings over a wide range of energies shows that the maximum value of the beam-beam tune shift, i.e. the beam-beam limit, is a good parameter with little variation between machines [4].
One power of $N$ can be eliminated from the luminosity formula by using the beam-beam tune shift $\delta y$:

$$ L = N f y^2 / 2 r e \delta y . $$

(3)

This expression is valid provided that the beam size and current are controlled such that the beam-beam limit is always reached. It is fairly easy to increase the beam size such that, with increasing current, $\delta y$ remains constant. This is possible up to the point where the required beam size fills the available aperture. It is more difficult to reduce the beam size. Therefore, below some current it will be impossible to reach the beam beam limit, and the luminosity will vary like $N^2$.

2.2 Beam physics

The critical synchrotron radiation energy

$$ E_c = (3/2) h c y^3 / \rho $$

(4)

describes the typical energy of synchrotron radiation quanta. Half the radiation power is in quanta with energy $E_y > E_c$. The dependence on $\gamma$ and the bending radius $\rho$ is given by the duration of the light pulse seen by a stationary observer [5]. At 55 GeV in LEP the critical energy is $E_c = 119$ keV.

The quantization of the synchrotron radiation causes an excitation of the particle oscillations in all three degrees of freedom: horizontal and vertical betatron oscillations, and synchrotron oscillations.

The synchrotron radiation loss per turn is given by:

$$ U_s = (4\pi/3) r_0 E_0 b^3 \gamma^4 / \rho $$

(5)

The rapid increase with $\gamma^4$ is responsible for the rather sharp limit on energy in LEP with a given RF system. At 55 GeV in LEP, the synchrotron radiation loss per turn is $U_s = 260$ MeV.

A circumferential RF voltage $V_{RF} = 400$ MV is needed to compensate the loss, and to ensure an adequate quantum lifetime of the beams.

The synchrotron radiation loss causes a damping of the oscillations in all three degrees of freedom. The sum of decrements per turn $\delta$ at energy $E$ is given by $\delta = 2 U_s/\rho$. Since the particle oscillations should be damped in all three degrees of freedom, the individual damping rates are all smaller than $\delta$. At 55 GeV in LEP the vertical damping time is $\tau_y = 37.8$ ms.
The damping times for horizontal betatron oscillations $\tau_x$ and synchrotron oscillation $\tau_S$ can be adjusted over some range by varying the radial position of the beams in LEP, using minute changes in the RF frequency. This is expressed in terms of the damping partition numbers which obey the sum rule $J_x + J_S = 3$. On the central orbit, $J_x = 1$, $J_S = 2$ (and $J_y = 1$).

The relative energy spread $\sigma_\varepsilon$ in the beams is due to the balance between the excitation due to the quantized emission of synchrotron radiation and synchrotron radiation damping. It can be calculated by describing synchrotron oscillations as a damped harmonic oscillator, driven by stochastic excitation. The result is, with the reduced Compton wavelength $\lambda_C$:

$$\sigma_\varepsilon^2 = (55/32\sqrt{3})\lambda_C \gamma^2/\rho J_S . \quad (6)$$

This result holds for a machine in which all bending magnets have the same bending radius $\rho$. It follows that the relative energy spread $\sigma_\varepsilon$ is proportional to $\gamma$. The energy distribution due to synchrotron radiation excitation and damping is a Gaussian.

Since particles with different energies oscillate around different closed orbits over most of the circumference of LEP, the instantaneous energy losses associated with the emission of synchrotron radiation quanta also cause an excitation of the horizontal betatron oscillations. This is best described by introducing the horizontal emittance $\sigma_x^2/\rho_x$, with horizontal rms beam radius $\sigma_x$ and horizontal amplitude function $\rho_x$, measured at the same position around the circumference. Both $\sigma_x$ and $\rho_x$ vary around the circumference, but $\sigma_x^2/\rho_x$ does not. The average value of $\rho_x = R/Q$ and the average value of the dispersion $R/Q^2$ are properties of the LEP lattice, with average radius $R$ and tune $Q$. Each time an $e^-$ or $e^+$ emits a quantum of energy $E_C$ it continues along its previous path because the transverse photon momentum is very small. But since the energy changes, it oscillates around a new orbit which is displaced from the previous one on average by an amount $(R/Q^2)(E_C/E)$. The result of the calculation is for a machine in which all bending magnets have the same bending radius $\rho$:

$$\sigma_x^2/\rho_x = (55/32\sqrt{3})\lambda_C \gamma^2 R/Q^3 J_x . \quad (7)$$

The emittance is proportional to the square of the energy, and inversely proportional to the cube of the tune $Q$.

The relative energy spread $\sigma_\varepsilon$ and the emittance $\sigma_x^2/\rho_x$ can be modified by wiggler magnets which are arrangement of three dipoles, such that the total integrated field vanishes in order to leave the LEP geometry undisturbed. The central positive field may reach 1 T, a factor of 15.5 higher than the normal dipole field at 55 GeV. It bends particles in the same direction as the normal bending magnets. The fields in the negative direction are at most 0.4 T in order not to reduce the asymptotic degree of polarization too much.
All wigglers enhance the synchrotron radiation loss, and therefore reduce the damping time and increase the relative energy spread \( \sigma_e \) and hence the bunch length \( \sigma_z \). Wigglers also reduce the polarization time.

Two types of wigglers may be distinguished, depending on their position in the LEP lattice. Damping wigglers are installed where the dispersion vanishes, and reduce the emittance in the same proportion as the damping time. Emittance wigglers are installed where the dispersion does not vanish, and increase the emittance.

The main uses of wigglers in LEP are (i) increasing the bunch length and therefore the circulating current at injection energy, (ii) reducing the polarization time around the \( Z_0 \) energy, (iii) adjusting the beam size around the \( Z_0 \) energy.

3. GOING FROM 55 GeV TO ABOUT 100 GeV

The discussion of the actions needed for raising the LEP beam energies to around 100 GeV is based on the following assumptions:

(i) The beam energies are 95 to 100 GeV which is close to the peak of \( W^+W^- \) production. Their precise value will be chosen such that it is convenient for the quantized superconducting (s.c.) RF system [1].

(ii) S.c. RF stations are installed around the 4 even interaction points of LEP. This provides enough space for the installation of the RF system, and a smaller energy variation of the beams around LEP. The performance estimates will be made with the assumption that the Cu RF system of Phase I has disappeared.

(iii) The circulating current in each beam is 3 mA which presumably is well known from Phase I and also matches well the quantized s.c. RF system [1]. In addition, the consequences of raising the current in each beam to 6 mA will be discussed as an optimistic goal.

3.1 Performance Scaling

The formulae for luminosity (Eq. (1)) and beam-beam tune shift (Eq. (2)) show that beam sizes \( \sigma_x \) and \( \sigma_y \), stored beam, and luminosity scale as follows, at constant \( k \), \( \beta_y \), \( Q \), \( J_x \) and \( \xi_y \):

\[
\sigma_x \sim \sigma_y \sim \gamma \quad N \sim \gamma^3 \quad L \sim \gamma^4.
\]

This is called the natural variation because no active steps are taken to modify this behaviour. Because of the increase in the beam sizes in proportion to the energy, these scaling laws only hold until the aperture limit is reached.
If the current cannot be increased, i.e. at constant $N$, the scaling laws are:

$$\xi \sim \gamma^{-2} \quad L \sim \gamma^{-2}.$$  \hfill (9)

This rapid decrease of the luminosity is undesirable. If the current can be increased in proportion to the energy, i.e. if $N \sim \gamma$, the scaling laws yield a constant luminosity:

$$\xi \sim 1/\gamma \quad L \sim \gamma^{0}$$ \hfill (10)

The luminosity scaling is improved further by controlling the beam sizes such that $\xi \gamma$ is constant, i.e. by taking active steps to modify the natural variation. At constant current, i.e. with $N \sim \gamma^{0}$, the following scaling laws apply:

$$\sigma \sim 1/\gamma \quad Q^{3}J_{X} \sim \gamma^{3} \quad L \sim \gamma^{3}$$ \hfill (11)

If the current increases in proportion to the energy, i.e. with $N \sim \gamma$, the luminosity improves even further:

$$\sigma \sim \gamma^{0} \quad Q^{3}J_{X} \sim \gamma^{2} \quad L \sim \gamma^{2}.$$ \hfill (12)

3.2 Lattice

The stronger focusing implied in Eqs. (11) and (12) can be achieved by increasing $Q$ by a factor 1.5, and/or varying $J_{X}$ between 0.5 and 2.5.

LEP lattices with 60° and 90° phase advance in the arc cells were studied in much detail for Phase I. It was found that the horizontal emittance and aperture of the 60° lattice were adequate to cover a range of currents around 3 mA at a beam energy of 55 GeV [6]. For operating LEP in the neighbourhood of 95 GeV beam energy, the 90° lattice is more appropriate. It covers a current range from a little more than 3 mA to over 6 mA per beam such that the luminosity is proportional to the circulating current [7].

At 95 GeV, the synchrotron radiation loss is 2.3 GeV per turn. With 4 RF stations around the even pits, this causes the beam energies to vary by ±0.3%, and the $e^{+}e^{-}$ orbits to be different. Since the reflection symmetry present in the LEP layout is destroyed by the closed-orbit distortions remaining after correction, all beam parameters (tunes, $\beta$-functions, dispersion, etc.) are slightly different for the $e^{+}$ and $e^{-}$ bunches which also would miss each other at the interaction points if their vertical positions were not adjusted by fine tuning of the electrostatic separators [8]. The computer simulation of the consequences of this effect for the 90° LEP lattice is under way [9]. This effect scales like $E^{3}/n_{RF}$, i.e. 2 RF stations at 80 GeV are as good/bad as 4 RF stations at 100 GeV.
3.3 Equipment Changes

Most LEP components are suitable for operation up to about 100 GeV. Therefore, most of the changes needed are additions to existing equipment rather than replacements. The most important changes are:

(i) Build klystron tunnels near Pits 4 and 8.
(ii) Install superconducting RF system, as discussed in detail in [1].
(iii) Install extra cooling on vacuum chamber for synchrotron radiation power going from 1.6 MW to 14 or 28 MW.
(iv) Install extra electric power supplies for LEP magnets.
(v) At 62.5 GeV, replace 24 concrete dipole cores in injection regions by laminated steel cores.
(vi) At about 65 GeV, replace some quadrupoles in all insertions by stronger ones, and rearrange others.
(vii) Modify straight section lattice near the even pits for better adaptation to length of superconducting RF cavity strings.

4. PERFORMANCE PREDICTION

4.1 Single-beam limits due to instabilities.

The most severe limit of the circulating current in LEP arises from the transverse mode-coupling instability. The threshold current $I_{th}$ per bunch is given by [10]:

$$I_{th} = \frac{8f_s(E/e)}{\sum \beta_i k_i(\sigma)}$$

The threshold is lowest at the injection energy $E = 20$ GeV, the synchrotron frequency $f_s$ is about 1.2 kHz or perhaps up to some 50% higher. The sum goes over all components with betatron amplitude function $\beta_i$ and transverse loss factor $k_i(\sigma)$ which decreases with the bunchlength $\sigma$. Therefore, wiggler magnets increase the bunchlength to $\sigma = 40$ mm at 20 GeV.

In order to reach the design current, 0.75 or 1.5 mA per bunch, the sum must be smaller than:

$$\sum \beta_i k_i(\sigma) < 2.5 \times 10^5 \text{ or } 1.25 \times 10^5 \text{ V/pC}$$

Components with small loss factors appearing in large numbers are as likely to contribute to the current limit as few components with large loss factors.
4.2 Transverse Loss Factors

The largest contributions to the product of amplitude function $\beta$ and transverse loss factor $k(\sigma)$ are listed in Table II, for a bunch length $\sigma_B = 40$ mm [11].

| 2880 bellows at $<\beta> = 60$ m | $0.4 \times 10^5$ V/pC |
| 128 Cu RF cavities of Phase I at $<\beta> = 40$ m | $1.1 \times 10^5$ V/pC |
| 256 superconducting RF cavities at $<\beta> = 40$ m | $0.2 \times 10^5$ V/pC |

Table II - Main contributions to $\beta k(\sigma)$ in LEP

The sum of the contributions of the bellows and the Cu RF cavities is larger than the permissible values for 6 mA. Therefore the Cu RF cavities must disappear before 6 mA are considered.

The loss factors of separator tanks, kicker tanks, etc. are comparable to those of a single Cu RF cavity cell, i.e. 1/5 of a Cu RF cavity. Since such tanks are much less numerous than RF cavities, their contribution to the loss factors are negligible.

4.3 Luminosity and Energy Spread

Table III shows the peak luminosity $L$, the absolute rms energy spread $\Sigma_e$ in each beam and the synchrotron radiation loss per turn $U_s$ at a few selected energies, assuming a current of 3 mA in each beam.

<table>
<thead>
<tr>
<th>E/GeV</th>
<th>L/cm$^{-2}$s$^{-1}$</th>
<th>$\Sigma_e$/MeV</th>
<th>$U_s$/MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>$1.6 \times 10^{31}$</td>
<td>51</td>
<td>263</td>
</tr>
<tr>
<td>75</td>
<td>$2.2 \times 10^{31}$</td>
<td>89</td>
<td>895</td>
</tr>
<tr>
<td>88</td>
<td>$2.5 \times 10^{31}$</td>
<td>156</td>
<td>1696</td>
</tr>
<tr>
<td>95</td>
<td>$2.8 \times 10^{31}$</td>
<td>196</td>
<td>2303</td>
</tr>
</tbody>
</table>

Table III - Performance parameters at several energies

The figures shown on Table III are based on the following assumptions:

- The transverse loss factors are small enough,
- The aperture of the LEP200 lattice are similar to that of Phase I lattices.
- The vertical amplitude function at the experimental interaction points, $\beta_y = 0.07$ m, remains as low as in Phase I although the LEP energy is nearly doubled [12].
- There is no significant reduction in aperture due to the energy variation around LEP.
- The beam-beam limit remains at $\xi_y = 0.03$.
- The same rules of the game as for Phase I apply.
4.4 Energy Uncertainty

The tolerances on the main contributions to the uncertainty of the mean energy of the LEP beams are listed in Table IV. Asymmetries in the RF system, i.e. differences in the accelerating voltages from the 8 accelerating stations on either side of the 4 even pits, cause energy errors at the interaction points which have opposite signs for e⁺ and e⁻, and therefore have no effect on the c.m. energy.

| Energy error from dipole field measurements: | ± 5 \times 10^{-4} |
| Power supply stability (long term) | ± 2 \times 10^{-4} |
| Power supply stability (short term) | ± 5 \times 10^{-5} |

Table IV - Tolerances on contributions to energy uncertainty

4.5 Lifetimes

Various phenomena cause particle losses and therefore a limitation of the beam lifetime. Estimates for the various lifetimes in hours are shown in Table V. The figures shown were obtained under the following assumptions:

- The quantum lifetime $\tau_q$ is due to the small probability that the emission of energetic synchrotron radiation photons causes the particle to leave the stable RF bucket. The value $\tau_q = 48$ h could be increased by a small increase in the RF voltage.

- The beam-gas lifetime $\tau_g$ is determined by the average pressure assumed to be 3 \text{ 10}^{-9} \text{ Torr}.

- The beam-photon lifetime is called $\tau_s$, it is due to the collision of particles with synchrotron radiation photons [13].

- Beam-beam bremsstrahlung [14] lifetime $\tau_{bb}$.

- The overall lifetime $\tau$ is calculated by adding all decay rates from $1/\tau = \sum 1/\tau_i$.

| Energy E GeV | 55 | 75 | 88 | 95 |
| Quantum lifetime $\tau_q$ | 48 h | 48 h | 48 h | 48 h |
| Beam-gas lifetime $\tau_g$ | 20 h | 20 h | 20 h | 20 h |
| Beam-photon lifetime $\tau_s$ | 13 h | 33 h | 31 h | 25 h |
| BB bremsstrahlung lifetime $\tau_{bb}$ | 13 h | 10 h | 10 h | 11 h |
| Overall lifetime $\tau$ | 6 h | 5 h | 5 h | 5 h |

Table V - Beam lifetimes
The lifetime due to the beam-beam effect is not listed in Table V. Experience with existing e⁺e⁻ storage rings shows that it drops very rapidly when the beam-beam tune shift $\delta_Y$ exceeds some value. Staying below that limit - the beam-beam limit for all practical purposes - ensures an adequate beam-beam lifetime.

5. BACKGROUND

We assume that the experiments are adequately shielded from the synchrotron radiation background from the LEP arcs by a system of collimators, as in Phase I [15]. However, two kinds of background for experiments originate in the straight sections and are therefore more difficult to eliminate:

(i) Synchrotron-radiation photons from insertion quadrupoles,
(ii) Low-energy e⁺ and e⁻ from beam-gas bremsstrahlung.

Both types of background are affected by the detailed lattice layout and the beam sizes. Quantitative answers can only be obtained by repeating earlier simulations [16] when the detailed lattice design is known.

5.1 Scaling law for synchrotron radiation

For photon energies $E_Y$ much smaller than the critical energy $E_C$, the photon flux per unit energy interval, per unit length of quadrupole, and per unit time scales like:

$$d^2N_Y/dE_Yd\sigma dt \sim (K\sigma/E_Y)^{2/3}.$$  \hspace{1cm} (12)

This expression contains no explicit dependence on the beam energy $\gamma$. If one assumes that the next generation of superconducting insertion quadrupoles will have about the same field gradient as the present ones [3,12], then their strength $K$ will vary like $1/\gamma$. The beam size $\sigma$ is roughly constant. Therefore the background due to photons with $E_Y \ll E_C$ will decrease with increasing energy.

On the other hand, the synchrotron radiation losses per unit length from the quadrupoles in the straight sections scale like $\gamma^3(K\sigma)^2$ while the critical energy scales like $\gamma^5K\sigma$. Hence, the total number of photons scales like their ratio:

$$d^2U_C/d\sigma dt = \gamma K\sigma \hspace{1cm} (14)$$

Again, the quadrupole strength $K$ is roughly proportional to $1/\gamma$. This variation cancels the explicit $\gamma$ dependence in Eq. (14) and therefore the background due to photons with $E_Y > E_C$ becomes independent of energy.
5.2 Scaling law for low-energy $e^+$ and $e^-$

The rate of $e^+$ and $e^-$ production is proportional to the current $I$ and the vacuum pressure which might well be less than in Phase I because by that time the LEP vacuum system will be well cleaned by synchrotron radiation photon desorption, and because superconducting RF cavities are good cryopumps.

6. POLARIZATION

Our plans for eventually providing longitudinal polarization in LEP include the following steps during the construction and early commissioning of LEP [17]:

(i) Equipment and procedures are foreseen which help transverse polarization, e.g. asymmetric wigglers and orbit correction schemes.

(ii) The polarization will be measured from startup with a polarimeter for the $e^-$ beam based on the backscattering of circularly polarized laser light [18], and destroyed at will by a depolarizer.

(iii) We do not knowingly build many things into LEP which are detrimental to polarization.

(iv) We shall simulate polarization effects before startup.

The reasons for this cautious course of action are budget and manpower limitations on one side, and spin physics on the other side.

As the energy of $e^+e^-$ storage rings was increased, their bending radius went up approximately like $\gamma^2$ [19]; therefore the absolute energy spread in the beam, which must be compared to the distance between depolarizing spin resonances, 440 MeV, still increases like $\gamma$, and becomes the critical issue in deciding whether the LEP beams will be polarized or not.

It will be possible in Phase I to experiment with transverse polarization during MD periods with the detector solenoids turned off, to compare predictions with observation, and to test correction procedures for machine errors. These experiments are complicated by the fact that at the $Z_0$ energy, 46.5 GeV, the polarization time with all wigglers excited is 80 minutes. If an adequate degree of polarization is achieved, the installation of appropriately matched spin rotators for obtaining longitudinal polarization might be decided at this stage, thus permitting polarization studies during colliding-beam physics run with the detector solenoids excited.

Results obtained in Phase I will show whether or not polarization at about 100 GeV is a realistic possibility. Extrapolating now from SPEAR or PETRA to LEP200 would be pure speculation.
7. **CONCLUSIONS**

Apart from the RF system, most LEP components are suitable for operation up to about 100 GeV. The modifications needed are mostly additions and rearrangements. Operating procedures are known in principle which optimize the luminosity up to energies around 100 GeV. Several detailed questions remain to be answered: (i) It must be found by computer simulation whether the (dynamic) LEP aperture is adequate. (ii) The detailed layout of the experimental insertions needs to be studied. (iii) Providing longitudinally polarized beams first around the Z0 mass and later around 100 GeV needs theoretical studies and equipment. Nowadays, all this work must be based on learned extrapolations from existing machines. Much better predictions can be made once LEP is operating, and the beam behaviour is well understood.

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UPGRADING OF LEP ENERGIES BY SUPERCONDUCTING CAVITIES

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ABSTRACT

The upgrading of LEP to energies above 90 GeV by superconducting cavities is considered. Development work at CERN has allowed to reach the design values for accelerating fields and quality factors, $E_{\text{acc}} = 5 \text{ MV/m}$ and $Q_0 = 3 \times 10^9$ respectively in 350 MHz, 4 cell cavities and the fabrication of a few cavities and cryostats by industry is prepared. Two possible scenarios of upgrading are discussed, involving installation of sc cavities in interaction regions 2 and 6 only or in all 4 interaction regions. The corresponding costs and time schedules are presented.

The installation of 128 copper accelerating cavities in LEP will allow to reach particle energies around 55 GeV [1]. Already at a very early stage of the planning for LEP it was considered to upgrade energies to the design value of LEP ($\sim 100$ GeV) by installing superconducting (sc) cavities. Recent progress [2] in the sc cavity development programme at CERN looks sufficiently promising to confirm this hope.

In the following the possibilities of the new technology are shortly discussed and the present status of development work at CERN is reviewed. Possible scenarios for the installation of sc cavities in LEP are presented and the costs and time schedules are explored. This presentation relies largely on ideas which have already been published in a 1985 LEP note [3].

1. BASICS OF RF SUPERCONDUCTIVITY [4]

1.1 rf losses

At high frequencies and for temperatures below the critical temperature $T_c$ the rf resistance of a superconductor decreases exponentially with temperature and its value can be made typically $10^4$ to $10^6$ times smaller than for copper at room temperature. The corresponding decrease of rf losses in sc cavities has attracted accelerator constructors because much higher acceleration efficiencies and higher CW accelerating fields than in Cu cavities can be reached.

Due to the Meissner effect the penetration depth of rf fields in a superconductor is much smaller than the normal skin depth and ranges in the region of 50-200 nm. rf superconductivity is therefore a surface effect. One may characterise rf losses of a superconductor by the surface resistance $R_s$ (in Ohm). For cavities, losses are generally expressed by the so-called quality factor $Q$. For a given cavity geometry and rf mode, quality factor and surface resistance are related by the relation

$$Q = G/R_s$$
where $G$ is a constant. For LEP accelerating cavities, one calculates $G = 278$ Ohm. It is possible to reach reliably in large multicell cavities $Q$-values well above $10^8$ which may be compared with typical $Q$ values for Cu accelerating cavities of $4 \times 10^6$.

At present the favourite material for cavity fabrication is pure niobium (Nb). It has mechanical properties which allow easy shaping and welding of cavities with complicated geometrical layouts.

1.2 Field limitations

In normal temperature cavities rf fields are limited either by the warming-up of walls due to the large rf losses or by the electric field component causing field emission, microdischarges or electron resonance phenomena (multipactor). For sc cavities one has to add the critical magnetic rf field $H_s$ as another limit. For ideal Nb surfaces one would expect to reach acceleration fields of the order of 50 MV/m. However, for real surfaces, fields are limited at much lower levels. There exist in cavities well-localised point-like defects with increased rf losses. These defects, which are not related to the sc properties of the cavity walls, heat up their surrounding and eventually drive the superconductor to temperatures above its critical temperature $T_c$ thereby inducing a thermally unstable process which finally leads to a fast field breakdown. These surface defects are mostly of trivial nature like e.g. cracks and holes in weldings, welding beads, inclusions of other materials, tooling marks, dust particles or residues from chemical treatments or rinsings.

Another cause of field limitations are point-like electron sources - similar to the DC field emission sources observed at large area high voltage electrodes - and located at regions exposed to high electric surface rf fields. The emitted electrons are accelerated in large cavities to energies in the 100 keV or MeV range and hit cavity surfaces causing heating and emitting bremsstrahlung X-rays. Field loading is produced not only by the acceleration of electrons but also by the increased rf losses at regions warmed up by electron impact.

Special diagnostic methods like e.g. temperature mapping of the cavity surfaces had to be developed to localise and to characterise surface defects and electron sources. They opened the way for the repair of such defects by local grinding and local chemical treatments. They also allowed to device more efficient inspection methods and to develop improved surface treatments and weldings. One of the most probable causes of electron emission are dust particles. Therefore clean room techniques for rinsing and assembling large cavities had to be applied. With the development of these methods fields could be gradually increased and 5-7 MV/m can now be reached reliably even in large multicell cavities.

The preparation of clean and defect free surfaces of many m$^2$ size remains one of the biggest technological challenge in the field. The progress in the performances of sc cavities with respect to rf losses and achievable fields will be bound to a more detailed understanding of the interaction between rf fields and real sc surfaces and to the transfer of this understanding to large scale cavity systems needed in accelerators and storage rings.

An important step towards higher gradients was the insight in the role of thermal conductivity $\lambda$ for stabilising localised defects [5]. Computer simulations have shown that breakdown levels scale with $\sqrt{\lambda}$. By now industry has produced Nb material whose $\lambda$ has been raised from an initial 10 W/m $\times$ K to over 30 W/m $\times$ K and cavity results have confirmed the predictions.
A follow-up of this idea is to replace the bulk niobium by copper with a high thermal conductivity (typically > 400 W/m x K) and to deposit a thin Nb layer of a few µm thickness on a Cu cavity [6]. A remarkable feature of Nb/Cu cavities is the absence of fast thermal breakdowns. This behaviour is particularly interesting for large storage rings where a string of cavities is powered by a common rf generator (for LEP e.g. 16 cavities). If a single cavity would deteriorate without producing a fast breakdown the additional cryogenic losses may be tolerable for some time and this may avoid switching off a whole array of sc cavities. For low frequency cavities as the ones foreseen for LEP the use of Cu instead of Nb cavities will also lead to a substantial reduction in cavity costs.

1.3 Advantages of sc cavities

The main advantage of sc cavities is their high acceleration field which can exceed the ones reached in Cu cavities (1.5 MV/m) by more than a factor 3-5. rf power, which can be produced in large CW klystrons with an efficiency of up to 70%, can be converted into acceleration with negligible losses. Even if one takes into account that a small part (≈ 10⁻³) of the rf power is dissipated in the cavity walls at LHe temperatures the overall efficiency is larger than for nc systems (cf. Appendix I).

For LEP the main current limits are given by transverse instabilities linked to the internal modes of individual bunches (short range wake fields). The main contribution [7] to the total transverse impedance causing this type of instability are stemming from the rf cavities and from the vacuum chamber bellows. For a bunch length of 40 mm the contributions are:

- 2880 bellows: $0.4 \times 10^5$ V/pC,
- 128 Cu cavities: $1.1 \times 10^5$ V/pC,
- 256 sc cavities: $0.2 \times 10^5$ V/pC.

The contribution of sc cavities is relatively small because their iris opening 2a can be made much larger (table I) and because the transverse impedances per unit length scale approximately like $a^{-2.9}$ [8].

Resonant built-up of higher order modes (hom) can also be kept sufficiently small in sc cavities because couplers have been developed which attenuate the most dangerous hom to levels even below the ones produced by the (natural) damping of Cu-cavities.

2. Status of SC Cavities at CERN

Up to 1983 the efforts in cavity development at CERN were mainly concentrated on 500 MHz cavities [9], leading to the successful test of a 5-cell, 500 MHz cavity at PETRA [10]. The results confirmed that the achievable accelerating fields do not decrease at lower frequencies as strongly as previously suspected. Therefore it was decided in 1984 to concentrate efforts on 352 MHz cavities [11]. This frequency choice is suggested by the fact that LEP will be equipped at the beginning with 128 Cu cavities at 352 MHz. There is an obvious interest to install at a later stage sc cavities with the same frequency and to use at maximum the existing installation of rf power sources. With the installed rf power of 16 MW, LEP can be upgraded to more than 90 GeV by using sc cavities.
Developments on 352 MHz I.EP cavities have been pushed along two lines: Nb cavities and Cu cavities coated by a thin layer of Nb.

**TABLE I**
A few I.EP cavity parameters

<table>
<thead>
<tr>
<th></th>
<th>Cu cavity</th>
<th>Sc cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency f</td>
<td>352.209 MHz</td>
<td>352.209 MHz</td>
</tr>
<tr>
<td>Wavelength λ</td>
<td>0.851 m</td>
<td>0.851 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Cavity active length</td>
<td>2.13 m</td>
<td>1.70 m</td>
</tr>
<tr>
<td>Iris hole diameter</td>
<td>100 mm</td>
<td>241 mm</td>
</tr>
<tr>
<td>Shunt impedance/quality factor r/Q</td>
<td>650 Ohm/m (a)</td>
<td>276 Ohm/m</td>
</tr>
<tr>
<td>Q (Cu, 300 K)</td>
<td>40000</td>
<td>-</td>
</tr>
<tr>
<td>Q (Nb, 4.2 K)</td>
<td>-</td>
<td>3 x 10¹⁰</td>
</tr>
<tr>
<td>2 (f₂ - f₁)/(f₂ + f₁)</td>
<td>1.28%</td>
<td>1.76%</td>
</tr>
</tbody>
</table>

(a) Accelerating cavity without storage cavity.

2.1 **Nb cavities**

With single cell cavities accelerating fields up to 10 MV/m and quality factors above 3 x 10⁹ at the design field of 5 MV/m have been reached. In some cases maximum fields were reached without rf- and He-processing. Exposures to dustfree, dry air and warm ups to room temperature did not degrade fields and Q values.

Based on the experience gained during the PETRA test the cavity design (fig. 1 and table 1) has been improved with respect to rf properties and manufacturing simplicity [12]. A first 4-cell cavity [2] using the new geometry and fabricated from Nb material of improved thermal conductivity (λ = 28 W/m x K) has reached \( E_{acc} = 7.5 \text{ MV/m} \) and a quality factor above 3 x 10⁹ at 5 MV/m (fig. 2). These performances remained unchanged after 2200 h of high field operation, after a vacuum failure whereby the vacuum degraded to \( \sim 10^{-9} \text{ mbar} \) and after an exposure to X-ray radiation of many tenths of krad.

2.2 **Nb coated copper cavities**

Two methods of coating Cu copper cavities with a thin (1-5 μm) layer of Nb have been developed and tested at CERN: diode sputtering [6] and magnetron sputtering [13]. Magnetron sputtering has the advantage of a less complicated sputtering layout and of faster deposition rates. Recently a few excellent results have been obtained with this method. Fields up to \( \sim 10 \text{ MV/m} \) with excellent \( Q \)-values have been reached in single cell 500 MHz cavities. There remain however still problems of \( Q \) degradation toward higher fields, and of blistering of the Nb layers [19]. At present efforts are pursued to overcome these problems and to get more information on the long-term behaviour of thin Nb layers. A first attempt for sputtering a 4-cell, 350 MHz cavity by the magnetron method has been performed. Fields up to 5 MV/m were reached but \( Q \) values are still limited by defects presumably due to inadequate cleaning of the underlying Cu surface. As expected these defects did not produce fast thermal breakdowns.
Fig. 1  Geometry of 350 MHz, 4-cell cavity for LEP with main coupler MC and higher order mode coupling ports (EHI, EH2).

Fig. 2  Quality factor as function of accelerating field for the first 350 MHz Nb 4-cell cavity for LEP. The design values are $Q_0 = 5 \times 10^8$ and $E_{acc} = 5$ MV/m.
2.3 Cryostats, couplers and tuners

Besides sc cavities the development of cryostats, couplers and frequency tuners has been pushed.

A first prototype LEP cryostat [14] of new and simplified design has been fabricated at CERN and tested successfully with a 4-cell sc cavity. Static cryogenic losses amount to 14 W. The cryostat has been designed as a module allowing the assembly of up to 8 cavities in a common vacuum vessel under clean and dust-free working conditions. Coaxial high power couplers [15] have been already operated successfully under warm and cold conditions and rf power levels foreseen for operation in LEP have been reached. The design of higher order mode couplers [16] has made great progress. Several types of compact horn couplers have been developed and tested. The damping provided to the most dangerous horn should be largely sufficient for LEP operation conditions. Frequency tuners of a new type [17] not involving moving mechanical parts are under development and first tests have already been performed.

It is at present intended to order from industry two sc Nb-cavities with cryostats and tuners of the type developed at CERN.

2.4 Long-term behaviour

Storage ring tests of sc cavities performed during the last years have shown that there are no major problems for operating such cavities in large storage rings [18]. However, some uncertainties concerning the degradation of cavity performances in a storage ring environment with respect to vacuum, dust transport and synchrotron radiation still remain. Therefore it is proposed to install a LEP sc cavity in the SPS accelerator-collider as soon as possible and to operate it with electron and proton beams for longer periods. In this way it will be possible to study the long-term behaviour in a well understood accelerator and well before such studies can be started inside LEP. This test will be backed up by systematic investigations on vacuum failures and air and dust exposures in test cavities.

Besides these tests, preparations are pursued for installing at an early stage of operation inside LEP a few sc cavities with all auxiliary items so that a maximum experience can be gained before upgrading at a larger scale will start.

3. Possible scenarios for the installation of sc cavities in LEP

For the first stage of LEP, 128 Cu-cavities with an effective length of 272.4 m and with $E_{\text{acc}} = 1.47$ MV/m are installed. They supply a circumferential voltage $U_{\text{rf}} = 402$ MV and a particle energy of $\sim 55$ GeV. Cavities will be installed in interaction regions 2 and 6 and will occupy on each side of the interaction point 4 rf-cells (fig. 3). Each rf cell contains eight 5-cell Cu cavities which are powered by one 1 MW klystron (or more precisely: 16 sc cavities and storage cavities located in adjacent rf cells are powered by a combined system of two 1 MW klystrons at slightly different frequencies. This layout of cavities and klystrons within two rf cells is shown in fig. 4.)
Fig. 3  Layout of one rf station containing 7 rf cells for installation of 8 rf cavities each.

Fig. 4  Layout of cavities and klystrons for 2 rf cells with Cu cavities. For sc cavities one klystron can be removed and the corresponding place can be occupied by the cold box of a refrigerator.

For the installation of sc cavities it is tried to keep changes of the existing layouts in the machine lattice, in the klystron and waveguide layout to a minimum and a number of eight sc cavities per half-cell has been adopted (table 2). With a given length of rf cells one can only locate eight 4-cell cavities in a rf-cell [3]. Even with 4-cell cavities the available space will be very marginal and it may turn out difficult to locate additional items like correction magnets and beam monitors. Therefore it would be highly desirable to increase the length of rf cells. The extra length of cell QS4–QS5 (fig. 3) would give a possibility for this.
The much higher accelerating efficiency of sc cavities allows to power 16 cavities with one 1 MW klystron as long as a beam current of 2 x 3 mA together with an accelerating field of 7 MV/m is not exceeded (table 2 and Appendix 1). Therefore the place foreseen in the Cu cavity layout for each second klystron will be free. It is proposed to install on each side of the interaction points and at one of the free klystron locations the cold box of the refrigerator (fig. 4). The available space allows installation of a cold box with up to 6 kW refrigeration power [20]. This should be sufficient to cool sc cavities in 6 rf cells up to a gradient of ~ 7 MV/m (table 2 and Appendix 1).

**TABLE 2**

A few parameters of rf cells equipped with sc cavities (cf. Appendix 1)

<table>
<thead>
<tr>
<th>Number of cavities</th>
<th>Eight 4-cell cavities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{eff}$</td>
<td>13.6 m</td>
</tr>
<tr>
<td>$E_{acc}$</td>
<td>5 MV/m</td>
</tr>
<tr>
<td>$U_{rf}$</td>
<td>60 MV</td>
</tr>
<tr>
<td>$P_{cry}$ (cavities + cryostats + transfer lines at 4.2 K)</td>
<td>544 W</td>
</tr>
<tr>
<td>Mains power for cryogenics</td>
<td>0.165 MW</td>
</tr>
<tr>
<td>$P_{rf}$ (2 x 3 mA)</td>
<td>355 kW</td>
</tr>
<tr>
<td></td>
<td>7 MV/m</td>
</tr>
<tr>
<td></td>
<td>95.2 MV</td>
</tr>
<tr>
<td></td>
<td>952 W</td>
</tr>
<tr>
<td></td>
<td>0.288 MW</td>
</tr>
<tr>
<td></td>
<td>496 kW</td>
</tr>
</tbody>
</table>

From the above considerations it turns out that a gradient of ~ 7 MV/m is a natural limit from the point of view of available rf generators and refrigerators. If currents above 2 x 3 mA are considered then either the accelerating fields have to be reduced or the number of klystrons has to be doubled. Similarly for gradients well above 7 MV/m the number of cold boxes has to be increased (unless $Q$ values well above $3 \times 10^8$ can be attained at those higher gradients). It should be mentioned that a cost optimization (Appendix 2) of the sc acceleration system for LEP including cavities, rf and cryogenic systems as well as operating costs shows a broad minimum for acceleration fields between 7 and ~ 15 MV/m. In order to keep the possibility for such upgrades it would be desirable to equip only six rf cells with sc cavities and to reserve the 7th rf cell for installation of more refrigeration capacity.

The possible installation of a proton ring is taken into account by leaving the space above the cryostats free from He transfer lines and from rf couplers and waveguides.

A crucial question for LEP upgrading will be whether at higher energies the concentration of acceleration in two interaction regions only will be possible [7]. As this question may be not answerable by simulation methods only we present two basic scenarios, one using only interaction regions 2 and 6 for accelerating and one using interaction regions 2, 4, 6 and 8.

In the following all LEP energies are calculated for an improved "90° phase advance" lattice [7]. Hom losses are sufficiently small to apply the simple extrapolation formula

\[ E = 13.23 \sqrt[4]{U_{\text{rf}}} \]  
(E in GeV, \( U_{\text{rf}} \) in MV)

where \( U_{\text{rf}} \) is the (peak) circumferential voltage supplied by the rf system.

All scenarios are based on accelerating fields of 5 or 7 MV/m. Intermediate steps of installation are of course always possible.

**Scenario 1: Installation in interaction regions 2 and 6 only**

Two stages can be considered and are detailed in table 3.

(a) **Installation of 8 rf cells with a total of 64 SC cavities**

One has to install four klystron systems and four 6 kW cold boxes (with 2 kW compressors). The energy can be upgraded to 73-77 GeV. The system of Cu cavities remains operational. This stage would be essentially a learning stage for producing and installing cavities at high rate and for operating LEP with two rf stations only at higher energies. A short shutdown will be sufficient for installation.

(b) **Removal of all Cu cavities and replacement by SC cavities**

This brings up the total number to 24 rf cells with 192 sc cavities. Klystron systems have to be reshuffled. Additional compressors and He transfer lines have to be installed. Energies can be brought up to 84-91 GeV and \( W^+ \) production will be possible. A very long shutdown will be needed for the removal of all Cu-cavities, for rearrangement of klystron systems and for installation of sc cavities.

We recall that this scenario is envisageable if LEP can be operated with two rf stations only at high energies. It has the drawback that Cu cavities have to be removed at a rather early stage.

For stage (b) installation of sc cavities in 6 rf cells on each side of the interaction points is assumed. Equipping the 7th cell also with sc cavities would bring up energies from

- 84 to 87.3 GeV for \( E_{\text{acc}} = 5 \) MV/m
- 91.5 to 95.1 GeV for \( E_{\text{acc}} = 7 \) MV/m.

An upgrading to even higher energies will need acceleration fields above 7 MV/m. This will ask for an increase of the number of klystron systems (if \( I_b \geq 2 \times 3 \text{ mA} \) has to be conserved) and of refrigerator units. Because of the quantization of systems this will be relatively costly.

**Scenario 2: Installation in all 4 interaction regions**

We consider 3 stages (cf. table 3):
(a) **Installation** (as in the first scenario) of 8 rf cells with 64 sc cavities in 2 and 6
Cu cavities remain operational.

(b) **Installation of 8 rf cells with 64 sc cavities in regions 4 and 8**
Klystron galeries and their access pits have to be constructed.

One has to install an additional 4 klystron systems and four 6 kW cold boxes with 2 kW compressors. Cu cavity systems can continue to operate. LEP will operate with 4 rf-stations which are only slightly different in acceleration voltage. An energy of 82-88 GeV can be reached. A short shutdown will be sufficient because the klystron systems can be prepared in advance in the klystron tunnels and no Cu cavities have to be removed.

(c) **Installation of another 16 rf cells with 128 sc cavities in regions 2, 4, 6 and 8**

One has to remove at least half of the Cu cavities. Klystron systems have to be reshuffled and the compressor power has to be upgraded everywhere to 4 kW. The 4 rf stations are now symmetric and energies can be upgraded to 90-98 GeV, largely sufficient for W⁺ production.

This scenario is more costly (see sect. 5), because of the additional civil engineering and installations in regions 4 and 8. It has nevertheless a few major advantages:

- LEP is operated with four rf stations.
- Cu cavities are removed at a much later stage (and 8 rf cells could even continue to operate if so wanted).
- Civil engineering, installation of klystron systems and refrigerators in 4 and 8 can be done without major interference with LEP operation. Shutdowns can be made shorter.

An upgrading of LEP to even higher energies could be achieved by increasing the number of sc cavities.

Passing from 32 to 48 rf cells would bring up energies from 90 to 100 GeV for $E_{\text{acc}} = 5 \text{ MV/m}$ and from 98 to 108 GeV for $E_{\text{acc}} = 7 \text{ MV/m}$.

4. PRODUCTION AND INSTALLATION OF SC CAVITIES

For the upgrading of LEP one can assume that the construction of cavities and cryostats will be on the critical path and that the budget situation is allowing production of a very high rate.

The construction, assembly and testing of many cavity-cryostat units with all auxiliary facilities could be performed at a rate of about 2 cavities/week. This would almost certainly imply at least two production chains and several independent installations for rf and cryogenic tests and for
the final assembly. From similar constructions by industry it is concluded that a period of about
2 years will be needed from the placing of fabrication orders to the moment where production at high
rate can be performed.

It is proposed to install cryostats and cavities in independent units of one cryostat containing
two cavities assembled and tested before their transport to the L.E.P tunnel. These units will have a
length of ~ 6 m so that the normal machine pits can be used for installation and so that no magnets
have to be removed for the transport of units to their final position. For the installation two
independent teams could be envisaged which would allow installation at a rate of ~ 8 cavities/week.

In fig. 5 we have illustrated a possible time schedule for production and installation. For
scenario 1 we assume a minimum duration based on the above assumptions. For scenario 2 an
additional decision point has been assumed after step 2(a) for the civil engineering in regions 4 and 8.
Some delay has been added, allowing to gain experience for the operation of L.E.P at higher energies
and with 2 rf stations only. It is assumed that the construction of klystron tunnels and access pits
will take one year. As can be seen, energies for $W^+$ production are reached for both scenarios at
about the same moment, i.e. about 4 years after the initial decision for upgrading.

![Graph showing time schedule for production and installation](image)

**Fig. 5** A possible production and installation time schedule and the L.E.P energies reached at each
step (for $E_{acc} = 7$ MV/m).
5. **COST ESTIMATES**

In table 3 and fig. 6 the costs for the sc cavity systems are given. For this estimate we have taken into account all information on costs of sc acceleration systems available to us (Appendix 2). They include:

- Cost of sc cavities and cryostats.
- Cost of complete rf system.
- Cost of complete cryogenic system.

There is some uncertainty in those cost estimates in particular for the costs of cryostats and cavities. We believe that the assumed costs are nevertheless realistic and may be somewhat reduced if mass production can be applied.

We have done a cost optimisation (Appendix 2 and fig. 7) for the accelerating field by taking into account in addition to the costs mentioned above the costs of electricity for 4000 h of operation during 5 years.

Additional costs directly related to the sc acceleration systems have also been estimated and are given in table 3. They include:

- Civil engineering of klystron tunnels and access pits in regions 4 and 8.
- Cooling and ventilation.
- Additional electricity installation.
- Surface buildings for cryogenics.
- Reshuffling of klystron systems.

For completeness we give also a rough estimation of additional costs for LEP upgrading from 65 to 100 GeV not directly related to the sc cavity systems. They include:

- Cooling and ventilation for magnets and synchrotron radiation.
- Additional electricity installations.
- Power converters for magnets.
- Low β systems.
- Additional magnets for injection.

An estimation of costs for energies above the ones of scenarios 1 and 2 are given in fig. 6.

6. **SUMMARY**

For the upgrading of LEP energies it is proposed to use superconducting cavities operated at the same frequency of 352 MHz as the existing Cu-cavities. In this way a maximum use of the already installed rf systems can be made. With the existing sixteen 1-MW klystrons foreseen for the first stage, energies can be upgraded well above 90 GeV.
### TABLE 3
Scenarios and costs of LEP upgrading (in MSF)

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1 2 and 6</th>
<th>Scenario 2 2, 4, 6 and 8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(a) + (b)</td>
</tr>
<tr>
<td>Number of rf cells/number of sc cavities</td>
<td>8/64 22.7</td>
<td>8/64 22.7</td>
</tr>
<tr>
<td>Costs of cavities and cryostats</td>
<td>24/192 68.5</td>
<td>16/128 45.4</td>
</tr>
<tr>
<td>Number of klystron systems to be bought</td>
<td>4 7.2</td>
<td>4 7.2</td>
</tr>
<tr>
<td>Costs of klystron systems to be bought</td>
<td>4 7.2</td>
<td>4 8 14.4</td>
</tr>
<tr>
<td>Number of refrigerators</td>
<td>4 20</td>
<td>4 20</td>
</tr>
<tr>
<td>Costs of refrigerators</td>
<td>4 29.6</td>
<td>4 40 8 51.2</td>
</tr>
<tr>
<td>Costs of cavities, rf, cryogenics</td>
<td>~ 50 ~ 105</td>
<td>~ 50 ~ 100 ~ 157</td>
</tr>
<tr>
<td>Additional costs related to sc cav. system</td>
<td>5 11</td>
<td>5 ~ 36 ~ 41</td>
</tr>
<tr>
<td>Additional costs of upgrading not related to sc cavity system</td>
<td>36 36</td>
<td>36 36 36</td>
</tr>
<tr>
<td>Total costs of LEP upgrading</td>
<td>91 152</td>
<td>91 172 234</td>
</tr>
<tr>
<td>LEP energy (GeV)</td>
<td>73-77 84-92</td>
<td>73-77 82-88 90-98</td>
</tr>
</tbody>
</table>

**Fig. 6** Total costs of upgrading for two different scenarios as a function of LEP energy (for $E_{\text{acc}} \approx 7$ MV/m). Costs are interpolated graphically inbetween the different scenario steps of table 3. Possible further upgrades are indicated by a broken line (increase of gradients from 7 to 10 MV/m for scenario 1(b); increase of number of rf-cells equipped with sc cavities from 32 to 48 for scenario 2(c)).
The first LEP 4-cell Nb cavities have been fabricated and tested at CERN and have exceeded the design values

\[ E_{\text{acc}} = 5 \, \text{MV/m} \quad \text{and} \quad Q_0 = 3 \times 10^8 \]

Cryostats, main couplers, horn couplers and frequency tuners for operation in LEP have been developed, constructed and tested successfully.

It is intended in near future to have some cavities and cryostats fabricated by industry in order to start the transfer of knowhow from CERN to industry and in order to get more precise information on costs. This should enable the installation of a few sc cavities in LEP at an early stage of operation.

For the upgrading two scenarios have been considered:

- Installation of sc cavities in interaction regions 2 and 6 only. If six rf cells on each side of interaction points will be equipped with sc cavities an energy of 84 GeV and \( \sim 92 \) GeV can be reached (for \( E_{\text{acc}} = 5 \) and \( 7 \) MV/m respectively). It has, however, to be checked whether L EP can be operated at higher energies with 2 rf stations only.

- Installation of sc cavities in all 4 interaction regions. This will enable an upgrading to 90 GeV and 98 GeV respectively but will need additional civil engineering and facilities in regions 4 and 8. A later additional upgrading up to 100 and 108 GeV respectively would be possible.

Total costs of upgrading including cavities and cryostats, rf systems, refrigerator systems and additional costs for civil engineering, surface buildings and upgrading of other machine components would amount to:

- 152 MSF (scenario 1, \( E_{\text{acc}} \leq 7 \) MV/m, \( E \leq 92 \) GeV) and
- 234 MSF (scenario 2, \( E_{\text{acc}} \leq 7 \) MV/m, \( E \leq 98 \) GeV).

Construction and installation time will extend over a period of 5–6 years. This includes a preparatory period of 2 years for transfer of knowhow and for preparing fabrication at high rate.

It has to be stressed that up to now no large accelerating system involving many sc cavities has ever been tested in a storage ring. Therefore it will be of utmost importance to install a string of sc cavities in LEP at an early stage of operation. In this way one should gain experience in the operation of cavities with all auxiliaries, study the influence of such systems with all their components on beam stability and higher order mode excitation. One has to study also the long terms effects of an accelerator environment on cavity performances. Therefore it is intended to install a sc cavity in the SPS and to operate it for long periods. This test will be backed up by systematic investigations on vacuum failures, air exposures and dust transport with test cavities.

Acknowledgements

Many aspects of the upgrading programme have been discussed with members of the rf and cryogenics groups of EF Division and with members of the LEP Division. We would like to thank all those who participated in these discussions.
A FEW FORMULAE AND SYSTEM PARAMETERS

Cryogenic losses

We assume in the following 4-cell cavities operated in the π mode at a frequency of 352.21 MHz (λ = 0.851 m). For the cavity geometry shown in fig. 2 this gives the parameters of Table 1. We assume a quality factor

\[ Q_0 = 3 \times 10^9 \text{ at } 4.2 \text{ K and at design field,} \]

a value which has been repeatedly reached in 350 MHz test cavities [2].

The (cryogenic) rf losses per unit length of the cavity are given by

\[ P_c = \frac{E_{\text{acc}}^2}{(r/Q)Q_0} \]

\[ = \begin{cases} 30 \text{ W/m for } E_{\text{acc}} = 5 \text{ MV/m} \\ \sim 60 \text{ W/m for } E_{\text{acc}} = 7 \text{ MV/m} \end{cases} \]  \hspace{1cm} (A1)

\( E_{\text{acc}} \): accelerating field
\( r/Q \): shunt impedance/quality factor

To these losses we have to add the cryostat and He-transfer line losses \( P_g \). Taking into account values obtained during the PETRA test and with other comparable cryostats and transfer systems we estimate for these additional losses 10 W/m. The total cryogenic losses per unit length at 4.2 K then amount to

\[ P_{\text{cry}} = \begin{cases} 40 \text{ W/m for } E_{\text{acc}} = 5 \text{ MV/m} \\ 70 \text{ W/m for } E_{\text{acc}} = 7 \text{ MV/m} \end{cases} \]  \hspace{1cm} (A2)

These values do not include synchrotron radiation losses and rf losses due to the vacuum tube tapers and vacuum valves located near the cavities which may be partly absorbed at 4.2 K walls. It is assumed that the overall technical efficiency for evacuating these losses at 4.2 K is \( \eta_{\text{cry}} = 0.33\% \) [21].

rf power requirements

For a sc cavity the rf losses can be completely neglected against the rf power transferred to the beam. Therefore the rf power per unit length needed is

\[ P_b = 2 \times i_b E_{\text{acc}} \times \sin \phi_s \]  \hspace{1cm} (A3)

\( i_b \): current per beam
\( \phi_s \): synchronous phase angle
For the LEP design current of 2 x 3 mA and for $\sin \phi_s = 0.87$ we get

$$P_b = \begin{cases} 26.1 \text{ kW/m} & \text{for } E_{\text{acc}} = 5 \text{ MV/m} \\ 36.5 \text{ kW/m} & \text{for } E_{\text{acc}} = 7 \text{ MV/m} \end{cases}$$

For one half-cell with \( L_{\text{eff}} = 8 \times 1.7 = 13.6 \text{ m} \) this amounts to 355 kW respectively 496 kW. It is thus clear that one 1-MW klystron can feed 2 half-cells. We note, that the bandwidth of the sc cavities may be somewhat increased for ease of operation above the optimum value. This leads to some additional reflection of rf power at the cavity entry.

For the scenario given above we assume always that one 1-MW klystron will feed 2 half-cells although this may ask in some cases for a reduction of klystron power. We also assume that the overall efficiency of rf production (including rf losses in the waveguides) is about $\eta_{rf} = 60\%$.

**Acceleration efficiency**

The acceleration efficiency $\eta_{\text{acc}}$ is given by

$$\eta_{\text{acc}} = \frac{P_b}{(P_b + P_c) \frac{1}{\eta_{\text{rf}}}} = \frac{P_b}{P_{\text{mains}}}$$

- $P_b$: power given to the beam.
- $P_c$: rf losses at cavity walls.
- $P_{\text{mains}}$: mains power needed for the production of $P_b$ and $P_c$.
- $\eta_{\text{rf}}$: overall efficiency for rf production, $\eta_{\text{rf}} \sim 60\%$.

For nc cavities one gets from (A1) and (A3)

$$\eta_{\text{acc}} = \frac{2 \frac{I_b}{E_{\text{acc}}} \times \sin \phi_s}{\frac{I_b}{E_{\text{acc}}} \times \sin \phi_s + \frac{E_{\text{acc}}^2}{(R/Q) \cdot Q_o \frac{1}{\eta_{\text{rf}}}}}$$

For a given cavity with fixed R/Q and $Q_o$ the efficiency depends on the gradient and on the beam current.

For sc cavities, $P_c \ll P_b$ and $P_c$ can be neglected in the rf power balance. However, it cannot be neglected for the electric power production because it is dissipated at low temperatures. In addition the static cryogenic losses of cryostats and He-transfer lines have to be taken into account.

One gets for sc cavities with (A1), (A2) and (A3)

$$\eta_{\text{acc}} = \frac{2 \frac{I_b}{E_{\text{acc}}} \times \sin \phi_s}{\frac{I_b}{E_{\text{acc}}} \times \sin \phi_s + \left[ \frac{E_{\text{acc}}^2}{(R/Q) \cdot Q_o \frac{1}{\eta_{\text{cry}}}} + \frac{P_s}{\eta_{\text{cry}}} \right] \frac{1}{\eta_{\text{rf}}}}$$

For the cavity parameters from table 1 and for LEP conditions one obtains the values given in table A.1.
### Table A.1
Accelerating efficiencies in LEP (for different beam currents and accelerating fields)

<table>
<thead>
<tr>
<th></th>
<th>nc cavities</th>
<th>sc cavities</th>
<th>sc cavities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.5 MV/m</td>
<td>5 MV/m</td>
<td>7 MV/m</td>
</tr>
<tr>
<td>2 x 3 mA</td>
<td>7.5%</td>
<td>47%</td>
<td>44%</td>
</tr>
<tr>
<td>2 x 6 mA</td>
<td>13.3%</td>
<td>53%</td>
<td>51%</td>
</tr>
</tbody>
</table>

The large difference in efficiency explains that a total rf power of 16 MW for LEP produces with nc cavities a voltage $U_{rf} = 402$ MV ($E = 55$ GeV), and with sc cavities a voltage $U_{rf} = 3044$ MV ($E = 98$ GeV).
APPENDIX 2

A COST OPTIMISATION FOR A SC ACCELERATION SYSTEM FOR LEP

We include in the cost optimisation:

- Cost of sc cavities and cryostats with all auxiliaries (like couplers, tuner, pumps).
- Cost of klystrons with circulators, power supplies, waveguide system, control, regulation and drive systems.
- Cost of refrigerators with He transfer lines and He storage.
- Cost of electricity for a given total number of operating hours.

With the length available to rf–cavities already fixed we exclude from the optimization civil engineering and possible machine component upgradings.

We assume a given particle energy of LEP which is related to the total accelerating voltage \( U_{rf} \) by the formula

\[
E = 13.23 \sqrt[4]{U_{rf}} \quad (E \text{ in GeV, } U_{rf} \text{ in MV})
\]

and we use the field gradient \( E_{acc} = U_{rf}/\eta_{eff} \) as an optimization parameter.

The total costs are then given by

\[
C = C_{cav} \times \eta_{eff} \\
+ C_{rf} \times 2l_0 \times U_{rf} \times \sin \phi_s \\
+ 2.5 \left( \frac{E_{acc}^2 \times \eta_{eff}}{r/Q x Q_0} + P_s \times \eta_{eff} \right)^{0.6} \\
+ N_h \cdot C_{kWh} \left( \frac{2l_0 \times U_{rf} \times \sin \phi_s}{\eta_{rf}} + \frac{E_{acc}^2 \times \eta_{eff}}{r/Q x Q_o} + P_s \times \eta_{eff} \right) \frac{1}{\eta_{cry}}
\]

- First term: cavity and cryostat costs. \( C_{cav} \): cost of cavities and cryostats per unit length; we have assumed as our best guess from presently known production costs

\[
C_{cav} = \begin{cases} 
0.21 \text{ MSF/m for 5 MV/m} \\
0.25 \text{ MSF/m for 10 MV/m}
\end{cases}
\]

which we interpolate linearly for other gradients.

- Second term: Cost of rf system. We assume that rf power is delivered by 1-MW klystrons. The cost for a complete rf system, \( C_{rf} \), is estimated to 1.8 MSF/MW [22]. We assume \( i_b = 3 \) mA and \( \sin \phi_s = 0.87 \).

- Third term: Cost of a complete refrigeration system. We assume that refrigerator costs including compressors and transfer lines are given by the formula [22]
\[ C_{\text{cry}} (\text{MSF}) = 2.5 \left( P_{\text{cry}} (\text{kW}) \right)^{0.5}. \]

\( P_{\text{cry}} \) is the sum of cavity losses, cryostat and He transfer line losses.

- Fourth term: Cost of electricity.
  
  \( N_h \): total number of operating hours.
  
  \( C_{\text{kWh}} = 0.07 \text{ SF/kWh} \): Mean cost of kWh at CERN.
  
  \( \eta_{\text{rf}} = 60\% \): total efficiency of rf production.
  
  \( \eta_{\text{cry}} = 0.33\% \): total technical efficiency of refrigerator system for cooling at 4.2 K.

The result of an optimization for \( E = 100 \text{ GeV}, \ U = 3264 \text{ MV}, \ N_h = 20000 \text{ h}, \ Q_o = 3 \times 10^8 \) and for three different sets of \( C_{\text{cav}} \) is given in fig. 7. There exists a broad minimum which shifts as expected towards higher accelerating fields for high \( C_{\text{cav}} \). It can be seen that a gradient of 7 MV/m corresponds to costs less than 10% above the minimum costs. The cost optimization does not take into account the “quantization” of klystron and cold box numbers in a real storage ring.

**Fig. 7** Cost optimization for a sc acceleration system as a function of accelerating fields and for three different values of \( C_{\text{cav}} \). Costs are normalised to the value for \( E_{\text{acc}} = 7 \text{ MV/m} \) and \( C_{\text{cav}} = 0.21 \text{ MSf/m} \).
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MEASUREMENT OF THE W MASS AT LEP 200

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1. INTRODUCTION

During the coming years, the mass of the $Z^0$ particle and the value of $\sin^2 \theta_W$ will be accurately measured at LEP I and SLC. For the first time, these measurements will be sensitive to the electroweak radiative corrections and any deviation from the values expected in the framework of the Standard Model can be interpreted in terms of the existence of heavy quarks, Higgs particles, other generations, ... But, to prove the real manifestation of the Standard Model there is a need for an other measurement at least with the same level of accuracy. We believe that the only place where this can be achieved in the foreseeable future is LEP 200. All the accurate methods we propose use the knowledge of the machine beam energy which has to be precisely measured and stable\(^1\).

2. **W** PRODUCTION AT $e^+e^-$ MACHINES

In an electron collider, with a beam energy of about 100 GeV, several mechanisms can contribute to the production of W and Z particles. The corresponding cross sections and the decay angular distributions of the produced particles can be accurately computed in the framework of the Standard Model.

2.1 **W** production in $\gamma\gamma$ collisions

Such a process has a very small cross section ($\approx 10^{-3}$ pb) and can be neglected in this energy domain.

\[ e^- \quad \gamma \quad W^+ \quad e^+ \]

\[ e^- \quad \gamma \quad W^- \quad e^+ \]

2.2 **Single W or Z** production\(^2\)

2.2.1 **Z** production

Radiative corrections to the $e^+e^-$ annihilation cross section into hadrons are the source of the process $e^+e^- \rightarrow \gamma Z^0 (\gamma)$. We note (\gamma) any additional photon produced in these events. This cross section is large and in practice, the "visible" cross-section depends on the acceptance cuts imposed on the high energy "monochromatic" photon. For instance $\sigma \approx 16$ pb for $E_\gamma > 10^0$ and $E_\gamma > 50$ GeV ($m_t > m_{Z^0/2}^2$) at $E_b = 95$ GeV.

In addition to this mechanism, photons radiated by one incoming beam particle can produce a $Z^0$ when they interact with the other beam particle.

\[ e^- \quad \gamma \quad Z^0 \quad e^+ \]

\[ e^- \quad \gamma \quad e^+ \]

The corresponding cross section is about 3 pb at $E_b = 100$ GeV. Low masses for the $Z_0 e_2$ system, close to threshold, are favoured. The $Z^0$ takes essentially all the beam...
energy and the remaining electron $e_2$ has a low momentum. This gives very asymmetric final states with large missing energy.

2.2.2 Single $W$ production

In the same way, single $W$ bosons can be produced by $\gamma e$ interactions. The cross section is much smaller than in the previous case, being $\sim 0.4$ pb ($E_D = 100$ GeV). In contrast to the previous reaction, this process favours large $W$-$\nu$ masses, close to the kinematical limit.

The direction of the produced $W$ is peaked along the beam.

2.3 $W$ and $Z$ pair production

The reaction $e^+e^- \rightarrow 4$ fermions, with intermediate formation of $W$ and $Z$ bosons is the dominant mechanism for $W$ and $Z$ production. Depending on the final state, $W$ and $Z$ mediated amplitudes can interfere. This effect is in practice very small and has been neglected in this study. At $E_D = 100$ GeV, the $Z^0$ pair production is about 6% of the $W$ pair cross-section which amounts to 15 pb. The rise of the cross section with energy is determined by the values of the $W$ mass and width and also by radiative corrections.

On fig. 1 we have shown the contribution of the various mechanisms for different beam energies and in the following we will consider only the pair production of $W$ bosons.

3. THEORETICAL $W$ MASS SPECTRUM AT THE PARTON LEVEL AND THE EFFECT OF RADIATIVE CORRECTIONS

3.1 $W$ line shape without radiative corrections

We forget for a moment the presence of radiative corrections and consider the $W$ production at $\sqrt{s} = 190$ GeV. Several effects can distort the $W$ mass line shape in comparison to a simple Breit-Wigner dependence of the form:

$$BW(M) \, dM = \frac{A}{(M^2 - M_0^2)^2 + \Gamma_0^2 M^2} \, dM$$  \hspace{1cm} (1)
3.1.1 The decay matrix elements of the W depend on the actual W mass
\[ A_{\text{decay}} \sim M f(\text{angles}) \]
and as the cross section varies like:
\[ d\sigma \sim |A|^2 dM^2 \]
we expect that the parameter \( A \) in (1) varies with \( M \) like:
\[ A = A_0 (M/M_0)^3. \]

In practice \( A = A_0 (M/M_0) \) gives the best adjustment of (1) to the generated parton-parton mass distribution. With this dependance, the fitted W mass is close to the input value within 4 MeV (with an accuracy of the similar order) whereas a cubic behaviour for \( A \) gives a displacement by 40 MeV. This result happens to be due to a cancellation mechanism generated by the interference of different W helicity amplitudes\(^4\).

3.1.2 Because of radiative corrections to the W propagator\(^5\) the W natural width depends on the actual vector boson mass. We expect to have \( \Gamma(M) = \Gamma_0 M/M_0 \). If we use this behaviour during the event generation and if we keep the expression (1) which neglects this effect, the fitted mass is lower than the nominal value by 20 MeV.

3.1.3 In the process of W mass reconstruction we will impose, as a constraint, the equality between the energies of the beam and of the W. In practice this is never true because the two emitted W’s have different masses. The adjustment of expression (1) gives a fitted value which is 25 - 35 MeV higher than the original mass.

3.2 Effect of radiative corrections

The complete hard photon emission cross-section, involving the diagrams shown on fig. 2, has been calculated\(^6\) and will be available as an event generator. These computations are needed for an accurate determination of the W mass using real data. In the following we have only considered the effect of the initial state radiation which represents the bulk of the contribution.

Radiative corrections decrease the slope of the energy variation near threshold of the W pair production cross-section.

They can also distort the W mass distribution by reducing the available phase space for W production. This effect is marked near threshold but is negligible for \( E_B = 95 \text{ GeV} \) (displacement \( \Delta M \sim \text{a few MeV} \)).

The largest effect of radiative corrections comes when we use the beam energy constraint (Cf. following paragraphs) because we overestimate the real available energy used to produce W pairs and thus we create a tail at high W masses. The fitted value is displaced by 470 MeV (± 40 MeV) and this value depends slightly on the interval used for the adjustment (fig. 3).

At this stage of the study we can reproduce, using a Monte Carlo program, the exact shape of the W mass distribution with an accuracy well below 50 MeV. This situation is a characteristic of \( e^+e^- \) machines.
4. BACKGROUND OF THE $W^+W^-$ CHANNEL AND ITS REJECTION

Cross sections for most of the processes occurring in the LEP II energy range, not including radiative corrections, are shown on fig. 4. The decays of pairs of $W$ or $Z$ bosons are not the main sources of hadronic events at $E_b = 95$ GeV. Hadrons are essentially produced in $e^+e^-$ annihilation events into $q\bar{q}$ pairs or in $\gamma\gamma$ collisions. Most of the events are coming from radiative corrections, this means that the mass of the hadronic system is smaller than the total available energy and that there is usually a bad energy balance between the initial and final measured states.

4.1 Cross sections (fig. 5)

4.1.1 Photon-photon interactions

The hadronic cross section in $\gamma\gamma$ collisions depends sharply on the mass of the produced hadronic system. For instance, if we require that $M_{had} \geq 0.4 \sqrt{s}$, the total cross section for $q\bar{q}$ production is of the order of 1 pb at $\sqrt{s} = 190$ GeV. Thus in the following, the contribution from this process will be neglected.

4.1.2 Annihilation events

At the same energy, the total hadronic annihilation cross-section is 23 pb in the absence of $t$ quark production and without including radiative corrections. The radiative corrections push this value up to about 100 pb. The cross section for events of the type $e^+e^- \rightarrow \gamma Z(\gamma)$ where the high energy photon is seen in the apparatus ($\theta_{\gamma} > 10^0$) amounts to about 16 pb.

4.2 Background rejection and selection of the $W^+W^-$ channel

Our aim being to measure the $W$ mass, we have only considered events in which, at most, one $W$ decays leptonically. These events can be characterized by the following properties:

- about equal energy sharing between the two $W$'s. Each $W$ has an energy close to the beam energy and large radiative emissions are depressed by the sharp energy variation of the production cross-section
- each hadronic decay of a $W$ gives at least two energetic jets
- a lepton decay generates a high energy charged lepton $E_L > 20$ GeV

(at $\sqrt{s} = 190$ GeV).

We have to consider two classes of events according to the presence or not of a $W$ lepton decay.

4.2.1 Background to the channel $W_{1,2}\rightarrow$ hadrons (fig. 6)

We have to eliminate most of the radiative events. An energy balance cut is not sufficient because a lot of the radiated energy can be seen in the apparatus. Such events can be characterized by the presence of an isolated $\gamma$ or by an $e^+e^-$ pair of high energy. Sometimes the photon is included in a hadronic jet by the cluster finding algorithm and this usually gives a jet with a leading high energy photon.
We have required a minimum number of 4 reconstructed jets. This notion of jet is
cutoff dependant during the generation of the events and also depends on the algorithm
during the reconstruction. Most of the events in the background have in fact three
generated QCD jets but the algorithm has splitted one of the jets in two parts providing a
four jets event. We have merged two jets if their relative angle satisfies \( \cos \theta_{jj} > 0.8 \).
This cut keeps more than 90% of the \( \WW \) events and removes 50% of the background.

We then ask for energy balance in two steps:

- we require a two jet combination with an energy close to the nominal beam energy
within 10 GeV
- we demand that the missing energy of the remaining system will be less than 30 GeV
to ensure a rough global energy balance.

These cuts, defined at \( \sqrt{s} = 190 \) GeV, have an efficiency of 75% on \( \WW \) events and
keep only 0.7% of the background.

4.2.2 Background to the channel \( W_1 \to \ell \nu, W_2 \to \text{hadrons} \) (fig. 7)

We ask for a high energy lepton (\( E_\ell \geq 20 \) GeV). This eliminates the main part of the
\( W \) decays into \( \ell \nu \) and a particular selection has to be defined to recover these events\(^7\).

The background is mainly coming from events in which a high energy photon is converted
inside the apparatus. For instance, the gamma produced in \( \gamma-Z_0 \) events. A good rejection
against electromagnetic pairs has to be provided by the detector, which can be achieved
easily with dE/dx information.

The remaining background comes from semi-leptonic decays of heavy flavours and can be
reduced by a cut on the lepton isolation. For instance, we can request that there is no
track with more than 2 GeV in the jet which contains the lepton. Such a cut is also
efficient against "muon" candidates from hadronic punch-through. On fig. 7 we have shown
the effect of a cut on energy balance between the beam and all the hadrons seen in the
apparatus. If events are selected using the condition \(-5 \) GeV \( \leq E_{\text{beam}} - E_{\text{hadrons}} \leq 10 \) GeV
we can remove essentially all the background.

The level of background for this channel becomes negligible, well below 1 pb.

5. \( W \) Mass Measurement from the Threshold Behaviour of \( \sigma(e^+e^- \to W^+W^-) \)

The lowest order cross section for \( e^+e^- \to W^+W^- \) grows very fast near threshold. The
finite width of the \( W \) will of course greatly reduce the sensitivity of the total cross
section to the \( W \) mass. However, a first study on the feasibility of the \( W \) mass determina-
tion from the measurement of the \( W^+W^- \) pair production cross section showed that a
statistical error on \( M_W \) below 100 MeV was within reach\(^8\).

In contrast to the other methods (where one would like to run at the maximum of the
production cross section), this method needs some luminosity around the threshold region.
However, in a scenario where the upgrading to full LEP II energy is done in steps, the
required lower beam energies will in any case be available for some time.

The method is based on a least squares fit of the expected cross sections at different
energies to the measured cross sections for \( W^+W^- \) production. A precise knowledge of the
theoretical cross section is therefore essential. In particular, radiative corrections
and finite width of the W have to be taken into account. From the experimental side, the
detector efficiency (as a function of the beam energy) has to be known with sufficient
precision.

5.1 Statistical Procedure

The excitation curves are calculated numerically. They can be compared to Monte Carlo
integration of the full matrix element for W pair production and their subsequent decay
into 4 fermions. This part of the event generator includes hard photon radiation by one
of the incoming electrons \( E_\gamma > 0.1 \text{ GeV} \) and the width of the W(\( \gamma = 2.65 \text{ GeV} \)). The results
are shown in figure 8, using \( M_W = 82 \text{ GeV} \) and \( \sin^2 \theta_W = 0.223 \).

It is now assumed that at N energies \( E_i \) the total W pair production cross section \( \sigma_i \)
is measured, from runs with integrated luminosity \( L_i \). The total integrated luminosity \( \sum_i L_i \)
was fixed at 500 pb\(^{-1}\). The values of \( \sigma_i \) were drawn from curves like in Figure 8 +
(optionally) a background contribution \( +bg \) parametrized as \( a+b/s \). These 'experimental'
cross section values \( \sigma_i \) were then smeared according to a rms spread of \( \sqrt{\sigma_i L_i} \). The N
values \( \sigma_i \) are then used in a least square fit to the theoretical excitation curve and a
'best' value of \( M_W \) is obtained. This 'experiment' is repeated say 1000 times. The expected
error \( \sigma_M \) on the mass measurement from a single experiment is then obtained from the rms
value of the distribution of these 1000 \( M_W \) values.

5.2 Optimization of data taking

Assuming an integrated luminosity of 100 pb\(^{-1}\) at each energy, figure 9.a shows the
absolute value of the difference between the cross sections obtained for two different
W masses (82 and 81.9 GeV) divided by the expected r.m.s. error. This quantity, called
resolving power, has a peak at about 82 GeV, is zero at 86 GeV where the two cross
sections are equal, and increases again for higher energies because the cross section
rises to a higher plateau for higher W masses. Therefore the energy settings should be
concentrated around the nominal W mass, \( M_W = 82 \text{ GeV} \), and at much higher energies, above
92 GeV. The cross over region between 84 and 88 GeV should be avoided.

This is demonstrated in figure 9.b. Data taking is always distributed over five
energies. The two points at 75 and 95 GeV are fixed. The low energy point is essential
for background determinations and the higher one is at the highest available energy.
Several ways to order the remaining three energies are shown in figure 9.b. The best
precision attainable for the W mass is \( \sigma_M = 90 \text{ MeV} \). This result holds for a variety of
energy settings as long as no luminosity is wasted in the region of 84 to 88 GeV.

The presence of a (small) constant background contribution of 0.2 pb would already
lead to a systematic shift on the W mass of \( \sim 130 \text{ MeV} \) if one would not allow for a
background term in the fit to the excitation curve. Considering 5 cross section measure-
ments at 75, 95, E and E \( \pm 1 \text{ GeV} \), with the same luminosity of 100 pb\(^{-1}\) at each point, we
have plotted in figure 10 the variation of \( \sigma_M \) with the energy E for different background
ccontributions:

1. solid curve: no background,

2. dashed curve: background = \( 2.0 \times \frac{10^4}{s} \text{ pb} \) (i.e. 0.74 pb at 82 GeV),
3. dashed-dotted curve: background = \( 4.0 \times 10^4 \left[ \text{pb} \right] \) (i.e. 1.5 pb at 82 GeV).

Figure 10a is the result from fits that include a background parametrization as \( b \), whereas Figure 10b corresponds to fits with background terms as \( a + \frac{b}{s} \).

From Figure 10a one can conclude that with reasonable assumptions about the background, a statistical error on \( M_w \) of 120 MeV can be obtained. The presence of an additional constant background term of 0.5 pb would give a systematic mass shift of -60 MeV.

5.3 Systematic errors

The statistical errors on the cross section measurements, even when using all the events, range between \( \sim 2.5\% \) at 95 GeV and \( > 10\% \) at 75 GeV. It is therefore clear that anticipated point-to-point uncertainties on the luminosity of 2% will have a negligible effect on the final mass resolution. An absolute uncertainty of 5% on the luminosity leads to a systematic error on the \( W \) mass of 120 MeV.

The same argument holds with respect to the knowledge of the detector efficiency. Its absolute value has to be known within a few percent to keep the resulting systematic error small compared to the expected statistical error. As the \( W \) production angular distribution becomes more peaked in the forward direction at higher energies, the loss of events due to the hole in the detectors around the beam direction is expected to be energy dependent. However, from Monte Carlo calculations it was found that this energy dependence over the range 75 to 100 GeV is small. When neglected in the fit, the additional systematic error on \( M_w \) was < 10 MeV.

5.4 Conclusions

A fit of the theoretical excitation curve for \( W \) pair production to the measured cross sections at 5 different energies with an integrated luminosity of 100 pb\(^{-1}\) at each energy, allows to extract the \( W \) mass with a statistical error between 100 and 130 MeV. This result is obtained using all events and under the assumption that the background has a known 1/s shape (one free parameter) and is < 1 pb at 83 GeV. If a second parameter is needed to describe the background, the error on the \( W \) mass becomes 170 - 190 MeV. It would then be better to use only events where one (or both) of the \( W \)'s decays leptonically. The background is expected to be much smaller. In that case, the statistical error on \( M_w \) will be \( \sim 130 \) MeV.

6. W MASS RECONSTRUCTION USING THE W DECAY PRODUCTS

During the LEP workshop in Les Houches in 1978\(^{10}\) it has been proposed to identify the \( W \) particles using the mass of the two jets produced in the final state. The peak obtained this way is broad and displaced towards low masses because not all particles are measured in the apparatus.

The accurate mass measurement of the \( D \) and \( B \) particles\(^{11}\) produced in pairs at \( e^+e^- \) colliders, is obtained after imposing the equality of the beam and particle energies. We will show in the following that a similar technique - quoted as "mass rescaling" - can be applied to \( W \) bosons.
After a study of the jet energy reconstruction in typical LEP detectors we discuss the details of the properties of "mass rescaling". The W mass measurement is then considered for hadronic and leptonic decays. In these Monte-Carlo studies events were always generated with a "true" W mass of 82 GeV. The understanding of systematic mass displacements is finally presented.

6.1 Measurement of the events

Particles produced in the final state are distributed among a few jets. The 4-vector energy-momentum of these jets can be obtained using the energy deposited in calorimeter cells (hadronic and electromagnetic) or by reconstructing individual particles making use of the fine granularity of LEP detectors. These detectors have been designed to do measurements at the Z⁰ pole. At LEP II the energy is 2 times higher but in case of W pair production we have also 2 times more jets in the final state which means that the detectors are also well conceived for this energy domain. Typical detector parameters used for this study are given in table I.

6.1.1 Calorimeter

We have considered the best calorimeter designed for LEP (L3 detector). Hadrons loose 40 ± 20% of their energy in the BGO with an e/m response of 1.4. No attempt has been made to distinguish electromagnetic from hadronic showers in the BGO. The total measured energy of the jets is then: \( E_{\text{meas}} = f_{\text{BGO}} \times E_{\text{BGO}} + E_{\text{HCAL}} \) with \( f_{\text{BGO}} = 1.15 \) determined empirically by Monte Carlo.

The detection threshold for photons in the BGO is very low (30 MeV) and the main distortion induced between the real and measured energy comes from:
- detector holes along the beam directions (2% of 4π)
- escaping neutrinos (\( E_\nu \sim 3\% \) of the jet energy)
- difference in response of the electromagnetic calorimeter to photons and hadrons.

6.1.2 Individual hadron reconstruction

Charged hadrons are measured in tracking devices with a better accuracy than in a hadronic calorimeter.

Photons and neutral hadrons are measured using electromagnetic and hadronic calorimeters. Some of them are lost because a charged hadron may have started a shower in their neighbourhood. These losses depend on the granularity of the calorimeters. With an appropriate algorithm we can reduce their importance. For instance we can measure the energy of a charged track in the following way:

\[ E = E_{\text{TPC}} \quad \text{if} \quad E_{\text{TPC}} > E_{\text{EM}} - 1.5 \sigma_E + E_{\text{HCAL}} - 1.5 \sigma_H \]

or

\[ E = E_{\text{EM}} + E_{\text{HCAL}} \quad \text{if this condition is not satisfied.} \]

Apart from the L3 BGO calorimeter, visibility thresholds for electromagnetic and hadronic showers are not usually negligible and we have done a study using 500 MeV for photons and 1 GeV for neutral hadrons.

6.1.3 Mean reconstructed energy \(<E_{\text{meas}}>\)

Whereas for the L3 calorimeter, the difference between the mean reconstructed energy and the jet energy can be attributed to escaping neutrinos (3%), for other detectors it
amounts to about 10%. The corresponding distribution is given in fig. 11 and is very asymmetric.

Two classes of events can be defined:
- those situated in the tail, for which usually a high energy particle has been lost (photon or neutral hadron in the jet core),
- those situated around the peak value (5% of energy loss) for which the energy loss is determined mainly by threshold effects and escaping neutrinos.

6.1.4 Energy resolution in $E_E$

In case of calorimetric measurements the jet energy resolution is given by the energy resolution of the hadronic calorimeter.

For granular devices the variance of the reconstructed energy distribution is determined by the tail and corresponds to a similar accuracy. If we consider only those events situated around the peak value and use the FWHM of the distribution to estimate the energy resolution we get an improvement by a factor $\sim 2$.

In fig. 12 we have shown the variation of $E_{\text{meas}}$ and $E_E$ with the jet energy. Apart from jets emitted in the very forward and backward directions, we have not observed a real variation of these quantities with the jet polar angle.

6.1.5 Reconstructed mass for hadronic decays

Two classes of events have to be considered:
- class 1: one $W$ decays leptonically
- class 2: the two $W$'s decay into hadrons.

These events have to fulfill the conditions imposed on background rejection.

For class-1 events all hadrons are supposed to come from the $W$ decaying non-leptonically. For class-2 events we have to use an algorithm to attribute the jets to a given $W$ decay. In this case we have only considered events with 4 or 5 jets reconstructed in the final state (using the standard LUND algorithm LUCUS)[13] and we have assumed that one of the $W$'s has decayed into 2 jets. This jet-jet combination is obtained requiring that the difference between the jet-jet energy and the beam energy is minimal. The remaining hadronic system is attributed to the other $W$.

The reconstructed mass-spectra are shown on fig. 13 and 14 for class-1 and class-2 events respectively. The peak position is lower than the monopole $W$ mass by a few GeV because of energy lost during the event reconstruction and the width of these distributions is of the order of 10 GeV.

6.2 Mass rescaling using the beam energy

We propose to scale the measured $W$ mass by the ratio of the beam and $W$ measured energies.

$$M = M_{\text{meas}} \times \frac{E_{\text{beam}}}{E_{\text{meas}}}$$

This expression can be justified by the following arguments. If we lose one particle belonging initially to one of the two jets from a $W$ decay: the difference between the measured $W$ mass and the initial value is coming from:
- a variation of the two jets opening angle,
- a decrease of the energy of one jet.

The relative jet-jet angle is very large (≈ 100°) and its variation will be of the order: \( \Delta \theta \sim P_j / P_{\text{jet}} \); thus \( \Delta \theta / \theta \) is very small and the corresponding variation of 
\( \Delta M \sim 100 \text{ MeV} \).

The effect of energy loss is much more important. If we define \( z = P / P_{\text{jet}} \) 
\((P = \text{particle momentum})\) the induced variation on the \( W \) mass will be: 
\( \Delta M \sim z \times \frac{M}{2} \sim P \).

The reconstructed \( W \) mass spectrum reflects the energy containment of the detector.

If we forget for a moment the presence of radiative corrections and the inequality 
of the two \( W \) masses we can impose the equality of the beam and \( W \) energies and, in the 
case of pure hadronic decays, the balance between the \( W \) momenta. Such a fitting procedure 
can be done, knowing the jet energy and angle reconstruction accuracies of the detector 
(cf. fig. 11). As the mass displacement is dominated by escaping energy we have only 
corrected for this effect and if we assume that the jet energy reconstruction obeys the 
relation:

\[
\frac{d \rho}{E} = \frac{A}{\sqrt{E}}
\]

we get the expression (2).

The mass distributions obtained this way are shown also on fig. 13 and 14 and have 
the following properties:

- the distributions are narrower by a factor 2, their width is about 2 times the \( W \) 
natural width
- they are about centered at the monomial \( W \) mass (if we do not consider for a moment 
the effect of radiative corrections)

\[
M = M_0 - 200 \text{ MeV (L3)}
\]
\[
M = M_0 - 300 \text{ MeV (Delphi)}
\]

In the following we show that the fitted position of the \( W \) reconstructed mass is not 
very sensitive to the expected detector properties and to the hypothesis used in the 
simulation.

A similar approach has been proposed to reconstruct the mass of heavy particles 
decaying into a few jet system produced in e^+e^- annihilation: top mass, gluino mass^{14}...

### 6.3 Reconstruction of the \( W \) mass when \( W \rightarrow \) hadrons

#### 6.3.1 Typical statistics and background

For class-1 events, after background rejection cuts, but without requiring energy 
balance and using an integrated luminosity of 500 events/pb we will get 3000 hadronic 
\( W \) decays.

For class-2 events, after all cuts we have 1700 events (in the mass peak). 
The background in the mass plots from non \( W \) events is negligible and structureless 
(fig. 15).

#### 6.3.2 Calorimetric approach

We use only class-1 events (fig. 16a).

The mass distribution is asymmetric with a tail towards high masses because of
radiative emissions. A Breit-Wigner distribution added to a second order polynomial background is adjusted on the "data". This gives

$$M = 82.273 \pm 0.065 \text{ GeV}$$

6.3.3 Granular detector approach

With class 1 events, a similar spectrum is obtained, the fitted mass being

$$M = 82.17 \pm 0.060 \text{ GeV}.$$ This value is 100 MeV lower than the previous one in agreement with the result given in 6.2 where, because of higher energy losses than in the calorimetric approach, the mass without radiative corrections is already lower by 100 MeV.

If we impose a cut on energy balance (at 10 GeV) the tail at high masses is reduced and we get:

$$M = 82.020 \pm 0.060 \text{ GeV} \quad (\text{fig. 16-b})$$

(this value is obtained adding the $W \rightarrow \text{hadrons} + \nu \ell$ mass distributions).

Using class 2 events (fig. 16-c) we get similar results, the signal is sitting on a flat "background" coming from wrong assignments of 2 or 3 jets combinations.

6.4 Reconstruction of the $W$ mass in case of leptonic decays

We consider class 1 events.

The direction of the neutrino was calculated from the momentum vectors of the lepton and the hadronic system

$$\mathbf{p}_\nu = \mathbf{p}_{\text{miss}} = - (\mathbf{p}_{\text{hadr}} + \mathbf{p}_Z)$$

In reconstructing the $\nu \ell$ mass two possibilities were tried to improve on the resolution. In the first method the $\nu$ energy was calculated from equation (5) imposing the beam constraint $E_{\text{hadr}} = E_b$. This results in mean mass values slightly below the nominal $W$ mass. In a second approach the $\nu$ energy was calculated constraining the $\nu \ell$ system to the beam energy using $E_\nu = E_b - E_\ell$.

The lepton energy was not rescaled since it is measured with much higher accuracy than the missing energy. This method gives mass values slightly above the nominal $W$ mass and also results in narrower ($\nu \ell$) mass distributions.

The statistics amounts to about 1200 - 1300 $W \rightarrow \nu \ell + \nu \ell$ events and we obtain the following results:

- method 1 : $M = 81.84 \pm 0.12 \text{ GeV}$
- method 2 : $M = 82.38 \pm 0.09 \text{ GeV}$

The corresponding mass distributions are given in fig. 17.

6.5 Stability of the $W$ mass measurement

The reconstructed $W$ mass has not exactly the nominal value. The largest displacement is due to radiative corrections and is well under control. The other source of displacements comes from the reconstruction procedure, their magnitude is at most 300 MeV and we have studied the dependence of these displacements on the hypothesis used in the simulation.

We have thus generated a large event sample to get a sufficient statistical accuracy. Using the nominal values of the expected detector performances we obtain a reconstructed $W$ mass of: $M = 82.037 \pm 0.030 \text{ GeV}$ and we study deviations from this value.
6.5.1 Bad knowledge of detector calibration

If we assume, in a completely unrealistic way, that the hadron energies are measured 3% higher than their exact values whereas photons have an energy 5% lower we obtain:

$$M = 82.137 \pm 0.031 \text{ GeV}.$$  

This effect has to be correlated to the displacements observed after mass rescaling where $\Delta M \sim -200 \text{ MeV}$ for a mean energy loss of 3% and $\Delta M \sim -300 \text{ MeV}$ for a loss of 5%.

If we want to keep these shifts below 100 MeV, the absolute detector calibrations must be known with an accuracy better than 2%.

We must note that, because of the rescaling procedure, our measurement is not affected by a common miscalibration of the hadronic and electromagnetic calorimeters.

6.5.2 Increase of the overlap probability between showers in case of granular detectors

The algorithm given in 6.1.2 is stable relative to this parameter. But if we consider that photon or neutral hadron initiated showers are lost because of the presence of a nearby hadronic shower we increase the mean energy loss.

For instance, if we multiply by 1.5 the minimum distances below which two showers are merged, we double the probabilities to lose such particles and their values become: 6% for photons and 38% for neutral hadrons.

The fitted mass is then: $M = 82.012 \pm 0.046 \text{ GeV}.$

6.5.3 Presence of the $\bar{t}b$ decays

Up to now we have assumed, in the case of pure hadronic final states, that the top quark mass is so heavy that no decay of the $W$ was possible into the $t\bar{b}$ channel. If we take a top quark mass of 35 GeV the main effect is to decrease by $\approx 25\%$ the number of reconstructed $W$'s appearing in mass peaks.

$$M = 81.936 \pm 0.036 \text{ GeV}.$$  

6.6 Need for a Monte Carlo program

This program has to correct for displacements of the mass of amplitude $\sim 200 - 300 \text{ MeV}$ with an accuracy better than 100 MeV.

It must incorporate radiative corrections and detector response. This program does not require a very high level of sophistication as the one needed to search for rare events which has to reproduce some marginal behavior of ordinary events. In our case we are concerned by the mean response of the apparatus.

The simulation of the energy reconstruction in the apparatus can be tuned using hadronic events registered at the $Z^0$ pole (a few $10^6$ events).

At LEP II the process $e^+e^- \rightarrow \gamma Z^0$ offers, with similar statistics as the production of $W$ pairs and in the same experimental conditions, a measurement of the $Z^0$ mass. In addition to Bhabha events the high energy "monochromatic" photon gives also an absolute calibration for the E.M. calorimeters.

The constraint $E_{Z^0} = 2 E_{\text{beam}} - E_{\gamma}$ can be imposed to rescale the $Z^0$ reconstructed mass and we obtain a statistical accuracy of $\sim 70 \text{ MeV}$ on the $Z^0$ mass (§16).
7. W MASS RECONSTRUCTION FROM THE END POINT OF THE LEPTON ENERGY SPECTRUM

The lepton energy spectrum from the two body decay \( W \rightarrow \nu \) can be directly related to the \( W \) mass. In the laboratory frame the kinematical limits \( \omega_\gamma < \nu < \omega_\tau \) are given by the Lorentz boost of the \( W \):

\[
\omega_\pm = E_W \times \frac{1 \pm \beta}{2}
\]

In addition to detector resolution effects the limits are smeared by the finite \( W \) width and QED photon radiation. This is particularly important for the low energy limit with additional background from \( \tau \) decays. To have the best sensitivity from this technique we need very good lepton energy resolution so that the smearing from the apparatus response will be smaller than the natural smearing. This imposes:

- \( \sigma_\nu/p < 2 \times 10^{-4} \) if tracks are measured by their curvature
- \( \sigma_\nu/E < 0.12/E \) if a calorimeter is used.

These conditions can be fulfilled only by the L3 BGO calorimeter for which the expected electron energy resolution is 0.6% at 70 GeV.

The simulated electron spectrum for \( M_W = 82.0 \) GeV is shown in fig. 19. The analysis was done by generating data sets with \( \sim 30 \) fold statistics for different \( M_W \) values in steps of 0.25 GeV and then performing a \( \chi^2 \) fit to the "measured" spectrum. To get a feeling of the sensitivity fig.20 shows a detailed view of the measured end point spectrum compared with the predicted spectra for 81, 82 and 83 GeV. The data start to become different only at \( E_\nu > 65 \) GeV, which contain only 20% of all electrons.

The result of the analysis gives \( M_W = 81.7 \pm 0.3 \) GeV, where the error is completely dominated by statistics (1000 events in the whole spectrum). If the energy resolution is assumed to be a factor 2 worse, then the result is \( M_W = 82.3 \pm 0.5 \) GeV.

In this kind of analysis a careful energy calibration is crucial. A calibration error directly reflects in the mass determination, e.g. a miscalibration by 1% induces a mass shift of \( \sim 0.7 \) GeV.

8. W WIDTH MEASUREMENT

Using class 1 events we studied the width of the fitted mass distribution as function of the intrinsic \( W \) width. As shown in figure 21 there is a linear relationship which can be approximated by:

\[
\Gamma_{\text{rec}} = \Gamma_W + 2.8 \text{ GeV}
\]

A determination of the \( W \) width seems possible with typical precision of 200 MeV, a value comparable to the contribution from one leptonic channel (e.g. \( W \rightarrow e \nu \)) in the standard model.

9. W MASS MEASUREMENT AT \( p\bar{p} \) COLLIDERS

P. JENNI has given at this workshop the expected accuracy on \( W \) mass determination after the upgrading of UA1 and UA2 experiments.

As in a \( p\bar{p} \) collider one cannot apply the "mass rescaling", the jet-jet decays cannot be used for an accurate mass measurement and only the ratio or the difference between the \( W \) and \( Z \) masses can be obtained with some accuracy.
In our group we have studied the theoretical uncertainties on the W mass reconstruction\textsuperscript{15).}

The W mass will be obtained from an adjustment of a theoretical expectation to the distribution of a measured variable which can be the transverse energy of the lepton. Such expectations require an elaborated QCD computation which will be tuned on other measurable quantities obtained in the same experimental conditions.

The theoretical uncertainty is actually estimated to be:

\[
\frac{\Delta M_W}{M_W} \sim \frac{300 - 400 \text{ MeV}}{M_Z}
\]

10. THEORETICAL INTEREST IN MEASURING $M_W$ AT THE $1^\circ/oo$ LEVEL

10.1 Introduction

A test of the electroweak theory beyond the tree level with high precision measurements at LEP-I, SLC, ACOL, Tevatron, HERA and LEP-II could confirm the range of validity of the standard model and/or could signal the need for significant modifications either in the Higgs-sector or in the fermion and gauge boson sector.

The experimental basis for such a quantitative test are (or will be) precise measurements of the ratio of neutral to charged $\nu\bar{\nu}$ and $e^\pm$ scattering, of the mass values of the Z and W bosons, their widths, and of various asymmetries (charge asymmetry, longitudinal and transverse polarization asymmetry) on the Z$^0$ resonances. At the one-loop level, all radiative corrections have been computed including the polarization asymmetries\textsuperscript{18).

It turns out that the bulk of the radiative corrections are contained in two easily calculable effects\textsuperscript{16):}

i) change in the scale in the definition of the renormalized coupling

\[\alpha(m_e) \rightarrow \alpha(m_W)\textsuperscript{25)}\]

ii) vacuum polarization effects for the massive gauge bosons (dominated by heavy fermions or scalars).

This allows to incorporate the radiative corrections in the following suggestive order\textsuperscript{16)}:

a) tree level results.

b) effective Born approximation: the effects i), ii) can be included by simple redefinition of the respective quantities in the (leading) tree results. This approximation is exact enough to discuss all measurements mentioned above, except the W-mass measurement proposed here.

c) inclusion of the remaining calculations of the Standard Model (Box-diagrams, etc). The effects of most particles beyond the standard model can be incorporated in b); their effects in c) are presumably always very small.

In models with non-standard Higgs content the so called $\rho$-parameter may be different from one already at tree level. Since, experimentally $\rho \approx 1$, and the radiative corrections to $\rho$ coming from vector boson self energies are already included under b), we can use the tree level expression, conveniently written as:
\[ \rho = 1 + \Delta \rho \quad |\Delta \rho| < 1 \]

to include such effects. Then, all data can be written, up to the corrections under c) in terms of \( \Delta \rho \) and the corrections of b).

10.2 Radiative corrections

10.2.1 Standard Model

The Standard Model contains three parameters in the gauge sector testable at LEP II.*

It is best to choose them to be the precisely known quantities:

\[ \alpha^{-1}(m_e^2) = 137.035963 (13) \quad \text{(from the Josephson effect)} \quad (1) \]

\[ G_F = (1.16634 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2} \quad \text{(from muon decay including QED corrections)} \quad (2) \]

along with the Z-mass which will be known with an accuracy of about 1/20%. Its present value is:

\[ M_Z = 93 \pm 3 \text{ GeV} \quad (3) \]

All other quantities are expressed in terms of these three, including \( S^2 = \sin^2 \theta_W \).

At tree level

\[ S^2 = \frac{1}{2} \left[ 1 + \sqrt{1 - \frac{4 \, \mu_B^2}{M_Z^2}} \right] \quad (4) \]

\[ M_W^2 = \frac{M_Z^2}{2} \left[ 1 + \sqrt{1 - \frac{4 \, \mu_B^2}{M_Z^2}} \right] \quad (5) \]

with

\[ \mu_B^2 = \frac{\mu_t \alpha(m_t)}{\sqrt{2} \, G_F} = (37.281 \text{ GeV})^2 \quad (6) \]

The one-loop radiative corrections modify these results. Since the quantity \( S^2 \) is not free anymore, it is expressible in terms of the basic three parameters. Its value depends on the particular experiment chosen to define it**, different definitions differ by radiative corrections which are small after the step b) above has been taken. We choose

\[ S^2 \equiv (1 - C^2) = (1 - \frac{\mu_B^2}{M_Z^2}) \quad (7) \]

advocated by SirLIn."\]

Now, all radiative corrections at the one-loop level can be parametrized by letting

\[ \mu_B^2 \rightarrow \mu_B^2 / (1 - \Delta r) \quad (8) \]

---

* Excluding the mass of the Higgs boson whose effects will be seen to be weak.

** \( S^2 \) is usually defined by an experiment in the same way as the tree level relation reads for the particular experiment.
where $\Delta r$ contains one-loop corrections due to known and unknown particles. $\Delta r$ has been evaluated in $^{17}$ in the Standard Model and by later authors also for other models $^{16}$. Setting the unknown parameters

$$m_t = 36 \text{ GeV}$$  
(9)

$$m_H = M_Z$$  
(10)

one gets

$$\Delta r = 0.0696 \pm 0.002$$  
(11)

with an uncertainty due to hadronic corrections $^{24}$. About 85% of (11) are accounted for by the change in $\alpha$.

10.2.2 Extensions of the Standard Model with $\rho \neq 1$

As mentioned, with an expanded Higgs sector, $\rho \neq 1$; instead

$$\rho = \frac{2 \sum_i \langle \nu_i^2 \rangle (i^2 + i - \frac{3}{4} i^2)}{\sum_i \langle \nu_i \rangle^2}$$  
(12)

at tree level. In (12), the $\nu_i$, $I_i$ and $Y_i$ are the v.e.v.'s, isospins and hypercharges of the Higgs fields $\phi_i$, over which the sum runs. For any number of doublets, $\rho = 1$; for one doublet $D$ and one triplet $T$;

$$\rho = \begin{cases} 
1 + \frac{4 \nu_T^2}{\nu_D^2} & Y_T = 0 \\
1 - \frac{2 \nu_T^2}{\nu_D^2 + 4 \nu_T^2} & Y_T = -2 
\end{cases}$$  
(13)

Experiments imply $\nu_T^2/\nu_D^2 \ll 1$.

Eqs (4) and (5) are now modified. Using the gauge invariant renormalization condition

$$M_W^2 = \rho c^2 M_Z^2$$  
(15)

where $\rho$ is set equal to its tree level value, one obtains

$$S^2 = \frac{1}{2} \left[ 1 - \sqrt{1 - \frac{4 \nu_B^2}{\rho M_Z^2 (1 - \Delta r)}} \right]$$  
(16)

$$M_W^2 = \frac{1}{2} \rho M_Z^2 \left[ 1 + \sqrt{1 - \frac{4 \nu_B^2}{\rho M_Z^2 (1 - \Delta r)}} \right]$$  
(17)

Writing, as suggested above, $\rho = 1 + \Delta \rho$, we see clearly how $\Delta r$ and $\Delta \rho$ enter in a similar way. Neglecting corrections c) (usually less than 10% of the total corrections), $\Delta r$ and $\Delta \rho$ are the same for all experiments analyzed in the effective Born approximation b)

* This condition generalizes (7) introduced by Sirlin$^{17}$.
and conveniently sum up the effects of $m_t$, $m_W$ and the new physics.

(15) and (16) now allow to test the Standard Model (with $\rho = 1$ and $\Delta r$ given in (11)); similarly new physics changes $\Delta r$ and $\Delta \rho$. Changing $\Delta \rho$ and $\Delta r$ by $\delta \Delta \rho$ and $\delta \Delta r$, we obtain for the shifts in the measured values of $S^2$ and $M_W^2$:

$$\delta S^2 = \frac{S^2 C^2}{1 - 2S^2} (\delta \Delta \rho - \delta \Delta r) \quad (18)$$

$$\delta M_W^2 = \frac{M_W^2 C^2}{1 - 2S^2} (C^2 \delta \Delta \rho - S^2 \delta \Delta r) \quad (19)$$

It is remarkable that in (18) und (19) $\delta \Delta \rho$ and $\delta \Delta r$ enter with different linear combinations. It is important to note that all the precision asymmetry measurements on the $Z^0$ resonance mentioned are sensitive only to $S^2$, that is to:

$$\delta \Delta \rho - \delta \Delta r \quad (20)$$

at the leading level (i.e. excluding corrections c) since they depend on $\rho$ only via $S^2$.

This is true also for the measurement of the ratio of the cross-sections for $\nu_e$ and $\bar{\nu}_e$ interactions.

On the other hand, the ratio of neutral to charged $\nu_\mu$ interactions on isoscalar targets has also a different dependence on $\Delta \rho$ and $\Delta r$.

On figure 22 we have drawn 1σ contour lines expected from the various experiments. The interest for a $W$ mass measurement at the 1σ/oo level is two fold:

- it provides an independent measurement with the same accuracy as LEPI on $\Delta r$,
- it is the most sensitive (5 times) to deviations from $\rho = 1$.

10.3 Effects of new particles on $M_W$

A shift $\delta \Delta r$ of $\Delta r$ (due to new physics) displaces $M_W$ by an amount $\delta M_W$ which, according to (19) is given by:

$$\frac{\delta M_W}{M_W} = - \frac{S^2}{2(1 - 2S^2)} \delta \Delta r \approx -0.2 \delta \Delta r \quad (23)$$

where we have assumed the new particles not to change $\rho = 1$ at the tree level. It is now interesting whether new particles can give rise to a measurable $\delta M_W/M_W$ (i.e. which exceeds 1%oo). The following results have been obtained:

10.3.1 Heavy top quark\(^{(19)}\)

If $m_t$ exceeds the "standard" value of 36 GeV, one obtains the following shifts in $M_W$ ($m_H = M_Z)$:

<table>
<thead>
<tr>
<th>$m_t$(GeV)</th>
<th>36</th>
<th>60</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta M_W$(MeV)</td>
<td>0</td>
<td>-30</td>
<td>220</td>
<td>500</td>
</tr>
<tr>
<td>$\delta M_W/M_W$</td>
<td>0</td>
<td>0.0004</td>
<td>0.003</td>
<td>0.006</td>
</tr>
</tbody>
</table>
If we assume, the measured value of $M_W$ to fall onto the "standard value", this allows a bound on $m_t << 80$ GeV in the experiment (assuming no other new physics!).

10.3.2 Additional heavy quarks and lepton doublets 18-19)

If the two members of the doublet have very different masses, the shift in $M_W$ is larger than 1 $^0/_{oo}$ for quarks and 0.0006 ($m^2_{lepton}/m^2_Z$) for leptons. In view of the asymptotic validity of these results, such doublets could be excluded. If the masses are degenerate, however, then one gets $\delta M_W = -14$ MeV and -42 MeV (leptons and quarks, respectively). A 1 $^0/_{oo}$ measurement would limit the number of additional generations to about 7 - 8 and 2 - 3.

10.3.3 Supersymmetric particles 18-20)

The effects of supersymmetric partner doublets of the quarks and leptons are the same as under 10.3.2 if the particles are heavy, as usually expected. Since in many models the squarks and sleptons have a large common mass $m_{3/2}$ with mass differences $\sim m_{3/2}$ ($\Delta m$ quark), the large effects due to very different masses may be damped and one expects a small effect, $<< 1^0/_{oo}$, ($\Delta m$ quark $\sim M_Z$). More details are found in ref (20).

10.3.4 The Higgs mass dependence is weak. For 30, 60, 90 GeV one has ($M_Z = 94$) 18)

<table>
<thead>
<tr>
<th>$\delta M_W$(MeV)</th>
<th>$M_H = 10$ GeV</th>
<th>100 GeV</th>
<th>1 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t = 30$</td>
<td>70</td>
<td>0</td>
<td>-160</td>
</tr>
<tr>
<td>$m_t = 60$</td>
<td>80</td>
<td>0</td>
<td>-160</td>
</tr>
<tr>
<td>$m_t = 90$</td>
<td>70</td>
<td>0</td>
<td>-160</td>
</tr>
</tbody>
</table>

for the shift $\delta M_W$ if $M_H$ deviates from 100 GeV (in this table we have taken $\delta M_W = 0$ for $M_H = 100$ GeV independently of $m_t$). A 1 $^0/_{oo}$ measurement thus may exclude a Higgs mass of $\approx 1$ TeV.

10.3.5 Several Higgs fields 21)

Adding any number of Higgs doublets does not change the-$p$-parameter; their effects on $M_W$ are in $\Delta r$. With two doublets one has 21)

<table>
<thead>
<tr>
<th>$m_1$ (GeV)</th>
<th>$m_2$ (GeV)</th>
<th>$m_p$ (GeV)</th>
<th>$\delta M_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$</td>
<td>100</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>5 $M_Z$</td>
<td>100</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>100</td>
<td>$M_Z$</td>
<td>$M_Z$</td>
</tr>
</tbody>
</table>

where $m_1$, $m_2$, $m_p$ are the masses of the charged, scalars 18) and pseudoscalar spinless bosons. $\delta M_W$ is the shift relative to the standard model with $M_Z = 94$ and $M_H = 100$ GeV.
More results are in ref.(21). If the additional scalars are triplets, they enter into $\Delta r$ (to our knowledge this has not been calculated, we are considering it); however their dominant effect is, at tree level already in $\rho \equiv M_Z^2 C^2 / M_W^2$, if they have a vacuum expectation value.

10.3.6 Two-loop effects

These have been recently considered for the case of a heavy degenerate fermion doublet. It turns out, that the small corrections (see 10.3.2) can only be overcome for very large masses ($\gtrsim$ few TeV), e.g. only for $m_t \gtrsim$ few TeV the shifts in $M_W$ are measurable.

10.4 Conclusion

The asymmetry measurement on the $Z^0$ resonance are dominated by radiative corrections of the type

$$\delta \rho - \Delta r + \text{subleading term}$$

If the correction is small (same as the subleading) it does not necessarily indicate small $\Delta r$ as long as a cancellation occurs in $\delta \rho - \Delta r$. Such a cancellation would be dramatically exhibited with the help of a precision measurement of the $W$-mass!

11. CONCLUSION

Each of the four LEP experiments can measure in at least three ways the mass of the $W$ boson at LEP 200 with an accuracy of the order of 100 MeV (or better). The integrated luminosity of 500 events/pb used in this study provides a better statistical accuracy (50 - 60 MeV) but it appears difficult to control the systematical uncertainties at such a level. All the methods proposed in this report which extends the work of (23) require the knowledge of the machine beam energy which gives in any case an absolute limit on the $W$ mass measurement accuracy.

12. ACKNOWLEDGEMENTS

During the course of this work we have benefited from the help and suggestions of many colleagues. We would like to thank especially: G. BARBIELLINI, M. DAVIER, R. KLEISS, R. MATTIG.
In this table you will find some general properties of the detectors used in our simulation. It gives some mean detector behaviour but the simulation programs are a lot more detailed.

<table>
<thead>
<tr>
<th>$\sigma_{E/E}$</th>
<th>Calorimetric measurement (L3)</th>
<th>Granular detector measurement (ALEPH - DELPHI - OPAL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.M.</td>
<td>$\frac{2.5%}{\sqrt{E}}$, &gt; 6°/oo</td>
<td>$\frac{17 - 20%}{\sqrt{E}}$</td>
</tr>
<tr>
<td>Had.</td>
<td>$\frac{55% + 5%}{\sqrt{E}}$</td>
<td>$\frac{60%}{\sqrt{E}}$, $\frac{100%}{\sqrt{E}}$</td>
</tr>
<tr>
<td>Muons</td>
<td>$0.4 \times 10^{-3}$ p</td>
<td>$10^{-3}$ p</td>
</tr>
</tbody>
</table>

Acceptances

| $|\cos \theta| \leq 0.984$ for $\begin{cases} e, \gamma \\ \mu \\ h \end{cases}$ | $|\cos \theta| \leq 0.98$ for $e, \gamma, h$ calorimetry |
| $p_{\mu}$ measured accurately over 70% of 4 $\pi$ | $|\cos \theta| \leq 0.965$ for charged track, momentum measurement (DELPHI) |

Typical Properties

| Energy loss of hadrons in BGO $40 \pm 20\%$ with $e/\pi = 1.4$ | DELPHI: |
| $E_{\text{meas}} = \frac{E_{\text{BGO}}}{E_{\text{BGO}} + E_{\text{HCAL}}}$ | - charged tracks measured in the central detector |
| $\Rightarrow 1.15$ (M.C.) | $\gamma$: $E_{\gamma} > 500$ MeV; lost if a hadron is "too close" |
| | $m_{h}$: $E > 1$ GeV; lost if a hadron is "too close" |
| | ALEPH: special algorithm to recover $\gamma$ or neutral hadrons when a charged hadron is in the vicinity. |

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FIGURE CAPTIONS

Fig. 1: W and Z production cross sections at LEP 200.

Fig. 2: Diagrams contributing to the radiative corrections in the process $e^+e^- \rightarrow W^+W^-$. 

Fig. 3: Effect of radiative corrections on a reconstructed W mass distribution when 
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Fig. 4: Integrated cross-sections of the various processes occurring at LEP 200 (without 
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Fig. 5: Hadronic production cross sections at LEP 200.

Fig. 6: Energy balance distributions in case of W pair events and other hadronic 
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   b) Background in the fit parametrized as a + b/s.
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Fig. 2: Diagrams contributing to radiative corrections

To eliminate dependence on $E_y$ cutoff also loop diagrams have to be calculated.
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Fig. 17: \( E_{\nu} = E_b - E_e \)

Fig. 17: \( z^{-}\) mass distributions

Fig. 18: Reconstructed \( Z^0 \) mass distributions
Fig. 19: Electron energy distribution

Fig. 20: End-point of electron energy spectrum
Fig. 21: $W$ width $\Gamma_W$ vs measured width $\Gamma_{\text{rec}}$

$\sqrt{\Delta r}$

Fig. 22: Expected accuracies on $\rho$ and $\Delta r$ from various experiments
1. Introduction.

In the second phase of its operation the large electron-positron collider LEP will reach an energy of nearly 100 GeV per beam. As suggested by the name of this Workshop we will refer to it as LEP200. At this energy the threshold for the production of W-pairs will be crossed and a copious sample of W's will allow a detailed exploration of W physics.

The purpose of this report is to examine in some detail the determination of the W decay branching ratios as a general test of the universality of the weak charged current, predicted by the standard model. To do this we discuss how to identify the three possible combinations of W decay modes,

\[ WW \rightarrow l\ell\ell \]
\[ WW \rightarrow l\nuqq \]
\[ WW \rightarrow qqqq \]

and how to separate the different flavour contributions, both in the lepton and in the quark sector.

This study is done in the framework of the predicted luminosities and considering the best expected performances of each of the four LEP experiments, in order to establish the impact of such a measurement on the present knowledge on the basis of the possibilities and the limitations related to the machine and to the detectors.

2. Charged current universality.

The basic vertex of the charged current is shown in fig. 2.1; the corresponding couplings can be written as \( g(U_{ij} f_i W f_j) \) where \( f_i \) and \( f_j \) represent a pair of fermion and antifermion helicity projected fields. Universality means that \( g \) is the same for any pair of fermions, while
$U_{ij}$ is equal to $\delta_{ij}$ when at least one of the fermions is a lepton, and is given by the $n \cdot n$ Kobayashi-Maskawa\(^1\) unitary matrix (where $n$ is the number of quark families) when both the fermion fields are quarks.

For leptons the immediate consequence of universality is that

$$\text{BR}(W \to e_\mu) = \text{BR}(W \to \mu_\mu) = \text{BR}(W \to \tau_\tau)$$

so that universality is directly tested by the equality of the decay rates in the three lepton families. For quarks, the decay rate of the $W$ into any possible quark pair will measure the corresponding element of $U_{KM}$, and the prediction of universality, including the colour factor, the QCD corrections of order $(\alpha_s)$ and the effect of the quark masses, is then

$$\text{BR}(W \to q_i q_j) = 3|U_{ij}|^2 \left(1 + \frac{\alpha_s}{\pi}\right) f(M_{q_i}, M_{q_j}) \cdot \text{BR}(W \to l\nu_l)$$

where $f(M_{q_i}, M_{q_j}) =$

$$= \left(\frac{(M_W^2 - (M_{q_i} + M_{q_j})^2)(M_W^2 - (M_{q_i} - M_{q_j})^2)}{M_W^4} \right)^{1/2} \left(1 - \frac{M_{q_i}^2 + M_{q_j}^2}{2M_W^2} - \frac{(M_{q_i}^2 - M_{q_j}^2)^2}{2M_W^4}\right)$$

is different from 1 only when one of the two quarks is the top, and in this case it simplifies to:

$$f(M_t) = \left(1 + \frac{M_t^2}{2M_W^2}\right) \left(1 - \frac{M_t^2}{M_W^2}\right)^2.$$ 

Assuming unitarity for $U_{KM}$ ($U^+U = 1$), the $n \cdot n$ elements of the matrix can be expressed in terms of $n_\theta = \frac{1}{2}n(n - 1)$ real parameters and $n_\delta = \frac{1}{2}(n - 1)(n - 2)$ phases. For $n = 3$ we deal with three rotation angles, $\theta_1, \theta_2$ and $\theta_3$ and one phase $\delta$. With $n = 4$ there are 6 rotation angles and 3 phases. As a consequence, from a given set of measured $W$ couplings, the unitarity assumption can also give limits on unmeasured elements if an assumption is made as to the number of quark families. We will discuss in next section what is the present status of these determinations.
3. Present measurements of universality.

In the lepton sector precise tests of universality come from the decay of pions to leptons\textsuperscript{2}) and from the decay $\tau \rightarrow \pi \nu$\textsuperscript{3)}, and can be expressed in terms of ratios between coupling constants as:

$$\frac{g_\pi}{g_\mu} = 0.9939 \pm 0.0057$$

$$\frac{g_\tau}{g_\mu} = 0.974 \pm 0.045.$$

These measurements refer to $Q^2$ values of the order of the square of the mass of the decaying particles. The $Q^2$ behaviour of the coupling constants would be sensitive to the appearance of unconventional virtual contributions. Recently the UA1 collaboration presented results\textsuperscript{4}) on coupling constant ratios at $Q^2 = M_W^2$ from an analysis of the $W$ decays at the CERN Collider, obtaining:

$$\frac{g_\mu}{g_e} = 1.05 \pm 0.07 \ (\text{stat.}) \pm 0.08 \ (\text{syst.}),$$

$$\frac{g_\tau}{g_e} = 1.01 \pm 0.09 \ (\text{stat.}) \pm 0.05 \ (\text{syst.}).$$

In the quark sector, as we saw, universality is directly reflected in the knowledge of the KM matrix. The directly measured elements are summarized in table 1a\textsuperscript{5}), while the 90% confidence intervals, determined assuming 3 or 4 quark families and unitarity, are given in table 1b\textsuperscript{6}) and 1c\textsuperscript{7}) respectively. On the measured elements one should note, apart from the obvious absence of the $U_{ts}$ elements, the rather large error on $U_{cs}$, measured from the semileptonic decays of the charged D mesons\textsuperscript{8}). The knowledge of the KM matrix is considerably improved by the use of the unitarity condition, but only under the assumption of 3 quark families. As it appears from table 1c, releasing this constraint the element $U_{ts}$ could be very large, and even the diagonality of the matrix can be questioned.

4. LEP200 scenario for $W$ pairs production.

In this study we will assume the machine running at $\sqrt{s} = 2 \times 95$ GeV (sometime we will consider also $2 \times 88$ GeV) with an integrated luminosity of 500 pb$^{-1}$, (optimistically) corresponding to a running period of one year. Unless stated otherwise, all the calculations are made in the framework of the Standard Model, with

$$M_W = 82 \ \text{GeV}$$

$$M_Z = 93 \ \text{GeV}$$

$$\sin^2 \theta_W = 0.223$$
Tab. 1 a: Measured elements of KM matrix.

<table>
<thead>
<tr>
<th></th>
<th>d</th>
<th>s</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>$0.9735 \pm 0.0015$</td>
<td>$0.231 \pm 0.003$</td>
<td>$&lt; 0.007$</td>
</tr>
<tr>
<td>c</td>
<td>$0.24 \pm 0.03$</td>
<td>$0.85 \pm 0.15$</td>
<td>$0.042 \pm 0.005$</td>
</tr>
<tr>
<td>t</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Tab. 1 b: KM matrix assuming 3 families and unitarity.

<table>
<thead>
<tr>
<th></th>
<th>d</th>
<th>s</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>$0.9742 \div 0.9756$</td>
<td>$0.219 \div 0.225$</td>
<td>$0 \div 0.008$</td>
</tr>
<tr>
<td>c</td>
<td>$0.219 \div 0.225$</td>
<td>$0.973 \div 0.975$</td>
<td>$0.037 \div 0.053$</td>
</tr>
<tr>
<td>t</td>
<td>$0.002 \div 0.018$</td>
<td>$0.036 \div 0.052$</td>
<td>$0.9986 \div 0.9993$</td>
</tr>
</tbody>
</table>

Tab. 1 c: KM matrix assuming 4 families and unitarity.

<table>
<thead>
<tr>
<th></th>
<th>d</th>
<th>s</th>
<th>b</th>
<th>b'</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>$0.9710 \div 0.9748$</td>
<td>$0.218 \div 0.224$</td>
<td>$0 \div 0.008$</td>
<td>$0.025 \div 0.093$</td>
</tr>
<tr>
<td>c</td>
<td>$0.175 \div 0.236$</td>
<td>$0.84 \div 0.975$</td>
<td>$0.039 \div 0.05$</td>
<td>$0 \div 0.5$</td>
</tr>
<tr>
<td>t</td>
<td>$0 \div 0.15$</td>
<td>$0 \div 0.48$</td>
<td>$0 \div 0.999$</td>
<td>$0 \div 0.999$</td>
</tr>
<tr>
<td>t'</td>
<td>$0 \div 0.16$</td>
<td>$0 \div 0.48$</td>
<td>$0 \div 0.999$</td>
<td>$0 \div 0.9995$</td>
</tr>
</tbody>
</table>

and

$$M_t = 40 \text{ GeV}.$$  

The bare cross section for $e^+e^- \to W^+W^- \to fff$ (fig. 4.1) at this $\sqrt{s}$ is $\sim 17 \text{ pb}$ but falls to $\sim 15 \text{ pb}$ including the W-width and the radiative corrections in the calculations. With the given $\int Ldt$, this corresponds to nearly 7500 W pairs events, or 15000 W's. Most of the results will refer for simplicity to $10^4$ events. The main sources of background to these events are:

a) $e^+e^- \to ZZ$ (fig. 4.2a) which is in total a few per cent of the WW signal and is precisely calculable within the Standard Model. Moreover these events are below threshold at $\sqrt{s} = 2 \times 88 \text{ GeV}$.

b) $e^+e^- \to \bar{f}f$ (fig. 4.2b), with a cross-section of 26 pb for quarks and 9.6 pb for leptons.

c) $e^+e^- \to \bar{f}f\gamma$ (fig. 4.2c), with a rather large cross section of 80 pb for quarks and
20 pb for leptons, mainly because of on-mass-shell $Z^0$ exchange, leading to a nearly monochromatic photon of $\sim 70$ GeV.

d) $e^+e^- \rightarrow e^+e^-e^+e^- (\mu^+\mu^-)$ (fig. 4.2d), with a huge cross section yielding events with very low $\theta$ leading leptons.

The topology of WW events is characterized by the angular distribution of the W's (fig. 4.3), dictated by the helicity properties of the W production\textsuperscript{9}). Because of the allowed W helicity states and of the V-A nature of their subsequent decay, these distributions reflect in those of the charged leptons coming from the decay, as shown in fig. 4.4, while those of the neutrinos are more isotropous. The four fermion energy distributions are given in fig. 4.5. All these plots are obtained using the MonteCarlo generator developed by R.Kleiss\textsuperscript{10}).

Fig. 4.2 Main backgrounds to $e^+e^- \rightarrow W^+W^-$. 
5. Identification of W decays

The W decay modes can be classified as pure leptonic,

\[ WW \rightarrow l\nu l\nu \]

corresponding to 7.5% of the total sample, "semi" leptonic (40%),

\[ WW \rightarrow l\nu q\bar{q} \]

and pure quark (52.5%),

\[ WW \rightarrow q\bar{q} q\bar{q}. \]
Fig. 4.4 Angular distributions of the four fermions respect to the $e^+$ beam line; index 1 refers to $W^+$ products, index 2 to $W^-$'s.
Fig. 4.5 Energy (in GeV) distribution for the four fermions; index 1 refers to $W^+$ products, index 2 to $W^-$'s.
The actual identification of the three modes is complicated by $\tau$ decaying into hadrons, by the possible presence of leptons in the quark jets and by the effects of gluon radiation making the number of jets in the final state different from the number of quarks in the decays. A more detailed scheme for classifying all the possible topologies of the final state is sketched in fig. 5.1, where an estimated number of events is attached to each case. Looking at this plot one can hardly imagine it will be possible to classify each mode on an event-by-event basis.

More realistic strategies can be envisaged, where for some channel a clear signature is defined, the corresponding acceptance is computed, the background contamination is subtracted yielding a determination of the branching ratio. However this way cannot be followed for all channels, because of the statistics or of missing signatures (as for the light quarks), and it has to be complemented with more inclusive classifications, where broad classes of events are compared (all leptonic vs. all hadronic decays, etc.). This global counting could allow more extended tests of the standard model predictions. An example will be given at the end of this section. We discuss now in some details the features of the three main decay modes.

5.1 Pure leptonic decays

The cross section for this mode is around 1 pb, yielding 750 events out of 10000. Signatures are:

a) two visible leptons only in the final state, or a single thin jet from the hadronic decay of the $\tau$ lepton.

b) acoplanarity and acolinearity of the two visible leptons, because of the missing neutrinos.

c) missing energy and missing $p_\perp$, for the same reason.

Background sources that contaminate this sample are:

a) $e^+e^- \rightarrow \gamma, Z \rightarrow ll$. This reaction has a cross section of 10 pb and is characterized by two back-to-back leptons with $E_l = \sqrt{s}/2$. Therefore it cannot be confused with the signal.

$e^+e^- \rightarrow \gamma, Z \rightarrow \tau\bar{\tau} \rightarrow l\nu l\nu$ (cross section $\sim 0.4$ pb). This is also a final state with back-to-back leptons, however in this case they are not monochromatic.

b) $e^+e^- \rightarrow \gamma + Z \rightarrow ll$, with the $\gamma$ escaping the detection (cross section $\sim 10$ pb). These events are easily identified by a 70 GeV/c missing momentum in the direction of the beam pipe; in addition the two leptons are coplanar.

$e^+e^- \rightarrow \gamma + Z \rightarrow \tau\bar{\tau}$ (cross section $\sim 0.4$ pb). The missing momentum due to the $\gamma$ is now washed out by that of the neutrinos from the $\tau$ decay, but the two leptons are still coplanar, because of the boost of the two $\tau$’s.

c) $e^+e^- \rightarrow e^+e^-e^+e^- (\mu^+\mu^-)$ with the two leading electrons lost in the beam pipe. This process has a total cross section $\sim 10^{-31}$ cm$^2$ but is reduced by several orders of magnitude$^{11}$ by requiring $\theta_l \geq 30^\circ$ or by a combined cut in missing $p_\perp$ and acoplanarity, in both cases with a large acceptance for the signal (see below).
Fig. 5.1 Tentative classification of the topology of 10000 WW final states, based on standard LUND decay probabilities (note that leptonic W decays are underestimated).
To identify this sample of events the experimental requirements for the detectors are then a good $e$ and $\mu$ identification and energy measurement with an angular acceptance close to $150^\circ > \theta_l > 30^\circ$. To identify and reject the QED background the capability of tagging electrons down to very small angles is crucial: for example, a minimum tagging angle of $1.5^\circ$ will limit to 5 GeV the maximum missing $p_{\perp}$ in $e^+e^- \rightarrow e^+e^-e^+e^-$. However a complete study of QED processes\(^\text{11}\) would require additional lepton coverage below $30^\circ$.

5.2 Pure quark decays

The cross section for this mode is about 8 pb, corresponding to 5250 events out of 10000. In principle these events should be characterized by four or more jets in the final state, but, given the high gluon multiplicity at the $Q^2$ values involved, the situation is not as clean as previously thought\(^\text{12}\). As firstly pointed out by P. Mättig\(^\text{13}\) less than 20% of $qqqq$ events appear as four jets. The natural background is given by $e^+e^- \rightarrow q\bar{q}$. In this case too, the larger cross section ($\sim 80$ pb) comes from events with the exchange of an on-mass shell $Z^0$ accompanied by a radiative $\gamma$ of nearly 70 GeV; these events can be rejected as before by looking at the momentum missing in the beam pipe; the events without $\gamma$'s (26 pb) are more dangerous, because their total energy is the same as the signal, and they have to be separated on topological basis.

A first glance to the problem can be given by the Lund predictions at the parton level. The fraction of events appearing with a given number of jets both for the signal ($e^+e^- \rightarrow WW \rightarrow qqqq$) and for the background ($e^+e^- \rightarrow qq$), obtained by the A.Blondel and P.Raimondi Lund-based code developed for this workshop\(^\text{14}\) is shown in table 2.

From this table, assuming "less than 3 jet" events easily attributed to the background and "more than 5 jet" events as a clean signal sample, we are left with the 4 jet case as an overlapping region. It seems then feasible to define an algorithm accepting more than 80% of the signal with a contamination that cannot be larger than few percent. Including the geometrical acceptance we cannot expect however to be able to collect more than 50% of this signal. Further studies are certainly required to clarify this point, but one should also realise that at LEP200 time large benefits will be derived from the analysis of the hadronization of the $Z^0$ at LEP phase one.

<table>
<thead>
<tr>
<th>n of jets</th>
<th>$e^+e^- \rightarrow WW \rightarrow qqqq$</th>
<th>$e^+e^- \rightarrow qq$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-</td>
<td>0.464</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>0.493</td>
</tr>
<tr>
<td>4</td>
<td>0.18</td>
<td>0.043</td>
</tr>
<tr>
<td>5</td>
<td>0.52</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>0.04</td>
<td>-</td>
</tr>
</tbody>
</table>
5.3 WW → ℓνqq

This sample of WW events is particularly clean, because no "natural" background can produce this combination of primary particles. In addition, the expected number of events is rather large (4000 out of 10000 WW pairs, coming from a cross section of 6 pb; this number falls to 3200 if we exclude the hadronic τ decays). Another advantage of such a sample is that it contains at the same time hadronic and leptonic W decays with common acceptance and statistics. The background to this sample is essentially due to semileptonic decays of quarks in pure quarks events.

We will show that the signal can be extracted in an easy fashion without using any algorithm such as number of jets, W-mass reconstruction, lepton isolation, event sphericity or other topological ones, that strongly reduce the acceptance (and rely on the details of the simulation). The signal can be isolated just looking at the scatter plot of the energy of the lepton against the total missing energy. For the signal, the missing energy is equal to the energy taken by unseen neutrinos which added to the visible lepton energy E_l, must be equal to the energy of the W, E_l + E_{miss} = E_W. For e or μ we have:

\[ 20 < E_l < 80 \text{ GeV}, \]
\[ 20 < E_{miss} < 80 \text{ GeV}, \]

while for τ

\[ 0 < E_l \lesssim 50 \text{ GeV}, \]
\[ 20 < E_{miss} \lesssim 100 \text{ GeV} \]

because of the additional neutrinos in the τ decay. The plots are shown in fig. 5.2 a, b, as produced by the Kleiss-Blondel-Raimondi generator. The contamination of the background over the signal region can result from the missing energy in q̅q jets or by the presence of unseen radiative γ's, but is not too large, as shown in fig. 5.2 c. The overlap is of course enlarged if one includes the effects of the detector acceptance and resolution for hadron jets.

The results of a simulation using the same generator coupled with the properties of LEP3 calorimetric detectors, represented by

\[ \frac{\sigma_{had}(E)}{E} = 0.5 \frac{0.5}{\sqrt{E}} \quad \text{and} \quad \theta_{had}^{acc} > 5^0 \]
\[ \frac{\sigma_{em}(E)}{E} = \max(0.02, 0.01) \quad \text{and} \quad \theta_{had}^{acc} > 5^0 \]

are shown for the signals and the background in fig. 5.3 a, b, c. The separation is still clean and will be quantified in sect. 6. We can anticipate here that it is possible to save ~ 85% of the signal, ending with a sample of 2700 events. This result can be achieved by any experiment providing good calorimetric measurement and coverage, and a reasonable identification of isolated leptons in high multiplicity events.
Fig. 5.2 Scatter plot of $E_t$ vs $E_{miss}$ (both in GeV) for $e/\mu\nu qq$ (a), $\tau\nu qq$ (b) and $qqqq$ (c) W decays.
Fig. 5.3 The same as Fig. 5.2 after acceptance and resolution.
5.4 An inclusive measurement.

We conclude this section giving an example of a simple handling of the three large classes of WW events described above. Considering the quark mass contribution in the $W \rightarrow q\bar{q}$ branching ratios given in sect. 2 it follows that $\text{BR}(W \rightarrow l\nu)$ decreases with $M_t$, while $\text{BR}(W \rightarrow \text{any } q\bar{q})$ increases. The effect enters twice in the ratio

$$ R' = \frac{N_{(WW \rightarrow l\nu\ell\nu)}}{N_{(WW \rightarrow l\nu q\bar{q})}} $$

and three times in the ratio

$$ R'' = \frac{N_{(WW \rightarrow l\nu\ell\nu)}}{N_{(WW \rightarrow q\bar{q}q\bar{q})}} ; $$

the behaviour of these ratios is shown in fig 5.4, where the errors are the statistical ones including the acceptances given before (not including the systematics in the background subtraction etc.).

Fig. 5.4 Behaviour of $R'$ (a), $R''$ for all leptons (b) and $R'''$ for light leptons only (c) as functions of the top mass.
We can now imagine two possible scenarios for the time when LEP200 will be operating:

a) The mass of the top quark is not yet known. Then the simple inclusive measurements here proposed will give the top mass within 5 to 10 GeV, without any identification of top events from topology etc. (Of course, this will give a better precision, see below, sect. 7.1).

b) Top is already observed (let us assume its mass to be 40 GeV). Then these inclusive measurements represent a general check of the standard model at the level of 5 to 10%; e.g. they are sensitive to any new heavy object (asymmetric in its quark/lepton decays) with $M_q \leq 60$ GeV.

6. Lepton universality tests

6.1 $l\nu l\nu$ events

This class of events provides a clean sample of leptonic W decays, when events are selected with $\theta_1 > 30^\circ$, as seen in the previous section. The double lepton acceptance for this cut, as computed from the Kleiss generator, is 72%. The separation of $e$ or $\mu$'s directly coming from the W decay from those coming from the $\tau$ decay can be obtained by comparing their energy spectra (see fig. 6.1). The selection $E_1 < 20$ GeV retains 60% of the $\tau$ decays with

![Energy distributions for light leptons coming from $\tau$ decays (continuous line) and from direct W decays (dashed line).](image-url)
no significant contamination from $e$ or $\mu$, while the selection $E_l > 45$ GeV retains 70% of $e$ or $\mu$ and rejects practically all of the $\tau$'s. Including the angular acceptance, the efficiencies are:

$$\epsilon_\tau = 0.6 \times 0.72 \simeq 40\%$$

$$\epsilon_{e,\mu} = 0.7 \times 0.72 \simeq 50\%$$

Moreover, as the two spectra of fig 6.1 may be calculated precisely, the unfolding of the two contributions would certainly be possible. In the overlap region, $20 < E_l < 45$ GeV, the absolute number of events, for $10^4$ W pairs is:

$$N_\tau = 175 \times 0.4 = 70 \pm 8$$

$$N_{e,\mu} = 1000 \times 0.3 = 300 \pm 18$$

Looking at the expected statistical fluctuations it appears that an unfolding of the two contributions can provide a precise determination of $N_{e,\mu}$, while the effect of its fluctuation on $N_\tau$ is expected to be large. We can then conclude that this unfolding (marginal for $\tau$ events) will bring the efficiency for electrons and muons to

$$\epsilon_{e,\mu} \simeq 70\%.$$

As shown in section 5.3 these events can be isolated with suitable cuts in the plane $E_l$ vs $E_{miss}$. A possible set of cuts ("lepton cut") is given by:

$$E_{miss} < 100 \text{ GeV}$$

$$E_{miss} > 90 \text{ GeV} - \frac{90}{80} \cdot E_l$$

$$E_l > 5 \text{ GeV}.$$

An additional cut on $E_l$ at 20 GeV ("$\tau$ cut") will separate the electron and muon decays from tau's. The arguments given in the previous section on the unfolding apply here as well. Table 3 summarizes the results for the $\tau$, the $e$ or $\mu$ and for the qqqq sample originating from 10000 W's. The figures, produced with the Kleiss-Blondel-Raimondi generator including initial state radiation, are given both for the LEP3 calorimetric detector (a) described in sect. 5.3 and for an 'average' detector (b) with

$$\sigma_{had}(E) = \frac{1.0}{\sqrt{E}} \quad \text{and} \quad \theta_{had}^{\text{rece}} > 5^0$$

$$\sigma_{em}(E) = \frac{0.2}{\sqrt{E}} \quad \text{and} \quad \theta_{had}^{\text{rece}} > 5^0$$
Tab. 3: Separation of $t\nu qq$ events.

<table>
<thead>
<tr>
<th></th>
<th>$r \rightarrow$ leptons</th>
<th>$e$ or $\mu$</th>
<th>$qqq \rightarrow$ leptons</th>
</tr>
</thead>
<tbody>
<tr>
<td>total accepted</td>
<td>470 ev.</td>
<td>2700 ev.</td>
<td>3100 ev.</td>
</tr>
<tr>
<td>lepton cut (a)</td>
<td>305 ev.</td>
<td>2400 ev.</td>
<td>62 ev.</td>
</tr>
<tr>
<td>$r$ cut (a)</td>
<td>160 ev.</td>
<td>0</td>
<td>30 ev.</td>
</tr>
<tr>
<td>lepton cut (b)</td>
<td>260 ev.</td>
<td>2150 ev.</td>
<td>65 ev.</td>
</tr>
<tr>
<td>$r$ cut (b)</td>
<td>130 ev.</td>
<td>0</td>
<td>46 ev.</td>
</tr>
</tbody>
</table>

From the table it appears that the efficiency for electrons and muons is high in both cases and the background contamination to this sample is negligible. The effect of good calorimetry is larger for $r$'s, leading to an efficiency of 35% with a background contamination not greater than 20% of the $r$ signal.

6.3 $r(\rightarrow$ hadrons)$\nu qq$ events.

This particular sample of "semi" leptonic decays has to be identified for at least two good reasons: firstly because is the largest $r$ sample, and secondly because it is a rather serious source of background for pure quark decays. On the other hand the hadronic $r$ decay is easy to identify because of its low multiplicity.

A recent analysis has been performed by UA1 collaboration in order to identify the $W \rightarrow r \rightarrow$ hadrons events at the SPS Collider. The experimental results from this analysis\textsuperscript{4) were presented in sect. 3. Their method can be directly transferred to LEP200 case, as the $W$ energy spectrum is rather similar in the two cases. It is based on a likelihood function $L$ obtained combining three different indicators:

a) fraction of jet energy contained in a cone defined by $\Delta r < 0.4$ where $\Delta r$ is the squared sum of the azimuthal angle and the pseudorapidity of each track;

b) angular separation between the leading track of the jet and the (calorimetric) jet axis;

c) charged particle multiplicity inside $\Delta r$.

A suitable cut on $L$ has an efficiency of 78% on hadronic $r$ decays with a background contamination equal to 10% of this sample. They also estimate a $\sim 9\%$ systematic error on this efficiency.

The background contribution at LEP200 is mostly due to $W \rightarrow q\bar{q}$ events, which are present in exactly the same relative amount at the SPS Collider, so that LEP200 situation cannot be worse. However, at LEP200, there is also the possibility that fragments of $q\bar{q}$ $W$ decay on the other side of the event are superimposed on the $r$ jet and the event does not
pass the cut in the likelihood function. The effect is shown in fig. 6.2, where are plotted the distributions of the angle between the original $\tau$ axis and any track coming from its decay (continuous line) or from the $qq$ jets (dashed line). We estimated the amount of this mixing by considering the charged particle multiplicity coming from $qq$ inside the cone $\Delta r = 0.4$ defined above; applying the selection criteria that identify isolated $\tau$'s, 10% of the sample is discarded. If we assume a conservative global loss of $\sim 15\%$ on the complete algorithm, the efficiency for $\tau$'s is then:

$$\epsilon_\tau = 0.78 \times 0.85 = 65\%.$$  

As a by-product we can also assume that for $\tau(\rightarrow \text{had.}) \nu \bar{\nu}$ we can reach a 90% efficiency with practically no background contamination.

![Fig. 6.2 Distribution of angles between the original $\tau$ axis and any track coming from its decay (continuous line) or from $qq$ jets (dashed line).](image)

6.4 Conclusions on leptonic decays

A summary of all the event numbers obtained from the efficiencies derived in the previous sections is given in table 4. From these numbers one can derive the errors on the branching ratios:
\( \Delta \text{BR}/\text{BR \ stat.} \quad \Delta \text{BR}/\text{BR \ syst.} \)

(W \to \mu \text{ or } e) \quad \text{in pure leptonic events} \quad \pm 0.06 \quad -

(W \to \mu \text{ or } e) \quad \text{in all the events} \quad \pm 0.025 \quad -

(W \to \tau) \quad \text{in clean events} \quad \pm 0.06 \quad -

(W \to \tau) \quad \text{in all the events} \quad \pm 0.03 \quad \pm 0.04

Clean events for \( \tau \) are those where no background subtraction is needed. For the other ones statistical errors include the statistical uncertainty from background subtraction. The systematic error is assumed to be equal to the statistical error for \( \tau(\to l\nu\nu)qq \) and equal to the one quoted by UA1 for \( \tau(\to \text{hadrons})qq \) case. Luminosity errors are not included, as they

Tab. 4: Summary of leptonic decays.

<table>
<thead>
<tr>
<th>decay mode</th>
<th>efficiency</th>
<th>events</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu/e\nu ) double decay</td>
<td>70 %</td>
<td>340</td>
</tr>
<tr>
<td>( \mu/e\nuqq )</td>
<td>90 %</td>
<td>240</td>
</tr>
<tr>
<td>( \tau(\to \text{had.})\nu\nu )</td>
<td>90 %</td>
<td>2700</td>
</tr>
<tr>
<td>( \tau(\to l\nu\nu)\nu\nu )</td>
<td>40 %</td>
<td>170</td>
</tr>
<tr>
<td>( \tau(\to l\nu\nu)qq )</td>
<td>35 %</td>
<td>470</td>
</tr>
<tr>
<td>( \tau(\to \text{had.})qq )</td>
<td>65 %</td>
<td>870</td>
</tr>
</tbody>
</table>

cancel out in the ratios of the coupling constants. Propagating the errors from the branching ratios, we obtain:

\[
\begin{align*}
g_\mu/g_e &= \ldots \pm 0.018(\text{stat.}) \pm \ldots(\text{syst.}) \\
g_\tau/g_e &= \ldots \pm 0.02(\text{stat.}) \pm 0.025(\text{syst.})
\end{align*}
\]

Comparing this with the results given in sect. 3, the improvement respect to the high \( Q^2 \) present determinations is substantial, while the \( \tau \) universality can be verified within even better limits than those at low \( Q^2 \).

Note that previous figures are given for 10000 WW events, which for a cross section of 15 pb correspond to \( \int L \, dt = 700 \, \text{pb}^{-1} \); if we assume \( \int L \, dt = 500 \, \text{pb}^{-1} \) they become \( g_\mu/g_e = \ldots \pm 0.020 \) and \( g_\tau/g_e = \ldots \pm 0.023 \) remaining still competitive, while for \( \int L \, dt = 100 \, \text{pb}^{-1} \) we get \( g_\mu/g_e = \ldots \pm 0.045 \) and \( g_\tau/g_e = \ldots \pm 0.050 \) that begins to be marginal.
7. Quark sector

The aim of this section is to discuss the possibility of identifying the flavour of the quark jets, in order to determine the elements of the K.M. matrix. Flavours may be separated by looking at the event shape or by using the vertex chamber to identify long living particles. The first method is only sensitive to top, while the second is sensitive to both top and bottom. Then a combination of the two can in principle be used to separate \( W \rightarrow tb \) from \( W \rightarrow ts \) or \( td \). In addition at least one experiment is able to identify the kaons, casting some light on the value of \( U_{cb} \).

7.1 Top tagging from event topology

A top jet should look different from others on several respects: the particle multiplicity is larger, the jet is wider because of the small \( \beta \) of the top quark, the number of leptons is higher and the hadron energy is lower, both because of the high probability of semileptonic decays, producing leptons and unseen neutrinos. The cleanest sample in which these features can be studied is \( WW \rightarrow l\nu qq \). We expect 2700 such events out of 10000 \( W \)'s including the acceptance given above. Among them, assuming \( M_t = 40 \) GeV in the formula given in sect. 2, there are 700 \( W \rightarrow t \) decays.

A simulation\(^{15}\) based on the Lund hadronization was adapted to a DELPHI-like detector, schematically described by:

- track detector for \( \theta > 15^\circ \)
- e.m. calorimeter \( \sigma(E) = 0.1 \cdot \sqrt{E} \) for \( \theta < 35^\circ \)
- e.m. calorimeter \( \sigma(E) = 0.2 \cdot \sqrt{E} \) for \( \theta > 35^\circ \)
- had. calorimeter \( \sigma(E) = 0.7 \cdot \sqrt{E} \) for \( \theta < 10^\circ \)

Many distributions have been compared for top and light quark jets:

- aplanarity
- thrust
- sphericity
- particle multiplicity
- number of reconstructed jets
- hadron energy
- number of leptons.

The most significant distributions (with acceptance and resolution applied) are plotted in fig. 7.1,2,3 and 4 for the two classes of quark pairs. For each distribution a suitable cut may be defined to optimize the acceptance on \( tq \) events with respect to the rejection on \( qq \) events. They are summarized in the following table:
Fig. 7.1 Distribution of the thrust for light quarks (continuous line) and top (dashed line) events.

Fig. 7.2 Distribution of the sphericity for light quarks (continuous line) and top (dashed line) events.
Fig. 7.3 Distribution of the hadronic energy for light quarks (continuous line) and top (dashed line) events.

Fig. 7.4 Distribution of the number of reconstructed jets for light quarks (continuous line) and top (dashed line) events.
<table>
<thead>
<tr>
<th>Quantity</th>
<th>cut applied</th>
<th>$\epsilon_{tq}$</th>
<th>$\epsilon_{qq}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>thrust</td>
<td>&lt; 0.83</td>
<td>70%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>&lt; 0.80</td>
<td>50%</td>
<td>6%</td>
</tr>
<tr>
<td>sphericity</td>
<td>&gt; 0.21</td>
<td>70%</td>
<td>17%</td>
</tr>
<tr>
<td></td>
<td>&gt; 0.31</td>
<td>50%</td>
<td>7%</td>
</tr>
<tr>
<td>had. energy</td>
<td>&lt; 82 GeV</td>
<td>70%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>&lt; 76 GeV</td>
<td>50%</td>
<td>8%</td>
</tr>
<tr>
<td>number of reconstructed jets</td>
<td>&gt;2</td>
<td>96%</td>
<td>64%</td>
</tr>
<tr>
<td></td>
<td>&gt;3</td>
<td>62%</td>
<td>7%</td>
</tr>
</tbody>
</table>

Most of the cuts are strongly correlated so it is difficult to find effective combinations of them. Examples of combinations are:

(more than 2 jets)×(thrust < 0.80), giving $\epsilon_{tq} = 50\%$ and $\epsilon_{qq} = 5\%$

combined cut in thrust vs. multiplicity plot (see fig 7.5), giving $\epsilon_{tq} = 70\%$ and $\epsilon_{qq} = 8\%$

Moreover we remark that a powerful parameter to tag the top decays would be the number of leptons observed inside the final state. However the lepton identification has to be done inside the hadron jets, so it will depend on the details of the detector and of the reconstruction program (and of the jet simulation). We do not yet have all the tools to make safe predictions at this level of detail.

From the derived efficiencies we can already conclude that it is possible to tag 70% of the top signal with a contamination of less than 8% of the background, or alternatively, to tag 50% of the signal with a contamination of less than 5% of the b.g. The first case, applied to our standard number of events, will select a sample of 500 top events with a background of less than 150 light quark events giving the best statistical error:

$$\Delta |U_{tq}|^2 = \pm 0.05$$

where $|U_{tq}|^2 = |U_{tb}|^2 + |U_{ts}|^2 + |U_{td}|^2$.

The second one will give a sample of 300 top's with a background of less than 100 light quarks ($\Delta |U_{tq}|^2 = \pm 0.06$) and it should be kept in mind because a background suppression as large as possible is required by further analysis (see sect. 7.3).

A larger acceptance given by a better combination of all the indicators cannot be excluded, but one should note that the ultimate limit from this sample, corresponding to 700 top events with no background is only 20% better:

$$\Delta |U_{tq}|^2 = \pm 0.04$$
Fig. 7.5 Scatter plot of particles multiplicity vs. thrust for light quarks (a) and top events (b).
Fig. 7.6 Behaviour of the mass factor in $\text{BR}(W \rightarrow tq)$. Statistical errors correspond to the number of identified top decays for each value of $M_t$.

To improve the $U_{tq}$ determination we must consider the large sample of 4 quark events containing 3500 tops. In principle the extension of the algorithms described before should not be difficult, the only obscure point being the multi-jet structure at the $M_W$ scale mentioned in sect. 5.2. Again, a complete understanding of the problem is deferred until the $Z^0$ jet analysis is completed. Once more, what we can give here is the ultimate but realistic limit based on the expected number of events and on a guessed acceptance of 50%. From 1750 events we can expect:

$$\Delta |U_{tq}|^2 = \pm 0.02$$

Before to proceed let us return to the scenario where top is not discovered before LEP200 is running. Then we have to assume the diagonality of the K.M. matrix and determine, under this hypothesis, the quark mass from the formula of sect. 2. Keeping the conservative efficiency of 50% for the $l\nu qq$ events we can draw the band of fig. 7.6 for the ratio of top over a light quark rate. We see that we can determine the top mass with an uncertainty of 1 to 2 GeV up to 70 GeV. This result can be compared with an uncertainty of 5 to 10 GeV from the inclusive rate determination in sect. 5.4.
7.2 Flavour tagging using vertex detectors

This method is based on the idea that long living particles present in the $b$ and $t$ jets will show up with a number of tracks not extrapolating back to the interaction vertex. This

![Diagram showing impact parameter and tracks](image)

Fig. 7.7 Relationship between the impact parameter $P_i$, the distance $l = \gamma r_c$ from the primary to the secondary vertex, the track momentum $p = m_o \beta \gamma$ and $p_\perp$.

can be quantified by measuring the impact parameter $P_i$ (illustrated in fig. 7.7), that results to be:

$$P_i = \frac{p_\perp}{m_o \beta r_c}.$$

To investigate this method a Montecarlo simulation has been developed, based on the Davier-Roudeau WW generator\(^{16}\), and considering the DELPHI vertex detector. The resolution in the impact parameter is given by:

$$\sigma \sim \sqrt{\left(\frac{81}{P_{xy}}\right)^2 + 27^2} \ \mu m \ \text{for} \ -60^0 < \theta < 60^0.$$

In this study we will also use the $\nu qq$ sample. The program generated 1970 light quark and 640 top decays. Of these, 1560 and 430 respectively have the interaction vertex reconstructed. For this reduced sample it is possible to classify events accordingly to the number $n$ of tracks whose impact parameter exceeds, say, $3\sigma$. The distribution of $n$ is given in fig. 7.8 for $W \rightarrow tb$ (the signal) and $W \rightarrow ud, cs, us, cd$ (the background) separately.

To show the effectiveness of different cuts in $n$, figure 7.9 gives the number of signal vs. the number of background events with $n$ larger than a given $n_0$, as a continuous curve connecting different values of $n_0$. The curve shows that it is possible to get an enriched sample of $W \rightarrow tb$ events, with different efficiencies and with different signal-to-background ratios. The relative statistical error on the signal determination (including the error from the background subtraction) changes smoothly along the curve, staying below 10% level with a minimum of 7% corresponding to a sample of $\sim 250$ signal events. This fact can be used to check the consistency of the method against the choice of the cut.
Fig. 7.8 Number of tracks with $P_t > 3\sigma$ for top (continuous line) and light quarks (dashed line).

Fig. 7.9 Signal and background composition for enriched (continuous line) and depleted (dotted line) top sample.
Using the same parameter it is also possible to invert the selection to deplete the top sample, isolating the light quark events. This is shown in the same figure 7.9 as a dotted line. All of the arguments given in the previous section on the extension of tagging to the larger sample of $W \rightarrow tqqq$ events apply here as well.

### 7.3 Combination of the two tagging methods

Top tagging using vertex detectors, as we saw in last section, allow us to determine $|U_{tq}|$ with

$$\Delta |U_{tq}|^2 = \pm 0.07$$

which is a little larger than the statistical error coming from event topology. However the two methods are to a large extent independent, so that their combination should reach the statistical limits indicated in sect. 7.1.

What is more interesting is that we are now in position to use the first method to extract as pure a top sample as possible. On this sample the impact parameter method can be used to tell $W \rightarrow tb$ from $W \rightarrow ts,td$. To check this, we consider once more the "semi"leptonic WW events, assuming to be able to select 50% of top events with negligible background. The additional requirement to have the event vertex reconstructed will yield 220 $W \rightarrow tb$ events, with a global efficiency of 38% for the rare $W \rightarrow ts,td$ events.

The distribution of the number of tracks with impact parameter larger than 2 $\sigma$ for $W \rightarrow tb$ and $W \rightarrow ts,td$ respectively is plotted in fig 7.10. The effectiveness of different cuts is shown in fig. 7.11, where the abscissa is the fraction of $W \rightarrow ts,td$ (to be multiplied by 0.38 for the selection efficiency) and the ordinate is the number of surviving $W \rightarrow tb$. From the plot one can extract an error on $(N_{ts} + N_{td})$ of $\pm 25$ events, or

$$\Delta (|U_{ts}|^2 + |U_{td}|^2) = \pm 0.08$$

and extending the method also to the $W \rightarrow tqqq$ events one can hope to bring this error to less than $\pm 0.05$.

### 7.4 $W \rightarrow cs$ tagging

The last point we want to discuss in this section is the tagging of $W \rightarrow cs$ decays by the $K^\pm$ identification. The $K^\pm$ identification can be easily done by DELPHI experiment using RICH detectors. Their expected efficiency is shown in fig. 7.12 together with the $\pi^\pm$ contamination. The experimental strategy is then:

a) select "semi"leptonic WW events,

b) for each jet containing at least one charged $K$ define the jet strangeness to be equal to that of the highest momentum $K^\pm$,

c) select events where the two highest energy jets have opposite strangeness

d) apply topology and impact parameter algorithms to remove top contribution
Fig. 7.10 Number of tracks with $P_i > 2\sigma$ for $W \rightarrow tb$ (continuous line) and $W \rightarrow ts, td$ (dashed line).

Fig. 7.11 $ts, td$ efficiency vs. number of $tb$ events.
Fig. 7.12 $K$ identification efficiency (continuous line) and $\tau$ contamination (dotted line) for DELPHI-RICH detectors.

Table 5 summarizes the results for different minimum momenta cuts for $K$'s. The first two columns refer to steps a) and b) with and without d), the second two to a) b) and c) with and without d). From the table it appears that it is possible to go from a sample with several hundreds signal events and a signal to background ratio close to one, down to a rather clean sample of $\sim 100 \ W \to cs$ events. This range should allow some control on the systematics.

From a statistical point of view we can conclude that the error on the number of $W \to cs$ can be better than 10% while a full understanding of the background could bring it to a minimum of $\sim 6\%$. In any case this determination of $U_{cs}$ will represent a consistent improvement with respect to the present 15% (see tab. 1a and ref. 8).

Tab. 5: Summary of $cs$ tagging as (number of signal events):(number of background events) for different cuts.

<table>
<thead>
<tr>
<th>Min. $p_K$</th>
<th>1 $K$ identified</th>
<th>top removing</th>
<th>2 $K$ identified</th>
<th>top removing</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 GeV</td>
<td>538 : 445</td>
<td>409 : 248</td>
<td>62 : 11</td>
<td>47 : 3</td>
</tr>
</tbody>
</table>
8. The knowledge of the weak quark mixing

The results expected from LEP200 which are described in the preceding sections can be used to determine the elements of the K.M. mixing matrix. As we discussed in section 2, there is no room for large improvements if one assumes the unitarity of the matrix and fixes the number of families to 3. We now concentrate our discussion on the four quark families scheme.

The analysis has been performed using the parametrization of Gronau and Schechter\(^1\)\(^7\)\(^1\), using as input the following experimental information\(^1\)\(^8\):

\[
|U_{ud}| = 0.9729 \pm 0.0012
\]
\[
|U_{us}| = 0.221 \pm 0.002
\]
\[
|U_{cd}| = 0.21 \pm 0.03
\]
\[
|U_{cs}|^2 / |U_{cd}|^2 \text{ as determined from } \nu \text{ dimuon events}
\]
\[
|f_P^P(0)|^2 \cdot |U_{cs}|^2 = 0.51 \pm 0.07
\]
\[
|f_P^P(0)| = 0.68 \pm 0.08
\]
\[
|U_{ub}| / |U_{cb}| < 0.19 \text{ at } 90\% \text{ C.L.}
\]
\[
\Gamma(b \to c \bar{c} \nu) = (9.6 \pm 1.4) \cdot 10^{10} \text{s}^{-1}
\]

The resulting matrix is given once more in table 6 a, in form of intervals of 90\% C.L. We may now add the information from the anticipated LEP200 results obtained from \(W\)-decays. To do this we take a "conservative" and an "optimistic" point of view, the former corresponding more or less to the results expected from the analysis of the \(WW \to l\nu qq\) sample, the latter referring to the additional use of the \(WW \to qqqq\) sample. The LEP200 new input for the "conservative" case is then:

\[
|U_{cd}|^2 = 1 \pm 0.1
\]
\[
|U_{ts}|^2 = 1 \pm 0.07
\]
\[
|U_{td}|^2 + |U_{ts}|^2 = 0 \pm 0.08
\]

and the resulting matrix is given in table 6 b. The "optimistic" input is:

\[
|U_{cd}|^2 = 1 \pm 0.05
\]
\[
|U_{ts}|^2 = 1 \pm 0.03
\]
\[
|U_{td}|^2 + |U_{ts}|^2 = 0 \pm 0.04
\]

and the resulting matrix is given in table 6 c.

From a comparison of the tables we conclude that the two cases are not qualitatively different, both showing that, in the eight quark scheme, LEP200 will give a considerable
Tab. 6 a: Present knowledge of KM matrix.

<table>
<thead>
<tr>
<th></th>
<th>$d$</th>
<th>$s$</th>
<th>$b$</th>
<th>$b'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>$0.9710 \div 0.9748$</td>
<td>$0.218 \div 0.224$</td>
<td>$0 \div 0.008$</td>
<td>$0.025 \div 0.093$</td>
</tr>
<tr>
<td>$c$</td>
<td>$0.175 \div 0.236$</td>
<td>$0.84 \div 0.975$</td>
<td>$0.039 \div 0.05$</td>
<td>$0 \div 0.5$</td>
</tr>
<tr>
<td>$t$</td>
<td>$0 \div 0.15$</td>
<td>$0 \div 0.48$</td>
<td>$0 \div 0.999$</td>
<td>$0 \div 0.999$</td>
</tr>
<tr>
<td>$t'$</td>
<td>$0 \div 0.16$</td>
<td>$0 \div 0.48$</td>
<td>$0 \div 0.999$</td>
<td>$0 \div 0.99995$</td>
</tr>
</tbody>
</table>

Tab. 6 b: KM matrix after "conservative" LEP200.

<table>
<thead>
<tr>
<th></th>
<th>$d$</th>
<th>$s$</th>
<th>$b$</th>
<th>$b'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>$0.9710 \div 0.9748$</td>
<td>$0.218 \div 0.224$</td>
<td>$0 \div 0.008$</td>
<td>$0.025 \div 0.093$</td>
</tr>
<tr>
<td>$c$</td>
<td>$0.19 \div 0.236$</td>
<td>$0.92 \div 0.975$</td>
<td>$0.039 \div 0.05$</td>
<td>$0 \div 0.32$</td>
</tr>
<tr>
<td>$t$</td>
<td>$0 \div 0.04$</td>
<td>$0 \div 0.10$</td>
<td>$0.95 \div 0.999$</td>
<td>$0 \div 0.31$</td>
</tr>
<tr>
<td>$t'$</td>
<td>$0 \div 0.14$</td>
<td>$0 \div 0.31$</td>
<td>$0 \div 0.32$</td>
<td>$0.93 \div 0.9995$</td>
</tr>
</tbody>
</table>

Tab. 6 c: KM matrix after "optimistic" LEP200.

<table>
<thead>
<tr>
<th></th>
<th>$d$</th>
<th>$s$</th>
<th>$b$</th>
<th>$b'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>$0.9710 \div 0.9748$</td>
<td>$0.218 \div 0.224$</td>
<td>$0 \div 0.008$</td>
<td>$0.025 \div 0.093$</td>
</tr>
<tr>
<td>$c$</td>
<td>$0.20 \div 0.236$</td>
<td>$0.95 \div 0.975$</td>
<td>$0.039 \div 0.05$</td>
<td>$0 \div 0.22$</td>
</tr>
<tr>
<td>$t$</td>
<td>$0 \div 0.028$</td>
<td>$0 \div 0.07$</td>
<td>$0.97 \div 0.999$</td>
<td>$0 \div 0.22$</td>
</tr>
<tr>
<td>$t'$</td>
<td>$0 \div 0.12$</td>
<td>$0 \div 0.21$</td>
<td>$0 \div 0.22$</td>
<td>$0.96 \div 0.9995$</td>
</tr>
</tbody>
</table>

Improvement on at least 8 of the 16 matrix elements, which is crucial in proving the diagonality of the mixing matrix for heavy quarks.

9. Conclusions

From the studies performed by this working group we can conclude that LEP200 will give a significant test of fermion universality, based on a clean and direct determination of the $W$ couplings to all the flavours. A remarkable feature will be that all the fermions will be measured at the same time in the same reaction, namely the $W$ decay, with the considerable advantage that most of the systematic uncertainties will tend to cancel out.
Due to the expected cross section of $\sim 15$ pb, the highest possible machine luminosity is welcome. We consider $\int \mathcal{L} dt = 500 \text{ pb}^{-1}$ as a reasonable level for almost all of the analysis presented, whereas if it were less than, say, $100 \text{ pb}^{-1}$, then most of the proposed measurement could be deemed marginal.

We are also convinced that the four approved experiments do not require major changes in their design principles. It has only to be stressed that in the second phase of LEP the missing energy will have a crucial role not only for the identification of new, non standard, particles, but also in the WW decays, to identify the unseen neutrinos. A good and hermetic calorimetry is important for the event signature. This argument, together with the need of a powerful electron tagging at very low angles respect to the beam line, will call for a detailed review of the forward regions of the detectors.

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We would like to thank A. Blondel, M. Davier, R. Kleiss, P. Roudeau, P. Raimondi, S. Ritz and J. Hilgart that kindly made their simulation code available to us (A. Blondel, R. Kleiss, P. Raimondi and S. Ritz also participated to several group meetings).

The group convenor feels personally in debt with R. Kleiss and C. Dionisi for many useful discussions, and with G. Salvini for detailed explanations of UA1 $W \rightarrow \tau$ analysis.

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14) A. Blondel, P. Raimondi, A W pair production generator, ALEPH-LEP200 note.
15) F. Cavallo, contribution to this Working Group.
16) Contribution to the Working Group on W dynamics.
18) From ref. 6, where a complete reference list can be found.
THE STUDY OF THE REACTION $e^+e^- \rightarrow W^+W^-$

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The reaction $e^+e^- \rightarrow W^+W^-$ provides a unique opportunity to test some important aspects of the Standard Model. After a review of the basic relevant measurements, data are presented which simulate the performance of LEP detectors for this process around 95 GeV beam energy. Of particular interest are measurements of the $W$ helicity providing new information on the process. The role of beam polarization is discussed in some detail.

1. INTRODUCTION

The prime motivations for the LEP design centre-of-mass energy of 200 GeV are: (1) the exploration of the fermion mass spectrum and their possible partners, (2) the search for Higgs bosons, both on a scale ~ 100 GeV typical of the electroweak symmetry breaking, and (3) the production of pairs of $W$ bosons. Whereas the first two topics are to some extent open-ended, the last one proceeds on a known energy scale and therefore can be considered as a safe ground for experimental planning. The production of $W$ bosons at LEP offers three areas of experimentation: (1) the study of the dynamics: cross section and underlying amplitudes, (2) the detailed investigation of $W$ decays and (3) precise determinations of the $W$ mass. Although all three aspects will be somewhat coupled in practice, we shall focus in this report on the first point, the production dynamics.

The subject of $W$-pair production in $e^+e^-$ annihilation

$$e^+e^- \rightarrow W^+W^-$$

(1)
has already received a very large amount of consideration. Besides the latest LEP workshop where one study group looked at the physics at the LEP highest energies\(^1\), a recent review\(^2\) lists up to 37 relevant papers considering one aspect or another of the physics to be studied in reaction (1). Therefore no great novel idea is expected to emerge here; however, the emphasis is more on trying to analyze in a realistic way the information content of reaction (1) which can be gathered in the LEP detectors under the projected machine performance. For the first time, the measurement of the \(W\) helicity has been simulated — a necessary step in trying to get to the full amplitude information. In the same spirit, we have reexamined in more detail the value of beam polarization.

2. THE PHYSICS INTEREST

2.1 \(W\)-pair production in the Standard Model

The 3 well-known contributions to \(e^+e^- \rightarrow W^+W^-\) are depicted in figure 1. Of these, neutrino exchange gives the largest amplitude, but it is the least interesting as the couplings involved are known. In the Standard Model (SM), large cancellations occur between the different amplitudes and the resultant cross section is "small" and well-behaved with increasing energies (Fig. 2). Small deviations from the SM are therefore expected to show up sensitively as, in general, the corresponding cross section will be larger than the SM one, as illustrated in figures 3 and 4.

The study of reaction (1) is a good overall test of the SM. The new aspects investigated in this reaction are the \(gW\) and the \(ZW\) couplings, which will be key issues in the SM still untested after LEP experiments at the Ze peak.

2.2 The 3-boson couplings

The 3-boson couplings, shown in figure 5, involve 2 x 7 independent form factors \(f_i\), where \(i\) runs from 1 to 7, defined by the interaction vertex\(^2\):

\[
i \, g_{\nu \nu \nu} \, f_1^{\alpha \beta \mu} \left(q,q,\bar{q}\right) = i \, g_{\nu \nu \nu} \, f_1^{\alpha \beta \mu} \left(q,q,\bar{q}\right) + \frac{f_2^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_2^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_3^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_3^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_3^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_3^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_3^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_3^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_3^\nu}{m_\nu^2} \left(q-q\right) + \frac{f_3^\nu}{m_\nu^2} \left(q-q\right)
\]
\[ g_{WW} = -e \]
\[ g_{WZ} = -e \cot \theta_W \]

(4)

The first 3 form factors can be expressed with 4 parameters \( \lambda_v \) and \( \kappa_v \)
\[ f^1_v = 1 + \frac{s}{2M^2_W} \lambda_v \]
\[ f^2_v = \lambda_v \]
\[ f^3_v = 1 + \kappa_v + \lambda_v \]

(5)

related to static properties of the \( W \), namely its magnetic moment \( \mu_W \) and its electric quadrupole moment \( Q_W \):
\[ \mu_W = \frac{e}{2M^2_W} (1 + \kappa_y + \lambda_y) \]
\[ Q_W = -\frac{e}{M^2_W} (\kappa_y - \lambda_y) \]

(6)

The other form factors involve some more parameters but should vanish under P and C invariance, which should certainly apply to the \( \gamma \) couplings. CP invariance also restricts the number of form factors:

\[ f^V_4 = f^V_5 = 0 \quad \text{C invariance} \]
\[ f^V_5 = f^V_6 = f^V_7 = 0 \quad \text{P invariance} \]
\[ f^V_4 = f^V_6 = f^V_7 = 0 \quad \text{CP invariance} \]

In the SM, we have at the lowest order
\[ \kappa_y = \kappa_z = 1 \]
\[ \lambda_y = \lambda_z = 0 \]
\[ f^{V,Z}_{4,5,6,7} = 0 \]

In order to test the validity of the SM for the 3-boson couplings, one is left with 9 parameters (Table 1). This is clearly too many to be experimentally determined and it is maybe reasonable, in a first step, to restrict oneself to the first 5, namely \( \kappa_v \), \( \lambda_y \), \( \kappa_z \), \( \lambda_z \) and \( g_{WZ} \). A discrepancy from the SM value for any of these parameter will result in an observable deviating from its standard value. The challenge will therefore be to discover where the discrepancy occurs. To put it another way, we must study the sensitivity and the selectivity of experimental observables to the 5 parameters defining the basic 3-boson interaction of the non-abelian gauge group.
Table 1

Minimum set of parameters in the 3-boson couplings

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SM value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_Y, \kappa_Z$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_Y, \lambda_Z$</td>
<td>0</td>
</tr>
<tr>
<td>$\delta_{WZ}$</td>
<td>$-e \cot \theta_W$</td>
</tr>
<tr>
<td>$f^Z_{i,s,s',s''}$</td>
<td>0</td>
</tr>
</tbody>
</table>

2.3 Physical observables

Production and decay angles, as well as helicities are defined in figure 6. Most of the time, $W$ bosons decay hadronically

$W \rightarrow q \bar{q'}$ (g) → jets

in which case it is not easy to determine the $W$ charge. This is straightforward with the leptonic decays

$W^- \rightarrow \ell^- \bar{\nu}_\ell$

$W^+ \rightarrow \ell^+ \nu_\ell$

Physical observables attainable with unpolarized beams are listed in Table 2. Any of these can also be measured with polarized beams with either transverse or longitudinal polarization. We shall come back on this later.

Table 2

Physical observables of the process $e^+e^- \rightarrow W^+W^-$

<table>
<thead>
<tr>
<th>final state*</th>
<th>experimental information needed</th>
<th>observable</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
<td>rate separation/background</td>
<td>$\sigma_T(s)$</td>
</tr>
<tr>
<td>$\ell h$</td>
<td>$W^-$ direction</td>
<td>$\frac{d\sigma}{d\cos \theta}$</td>
</tr>
<tr>
<td>$\ell h$</td>
<td>$W$ decay</td>
<td>$\frac{d\sigma}{d\cos \theta} \rightarrow \frac{d\sigma}{d\ell}$</td>
</tr>
<tr>
<td>$\ell h$</td>
<td>&quot;</td>
<td>$\frac{d\sigma}{d\cos \theta} (\lambda, \lambda')$</td>
</tr>
</tbody>
</table>

* $\ell h$ corresponds to the final state $W \rightarrow \ell \nu, W \rightarrow q \bar{q}'$
3. SIMULATION OF $ee \rightarrow WW$ DATA

3.1 Monte-Carlo generators

Two generators (at least) exist to produce events containing 4 fermions according to

$$e^-e^+ \rightarrow W^-W^+ \rightarrow f_1 \bar{f}_1 \ f_2 \bar{f}_2$$

(1) the Kleiss program\(^3\) generates directly the 4-fermion final state through calculation of the Feynman diagrams with the $W$ propagators. It also includes interference with the $ZZ$ production amplitude (Fig. 7). The hadronization of quark jets has been introduced using the Lund prescription, including gluon radiation, which turns out to be important\(^1\). Since gluon radiation cannot be incorporated in this generator in a correct way, some thought has to be given to arrive at an acceptable procedure\(^8\).

(2) the Bilal-Davier program\(^6\) starts with helicity amplitudes for $ee \rightarrow WW$ and incorporates the decay tensors for $W \rightarrow ff'$, Lund hadronization has also been implemented\(^7\), taking into account gluon emission.

Both programs include initial state photon radiation and finite $W$ width. It is pleasing to observe that both sets of simulated data are in close agreement with one another, despite the fact that they were produced via radically different approaches.

3.2 Simulation of experiment

No full simulation of the experiments has been accomplished yet. Also the problems of reconstruction (tracks, showers) have been excluded so far. The experimental effects are lumped into overall resolutions in angles and energy, and acceptance cuts are applied.

Since the angular distribution of the $W$ pair is an important observable, the precision which can be achieved on the $W$ direction has been examined. The actual measurement deals with 2 jets (Fig. 8) and the $W$ angular resolution function has been simulated in two situations of "low" and "high" resolutions (Fig. 9). The distortion of the angular distribution is presented in figure 10: even in the case of "low" resolution, it does not affect too much the distribution and it can be corrected for. The most important effect is the reduction in mass acceptance, since a fixed cut 76-88 GeV was applied to validate a jet-jet mass combination. In order to get a feeling for the effect of resolution in the angular distribution, we can compare it to a possible deviation in the couplings: it is roughly equivalent to a shift in the $\kappa$ parameter less than 0.1, which is less than the statistical uncertainty achieved on this quantity.

Therefore, it seems that the foreseen resolutions of the LEP detectors will be quite appropriate to measure sensitively the $WW$ observables.

3.3 Useful event sample

A total integrated luminosity of 500pb\(^{-1}\) will produce 8000 events at 95 GeV centre-of-mass energy. This sample will be spread out in 3 final states
2 leptons + missing $\not{p}$
1 lepton + hadrons + missing $\not{p}$
hadrons

Except for the total rate, where all events are useful, most physics investigations rely on identifying the $W$ charge; hence the importance of the lepton hadron sample, where both the charge and direction can be reconstructed. Taking into account cuts in lepton energy and angles, jet angles, isolation cuts for lepton identification and $W$ mass cuts, the lepton-hadron sample reduces to about 2000 events or even a bit less. This is the physics sample for most of the analysis.

3.4 Study of the total cross section and the angular distribution

Figure 11 shows the total cross section within a typical acceptance: $\theta_{\text{jet}}, \theta_{\text{lepton}} > 20^\circ$. Beyond the sharp rise governed by $v$ exchange, it reaches a maximum for a beam energy of 95 GeV; then it falls off as a consequence of the large cancellations between the contributing amplitudes. It is therefore important to have a good handle on this behaviour. Clearly the higher the energy, the better. The measurement of $\sigma$ is limited by systematic effects (luminosity, acceptance calculation, background subtraction). An uncertainty of 5% is typical and it allows at 95 GeV beam energy a meaningful test of the SM: for example, $\kappa_v$ can be determined with an accuracy of $\sim 40\%$. Increasing the beam energy to 110 GeV would decrease this figure by about a factor of 2.

Of course, some care should be given to the treatment of radiative corrections and one should also include finite width effects\textsuperscript{4} as illustrated in figure 12. These effects are important near threshold and they displace the cross section maximum. However, they can be reliably calculated.

The angular distribution (determined with the lepton + hadrons sample) contains a lot of information about the basic couplings as demonstrated in figure 13 through 15 for a beam energy of 95 GeV. The backward region is particularly sensitive to possible deviations from the SM. In general, it will be impossible to determine the cause of the deviation, as any departure, whether it applies to $\kappa_v$, $\kappa_{\gamma}$, $\lambda_\gamma$ or $\lambda_{\gamma}^{-1}$, tends to produce similar effects: increase in the forward region, decrease in the backward region.

The sensitivity of the angular distribution to departures from the SM has been studied previously\textsuperscript{3} and we have not much to add. As an example, it will be possible to measure $\kappa_\gamma$ with an accuracy of 15% if it is kept as the only parameter. Assuming all the $\kappa_{v,z}$ and $\lambda_{\gamma,z}$ are correctly given by the SM, it is possible to check the overall $ZW$ coupling and therefore measure $\sin^2\theta_W$ in a totally independent way. The expected accuracy is only about 0.03 but the agreement with the much more precise value from the $Zff$ couplings or the $W$ and $Z$ masses would be a new test of the theory.
4. MEASURING THE W POLARIZATION

4.1 The lepton energy spectrum

The lepton energy spectrum in the W decay reflects the W decay angular distribution in the W centre-of-mass. Therefore, it carries some information on the W polarization, which is another observable we can use to test the SM. Such a test is shown in figure 16. Let us look at W polarization in a more systematic way.

4.2 Helicity amplitudes

Introducing the helicities involved

\[ e^- e^+ ightarrow \nu^- \nu^+ \]
\[ \sigma \quad \lambda \quad \lambda \]

the process is then described by helicity amplitudes \( F_{\sigma \lambda \alpha} \). Table 3 shows the different contributions for unpolarized beams (averaging over \( \sigma \) and \( \overline{\sigma} \) helicities) as labelled by \( \Delta \lambda = \lambda - \overline{\lambda} \). The dominant amplitude corresponds to \( (\lambda \overline{\lambda}) = (- +) \) and is generated by \( \nu \) exchange.

Table 3

Amplitudes for \( W^- \nu^+ \) of a given helicities

(unpolarized beams)

<table>
<thead>
<tr>
<th>( \Delta \lambda )</th>
<th>( (\lambda \overline{\lambda}) )</th>
<th>exchange</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 2</td>
<td>(- +)</td>
<td>dominant</td>
</tr>
<tr>
<td>+ 2</td>
<td>(+ -)</td>
<td>only ( \nu )</td>
</tr>
<tr>
<td>1</td>
<td>(+ 0) + (0 -)</td>
<td>( \nu, \gamma, Z )</td>
</tr>
<tr>
<td>0</td>
<td>(+ +) + (- -)</td>
<td></td>
</tr>
<tr>
<td>- 1</td>
<td>(- 0) + (0 +)</td>
<td></td>
</tr>
</tbody>
</table>

How these various amplitudes construct the final angular distribution is displayed in figure 17. It allows one to understand why the backward region is more sensitive to the gauge couplings than the forward one, since the dominant \( \Delta \lambda = - 2 \) amplitude from \( \nu \) exchange vanishes there.

Transverse \( W (\lambda = \pm 1) \) production dominates over longitudinal \( W (\lambda = 0) \) everywhere, as shown in figures 18 and 19.
4.3 An exercise in reconstructing W helicities

The normalized V-A angular distribution for the W-decay fermion is

\[
\frac{dN}{d\cos^*} = \begin{cases} 
\frac{3}{8} (1 - \cos^*)^2 & \text{for } \lambda = \pm 1 \\
\frac{3}{4} \sin^2 \theta^* & \text{for } \lambda = 0
\end{cases}
\]

(7)

where \(\theta^*\) is the lepton angle in the W rest frame with respect to the W line of flight.

From an experimental study of the doubly differential cross section

\[
\frac{d^2\sigma}{d\cos \theta \, d\cos^*} = \frac{3}{8} (1 - \cos^*)^2 \left( \frac{d\sigma}{d\cos \theta} \right)_{\lambda=\pm 1} + \frac{3}{8} (1 + \cos^*)^2 \left( \frac{d\sigma}{d\cos \theta} \right)_{\lambda=-1} + \frac{3}{4} \sin^2 \theta^* \left( \frac{d\sigma}{d\cos \theta} \right)_{\lambda=0}
\]

(8)

it is hence possible to determine the inclusive differential cross sections for W of given helicities \(\left( \frac{d\sigma}{d\cos \theta} \right)_{\lambda=0, \pm 1}\) and compare the experimental results to theoretical predictions (the SM expressions are given in Ref. 1).

The W charges are determined from the leptonic decay of one of the W bosons and the total momentum vector of the products of the hadronic decay of the other W fixes the W\(^+\)W\(^-\) line of flight in the e\(^+\)e\(^-\) centre-of-mass. The decay angle \(\theta^*\) can be measured independently from the lepton energy \(E_\ell\)

\[
\cos^* = \frac{1}{\beta} \left( \frac{2 E_\ell}{E_B} - 1 \right)
\]

where \(\beta\) is the W velocity.

A sample of 3000 events has been generated at \(E_B = 100\) GeV according to the SM. Estimates were obtained by either \(\chi^2\) fits of the cross sections for W\(^-\) with fixed helicities or by taking the expectation values of the functions

\[
\frac{1}{2} (-1 \pm 2 \cos^* + 5 \cos^2 \theta^*)
\]

and

\[
(2 - 5 \cos^2 \theta^*)
\]

(9)

to project out \(\lambda = \pm 1\) and \(\lambda = 0\), respectively. The two methods are statistically equivalent.
As a test of the procedure a sample of 3000 \((\cos\theta, \cos^\theta)\) values has been generated according to expression (9). This sample was analysed and resulting cross sections for the production of \(W\)'s of given helicity could be compared to the input cross sections. The agreement is shown in figure 20.

Next, a sample of 3000 completely simulated events has been filtered and "measured" in a typical LEP detector. Particles at angles \(\theta < 15^\circ\) were assumed to be lost and a measured total hadronic energy of 90 GeV was required in order to determine \(\cos\theta\) reliably. Only \(e^+\bar{\nu}\) and \(\mu^+\bar{\nu}\) were retained for leptonic decays. The average accuracy on \(\cos^\theta\) was .08. The final sample of 1410 events gives a realistic impression of the results that can be achieved at LEP 200 (Fig. 21): it shows that it is possible to determine cross section components which are at the 10% level or even smaller. The measurement of the \(W\) polarization will provide a valuable test of the predictions of the SM.

4.4 Angular decay correlations

More information can be obtained from the correlation between decay angles \((\theta^*, \phi^*)\) and the production angle \(\theta\) expressed in the form\(^{17}\)

\[
\frac{d\sigma (W^+ \to Z^-)}{d\cos\theta \, d\cos^\theta \, d\phi^*} = \sum_{i=1}^{9} F_i (\cos\theta) \, L_i (\theta^*, \phi^*)
\]

\[
\frac{d\sigma (W^- \to Z^+)}{d\cos\theta \, d\cos^\theta \, d\phi^*} = \sum_{i=1}^{9} F_i (\cos\theta) \, L_i (\theta^*, \phi^*)
\]

where the sum runs over the 9 independent helicity states and the \(L_i\) is a known set of orthogonal functions.

The functions \(F_i\) and \(F_j\) can be extracted from the data with suitable projectors dependent on \(\theta^*, \phi^*\), in a manner identical to that of the previous section. Figure 22 shows such a determination for the quantity

\[
\frac{F_j - F_i}{F_i}
\]

which turns out to be one of the most sensitive and complementary to the differential cross section. Nevertheless, it will be difficult to always separate the effect of different couplings.

5. IS BEAM POLARIZATION USEFUL FOR ee \(\to WW\) ?

5.1 Longitudinal beam polarization

A clean way in principle to separate out \(v\) exchange is to use \(e^+e^-\) beams with definite helicities\(^{11}\), as can be seen in Table 4. Using helicity indices refering to the \(e^-e^+\) initial state, we have the following amplitudes
Table 4

Contributing amplitudes for $e^+e^-$ with definite helicities

\[
\begin{array}{c|cc}
| & e^- & e^+ \\
\hline
L & 0 & \gamma, Z \\
R & \nu, \gamma, Z & 0
\end{array}
\]

\[A_{LR} = M^{-}_\gamma + M^{-}_Z + M^{-}_\nu\]

\[A_{RL} = M^+_\gamma + M^+_Z\]  \hspace{1cm} (11)

The corresponding cross sections are displayed in figure 23. It is seen that, after integrating over angles

\[|A_{RL}|^2 \sim 10^{-2} |A_{LR}|^2\]

and therefore the rate in the interesting RL state is discouragingly small. Another problem is how to achieve polarization of the beams. Even if it can be solved, the smallness of the rate remains a big concern, at least if the process follows the SM. As an example, let us consider the rates that could be achieved in one year for two values of the longitudinal polarization

\[
\begin{array}{c|c|c|c|c|c}
P_L & .8 & 90 \text{ events} & 40 & e^{-}_R e^{+}_L & \gamma^- \gamma^+ \\
 & .5 & 325 \text{ events} & 25 & e^{-}_R e^{+}_L & \gamma^- \gamma^+ \\
\end{array}
\]

It is clear from these numbers that a good signal-to-background ratio for the RL state requires a $P_L$ value which is not realistic. Even if it could be achieved, the rates are such that one would normally not spend one year running in these conditions.

In general, longitudinal beam polarization does not seem to be a profitable business as far as $ee + \gamma\gamma$ is concerned, even not taking into account the enormous difficulties to obtain it. Now, it could be that deviations from the SM do occur and, in that case, larger rates could be achieved, as indicated in figure 24 for some particular values of the parameters.
5.2 Transverse beam polarization

Since $A_{RL}$ is an order of magnitude smaller than $A_{LR}$, it may be advantageous to measure their interference rather than the squares. This is achieved using transversely polarized beams\(^2\).

If $\phi$ is the angle between the transverse polarization $\hat{p}_T$ and the normal to the scattering plane (Fig. 25) we expect to detect such an interference as a $2\phi$ modulation:

$$
\sigma \sim |A_{LR}|^2 + |A_{RL}|^2 + 2 P_T^2 \Re \left( A_{LR}^* A_{RL} \right) \cos 2\phi + \Im \left( A_{LR}^* A_{RL} \right) \sin 2\phi
$$

(12)

Since $A_{LR}$ is mostly $\nu$ exchange and $A_{RL}$ has only contributions from $\gamma$ and $Z$ channels, the situation is, in principle favourable: the large interfering amplitude $A_{LR} \sim H_\nu^2$ is large and well-known and we expect a 10 % effect rather than a 1 % effect in the unpolarized case or with longitudinally polarized beams.

In practice, the situation is somewhat less promising. After summing over $W$ helicities, the interference is quite small (Fig. 26), for the obvious fact that the dominant ($- +$) $W$ helicity amplitude has nothing to interfere with and therefore only the less populated helicity states have a $2\phi$ modulation.

Now, again, this is the situation in the SM. If deviations occur, especially if they are large, significant effects could be observed and disentangled using polarization. In particular, transverse polarization is very efficient to separate effects coming from $\gamma$ or $Z$ couplings. This is apparent in figure 27 as far as $\kappa_\gamma$ and $\kappa_Z$ couplings are concerned.

5.3 Combining all observables: the merit of beam polarization\(^{10}\)

For a more systematic treatment, the significance of deviations from the SM was calculated in the ($\kappa_\gamma$, $\kappa_Z$) and ($\lambda_\gamma$, $\lambda_Z$) planes. Contour lines are obtained expressing any deviation from the SM expectation in terms of the statistical uncertainty in the "data" distributions for a typical LEP 200 experiment (based on a somewhat optimistic sample of 4000 lepton-hadrons events) folded with a 5 % normalization uncertainty. Observables used include the $W$ differential cross section, the $W \rightarrow e\nu$ inclusive distributions and all azimuthal angle correlations (around the $W$ axis) of the decay fermions.

The resulting contour lines for a deviation of either $\kappa_{\gamma,Z}$ or $\lambda_{\gamma,Z}$ from their SM values are given in figures 28 and 29. Figures 28a and 29a assume no beam polarization, while the contours in figures 28b and 29b are obtained from the $2\phi$ modulation alone with a 70 % transverse polarization. Figures 28c and 29c combine the two approaches, $\phi$-integrated variables and $2\phi$ modulation. Transverse polarization clearly helps: it is a complementary tool, as clearly indicated in figures 28a-b and 29a-b and the resultant accuracy is significantly improved.

To compare the respective merits of transverse and longitudinal beam polarization, a similar analysis was performed, assuming a 50 % longitudinal polarization of one of the
two beams. Combining the results obtainable when running half of the time with left-handed and right-handed beams each, the contours in figures 28d and 29d were obtained.

The usefulness of transverse and longitudinal beam polarization is therefore comparable. This is to be contrasted with the effort involved in providing longitudinal polarization. If transverse polarization indeed builds up to a significant value at LEP 200 (which is presently far from being a settled issue, at least in the positive sense) then it can be used freely with no more experimental investment. The only compromise might be with luminosity.

In either case, substantial polarizations, $P_L$ or $P_T$, are needed to significantly improve the capabilities of LEP 200 for the measurement of the gauge couplings. This is obvious for transverse polarization where the size of the effect is proportional to $P_T^2$. A transverse polarization of 50% is the minimal requirement. Below this value, essentially no new information is gained, if the $ee \rightarrow WW$ process is close to the SM expectation.

6. $ee \rightarrow WW$ VERSUS OTHER INVESTIGATIONS IN THE STANDARD MODEL

6.1 Electroweak radiative corrections: sensitivity to the Higgs mass

Radiative corrections to the process $ee \rightarrow WW$, including the effect of virtual Higgs bosons, have been calculated$^{11,12}$. Their results do not quite agree, but the differences seem to be understood because different input parameters and different renormalization schemes were used.

Some results of the calculations done by Lemoine and Veltman$^{11}$ are given in figure 30. Only two quark families are taken into account and the radiated photon energy cut is not convenient for actual experiments. Nevertheless, the effect of the Higgs mass on the $ee \rightarrow WW$ cross section is deceptively small: typically, $\frac{d\sigma}{d\theta}$ is about 4% when the Higgs mass goes from 10 GeV to 1 TeV. Even this result is not guaranteed as an independent conclusion: indeed, it is not clear to what extent the predicted change is not just a reflection of the changing $W$ mass. This point has to be checked.

The same calculation shows a similar expectation for the cross section for longitudinally polarized $W (W_L)$, contrary to some expressed hope$^1$ that the behaviour of the $W_L$ state is somehow related to the Higgs mechanism in a preferred fashion.

6.2 Gauge couplings at the $Z^*$ peak

The $ZWW$ coupling can play a role in $Z^*$ decays, via the diagram in figure 31 where one of the two $W$ bosons is virtual and "decays" into a fermion-antifermion pair

$$Z^* \rightarrow WW' \rightarrow Wff'. $$

The branching fraction is expected$^{13}$ to be $3 \times 10^{-7}$ in the SM. Experimental detection requires the on-mass-shell $W$ to decay leptonically and the observable rate is too small, unless the $ZWW$ coupling is an order of magnitude larger at least than the SM value.
6.3 Gauge couplings from hadron colliders

Quark-antiquark annihilation probes the same physics as $e^+e^- \rightarrow W^+W^-$, with more flexibility. The following processes can, in principle, be studied:

$$q\bar{q} \rightarrow W^+W^-$$
$$WZ$$
$$W\gamma$$

At present energies and luminosities, the rate for $W$ pairs is too small: asking for one $W$ to decay into $e\nu$ or $\mu\nu$ produces a rate below 1 event/year. At higher energies, the cross section increases and with possibly much larger luminosities (with pp colliders) the rate becomes interesting. However, one has to cope with the much more frequent process

$$q\bar{q} \rightarrow W + 2 \text{ jets}$$

where additional gluons and $qq^-$ pairs are produced together with the usual $W$ production. Even restricting the di-jet mass to lie in the $W$ mass band ($\pm 10$ GeV), the background is prohibitive\textsuperscript{14}) (Fig. 32). The total rate can probably be measured, but it is unlikely that a detailed study of the process can be performed.

The reaction $q\bar{q} \rightarrow WW$ may be more interesting\textsuperscript{15}), both experimentally and theoretically since it only involves the $W$ magnetic moment. The rate is too small for ACOL, but it could be handled in a future high luminosity collider. A realistic simulation of such an experiment is lacking and its sensitivity to $k_7$ remains to be determined, although it appears to be significantly worse than could be achieved at LEP 200.

7. CONCLUSIONS

From this study and previous ones on the same subject, we can draw the following conclusions.

(1) The study of $e^+e^- \rightarrow W^+W^-$ is the only practical way to investigate the gauge boson interaction.

(2) The 3-boson interaction, characteristic of the non-abelian gauge group, is a very important test of the Standard Model, independent of other standard tests.

(3) Therefore it is a unique opportunity to explore the validity of the Standard Model and possibly discover a departure from it. To achieve this goal, high energies are important: a beam energy of 95 GeV is a minimum to achieve a good sensitivity.

(4) Many analysis tools exist to determine the origin of a possible deviation. However, this is hard: up to 9 parameters can differ from their SM values.

(5) On any single parameter, accuracy can reach the 10-20 % level, as for example the $W$ magnetic moment. In general, one can only limit some domains in a multi-parameter space.

(6) Realistically, polarized beams will not provide a substantial gain in sensitivity. However, if the observed deviations are large, they could help disentangling different contributions. Therefore the possibility of polarized beams should be kept open, to be available in a further stage.

(7) LEP detectors, as presently being built, will be suitable to carry out this physics programme.
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FIGURE CAPTIONS

Fig. 1  Lowest order contributions to the process $e^+e^- \rightarrow W^+W^-.$

Fig. 2  Total cross section for $e^+e^- \rightarrow W^+W^-$ in the standard model as a function of beam energy for $M_W = 82 \text{ GeV}.$

Fig. 3  Total cross section for $e^+e^- \rightarrow W^+W^-$ as a function of c.o.m. energy in the standard model (solid line) and assuming $g_{ZWW} = 0$ (dashed line).

Fig. 4  Total cross section for $e^+e^- \rightarrow W^+W^-$ as a function of beam energy in the standard model (solid line) and for pure $\nu$-exchange (dashed line).

Fig. 5  The three-boson coupling $\gamma_{ZWW}.$

Fig. 6  Production angle $\theta$ in the over-all c.m.s. system and decay angle $\theta^*$ in the $W$ centre of mass frame, for electrons and $W$'s with helicities $\sigma$ and $\lambda$, respectively.

Fig. 7  Graphs contributing to four fermion final states.

Fig. 8  The production angle $\theta$ reconstructed from the two jet decay of a $W$.

Fig. 9  $W$ angular resolution for two detector resolutions.

Fig. 10 Distortion of the angular distribution for $W$ production for a high and a low resolution detector respectively.

Fig. 11 Total cross section for $e^+e^- \rightarrow W^+W^-$ as a function of beam energy for different values of the parameter $\kappa_\gamma$, $\kappa_\gamma = 1$ in the standard model.

Fig. 12 Effect of the finite $W$-width on the threshold behaviour of the cross section.

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Fig. 16 Effect of changing the parameter $\kappa_\gamma$ on the lepton energy spectrum.

Fig. 17 Contribution of various helicity amplitudes to the production angular distribution of the $W$.

Fig. 18-19 Longitudinal and transverse cross sections as a function of $W$ production angle for a beam energy of 100 GeV.

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Fig. 26 Different helicity contributions to the differential cross section $d\sigma/d \cos \theta d\phi$ as a function of the angle $\phi$ defined in fig. 25 and for $\cos \theta = 0.8$ in case of 100% transverse polarization.

Fig. 27 Different helicity contributions to the differential cross section $d\sigma/d \cos \theta d\phi$ in case of deviations from the SM for 100% transverse polarization.

Fig. 28-29 Contour lines indicating the statistical uncertainty in the "data" distribution (number of standard deviations) for a typical LEP200 experiment from the SM expectation in the $(\kappa_2, \kappa_2)$ and $(\lambda_2, \lambda_2)$ planes respectively.

Fig. 28a and 29a assume no beam polarization.

Fig. 28b and 29b are obtained from the $2\phi$ modulation alone with a 70% transverse polarization.

Fig. 28c and 29c combine the two approaches, $\phi$-integrated variables and $2\phi$ modulation. Fig. 28d and 29d are obtained when assuming a 50% longitudinal polarization of one of the two beams and running half of the time with left-handed and right-handed beams respectively.

Fig. 30 Effect of radiative corrections to the process $e^+e^- \rightarrow WW$ as calculated by Lemoine and Verltman.

Fig. 31 $Z^0$ decay in a W and two fermions through a virtual W.

Fig. 32 Cross section for W-pair production in pp collisions with one W decaying into $e\nu$ and the other one in two jets (solid line) and the background cross section for $W \rightarrow$ jet-jet production, assuming the dijet mass to lie in the W mass band ($\sim 10$ GeV).
Figure 1

\[ \begin{align*}
  e^- & \rightarrow W^- + e^+ \\
  e^+ & \rightarrow Z + W^+ \\
  e^- & \rightarrow e^- \\
\end{align*} \]

Figure 2

\[ \sigma (\text{pb}) \]

\[ E_b (\text{GeV}) \]

SM
\[ M_W = 82 \text{ GeV} \]

Figure 3

\[ \sqrt{S} (\text{GeV}) \]

\[ E_b (\text{GeV}) \]

Figure 4
W angular resolution

jet resolution

"LOW"
\[ \sigma_E = 100\% \sqrt{E} \]
\[ \sigma_\theta = 5^\circ \]

"HIGH"
\[ \sigma_E = 50\% \sqrt{E} \]
\[ \sigma_\theta = 1^\circ \]

\( \Delta \cos \theta \)

Figure 9
$76 < M_W < 88$ GeV reconstructed

Figure 10
Figure 11

Figure 12

\[ \sigma (pb) \quad e^+ e^- \rightarrow W^+ W^- \]

within acceptance

\[ E_B (GeV) \]

\[ K_Y \]

\[ \Gamma_W = 0 \quad \Gamma_W \neq 0 \]

\[ M_W = 82.5 \text{ GeV} \]

\[ \Gamma_W (M_W) = 2.8 \text{ GeV} \]
Figure 13
UNPOLARIZED

2000 events

\( \lambda_Z = 1 \)
\( \lambda_Y = 1 \)
SM \( \lambda_Y = \lambda_Z = 0 \)

\( \frac{d\sigma}{d\cos\theta} (\text{pb}) \)

\( \cos\theta \)

Figure 14
Figure 15
$e^+ e^- \rightarrow W^+ W^-$

$E_B = 110 \text{ GeV}$

$100 \text{ pb}^{-1}$

700 events

$\kappa_Y$

$\frac{d\sigma}{d\omega_{SW}}$

$E_\ell (\text{GeV})$

Figure 16
Figure 17
Figure 18

Figure 19
Figure 20
Figure 21
Figure 22
Figure 23
Figure 24
Figure 27

Transverse Polarization
($P_T = 1$)

$\cos \theta = 0.8$

$\frac{d\sigma}{d\cos\theta d\phi}$ pb/rad

$\kappa_{T} = 2$

$\kappa_{Y} = 2$

SM

(0, 0)

(0-) + (+0)

$\frac{\pi}{2} \leq \phi \leq \pi$
Figure 28
Figure 50

Figure 31

Figure 32
ELECTROWEAK RADIATIVE CORRECTIONS AT LEP ENERGIES


PREFACE

In the frame of the LEP 200 Workshop, a study group was set up to look into the problem of electroweak radiative corrections. Since these corrections were dealt with at the earlier LEP Workshop [LEP 86] only as a side issue, and since their relevance extends to the entire energy range of LEP, the study group decided to have a global view of the matter, not confined to the high-energy regime alone.

The study group set itself three tasks: first, to review and discuss the physics reward of electroweak radiative corrections; second, to attempt to unify the language and concepts in the calculation of electroweak radiative corrections; third, to make progress in the practical applications of electroweak radiative corrections—in their availability, in their technical implementation in the analysis of e+e− data, and in the way that data should be presented.

This report summarizes the work of the study group. It also aims at familiarizing the non-expert reader with the field. Several contributions to the subject, which are of a more technical nature, are appended as annexes.

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1. INTRODUCTION

1.1 Overview of electroweak radiative corrections

One of the reasons for the current belief that the electroweak interactions are described by a spontaneously broken gauge theory is the fact that such theories are renormalizable: for any observable quantity its theoretical prediction can be calculated to — in principle — an arbitrary order of perturbation theory, in terms of a finite set of input parameters. The predictions to ‘first non-trivial order of perturbation theory’, the ‘Born approximation’ predictions, can usually be calculated fairly simply. In contrast, the effects of the higher orders of the perturbation expansion, called ‘radiative corrections’ are usually complicated to calculate, and smaller in size. On the other hand, it is the existence and consistency of the radiative corrections which give the Standard Electroweak SU(2)_L ⊗ U(1) Model the character of a quantum field theory.

The physics motivation for the study of electroweak radiative corrections (hereafter called EWRC) is twofold:

i) High-precision measurements of electroweak observables can test the validity of EWRC as implied by the Standard Electroweak Model, and hence test the model at the quantum level. An experimental verification of EWRC would constitute an important milestone in the tests of SU(2)_L ⊗ U(1). In principle, although not comparable in precision, this programme is analogous to the measurement of, for example, the Lamb shift in the hydrogen atom, as a test of QED. Alternatively, while leaving the Standard Electroweak Model intact, a discrepancy between measured and calculated EWRC may signify the existence of heavy unknown particles (see Section 7).

ii) Possible ‘new physics’ in the sense of departures from the Standard Electroweak Model as a spontaneously broken gauge theory of the electroweak interaction of fundamental fermions (e.g. compositeness, technicolour), or in the sense of the appearance of new particles and/or symmetries, will probably manifest itself as small deviations from the predictions of the model. Hence the latter have to be known accurately, including EWRC.

1.2 Why worry about electroweak radiative corrections at LEP?

The reason why we should worry about EWRC at LEP energies is simple: EWRC can be large, of O(1). Any quantitative measurement at LEP energies, even at moderate precision, faces the problem of EWRC.

In e⁺e⁻ → ab physics, a sample of events of the type e⁺e⁻ → ab invariably entails another sample of bremsstrahlung events of the type e⁺e⁻ → aγ, abγγ, .... These radiative events constitute part of the EWRC. The trouble is their frequent occurrence, and that their relative amount strongly depends on the cuts applied to the data. The importance of EWRC is shown in Figs. 1a,b, which compare the ratio R_{ep} = σ(e⁺e⁻ → μ⁺μ⁻)/σ_{points}, and the forward–backward asymmetry A_{FB}(e⁺e⁻ → μ⁺μ⁻), as a function of √s, in Born approximation and with O(α) EWRC included. No experimental cuts are applied. The importance of taking EWRC into account for any measurement at LEP seems apparent, although this argument chiefly applies to photon bremsstrahlung.

Another demonstration of the need of EWRC can be found in the Z energy region: the large peak cross-section enables measurements with unparalleled precision: m_Z to ±0.20 MeV [Altarelli 86], the polarized left–right asymmetry A_{LR} to ±0.003 [Blockus 86]. It is a challenge to the computation of EWRC to match such splendid experimental precision.

1.3 A guided tour through the terminology

This subsection recalls the main vocabulary used in EWRC, with the aim of arriving at a more uniform language. This seems the more appropriate when looking at the somewhat confusing terminology which is used in the literature.

Physical observables are calculated in perturbation expansion. The lowest non-trivial order of perturbation expansion is called ‘Born approximation’, also ‘tree level’ or ‘zero-loop level’. We prefer the term ‘Born approximation’. The lowest non-trivial order of QED cross-sections is typically O(α²) but not necessarily so.
Fig. 1 The ratio $R_{\mu\mu} = \sigma(e^+e^- \rightarrow \mu^+\mu^-)/\sigma_{\text{point}}$, and the forward-backward asymmetry $A_{FB}(e^+e^- \rightarrow \mu^+\mu^-)$, as a function of $\sqrt{s}$, in Born approximation and with $O(\alpha)$ EWRC included. The input parameters are $\alpha$, $G_F$, $\sin^2 \theta_W = 0.225$, $m_H = 100$ GeV, and $m_t = 35$ GeV. No experimental cuts are applied.

If calculations include higher orders of the perturbation expansion they 'include EWRC'. Rather than the absolute order (because of the dependence of the meaning on the process under consideration) we prefer to give the relative order with respect to the lowest non-trivial order:

'O(\alpha) corrected' = Born approximation + $O(\alpha)$ corrections;
'O(\alpha^2) corrected' = Born approximation + $O(\alpha)$ corrections + $O(\alpha^2)$ corrections;
etc.
The orders are always counted in powers of $\alpha$ (in contrast to older papers on QED radiative corrections where the counting is done in powers of $e$). Virtual $O(\alpha)$ corrections are also called one-loop corrections; virtual $O(\alpha^2)$ corrections are also called two-loop corrections, and so on.

We now concentrate on $O(\alpha)$ EWRC. During the last few years it has become customary to divide them into two classes: the first group contains those diagrams which involve an extra photon which has been added to the Born diagrams, either in the form of a real bremsstrahlung photon, or as a virtual photon loop. They are called 'QED corrections' or else 'photonic corrections'. The other group contains all other diagrams. We call them 'non-QED corrections' or else 'weak corrections'. The subset of those diagrams which involve corrections to the self-energy of the gauge bosons, is frequently referred to as 'oblique corrections'. This division is also shown in Table 1, which lists in more detail the categories of radiative corrections within each class.

### Table 1

**Preferred classification of $O(\alpha)$ electroweak radiative corrections**

<table>
<thead>
<tr>
<th>EWRC</th>
<th>Non-QED RC (Weak RC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED RC (Photonic RC) [diagrams involving an extra real or virtual $\gamma$]</td>
<td>Non-QED RC (Weak RC) [all other diagrams]</td>
</tr>
<tr>
<td>Real $\gamma$ (Hard bremsstrahlung, Soft bremsstrahlung)</td>
<td>$\gamma$ self-energy, $Z$, $W$ self-energy, $\gamma$--$Z$ mixing [Oblique corrections]</td>
</tr>
<tr>
<td>Virtual $\gamma$ (Fermion self-energy, Vertex corrections, Box diagrams)</td>
<td>Virtual $Z$, $W$, etc.</td>
</tr>
</tbody>
</table>

There is a theoretical and an experimental reason for the above classification. Firstly, the QED radiative corrections form a gauge-invariant subset within the totality of EWRC. Secondly, the QED radiative corrections are dependent on the cuts applied to the data, whereas the non-QED radiative corrections are not.

While the classification shown in Table 1 describes the consensus achieved within our study group we are quite aware that not everybody will like it. The main point of disagreement concerns the photon self-energy which we classify as a 'non-QED correction', whereas it is traditionally classified as a 'QED correction'. For those who prefer to stick to the historical classification, and consider the photon self-energy as part of the QED correction, we suggest the term 'purely weak corrections' (comprising $Z$ and $W$ self-energy, $\gamma$--$Z$ mixing, virtual $Z$ and $W$, etc.) for all corrections other than QED.

The QED corrections (in our preferred sense) are considered well known and not very interesting since they contain no 'new physics'. On the other hand, they give the largest contribution to the total EWRC. Once a suitable renormalization scheme has been adopted (see Section 2) the effects of the non-QED corrections are typically much smaller than those of the QED corrections. This experience motivates a simpler and restricted treatment of EWRC, in which only QED corrections are applied (see Section 6).

For certain high-precision measurements, $O(\alpha)$ EWRC are considered inadequate, and $O(\alpha^2)$ EWRC have to be applied. The calculation of $O(\alpha^2)$ EWRC is a rather tedious enterprise because of the rapidly growing number of diagrams to be considered. For practical purposes, no complete calculation of all $O(\alpha^2)$ diagrams is made but an approximation as shown in Table 2 is done which is considered good enough for all practical purposes. The approach adopted in practice is that to $O(\alpha^2)$, only the QED corrections are explicitly calculated.
2. THE 'BEST' RENORMALIZATION SCHEME

The calculation of EWRC involves the choice of a set of independent input parameters, and the choice of a renormalization scheme in order to deal, in a well-defined way, with various divergences (ultraviolet and infrared divergences, mass singularities). Although all renormalization schemes are in principle equivalent, the results in a given order of perturbation in different schemes will deviate from each other, because of higher-order contributions. From a practical point of view it is preferable to choose a scheme where the $O(\alpha)$ corrections are small, though this is no small higher-order effects.

In QED, the favoured scheme is the 'on-shell (OS) scheme': the physical fermion ($e, \mu, \ldots$) mass is defined as the pole position of the fermion propagator, and the only coupling constant $\alpha = e^2/4\pi$ is defined in the Thomson limit ($Q^2 = 0$) where the photon and the fermions are on mass shell.

In the SU(2)$_L$ $\otimes$ U(1) Standard Model the situation is more complex. The Lagrangian has in its manifest SU(2)$_L$ $\otimes$ U(1) symmetric form the input parameters $g_2, g_1$ (the SU(2) and U(1) gauge coupling constants), $\mu^2, \lambda$ (the second- and fourth-order coefficients in the Higgs potential), and $g_t$ (the fermion–Higgs Yukawa coupling constants). Other than in QED, none of these parameters can be directly measured in present-day experiments. An alternative set of independent parameters,\n
\[ \alpha, m_W, m_Z, m_H, m_t \]  

has the advantage that each quantity can—in principle—be measured. The so-far unknown Higgs mass $m_H$ is treated as a free parameter. The fermion mass parameter $m_t$ is a shorthand notation for the fermion masses and the weak quark mixing angles.

The renormalization scheme which makes use of set (1) as parameters, and establishes their physical meaning by appropriate renormalization conditions, is called the 'on-shell (OS) scheme', in analogy to QED. It is the most direct and natural extension of the QED renormalization scheme. All parameters are defined as on-shell quantities: $\alpha$ in the Thomson limit, and the masses as pole positions of the corresponding propagators.

What is then the role of the popular electroweak mixing parameter $\sin^2 \theta_W$? The simplest and most natural definition of this parameter is in terms of the physical $W$ and $Z$ masses:

\[ \sin^2 \theta_W = 1 - m_W^2/m_Z^2. \]
By definition, relation (2) is valid to all orders of perturbation theory. The parameter $\sin^2 \theta_w$ is not an independent quantity and can, in principle, be avoided completely. However, for historical and practical reasons, $\sin^2 \theta_w$ is retained as a bookkeeping device in the neutral-current phenomenology.

The advantages of the OS scheme are:

i) The input parameters have a clear physical meaning, and can be measured directly. Except $m_H$ and the top quark mass $m_t$, all parameters are known.

ii) The Thomson cross-section formula from which $\alpha$ is obtained is exact to all orders of perturbation theory.

iii) The separation between the QED corrections and the non-QED corrections (which is important for the Monte Carlo implementation of EWRC, see Section 4) is automatic.

The OS scheme also has drawbacks:

i) The experimental precision of $m_W$ is not as good (even assuming an error of $\pm 100$ MeV as can be achieved at LEP 200) as to make the uncertainty in EWRC negligible.

ii) The fine-structure constant $\alpha$ is defined as a low-energy parameter, at $Q^2 = 0$, whereas $m_W$ and $m_Z$ are high-energy parameters. The renormalization of the Thomson $\alpha$ therefore induces large logarithmic corrections from the photon self-energy, typically $(\alpha/\pi) \log (m_W/m_\gamma)$, in the EWRC at the W mass scale.

The first drawback can easily be overcome: employing the OS scheme one can calculate the muon decay width $\Gamma_\mu$ in terms of the parameter set (1), including $O(\alpha)$ EWRC. On the other hand, $\Gamma_\mu$ is related to the Fermi coupling constant $G_F$ by the following relation which includes — for historical reasons — $O(\alpha)$ QED corrections:

$$\Gamma_\mu = \left( G_F^2 m_\mu^3 / 192\pi^3 \right) \left( 1 - 8m_\mu^2 / m_\pi^2 \right) \left[ 1 + (25/4 - \pi^2) \alpha/2\pi \right].$$

(3)

Utilizing the thus-determined coupling constant,

$$G_F = (1.16637 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2},$$

one obtains from

$$\Gamma_\mu = \Gamma_\mu(\alpha, m_W, m_Z, m_H, m_t)$$

the relation

$$G_F = (\pi\alpha/\sqrt{2}) [m_\mu^2 (1 - m_\mu^2 / m_\pi^2) (1 - \Delta r)]^{-1},$$

(4)

where $\Delta r = \Delta r(\alpha, m_W, m_Z, m_H, m_t)$ is the $O(\alpha)$ non-QED correction of muon decay in the OS scheme. Relation (4) allows replacement of $m_W$ by the precisely measured $G_F$.

We denote the Fermi coupling constant by $G_F$, rather than $G_\gamma$, in order to underline the origin of its numerical value from the muon-decay width. There are minor ambiguities in the literature about the extraction of $G_F$ from the muon lifetime, owing to neglect of higher-order terms in the QED correction. The differences are, however, numerically unimportant at the present level of precision.

The parameter set $\alpha, G_F, m_Z, m_H, m_t$ then comprises the best measured quantities, and only $m_H$ and the quark masses give rise to a theoretical uncertainty in EWRC.

The second drawback of the OS scheme seems to be more controversial. On the one hand, the large logarithmic corrections which are due to the definition of $\alpha$ at $Q^2 = 0$, can be 'absorbed' by replacing the Thomson $\alpha$ by a running QED coupling constant, evaluated at the W mass scale:

$$\alpha(m_\mu^2) = \alpha(0) / [1 - \Pi_{\text{QED}}(m_\mu^2)],$$

where $\Pi_{\text{QED}}(Q^2)$ denotes the fermion-loop corrections to the photon self-energy. On the other hand, this introduces an uncertainty in the parameter $\alpha(m_\mu^2)$ from the quark loops in the photon self-energy, mostly from the unknown $m_t$. The contribution from light quarks is also uncertain because of the unknown quark masses and QCD radiative
corrections. Using a dispersion relation, it is customarily related to available data on $e^+e^- \rightarrow$ hadrons at low energies. The experimental error of these data may well be the ultimate limit of the precision with which EWRC can be calculated.

Another problem occurs in QED corrections where large corrections arise from the soft real and virtual photon emission. Here the Thomson $\alpha$ is more adequate than $\alpha(m_W)$. Also in Bhabha scattering at small angles the Thomson $\alpha$ is more appropriate.

We favour the following procedure as a viable way out of this dilemma:

i) Calculate within the OS scheme the $O(\alpha)$ corrections to the gauge boson propagators,

\[
\begin{align*}
\gamma &: \frac{1}{q^2} \rightarrow \frac{1}{[q^2 + \Sigma(q^2)]} \\
W &: \frac{1}{[q^2 - m_W^2 + i m_W \Gamma_W]} \rightarrow \frac{1}{[q^2 - m_W^2 + \Sigma_W(q^2)]} \\
Z &: \frac{1}{[q^2 - m_Z^2 + i m_Z \Gamma_Z]} \rightarrow \frac{1}{[q^2 - m_Z^2 + \Sigma_Z(q^2)]},
\end{align*}
\]

(5)

with the renormalized self-energies $\Sigma(q^2)$ in terms of $\alpha$, $m_W$, $m_Z$, $m_H$, and $m_t$.

ii) Replace in the Born approximation expressions each boson propagator by the expressions (5).

iii) Use the relation (4) to replace $m_W$ by $\alpha$, $G_\mu$, and $m_Z$.

iv) Add the remaining non-QED corrections.

v) Add the QED corrections.

This procedure leaves the non-QED corrections small while retaining the Thomson $\alpha$ and $G_\mu$ as input parameters.

The effect of employing the 'wrong' renormalization scheme can be seen in Fig. 1a, where $\alpha$, $G_\mu$, $\sin^2 \theta_W$, $m_H$, and $m_t$ have been used as input parameters: notice the large displacement of the $Z$ peak due to $O(\alpha)$ non-QED corrections.

3. QED RADIATIVE CORRECTIONS

As already mentioned above, the QED radiative corrections form a gauge-invariant subset of all EWRC. They are considered not very interesting but are at LEP energies large in magnitude, and hence must receive a lot of attention.

In calculating QED radiative corrections one usually restricts oneself to the $O(\alpha)$ correction. When the result of this calculation is not too large (say, 10% of the Born value), the $O(\alpha^2)$ corrections are in general considered negligible. It is known though that stringent cuts applied to the data, which leave little room for the emission of bremsstrahlung photons, imply large and negative corrections. In such a situation one has to calculate the $O(\alpha^2)$ corrections. In the particular case of bremsstrahlung emission, there is another way of incorporating higher-order corrections, namely by exponentiating a certain part of the $O(\alpha)$ QED correction, a technique pioneered by Yennie, Frautschi and Suura [Yennie 61].

The ultraviolet divergences in EWRC, which arise from the high-momentum domain in loop integrals, are removed by the renormalization procedure. The virtual QED and the non-QED corrections then modify the Born cross-section (which we take as an example of a physical observable) as follows: $d\sigma/d\Omega = (d\sigma_{\text{Born}}/d\Omega)(1 + \delta_{\text{QED}} + \delta_{\text{non-QED}})$. Both corrections are ultraviolet-finite. However, $\delta_{\text{QED}}$ still contains an infrared divergence which has its origin in the low-energy region. The Bloch-Nordsieck theorem [Bloch 37], generalized by Kinoshita [Kinoshita 62] and Lee and Nauenberg [Lee 64], ensures that the infrared divergences from the emission of real and virtual photons cancel each other.

In practice, the bremsstrahlung emission of photons is divided into two classes: 'soft' bremsstrahlung ($E_\gamma < k_i$) and 'hard' bremsstrahlung ($E_\gamma > k_i$), where $k_i$ is an arbitrary cut-off energy chosen smaller than $E_{\gamma}^{\text{min}}$, the detection threshold of the energy of photons in the experimental apparatus employed. The soft bremsstrahlung correction, together with the virtual photon correction, leads to a finite correction of the cross-section. The hard bremsstrahlung leads to events with additional photons in the final state. In the following, we concentrate on the
bremsstrahlung part of the QED radiative correction. For the discussion, we refer to the specific case of muon-pair creation: $e^+e^- \rightarrow \mu^+\mu^-$. 

In the Standard Electroweak Model the bremsstrahlung matrix element is described by eight Feynman graphs, obtained by attaching photons to each $e^+$ or $\mu^+$ line in the $\gamma$ and $Z$ exchange diagrams. The differential cross-section is a complicated expression. For two specific kinematical situations, however, the expressions become much simpler: in the 'soft photon limit' ($E_\gamma \ll E_e$), where the single bremsstrahlung cross-section becomes proportional to the Born cross-section, and for hard photon emission in the 'ultrarelativistic limit' ($E_e \gg m_e, m_\mu$). Calculation techniques have been given in the literature [Berends 82] for these cases, and compact formulae exist for all standard reactions.

Other than in the case of soft photon emission where the photon is emitted nearly isotropically, the analytical integration of the differential cross-section for hard photon emission is only sometimes possible. When all kinds of cuts are applied to the data the integration boundaries become too complicated for an analytical calculation of the total cross-section.

4. THE IMPLEMENTATION OF ELECTROWEAK RADIATIVE CORRECTIONS

There are two approaches to obtaining the hard bremsstrahlung correction. The first is to perform a numerical integration over the phase space allowed for by the experimental cuts applied to the data, by means of a standard multidimensional integration routine [Berends 79]. Care must be taken that strong peaks in the differential cross-section are adequately dealt with. The other approach is the use of a Monte Carlo event generator [Berends 81, 81a, 82a, 83]: a program generates a set of four-momenta of the final-state particles, including the real photon(s), in such a way as to reproduce the differential cross-section for hard bremsstrahlung. The events are either generated in the full phase space, or in a large part of the phase space which is restricted by one or two 'natural' variables only, such as a minimum scattering angle, or a maximum photon energy. Besides generating a large number of events, the program should also yield the bremsstrahlung cross-section for the phase space covered by the events. Since the events are not weighted the application of cuts on the event sample is simple: the events not satisfying the selection criteria are rejected.

In practice, one not only considers an event generator for photons with $E_\gamma > k_1$ but one also generates a set of four-momenta of the final-state particles without a bremsstrahlung photon when $E_\gamma < k_1$. The generation is done according to the analytical expressions which result after the $O(\alpha)$ non-QED, virtual QED, and soft bremsstrahlung ($E_\gamma < k_1$) corrections have been applied. The event generator is normalized such that the ratio of soft- to hard-photon events is given by the theoretically-known respective cross-sections.

Although an event generator allows a great flexibility in the implementation of EWRC, it has one disadvantage with respect to a numerical integration program. In the latter there is more freedom to vary the cut-off energy $k_1$. The larger $k_1$ becomes, the less good is the soft-photon approximation of the bremsstrahlung cross-section. On the other hand, the smaller $k_1$ becomes, the larger becomes the sum of the $O(\alpha)$ virtual QED and soft-photon corrections. The latter sum is a (potentially) large negative correction, which is compensated by the positive correction due to hard bremsstrahlung, so as to yield an overall QED correction of reasonable size. It may even happen that if the cut-off energy $k_1$ becomes too small, the $O(\alpha)$ QED corrected cross-section becomes negative in some part of the phase space. In order to generate events, however, a positive cross-section is mandatory. If this problem occurs and no suitable $k_1$ can be found, one has to go beyond the $O(\alpha)$ approximation in the Monte Carlo generator.

Another problem related to the introduction of $k_1$ can arise when certain distributions are generated where soft bremsstrahlung events play a dominant role. For example, consider the study of $d \sigma / d \xi$ in $e^+e^- \rightarrow \mu^+\mu^-$, where $\xi$ is the acollinearity angle of the muon pair. For very small $\xi$ (i.e. nearly back-to-back muons) soft bremsstrahlung events become predominant, and the $O(\alpha)$ approximation may not apply any longer. Since the sum of the $O(\alpha)$
virtual- and soft-photon corrections is negative, the number of events in the interval \([0, \pi]\) is underestimated. This means that in the angular interval \([\pi, 2\pi]\) the number of events is overestimated, since the total number of events in \([0, \pi]\) is adequate. In the case of a particular interest in the bin \([3, 4\pi]\), higher-order corrections should be taken into account in the event generator, for example by extending the whole procedure to \(O(\alpha^3)\), including the generation of double bremsstrahlung events.

As an alternative to calculating the QED corrections to \(O(\alpha^2)\), techniques exist to re-sum the leading logarithmic QED corrections to all orders of perturbation. It was noticed by Greco and Rossi [Greco 67] that the so-called ‘coherent state formalism’ not only reproduces the exponentiated form of the soft bremsstrahlung photon correction [Yennie 61], but also yields a correction for hard collinear photons emitted from any external leg of a diagram, in exponentiated form [Caffo 85]. The latter feature is important because the emission of hard collinear photons is steeply peaked under zero angle, which is difficult to deal with in a Monte Carlo generator with adequate precision.

5. HOW TO PRESENT THE DATA?

Once the theoretical calculation of EWRC is available, incorporating the special experimental conditions, theoretical expectations can be compared with experimental measurements. If several experiments perform the same measurement, there will be a strong interest not only in comparing the physics conclusions but also the data from which they have been obtained. This will be the case, in particular, when experimental results do not agree: is the source of the disagreement in the data, or in the radiative corrections, or in the analysis of the data? So one wants to see the data themselves published as has always been the case. The only question is: before or after radiative corrections?

Before we turn to the situation at LEP energies, we recall the situation at PETRA energies. The experimenters chose to publish data after the subtraction of \(O(\alpha)\) QED corrections. That is the event sample was the one which one would find if the photon radiation had been switched off. The theory could be compared with the thus obtained ‘data’ at the level of the Born approximation. Non-QED radiative corrections were too small to be considered if the OS scheme was employed. The argument in favour of this procedure was that at the level of the Born approximation the dependence of the data on the selection criteria was eliminated, and hence data from different detectors could be directly compared. Moreover, the ‘interesting’ physics could be studied without disturbance from the ‘known’ physics of the QED radiative corrections.

While this is a valid point of view it is not easy to see how this procedure could be continued at LEP energies because there the QED corrections can be of \(O(1)\)! Are ‘data’ after 100% corrections still ‘data’? Secondly, the radiative corrections applied by one experiment may for some reason be different from those applied by another experiment although ‘they used the same program’. Needless to say, the problem of the correctness of the radiative corrections is much aggravated by their magnitude.

We list in Table 3 possible ways of presenting data together with their pros and cons. The first column suggests the presentation of ‘raw’ data, i.e. of data corrected for apparatus acceptance, bad channels, and insensitive regions, but before any radiative correction. Although the data are model independent, they are strongly dependent on the experimental cuts via the QED radiative effects, and hence a comparison cannot be made between one experiment and another. We discard the option to present raw data.

The last column suggests the presentation of data after the totality of the EWRC has been applied, i.e. in the form to be compared with the Born approximation prediction. Whilst this procedure is transparent and hence attractive, the fact remains that the applied radiative corrections are large and, chiefly via the non-QED corrections, model dependent. We feel that data should not be published with EWRC applied which are calculated in today’s accepted framework of the Standard Electroweak Model, with everyone’s preferred values for \(m_W\) and \(m_t\). We also discard this option.

The third column, QED removed, is in essence the extrapolation of the PETRA procedure to LEP energies. A slight model dependence would arise from QED corrections to \(Z\) exchange diagrams which rely on the Standard
Table 3
Options of presenting data at LEP energies

<table>
<thead>
<tr>
<th></th>
<th>'Raw' data</th>
<th>Canonical cuts applied(^a)</th>
<th>QED removed</th>
<th>Born approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pros</strong></td>
<td>Model independent</td>
<td>Model independent</td>
<td>Insensitive to non-QED corrections, hence largely model independent</td>
<td>Clear, simple, independent of exp. cuts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Analytical formulae for EWRC possible?</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cons</strong></td>
<td>Dependent on exp. cuts</td>
<td>Difficult since consensus required</td>
<td>QED corrections are large, slight model dependence</td>
<td>Large, model-dependent corrections</td>
</tr>
</tbody>
</table>

\(^a\) Recommended procedure

Electroweak Model. Yet it would be a viable solution, in principle, if the QED corrections were not so large. Still, if we adopted this procedure for LEP, could we accept even larger corrections at future e\(^+\)e\(^-\) colliders in the TeV energy region? Since we feel that published ‘data’ should be close to what has actually been measured, we also discard this option.

The consensus within the study group was a preference for the use of ‘canonical cuts’. Data should be presented (see, however, the clarification below!) before EWRC but after an agreed set of canonical cuts have been applied. Canonical cuts are thought to be cuts which satisfy the following criteria:

i) the phase space within the cuts is well within the acceptance of all relevant detectors;

ii) the cuts have a clear and unambiguous physical meaning;

iii) the cuts should be chosen such as not to cause \(O(\alpha)\) QED corrections to become too large, so that the \(O(\alpha^2)\) corrections may be assumed to be small and hence negligible.

The comparison between various experiments will be quite simple. Also the comparison between theory and experiment may be facilitated: if a set of canonical cuts exists it becomes worthwhile to construct a numerical integration program specialized to these cuts.

Our recommendation of the use of canonical cuts does not, of course, restrict anybody’s freedom to publish whatever is deemed appropriate, with any cuts applied or with any method of analysis preferred. We only urge the experimental teams to publish in addition their data with canonical cuts applied. This implies no restriction whatsoever to employing all the strong features of each detector in the main analysis.

Returning once again to the specific case of muon-pair production it seems that good candidates for a set of canonical cuts are the acollinearity angle \(\gamma\), the muon momentum \(p_\mu\), and the scattering angle \(\theta\) with respect to the beam line. Thus all events where the muons have a scattering angle \(\theta > \theta^\text{cut}\), momentum \(p_\mu > p_\mu^\text{cut}\), and acollinearity angle \(\gamma < \gamma^\text{cut}\), fall within the canonical region.

6. **APPROXIMATIONS AND ‘RULES OF THUMB’**

Within the complete set of EWRC, the numerically most important contributions arise from photon bremsstrahlung from the initial electron and positron legs (assuming that a convenient renormalization scheme has been chosen such as to minimize the non-QED corrections). Hence the obvious approximation to calculating the
entire set of corrections is to restrict oneself to the initial-state bremsstrahlung correction only. It depends on the experimental precision of the particular experiment whether or not more elaborate radiative corrections are needed.

The photon radiation from the initial legs gives rise to a large logarithmic correction of the size ($\alpha/\pi)t = 0.056$, with $t = \ln m_Z^2/m^2 = 24.2$. Other terms in the perturbative series are of the size $(\alpha/\pi)t^{1/2}$. Since $t \gg 1$, the effective expansion parameter $(\alpha/\pi)t$ is also large, and the influence of orders beyond $O(\alpha)$ are to be considered in experiments which measure quantities which vary strongly with $\sqrt{s}$. A typical experiment of this type is the measurement of the line-shape of the $Z$ resonance.

The ‘leading log approximation’ just retains the leading logarithmic terms, often summed over all orders of perturbation. The summation techniques are either based on the exponentiation of the $O(\alpha)$ QED correction (excluding the photon self-energy part, according to our definition given above), or on QCD inspired approaches employing the Altarelli–Parisi evolution equations for the initial lepton states [Greco 67; Tsai 83; Kuraev 85; Altarelli 86; Nicrosini 86].

In the following, we give a few ‘rules of thumb’ on the approximate use of EWRC. Their justification is a posteriori, i.e. from comparison with the result of exact calculations. Their numerical accuracy is at the level of 20%.

**Rule of Thumb No. 1**

Non-QED radiative corrections will be largely absorbed by the use of $\alpha(m^2)$, $G^s$, $m_Z$ (or, equivalently: $G^v$, $m_W$, $m_Z$) in the Born approximation expressions. Remaining non-QED corrections are of the order of 1%.

**Rule of Thumb No. 2**

For QED radiative corrections, use the Thomson $\alpha$ in the QED formulae. If the $O(\alpha)$ QED correction changes a cross-section by a fraction $x \approx 0.3$ then the $O(\alpha^2)$ QED correction will cause a further change by a fraction $= x^2/2$.

**Rule of Thumb No. 3**

The peak cross-section at the $Z$ pole gets reduced by QED corrections by a factor $f = (1/\sqrt{\beta^2 + 1}) = 0.7$, where $\beta = (2\alpha/\pi)(\ln m_Z^2/m^2 - 1) \approx 0.1$. The position of the maximum of the cross-section is shifted to $\sqrt{s} = m_Z + \Delta E$, where $\Delta E = \sqrt{\beta}\Gamma_Z/2 = 120$ MeV.

**Rule of Thumb No. 4**

An observable $\xi$ changing linearly with $\sqrt{s}$ around the $Z$ pole will change by initial-state QED corrections from $\xi(m_Z)$ to $\xi(m_Z - 2\Delta E)$, where $\Delta E = 120$ MeV (see rule of thumb No. 3).

7. **THE PHYSICS REWARD OF ELECTROWEAK RADIATIVE CORRECTIONS**

The EWRC constitute a considerable obstacle between data taking and physics analysis at LEP energies. They also merit attention in their own right because of their physics content. This latter feature has been highlighted already at the LEP I Physics Workshop, in particular in the contribution of Lynn, Peskin and Stuart [Lynn 86]. Because of the importance of the subject, but also because of its relation to a precise measurement of the $W$ mass, accessible only at LEP 200, we recall here the main arguments.

The $W$ and $Z$ bosons couple to all particles which take part in the weak interaction. There is a fair chance that expected but hitherto unobserved particles (top quark, Higgs boson) will be produced directly at LEP. However, if these or other novel particles are too heavy to be directly produced, they might still be ‘observable’ through their contributions to the EWRC. Actually the heavier the respective particles, or the larger the mass splitting within isospin multiplets, the larger their effects. Hence the experimental measurement of EWRC provides a window on ‘new physics’ in the mass range above 100 GeV, which is otherwise not accessible at LEP and is complementary to the searches for the direct production of new particles.

In order to profit from the physics reward of EWRC there is a price to pay: not the large QED radiative corrections but the small non-QED corrections are the interesting part, with effects of relative size $\alpha/\pi = 0.002$. 
This means first that precision experiments are called for, and second that the logarithmically enhanced QED radiative corrections of size \((\alpha/\pi) \ln m_Z^2/m_t^2 = 0.056\) have to be handled with care. In the following it will be argued that the measurement of the left–right asymmetry \(A_{LR}\), at the Z pole, and the measurement of \(m_w\) have a considerable potential for insight into ‘new physics’.

In a renormalization scheme with \(\alpha, G_F, m_Z, m_H,\) and \(m_t\) as the input parameters, other precisely measurable quantities such as \(A_{LR}\) and \(m_w\) can be calculated. As is shown elsewhere in these Proceedings [Roudeau 87], \(m_w\) can be measured at LEP 200 to \(\pm 100\) MeV. The quantity \(A_{LR} = (\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2)\) is the asymmetry of the annihilation cross-sections of left- and right-handed electrons colliding with unpolarized positrons, and hence is accessible with longitudinally polarized beams only. The feature which singles out \(A_{LR}\), as compared to other measurable asymmetries, is that it is independent of the fermion type in the final state, at the Z pole:

\[
A_{LR}^{\text{exp}}(m_Z) = 2\nu_{\text{atm}}/(\nu_1^2 + \nu_2^2).
\]

Hence \(A_{LR}\) can be measured in an inclusive mode, rendering the statistical precision superior to any other asymmetry measurement. Experimentally, an overall precision of \(\Delta A_{LR} = \pm 0.003\) seems within reach [Blockus 86]. QED radiative corrections—which are almost negligible for \(A_{LR}\) [Böhm 82] because of its very mild dependence on \(\sqrt{s}\)—are ignored in the following.

In terms of the precisely measured input parameters \(\alpha, G_F,\) and \(m_Z\)—the Z mass is assumed to be ultimately measured to \(\pm 20\) MeV [Altarelli 86]—the predictions for \(A_{LR}\) and \(m_w\) depend on \(m_t\) and \(m_H\) when \(O(\alpha)\) non-QED radiative corrections are included, in the framework of the Electroweak Standard Model. The dependence of the predictions on \(m_t\) and \(m_H\) within the presently favoured domain are shown in Fig. 2, together with the expected

---

Fig. 2. Variation of the predictions for the left–right asymmetry \(A_{LR}\), and for \(m_w\), as a function of \(m_t\) and \(m_H\), in comparison with the expected experimental precision of \(A_{LR}\) and \(m_w\).
experimental error. One concludes that each $A_{LR}$ and $m_W$ restricts the allowed range of $m_t$ and $m_H$ considerably, and even more so if both quantities are measured. Optimistically, $m_W$ 'measures' $m_t$ to ± 20 GeV, and $A_{LR}$ narrows the allowed range of $m_H$ to ± 200 GeV. All this is made possible by experimental errors which are expected to be small as compared to the $O(\alpha)$ non-QED radiative corrections caused by the top quark and the Higgs boson.

Similarly, the effects of new heavy quarks and leptons, or of supersymmetric particles, would show up in the radiative corrections of $A_{LR}$ and $m_W$ [Lynn 86]. This discovery potential, however, can be fully exploited only if polarized beams at $\sqrt{s} = m_Z$ become available at LEP.

8. INVENTORY OF EXISTING CALCULATIONS

The study group saw as one of its main tasks a review of the likely measurements at LEP together with the expected precision, and the preparation of Monte Carlo event generators for the various processes including EWRC with adequate accuracy. Table 4 gives an overview of the availability of EWRC, as of October 1986.

For the process $e^+e^- \rightarrow \ell\bar{\ell}$, where $\ell$ is a light fermion other than an electron, many authors (including Berends, Böhm, Greco, Hollik, Jadach, Kleiss, Lynn, Pancheri, Srivastava, and Stuart) have contributed to the complete EWRC at $O(\alpha)$. The most versatile Monte Carlo generator which is available at present is BREMMUS. Beyond $O(\alpha)$, analytic expressions exist for the distortion of the Z resonance shape due to initial-state QED radiation (in leading log approximation at $O(\alpha^2)$ by Kuraev and Fadin [Kuraev 85], and Altarelli and Martinelli [Altarelli 86]; in leading log approximation to all orders by exponentiating the infrared divergent $O(\alpha)$ correction, by Greco [Greco 86]; complete $O(\alpha^2)$ by Berends, Burgers and van Neerven, see Annex A, and by Nicrosini and Trentadue [Nicrosini 86]).

For Bhabha scattering, $e^+e^- \rightarrow e^+e^-$, analytic formulae exist which are complete to $O(\alpha)$ [Böhm 86], and have large logarithmic contributions resummed to all orders [Greco 86a, and references quoted therein]. The formulae, however, do not include hard bremsstrahlung, so their use is limited to essentially collinear $e^+e^-$ pairs. A Monte Carlo generator, comprising the complete EWRC at $O(\alpha)$, is available (Berends, Hollik and Kleiss). At present, the option of initial-state longitudinal polarization is being implemented in this Monte Carlo generator.

The process $e^+e^- \rightarrow \tau^+\tau^-$ is of interest because it offers an easy way to measure the final-state polarization asymmetry $A_{pol}$. A Monte Carlo generator is prepared by Jadach, Stuart and Was, which will include the complete $O(\alpha)$ EWRC, and all major $\tau$ decay modes [Jadach 84; see also Annex B]. The attainable experimental precision is such that ultimately $O(\alpha^2)$ QED corrections may have to be included.

Electroweak radiative corrections to heavy fermion production, $e^+e^- \rightarrow FF$, are being calculated by Beenakker and Hollik (see Annex C). Analytical formulae for the one-loop corrections and soft bremsstrahlung exist. A Monte Carlo generator, TITTOP, which simulates $O(\alpha)$ initial-state bremsstrahlung effects in heavy fermion production, has recently been made available by Jadach and Kühn [Jadach 86].

The $O(\alpha)$ EWRC for W production, $e^+e^- \rightarrow WW^-$, is being worked upon by Gaemers and Kunszt, making use of earlier work by Lemoine and Veltman [Lemoine 80], and Phillips [Phillips 82]. At present, only $O(\alpha)$ hard bremsstrahlung corrections are available in the form of a Monte Carlo generator.

Higgs particle creation via $e^+e^- \rightarrow H^0\gamma$ has been studied by Fleischer and Jegerlehner. A Monte Carlo generator including complete $O(\alpha)$ EWRC is in preparation. Figure 3 shows the cross-section for Higgs-boson production ($m_H = 50$ GeV) before and after $O(\alpha)$ EWRC, where a cut-off $E_\gamma < 0.25 E_\gamma$ for bremsstrahlung photons has been applied. A Monte Carlo generator including $O(\alpha)$ initial-state QED radiation has been used by Berends and Kleiss [Berends 85].

Higgs particle creation via $e^+e^- \rightarrow H^0\gamma$ has been studied by Barroso, Pulido and Romão, including one-loop non-QED corrections [Barroso 86].

Radiative Z production, $e^+e^- \rightarrow Z\gamma$, has attracted a lot of interest. This process was originally suggested as a means of 'counting' the number of neutrino families [Ma 78, Barbiellini 81], via the cross-section for single-photon production in $e^+e^- \rightarrow Z\gamma$, with $Z \rightarrow \nu\bar{\nu}$. The question of $O(\alpha)$ QED corrections of the cross-section has been
## Table 4

Status of EWRC of various processes\(^a\)

<table>
<thead>
<tr>
<th>Process</th>
<th>O(α) EWRC</th>
<th>O(α(^2)) EWRC</th>
<th>Comments</th>
</tr>
</thead>
</table>
| e\(^+\)e\(^-\) → ff \((f = \text{light fermion other than } e; \text{ initial-state } e \text{ can be longitudinally polarized})\) | Done\(^b\) | At work | O(α): non-QED part needs to be checked.  
O(α\(^2\)): QED part only; analytic formulae for the Z resonance shape; a MC generator including O(α\(^2\)) QED should exist by the end of 1987. |
| e\(^+\)e\(^-\) → e\(^+\)e\(^-\) \((\text{initial-state } e \text{ can be longitudinally polarized})\) | Done | Needed (?) | Also existing are analytic formulae in leading log approximation. |
| e\(^+\)e\(^-\) → τ\(^+\)τ\(^-\) \((\text{initial-state } e \text{ can be longitudinally polarized})\) | At work | Needed (?) | O(α): QED part complete; non-QED part at work; complete O(α) should exist by the end of 1986.  
One-prong τ decays included.  
O(α\(^2\)) QED corrections from e\(^+\)e\(^-\) → ff applicable. |
| e\(^+\)e\(^-\) → F\(^+\)F\(^-\) \((F = \text{heavy fermion other than } τ)\) | At work | Not needed | One-loop corrections and soft bremsstrahlung available in analytical form; hard bremsstrahlung missing.  
A MC generator with initial-state bremsstrahlung is available. |
| e\(^+\)e\(^-\) → W\(^+\)W\(^-\) | At work | Not needed | Hard bremsstrahlung done; one-loop corrections and soft bremsstrahlung in analytical form; radiative corrections for on-shell W’s only. |
| e\(^+\)e\(^-\) → H\(^0\)μ\(^+\)μ\(^-\) | At work | Not needed | Hard bremsstrahlung done for the initial state; one-loop corrections and soft bremsstrahlung done; a MC generator should exist by the end of 1986 |
| e\(^+\)e\(^-\) → H\(^0\)γ | At work | Not needed | One-loop non-QED corrections exist in analytical form. |
| e\(^+\)e\(^-\) → Zγ | At work | Needed (?) | One-loop electroweak corrections as well as soft bremsstrahlung exist in analytical form; an estimate of hard bremsstrahlung is available in the form of a MC generator. |
| e\(^+\)e\(^-\) → e\(^+\)e\(^-\)γ | At work | Not needed (?) | |

\(^a\) The initial-state electrons and positrons are unpolarized unless stated otherwise.

\(^b\) ‘Done’ means ‘available in the form of a Monte Carlo generator’.
Fig. 3  Cross-section of $\sigma(e^+e^- \rightarrow H^0\mu^+\mu^-)$ as a function of $\sqrt{s}$. The dotted line shows the Born approximation; the full line includes O($\alpha$) EWRC, with a cut-off $E_{\gamma} < 0.25 E_{\mu}$ for bremsstrahlung photons. The Higgs boson mass is $m_H = 50$ GeV.

addressed by Berends, Burgers and van Neerven [Berends 86], Boudjema, Domby and Cole [Boudjema 86], Igarashi and Nakazawa [Igarashi 86], Mana, Martinez and Cornet [Mana 86], and Bento, Romão and Barroso [Bento 86, Romão 87]. An estimate of O($\alpha$) hard bremsstrahlung corrections to the (dominant) $Z$ exchange graphs of $e^-e^+ \rightarrow \gamma\gamma$ is shown in Fig. 4: the cross-section for single-photon production is reduced by a significant amount,

Fig. 4  Photon energy spectrum from $e^+e^- \rightarrow \gamma\gamma$, in Born approximation (dotted line) and after O($\alpha$) hard bremsstrahlung corrections.
on the scale of the 30% variation from three to four neutrino generations. The one-loop QED and soft bremsstrahlung corrections exist in analytical form only. Also, the $O(\alpha)$ non-QED corrections as published by Böhm and Sack [Böhm 86a], and discussed by Boudjema, Coè and Dombey (see Annex D), exist in analytical form only. The inclusion of $O(\alpha)$ EWRC to $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ in a Monte Carlo generator is an urgent task. Also, in view of the large $O(\alpha)$ QED correction, $O(\alpha^2)$ QED corrections may have to be considered.

The dominant background of the neutrino counting process is radiative Bhabha scattering, where both final-state electrons remain undetected: $e^+e^- \rightarrow (e^-e^+)\gamma$. A Monte Carlo generator for the process $e^+e^- \rightarrow e^-e^+\gamma$ including $O(\alpha)$ EWRC is under preparation by Kleiss. Calculations of the cross-section of radiative Bhabha scattering at the Born approximation level have recently been performed by Caffo, Gatto and Remiddi (see Annex E). Mana and Martinez [Mana 86a], and Karlen [Karlen 86]. Mana and Martinez, and Karlen, have developed a Monte Carlo generator which is particularly designed to handle radiative Bhabha scattering at very small electron scattering angle. At the Born approximation level, there is good numerical agreement between the three calculations. However, Karlen points out that the $O(\alpha)$ QED corrections to radiative Bhabha scattering are sizeable, and have to be taken into account in a quantitative analysis of the neutrino counting experiment. This claim is not supported by Mana and Martinez, and obviously deserves clarification. Notice that radiative Bhabha scattering might also be of interest for a luminosity measurement.

Finally, there is good news from a sector which has particular relevance to the energy range of LEP 200. Lynn, Kennedy and Verzegnassi (see Annex F) have studied the shift in the W mass due to a very heavy top quark, at $O(\alpha^3)$. This question was triggered by the large influence of $m_t$ on the W mass at $O(\alpha)$, see Section 7. For $m_t = 200$ GeV, the calculation yields an $O(\alpha^3)$ correction of 18 MeV only, so that $O(\alpha^3)$ corrections from $m_t$ can be safely neglected, compared to the expected experimental precision of $\pm 100$ MeV.

9. CONCLUSION AND RECOMMENDATIONS

Electroweak radiative corrections, in particular their QED part, are large at LEP energies. Their application by means of well-tested Monte Carlo generators is inescapable for any quantitative analysis.

Electroweak radiative corrections not only allow significant tests of the Standard Electroweak $SU(2)_L \otimes U(1)$ Model beyond the Born Approximation level, but also offer an opportunity to spot particles which are too heavy to be observed directly, through their virtual effect in loop diagrams.

Although a great deal of work has been done already to calculate $O(\alpha)$ [and $O(\alpha^2)$ where needed] electroweak radiative corrections, and to make them available in the form of Monte Carlo generators, a lot remains to be done. This includes cross-checking of the results of different authors.

In order to make the comparison of the results of calculations easier, the use of the on-shell renormalization scheme is recommended, with $\alpha$, $G_F$, $m_Z$, $m_W$, and $m_t$ as input parameters.

Experimental teams should feel committed to also present their data with canonical cuts applied. We urge a Workshop of experts to be held, in order to define such canonical cuts.

Finally, we urge the creation of a library of ‘standard’ Monte Carlo generators for events which have electroweak radiative corrections included.
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ANNEX A
ON SECOND ORDER QED CORRECTIONS TO THE $Z^0$ RESONANCE SHAPE
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Since the $O(a)$ correction to the $Z$ resonance shape is sizeable, higher order corrections have to be considered. This had been done in the Physics at LEP report by Altarelli and Martinelli$^1$ and by Greco$^2$. The former authors present results for the first order corrections plus second order leading logarithmic correction whereas the latter takes higher order effects into account by exponentiating the infrared singular part of the first order correction.

Here we present an exact $O(a^2)$ calculation of the initial state radiative corrections. Results are given for the total cross section of the reaction

$$e^+ e^- \rightarrow \gamma, Z + \mu^+ \mu^-.$$  \hfill (1)

Once an event generator is available experimental cuts can be considered in the calculation.

Since we want to be able to make eventually an event generator for $O(a^2)$ corrections we have to improve on the ingredients of the $O(a)$ event generators. More specifically we need the following information:

1. The total cross section for (1) is needed with $O(a^2)$ virtual corrections and including single or double soft photon emission:

$$\sigma = \sigma^0 \left[ 1 + \delta^1(\epsilon) + \delta^2(\epsilon) \right]$$  \hfill (2)

where

$$\sigma^0(s) = \frac{4\pi a^2}{3s} \left[ 1 + \frac{2(s-m^2)sc}{|z(s)|^2} + \frac{s^2(c_v A^2 + c_s c_v^2)^2}{|z(s)|^2} \right]$$  \hfill (3)

$$z(s) = s-M^2 + iM\Gamma$$  \hfill (4)

$$\delta^1(\epsilon) = \beta \ln \epsilon + \delta^1_v$$  \hfill (5)

$$\beta = \frac{2\alpha}{\pi} \left( \ln \frac{s}{m^2} - 1 \right)$$  \hfill (6)

$$\delta^1_v = \frac{a}{\pi} \left( \frac{3}{2} \ln \frac{s}{m^2} + \frac{\pi^2}{3} - 2 \right)$$  \hfill (7)

The invariant mass of the $\mu$ pair defines $\epsilon$:

$$s' = m_{\mu \mu}^2 = s(1-\epsilon)$$  \hfill (8)
For single soft photon emission $\epsilon = k_1 / E$, where $k_1$ is the maximum soft photon energy. For double soft photon emission the maximum total photon energy is $\epsilon E$, neglecting $\epsilon^2$ terms.

The $O(\alpha^2)$ correction is

$$
\delta_2^v(\epsilon) = \frac{1}{2} \beta^2 \ln^2 \epsilon \ln \epsilon + \beta \delta_1^v \ln \epsilon + \delta_2^v
$$

where (the results originate from ref. 3)

$$
\delta_2^v = \frac{\alpha^2}{\pi^2} \left[-\frac{1}{16} \ln^3 \frac{s}{m^2} + \left[-\frac{119}{72} - 2\zeta(2)\right] \ln^2 \frac{s}{m^2}
\right. \\
\left. + \left[-\frac{2275}{432} + \frac{37}{6} \zeta(2) + 3\zeta(3)\right] \ln \frac{s}{m^2} - \frac{6}{3}[\zeta(2)]^2
\right. \\
- \frac{9}{2} \zeta(3) - 6\zeta(2) \ln 2 - \frac{79}{24} \zeta(2) + \frac{1279}{108}
$$

with $\zeta(2) = \pi^2/6$

$\zeta(3) = 1.20205$

For small $\epsilon$ it has been shown that the corrections can be summed into the form:

$$
\sigma = \sigma^0 (1 + \delta_1^v + \delta_2^v + \ldots) \epsilon^\beta
$$

Here we know $\delta_1^v$ and $\delta_2^v$, but not the higher order terms. We assumed $\epsilon$ to be so small that the energy loss does not effect $\sigma^0(\epsilon)$.

When the energy loss matters other formulae can be used. Since we will also consider hard bremsstrahlung, we can take $\epsilon$ as small as we want.

2. The second ingredient is $\frac{d\sigma}{d\Omega}$ in some form.

In lowest order it is just another form of the single bremsstrahlung spectrum

$$
\frac{d\sigma}{dk} = \beta \frac{1+(1-k)^2}{2k} \sigma_0(s^1) = \beta \frac{1+z^2}{2v} \sigma_0(s^1)
$$

where $kE$ is the photon energy.

Considering virtual corrections to single bremsstrahlung and adding the effect of the emission of a second photon, we get another distribution $\frac{d\sigma}{d\Omega}$, where $s^1 / s = 1 - v = z$, which is an order in $\alpha$ higher than eq. (12):
\[
\frac{d\sigma}{dv} = \left( \frac{1+4z^2}{v} \right) \left[ \frac{1}{2} \beta^2 \ln v + \frac{1}{2} \beta \delta_1 v + \left( \frac{z}{x} \right)^2 A \right] \\
+ (1+z) \left( \frac{z}{x} \right)^2 B + z \left( \frac{x}{z} \right)^2 C \sigma_0(s')
\]

(13)

The full distribution \(d\sigma/dv\) (lowest order plus first order correction) is the sum of eqs. (12) and (11). In eq. 13 we introduced the quantities:

\[
A = -\ln^2 \frac{x}{m} \ln z + \ln \frac{x}{m} \left[ \mathcal{L}_{12}(v) - \frac{1}{2} \ln^2 z \right] \\
+ \ln z \ln v + 7/2 \ln z + \frac{1}{2} \ln^2 z \ln v \\
+ \frac{1}{2} \mathcal{L}_{12}(v) \ln z - \frac{1}{6} \ln^3 z + \zeta(2) \ln z - \frac{3}{2} \mathcal{L}_{12}(v) \\
- \frac{3}{2} \ln z \ln v - \frac{17}{6} \ln^2 z + \frac{1}{6} \ln^2 z - \frac{1}{v} \ln^2 z \\
- \frac{1}{3} - \frac{2}{3v^2} \ln z - \frac{1}{3v^2} \ln^2 z
\]

(14)

\[
B = \ln^2 \frac{x}{m} \left( \frac{1}{2} \ln z - 1 \right) + \ln \frac{x}{m} \left[ \frac{1}{4} \ln^2 z - \ln z + \frac{7}{2} \right] \\
+ \frac{3}{2} \mathcal{L}_{12}(v) - 2 S_{12}(v) - \ln v \mathcal{L}_{12}(v) - \zeta(2) \\
+ \frac{1}{6} \ln^2 z + \frac{1}{2} \ln z \ln v - \frac{1}{4} \ln^2 v + \frac{5}{2} \ln z \\
+ \frac{3}{2} \ln v - \frac{1}{6}
\]

(15)

\[
C = 2 \ln^2 \frac{x}{m} + \ln \frac{x}{m} \left( \ln z - \frac{13}{2} \right) + \frac{16}{3} \zeta(2) \\
- \frac{25}{6} \mathcal{L}_{12}(v) - \frac{13}{12} \ln^2 z - 4 \ln z \ln v \\
+ \frac{3}{2} \ln^2 v - \frac{5}{6} \ln z - \frac{15}{6} \ln v - \frac{2}{3}
\]

(16)

These formulae are based amongst others on results of ref. 4. When \(v\) is small, or \(z = 1\) the \(1/v\) term dominates and in the coefficient of \(1/v\) \(A\) can be neglected. This result is the same as differentiating eq. (11) with respect to \(z\) and expanding the result up to \(O(x^2)\). The \(O(x)\) term is the same as eq. (12), the \(O(x^2)\) as eq. (13) in the \(z = 1\) limit.

We see however from eq. (13) that for larger \(v\) values the distribution deviates from the one implied by soft photon emission alone c.f. eq. (11).
By exponentiating the infrared parts we may write for \( \frac{d\sigma}{dv} \)

\[
\frac{d\sigma}{dv} = \beta \nu^2 \frac{1+\varepsilon}{2\nu} \left[ 1+\delta_1 + \delta_2 + \ldots \right] \sigma_0(s')
+ \left( \frac{\alpha s}{\pi} \right)^2 \left[ \frac{1+\varepsilon^2}{\nu} A + (1+\varepsilon)BzC \right] \sigma_0(s')
+ \left( \frac{\alpha s}{\pi} \right)^3 \ldots + \ldots
\]

(17)

By expanding eq. (17) in \( \alpha \) the 0(\( \alpha \)) term equals eq. (12) and the 0(\( \alpha^2 \)) equals eq. (13).

We now have the ingredients to calculate the radiative corrections to \( \sigma^0(s) \) for some energy \( s = 4g^2 \).

Up to 0(\( \alpha^2 \)) the corrected cross section is given by

\[
\sigma = \sigma^0 \left[ 1+\delta_1(\varepsilon) + \delta_2(\varepsilon) \right] + \int_{\varepsilon}^{1} \frac{d\sigma}{dv} dv,
\]

(18)

where \( \frac{d\sigma}{dv} \) is given by the sum of eqs. (12) and (13). The result of (18) is the cross section including an exact evaluation of 0(\( \alpha \)) and 0(\( \alpha^2 \)) initial state corrections, due to virtual and real photons. It should be noted at this point that in principle there is another 0(\( \alpha^2 \)) radiative correction namely due to the emission of an \( e^+e^- \) pair instead of a \( \gamma\gamma \) pair. We have omitted this contribution for the moment since it is small with respect to the double bremsstrahlung correction.

We evaluate (18) numerically by subtracting in the integrand \( d\sigma/dv \), a distribution which behaves like \( d\sigma/dv \) for small \( \nu \) and which can be easily integrated analytically.

In fact we evaluate (18) for a number of cases:

a. Only 0(\( \alpha \)), i.e. \( \delta_2(\varepsilon) = 0 \) and \( \frac{d\sigma}{dv} \) given by (12).

b. 0(\( \alpha \)) and 0(\( \alpha^2 \)), i.e. \( \delta_1 \) and \( \delta_2 \), \( \frac{d\sigma}{dv} \) given by the sum of eqs. (12) and (13).

c. An exponentiated form of 0(\( \alpha \)), i.e. the first term of eq. (18) is replaced by eq. (11) with \( \delta_2^\nu = 0 \) and \( \frac{d\sigma}{dv} \) is given by (17) with

\( \delta_2^\nu = 0 \) and the explicit \( \left( \frac{\alpha}{\pi} \right)^2 \) term omitted. In this case one could directly take \( \varepsilon = 0 \) because (17) is integrable.

d. An exponentiated form of 0(\( \alpha^2 \)), i.e. the first term of (18) is replaced by (11) and \( d\sigma/dv \) is replaced by (17).
Fig. 1: The total cross section for $e^+ e^- \rightarrow \mu^+ \mu^-$ as a function of the c.m.s. energy. The dash-dotted, dashed, and solid lines represent the Born approximation, the $O(\alpha)$ corrected, and up to $O(\alpha^2)$ corrected calculations.

Fig. 2: The total cross section for $e^+ e^- \rightarrow \mu^+ \mu^-$ around the top. The dotted line represents the $O(\alpha)$ corrected cross-section. The curve with large dashes represents the up to $O(\alpha^2)$ corrected cross section. The other dashed line is the $O(\alpha)$ exponentiated form (case c) whereas the solid line represents case d, the $O(\alpha^2)$ exponentiated expression.
The numerical results are shown for the resonance region in Figs. 1 and 2. Also results are given for an energy above resonance. Here $\frac{d\sigma}{dv}$ in lowest order is compared with a first order corrected $\frac{d\sigma}{dv}$ (Fig. 3).

In the region of the resonance the small $v$ region is most important in the integral, since $\sigma(s')$ falls off sharply with decreasing energy. So in this case the most important terms in (13) are the $\beta^2$ and $\beta$ terms. Since the first order exponentiated expression, listed under case c, takes these terms into account it is not surprising that the results do not differ very much quantitatively.

![Graph](image)

Fig. 3: The energy loss spectrum $\frac{d\sigma}{dv}$ in Born approximation (dashed line) and with $O(\alpha)$ corrections (solid line), at c.m.s. energy 100 GeV.

In fact a comparison between cases b, c and d gives an indication of the theoretical accuracy of the radiative corrections.

We find that for a peak value in nb. of 1.38 the uncertainty due to radiative corrections is 0.01 nb. For the position of the peak we find similarly an uncertainty of 15 MeV. The peak values for various assumptions are (in nb):

- lowest order: 1.86
- $O(\alpha)$: 1.32
- $O(\alpha^2)$: 1.38

The position is in these three cases respectively (in GeV): 93.020, 93.204, 93.116. All calculations were performed with $M_z = 93$ GeV, $\Gamma_z = 2.5$ GeV and $\sin^2\theta = 0.223$. 
REFERENCES

ANNEX B
SYSTEMATIC UNCERTAINTIES IN THE MEASUREMENT OF $A_{pol} (m_Z)$ WITH $\tau$ PAIRS
S. Jadach and Z. Was

The strategy of precision tests of the Standard Electro-weak Model is to predict theoretically the value of some precisely measurable quantities—including electroweak radiative corrections—using for example $\alpha$, $G_{\mu}$, and $m_\tau$ as input parameters, and to test their consistency with the measured values. Besides the left-right asymmetry $A_{LR}$, which offers the best sensitivity but requires polarized beams, and the forward-backward asymmetry $A_{FB}$, the $\tau$ spin polarization asymmetry $A_{pol}$ in the reaction $e^+e^- \rightarrow \tau^+\tau^-$ is of particular interest [1]. It offers the same sensitivity to ‘new physics’ as $A_{LR}$, and requires no beam polarization. Hence the measurement of $A_{pol}$ is of great importance although the precision of its measurement cannot match that of $A_{LR}$. All the quantities discussed are measured on top of the $Z$ resonance. This note discusses briefly the systematic uncertainties of the measurement of $A_{pol} (m_Z)$, in particular $O(\alpha)$ QED radiative corrections. The systematic uncertainties may be divided into three categories: i) QED bremsstrahlung, ii) imprecise knowledge of the physics of $\tau$ decays, and iii) contamination due to background from unwanted $\tau$ decay modes or other processes.

It was shown [2] that the value of $|P_{\beta}|$, in the case of unpolarized beams, is not directly affected by QED bremsstrahlung. In fact $|P_{\beta}|$ is changed by less than 0.005, very little in comparison with its value $|P_{\beta}| \approx 0.16$ and very little in terms of $\sin^2 \theta_{ew}$. One could expect a larger effect due to initial-state bremsstrahlung and the related reduction of $\sqrt{s}$ if $P_{\beta}$ was strongly dependent on $\sqrt{s}$. This is, however, not the case here. To some extent, however, this happens for polarized beams, see Ref. [2]. Let us note that this mechanism, owing to the strong dependence of $A_{FB}$ on $\sqrt{s}$, shifts $A_{FB}$ by $-3.5\%$, a very large amount as compared to $A_{FB} \approx 2\%$, and in terms of $\sin^2 \theta_{ew}$. The experimentally measured value of $|P_{\beta}|$ is influenced by QED bremsstrahlung indirectly, and in a different way from $A_{FB}$. The $|P_{\beta}|$ is measured ($|P_{\beta}|_{\text{meas}}$) using the slope of the pion energy distribution in the $\tau \rightarrow e\nu\bar{\nu}$ decay channel; owing to the loss in the c.m.s. energy $\sqrt{s}$ (both the initial- and the final-state bremsstrahlung contribute), the $\pi$ momentum distribution gets distorted (more steep) leading to $|P_{\beta}|_{\text{meas}}$ different from $|P_{\beta}|$ by about $-0.03$. This effect, when compared with $|P_{\beta}| = 0.16 (\sin^2 \theta_{ew} = 0.23)$, and in terms of $\sin^2 \theta_{ew}$, is much less dramatic than the shift of $A_{FB}$.

In conclusion, we would like to stress that in order to obtain $\sin^2 \theta_{ew}$ with comparable precision from $|P_{\beta}|$ and $A_{FB}$, the QED bremsstrahlung effect must be known in the case of $A_{FB}$ with much higher precision than for $|P_{\beta}|$. So far, no study has been done on the dependence of the QED bremsstrahlung effects on $|P_{\beta}|$ on kinematical cut-offs, but it seems that they should diminish with stronger cut-offs, as opposed to the situation in the case of $A_{FB}$. As for radiative QED corrections to the $\tau \rightarrow e\nu\bar{\nu}$ decay, to our knowledge there is nothing in the literature, but generally these effects are expected to be of the order of $\alpha \ln (m_\tau/m_e)$ and therefore very small. The strongest QED bremsstrahlung effect may show up in $\tau \rightarrow e\nu\bar{\nu}$ decay. Here many analytical results from $\mu \rightarrow e\nu\bar{\nu}$ may be used, but nothing in the form of a Monte Carlo event generator exists as yet.

Figure 1 shows a comparison of the $\pi$ momentum spectrum in $\tau \rightarrow \pi\nu$ decay, in Born approximation and after $O(\alpha)$ EWRC are applied. The cut-offs used in the latter case are rather strong, yielding closely similar values for $P_{\beta}$, if analysed in Born approximation, or after $O(\alpha)$ EWRC.

The main uncertainty due to the unknown aspects of the $\tau$ decay mechanism is related to the fact that the $V-A$ nature of $\tau$ decay is not confirmed by experiment precisely enough. Let us note that if $\tau$ decay is used as a spin polarimeter then one obtains as a result not really $|P_{\beta}|$ but rather $q/P_{\beta}'$ where $q = -2g_{V,A}/(g_V^2 + g_A^2)$, $g_{V,A}$ being charged-current couplings in $\tau$ decay. If $|P_{\beta}|$ is to be measured model independently then the value of $q$ must be

*) This is countered by another contribution from the interference of initial- and final-state bremsstrahlung, which changes $A_{FB}$ typically by $+1.5\%$ to $+3\%$, depending on cut-offs.
known from independent sources. Most probably the experimental error on $q$ will still be large by the time of the LEP experiments, and we will have to assume $q = 1$.

The background problems depend on the quality of the detector and we are not entering into these questions. Here we limit ourselves to some rather general and detector-independent aspects of the background problems. First, it should be noted that, in comparison with PETRA/PEP experiments, backgrounds from other processes, namely from Bhabha and $\gamma\gamma$ physics, are fortunately much less important. On the other hand, the problem of the contamination of one $\tau$-decay channel by another will look similar. The common assumption that it is not worth including the decays $\tau \to e\nu\bar{\nu}$, $\mu\nu\bar{\nu}$, $q\bar{q}$ as spin analysers, because the reduction of the statistical error on $<P_0>$ is less than a factor of 2, might have to be reconsidered. The inclusion of these other $\tau$ decay modes in the $<P_0>$ measurement may significantly reduce not only the statistical error but also the overall systematic error [3].

For the analysis of $\tau$ pair events, a Monte Carlo generator is required which includes all the features discussed above. A good step in this direction has already been made [4], and some results presented here were based on the results from this program. However, it is still not complete and requires inclusion of some missing $O(\alpha)$ non-QED corrections, of some multipion decay modes of $\tau$, and a more refined QED bremsstrahlung. A study of the impact of changes of the Higgs mass, etc., on $<P_0>$ is in progress.

REFERENCES


ANNEX C
ELECTROWEAK ONE-LOOP CORRECTIONS TO HEAVY-FERMION PAIR PRODUCTION
W. Beenakker and W. Hollik

The one-loop corrections to \( e^+ e^- \rightarrow \bar{F} F \) for charged heavy fermions \( F \) can be classified in the following way:

i) QED corrections, consisting of initial- and final-state bremsstrahlung, together with the virtual photon corrections: vertex corrections and box diagrams with internal photon lines. It is sometimes convenient to include the fermionic part of the photon vacuum polarization in this subclass. These corrections have been calculated in Refs. [2-4] and, partly, in Ref. [1].

ii) Non-QED corrections (sometimes also referred to as 'purely weak corrections'), consisting of the Z boson self energy, the \( \gamma-Z \) mixing energy, vertex corrections not from virtual photon exchange, and massive ZZ and WW box diagrams [5-10].

For a completion of the fermionic sector in the Electroweak Standard Model the top quark has still to be discovered experimentally. In the case of a fourth generation also a new sequential heavy lepton may be around in the energy range covered by LEP II. For the experimental investigation of cross-sections, forward–backward asymmetries, polarization asymmetries, etc., and for comparison with the model predictions, it will become necessary to extend the radiative-correction calculations to the case of a heavy fermion pair in \( e^+ e^- \rightarrow \bar{F} F \).

Finite mass effects give drastic reductions of cross-sections and asymmetries if one is near to threshold. But also the calculation of radiative corrections becomes more cumbersome for the following reasons:

i) The evaluation of the vertex and box diagrams with heavy fermions yields lengthy expressions which are less transparent than in the light fermion case, where one has only vector and axial-vector form factors in a compact and handy form (see, for example, Ref. [11]).

ii) The coupling of Higgs bosons to heavy fermions is no longer negligible. Therefore one has to respect the Higgs contributions in the vertex corrections and fermion self energies. For a renormalizable gauge also with unphysical Higgs bosons \( \phi^* \), \( \chi \), the additional vertex and fermion self-energy diagrams are depicted in Figs. 1

\[ \text{Fig. 1} \] Vertex corrections from the Higgs bosons H, \( \phi^* \), and \( \chi \).
Fig. 2  Fermion self-energy diagrams from the Higgs bosons H, φ⁺, and χ.

and 2. In box diagrams the exchange of virtual Higgs bosons can again be neglected since they have to couple also to the electron line leading to a suppression factor m_e/m_w. The renormalization is performed in the on-shell scheme, as described in detail in Ref. [11] for the light fermion case, now extended to the heavy fermion one [12].

The renormalization conditions are the on-shell subtractions of the self energies, the vanishing of the γ–Z mixing for on-shell photons, and the definition of the electric charge in the Thomson limit. The input quantities are therefore the physical masses

m_w, m_z, m_H, m_f

and the fine structure constant α in the Thomson limit.

In Figs. 3–5 we display the case of ττ productions with either m_τ = 40 GeV or m_τ = 50 GeV in terms of the integrated cross-section σ and the forward–backward asymmetry

\[ A_{FB} = \frac{1}{σ} \left[ ∫_{cosθ > 0} dσ - ∫_{cosθ < 0} dσ \right], \quad θ = θ(τ^+, τ^-). \]

Fig. 3 Cross-section ratio \( σ(e^+e^- → ττ)/σ_e \), \( σ_e = \frac{4πα^2}{3s} \), \( m_τ = 40 \text{ GeV} \).

--- Born

O(α) corrected
Fig. 4 Cross-section ratio $\sigma(e^+e^- \rightarrow \bar{t}t)/\sigma_{\text{point}}$, as a function of $\sqrt{s}$, in Born approximation and after $O(\alpha)$ corrections are applied ($m_t = 50 \text{ GeV}$).

Fig. 5 Forward-backward asymmetry of $e^+e^- \rightarrow \bar{t}t$, as a function of $\sqrt{s}$, in Born approximation and after $O(\alpha)$ are corrections applied, for $m_t = 40 \text{ GeV}$ and $m_t = 50 \text{ GeV}$.
As in the case of a light fermion pair the weak corrections are dominated by the $Z$ boson self energy, where the largest part could be absorbed by defining a running $\alpha(s)$

$$\alpha(s) = \alpha(0)/(1 - \Pi_\text{QED}(s))$$

with the QED photon vacuum polarization $\Pi_\text{QED}$. The residual corrections are of the order of 1–2% depending slightly on the Higgs mass. They can best be exhibited on top of the $Z$ peak (if $m_t < m_Z/2$), where the large $Z$ boson self energy vanishes owing to the on-shell renormalization condition.

Large corrections arise from the QED subclass, in particular from initial-state bremsstrahlung together with the virtual photonic vertex correction. The dominant initial-state radiation is the same as in the case of light fermion production. Since the QED corrections depend on the experimental cuts they have not been included in Figs. 3–6.

REFERENCES

ANNEX D
RADIATIVE Z PRODUCTION AT LEP I
F. Boudjema, J. Cole and N. Dombey

1. BORN APPROXIMATION

At tree level the $e^+e^- \rightarrow Z\gamma$ cross section is given by

$$\frac{d\sigma}{dx} = \frac{s^2 F(s_u)}{s^2 (s - M_Z^2)^2} \left[ 1 - \frac{1}{s} x^2 - \frac{1}{2} \frac{2s_u^2 - s_u + 1}{s_\nu (1 - s_u)} \right]$$

where $x = \cos^2 \theta$, $s_u = \sin^2 \theta_w$, $F(s_u) = \frac{2s_u^2 - s_u + 1}{s_\nu (1 - s_u)}$

and $v = -(1 - 4m_e^2/s)$, neglecting the electron mass in the denominator.

2. RENORMALISATION SCHEME

We outline the renormalisation scheme (RS) we have used in the calculation of the radiative corrections (RC) to the process $e^+e^- \rightarrow Z\gamma$.

In this paper we use the RS based on Ref.[1]. The merits of this scheme are

(i) it is an on-shell, gauge invariant scheme,

(ii) all Green's functions calculated in this scheme are finite, enabling their values to be taken from previous calculations,

(iii) the counterterms are readily divided into photonic and weak parts enabling the corresponding Green's functions to be similarly divided.

The outline of the scheme is as follows. First the parameters of the electroweak Lagrangian are rescaled, generating counterterms in the usual way. Then renormalisation conditions are chosen such that the renormalised masses are equal to the physical masses. The value of the Weinberg angle is defined by $\cos^2 \theta_w = M_\nu / M_Z$.

The other coupling is the renormalised electric charge is equal to the physical value determined at low energy by Thomson scattering. In other words the values of $M_\nu$, $M_Z$ and $\alpha$ are used as input parameters. In addition the scheme gives us rules for renormalising external lines to any diagram. The details may be found in Ref.[1].

The RC for the processes we are considering can be divided into several groups. First we can separate the photonic and weak corrections defined in the following way: photonic RC are those diagrams involving either virtual photons or Bremsstrahlung diagrams whereas weak RC are any other diagram. Our scheme enables each set to be calculated as both infrared (IR) and ultraviolet (UV) finite.
Next the virtual RC of each of these groups is divided into subgroups consisting of

(i) electron self energy corrections (Fig.1).
(ii) vertex corrections (Fig.2)
(iii) box diagrams (Fig.3)
and two groups which apply only to the weak corrections
(iv) boson self energy corrections and the $Z\gamma$ mixing amplitude (Fig.4)
(v) fermion triangle diagrams (Fig.5)

Fig. 1: Electron self energy corrections
Fig. 2: Vertex corrections
Fig. 3: Box diagrams
Fig. 4: Boson self energy corrections, and the $Z\gamma$ mixing amplitude
Fig. 5: Fermion triangle diagrams

Fig. 6: W self energy diagram

--- $Z$
--- $\gamma$
--- $e$
Unless marked otherwise
This last group of diagrams requires additional consideration since they are diagrams which generate the well-known ABJ anomalies, although they are finite and so are not scheme dependent in the usual way. The treatment of such diagrams is given in Ref.[2] using symmetry arguments and in Ref.[3] using dimensional regularisation.

3. WEAK CORRECTIONS

A large RC comes, perhaps surprisingly, from the weak sector. This is the contribution of the $Z^0$ self-energy (see Ref.[1]) which is of order 7%. This diagram has a large value because of the fact that its counterterm contains the value of the unrenormalised photon self energy evaluated at $q^2 = 0$. This is essentially because the $Z^0$ and the photon are renormalised at different scales and yet are components of the same boson in the SU(2)xU(1) Lagrangian. Since $M_z$ is taken as an input parameter the $Z^0$ is renormalised at the energy scale $M_z$ whereas $\alpha$ is renormalised at zero energy.

Should we have a scheme in which $\alpha$ was renormalised at a scale $M_z$, the calculated value of the $Z^0$ self energy would not be large. Thus, as emphasised by Altarelli [4], this large weak correction is equivalent (at least to leading logarithm) to the running of the electromagnetic coupling constant $\alpha$. Calculation of the RC to muon decay gives the same term in the W self energy diagram of Fig. 6. This in turn affects the calculation of the Fermi constant $G_{\nu}$ (cf the err of Sirlin [5]). So, if the formula for the differential cross section is written down in terms of $G_{\nu}$, for example in $e^+e^- \to Z\gamma$ we have

$$\frac{d\sigma}{dx} = \frac{G_{\nu} M_z^2}{s^2} \left[ \frac{F(s_{\nu})}{s^2(s-M_Z^2)} \right] \left[ \frac{2s^2 + M_\pi^4}{1 - v^2 x^2} - (s-M_Z^2)^2 \right]$$

where $F'(s_{\nu}) = \sqrt{2} F(s_{\nu}) s_{\nu} (1-s_{\nu})$

We have also calculated the weak corrections to $e^+e^- \to Z\gamma$. Apart from the scheme dependent large correction mentioned above (which becomes small if $G_{\nu}$ is used for the differential cross section formula) the weak corrections are small (of order 1%) in the standard model.

4. ELECTROMAGNETIC CORRECTIONS

We show in Ref.[6] that there is a low energy theorem for this process. This allows for a simple derivation of the IR contribution near threshold (where LEPI will be operating) as well as giving the
dependence of the threshold amplitudes in $\ln(M_g/m)$ where $m$ is the electron mass. Adding the soft bremsstrahlung contribution in the usual way leads to the full one loop radiative correction

$$d\sigma/dx = d\sigma/cd(1-\alpha/m)(\ln(s/m^2)\ln(s/4\gamma_0)-3\ln(M_g/m))$$

where $\gamma_0$ signifies the photon energy resolution.

REFERENCES

ANNEX E
EXPECTED COUNTING RATES FOR $e^+e^- \rightarrow (e^+e^-)\gamma$
M. Caffo

The availability of the neutral vector boson $Z^0$ has open the possibility of counting the number of light neutrinos. A definite measurement of this number could be done in LEP stage 1 experiments through the investigation of the process

$$(1) \quad e^+e^- \rightarrow \nu\nu\gamma$$

whose cross section is dominated by the $Z^0$ diagrams [1]. Signal (1) has to compete with a large background due to the process

$$(2) \quad e^+e^- \rightarrow (e^+e^-)\gamma$$

for the kinematical configuration in which the final positron and electron go undetected in forward and backward opening cone around the beam line, and the emitted photon is detected in an angular range $\theta_{\gamma}^{\text{min}} < \theta_{\gamma} < (180^\circ - \theta_{\gamma}^{\text{min}})$, where $\theta_{\gamma}^{\text{min}} > \theta_{\text{cone}}$.

The knowledge of this background could be relevant also at higher energies where processes of the kind

$$(3) \quad e^+e^- \rightarrow X\gamma$$

where $X$ is missing energy-momentum, could take place.

An accurate description of the kinematics and of the cross section for the process (2) is therefore given and recent results are reviewed.

In the following we shall use the notation:

$$(4) \quad e^-(p_1)e^+(p_2) \rightarrow e^-(q_1)e^+(q_2)\gamma(k)$$

electron mass: $m_e$;

electromagnetic coupling constant: $\alpha$;

beam energy: $E$, $s = -(p_1 + p_2)^2 = 4E^2$, $p^2 = E^2 - m^2$;

photon energy: $\omega$, $x = \omega/E$, $s_1 = -(q_1 + q_2)^2 = s(1-x)$;

photon angle with the direction of the incident electron: $\theta_{\gamma}$;

y = $\cos\theta_{\gamma}$;

half opening angle of desapparance cone: $\theta_{\text{cone}}$;

minimum angle of photon detection: $\theta_{\gamma}^{\text{min}}$;

final electron energy: $E_e$, $p_e^2 = E_e^2 - m_e^2$;

final electron angle with the direction of the incident electron: $\theta_e$;

azimuthal angle of final electron: $\phi_e$;

angle between photon and final electron directions: $\theta_{e,\gamma}$;

other invariant variables:

$t = -(p_2 - q_2)^2$, $t_1 = -(p_1 - q_1)^2$, $u = -(p_2 - q_1)^2$, $u_1 = -(p_1 - q_2)^2$,

$k_+ = -p_1$, $k_+ = -p_2$, $k_+ = -q_1$, $k_+ = -q_2$.

In the kinematical configuration relevant for the discussion of the background the invariant quantities $s, s_1$ and the absolute values of $u, u_1$ are always large and the Fermion propagators $k_{\pm}, h_{\pm}$ are never too small in absolute value because of the restriction on the photon angle. On the contrary the other two invariant variables $t$ and $t_1$ can reach very small values: in a typical configuration they can run over 15 orders of magnitude. So an accurate study of their behaviour is crucial for a correct integration. The variables $t$ and $t_1$ are prevented from vanishing by the non zero values of the photon energy $\omega$ and angle $\theta_{\gamma}$ and can be appropriately written as function of the variables $x, \theta_{\gamma}, \theta_{e}$ and $\phi_e$. The expression for $t_1$ is
\[ t_1 \approx -m_e^2 \frac{x^2 E(1 + \cos \theta_{e,\gamma})^2}{4 E_e (1 - \frac{x}{2}(1 - \cos \theta_{e,\gamma}))^2} - 4E E_e \sin^2 \left( \frac{\theta_e}{2} \right) \]

where

\[ E_e \approx \frac{E(1 - z)}{1 - \frac{x}{2}(1 - \cos \theta_{e,\gamma})} + \frac{m_e^2 \cos \theta_{e,\gamma}}{4E(1 - z)} \]

\[ \cos \theta_{e,\gamma} = \cos \theta_e \cos \theta_\gamma + \sin \theta_e \sin \theta_\gamma \cos \phi_e \]

The very steep dependence on \( \theta_e \) in the second term of \( t_1 \) is very clear, but the first term, although very small is never vanishing for the indicated conditions. A corresponding expression holds for \( t \), when the electron variables are consistently replaced by the positron ones. Therefore, when the photon is radiated in the positron hemisphere (\( \cos \theta_{e,\gamma} < 0 \)) the minimum of the absolute value of \( t_1 \) is much lower than the minimum of the absolute value of \( t \), although for values of the electron angles

\[ \theta_e \approx 2 \arctg \frac{x \sin \theta_\gamma}{2 [1 - \frac{x}{2}(1 + \cos \theta_\gamma)]}, \quad \phi_e = \pi \]

the positron lies on its initial direction. Obviously, when the photon is radiated in the electron hemisphere (\( \cos \theta_{e,\gamma} > 0 \)) the roles of \( t \) and \( t_1 \) are exchanged.

As a last observation the value of \( \theta_e \) in Eq. (7) shows that, for not too small values of \( x \) and \( \theta_\gamma \), the absolute values of \( t \) and \( t_1 \) are not simultaneously pathologically small.

In this kinematical configuration, therefore, the dominant contribution is given by the t-channel with virtual photon exchanged, so that the calculation for the background actually reduces to the evaluation of the QED part only. Indeed the contributions coming from \( Z^0 \) exchange have been computed and found to be negligible [7,9]. For hard (\( \omega > m_e \)), non collinear (\( \theta_\gamma >> m_e/E_e \)) photons the QED bremsstrahlung formula simplifies into [2,3]

\[ \frac{d^2 \sigma_{QED}}{dx dy} = \frac{\alpha^3}{\pi s} \int_{	heta_{e,\gamma}}^{\theta_{e,\gamma}} \sin \theta_e d\theta_e \int_{0}^{2\pi} d\phi_e \frac{E E_e x}{2 - x(1 - \cos \theta_{e,\gamma})} X_0, \]

\[ X_0 = \frac{1}{4} A \ W_{IR} \]

\[ A = \frac{ss_1(t^2 + t_1^2) + tt_1(t^2 + t_1^2) + uu_1(u^2 + u_1^2)}{ss_1tt_1} \]

\[ W_{IR} = \left[ \frac{s}{k_+ k_-} + \frac{s_1}{h_+ h_-} - \frac{t}{k_+ h_+} - \frac{t_1}{k_- h_-} + \frac{u}{k_+ h_-} + \frac{u_1}{k_- h_+} \right] \]

Note the lucky absence of double poles in \( t \) and \( t_1 \), which should be expected for a square matrix element. To obtain the background the integration of Eq. (8) has to be done also for the above specified range of \( y = \cos \theta_\gamma \) and for some chosen range of \( x = \omega/E \).

The whole operation requires a numerical treatment and much caution, even if the angle of the emitted photon is large enough for avoiding collinear singularity problems.

First attempts used an available Bhabha scattering Monte Carlo program [3], which requires a minimum scattering angle of both the final states \( e^+ \) and \( e^- \), whereas the photon angle is unrestricted. The program is claimed to work correctly down to minimum scattering angles of 0.001 degrees. However, since the cross section is peaked very much near zero angle, the Monte Carlo generation technique becomes extremely inefficient and is limited by the available computer time. While for large photon energies results [4] in agreement with the more recent ones were obtained, for small photon energies such results [5] turned out to be too small. In Ref. [5] a sequence of minimum scattering angles were chosen and plotted against the number of accepted single photon events in order to obtain an extrapolation to zero minimum scattering angle. While this procedure is in principle correct, it suffers very much from lack of statistics and hence does not allow a precise extrapolation.
A different attempt with a newly written Monte Carlo program and using a square matrix element keeping also the electron mass gave too large values, but a revised version seems to give results more in shape [6].

An approach that overcomes all such difficulties has been developed [7] using the Monte Carlo program RIWAD [8]. It consists on integrating only on a reduced region of the phase space, where the photon energy \( \omega \), the photon angle \( \theta_\gamma \) and the electron angles \( \theta_e \) and \( \phi_e \) are good integration variables, and then to exploit the symmetry of the integrand to recover the full value of the integral. In fact, as the differential cross section for photon emission is symmetric for \( \theta_\gamma \rightarrow (180^\circ - \theta_\gamma) \), the integral over \( \theta_\gamma^\text{min} < \theta_\gamma < 90^\circ \) is equal to the integral over \( 90^\circ < \theta_\gamma < (180^\circ - \theta_\gamma^\text{min}) \). In this second range of \( \theta_\gamma \) the integration variables accurately map the peak due to \( \frac{1}{t_{11}} \); less well they map the secondary peak due to \( \frac{1}{t_{11}} \), but this contribute much less to the integral. One integrates only in the region \( 90^\circ < \theta_\gamma < (180^\circ - \theta_\gamma^\text{min}) \) and then one multiplies the result by a factor of two. With this prescriptions the indicated integration variables are suitable for properly accounting for the dominant contribution of the region of very small values of \( \theta_e \).

Furthermore Eq. (8) is so compact that numerical contributions can be kept under control in every situation. In the expression for \( X_0 \) the square bracket in \( A \) is composed of terms which are always positive and so can not originate loss in precision. On the contrary the expression for \( W_{\nu \gamma} \) in the very peaked region of small \( t \) and \( t_1 \) exhibits quite large cancellations, checked to be of 8 digits at most. Since VAX double-precision arithmetic with 15 decimal digits has been used, this can not spoil the requested precision of one percent.

All \( m_e^2 \) terms have been dropped from the numerator in Eq. (8), but they were kept in the careful evaluation of the kinematical variables. It was also observed that no term with \( \frac{1}{t_{11}^2} \), which has the leading behaviour in the chosen configuration, is present in \( X_0 \). An explicit calculation, done by subtracting from the square matrix element exact in \( m_e \) the value of \( X_0 \), shows that, in the considered kinematical region (where the electron is almost aligned with the beam line and the photon in the positron emisphere) the leading \( m_e^2/t_{11}^2 \) term is given by

\[
X_m = -\frac{m_e^2}{2t_{11}^2} \left[ \frac{s^2 + u^2}{k_+^2} + \frac{s^2 + u^2}{k_-^2} + \frac{t^2 - 2s_1 - 2u_1}{k_+^2} \right],
\]

an analogous formula holding for the leading \( m_e^2/t_{11}^2 \) term in the symmetric situation with the gamma in the electron emisphere.

\( X_m \) is very strongly peaked at \( \theta_e = 0 \), where it almost compensate the \( X_0 \) peak, and its contribution to the cross section has been approximately evaluated analytically and is always negative, usually very small (a few percent at most) against that of \( X_0 \), to which it has to be summed, and its contribution to the cross section is

\[
\sigma_m^{QED}(x_{\text{max}}) - \sigma_m^{QED}(x_{\text{min}}) = \frac{\alpha^2}{s} 2 \left\{ \frac{2\pi(x - 4) + 8 \ln(x)}{y_{\text{max}}(y_{\text{max}} - 1)} \right\}^{x_{\text{max}}} - x(x - 2) \ln(1 - y_{\text{max}}) + 2Li_2 \left( \frac{1 + y_{\text{max}}}{1 - y_{\text{max}}} \right) - 2Li_2(x - 1) \right\}^{x_{\text{min}}},
\]

with \( y_{\text{max}} = \cos \theta_\gamma^\text{min} \) and \( Li_2 \) the Euler dilogarithm, the limit of applicability being

\[
\omega < \frac{\sin \theta_\gamma^\text{max}}{\sin \theta_\gamma^\text{min}}.
\]

For values of \( \omega \) larger than in Eq. (11), the value of Eq. (10) is however an upper limit. Numerically the \( X_m \) contribution amounts to a few percent of the \( X_0 \) contribution at most.

One has to note that this method uses very simple formulae, some known for a long time and all very easy to check. In the numerical treatment the region of integration is accurately described and the very limited number of terms allows a good control of precision loss.
For annihilation-channel dominated processes like $e^+e^- \rightarrow \nu\nu\gamma$ the choice of $\alpha$ renormalized at the square $Z^0$ mass ($\alpha = 1/128.5$) is the obvious one. On the contrary for the amplitude of the process $e^+e^- \rightarrow (e^+e^-)\gamma$, strongly peaked corresponding to extremely small values of the momentum transfers $t,t'$ (exceptional momenta in the terminology of renormalization group), the value $\alpha = 1/137$ seems to be more natural. In the first of Refs. [7] the value $\alpha = 1/128.5$ was used for simplicity for both the neutrino counting reaction and the electromagnetic background. Guessing that the use of $\alpha = 1/137$ is more appropriate for the electromagnetic background, due to the dominance of the lowest transferred momenta, one has only to uniformly decrease the cross-section by the constant factor 0.825.

Recently some other calculations of the process $e^+e^- \rightarrow (e^+e^-)\gamma$ have been completed. One [9] is done with the helicity amplitude technique and with a mapping of the phase space variables in order to absorb the peak of the square matrix element. According to Ref. [9] a discrepancy of 10-20 % for photon energy over 1 GeV and larger for smaller photon energy exists with the values reported in the first of Refs. [7]. A part from the different choice for the value of $\alpha (\alpha = 1/137$ in Ref. [9]), the discrepancy is no longer claimed after the Aachen Conference. Finally an event generator program [10] and a Monte Carlo program [11] have provided results in agreement with Ref's [7].

In conclusion the results of Refs [7], confirmed by Refs [9,10,11] fix the values for the background process $e^+e^- \rightarrow (e^+e^-)\gamma$ in Born approximation.

However in Ref. [10] is given also an estimation for the order $\alpha$ correction which comes out to be larger than the lower order. As this estimation is obtained by keeping only the contributions supposed to be dominant, a complete calculation is required and a cross-check would not be superfluous. A confirmation of this result implies a reevaluation of the proposed experiments on neutrino counting.

References.
ANNEX F

O(α3)W MASS SHIFT FROM A VERY HEAVY TOP QUARK

B.W. Lynn, D. Kennedy and C. Verzegnassi

One of the most sensitive tests of the Standard Model\(^1\) (and of electroweak theories in general) to one loop level will be the precision measurement of the W mass to better than 1% accuracy. As is known, the latter is related to the Fermi constant, the Z0 mass and the electric charge by Sirlin’s one-loop formula:\(^2\)

\[
M_W^2 \left[ 1 - \frac{M_W^2}{M_Z^2} \right] = \frac{\pi \alpha}{\sqrt{2} G_F} \frac{1}{1 - \Delta_r},
\]

where \(\Delta_r\) is the radiative correction, evaluated to one loop. \(\Delta_r\) contains the still unknown parameters \(M_{Higgs}\) and \(M_{top}\), so that its numerical value can only be given for fixed values of these quantities. Normally, one assumes \(M_t \simeq 30\) GeV, \(M_H \simeq 100\) GeV and finds\(^3\)

\[
\Delta_r (M_t = 30\text{ GeV}, M_H = 100\text{ GeV}) \simeq 0.07.
\]

In practice, this important correction stems mostly from oblique corrections, particularly fermionic vacuum polarization diagrams. More precisely, the value of Eq. (2) is mainly determined by renormalization of the running electric charge where in Euclidean metric with \(q^2 = q^2 - q_0^2 = -M_Z^2\)

\[
\alpha_{em}(-M_Z^2) = \frac{\alpha(0)}{1 - \Delta_\alpha(-M_Z^2)},
\]

with \(\alpha^{-1}(0) \simeq 137.036\) and \(\Delta_\alpha(-M_Z^2) \simeq 0.06\). Actually, one can write

\[
\Delta_r = \Delta_\alpha(-M_Z^2) - \frac{c_\rho^2}{s_\rho^2} \Delta_\rho(0) + \text{small contributions},
\]

where \(c_\rho = M_W/M_Z, s_\rho^2 = 1 - c_\rho^2\). The parameter \(\Delta_\rho(0)\) gives the correction to the \(\rho\) parameter

\[
\rho = 1 + \Delta_\rho(0),
\]

and, if the top mass is equal to 30 GeV, \(\Delta_\rho(0)\) is sensibly smaller than \(\Delta_\alpha(-M_Z^2)\).

In fact, \(\Delta_\alpha(-M_Z^2)\) gives the leading logarithmic contribution \(\sim \ln(M_Z^2/m_t^2)\) to \(\Delta_r\). This is not the case of \(\Delta_\rho(0)\), which is quadratic in the fermionic mass and proportional to \(m_t^2/M_Z^2\). Thus for \(m_t^2/M_Z^2 \ll 1\) one can discard \(\Delta_\rho(0)\) and approximate \(\Delta_r\) by its leading logarithmic term \(\Delta_\alpha\). In this case, renormalization
group arguments first introduced by Marciano and Sirlin\cite{Marciano} allow us to compute next order effects in Eq. (1) by simply expanding the $\Delta_\alpha$ content of $\Delta_r$ through the related geometrical series. Thus, one easily computes the contribution to leading log to Eq. (1) from $O(\Delta_\alpha^2)$ and finds that it is small; \textit{i.e.}, much smaller than the $O(\Delta_\alpha)$ term. This is a welcome indication that, as far as Eq. (1) is concerned, assuming $m_t \simeq 30$ GeV, higher order effects can probably be neglected.

The situation might be rather different if the top quark turned out to be substantially heavier; \textit{e.g.}, of the order of $\simeq 200$ GeV. This is still not ruled out by the existing experimental evidence. A straightforward computation shows that in that case the numerical contribution of $\Delta_\rho(0)$ to Eq. (4) becomes almost of the same size (and opposite) to that of $\Delta_\alpha(-M_Z^2)$:

$$-\frac{c_\beta}{s_\beta} \Delta_\rho(0) \bigg|_{m_t=200 \text{ GeV}} \simeq \frac{c_\beta^2}{s_\beta^2} \left[ \frac{3\alpha}{16\pi s_\beta^2 c_\beta^2} \frac{m_t^2}{M_Z^2} \right] \simeq -0.05 \quad . \quad (6)$$

If this were the case, one would have strong motivation to fear that next order contributions to $\Delta_r$, \textit{e.g.}, of the kind $\simeq \Delta_\rho^2(0)$ and $\Delta_\alpha(-M_Z^2)\Delta_\rho(0)$ might be relevant. Since these contributions are not of the leading logarithmic kind, their coefficient will differ from that of $\Delta_\rho^2$. In this case it is not correct to expand Eq. (1) including terms $\simeq (\Delta_r)^2$ with $\Delta_r$ given in Eq. (4). The relevant terms must be evaluated by application of perturbation theory to the proper oblique corrections contributions involving the various vacuum polarizations in a renormalization scheme independent way. We have done this starting from a general approach which evaluates higher order corrections which will be illustrated elsewhere.\cite{Sirlin} Here we only deal with the specific case of the $O(\alpha^2)$ heavy top corrections to the precise $W^\pm$ mass which will be of special interest for the $W$ mass measurement to be carried through at LEP II.

Here we work in the renormalization scheme which uses $\alpha(0)$, the muon lifetime coefficient, $G_\mu(0)$ and the physical $Z^0$ mass $M_Z$ as physical input parameters and start from the coupled Dyson's equations for the various gauge bosons propagators:

$$G_{WW} = \frac{1}{M_W^2 + q^2 - \pi_{WW}(q^2)} ,$$

$$G_{ZZ} = \frac{1}{M_Z^2 + q^2 - \pi_{ZZ}(q^2) - \frac{\pi_{AA}(q^2)}{q^2 - \pi_{AA}(q^2)}} ,$$

(7)
\[ G_{AA} = \frac{1}{q^2 - \overline{\pi}_{AA}(q^2) - \frac{q^2 \overline{\pi}_{AA}(q^2)}{M_Z^2 + q^2 - \overline{\pi}_{ZZ}(q^2)}} , \]

\[ G_{ZA} = \frac{\overline{\pi}_{ZA}(q^2)}{[q^2 - \overline{\pi}_{AA}(q^2)] [M_Z^2 + q^2 - \overline{\pi}_{ZZ}(q^2)] - \frac{1}{2} \overline{\pi}_{ZA}(q^2)} , \]

where the \( \overline{\pi}_{ij} \)'s are the 1PI vacuum polarizations for vector bosons \( i,j = W^\pm, Z, A \) (photon) which we write as

\[ \overline{\pi}_{ij} \equiv \pi_{ij} + \text{counterterms} , \tag{8} \]

with \( \pi_{ij} \) calculated with the bare coupling constants. The specific choices of physical parameters are then used to fix the numerical value of different quantities which enter the oblique radiative corrections. In particular, we find:

\[ \text{Re} \left( \frac{\overline{\pi}_{ZA}(-M_Z^2)}{M_Z^2} \right) = \frac{\Delta_\rho(-M_Z^2)}{s_\rho c_\rho} \left[ 1 + \frac{\Delta_\rho(-M_Z^2)}{1 - 2s_\rho^2} + O(\alpha^2) \right] ; \]

\[ \Delta_\rho(-M_Z^2) \approx \frac{s_\rho^2 c_\rho^2}{1 - 2s_\rho^2} \left[ \Delta_\rho(-M_Z^2) - \Delta_\rho(0) \right] . \tag{9} \]

Defining the \( W \) mass as the pole of the \( W \) propagator and using consistently Eq. (7) leads us then to the following result:

\[ M_W^2 \approx \hat{s}_\rho \frac{M_Z^2}{2} \left\{ 1 - \frac{s_\rho^2}{1 - 2s_\rho^2} \left[ \Delta_\rho(-M_Z^2) - \frac{c_\rho^2}{s_\rho^2} \Delta_\rho(0) \right] \right. \]

\[ - \frac{s_\rho^2 (1 - 3s_\rho^2 + 3\hat{s}_\rho^2)}{(1 - 2s_\rho^2)^3} \Delta_\rho^2(-M_Z^2) + \frac{c_\rho^2 (1 - 3\hat{s}_\rho^2)}{(1 - 2s_\rho^2)^3} \Delta_\rho(0) \]

\[ + \frac{2s_\rho^2 \hat{s}_\rho^2 \Delta_\rho(-M_Z^2) \Delta_\rho(0)}{(1 - 2s_\rho^2)^3} \right\} ; \tag{10} \]

\[ \hat{s}_\rho \equiv 1 - \delta_\rho^2 = \frac{1}{2} \left( 1 + \sqrt{1 - \frac{4\alpha \pi}{\sqrt{2} G_\mu M_Z^2}} \right) . \tag{11} \]

which allows us to compute those effects coming from a heavy top quark to \( O(\alpha^2) \) (one loop \( \times \) one loop) terms.\(^{11}\)

Note that the coefficient of \( \Delta_\rho^2 \) in Eq. (10) is, as we expected from Marciano and Sirlin's arguments, that which corresponds to the geometrical series

\(^{11}\) One particle irreducible two-loop effects within \( \Delta_\rho(0) \) have been computed and found to be negligibly small.
expansion of the $\Delta_{\alpha}$ content of $1/(1 - \Delta_{\tau})$. But the coefficients of $\Delta_{\alpha}^2$ and of $\Delta_{\alpha} \Delta_{\tau}$ are, as one might expect, quite different. For a top quark mass of 200 GeV, we find from Eq. (11)

$$\begin{align*}
\Delta M_{W}^{\text{top; } \mathcal{O}(a^2)} & \approx +18 \text{ MeV ,}
\end{align*}$$

and of these $\sim 18$ MeV, $\sim 10$ come from the interference $\sim \Delta_{\alpha} \Delta_{\tau}$, while $\sim 8$ come from $\Delta_{\tau}^2$. This $\mathcal{O}(a^2)$ contribution should be compared to that coming, for the same value of $m_t = 200$ GeV, from the $\mathcal{O}(\alpha)$ term, which is of approximately $+1$ GeV.\textsuperscript{2,3,7} Thus we conclude that such $\mathcal{O}(a^2)$ effect is completely negligible even at the required level of accuracy, which we assume to be of the order of $\sim 50$ MeV. This result is rather important since, a priori, a larger effect might have been found\textsuperscript{22} and thus it may be feared that a large uncertainty in the Standard Model prediction for the $W^\pm$ mass could come from higher order effects.

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\textsuperscript{22} An incorrect calculation done by expanding Eq. (1) including terms $\sim (\Delta_{\tau})^2$ with $\Delta_{\tau}$, given in Eq. (4) would have yielded the incorrect result $\Delta M_{W}^{\text{top; } \mathcal{O}(a^2)} \approx -40$ MeV.
References


GAMMA-GAMMA PHYSICS

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1. Convener's Review

1.1 INTRODUCTION

There are four detailed sections written by members of the working group:-

Section 2. F.Erne
"Resonance Excitation in \( \gamma \gamma \) Collisions at LEP200"

Section 3. S.J.Brook, F.Erne, P.H.Damgaard and P.M.Zerwas
"Two Photon Exclusive Meson Production at Large Angles"

Section 4. P.Aurenche, R.Baier, A.Douiri, M.Fontannas and D.Schiff
"High \( p_T \) Inclusive Hadron Spectrum at LEP200"

Section 5. A.Cordier and P.Zerwas
"The Photon Structure Function at Large \( Q^2 \)"

Work by these and other members of the group, and by the convener, is discussed in section 1, with overall conclusions in 1.6.

1.2 RESONANCE PRODUCTION

1.2.1 Hadronic Resonances

Experience from the "TPC/2-GAMMA" experiment (see PEP9/PEP14), working with beams of around 14.5 GeV at PEP, was used by F.Erne (see section 2,below) to make estimates and extrapolations of the production rate and measurability of resonant states. Cross sections and rates for gamma-gamma processes rise logarithmically with beam energy, although the benefit of this increase is not all accessible to experiment. Most of the resonant cross section is at low
values of the resonance mass $W_{\gamma\gamma}$, but at higher energy there is more longitudinal momentum available to boost final-state particles along the beam direction where they cannot be seen by the detectors.

Where LEP200 should have an advantage over all other machines is in the study of high mass states. But two more problems arise:

i) Hadron resonances are normally produced in gamma-gamma scattering by a quark-loop diagram for which the amplitude drops like $1/W_{\gamma\gamma}$.

ii) High mass hadronic final states fragment with large multiplicities, and only a few of the channels decay exclusively to measurable charged particles. PLUTO was lucky in finding a fully measurable final state for $\Upsilon_c$ with a 5% branching ratio. Both Ernö and Finch have calculated that only about 3% of all the final states from $\Upsilon_b$ will be fully measurable.

The curves on Ernö's "discovery plot" (figure 5, section 2) represent numbers of events produced and reconstructed in a few years running at LEP200 (integrated luminosity 1 fb$^{-1}$) as a function of the mass $W_{\gamma\gamma}$ and partial width $\Gamma_{\gamma\gamma}$ for gamma-gamma decay of hypothetical resonances. Decays via two quarks have been fragmented into a fraction of fully measurable (2, 4, or 6 body) final states which decreases as $W$ increases. The points labelled $\Upsilon_c$ and $\Upsilon_b$ in the lower left hand corner represent the numbers of these resonances which would be produced and reconstructed, given quark model couplings and fragmentation. Normal gamma-quark couplings cannot produce useful numbers of events via resonances much heavier than the $\Upsilon_c$, and in this mass region PEP is as competitive as LEP200. The discovery plot tells us that the advantage of LEP200 will lie in its sensitivity to new massive resonances with exotic couplings such as Renard's $b$ particle (see 2.7, below).

1.2.2 Excited Electron

(Also discussed by D.Treille in "Compositeness" working group.)

The process $e^+e^- \rightarrow e^* \rightarrow e\Upsilon$. 
if it occurs, will be very easy to recognise and reconstruct (see Erne's discussion in 2.9 below). The LEP200 experiments will be able to see any such state with a mass up to $\sim 90\%$ of the available centre of mass energy and with a sensitivity corresponding to the coupling expected from a compositeness scale $\Lambda$ of several TeV.

1.3 LARGE-ANGLE EXCLUSIVE HADRON PRODUCTION

In section 3 (below) Brodsky, Damgaard, Erne and Zerwas present new calculations of the rates for the production of two pions or two rho-mesons by perturbative hard gamma-gamma scattering. The region of sensitivity to the interesting physics is the same ($W_{\gamma\gamma} \sim 4 \text{ to } 5 \text{ GeV}$) as for resonance production, so many events are boosted out of the acceptance of the detectors. Nevertheless, a worthwhile rate of both two pion and two rho final states is predicted at up to $\sim 5 \text{ GeV}/c^2$, well beyond the resonance-dominated region. There is, therefore, hope that a check may be made on the validity of models which describe higher twist processes by combining perturbative QCD with lightcone parton wavefunctions for the hadrons.

1.4 HIGH P_T INCLUSIVE PROCESSES

1.4.1 New Gamma-Gamma Monte Carlo

S. Grayson has written a program which generates gamma-gamma to hadron events, including all orders of Q.C.D. It avoids singularities by using experimentally determined structure functions. Hard scattering off partons in a vector-dominated soft photon is automatically included. Because it is a Monte Carlo program it is easy to use realistic experimental cuts. Predictions have been made both for inclusive single jet $p_T$ distributions and for inclusive single hadron $p_T$ distributions. Jets and single hadrons are derived from the original partons by using the LUND fragmentation routines. The results have been compared with the work of Aurenche et al (next paragraph) and will be published as a Durham University report, to be written jointly with J. Stirling.
1.4.2 Inclusive QCD Calculation

Aurenche et al (section 4, below) have updated to LEP200 Energy the work (order $\alpha_\text{s}$ exact calculation) done for the LEP Physics Jamboree. It is encouraging that their predictions for the single particle inclusive distributions agree with Grayson's to within 20%.

The method of calculation is quite different and one may regard these predictions as noncontroversial. (c.f. Stirling's talk at the 1986 Paris Gamma-Gamma workshop). It can be seen from figure 10, section 4 that there are measurable rates for single pion production at $p_T$ values up to $\sim 16$ GeV/c; $500$ pb$^{-1}$ at 200 GeV gives $\sim 15$ events per GeV/c. At this value of $p_T$ the "vector-dominance" contribution has fallen away, so a good measurement will be a direct test of QCD.

A question was raised by N. Wermes at the Aachen meeting concerning the background to these events at electron-positron energies above the $Z^0$ due to $e^+e^-\rightarrow Z^0\gamma$. This is discussed in 4.3, and figure 4.9 shows how the hadrons from $Z^0$ decay would swamp the QCD effects if no cuts were made. In figure 4.10 the QCD prediction is modified by requiring $W_{\text{vis}} < 70$ GeV (for $100 + 100$ Gev beams). Aurenche et al estimate that this causes the background to "vanish", but they acknowledge that their calculation can only be regarded as a first try. They have not used a Monte Carlo for $Z^0$ decay and, in particular, they have not considered the effects of large missing energies carried by neutrinos from leptonic decays of heavy quarks. The high $p_T$ background from $Z^0$ in Fig. 4.9 needs to be reduced by a factor of $\sim 10^{-3}$ before it ceases to be serious. But $b\bar{b}$ (and $t\bar{t}$?) will constitute 10s of percent of the $Z^0$ hadronic final states. They have $\sim 10\%$ leptonic decay rates, and their neutrinos can carry 10s of GeV. The hard $\gamma$ ray will often be lost down the beampipe and it is not so difficult for the visible energy from $Z^0$ decay to drop below 70 GeV. More work is needed to model this.
1.4.3 Higher Twist Single Particle Inclusive Rates

Donnachie gave estimates of high $p_T$ inclusive single pion production via a higher twist process. When Erne's effective luminosity (table I, section 2, below) was folded in, the rate was three or four orders of magnitude lower than Aurene's prediction for fully inclusive pion production over the same range of $p_T$: approximately one event in 500 pb$^{-1}$ with $4 < p_T < 6$ GeV/c: not enough to consider further.

1.5 STRUCTURE FUNCTIONS

The differential cross section for single-tagged deep inelastic scattering of an electron from a soft photon can be written

$$\frac{d^2\sigma}{dx dy \cos \theta_E} = \frac{4\pi^2E_E}{Q^2y}(1 + (1-y)^2)F_2(x,Q^2) - y \frac{x}{x}F_1(x)$$

$$Q^2 = 4EE_x(1 - \cos^2 \frac{\theta_E}{2}); E_x \quad \text{and} \quad \theta_E \quad \text{are for tagged electron.} \quad Q^2 = \frac{Q^2}{Q^2 + W^2}$$

This is equivalent to equation 1 in 5.2 (below). Most of the rate is given by the nonscaling structure function $F_2$.

1.5.2 $F_2$ and Q.C.D.

The basic predictions of Q.C.D. for $F_2$ are uncontroversial, and can be tested very thoroughly at LEP200 (see section 5 below for a more detailed discussion by Cordier and Zerwas). They are:

a) $F_2(x,Q)$ rises linearly with log ($Q^2/A_0^2$) at all x. Cordier (checked by D.J.M) calculates substantial rates at large $Q^2$, well in excess of what can be achieved at lower energy machines. In 500 pb$^{-1}$ of running at 200 GeV there will be more than 2000 events with $Q^2$ greater than 100 GeV$^2$; one hundred of which will be above 1000 GeV$^2$.

b) The shape of $F_2(x)$, at a given $Q^2$, is modified from the quark-parton model, primarily by gluon bremsstrahlung effects which damp down the growth of $F_2(x)$ as x tends to 1. Current experimental data seem to agree with this.
prediction, but experiments have not been done at high enough $Q^2$ to be sure. High $x (= Q^2/(Q^2 + M^2))$ at low $Q^2$ means relatively low $w^2$. To make a reliably "asymptotic" measurement up to $x = 0.9$ needs $W$ greater than 10 GeV, which means $Q^2$ must be more than 100 GeV — only available at LEP200.

What is not so clear is whether the Q.C.D. scale parameter $\Lambda_{\overline{MS}}$ will be reliably determined, as Witten suggested. The problem has been extensively discussed in the literature, and different members of the working group had different opinions. Storrow and Daluz-Vieira are working on a fitting technique which they claim will be sensitive to the value of $\Lambda_{\overline{MS}}$. They draw attention to new data from TPC/2-GAMMA (1) which have confirmed the previous conjecture that $F_2(x)$ at $Q^2 \gtrsim 1$ GeV goes like $0.2(1-x)$, a recipe derived from the vector dominance model of the soft photon, with the structure functions of the rho etc. taken as behaving like the pion, as measured in Drell Yan processes. This removes some of the uncertainties in setting up a fit to the $Q^2$ evolution of $F_2$. Unfortunately $\Lambda_{\overline{MS}}$ can only be obtained from a $\log(Q^2/\Lambda^2)$ dependence by looking at the behaviour of $F_2$ in the $Q^2 \gtrsim 1$ region, and this is where an extra parameter has to be introduced to allow for the "hadronic" component (including vector-dominance) of the photon from which the evolution to higher $Q^2$ begins.

1.5.3 Can $F_2$ be Measured ?

The longitudinal component of the photon does not couple to the leading Q.P.M. box graph, so $F_2$ does not develop a scale-breaking $\log Q^2$ term. It should have "hadronic" contributions, but without VDM. If it can be measured it gives an independent check on models for the "hadronic" part of $F_2$. But it only contributes a small part of the total rate, due to the $y^2$ weight in equation (1.1) and because it does not grow with $Q^2$.

A calculation by D.J.M. has shown that there are regions where single-tagged events might be used to measure $F_2$. Table I shows the number of events $N_2$.
generated by $F_2$ compared with the number $N_L$ generated by $F_L$ (actually a negative number because we have used $F_2 = 2x^2 F_L$). Note that in order to enhance the effect of $F_L$ it is necessary to go to very small values of the tagged electron energy $E'$ and to the smallest available tagging angles. There will be a serious background to tagging in this kinematic region caused by fast pions close to the beam direction, coming from the vector meson dominated dissociation of soft photons from beam particles. The tagging detector must therefore be capable of detecting electrons and rejecting pions down to $\sim 15\%$ of the beam energy.

The planned angular acceptance of the OPAL (from D.J.M.) and L3 (from A.Engler) forward detectors have been used for LEP1 and the published acceptance of TPC/2-GAMMA for PEP. Acceptances for TRISTAN and LEP200 are entirely conjectural. With these acceptances (see Table I), at LEP200 the mean $Q^2$ will be a few GeV$^2$, sufficient to check the predicted value of $F_L$ in the scaling region. High-luminosity running at PEP will give a good rate, but the mean $Q^2$ is too low for scaling to be assumed.

<table>
<thead>
<tr>
<th>MACHINE</th>
<th>$E'$ Cut (GeV)</th>
<th>$\theta'$ Cut (mr.)</th>
<th>$L dt$ (pb)$^{-1}$</th>
<th>$N_L$ Events</th>
<th>$N_L$ Events</th>
<th>Mean $Q^2$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP1 (50+50 GeV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OPAL</td>
<td>7 to 15</td>
<td>47 to 80</td>
<td>100</td>
<td>730</td>
<td>-110</td>
<td>1.8</td>
</tr>
<tr>
<td>L3</td>
<td>10 to 20</td>
<td>30 to 40</td>
<td>100</td>
<td>1365</td>
<td>-170</td>
<td>0.9</td>
</tr>
<tr>
<td>LEP200 (100+100 GeV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;Better tagger&quot;</td>
<td>14 to 30</td>
<td>25 to 60</td>
<td>500</td>
<td>4750</td>
<td>-700</td>
<td>2.7</td>
</tr>
<tr>
<td>PEP</td>
<td>(14.5+14.5 GeV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPC/2-GAMMA</td>
<td>4 to 7</td>
<td>30 to 200</td>
<td>50</td>
<td>8250</td>
<td>-1305</td>
<td>0.24</td>
</tr>
<tr>
<td>TRISTAN (70+70 GeV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;Good tagger&quot;</td>
<td>5 to 16</td>
<td>30 to 100</td>
<td>100?</td>
<td>7170</td>
<td>-1080</td>
<td>0.73</td>
</tr>
</tbody>
</table>

It will be difficult to measure $F_L$. All that can be attempted is an average measurement over a range of $x$ and $Q^2$. The method is statistically unreliable.
N_2 must be predicted accurately, using the value of F(x,Q^2) from other lower energy experiments where the contamination from F_L is small because the tagged electron is close to beam energy. Then N_R is estimated by subtracting N_2 from the observed number of events in the selected tagging region, giving a difference which is never as big as 20% of N_2. And the kinematics of events with a low tagged electron energy E_T^t are such that the hadronic fragments are boosted strongly into the same direction as the tagged electron (this effect is being studied further in Monte Carlo by P.Kyberd and D.J.M.).

Realistic ambitions for F_L may be:

i) At LEP1. To put bounds on the maximum size of F_L at Q^2 \sim 1 \text{ GeV}^2.

ii) At LEP200. To measure the average value of F_L to within 30% at Q^2 \sim 2.5 \text{ GeV}^2.

These modest goals will be worth achieving. There is little direct knowledge of the nonvector hadronic coupling of the photon, and it will be well worthwhile to test the QCD predictions.

1.5.4 Double-Tagging

No special study has been made, but there are two good reasons for trying to double-tag gamma-gamma events. It is the only way to measure the F_X structure function (2) which depends on the distribution in the azimuthal angle between the two tagged electrons. It also takes both photons far from the mass-shell into a region where QCD predictions for F_2 are less bedevilled with singularities (3). Both ALEPH and OPAL plan to provide "far-forward" luminosity monitors, beyond the mini-beta quadrupoles. If they work well at LEP1, without being swamped by synchrotron radiation and off momentum electrons, then at LEP200 they may be used to double-tag up to quite large values of Q^2 and S^4.
1.6 CONCLUSIONS

LEP200 will bring clear benefits to some parts of gamma-gamma physics.
- It will extend the range of $Q^2$ from around 100 GeV$^2$ towards 2000 GeV$^2$ for the study of the QCD evolution of structure functions in deep inelastic e-gamma scattering.
- QCD predictions for jet and particle inclusive production can be tested for $p_T$ greater than 10 Gev/c where the vector dominance background is no longer important.
- New resonant states with exotic couplings to gamma-gamma can be searched for over a wide range of masses. Exotic excited electron states will also be accessible with high sensitivity up to a mass of almost 200 GeV/c$^2$.
- Exclusive meson pair production can be extended beyond the resonance region to where the predictions of "QCD+wavefunctions" theory may be studied.

All of these benefits come from the anticipated combination of higher energy and higher luminosity at LEP200. If the integrated luminosity at LEP200 is significantly less than 500 pb/year, above the W W threshold, then the benefit to gamma-gamma physics will be much less.

Special new equipment will not be needed, though it would help to be able to tag electrons to the smallest possible angles; certainly to less than 30 milliradians. This requires the smallest possible radius of beampipe, consistent with manageable synchrotron-radiation backgrounds.

REFERENCES

2. RESONANCE EXCITATION IN $\gamma \gamma$-COLLISIONS AT LEP200

F.C. Erné, NIKHEF-H, Amsterdam

Cross sections and rates for observing charmonium and heavier states in exclusive channels are estimated for LEP200. A resonance discovery plot is presented. Searches for excited electrons are discussed.

2.1 CROSS SECTIONS AND RATES

The Cross Section for a $\gamma \gamma$ collision leading to a final state $X$ can be written as

$$\frac{d\sigma}{d\Omega}(e^+e^- \rightarrow e^+e^-X) = \int \left[ \frac{dL_{\gamma\gamma}}{dW_{\gamma\gamma}} / L_{e^+e^-} \right] \frac{d\sigma}{d\Omega}(\gamma\gamma \rightarrow X)dW_{\gamma\gamma}. \quad (1)$$

Here the quantity $\Omega$ is generic for any number of phase space variables describing the state $X$, while the quantity in square brackets is commonly referred to as the $\gamma\gamma$-luminosity function; it is determined entirely by QED.

The collected rate is then found from:

$$\frac{dN_X}{dt} = L_{e^+e^-} \int \frac{d\sigma}{d\Omega}(e^+e^- \rightarrow e^+e^-X) A(\Omega) d\Omega. \quad (2)$$

Here $L_{e^+e^-}$ is the machine luminosity for $e^+e^-$-collisions, and $A(\Omega)$ the detector acceptance.

In case that the final state $X$ results from a decaying resonance $R$, the integrated $\gamma\gamma$ cross section is, in the narrow resonance approximation:

$$\int \sigma(\gamma\gamma \rightarrow X)dW_{\gamma\gamma} = \frac{4\pi^2(2J + 1)}{m_R^2} \Gamma_{R \rightarrow \gamma\gamma} Br(R \rightarrow X)(\hbar c)^2. \quad (3)$$

Here $J$ is the resonance spin, $m_R$ its mass, $\Gamma_{R \rightarrow \gamma\gamma}$ its partial width for decay into $\gamma\gamma$ and $Br(R \rightarrow X)$ its branching ratio for decay into the final state $X$.

2.2 $\gamma\gamma$ LUMINOSITIES USABLE FOR OBSERVING EXCLUSIVE FINAL STATES

To identify the final state $X$, and determine its mass, the identities and momenta of all particles belonging to $X$ have to be known from observation in the central detector, as in most cases the scattered electrons escape detection into the extreme forward directions and carry away large and unknown momenta.
This requirement imposes a low longitudinal momentum on the state to be observed. Below we assume that the following relation holds:

$$|p_{\gamma\gamma}| < W_{\gamma\gamma}.$$  \hspace{1cm} (4)

This relation is approximately valid for four-prongs in the low-mass region ($W_{\gamma\gamma} \leq 4$ GeV) in the TPC-twogamma experiment at PEP. The effect of this requirement on the allowed energies of the colliding photons (with $E_{\gamma,\gamma} \approx E_{\text{beam}} - E_{e\pm}$) can be seen in fig.1 for an example at $W_{\gamma\gamma} = 10$ GeV and for various beam energies. Only a fraction of the $x_1, x_2$ hyperbolae contributes. One observes that the cut becomes increasingly severe at higher beam energies.

In fig.2 we show the reduction in the available $\gamma\gamma$-luminosity that results from this central-detector selection factor $F_{cd}$. By introducing this factor we have reduced the problem of the estimation of the acceptance at LEP energies to a problem that has been solved already for detectors at PEP and PETRA energies, where rapidity-ranges comparable to those of LEP-detectors are covered.

The $\gamma\gamma$-luminosity functions multiplied by this factor $F_{cd}$ are given in fig.3. The curves result from a 5-dimensional numerical integration of the exact lowest order differential expression for the $\gamma\gamma$-luminosity \(^1\) over the electron variables. The numbers used in fig.3 and numbers from integration without the central detector requirement are given in table I.

It is interesting to multiply the above functions by the expected $e^+e^-$ luminosities for machines that will be soon in operation. This is shown in fig.4. We use the following numbers for $L_{e^+e^-}$:

<table>
<thead>
<tr>
<th>Collider</th>
<th>$E_3$(GeV)</th>
<th>$L_{e^+e^-}$ (cm(^{-2})sec(^{-1}))</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upgraded PEP</td>
<td>14.5</td>
<td>1.2 \times 10^{32}</td>
<td>2)</td>
</tr>
<tr>
<td>TRISTAN</td>
<td>30</td>
<td>2 \times 10^{31}</td>
<td>3)</td>
</tr>
<tr>
<td>LEP100(<em>v</em>)</td>
<td>50</td>
<td>1.2 \times 10^{31}</td>
<td>4)</td>
</tr>
<tr>
<td>version 13, 3mA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEP200</td>
<td>100</td>
<td>5 \times 10^{31}</td>
<td>5)</td>
</tr>
</tbody>
</table>

As these numbers represent expectations, the comparison in fig.4 can only be tentative. It points, however, to competition between LEP200 and PEP in the low-mass region ($W_{\gamma\gamma} \leq 10$ GeV), while the upgraded PEP may be on the air a few years earlier than LEP200. However, for $W_{\gamma\gamma} > 10$ GeV, LEP200 will be unique.
2.3 ACCEPTANCE ESTIMATES

In addition to the momentum cut evaluated above, several other factors reduce the useful counting rate: trigger efficiencies, reconstruction efficiencies, absorption inside the detector, particle decays, particle identification, and the cut that verifies that we have to do with an exclusive two-photon process: the sum of the transverse momenta w.r.t. the beams should be below a value of the order of 0.2 GeV. These factors depend on the detector and on the physics interest expressed in the collaboration: the trigger thresholds may be high if they are primarily meant to enable the recording of annihilation events with a total energy of 200 GeV.

To have some estimates we extrapolated known acceptance figures from the TPC-Twogamma Collaboration at PEP. Typically one looses about an order of magnitude in rate w.r.t. a 4π-detector with ideal identification, and in the identification of kaons one looses another factor two because of background subtractions, decay and ambiguities.

2.4 EXPECTED $\Gamma_{\gamma\gamma}$ FOR CHARMONIUM RESONANCES AND BEYOND

For Charmonium and Bottomonium we resort to a few simple relations given by Close 6. For S-wave $J^P = 0^-$-resonances one relates the $\gamma\gamma$-width to the leptonic width of the corresponding vector particle. At the right-hand side below we give the expected number:

$\Gamma_{\gamma\gamma}$ (keV)

\begin{align*}
\Gamma(\eta_c \to \gamma\gamma) & \approx \frac{4\pi}{3} \Gamma(J/\Psi \to e^+ e^-) M_{J/\Psi}^2 / M_{\eta_c}^2 & 6.8 \\
\Gamma(\eta_c' \to \gamma\gamma) & \approx \frac{4\pi}{3} \Gamma(\Psi' \to e^+ e^-) M_{\Psi'}^2 / M_{\eta_c'}^2 & 2.7 \\
\Gamma(\eta_b \to \gamma\gamma) & \approx \frac{2\pi}{3} \Gamma(T \to e^+ e^-) M_T^2 / M_{\eta_b}^2 & 0.4
\end{align*}

Similar relations connect the $\gamma\gamma$-width of P-wave resonances to their own hadronic width:

\begin{align*}
\Gamma(\chi_c^0 \to \gamma\gamma) & \approx \Gamma(\chi_c^0 \to \text{hadrons}) \frac{128}{1271} \alpha^2 / \alpha_s^2 & 3.3 \\
\Gamma(\chi_b^0 \to \gamma\gamma) & \approx \Gamma(\chi_b^0 \to \text{hadrons}) \frac{128}{1271} \alpha^2 / \alpha_s^2 & 3.3
\end{align*}

An estimate for the $\gamma\gamma$-width of a Higgs particle, via a triangular diagram is given by Okun 7. If there is only one Higgs, one has the relation:

$$\Gamma(H \to \gamma\gamma) = \left( \frac{\alpha \left( \sum_i q_i^2 F_i(\beta) \right)}{4\pi} \right)^2 \frac{Gm_H^3}{8\pi\sqrt{2}} \quad (5)$$
The factors $F_i(\beta)$ for spin $i = 0, \frac{1}{2}, 1$ are: $F_0 = \beta(1 - \beta x^2)$, $F_{\frac{1}{2}} = -2\beta(1 - \beta x^2 + 1)$, $F_1 = [2 + 3\beta + 3\beta(2 - \beta)x^2]$ with $\beta = 4m^2/m_H^2$ and $x = \arctan(1/\sqrt{\beta - 1})$ for $\beta > 1$ and $x = \frac{1}{2}(\pi + i\ln \frac{1 + \sqrt{1 - \beta}}{1 - \sqrt{1 - \beta}})$ for $\beta < 1$; here $m$ is the mass of the virtual particle. For example for $m_H = 50$ GeV, $m = 80$ GeV, $i = 1$ we obtain $\Gamma_{\gamma\gamma} \approx 0.7$ keV.

Certain composite models of leptons, quarks and weak bosons, suggest a more sizable $\gamma\gamma$-width for spin-zero partners of the W-boson. Renard\(^6\) estimated from pointlike photon-coupling with subconstituents for a pseudoscalar particle $H$ (not necessarily a Higgs) with a mass of 50 GeV a $\Gamma_{\gamma\gamma}$ ranging between 5 keV and 0.8 MeV, and from W-dominance models a $\Gamma_{\gamma\gamma}$ ranging between 5 and 70 MeV. The state may be several GeV wide and therefore not readily observable, if the subconstituents are coloured. Baur, Fritzsch and Faisser\(^9\) discuss the case of a neutral isoscalar boson $U$ with spin-$\frac{1}{2}$ haplons as constituents. For $M_U = 50$ GeV they find $\Gamma_{\gamma\gamma} \approx 17$ MeV.

2.5 DECAY-CHARACTERISTICS OF $\bar{c}c$ AND $\bar{b}b$ STATES

The $\eta_c$, $\eta_b$- and $\chi_c$ - states mentioned above, decay mainly via two gluons with flavour-independent couplings into any number of mesons. To obtain an estimate of the fractional decay into states that can be experimentally reconstructed with some accuracy, we take a Poisson-distribution for the number of $\pi^+\pi^-$ or $K^+K^-$ pairs, and independently for the number of $\pi^0$s, with the average value $\bar{n} = 1 + 0.35\ln M^2$. This gives for example the probabilities $P_n$ for:

<table>
<thead>
<tr>
<th>$M$ (GeV)</th>
<th>$\bar{n}$</th>
<th>$P_0$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.77</td>
<td>0.170</td>
<td>0.300</td>
<td>0.267</td>
<td>0.157</td>
</tr>
<tr>
<td>10</td>
<td>2.61</td>
<td>0.0735</td>
<td>0.192</td>
<td>0.250</td>
<td>0.218</td>
</tr>
</tbody>
</table>

If we consider only final states with 2 or 4 charged particles, and no $\pi^0$s, then the branching ratio is $P_0(P_1 + P_2)$. For pseudoscalars the decay into $\pi^+\pi^-$ and $K^+K^-$ is forbidden and we have $P_0P_2$. It amounts to 4.5% for $M = 3$ GeV and 1.8% for $M = 10$ GeV. The above estimate at 3 GeV can be confronted with the experimental situation as is known for all-charged final states in radiative $J/\Psi$ decay into the $\eta_c$, (using the Crystal Ball value $^{10}$)

$$Br(J/\Psi \rightarrow \eta_c) = (1.27 \pm 0.36) \times 10^{-2}.$$ 

The agreement is satisfactory as can be seen from table II.
<table>
<thead>
<tr>
<th>$X$</th>
<th>MKIII$^{(1)}$</th>
<th>DM2$^{(2)}$</th>
<th>MKII$^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \to K^+K^- \phi \to K^+K^-$</td>
<td>$0.19 \pm 0.07$</td>
<td>$0.08 \pm 0.03$</td>
<td></td>
</tr>
<tr>
<td>$2(\pi^+\pi^-)$</td>
<td>$1.3 \pm 0.6$</td>
<td>$2.0 \pm 1.5$</td>
<td>$1.0$</td>
</tr>
<tr>
<td>$K^{*0}<em>{\to K^+\pi^-} \bar{K}^{*0}</em>{\to K^-\pi^+}$</td>
<td>$0.23 \pm 0.13$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^{*0}_{\to K^+\pi^-} K^-\pi^+ + c.c.$</td>
<td>$2.0 \pm 0.5$</td>
<td>$1.4 \pm 2.1$</td>
<td>$1.0$</td>
</tr>
<tr>
<td>$K^+ K^- \pi^+ \pi^-$</td>
<td>$2.1 \pm 0.3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^{*0}_{\to K^+\pi^-} K^{\mp}\pi^\pm$</td>
<td>$1.0 \pm 0.4$</td>
<td>$1.2 \pm 0.6$</td>
<td>$5.2 \pm 3.1$</td>
</tr>
<tr>
<td>$p\bar{p}$</td>
<td>$0.11 \pm 0.06$</td>
<td>$0.10 \pm 0.04$</td>
<td></td>
</tr>
</tbody>
</table>

2.6 EXPECTED RATES IN 2-, 4-, AND 6-BODY CHANNELS: DISCOVERY PLOT

The procedures outlined above, can be used to calculate the number of events expected in approximately two years of LEP200 running with an integrated $e^+e^-$ luminosity of $1 fb^{-1}$. The result is shown in fig.5 in the form of lines relating the mass and width of $J = 0$ resonances for a fixed number of observed and reconstructed events. Also indicated are the $\eta_c, \eta'_c$ and $\eta_b$ with their expected $\gamma\gamma$-widths.

One sees that exploration of the Charmonium region is relatively comfortable, while $\eta_c$ observation is out of the question. Also for a 50 GeV Higgs particle one expects less than 1 event, even if the branching ratio into the observed final state is 100%. On the other hand, the $\gamma\gamma$-decays of composite scalar bosons might be observable. The 50 GeV U-particle$^{(9)}$ discussed would produce $\approx 20$ observable events, and Renard’s 50 GeV H-particle$^{(8)}$ $\approx 70$. Fig.5 can also be used to predict rates for any new states decaying into heavy quark pairs that might be suggested.

The Charmonium states may be accessible to PEP in 1987-1988. Fig.6 shows a Monte Carlo calculation of the expected number of Charmonium events in the $2(\pi^+\pi^-)$ channel at PEP.

2.7 TAGGING

In principle tagging by observation and momentum measurement of one of the electrons can help in distinguishing heavy-flavour excitation from $\rho$-dominance background, because form factors go approximately as $1/(1 - Q^2/W^2)$ for $\eta_c$, $\eta_b$, ..., and as $1/(1 - Q^2/m_b^2)$ for $\rho$-dominance. Here $Q_i^2 = 4E_{beam}E_i \sin^2(\theta_i/2)$, with $E_i$ and $\theta_i$ the energy and angle w.r.t. the beam, of the observed electron. As $E_i \approx E_{beam}$, and as tagging devices have a substantial minimum angle, the $Q^2$ of the tagged photon becomes considerable at LEP energies, and the luminosity fraction seen by the tagging devices is correspondingly small. Even the reduction
from a Charmonium form factor becomes substantial. This is shown in Table III for a tagging region between 40 and 180 mrad at both sides of the central detector and for $W = 3$ GeV.

Table III ($W = 3$ GeV, $0.04 < \theta_{tag} < 0.180$)

<table>
<thead>
<tr>
<th>Ebeam (GeV)</th>
<th>PEP LEP100 LEP200</th>
</tr>
</thead>
<tbody>
<tr>
<td>- $Q^2 (GeV^2)$</td>
<td>0.34 - 6.8</td>
</tr>
<tr>
<td>tagging fraction</td>
<td>0.23</td>
</tr>
<tr>
<td>$\eta_e$ form-factor reduction</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The number of tagged events at LEP200 would be very small in the Charmonium region. Here PEP is in a better position.

2.8 SEARCH FOR EXCITED ELECTRONS IN QUASI-REAL COMPTON SCATTERING

Previous searches for excited electrons ($e'$) in quasi-real Compton scattering at ACO$^{15}$, PETRA$^{16}$ and PEP$^{17}$ can be significantly extended in mass up to $\approx 90\%$ of the available cm energy at LEP200. In the process a quasi-real photon emitted by one of the incoming particles is elastically scattered by the other. The spectator electron, which radiates the photon, usually escapes detection along the beam direction. The electron-photon mass can then be determined most accurately from a combination of the c.m. energy and a measurement of the angles $\theta_\gamma$ and $\theta_\gamma$ of the scattered electron and photon: $W^2 = s(1 - \beta)/(1 + \beta)$, with the velocity $\beta = \sin(\theta_\gamma + \theta_\gamma)/\sin \theta_\gamma$. Most of the LEP detectors have an angular resolution which is superior to that of earlier detectors, of the order of 1.5-5 mrad for photons, and much better for charged particles. This leads to a mass resolution of typically $\approx 0.5$ GeV for $W \approx 150$ GeV at LEP200. Several detectors moreover measure electron- and photon-energies to the 1% level (OPAL,L3), which provides precise additional constraints. The mass resolutions are comparable to those achieved in earlier searches at lower-energy machines. One can write a gauge-invariant effective magnetic $e' e\gamma$ interaction for an $e'$ of spin $\frac{1}{2}$ as:

$$I_{eff} = \frac{e f_{\gamma}}{2M_{e'}} \overline{\Psi} e \sigma_{\mu \nu} \Psi e F^{\mu \nu} + h.c.$$  \hspace{1cm} (6)

In a fixed range of $\delta W$ the cross section for the possible signal then scales as
$f^2_\gamma/M_c^2$, while the continuum scales as $1/W^2$. Upper limits on $f_\gamma$ are at present of the order of 0.01 for $m^*_\gamma < 30$ GeV. At LEP200 one can expect limits on $f_\gamma$ of the order of 0.02-0.05, with integrated luminosities comparable to those of present experiments ([200$pb^{-1}$ for DELCO $^1$]). If one assumes $f_\gamma = M_\gamma/\Lambda^{18}$, this is sufficient to be sensitive to a compositeness scale $\Lambda$ of several TeV.

Table I

<p>| Integrated over the electron variables | Integrated with the requirement $|p_{E\gamma}| &lt; W_{E\gamma}$: |
|--------------------------------------|--------------------------------------------------|
| $d\sigma_{e\gamma}/dW_{E\gamma}/L_{e^+e^-}$ | $F_{e\gamma} d\sigma_{e\gamma}/dW_{E\gamma}/L_{e^+e^-}$ |</p>
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<th>$W_{\gamma\gamma}$</th>
<th>Ebeam (GeV)</th>
<th>Ebeam (GeV)</th>
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Figure 1. The range of $\gamma$-energies selected by observation of exclusive final states in a central detector, for $W_{\gamma\gamma} = 10$ GeV, by the requirement $|p_{\gamma\gamma}| < W_{\gamma\gamma}$.

Figure 2. The central-detector selection factor $F_{cd}$, resulting from the requirement $|p_{\gamma\gamma}| < W_{\gamma\gamma}$ for various beam energies.
Figure 3. The $\gamma\gamma$ luminosity function multiplied by $F_{cd}$ for various beam-energies.

Figure 4. Projected $\gamma\gamma$ luminosities (see text) multiplied by $F_{cd}$ for various $e^+e^-$ colliders.
Figure 5. Discovery limits on $\Gamma_{R\rightarrow \gamma\gamma}$ vs $W$ for narrow $J=0$ resonances decaying into heavy-quark pairs in the sum of exclusive $2-, 4-$ and $6-$ body channels at LEP200 for $1/fb^{-1}$ integrated $e^+e^-$-luminosity. Quark-model predictions for some known states are indicated. Also a recent result \cite{14} for 95% confidence limits on new narrow states from the TPC-twogamma experiment at PEP with double tagging and the missing mass technique, is indicated.

Figure 6. Monte Carlo calculation of the expected number of Charmonium events into the $2(\pi^+\pi^-)$ channel at PEP on top of the expected vector-dominance background.
3. TWO-PHOTON EXCLUSIVE MESON PRODUCTION AT LARGE ANGLES

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P.H. Damgaard (c) and P.M. Zerwas (d)

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ABSTRACT

QCD predictions for the production of meson pairs at large angles in \( \gamma\gamma \) collisions are summarized, and production rates for LEP200 energies are investigated.

3.1 INTRODUCTION AND THEORETICAL SET-UP

Exclusive hadron production in two-photon processes can provide us with very interesting tests of QCD. By a comparison between several different exclusive processes one can explore not only the (obvious) high-energy limit of perturbative QCD, but also gain knowledge about such low-energy hadronic structures as mesonic and baryonic wave functions. In this way the study of exclusive two-photon processes can show us the interplay between short-distance and long-distance strong interaction physics.

Apart from hadronic form factors, two-photon processes leading to exclusive hadron production are also the simplest exclusive processes to analyze theoretically. As we shall review in this study, one can obtain several parameter-independent tests of perturbative QCD in the very high \( Q^2 \) regime. Some of these parameter-independent statements, such as scaling laws and asymptotic helicity selection rules, need no specific computation at all, but simply follow as immediate by-products of the theoretical framework. These predictions are valid to all orders in perturbation theory. Others, such as an essentially parameter independent expression of the cross section for \( \gamma\gamma \to \pi^+\pi^- \) in terms of the pion form factor \( F_\pi(Q^2) \), require detailed calculations.

We now turn to a brief description of the basic formalism. Using the most convenient terminology \(^{1,2}\), the crucial step in the theoretical analysis consists in a demonstration of the factorization of an exclusive scattering amplitude into
a hard scattering amplitude \( T_H \) and "soft", in perturbation theory incalculable, distribution amplitudes \( \Phi(x_i) \). To get an intuitive feeling for this factorization, consider first the two-photon process in the center-of-mass frame. The two photons collide, and in the interaction region create a set of quark-antiquark pairs. Demanding that these quark-antiquarks combine into just the set of hadrons at large angles required for the exclusive process, forces large \( p_T \) to be routed through all constituents of the hadrons. This is illustrated in fig.1. For such a wide-angle scattering the full amplitude \( M \) can therefore, as suggested by fig.1, be written in the form

\[
M = \int [dx|dy| \Phi^*(y_i, p_T) T_H(x_i, y_i, p_T) \Phi(x_i, p_T) \tag{1}
\]

where \( x_i \) and \( y_i \) denote the fractional longitudinal momenta of the constituents in the outgoing hadrons. The distribution amplitudes \( \Phi(x_i, p_T) \) give the probability amplitude for finding the constituents of the hadrons with longitudinal momenta \( x_i \) and with transverse momenta less than \( p_T \).

To lowest order in \( \alpha_s(p_T^2) \), the Feynman diagrams contributing to the hard scattering amplitude \( T_H(x_i, y_i) \) of eq.(1) are precisely those diagrams which produce the right number of quarks and antiquarks moving parallel (along the \( x \)-axis), but with arbitrary relative longitudinal momenta \( x_i \). For pair production of mesons, \( \gamma \gamma \to M\overline{M} \), it is clear that this can, to lowest order in \( \alpha_s(p_T^2) \), be accomplished by just one-gluon exchange. This scattering amplitude is hence of order \( \alpha_s EM \alpha_s(p_T^2) \).

In general, scaling laws for wide-angle scattering amplitudes follow from the dimensional counting rule\(^3\)

\[
M \approx \frac{1}{(p_T^2)^{(n-4)/2}} f(\theta_{c.m.}), \quad p_T^2 \to \infty \tag{2}
\]

where \( n \) equals the total number of scattered quanta, i.e. in the case of \( \gamma \gamma \to M\overline{M} \) (with 2 quarks, 2 antiquarks and 2 photons) \( n = 6 \). The amplitude therefore scales as \( M \approx 1/p_T^2 \). Similarly for baryon production \( \gamma \gamma \to B\overline{B} \) we have \( n = 3+3+2 = 8 \) and hence \( M \approx 1/p_T^4 \). These scaling laws are fundamental predictions of QCD perturbation theory (arising because of \( \alpha_s(p_T^2) \) being dimensionless), and it is important to have them tested experimentally. They are (weakly) modified by the \( p_T^2 \)-evolution of the coupling constant \( \alpha_s(p_T^2) \), and by the weak (logarithmic) \( p_T^2 \)-evolution of the distribution amplitudes\(^1,2\). Ignoring these logarithmic corrections, the dimensional counting rules predict the following power-law behaviour.
of the cross sections at fixed angles

\[
\left(\frac{d\sigma}{dt}\right)_{\gamma\gamma\rightarrow MM} \sim s^{-4} \quad \text{and} \quad \left(\frac{d\sigma}{dt}\right)_{\gamma\gamma\rightarrow BB} \sim s^{-6}.
\]  \hspace{1cm} (3)

These scaling laws are valid at sufficiently large \( p_T^2 \) (when \( \alpha_s(p_T^2) \) is small enough to make the Feynman diagram expansion meaningful). As \( p_T^2 \) becomes smaller, nuclear binding effects can become important, and one must expect in addition to have contributions from vector meson resonances. Similarly, higher Fock state contributions to the hadronic wave functions can be important at low \( p_T^2 \). At large \( p_T^2 \), these additional contributions can all be shown \(^{1,2}\) to vanish faster than the dominant contribution \((3)\).

One more prediction follows immediately once one assumes large enough \( p_T^2 \) to make QCD perturbation theory valid, this is the one of hadronic helicity conservation. Since at large \( p_T^2 \) we can ignore \( u \) and \( d \) quark masses (and to some approximation also the \( s \)-quark mass), the constituent quarks should all scatter as massless fermions. As these fermions all couple minimally to the gauge fields, and since fermion helicity is conserved at vector couplings in a massless theory, it follows that quark helicity is preserved throughout the process. Since the hadrons are built out of these constituents, it follows that the hadronic helicity itself is preserved as well.

Other predictions require more detailed evaluation of the set of Feynman diagrams contributing to the various processes. We shall now turn to a review of some of the results which have been obtained in such computations.

The Feynman diagrams which to lowest order in \( \alpha_s(p_T^2) \) contribute to the hard scattering amplitude \( T_H \) for meson-antimeson production, \( \gamma\gamma \rightarrow MM \), have been computed in ref.4. In order to give actual predictions for the cross sections themselves one needs, however, to know both the normalization and functional form of the distribution amplitudes \( \Phi(x, p_T) \) at some given \( p_T^2 \). Fortunately, in the case of mesons the overall normalization is fixed by the "sum rule"

\[
\int_0^1 dx \Phi_M(x, p_T) = \frac{f_M}{2\sqrt{3}} \hspace{1cm} (4)
\]

where \( f_M \) is the meson decay constant. Furthermore, in the limit of infinitely high \( p_T \) the functional form of \( \Phi_M(x, p_T) \) is fixed as well \(^{1,3}\), \( \Phi_M(x, p_T) \rightarrow \sqrt{3} f_M x (1-x) \). At moderate energies the functional form of \( \Phi_M(x, p_T) \) can not
be computed within this formalism itself. However, once determined at some fixed $p_T^2$, it is given at any other $p_T$ through an evolution equation $^{1,3}$.

We shall now present the results for two different parametrizations of the distribution amplitudes. The first, which we shall denote by "weak binding", corresponds to the quark constituents sharing the longitudinal momentum equally:

$$
\Phi_{M}^{W,B}(x, p_T) = \frac{f_M}{2\sqrt{3}} \delta(x - \frac{1}{2})
$$

(5a)

whereas the other is the one extracted from QCD sum rules $^{5}$

$$
\Phi_{M}^{C,Z}(x, p_T) = 5\sqrt{3} f_M x (1-x)(2x-1)^2.
$$

(5b)

Shown in fig. 2a) are the differential cross sections for $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\gamma\gamma \rightarrow \pi^0\pi^0$ for these two distribution amplitudes. As can be seen, the cross section for $\gamma\gamma \rightarrow \pi^+\pi^-$ is practically independent of our two choices for quark distribution amplitudes $\Phi_{M}^{W,B}$ or $\Phi_{M}^{C,Z}$. This result, which is valid for all wave functions tried to date $^{4}$, means that the cross section for $\gamma\gamma \rightarrow \pi^+\pi^-$ is essentially parameter-independent, and can be expressed solely in terms of the (measured) pion form factor $F_\pi$. In detail $^{4}$,

$$
\frac{d\sigma}{dx}(\gamma\gamma \rightarrow \pi^+\pi^-) \approx \frac{4|F_\pi(s)|^2}{1 - \cos^2 \theta_{c.m.}}.
$$

(6)

In contrast to this, the differential cross section for $\gamma\gamma \rightarrow \pi^0\pi^0$ shows a clear dependence on the choice of distribution amplitude. The "weak-binding" approximation yields a differential cross section which, at least to this order in perturbation theory, is completely flat, whereas $\Phi_{M}^{C,Z}$ gives a cross section whose dependence on $\theta_{c.m.}$ more resembles the one of $\gamma\gamma \rightarrow \pi^+\pi^-$. For both choices of distribution amplitudes the differential cross section for $\gamma\gamma \rightarrow \pi^0\pi^0$ is about one order of magnitude smaller than the cross section for $\gamma\gamma \rightarrow \pi^+\pi^-$, a suppression one should expect due to charge cancelations.

Predictions of $\gamma\gamma \rightarrow \rho^+\rho^-$ and $\gamma\gamma \rightarrow \rho^0\rho^0$ have been given in the case of $\Phi_{M}^{C,Z}$ in ref.6. The diagrams contributing to these processes are the same as shown in fig. 1 for pions. In the special case $\gamma\gamma \rightarrow \rho^0_L\rho^0_L$ (longitudinally polarized), a set of 2-gluon exchange diagrams which are formally of higher order in $\alpha_s(p_T^2)$, have been included. These diffractive diagrams turn out to modify the result substantially $^{6}$. Results of the calculations of both transverse and longitudinal mesons are summarized in fig. 2b). Note again that charged $\rho$'s are more easily produced than
neutrals. The exception occurs for $p_L^0 p_L^0$, due to the extra diffractive diagrams with gluon exchanges for the neutral vector mesons.

Results for $\gamma\gamma \rightarrow M\bar{M}$ for other mesons can be computed analogously, again with the approximation that all quark masses are set equal to zero. The overall normalizations thus only differ through the different decay constants (c.f. eq. (4)). In this way one obtains the results of table 1, which shows relative cross sections. Flavour singlet mesons have not been included in this table, because they require a separate analysis due to the pure gluon component of the relevant distribution amplitudes and the associated larger number of diagrams in the hard scattering amplitudes. These have been computed in ref. 7 for most wave functions of interest.

We next turn to the case of baryon-antibaryon production in two-photon processes. The theoretical formalism is here essentially the same as described for $\gamma\gamma \rightarrow M\bar{M}$, the main difference being the larger number of diagrams contributing to the hard scattering amplitudes. Another important difference comes from the fact that in the case of baryons we do not have a "sum rule" (as eq. (4)) which can determine the overall normalization of the baryonic distribution amplitudes. Instead one must rely on an indirect method for fixing the normalization: one can compute another exclusive process involving the same baryons, and then extract the normalization factor for any given amplitude. Normally the "reference process" $J/\Psi \rightarrow p\bar{p}$ is chosen. Results for $\gamma\gamma \rightarrow p\bar{p}$ and $\gamma\gamma \rightarrow n\bar{n}$, using Chernyak-Zhitnitsky wave functions, are presented in ref.8. As expected, the cross section for $\gamma\gamma \rightarrow n\bar{n}$ is substantially smaller than the $\gamma\gamma \rightarrow p\bar{p}$ cross section. Both processes depend rather strongly on the particular wave function chosen.

Similar predictions can be derived for exclusive processes involving other baryons. Since most other baryons are rather heavy, the massless approximation may be somewhat drastic in these cases.

The main difficulty with exclusive two-photon processes involving baryons comes from the fact that all cross sections are predicted to fall dramatically with increasing $p_T^2$. Thus, from the dimensional counting rules we already saw that $d\sigma(\gamma\gamma \rightarrow B\bar{B})/dt \approx s^{-6}$. This makes these processes very difficult to detect at energies where the approximations are clearly under control.

3.2 EXPERIMENTAL PERSPECTIVES

Meson-pair production can be studied experimentally with sufficient counting rates at LEP200, provided the triggers of the LEP experiments are sensitive
to charged particles of low momentum \((p \gg 1 \text{ GeV})\). For \(\pi^+\pi^-\) and \(K^+K^-\) pairs one needs good particle identification and a good discrimination against an overwhelming background from \(\mu^+\mu^-\) pairs. In the case of vector-meson pair production a reconstruction takes place from decay products, and besides a non-resonant background also a combinatorial one has to be subtracted.

The number of events per mass-interval can be expressed as

\[
\frac{dN}{dW} = L_{e^+e^-} \left[ \frac{dL_{\gamma\gamma}}{dW} / L_{e^+e^-} \right] \int \frac{d\sigma(\gamma\gamma \to m_1 m_2)}{d\cos \theta} A(\cos \theta) d\cos \theta.
\]

(7)

Central detectors which are primarily designed for observation of annihilation events, are only sensitive to two-photon events with a limited longitudinal momentum, roughly \(|p_{\gamma\gamma}| < W_{\gamma\gamma}\). The acceptance \(A\), therefore, depends only weakly on the machine energy; in first approximation it can be taken from known acceptances for existing detectors, provided the above limitation in longitudinal momentum is incorporated in the quantity in square brackets, the luminosity function.

Examples of expected event-rates are given in figs. 3 and 4. The acceptance is that of the TPC-detector at PEP. The luminosity function is calculated by numerical integration of the lowest order QED expression. The figures indicate that \(\pi^+\) and \(\rho^0\) pair production can be studied experimentally at LEP200 with sufficient counting rate up to \(W \approx 5-6 \text{ GeV}\), well beyond the resonance region.

The case for \(\pi^0\) pair production is considerably less favourable, as both the expected cross section and the reconstruction efficiency from photon pairs are considerably smaller than for charged pions.

REFERENCES


Table 1 Wide-angle high-energy relations for $\gamma\gamma$ annihilation into two helicity 0 ($h=0$) or helicity $\pm 1$ ($h = \pm 1$) mesons.

$\eta$-$\eta'$ mixing is neglected and $f_\eta \approx f_\pi \approx 93$ MeV.

The $\phi$ is assumed to be an $s\bar{s}$ state.

(From ref.4, modified).

<table>
<thead>
<tr>
<th>Process</th>
<th>Cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 0$</td>
<td></td>
</tr>
<tr>
<td>$\gamma\gamma \to K^+K^-$</td>
<td>$2\frac{d\sigma}{d\Omega}(\gamma\gamma \to \pi^+\pi^-)$</td>
</tr>
<tr>
<td>$\gamma\gamma \to K_LK_S$</td>
<td>$0.3\frac{d\sigma}{d\Omega}(\gamma\gamma \to \pi^0\pi^0)$</td>
</tr>
<tr>
<td>$\gamma\gamma \to \pi\eta$</td>
<td>$0.1(f_\eta/f_\pi)^2\frac{d\sigma}{d\Omega}(\gamma\gamma \to \pi^0\pi^0)$</td>
</tr>
<tr>
<td>$\gamma\gamma \to \eta\eta$</td>
<td>$0.4(f_\eta/f_\pi)^2\frac{d\sigma}{d\Omega}(\gamma\gamma \to \pi^0\pi^0)$</td>
</tr>
<tr>
<td>$\gamma\gamma \to \rho^0\omega$</td>
<td>$0.4\frac{d\sigma}{d\Omega}(\gamma\gamma \to \rho^0\rho^0)$</td>
</tr>
<tr>
<td>$\gamma\gamma \to \omega\omega$</td>
<td>$1.1\frac{d\sigma}{d\Omega}(\gamma\gamma \to \rho^0\rho^0)$</td>
</tr>
<tr>
<td>$\gamma\gamma \to \phi\phi$</td>
<td>$0.2\frac{d\sigma}{d\Omega}(\gamma\gamma \to \rho^0\rho^0)$</td>
</tr>
<tr>
<td>$h = \pm 1$</td>
<td></td>
</tr>
<tr>
<td>$\gamma\gamma \to \rho^0\omega$</td>
<td>$0.4\frac{d\sigma}{d\Omega}(\gamma\gamma \to \rho^0\rho^0)$</td>
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<td>$0.2\frac{d\sigma}{d\Omega}(\gamma\gamma \to \rho^0\rho^0)$</td>
</tr>
</tbody>
</table>
Figure 1. Generic diagram for $\gamma\gamma \rightarrow M\bar{M}$ at large angles.
Figure 2a). Cross sections for wide-angle p-pur production in $\gamma\gamma$ collisions.

Figure 2b). Cross sections for wide-angle p-pair production in $\gamma\gamma$ collisions.

0.2 0.4 0.6 0.8
$\cos^2\theta$

10 100 10 100 $w_{\gamma\gamma}$ $w_{\gamma}$ $w_{\gamma\gamma}$ $w_{\gamma}$

Section 3
Figure 3. Number of $\gamma\gamma \rightarrow \pi^+\pi^-$ events per GeV mass interval vs $W$ expected at LEP200 for 1 $fb^{-1}$ integrated luminosity. The calculation includes an acceptance estimate.

Figure 4. Number of $\gamma\gamma \rightarrow \rho_L\rho_L^*$ events per GeV mass interval vs $W$ expected at LEP200 for 1 $fb^{-1}$ integrated luminosity. The calculation includes an acceptance estimate.
4. HIGH \( P_T \) INCLUSIVE HADRON SPECTRUM AT LEP 200

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4.1 INTRODUCTION

This contribution is an update of our work which appeared in Z. Phys. C and in the yellow report "PHYSICS AT LEP\(^{1}\)" to which we refer for further details and references. It is concerned with the calculation of the single charged hadron inclusive spectrum via the process

\[ e^+ + e^- \rightarrow e^+ + e^- + \gamma + \gamma \rightarrow e^+ + e^- + \text{hadron} + X. \]  

(1)

Two cases will be considered: either both photons are quasi-real (untagged experiment) or one of them is off-shell (single tag experiment).

The general structure of the reaction is shown in Fig. 1. The diagrammatic content of the process \( \gamma + \gamma \rightarrow h + X \) consists of:

a) the BORN term: \( \gamma + \gamma \rightarrow q + \bar{q} \) (Fig. 2);

b) LEADING LOGarithm QCD contributions where at least one of the photons breaks up into collinear partons (quarks or gluons), one of these partons undergoing a hard collision to produce final state hadrons (Fig. 3). In the case, e.g. where one photon only breaks up the cross-section takes the form (cf. Fig. 3a):

\[ \frac{d\sigma^{YY \rightarrow hX}}{d^2z} = \sum_{i,j} \int F_{i/\gamma}(x, \Lambda^2) \, dx \, d\sigma_{i-j}^{\gamma \rightarrow h} \, D_{h/j}(z, M^2) \, \frac{dz}{2z}. \]  

(2)

where \( F_{i/\gamma}(x, \Lambda^2) \) is the anomalous photon component (\( \sim \ln(M/\Lambda) \)) which describes the perturbative break up of the photon into parton \( i \), and \( M \) is as usual a large scale characteristic of the process, with \( M \) of order \( P_T \).

c) QCD corrections of order \( \alpha_s \) (Fig. 4);

d) a particular \( O(\alpha_s^2) \) correction (the BOX) which may be important because of the \( 2-2 \) kinematics (Fig. 5). The above contributions account for the point-like photon interaction including perturbative QCD corrections. These contributions are aimed to be experimentally isolated in order to test rather firm perturbative QCD predictions. One has also to add:
e) the hadron-like part where the photon interacts as a hadron; the hard, large 
\( p_T \) tail, will be taken into account.

Following standard assumptions supported by experiment, one simply adds to the 
contributions a) to d) the cross-section for the photon to interact as a vector 
meson (according to the vector dominance model - VDM - hypothesis) as shown in 
Fig. 6. The conventions and structure/fragmentation functions parametrization are 
as in ref. 1). The scale \( M \) is set equal to \( p_T \), where \( p_T \) is the 
transverse momentum of the observed hadron (in case of the single tag the axis of 
reference is the \( \gamma - \gamma \) CMS). The scale factor in the strong coupling \( \alpha_S \) is also 
taken as \( p_T \). The production of three light flavours (u, d, s) and of the charm 
quark, and the subsequent hadronization are considered separately.

In order to isolate the photon-photon interactions a) - d) one has to consider 
in addition to e) different sources of background for high \( p_T \) jet/inclusive 
hadron production. The main source is the annihilation process: \( e^+e^- \rightarrow \text{hadrons} \), 
which has the characteristics that the total hadronic mass \( W \) is peaked at \( \sqrt{s} \), 
the \( e^+e^- \) CMS energy, whereas for the process (1) the hadronic energy \( W \) (vis) 
is less than \( \sqrt{s} \): it has a rapidly falling spectrum with respect to the \( \gamma - \gamma \) CMS 
ergy. However, hard photon radiation off one of the initial electrons

\[
e^+e^- \rightarrow \gamma + \text{hadrons} \quad , \tag{3}
\]

and especially at LEP 200 energies the reaction (Fig. 7)

\[
e^+e^- \rightarrow Z^0 \gamma \rightarrow q \bar{q} \gamma \quad , \tag{4}
\]
can give rise to a pair of hadronic jets with \( W \) (jets) less than \( \sqrt{s} \); therefore (3) and (4) are contributing to the untagged case.

In general the background has to be studied by careful Monte-Carlo simulations. In the following, however, we instead give an estimate of the contributions from (3) and (4) in the approximation of collinear photon emission (Weisz- 
Säcker-Williams) - i.e. we assume that the photon completely escapes the detection. For the inclusive jet cross-section (for the light - massless - quarks 
\( u, d, s \)) integrated with respect to the jet rapidity the result is the following:

i) for the process (3)

\[
d\sigma / dp_T(\text{jet}) = 12\alpha^3 \sum_{u, \ldots, \bar{q}} e^2 q \ln s/m_e^2 s^{3/2} \int_{x_{\min}}^{x_{\max}} \frac{dx}{x^2} \left[ \frac{1 + (s/s)^2}{1 - s/s} \right] \left( 1 - \frac{x_T^2 s}{2s} \right) , \tag{5}
\]

where \( \sqrt{s} \) is the invariant mass of the \( q\bar{q} \) system, which should correspond 
to \( W \) (vis). The boundaries are given by \( x_{\max} = (1 + \sqrt{1 - x_T^2})/x_T = 1/x_{\min} \), 
\( x_T = 2p_T(\text{jet})/\sqrt{s} \). \( m_e \) is the mass of the electron.
ii) for the process (4) the standard model (with $\sin^2\theta_W = 1/4$) yields

$$
\frac{d\sigma}{dp_T^2(\text{jet})} = \frac{27\alpha}{4\sqrt{2}} \frac{G}{N_G} \frac{p_T^2(\text{jet}) \ln s/m_e^2}{s} \frac{(1 + M_Z^2/s)^2}{(1 - M_Z^2/s)} \times \frac{1 - 2p_T^2(\text{jet})/M_Z^2}{\sqrt{1 - 4p_T^2(\text{jet})/M_Z^2}} ,
$$

(6)

where $N_G$ denotes the number of families (for the curves $N_G = 2$ is taken).

In deriving eq. (6) it is assumed that the $q\bar{q}$ system is produced at the resonance peak, i.e. the hadronic energy = $W$ (vis) = $M_Z$. The quark jets are expected to peak at rapidities (in the $e^+e^-$ CMS) near

$$
y = \ln \left[ \frac{M_Z^2}{2\sqrt{s} p_T(\text{jet})} (1 \pm \sqrt{1 - 4p_T^2(\text{jet})/M_Z^2}) \right] .
$$

(7)

For charm production (in the approximation $m_c = 0$) the cross-sections are

for i) $d\sigma(\text{charm}) = 2/3 \, d\sigma (u+d+s)$ ,

for ii) $d\sigma(\text{charm}) = 5/18 \, d\sigma (u+d+s)$ .

(8)

In order to obtain the inclusive charged hadron cross-section $d\sigma/dp_T$, one has to fold the expressions (5,6) with the corresponding fragmentation functions. For the charm fragmentation we take simple parametrizations of the Lund model for charm decay$^2$ in order to obtain rough estimates: a hard distribution (actually corresponding to $p_{\text{charm}} = 2.5$ GeV)

$$
D(\text{charged hadrons}) = 10 (1 - z)^{4.1} + 1.6 (1 - z)^{2.5} ,
$$

(9)

and a softer one (corresponding to $p_{\text{charm}} = 20$ GeV)

$$
D(\text{charged hadrons}) = 8.2 (1 - z)^{4.5} , \quad z > 0.2 .
$$

(10)

Bottom flavours are neglected in the calculation, since their contribution should be insignificant, because of the charge suppression factor $e^{-}\tau_q$ and the heavy mass of the bottom quark.

4.2 JET PRODUCTION - UNTAGGED CASE

In Fig. 8 we plot the different jet cross-sections resulting from the different contributions: the dashed line represents the $(u, d, s)$ jet cross-section according to the BORN (Fig. 2) term. Adding the LEADING LOGarithmic terms (Fig. 3a) and the charm quark the solid line results (for the fragmentation of the initial photon the parton model is used,

$$
F_{c/g}(x, M^2) = \frac{\alpha}{2\pi^2} \left( \frac{Z}{3} \right)^2 N_c \left[ x^2 + (1-x)^2 \right] \ln \frac{M^2}{m_c^2} ,
$$

$$
F_{G/g}(x, M^2) = 0 .
$$

(11)
The value for the mass of the charm quark is $m_c = 1.5$ GeV. One remarks that the jet production cross-section is, to a very good approximation, dominated by the BORN term. The charm quark jet cross-section is about 60% of the light quark jet cross-section at large $p_T$, $p_T(jet) > 10$ GeV/c. The dotted line shows the contribution from the process (3), and the dashed-dotted line is the $Z^0$ contribution (4) (in these curves the charm quark is included, however, $m_c = 0$ is taken). Indeed one observes that especially the $Z^0$ "background" dominates the $\gamma\gamma$ process for $p_T$ larger than 10 GeV. In this Fig. no cuts with respect to $W$ (vis) are applied.

4.3 PRODUCTION OF CHARGED HADRONS - THE UNTAGGED CASE

The inclusive single charged hadron cross-sections $d\sigma/dp_T$ are shown in Fig. 9 for the LEP 200 energy. The thin solid line describes the BORN contribution for the light quarks ($u, d, s$) - (Fig. 2 only) -, whereas the thick solid one shows the sum of all perturbative contributions a) - d) (Figs. 2-5): the corrections are dominated by the LEADING LOGarithmetic QCD terms (Fig. 3); the gluon content of the photon is rather important, since it gives a large contribution to these terms, namely 80% (28%) of the BORN term at $p_T = 6$ GeV (10 GeV). The order $\alpha_s$ terms (Fig. 4) are negative, but small i.e. $-10\%$ of the BORN term; the BOX (Fig. 5) is of the same order of magnitude. The VDM component is plotted as a hatched band: the lower line neglects the effects of the primordial $K_T$ of the partons and the higher one includes a $K_T$ value of 400 MeV/c. The higher curve should be used when comparing with forthcoming experiments. The full hadronic spectrum for $\gamma\gamma$ into charged hadrons is thus the sum of the QCD and the VDM components. The dashed line adds to the $u, d, s$ contributions the charm jet, fragmenting into charged hadrons according to the fragmentation functions given above - the band represents the result for the two different fragmentation functions, eqs. (9, 10). It is worth to remark that the alternative approach to describe hadron production, namely evaluating by Monte-Carlo methods the production of exclusive events - corresponding to some of the diagrams of Figs. 2-6 -, is in good agreement with the inclusive single hadron spectrum (denoted by QCD) in Fig. 9.

For comparison the large "background" $p_T$-spectra derived from the processes (3 and 4) are included in Fig. 9: the dashed-dotted line is the $Z^0$ contribution, the dotted one is the radiative annihilation contribution. The charm quark is included (also here the two lines correspond to the hard (soft) charm fragmentation functions given above). Because of the underlying flat jet cross-sections (Fig. 8) - the diagram Fig. 7 results, in the approximation considered, in a $p_T(jet)$ spectrum peaking at $p_T(jet) = M_c/2$ - the hadronic $p_T$-spectra due to (3) and (4) are rather flat too, dominating the $\gamma\gamma$ rates completely.
in the $p_T$-range of Fig. 9. Although it is interesting by itself to study the process (4) at LEP 200 - e.g. to observe the quark jets from $Z^0$ - it is crucial for testing QCD in the $\gamma\gamma$ channel to suppress the huge "background" shown in Fig. 9 for the untagged case. The standard way to suppress background contributions is to apply a cut on the visible hadronic energy $W(\text{vis})$. In the following, we estimate the effect of this cut by identifying $W(\text{vis})$ by the invariant mass of the $\gamma\gamma$ system for the contributions a) - e), and of the $qq$ system in the case of processes (3) and (4), which however amounts to detect all charged and neutral hadrons. For the estimates $W(\text{vis})$ is taken to be less than 70 GeV - this value corresponds to the fraction 0.35 of the available $e^+e^-$ CMS energy, a fraction used by PLUTO and TASSO in analyzing $\gamma\gamma$ induced hadronic events. This cut should reduce the $Z^0$ background considerably - in our approximation it vanishes. The non-vanishing spectra $d\sigma/dp_T$ for charged hadrons are summarised in Fig. 10. The curves denoted by QCD are the $\gamma\gamma$ produced rates via $u, d, s$ quarks (solid line) and $u, d, s, c$ quarks (dashed line, with hard charm fragmentation). The VDM part is as in Fig. 9 plotted as the hatched line. The dotted line indicates the background left from process (3) in the Weizsäcker-Williams approximation. Because of the $W(\text{vis})$ cut the "QCD" spectra are becoming steeper than without imposing the cut (cf. Fig. 9).

4.4 PRODUCTION OF CHARGED HADRONs - THE SINGLE TAG CASE

We briefly discuss the results for single tag experiments in which one of the photons is off-shell. For LEP 200 the range $5 \text{ GeV}^2 < Q^2$ - corresponding to a minimum detection angle of 23 mrad - is particularly interesting. For the curves shown in Fig. 11 the following ansatz/input is used: the no-tag expressions are kept for the perturbative sector (a-d), only the logarithmic factor appearing in the photon flux, namely $\ln p_{\text{beam}}^2/m_c^2$ is replaced in the case of a virtual photon by $\ln Q_{\text{max}}^2/Q_{\text{min}}^2$, where $Q_{\text{max}}$ ($Q_{\text{min}}$) are defined by the tagging condition. Therefore small contributions from the longitudinally polarized virtual photon are neglected. Concerning the anomalous part the factor $\ln(p_T^2/Q^2)$ which enters the definition of $F_{1/2}(x, p_T^2)$ - cf. eq. (2) - is replaced by $\ln p_T^2/Q_{\text{max}}^2$, with $Q_{\text{max}}^2 = Q_{\text{min}}^2/4$ in the expression for $F_{1/2}(x, p_T^2)$; the $x$-dependence being otherwise unchanged (this result can be obtained from ref. 4)). As for the VDM part, we take into account the damping of the virtual photon coupling to the vector meson:

$$4.5 \times 10^{-3} \text{ m}^4 / (m^2 + Q^2)^2$$

where we choose $m^2 = 0.6 \text{ GeV}^2$. The results are shown in Fig. 11 for the tagging range $5 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$. The solid curves QCD and VDM do not include a cut on the visible energy; the dashed one estimate the cut-effect for $W(\text{vis}) < 70 \text{ GeV}$, in the same way as described for the no-tag case, although

* See discussion by convenor in section 1.4.2 above
here this cut is not crucial for suppressing the background. As expected, going off-shell the VDM component is reduced more than the perturbative QCD part. Moreover the background due to processes (3) and (4) is vanishing. Therefore the theoretical predictions should be quite reliable in the $p_T$-range covered in Fig. 11.

However, based on the integrated luminosity of 500 pb$^{-1}$, the single tag rates may be too low, i.e. at $p_T = 8$ GeV - where the VDM background is already negligible - only 15 events/GeV are to be expected. Therefore, at LEP 200 the no-tag experiment may be favoured, since more events will be observed; from Fig. 10 - including the cut on $W$ (vis) - in the interesting range $p_T = 8$-14 GeV the expected number of events is about 450, which should be enough to test QCD in $\gamma\gamma \rightarrow$ hadrons at $p_T$'s not accessible to present experiments.

References


2) G. Ingelman, private communication; we kindly acknowledge the histograms for the charm fragmentation function at $p_{\text{charm}} = 20$ GeV, from which the parametrisation (eq. 10) in the text is obtained.

3) S.L. Grayson, $\gamma\gamma$ subgroup for LEP 200.

Figure Captions

Figure 1 : Two-photon reaction $e^+e^- \rightarrow e^+e^-$ hadrons.

Figures 2-5: Diagrams contributing to the process $\gamma\gamma \rightarrow hX$ corresponding to (a-d) described in the text. Solid lines denote quarks, curly ones are gluons, and the wavy ones are photons.

Figure 6 : VDM diagrams corresponding to e).

Figure 7 : Diagrams for the $Z^0$ $\gamma$ "background".

Figure 8 : Single jet cross-sections as function of $p_T^{(\text{jet})}$ for LEP 200. Solid (dashed) line is the light and charm quark (u, d, s-BORN) jet spectrum. Dashed-dotted and dotted lines are from process (4) and (3), respectively, as described in the text.

Figure 9 : Single charged ($\pi^+$ and $K^+$) hadron cross-sections as function of $p_T$ for the untagged experiment. The different curves are described in the text.

Figure 10 : Same as in Fig. 9, but with the cut on $W (\text{vis})$, $W (\text{vis}) < 70$ GeV.

Figure 11 : Single charged hadron cross-section as a function of $p_T^{\gamma\gamma}$ - with respect to the $\gamma-\gamma$ CMS - for the single tag experiment. The lines denoted by QCD are from contributions (a-d) including charm (with the hard decay), the ones denoted by VDM correspond to e). The solid (dashed) lines are for no cut in $W (\text{vis})$ (for $W (\text{vis})$ less than 70 GeV).
Section 4

Fig. 4

Fig. 5

Fig. 6

Fig. 7
5. THE PHOTON STRUCTURE FUNCTIONS AT LARGE $Q^2$.

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5.1 INTRODUCTION

Quantum chromodynamics is supported by experimental results of many short-distance processes. Among the processes that have been thoroughly investigated, deep-inelastic electron-photon scattering (Fig. 1) is of special interest. This reaction has had a long theoretical history\cite{1-8} before first measurements were reported\cite{9}. The physical interest in the analysis of the photon structure function with regard to QCD is based on the following key points:

i) As a consequence of asymptotic freedom, the structure function rises linearly with \( \log Q^2 \) in leading order, the slope being predicted by QCD.

ii) In next-to-leading order, the absolute size of all moments of \( F_2 \) with \( N > 2 \) is asymptotically fixed by the \( A \) parameter.

A spurious singularity at \( N = 2 \) is not expected to spread to larger \( N \) values as can be inferred from electron scattering off off-shell photons that is completely calculable perturbatively\cite{5}. Higher orders can only be calculated for yet larger \( N \) values since the residual non-perturbative contributions have to fall off faster than the perturbative component.

Applying such a QCD analysis to medium range \( Q^2 \) values, supplementary assumptions or the residual non-perturbative part of the structure function are needed. These assumptions are beyond the realm of perturbative QCD calculations, and they must be subject to experimental scrutiny. The \( e\gamma \) experiments carried out till now, though limited in statistics, provide in fact a consistent picture.

LEP I\cite{8} and still better LEP II with a total CM \( e^+e^- \) energy of approximately 200 GeV offers the unique opportunity to measure the photon structure function over a wide \( Q^2 \) range from 10 to 2000 GeV\(^2\) (Fig. 2). The large lever arm in \( Q^2 \) can be exploited to study carefully the rise in \( \log Q^2 \):

i) The linear rise in \( \log Q^2 \) is a consequence of the running coupling constant. Freezing the coupling constant at an initial value of \( Q_0^2 = 5 \) GeV\(^2\), for instance, bends \( F_2(x, Q^2) \) asymptotically to a scale invariant function that is independent of \( Q^2 \)[10]. This effect can show up only in the highest \( Q^2 \) bins accessible in LEP II.
ii) The slope parameter is reduced in QCD by an amount of $O(\alpha)$ compared with the parton model \cite{4}. This unique prediction of QCD can experimentally be clearly proved at an integrated luminosity of $\int \mathcal{L} dt=500 \text{ pb}^{-1}$, unperturbed by higher-twiss effects at large $Q^2$.

5.2 STRUCTURE FUNCTIONS

The cross-section for deep inelastic electron scattering off a photon target (Fig. 1), is parametrized by 2 structure functions,

$$\frac{d\sigma}{dx dy} = \frac{4\pi^2 \alpha^2 s (e^2)}{Q^4} \left[(1-y) F_2(x, Q^2) + y^2 x F_1(x, Q^2)\right] \quad (1)$$

$F_1$ and $F_2$ are proportional to the cross sections for transversely and longitudinally polarized virtual photons,

$$F_1 = F_T \quad (2a)$$

$$F_2 = 2x F_T + F_L \quad (2b)$$

The Bjorken variable $x$ and $y$ can be expressed in terms of momentum transfer, energies and scattering angle,

$$x = Q^2/2q.p_y = Q^2/(Q^2 + \nu^2) \quad (3a)$$

$$y = q.p_y/k_{e^2} p_y = 1-E^e/E\cos^2\theta/2 \quad (3b)$$

The coefficient $y^2 x$ is small under normal experimental conditions and only $F_2$ can be measured easily (see 1.5.3, above, for discussion of $F_L$).

In the quark parton model the structure functions are calculated in a way similar to QED, and quarks are treated as free particles without strong interactions. In this approach, the leading part of $F_2$ is linear in $\log Q^2$ for light quarks while $F_L$ is asymptotically finite and scale invariant,

$$F_{2,\text{QPM}} = \frac{\alpha e^4}{\pi} x \left[x^2 + (1-x)^2\right] \log Q^2 + \ldots \quad (4a)$$

$$F_{L,\text{QPM}} = \frac{4\alpha e^4}{\pi} x^2 (1-x) \quad (4b)$$
Virtual photon exchange dominates at low $Q^2$ and $\langle e'^4 \rangle$ is a sum over the fourth power of all quark charges involved,

$$\langle e'^4 \rangle = 3 \sum_{f} \frac{e_f^4}{e_q}.$$ \hspace{1cm} (5a)

For $Q^2 \gg 1000 \text{ GeV}^2$, Z exchange becomes increasingly important so that:

$$\langle e'^4 \rangle \to 3 \sum_{f} \frac{1}{4} \sum_{i,j=L,R} e_i^2 e_j^2 \left( e_q - \frac{Q^2}{Q^2 + m^2} \frac{Z_i(e)Z_j(q)}{\sin^2\theta_W \cos^2\theta_W} \right)^2$$ \hspace{1cm} (5b)

where the electroweak Z quark charges are given in the Glashow-Salam-Weinberg model as

$$Z_L(F) = I_{3L}(F) - e(F) \sin^2\theta_W$$

$$Z_R(F) = - e(F) \sin^2\theta_W$$

for left and right-handed couplings (Fig. 3). The different behavior of the $\gamma$ structure functions in $x$ and $Q^2$ compared to hadronic targets is a consequence of the unlimited transverse momentum in the pointlike splitting $\gamma \to q\bar{q}$. For heavy quarks the finite mass corrections are given by\cite{1}:

$$F_2^0 = \frac{3\alpha e_0^4}{\pi} \left[ \frac{1}{x} \left\{ 4x^2(1-x) \left( 2 - \frac{m_0^2}{Q^2} \right) - 1 \right\} 
+ x \left\{ x^2 + (1-x)^2 + \frac{4m_0^2}{Q^2} x(1-3x) - \frac{8m_0^4}{Q^4} x^2 \right\} \log \frac{1+x_0}{1-x_0} \right]$$ \hspace{1cm} (6a)

$$F_L^0 = \frac{12\alpha e_0^4}{\pi} \left[ \frac{1}{x^2(1-x)} - \frac{2m_0^2}{Q^2} x^3 \log \frac{1+x_0}{1-x_0} \right]$$ \hspace{1cm} (6b)

where $x_0$ is the quark velocity in the $Q\bar{Q}$ rest frame and the Bjorken variable is restricted to $x < Q^2/(Q^2 + 4m_0^2)$. Figure 4 shows the charm contribution for various values of $Q^2$.

After switching on perturbative gluon radiation, three mechanisms compete with each other in the final build up of the structure function $F_2$ for rising $Q^2$:

i) the number of quarks rises uniformly through the increasing $\gamma \to q\bar{q}$ splitting probability,

ii) at $x > 0.4$, quarks are lost through increasing gluon radiation (accumulating at small $x$),

iii) gluon radiation is damped due to the logarithmically decreasing coupling constant.
The net effect after solving the Altarelli-Parisi equations asymptotically, is a structure function that keeps rising linearly in \( \log Q^2 \) but with an \( O(1) \) change of the coefficient. (QCD corrections to \( F_L \) turn out to be numerically small.) A comparison between the Born term prediction of the \( \gamma \) structure function and the \( O(1) \) QCD correction is presented in figure 5. Since the structure function can be well determined at \( Q^2 > 200 \text{ GeV}^2 \) in LEP II, this qualitative difference can be established unperturbed by higher-twist effects.

The kinematical increase of gluon bremsstrahlung with \( Q^2 \) is just balanced in QCD by the decrease of the running coupling constant, resulting in a uniform rise of \( F_2 \) with \( \log Q^2 \). In a theory with a small but fixed coupling constant \( \alpha_s \), gluon bremsstrahlung moves the increasing number of quarks (due to \( \gamma + q\bar{q} \) splitting for rising \( Q^2 \)) all down to small \( x \) values. For finite \( x > 0 \) the structure function becomes asymptotically scale invariant at a size \( (\alpha/\alpha_s) \). This is illustrated in figure 6 for a toy model in which the coupling constant \( \alpha_s \) is frozen at \( Q^2 = 5 \text{ GeV}^2 \) with \( \Lambda = 200 \text{ MeV} \). The comparison with the expected behavior of \( \int x^2 F_2(x,Q^2) \) in QCD proves that LEP II provides a lever arm in \( Q^2 \) large enough to observe direct consequences of the running QCD coupling.

Eventhough the high \( Q^2 \) data at LEP II cannot shed any new light on the problem of the absolute normalization of the structure function and the QCD \( \Lambda \) parameter, that can be studied equally well at medium \( Q^2 \) accessible through PETRA and the (upgraded) PEP, the main points should be summarized for completeness.

In next-to-leading order, needed to fix the absolute scale of the structure function, light-cone expansion plus renormalization group result in the following expansion of the moments of the \( \gamma \) structure function\(^5\):\(^6\)

\[
F^N_2(Q^2) = \sum_{\lambda=NS} A^N_\lambda \left[ a_\lambda(Q^2) \right] \frac{d^4_\lambda}{a_\lambda(Q^2)} + \frac{1}{a_\lambda(Q^2)} \sum_{\lambda=NS} \frac{a^{-1}_\lambda}{d^4_\lambda} + \sum_{\lambda=NS} \frac{b^4_\lambda}{d^4_\lambda} + C_N + O(\alpha_s) \quad (7)
\]

The anomalous dimensions \( d^4_\lambda \), the \( a^{-1}_\lambda \), etc. are calculable numbers in perturbation theory whereas the \( A^N_\lambda \) are remnants of non-perturbative long-distance QCD. They must also contain contributions that remove the singularities induced when \( d^4_\lambda \to 0 \) for \( N \to 2 \), etc. Two approaches have been proposed to solve this problem:

i) Inventing a parametrization of quark and gluon densities to describe the \( \gamma \) structure function at moderate \( Q^2 \), allows the calculation of the \( Q^2 \) evolution in a way analogous to deep inelastic scattering on hadronic targets\(^{10-11}\). In this approach no more singularities are encountered. The slope of the structure function in \( \log Q^2 \) can well be predicted. The sensitivity on \( \Lambda \), however, is lost since \( \alpha_s^2(Q^2) - \alpha_s^-(Q^2) \alpha \log Q^2/Q^2 \) does not depend on \( \Lambda \) in leading order.

ii) In a different approach that does not merely copy well-known patterns in deep-inelastic lepton-nucleon scattering, the singularities in the point-like piece are isolated and canceled against pole terms with properly fixed residue \( A^{-1}_N \). The remnants of
the expansion about the poles are summarized in a free parameter $\lambda$. Besides this non-perturbative parameter one must expect, on general physical grounds, a VDM type contribution to be present in $\Lambda_N^l$ \cite{10}. If the transverse momentum in $\gamma + q\bar{q}$ is small, the lifetime of the quark-antiquark pair is long, and the overlap with the low-lying resonances $\rho$, $\omega$, $\phi$ is large. The $x$ dependence of this component drops as $x H (1-x)$ for $x \to 1$, and since this contribution comes with a coefficient proportional to $[\alpha_s(Q^2)]^4 N$, it will die out for $Q^2 \to \infty$. Two remarks are mandatory: (a) The structure function is not affected by the regularization procedure beyond $x > 0.2$ \cite{11}, (b) The strength $H < 0.2$ of the hadronic component can be traded for a change of the $\lambda$ parameter, with moderate impact on $\Lambda$ \cite{12}. Duality arguments might be invoked to explain this rather natural observation. Adopting the PLUTO values for these two free parameters, the prediction for the structure function at $Q^2 = 200$ GeV$^2$ is shown in figure 7 for $\Lambda_{MS} = 183_{+80}^{-60}$ MeV. The errors correspond to an integrated luminosity of 500 pb$^{-1}$ and 100 $< Q^2 < 500$ GeV$^2$. The figure proves that the photon structure function can accurately be measured at large $Q^2$ in LEP II.

In conclusion, LEP II offers the unique opportunity to measure the photon structure function over a large $Q^2$ range up to $\sim 2000$ GeV$^2$. Two crucial predictions of QCD can be tested in this experiment: the linear rise in log $Q^2$ as a consequence of asymptotic freedom, and the large renormalization $O(1)$ of the shape of the structure function due to gluon bremsstrahlung, unperturbed by higher-twist effects.

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FIGURE CAPTIONS


2. $Q^2$ distributions for PEP, LEP I and LEP II.

3. Effective quark charges in electron-photon scattering at high energies.

4. Contribution of charmed quarks to the photon structure function for various $Q^2$ values.

5. $O(1)$ change of the shape of the photon structure function after switching on gluon Bremsstrahlung.

6. Comparison of the $Q^2$ evolution of the photon structure function in QCD with a theory in which the coupling constant is frozen.

7. Regularized QCD prediction for the photon structure function at $Q^2 = 200 \text{ GeV}^2$ and sensitivity to the QCD $\Lambda$ parameter. Errors bars correspond to an integrated luminosity of 500 pb$^{-1}$ and $100 < Q^2 < 500 \text{ GeV}^2$.

Figure 1
Section 5

Figure 2

Figure 3
Figure 6

Figure 7

$Q^2 = 200 \text{ GeV}^2$

$\Delta_{\overline{\text{MS}}} = 183^{+60}_{-60} \text{ MeV}$

$F_2(x, Q^2/\alpha) = ...$