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GAUGE FIXING IN FULLY GAUGE-INVARIANT FIELD THEORY
OF CLOSED BOSONIC AND NEVEU-SCHWARZ STRINGS

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A B S T R A C T

The gauge fixing procedure is reanalyzed in our fully gauge-invariant closed bosonic and Neveu-Schwarz string field theory which does not have any constraint on string functionals and gauge parameters. The relations to other formulations are clarified; in particular, it is shown that our theory recovers other formulation with constraints $L_+ - L_- = 0$ after partial gauge fixing but without any truncation. Complete gauge fixing is also made and the expected propagator is obtained. The constraint $L_+ - L_- = 0$ appears as a field equation in our formulation.

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There have been various attempts to construct gauge-invariant field theory of closed bosonic and Neveu-Schwarz strings¹⁻⁹⁾. However, the formulations in Refs. 1-6) have required the constraint $L_+ - L_- = 0$ on the string functionals and/or gauge parameters, which may be considered an unsatisfactory feature of these formulations. It is then desirable to have a formulation without this constraint, i.e., fully gauge-invariant theory.

Up to now, there appear to be several proposals to circumvent this problem. In our previous papers⁷⁾, we have proposed a fully gauge-invariant formulation of closed string field theory without any constraint and have shown that the theory reproduces the formulations with constraints¹⁻⁶⁾ upon partial gauge fixing and consistent truncation. However, the simplified procedure presented in Ref. 7) involved truncation of some string fields and is not completely satisfactory. Later, a seemingly different proposal⁸⁾ has been made for fully-invariant theory with "larger" invariance than BRST and it was shown that the theory describes the correct spectrum. More recently, the same action as ours in Ref. 7b) has been proposed^{*)} and it has been shown to reproduce the closed string spectrum¹⁰⁾. The situation in the closed string field theory thus looks complicated with various proposals.

The purpose of this paper is to clarify the relation of our original formulation⁷⁾ without constraints to other formulations^{1-6), 8-10)} and further examine our theory. Namely, we show that actually all other proposals are equivalent to ours or can be obtained by partial gauge fixing but without any truncation. An important point in our formulation is that the constraint $L_+ - L_- = 0$ appears as a field equation. We also carry out a complete gauge fixing in our fully-invariant theory and show that it gives the expected propagator $1/(L_+ + L_-)$ with two line integrals $\oint b_{zz}$ and $\oint b_{\bar{z}\bar{z}}$ of the antighost around it, as expected¹¹⁾.

Before discussing the relations of our theory to other proposals, let us briefly summarize our formulation⁷⁾. The Fock space of the closed string is a direct product of two spaces for right-moving (+) and left-moving(-) modes. The BRST charge on the full space is given by

*) The action is also considered in Ref. 9).

$$Q = c_0^+ L_+ + c_0^- L_- + b_0^+ M_+ + b_0^- M_- + \tilde{Q}, \quad (1a)$$

$$= c_0 H + \tilde{c}_0 N + b_0 M + \tilde{b}_0 P + \tilde{Q}, \quad (1b)$$

with

$$c_0^\pm = (c_0 \pm \tilde{c}_0)/\sqrt{2}, \quad b_0^\pm = (b_0 \pm \tilde{b}_0)/\sqrt{2},$$

$$L_\pm = (H \pm N)/\sqrt{2}, \quad M_\pm = (M \pm P)/\sqrt{2}. \quad (2)$$

On the basis in (1b), we define ghost zero-mode vacua $|>$ and $|\sim\rangle$ by

$$b_0 |> = \tilde{b}_0 |\sim\rangle = 0,$$

$$\langle |c_0| \rangle = \langle \sim | \tilde{c}_0 | \sim \rangle = 1. \quad (3)$$

Our action is then given by⁷⁾

$$S = i \langle \sim | \langle | \tilde{c}_0 \Phi^\dagger Q \Phi | \rangle | \sim \rangle + h.c. \quad (4)$$

An important point to be noticed is that the string functional Φ is an operator in the ghost zero-modes, depending on b_0 and $\tilde{b}_0$ as well. It cannot be overemphasized that this is necessary to have full gauge-invariance under

$$\delta \Phi = Q \Lambda, \quad (5)$$

without any restriction on Λ , which is also an operator in ghost zero-modes, because of the ghost insertion factor $\tilde{c}_0$ in (4). As a consequence, the b_0 and $\tilde{b}_0$ coefficients in the expansion of Φ also appear in the action, in contrast to the usual formulation¹⁻⁶⁾.

We are now ready to show that this theory is equivalent to that of Ref. 8). To this aim, we simply expand the string functional in $\tilde{b}_0$ and write

$$\Phi = \varphi + \eta \tilde{b}_0. \quad (6)$$

Substituting (6) into (4) and using (3) then yields^{*)}

*) The equivalence of our formulation to that in Ref. 8) was first pointed out by Maina.

$$S = i \langle \sim | \langle 1 \{ \varphi^\dagger [\tilde{c}_0, Q] \varphi + \eta^\dagger Q \varphi - \varphi^\dagger Q \eta \} | 1 \rangle | \sim \rangle, \quad (7)$$

which is precisely the action in Ref. 8). The full invariance (5) is reduced to

$$\begin{aligned} \delta \varphi &= Q \varepsilon, \\ \delta \eta &= -N \varepsilon + Q \varepsilon', \end{aligned} \quad (8)$$

in agreement with Ref. 8b), where ε and ε' are defined by

$$\Lambda = \varepsilon + \varepsilon' \tilde{b}_0. \quad (9)$$

Thus the invariance under (8) is nothing but the BRST invariance.

For our purposes, it will be more convenient to use the basis in (1a). Let us then rewrite our action in a basis-independent way. As we discussed in Ref. 7b), the ghost zero-mode vacua $|>|\sim\rangle$ and $\tilde{c}_0|>|\sim\rangle$ pick up independent components of Φ . Hence, defining $A_0 = \tilde{\Phi} \tilde{c}_0|>|\sim\rangle$, $A_{-1} = \Phi|>|\sim\rangle$, our action (4) can be rewritten as⁷⁾

$$S = (A_0 | Q | A_{-1}), \quad (10)$$

where the action is written in the bilinear product $(|)$, assuming the reality of the string functional^{12)*)}. Thus our action is the same one as proposed in Refs. 9,10).

Now we choose the ghost zero-mode vacuum $| \downarrow \downarrow \rangle$ with ghost number -1 defined by^{**))}

$$\begin{aligned} b_0^+ | \downarrow \downarrow \rangle &= b_0^- | \downarrow \downarrow \rangle = 0, \\ (\downarrow \downarrow | c_0^+ c_0^- | \downarrow \downarrow) &= 1, \end{aligned} \quad (11)$$

on the basis in (1a). If we expand our string functionals as

$$\begin{aligned} |A_{-1}\rangle &= (c_0^+ c_0^- \eta_{-2} + c_0^+ \eta_{-1} + c_0^- \tilde{\eta}_{-1} + \eta_0) | \downarrow \downarrow \rangle, \\ |A_0\rangle &= (c_0^+ c_0^- \phi_{-1} + c_0^+ \phi_0 + c_0^- \tilde{\phi}_0 + \phi_1) | \downarrow \downarrow \rangle, \end{aligned} \quad (12)$$

*) Actually, as Witten points out¹²⁾, the bilinear form (10) is more fundamental.

**) The choice of the ghost zero-mode vacuum does not affect our theory, since we expand string functionals in $b_0^\pm$ as well as $c_0^\pm$.

our action (10) takes the form^{*)}

$$\begin{aligned}
 S = & -(\phi_{-1} | [M_{+} | \eta_{-1}) + M_{-} | \tilde{\eta}_{-1}) + \tilde{Q} | \eta_0]) \\
 & + (\phi_0 | [L_{-} | \eta_0) + M_{+} | \eta_{-2}) - \tilde{Q} | \tilde{\eta}_{-1})] \\
 & - (\tilde{\phi}_0 | [L_{+} | \eta_0) - M_{-} | \eta_{-2}) - \tilde{Q} | \eta_{-1})] \\
 & + (\phi_1 | [L_{+} | \tilde{\eta}_{-1}) - L_{-} | \eta_{-1}) + \tilde{Q} | \eta_{-2})] .
 \end{aligned} \tag{13}$$

The gauge transformation (5) is reduced to

$$\begin{aligned}
 \delta \eta_{-2} &= L_{+} \tilde{\lambda}_{-2} - L_{-} \lambda_{-2} + \tilde{Q} \lambda_{-3} , \\
 \delta \eta_{-1} &= L_{+} \lambda_{-1} - M_{-} \lambda_{-3} - \tilde{Q} \lambda_{-2} , \\
 \delta \tilde{\eta}_{-1} &= L_{-} \lambda_{-1} + M_{+} \lambda_{-3} - \tilde{Q} \tilde{\lambda}_{-2} , \\
 \delta \eta_0 &= M_{+} \lambda_{-2} + M_{-} \tilde{\lambda}_{-2} + \tilde{Q} \lambda_{-1} , \\
 \delta \phi_{-1} &= L_{+} \tilde{\varepsilon}_{-1} - L_{-} \varepsilon_{-1} + \tilde{Q} \varepsilon_{-2} , \\
 \delta \phi_0 &= L_{+} \varepsilon_0 - M_{-} \varepsilon_{-2} - \tilde{Q} \varepsilon_{-1} , \\
 \delta \tilde{\phi}_0 &= L_{-} \varepsilon_0 + M_{+} \varepsilon_{-2} - \tilde{Q} \tilde{\varepsilon}_{-1} , \\
 \delta \phi_1 &= M_{+} \varepsilon_{-1} + M_{-} \tilde{\varepsilon}_{-1} + \tilde{Q} \varepsilon_0 .
 \end{aligned} \tag{14}$$

We note that the only propagating mode that cannot be gauged away completely is η_0 , since $M_{\pm}$ is invertible on fields with non-zero ghost number²⁾.

The field equations for our system are then derived from (13). Using the full gauge transformation (14), it is not difficult to show that, after gauging away the pure gauge states like η_{-2} , one is left with the equations

$$\begin{aligned}
 (L_{+} + L_{-}) \eta_0 &= 0 , \\
 (L_{+} - L_{-}) \eta_0 &= 0 , \\
 \tilde{Q} \eta_0 &= 0 .
 \end{aligned} \tag{15}$$

This implies that the original theory has the right spectrum. This analysis has recently been done also in Ref. 10) and our conclusion is in agreement with the Hamiltonian analysis in Ref. 8b). Note that the physical string functional is η_0 in the expansion (12), in accordance with the obvious generalization of the

^{*)} Note that the theory contains the same number of string functionals as in Ref. 7a) on a different basis and our action is the same.

observation by Thorn¹³⁾ for the open string that the physical states are identified with those constructed on the down vacuum $|\downarrow\rangle$ ($b_0|\downarrow\rangle = 0$) in the first quantized theory.

Since we now know that our theory describes the correct spectrum, it may not be necessary to show how our theory reproduces other formulations with constraints¹⁻⁶⁾. However, it is useful to see explicitly that this can be done only by partial gauge fixing without truncation of any string functionals. This also gives a double check on the correctness of our spectrum.

Our partial gauge fixing is as follows. First, it is possible to put $\eta_{-2} = 0$ by using λ_{-2} or $\tilde{\lambda}_{-2}$. Also use of ε_0 and $\tilde{\varepsilon}_{-1}$ allows to set $\phi_0 - \tilde{\phi}_0 = \eta_0$ and $\eta_{-1} + \tilde{\eta}_{-1} = 2\phi_{-1}$. Finally, noting again that $M_{\pm}$ is invertible on fields with non-zero ghost number²⁾, λ_{-3} or λ_{-1} can be used to make $\eta_{-1} = \tilde{\eta}_{-1}$. This gauge choice is concisely expressed as

$$|A_{-1}\rangle = (b_0^+ - b_0^-)|A_0\rangle. \quad (16)$$

What we have to show next is that this partial gauge fixing does not introduce additional propagating ghost modes. This gauge choice will be one of the conditions we will impose in our full gauge-fixing, and there it will be shown that this does not produce any dynamical modes. After this procedure, one finds

$$S = \frac{1}{2}(\phi|(L_+ + L_-)|\phi) - (\psi|(M_+ + M_-)|\psi) - (\phi|\tilde{\alpha}|\psi) - (\psi|\tilde{\alpha}|\phi) - (\lambda|(L_+ - L_-)|\phi) + (\chi|(L_+ - L_-)|\psi), \quad (17)$$

where we have defined new fields by $\phi \equiv \eta_0$, $\lambda \equiv \frac{1}{2}(\phi_0 + \tilde{\phi}_0)$, $\chi \equiv \phi_1$ and $\psi \equiv \phi_{-1}$. Thus our action precisely reproduces the formulation in Refs. 1-6) only by gauge fixing, but it should be noted that the constraint $L_+ - L_- = 0$ is here imposed on string functionals by the Lagrange multipliers λ and χ . In this theory (17), the gauge parameters must satisfy the constraint $L_+ - L_- = 0$. This corresponds to the existence of second-class constraint in the Hamiltonian analysis, which as a result becomes complicated for the system (17)^{*)}.

Let us now turn to the problem of complete gauge fixing and derivation of the propagator in our theory. A natural choice suggested from our previous analysis of partial gauge fixing is

*) We thank C. Battle and J. Gomis for comments on this point.

$$(b_0^+ + b_0^-) |A_{-1}\rangle = 0, \quad (18a)$$

$$|A_{-1}\rangle = (b_0^+ - b_0^-) |A_0\rangle. \quad (18b)$$

The second condition implies $(b_0^+ - b_0^-) |A_{-1}\rangle = 0$, which together with the first condition gives

$$|A_{-1}\rangle = \eta_0 | \downarrow \downarrow \rangle. \quad (19)$$

The second condition is solved to yield

$$|A_0\rangle = (c_0^+ \alpha_0 + c_0^- \beta_0 + \gamma_1) | \downarrow \downarrow \rangle, \quad (20a)$$

with

$$\alpha_0 - \beta_0 = \eta_0. \quad (20b)$$

The gauge fixing and Faddeev-Popov ghost terms are then given by

$$\begin{aligned} & (B_{+2} | (b_0^+ + b_0^-) | A_{-1}) + (B_{+1} | [(A_{-1}) - (b_0^+ - b_0^-) | A_0]) \\ & + (\bar{C}_{+2} | (b_0^+ + b_0^-) Q | C_{-2}) + (\bar{C}_{+1} | [Q | C_{-2}) + (b_0^+ - b_0^-) Q | C_{-1}]) \end{aligned} \quad (21)$$

The relative + sign of $|C_{-2})$ and $|C_{-1})$ in the last term is due to the anticommuting property of the BRST transformation.

We now find that the system (21) is again invariant under the gauge transformation

$$\delta C_{-2} = Q \Lambda_{-3}, \quad \delta C_{-1} = Q \Lambda_{-2}, \quad (22)$$

which is a phenomenon known as the ghosts for ghosts^{14),2)}. We then make the same gauge choice for these ghosts as (18), and it would be clear that this process continues at all higher levels. It is then not difficult to see that this produces finally the following ghost action:

$$\sum_{n=1}^{\infty} \{ (\bar{C}_{n+1} | (b_0^+ + b_0^-) Q | C_{-n-1}) + (\bar{C}_n' | [Q | C_{-n-1}) + (b_0^+ - b_0^-) Q | C_{-n}') \} \quad (23)$$

where all fields are constrained as

$$(b_0^+ + b_0^-) |C_{-n-1}\rangle = 0, \quad (24)$$

$$|C_{-n-1}\rangle = (b_0^+ - b_0^-) |C_{-n}'\rangle.$$

Equations (24) have the solution

$$|C_{-n-1}\rangle = \eta_{-n} |\downarrow\downarrow\rangle, \quad (25a)$$

$$|C_{-n}'\rangle = (c_0^+ \alpha_{-n} + c_0^- \beta_{-n} + \gamma_{-n+1}) |\downarrow\downarrow\rangle,$$

with

$$\alpha_{-n} - \beta_{-n} = \eta_{-n}. \quad (25b)$$

We then find that the first term in (23) gives

$$\sum_{n=1}^{\infty} \rho_n (L_+ + L_-) \eta_{-n}, \quad (26)$$

and the second term, corresponding to the gauge fixing (18b), may be rewritten as

$$\sum_{n=1}^{\infty} \{ (\bar{C}_{-n}' | Q [|C_{-n-1}\rangle - (b_0^+ - b_0^-) |C_{-n}'\rangle] + (\bar{C}_{-n}' | (L_+ - L_-) |C_{-n}'\rangle) \}$$

$$\approx \sum_{n=1}^{\infty} \rho_n' (L_+ - L_-) \eta_{-n}, \quad (27)$$

using $\{b_0^+ - b_0^-, Q\} = L_+ - L_-$, where ρ_n and ρ_n' are string functionals for $\bar{C}_{n+1}$ and $\bar{C}_n'$ defined as in Eq. (12). In Eq. (27), the first term on the left-hand side is zero because of the constraint (24) and we have neglected non-dynamical terms involving only non-propagating modes. Thus, as announced in our partial gauge fixing, the ghost action (27) corresponding to (16) or (18b) does not produce any dynamical mode. Our fully gauge-fixed action thus turns out to be

$$S_{\text{fixed}} = \sum_{n=0}^{\infty} [\rho_n (L_+ + L_-) \eta_{-n} + \rho_n' (L_+ - L_-) \eta_{-n}], \quad (28)$$

with $\rho_0 \equiv \eta_0$ being the original physical field. Note that the constraint $L_+ - L_- = 0$ is imposed automatically.

$$\text{If we write } A = \sum_{n=1}^{\infty} \frac{1}{\sqrt{2}} (\rho_n + \eta_{-n}) + \eta_0, \quad E = \sum_{n=0}^{\infty} \rho_n',$$

Eq. (28) can be written as

$$S_{\text{fixed}} = (A | (L_+ + L_-) | A) + (E | (L_+ - L_-) | A), \quad (29)$$

using the fact that the bilinear product is non-zero only when ghost number matches. If the antighost $(\bar{C}_{n+1}' |$ or ρ_n is chosen to obey the constraint $L_+ - L_- = 0$, (29) can also be written as

$$(A|Q|A') + (D|A) + (C|A'), \quad (30a)$$

with

$$D = (b_0^+ + b_0^-)B - C', \quad C = (b_0^+ - b_0^-)C', \quad (30b)$$

where all fields are now without restriction on ghost numbers and include zero-modes too. By the same analysis as above, it is easy to show that (30) reproduces (28). Written in this form, our action (30) has the BRST invariance under

$$\begin{aligned} \delta A' &= QA' + D, & \delta A &= -QA - C, \\ \delta D &= 0, & \delta C &= 0, \end{aligned} \quad (31)$$

which will have a non-linear extension and is useful in deriving Ward identities.

If one includes the ghost zero-mode vacuum in (29), its metric $c_0^+ c_0^-$ must also be included in order to give non-zero result. By introducing a source term for string functional, one can derive the propagator that should be used in our string Feynman rules, as for the open bosonic case^{11),15)}. It is easy to show that, because of the metric $c_0^+ c_0^-$ in the kinetic term, the propagator is given by $1/(L_+ + L_-)$ with two line integrals $\oint b_{zz}$ and $\oint b_{\bar{z}\bar{z}}$ of the antighost around it, corresponding to the two moduli of each internal propagator¹¹⁾.

In summary, we have shown that our previous formulation of fully gauge-invariant field theory of closed bosonic and Neveu-Schwarz strings can be reduced to other formulations by partial gauge fixing, and satisfactorily gives the correct spectrum and the expected propagator. The constraint $L_+ - L_- = 0$ follows as a field equation in our theory. It would be interesting to reconsider the interacting case⁷⁾ in the light of the present analysis.

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REFERENCES

- 1) W. Siegel and B. Zwiebach, Nucl. Phys. B263 (1986) 105.
- 2) T. Banks, M.E. Peskin, C.R. Preitschopf, D. Friedan and E. Martinec, Nucl. Phys. B274 (1986) 71.
- 3) A. Neveu and P.C. West, Nucl. Phys. B278 (1986) 601.
- 4) D. Pfeffer, P. Ramond and V.G.J. Rodgers, Nucl. Phys. B276 (1986) 131.
- 5) H. Hata, K. Itoh, T. Kugo, H. Kunitomo and K. Ogawa, Nucl. Phys. B283 (1987) 433.
- 6) H. Terao and S. Uehara, Phys. Lett. 173B (1986) 409.
- 7) a) S.P. de Alwis and N. Ohta, Phys. Lett. 174B (1986) 388;
b) Phys. Lett. 188B (1987) 425.
- 8) a) A. Ballestrero and E. Maina, Phys. Lett. 180B (1986) 53;
b) C. Battle and J. Gomis, Phys. Lett. 187B (1987) 61.
- 9) N. Marcus and A. Sagnotti, Phys. Lett. 178B (1986) 343.
- 10) P. Ramond, V.G.J. Rodgers and R.R. Viswanathan, Florida preprint, UFTP-86-22 (1987).
- 11) S.B. Giddings and E. Martinec, Nucl. Phys. B278 (1986) 91.
- 12) E. Witten, Nucl. Phys. B276 (1986) 291.
- 13) C.B. Thorn, Nucl. Phys. B286 (1986) 291.
- 14) W. Siegel, Phys. Lett. 93B (1980) 170;
T. Kimura, Prog. Theor. Phys. 65 (1981) 338;
H. Hata, T. Kugo and N. Ohta, Nucl. Phys. B178 (1981) 527.
- 15) C.B. Thorn, Nucl. Phys. B287 (1987) 61;
M. Bochicchio, Phys. Lett. 188B (1987) 330.