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**SIGNALS FROM NON-STRONGLY INTERACTING
SUPERSYMMETRIC PARTICLES AT LEP ENERGIES**

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ABSTRACT

The production and detection of non-strongly interacting supersymmetric particles at LEP energies using supergravity models have been investigated. It is feasible to detect most of the proposed new particles up to masses of about 90% of the beam energy using two-body final states. Moreover, particular three-body reactions such as $\gamma\tilde{\gamma}\tilde{\gamma}$, $\gamma\tilde{\nu}\tilde{\nu}$, $\tilde{e}e\tilde{\gamma}$, and $\tilde{W}e\tilde{\nu}$ make the detection possible even for masses well above the beam energies. From these studies we conclude that LEP should provide excellent opportunities to search for SUSY particles.

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1. AN OVERVIEW OF THE SUPERSYMMETRIC EXTENSION OF THE STANDARD MODEL

1.1 Introduction

The Large Electron-Positron (LEP) Collider at CERN and the Stanford Linear Collider (SLC) at SLAC are currently being built. At the same time six large detectors (four at CERN and two at SLC) are under construction. The SLC is scheduled to have beam in 1987, while the first phase of LEP (LEP I) is expected to be completed by 1989. Both colliders will give the first collisions at a centre-of-mass energy $\sqrt{s} = m_Z^0$. The luminosity at LEP ($L \approx 10^{31} - 0.5 \times 10^{32} \text{ cm}^{-2} \cdot \text{s}^{-1}$) is expected to be higher than at the SLC ($L \approx 10^{27} - 10^{29} \text{ cm}^{-2} \cdot \text{s}^{-1}$). Since searching for supersymmetry needs very high luminosity, in the following we concentrate on LEP with an integrated luminosity of about 100 to 500 pb^{-1} per year of running. It is expected that the energy will be subsequently increased to beyond W-pair threshold during the second phase (LEP II). This would obviously provide for the possibility of precision tests for the gauge structure of the Standard Model [1.1].

Electron-positron colliders are ideal for the study of the production of new particles since the production mechanism entails just the electroweak interaction. Furthermore, the initial state is very simple so that rather reliable estimates of the cross-sections and final-state distributions are possible. Thus, at these machines, apart from the possibility of discovering new particles, it may also be possible to obtain the nature of their couplings, thereby enabling one to really test whether or not these (if discovered!) fit into any of the theoretical schemes that have been proposed. One such scheme is supersymmetry [1.2]. It is a discussion of the prospects of discovering supersymmetry at LEP that forms the subject of this report.

The Standard $\text{SU}(2)_L \times \text{U}(1)$ Model has been very successful in its description of all the present data on the electroweak interactions. However, it leaves many fundamental parameters unexplained, such as the magnitudes of particle masses, charged-current mixing angles, the number of quark and lepton generations, the neutral weak mixing angle $\sin^2 \theta_w$, etc. To find origins for these parameters, one must go beyond the Standard Model. One popular extension of it, which attempts to understand the hierarchy of the different mass scales in physics, invokes supersymmetry (SUSY), a new type of symmetry which interrelates bosons and fermions.

Supersymmetric theories [1.3] predict many new particles with spins differing from those of the known particles by half a unit. In the limit of exact SUSY, the masses of particles and their supersymmetric partners would be equal. However, the negative experimental results from PETRA, PEP, and elsewhere (see section 1.8) tell us that SUSY must be badly broken. LEP will offer experimentalists a new opportunity to find supersymmetric particles if they exist, or at least improve dramatically the present limits on their masses, since the coupling constants for these particles can be well predicted.

1.2 Organization of the report

Some motivations for SUSY are discussed in section 1.3. General aspects of the particle content of supersymmetric theories are presented in section 1.4, while details of the supersymmetric model studied in this report are described in section 1.5. We present explicitly the couplings for the mass eigenstates and elucidate the assumptions made in deriving these.

Section 1.6 sets out the strategy for finding sparticles advocated in this report. We concentrate only on non-strongly interacting sparticles, such as sleptons, $\tilde{\ell}$, the photino $\tilde{\gamma}$, winos $\tilde{W}$, and the neutral zino $\tilde{Z}$. Section 1.7 describes the experimental signatures expected for SUSY and summarizes the relevant characteristics of a representative detector that we have assumed in our estimates of signal selection, backgrounds, etc. Finally, the present experimental limits obtained from the non-observation of SUSY particles are summarized in section 1.8.

The search is divided into two parts: two-body final states (chapter 2) and three-body final states (chapter 3). In these chapters the relevant cross-sections for various processes are presented together with the results for their rates and distributions.

We conclude in chapter 4 with a general discussion of the prospects of observing SUSY at LEP energies. All the details of our calculations are concentrated in nine appendixes.

1.3 Motivation for supersymmetry

Ever since its discovery in the early seventies, SUSY [1.2] has been the focus of considerable attention. Although no compelling supersymmetric model has yet emerged, and in spite of the fact that there is no experimental evidence for SUSY, its remarkable theoretical properties have provided sufficient motivation for its study.

Supersymmetry is a novel symmetry that interrelates bosons and fermions, thereby providing a new level of synthesis. It is the most general (known) symmetry of the S-matrix consistent with Poincaré invariance [1.4]. Supersymmetry leads to an amelioration [1.5] (and sometimes, even elimination) [1.6] of divergences that occur in quantum field theory (in particular, quadratic divergences are absent; this feature will play an important role in our subsequent discussion). Since two successive SUSY transformations involve a space-time translation, local SUSY theories [1.7] (and past experience shows that nature prefers local symmetries!) necessarily include gravitation with the gauge fermion (the gravitino) being the supersymmetric partner of the graviton. Further, because supersymmetric quantum field theories exhibit a better ultraviolet behaviour, they may provide a hope of eventually obtaining a consistent quantum theory that includes gravitation. Finally, SUSY is an indispensable ingredient in the construction of the most recent candidate for a ‘theory of everything’, the superstring [1.8].

At this point, one may ask why any extension of the Standard Model need to be considered. After all, as stated in the introduction, the Glashow–Salam–Weinberg (GSW) theory [1.9] seems to account for all known electromagnetic and weak phenomena, and quantum chromodynamics (QCD) [1.10] is generally accepted to be the theory of strong interactions. Indeed, it appears that all known experiments are consistent with a gauge theory based on $SU(3)_c \times SU(2)_L \times U(1)_Y$, with the $SU(2)_L \times U(1)_Y$ being spontaneously broken to $U(1)_{em}$.

In the GSW model, the spontaneous breakdown is brought about by the introduction of an elementary scalar field. This leads to the prediction of (at least) one additional spin-zero particle, the Higgs boson. With the discovery of $W^\pm$ [1.11] and Z^0 [1.12] bosons at the CERN $p\bar{p}$ Collider, only the Higgs boson (and the t-quark) remains to be discovered to complete the particle content of the GSW electroweak theory. It should be pointed out that no elementary spin-zero particles have ever been found. In fact, the *ad hoc* introduction of these is considered by many theorists to be an unpleasant feature of the Standard Model.

The problem [1.13] is the instability of scalar particle masses under radiative corrections. For example, one-loop radiative corrections to these diverge quadratically, leading to corrections of the form:

$$\delta m^2 = O(\alpha/\pi)\Lambda^2 \quad , \quad (1.1)$$

where Λ is a cut-off parameter representing the scale of the theory and $\alpha = e^2/\hbar c = 1/137$. The parameter Λ may be the grand unified theory (GUT) scale $O(10^{15} \text{ GeV}/c^2)$, or the Planck scale $O(10^{19} \text{ GeV}/c^2)$ if we believe there is no new physics all the way up to the scale associated with quantum gravity. On the other hand, we know that for the scalar self couplings to be sensibly treated within perturbation theory, the scalar mass:

$$m^2 \leq O(m_W^2/\alpha) \sim (1 \text{ TeV}^2/c^2) \quad . \quad (1.2)$$

In other words, either the Higgs sector is strongly interacting, or $\delta m^2 \gg m^2$. Such theories (where $\delta m^2 \gg m^2$) have been technically referred to as ‘unnatural’, because the parameters have to be tuned with uncanny precision in order to preserve the lightness of the Higgs mass compared to the GUT or Planck scales. In order to maintain naturalness, i.e. $\delta m^2 \leq m^2$, we see that new physics must set in at a scale $\Lambda \sim O(1 \text{ TeV}/c^2)$.

One possible solution to the problem of naturalness is to imagine that the quarks, leptons, and gauge bosons are all composites [1.14] with an associated scale $\Lambda \sim O(1 \text{ TeV}/c^2)$. While this alleviates the problem at present energies, it does not really represent a solution as we could ask the same questions of any underlying theory. Moreover, it would be difficult to understand why the gauge principle seems to work so well at least up to the highest energies we have achieved up to this point.

A different approach [1.15] would be to eliminate the fundamental scalars and imagine that the Higgs is a composite of new fermions bound by a new force—the technicolour force—that becomes strong at a scale of $O(1 \text{ TeV}/c^2)$. While this is an appealing idea, and while the composite Higgs boson can indeed lead to masses for the $W^\pm$ and Z^0 , it does not account for quark and lepton masses. This led to the introduction of yet another interaction, the extended technicolour. This appears to have phenomenological problems, particularly with flavour-changing neutral currents. Although there appears to be no reason for the technicolour idea not to work, it seems fair to say that no viable model has yet emerged.

Supersymmetry provides yet another approach to the naturalness question [1.16]. This may be simply understood by recalling that fermion masses are protected from large radiative corrections by chiral symmetry. The analogue of eq. (1.1) for fermions is

$$\delta m_f = O(\alpha/\pi) m_f \ln \Lambda/m_f \quad . \quad (1.3)$$

Thus, massless fermions do not acquire masses via radiative corrections. This is a manifestation of the chiral symmetry. The naturalness problem arises because, unlike the case of fermions, there is no symmetry to keep massless scalars from acquiring large masses via radiative corrections. But SUSY interrelates the scalars with their fermionic partners. It then seems reasonable that if fermion masses are protected from quadratic divergences by chiral symmetry, SUSY will protect the supersymmetric partners of these fermions, if they are scalars, from these unwanted radiative corrections.

In practice, at the one-loop level this works because boson and fermion loops both enter the scalar mass correction, but with a relative minus sign. For supersymmetric theories, eq. (1.1) takes the form:

$$\delta m^2 \simeq O(\alpha/\pi) \Lambda^2 - O(\alpha/\pi) \Lambda^2 \quad . \quad (1.4)$$

The exact cancellation requires that the bosons and fermions enter with the same quantum numbers (this is ensured by SUSY) so that their couplings are the same except for SUSY Clebsch–Gordan factors, and also that they have the same masses. Spontaneous breaking of SUSY, as for any other symmetry, maintains the relations between the couplings but breaks the mass relations, and eq. (1.4) takes the form

$$\delta m^2 \simeq O(\alpha/\pi) |m_B^2 - m_f^2| \quad . \quad (1.5)$$

We see from expressions (1.2) and (1.5) that for SUSY to solve the naturalness problem,

$$|m_B^2 - m_f^2| \leq O(1 \text{ TeV}/c^2)^2 \quad ,$$

where m_B^2 (m_F^2) is the boson (fermion) mass squared. In this case, we expect the supersymmetric partners of quarks and leptons to have masses $\leq O(1 \text{ TeV}/c^2)$ and hope that these may show up at LEP energies.

We emphasize that not one of all these reasons for considering SUSY, no matter how compelling it may appear, requires any particular mass scale for supersymmetric particles (sparticles). It is only if we require SUSY to address the naturalness question that the mass scale is fixed. We note that SUSY does not address the question [1.17] ‘Why is the scalar mass 12 (16) orders of magnitude smaller than the GUT (Planck) scale in the first place?’. But in supersymmetric theories, once this value has been set, either ‘by hand’ or by any other mechanism, radiative corrections preserve this hierarchy of scales [1.5, 1.16].

It is not the purpose of this report to discuss the details of the cancellations that lead to the naturalness of supersymmetric theories. For this, we refer the reader either to the original literature or to the several excellent reviews [1.18] of SUSY that already exist. Instead, we now turn our attention to the survey of the supersymmetric partners of the quarks, the leptons, and the gauge and Higgs bosons that need to be introduced to bring about the aforementioned cancellations.

1.4 A brief survey of the sparticle spectrum

Since the generator of SUSY transformations is a spin-half operator, SUSY relates particles differing by one half-unit of angular momentum. The supersymmetric partners of the quarks and leptons can thus have spin zero or spin one. But, since the only way we know of consistently incorporating spin-one particles in an interacting quantum field theory is when they correspond to gauge bosons, the superpartners of the quarks and leptons cannot have spin one. If this was the case, the fact that the supercharge commutes with internal symmetries would lead us to conclude that the quarks and leptons are themselves gauge particles. The superpartners of these must therefore have spin zero. Furthermore, SUSY ensures that these have the same quantum numbers as the usual quarks and leptons. Therefore, for example, corresponding to the left-(right-)handed electron, there is a negatively charged, colour-singlet scalar particle that belongs to the doublet (singlet) representative of the weak SU(2), and likewise for the other particles. We note that there is a scalar particle for each helicity of fermion. There are therefore the same number of Bose and Fermi degrees of freedom. Each pair of these can be grouped together into a ‘chiral superfield’. (In this context, chiral means that the fermionic component of the superfield is chiral.)

In contrast, the gauge fields and their spin-half superpartners, the gauginos, group together to form the ‘vector (or real) superfield’ — the gauge fermions are Majorana spinors. Again, we see that the two bosonic degrees of freedom of the gauge field match those of the gauge fermion. There is thus a colour octet of spin-half fields, the gluinos, and four colour singlet spinor fields corresponding to the gauge fields of $SU(2)_L \times U(1)_Y$. As will be discussed shortly, the weak gauge fermions mix with the superpartners of the Higgs bosons once $SU(2)_L \times U(1)_Y$ is broken.

Finally, we turn to the Higgs content of the theory. The Higgs mechanism, by which the vector bosons $W^\pm$ and Z^0 acquire masses, works as in the Standard Model, i.e. via the introduction of a Higgs doublet which acquires a vacuum expectation value. Supersymmetry, however, precludes the possibility of using a single Higgs doublet to give masses to both the up and the down quarks via Yukawa couplings [1.16]. In the minimal version of the Standard Model, there are, therefore, two doublets. Thus, after the Higgs mechanism, one charged and three neutral scalars remain in the spectrum. Similarly to matter fields, the scalars and their spin-half partners, the higgsinos, form chiral supermultiplets. After the breakdown of $SU(2)_L \times U(1)_Y$ all the weakly interacting fermions (the gauginos and higgsinos) of the same charge mix to form the mass eigenstates. The details of the mixing will be discussed in the next section.

This completes the particle content of the minimal supersymmetric Standard Model. In the case of local SUSY we have, in addition, the supergravity multiplet that contains the graviton and the gravitino. Since we are not going to be concerned with these particles in our discussion of 'SUSY at LEP', we will not elaborate any further on this point.

As has already been mentioned, the particles have the same quantum numbers as their ordinary partners. Thus, any initial state that can lead to the final state $p_1 + p_2 + p_3 \dots p_n$ of conventional particles can also lead to final states where the conventional particles are *pairwise* replaced by their superpartners, e.g. the state $\tilde{p}_1 + \tilde{p}_2 + p_3 \dots p_n$. Here, and in the rest of this report, $\tilde{p}$ will denote the superpartner of the conventional particle p . The replacement is done pairwise so as to conserve angular momentum. Examples of allowed processes are

$$\begin{aligned} e^+ e^- &\rightarrow e^+ e^- & \text{or} & \quad \tilde{e}^+ \tilde{e}^- \\ e^+ e^- &\rightarrow e^+ e^- \mu^+ \mu^- & \text{or} & \quad e^+ e^- \tilde{\mu}^+ \tilde{\mu}^- \\ & & \text{or} & \quad \tilde{e}^+ \tilde{e}^- \tilde{\mu}^+ \tilde{\mu}^- \\ & & \text{or} & \quad e^+ \tilde{e}^- \tilde{\mu}^+ \mu^- \end{aligned} \quad (1.6)$$

At the level of the Lagrangian, this feature arises owing to the structure of the vertices. The trilinear gauge vertex is

$$f \tilde{f} G \rightarrow f \tilde{f} \tilde{G} \quad \text{or} \quad \tilde{f} \tilde{f} G \quad \text{or} \quad \tilde{f} f \tilde{G} , \quad (1.7a)$$

with f being a quark or lepton and G a gauge boson. Similarly, the Yukawa couplings to the Higgs scalars H are

$$\tilde{f}_1 f_2 h \rightarrow \tilde{f}_1 \tilde{f}_2 \tilde{h} \quad \text{or} \quad \tilde{\tilde{f}}_1 f_2 \tilde{h} \quad \text{or} \quad \tilde{\tilde{f}}_1 \tilde{f}_2 h . \quad (1.7b)$$

One can check that angular momentum conservation and baryon (lepton) number conservation exclude other three-point vertices. The resulting vertices are shown in fig. 1.1. Notice that all the vertices involve pairs of sparticles. This is a feature of a wide class of SUSY models. One can introduce a multiplicatively conserved quantum number [1.19] (R-parity) that takes values $+1$ or -1 for conventional and supersymmetric particles respectively. The conservation of R implies that the lightest SUSY particle is stable since it is forbidden from decaying into conventional particles. It is unlikely that this particle would be electrically charged or strongly interacting. If this were the case, it would be bound in atoms or nuclei and would show up as exotic heavy isotopes. For masses up to about $1 \text{ TeV}/c^2$, estimates [1.20] of their abundance are in the range $O(10^{-10}-10^{-6})$, whereas the empirical abundance [1.21] is $\leq O(10^{-20})$. Thus the candidates for this particle are the neutral higgsinos, the sneutrino or the photino (and their linear combinations). Whichever it is, this particle will behave like the neutrino in the experimental apparatus: it will escape unseen, leading to missing energy $\cancel{E}$ and missing transverse momentum $\cancel{p}_T$ in supersymmetric events. Thus $\cancel{E}$ and $\cancel{p}_T$ would be the footprint of SUSY.

The conventional particles of the Standard Model and their supersymmetric partners are listed in table 1.1.

1.5 Theoretical framework

1.5.1 Supersymmetric models

The realization [1.16] that SUSY may provide a solution to the naturalness problem discussed in section 1.3 aroused a great deal of interest in SUSY model building [1.18]. The first models to be

considered had one global SUSY, i.e. $N = 1$ (models with $N \geq 2$ SUSYs do not lead to chiral fermions), with SUSY broken at a scale of $O(1 \text{ TeV}/c^2)$ [1.22] (see section 1.3). It was soon recognized [1.23] that models with such a low SUSY-breaking scale had serious difficulties if SUSY was broken spontaneously. Models with the so-called D-type breaking require the introduction of an extra $U(1)$ group. This leads to new triangle anomalies which can only be cancelled by the introduction of a number of new chiral supermultiplets. This leads to one (or both) of two problems: either new supersymmetric minima appear (these are energetically preferred), or incorporating the new multiplets into the renormalization group equations for the running gauge couplings causes them to become strong before the unification scale, making perturbative unification impossible. The F-type breaking models require the introduction of super-renormalizable terms (with small masses) and a rather *ad hoc* set of fields and couplings.

These difficulties led to the development of the models with a ‘geometric hierarchy’ [1.24]. In these models, SUSY is broken at a scale $\mu_s \gg m_W/\alpha$. In general, this would lead to a mass gap of $O(\mu_s)$. On the other hand, our discussion of section 1.3 would imply that the mass gap is less than $1 \text{ TeV}/c^2$ for the solution of the naturalness problem. The new ingredient in this class of models is that the goldstino couples to the known particles only very weakly, leading to an effective scale of SUSY breaking $\mu \approx (\mu_s \times \text{small coupling})$, which may be then of $O(1 \text{ TeV}/c^2)$ even if μ_s is large. In the earliest models, this was arranged by having a ‘hidden sector’ that communicated with the usual sector of known particles only via the exchange of superheavy particles. Supersymmetry is broken in the hidden sector (so that the goldstino belongs there) and the effect of this is felt in the ‘observable’ sector only through the interactions with the superheavy particles, leading to an effective SUSY-breaking scale $\mu \approx \mu_s^2/m_X$ (m_X is the superheavy particle mass).

A particularly attractive possibility is that the hidden sector interacts with the quarks and leptons only via gravitational interactions. This naturally led to the development of supergravity models [1.25] in which the scale m_X is replaced by the Planck scale m_P , so that $\mu_s \sim O(10^{11} \text{ GeV}/c^2)$ for the effective $\mu \sim O(1 \text{ TeV}/c^2)$. In these models, the breaking of SUSY in the hidden sector at a scale μ_s , because of gravitational interactions, drives the breaking of $SU(2)_L \times U(1)$ at a scale $\mu \approx \mu_s^2/m_P$. Supergravity, of course, is a non-renormalizable theory. Thus, the philosophy is that the model with the hidden and observable sectors coupled via $N = 1$ supergravity represents an effective theory (that may one day be obtained from a more complete theory) valid at scales below m_P in much the same way as chiral dynamics represents the theory of pions at low energy resulting from QCD. We will say a few more words about this later.

Before closing this discussion, one important and rather non-trivial feature of the geometric hierarchy must be stressed. Radiative corrections that appear in the observable sector correspond to a mass scale μ and not μ_s . This was first analysed by Polchinski and Susskind [1.24] in a toy model and has been subsequently analysed in the supergravity context by various authors. It is this remarkable fact that enables this approach to work. Otherwise induced terms of $O(\mu_s^4, \mu_s^3\mu, \mu_s^2\mu^2, \mu_s\mu^3)$ would have rendered the theory inconsistent and probably caused the whole approach to be abandoned.

It is not our purpose to discuss the various models. A survey of SUSY models may be found in the review by Nilles [1.18], which also contains an extensive list of references. Instead, we will directly proceed with setting up the framework in which we work and derive the relevant couplings before going on to expressions for the cross-sections for various processes.

1.5.2 The Supersymmetric Standard Model

In this subsection, we explain our assumptions, describe our general guidelines and derive the couplings of the various particles and sparticles relevant for physics at LEP.

We work within the framework of a supergravity GUT. In other words, we introduce gauge, matter, and Higgs supermultiplets in appropriate representation of a GUT group G as we would in

any SUSY GUT. Supersymmetry is broken in a different sector (the hidden sector referred to earlier [1.24]) which couples to known particles only via gravity [1.25]. The goldstino couplings to the matter multiplets are, therefore, very weak. Thus, the effective SUSY-breaking scale in the observable sector can be adjusted to be $O(1 \text{ TeV}/c^2)$, as was required for the solution of the naturalness problem. Finally, we remark that in this class of models SUSY breaking also drives the breaking of $SU(2)_L \times U(1)_Y$, once the hidden and observed sectors are coupled by gravity.

It is clear that, for our purpose, the relevant theory is obtained by integrating out all the heavy degrees of freedom in the above-mentioned supergravity GUT. One is then effectively left with an $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant globally supersymmetric Lagrangian, along with additional soft SUSY-breaking terms induced by the super-Higgs mechanism [1.25]. In this sense, *local* SUSY plays no direct role in the determination of couplings. Where it enters is in the determination of the masses of some of the sparticles. For instance, all the scalar fermions have a common mass term in the low-energy Lagrangian (this may [1.25] or may not [1.26] be equal to the gravitino mass). According to our philosophy, the parameters in this effective Lagrangian (and the mass term is just such a parameter) must be considered as renormalized at the unification scale m_P . In order for us to be able to use this Lagrangian for perturbative calculations of processes at LEP energies [$O(10^{-17} m_P)$], one must sum the large logarithms resulting from the vast disparity of the two scales. This is done using the renormalization group—the calculation [1.27] is analogous to that of running couplings or the mass ratio m_b/m_t in usual GUTs—and the degeneracy between the various sfermion masses is lifted. We will present explicit mass formulae later in this section. Soft SUSY-breaking gaugino masses are treated in a similar fashion.

To recapitulate, the couplings of the current eigenstates are all fixed by $SU(3)_c \times SU(2)_L \times U(1)_Y$ and SUSY. The couplings of the mass eigenstates are then determined once the mixing matrix is known. This is, in general, model-dependent but, as we will see, it is possible to parametrize all the mixing angles in terms of relatively few parameters (some of which are determined in terms of the sparticle masses). Thus, it is possible to use the couplings obtained herein to put limits on sparticle masses in a relatively model-independent fashion.

Apart from the parameters that occur in the non-SUSY Standard Model, the effective low-energy theory derived as described above can be parametrized in terms of the following:

- i) m_0 , the common sfermion mass at the unification scale,
- ii) $\mu_{1/2}$, the common gaugino mass at the unification scale,
- iii) v'/v , the ratio of the vacuum expectation values of the two Higgs fields, h' and h , that give masses to the $T_{3L} = 1/2$ and $T_{3L} = -1/2$ fermions respectively,
- iv) $2m_1$, the supersymmetric Higgsino mass term,
- v) the trilinear scalar coupling A and its bilinear counterpart B that is equal to $(A - 1)$ in a class of models; these will not concern us as they modify only the masses of those sfermions whose spin-1/2 partners are heavy (such as the top quark).

Now we turn to a discussion of the mixing matrices that determine the mass eigenstates. The supersymmetric partners of the left- and right-handed quarks (leptons) $\tilde{q}_L$ ($\tilde{\ell}_L$) and $\tilde{q}_R$ ($\tilde{\ell}_R$), in general, are not exact mass eigenstates. The mass matrix can be conveniently parametrized as [1.28]:

$$(\tilde{q}_L, \tilde{q}_R) = \begin{pmatrix} L^2 m_0^2 + m_q^2 & A m_0 m_q \\ A m_0 m_q & R^2 m_0^2 + m_q^2 \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} \quad (1.8)$$

with a similar matrix for the sleptons. In eq. (1.8) L and R are model-dependent constants. We see that if the quark (lepton) masses can be neglected, as is the case for all known quarks and leptons (other than the t -quark), the mixing between $\tilde{q}_L$ ($\tilde{\ell}_L$) and $\tilde{q}_R$ ($\tilde{\ell}_R$) is negligible and these are (almost

exactly) mass eigenstates. We will refer to these as left-(right-)handed squarks and sleptons. (We have implicitly assumed $[m_{\ell,q}/m_0(L^2 - R^2)] \ll 1$, i.e. $L \neq R$. This can only be avoided by fine tuning v and v' to be very exactly equal since $m_{\ell,q}/m_0$ is, in general, very small.)

We now turn to the diagonalization of the weak gaugino–higgsino sector. As already mentioned, apart from the $SU(2)_L$ and $U(1)_Y$ gaugino, $\tilde{\lambda}$ and $\tilde{\lambda}_0$, the minimal supersymmetric model contains at least two Higgs supermultiplets h and h' whose scalar components give masses to the $T_{3L} = 1/2$ and $T_{3L} = -1/2$ fermions, respectively [1.16]. The gauginos and higgsinos of the same charge mix to form the mass eigenstates once $SU(2)_L \times U(1)_Y$ is broken.

The mass terms of the $SU(2)_L$ and $U(1)_Y$ gauge and Higgs fermions can be written in the form [1.29, 1.30]:

$$(\tilde{\lambda}_-, \tilde{\chi}_-) = [M_{(\text{charge})}P_L + M_{(\text{charge})}^T P_R] \begin{pmatrix} \tilde{\lambda}_- \\ \tilde{\chi}_- \end{pmatrix} \quad (1.9a)$$

for the charged sector, and

$$1/2(\tilde{h}^0, \tilde{h}^{0'}, \tilde{\lambda}_3, \tilde{\lambda}_0) = [M_{(\text{neutral})}P_L + M_{(\text{neutral})}^T P_R] \begin{pmatrix} \tilde{h}^0 \\ \tilde{h}^{0'} \\ \tilde{\lambda}_3 \\ \tilde{\lambda}_0 \end{pmatrix} \quad (1.9b)$$

for the neutral sector. In eq. (1.9a), the Dirac spinors $\tilde{\lambda}_-, \tilde{\chi}_-$ are defined by

$$\begin{aligned} \tilde{\lambda}_- &\equiv (\tilde{\lambda}_1 + i\tilde{\lambda}_2)/\sqrt{2} \\ \tilde{\chi}_- &\equiv P_L \tilde{h}' - P_R \tilde{h} \end{aligned}$$

with $\tilde{h}$ ($\tilde{h}'$) and $\tilde{h}^0$ ($\tilde{h}^{0'}$) being a shorthand notation for the charged and neutral components of the fermion doublet $\tilde{h}$ ($\tilde{h}'$), respectively. The mass matrices that appear in eqs. (1.9a) and (1.9b) are given by

$$M_{(\text{charge})} = \begin{pmatrix} \mu_2 & gv' \\ gv & 2m_1 \end{pmatrix} \quad (1.10a)$$

$$M_{(\text{neutral})} = \begin{pmatrix} 0 & -2m_1 & gv/\sqrt{2} & -g'v/\sqrt{2} \\ -2m_1 & 0 & -gv'/\sqrt{2} & g'v'/\sqrt{2} \\ gv/\sqrt{2} & -gv'/\sqrt{2} & \mu_2 & 0 \\ -g'v/\sqrt{2} & g'v'/\sqrt{2} & 0 & \mu_1 \end{pmatrix} \quad (1.10b)$$

P_L and P_R are the left and right chirality projectors $(1 - \gamma_5)/2$ and $(1 + \gamma_5)/2$, respectively.

In eqs. (1.10), $2m_1$ is the supersymmetric Higgsino mixing mass term; μ_2 and μ_1 are the soft SUSY-breaking $SU(2)_L$ and $U(1)_Y$ gaugino masses; v and v' are as defined previously. The parameters μ_1 and μ_2 are obtained from $\mu_{1/2}$, the common gaugino mass at the unification scale using renormalization group equations. They are then related according to

$$\mu_1/\mu_2 = (5/3) \tan^2 \theta_w . \quad (1.11)$$

In the class of supergravity models in which the electroweak breaking is radiatively driven [1.31] by the Yukawa couplings of a top quark of mass 40–50 GeV, or in the simple model where $SU(2)_L \times U(1)_Y$ is broken at the tree level [1.32]*), $v'/v \simeq 1$. In this case the mass matrix (1.9a) takes a particularly simple form and can be easily diagonalized. The eigenstates $\tilde{W}_{(+)}$ and $\tilde{W}_{(-)}$ with eigenvalues $m_{(+)}$ and $m_{(-)}$, respectively, are given by

$$\begin{pmatrix} \tilde{W}_{(+)} \\ \gamma_5 \tilde{W}_{(-)} \end{pmatrix} = \begin{pmatrix} f_- & f_+ \\ f_+ & -f_- \end{pmatrix} \begin{pmatrix} \tilde{\chi}_- \\ \tilde{\chi}_- \end{pmatrix} \quad (1.12a)$$

where

$$f_{\pm} = \sqrt{(m_{\pm} \mp \mu_2)/(m_+ + m_-)} . \quad (1.12b)$$

For the sake of definiteness, we will assume that $v = v'$ throughout most of this report. The only time we will consider $v'/v \neq 1$ in any detail, is when we consider the possibility of sneutrinos that are much lighter than selectrons, smuons or staus [see eqs. (1.21)]. This has been studied in the case of $\tilde{W}^+ \tilde{W}^-$ production, and when the $\tilde{\nu}$ is produced in association with the $\tilde{W}$ and an electron, or in pairs in association with the photon (see section 1.3). For this reason we are interested in the chargino eigenstates even when $v'/v \neq 1$.

In this general case, the mass matrix (1.9a) is diagonalized by a biunitary transformation. The eigenvalues $m_{\pm}$ are given by

$$m_{\pm}^2 = 1/2 \{ (4m_1^2 + 2m_W^2 + \mu_2^2) \pm \zeta \} \quad (1.13)$$

with the mixing angles being determined in terms of the angles γ_L, γ_R (between 0° and 180°) defined by

$$\tan \gamma_L = x_-^{-1} \quad (1.14a)$$

$$\tan \gamma_R = y_-^{-1} , \quad (1.14b)$$

where

$$x_- = \frac{(4m_1^2 - \mu_2^2 - 2m_W^2 \cos 2\alpha) - \zeta}{2\sqrt{2} m_W (\mu_2 \sin \alpha + 2m_1 \cos \alpha)} \quad (1.15a)$$

*) These models contain an extra (gauge) singlet field U . The neutralino mass matrix is then 5×5 with off-diagonal $\tilde{h}^0 \tilde{U}$ and $\tilde{h}^{0'} \tilde{U}$ mass terms. Such models seem to have problems with radiative corrections that tend to upset the hierarchy. These can be fixed, at least to leading order, by introducing an extra pair of doublets [1.33].

and

$$y_- = \frac{(4m_1^2 - \mu_2^2 + 2m_W^2 \cos 2\alpha) - \zeta}{2\sqrt{2} m_W (\mu_2 \cos \alpha + 2m_1 \sin \alpha)} \quad (1.15b)$$

with

$$\zeta^2 = (4m_1^2 - \mu_2^2)^2 + 4m_W^2(m_W^2 \cos^2 2\alpha + 4m_1^2 + \mu_2^2 + 4m_1\mu_2 \sin 2\alpha) \quad (1.15c)$$

Here, $\tan \alpha = v'/v$. Unlike for the $v' = v$ case, we have not been able to explicitly eliminate m_1 in favour of the mass of the eigenstates. For $v' = v$, we have $\gamma \equiv \gamma_L = \gamma_R$, with $f_- = -\cos \gamma$ and $f_+ = \sin \gamma$. Further details about the diagonalization may be found in ref. [1.30].

The neutralino mass matrix (1.10b) is more tedious to diagonalize. As already stated in section 1.1 we have assumed that the photino mass (and hence, as we will see, μ_1 and μ_2) is small compared with m_W . We proceed by first diagonalizing the neutralino system for $\mu_1 = \mu_2 = 0$ and then incorporating the effects of SUSY-breaking gaugino masses in first-order perturbation theory. For $\mu_1 = \mu_2 = 0$,

$$-i\gamma_5 \tilde{\gamma}^{(0)} \equiv \cos \theta \tilde{\lambda}_0 + \sin \theta \tilde{\lambda}_3 \quad (1.16a)$$

is the superpartner of the photon and is a zero-mass eigenstate. For $v' = v$, the other mass eigenstates are

$$\begin{aligned} -i\gamma_5 \tilde{Z}_{(-)}^{(0)} &= \sqrt{\mu_-/[2(\mu_+ + \mu_-)]} \\ &\times [-\tilde{h}^0 + \tilde{h}^{0'} + \sqrt{2}m_Z(\cos \theta_w \tilde{\lambda}_3 - \sin \theta_w \tilde{\lambda}_0)/\mu_-] \end{aligned} \quad (1.16b)$$

$$\begin{aligned} \tilde{Z}_{(+)}^{(0)} &= \sqrt{\mu_+/[2(\mu_+ + \mu_-)]} \\ &\times [\tilde{h}^0 - \tilde{h}^{0'} + \sqrt{2}m_Z(\cos \theta_w \tilde{\lambda}_3 - \sin \theta_w \tilde{\lambda}_0)/\mu_+] \end{aligned} \quad (1.16c)$$

and

$$-i\gamma_5 \tilde{h}_1 = (\tilde{h}^0 + \tilde{h}^{0'})/\sqrt{2} \quad (1.16d)$$

with the eigenvalues for $\tilde{Z}_{(-)}^{(0)}$, $\tilde{Z}_{(+)}^{(0)}$, and $\tilde{h}_1$ being μ_- , μ_+ , and $2m_1$, respectively. In our analysis, we have tacitly assumed $m_1 > 0$. The reason for the ' $i\gamma_5$ ' transformation on $\tilde{Z}_{(-)}^{(0)}$ and $\tilde{h}^0$ is that these eigenvectors have a negative eigenvalue for the mass matrix (1.10b). The γ_5 flips the sign of the mass term and the i ensures that the state is self-conjugate. The superscript (0) denotes that these are eigenstates only for $m_{\tilde{\gamma}} = 0$.

The eigenvalues $\mu_{\pm}$ are not independent but satisfy

$$\mu_+ \times \mu_- = m_Z^2 \quad (1.17a)$$

Furthermore, for $v = v'$, the chargino and neutralino eigenvalues are related according to

$$m_{\pm} = |\sqrt{(m_1 - \mu_2/2)^2 + m_W^2} \pm (m_1 + \mu_2/2)| \quad (1.17b)$$

and

$$\mu_- = \sqrt{(m_1^2 + m_Z^2)} - m_1 \quad (1.17c)$$

We see that if μ_2 (and hence $m_{\tilde{\gamma}}$) is zero, $\tilde{W}_{(-)}$ and $\tilde{Z}_{(-)}^{(0)}$ are respectively lighter than the W and Z bosons. This is a general result [1.34]—all SUSY models with $\mu_2 = 0$ contain supersymmetric fermions which are lighter than the W and Z^0 bosons. We will refer to these eigenstates as the wino and the zino.

Before considering the effects of SUSY-breaking gaugino masses, we briefly describe the situation for tree-breaking models. They contain an extra gauge singlet field U that modifies the mass matrix in the neutral sector. Its couplings are such that it leaves the $\tilde{\gamma}^{(0)}$ and $\tilde{Z}_{\pm}^{(0)}$ states unchanged but modifies only the pure higgsino sector. Since we are not greatly concerned about the higgsino sector (because this is the most model-dependent), we will not make the distinction between radiative and tree-breaking models any more. Our results apply for both classes of models as far as photinos, winos, and zinos are concerned.

We now consider the effects of turning on the gaugino masses μ_1 and μ_2 . Since these enter only the gaugino sector, the states $\tilde{\gamma}^{(0)}$, $\tilde{Z}_{(-)}^{(0)}$, and $\tilde{Z}_{(+)}^{(0)}$ further mix to form the mass eigenstates $\tilde{\gamma}$, $\tilde{Z}_{(-)}$, and $\tilde{Z}_{(+)}$. Treating μ_1 and μ_2 to lowest order in perturbation theory, the mass eigenstates are related to $\tilde{\gamma}^{(0)}$, $\tilde{Z}^{(0)}$ by

$$\begin{pmatrix} \tilde{Z}_{(-)}^{(0)} \\ \tilde{Z}_{(+)}^{(0)} \\ \tilde{\gamma}^{(0)} \end{pmatrix} = \begin{pmatrix} 1 & \delta & \epsilon_1 \\ -\delta & 1 & \epsilon_2 \\ -\epsilon_1 & -\epsilon_2 & 1 \end{pmatrix} \begin{pmatrix} \tilde{Z}_{(-)} \\ \tilde{Z}_{(+)} \\ \tilde{\gamma} \end{pmatrix} \quad (1.18)$$

where

$$\epsilon_1 = \sqrt{2} N_1 (m_Z/\mu_-^2) \sin \theta_w \cos \theta_w (\mu_2 - \mu_1) \quad (1.19a)$$

$$\epsilon_2 = \sqrt{2} N_1 (m_Z/\mu_+^2) \sin \theta_w \cos \theta_w (\mu_2 - \mu_1) \quad (1.19b)$$

and

$$\delta = [2N_1 N_2 / (\mu_+ + \mu_-)] \times (\mu_2 \cos^2 \theta_w + \mu_1 \sin^2 \theta_w) \quad , \quad (1.19c)$$

with $N_{1,2} \equiv [(\mu_- , \mu_+)/2(\mu_+ + \mu_-)]^{1/2}$. The corresponding eigenvalues are

$$m_{\tilde{Z}_{(-)}} = \mu_- + \mu_+ |\mu_2 \cos^2 \theta_w + \mu_1 \sin^2 \theta_w| / (\mu_+ + \mu_-) \quad (1.20a)$$

$$m_{\tilde{Z}_{(+)}} = \mu_+ - \mu_- |\mu_2 \cos^2 \theta_w + \mu_1 \sin^2 \theta_w| / (\mu_+ + \mu_-) \quad (1.20b)$$

and

$$m_{\tilde{\gamma}} = |\mu_2 \sin^2 \theta_w + \mu_1 \cos^2 \theta_w| \quad . \quad (1.20c)$$

We note that since we have chosen to define $\tilde{\gamma}^{(0)}$ with a ' γ_5 ' [see eq. (1.16a)], we choose μ_1 and μ_2 as negative numbers.

As already mentioned, we will refer to the state $\tilde{Z}_{(-)}$ as the 'zino' (denoted by $\tilde{Z}$), the state $\tilde{Z}_{(+)}$ as the 'heavy zino' ($\tilde{Z}_h$) and $\tilde{\gamma}$ as the photino. The other neutralino states contain only Higgsino

components. Similarly, we refer to $\tilde{W}_{(-)}$ and $\tilde{W}_{(+)}$ as the ‘wino’ ($\tilde{W}$) and the ‘heavy wino’ ($\tilde{W}_h$), respectively. The coupling of the winos and zinos to gauge bosons and matter fermions (and sfermions) are completely determined now that the mixing angles are known.

The couplings of the current eigenstates, as has been emphasized, are fixed by $SU(3)_c \times SU(2)_L \times U(1)_Y$ and SUSY. We will not list these here (they may be found in ref. [1.18]), but using them in conjunction with the mixing angles determined above leads to the Lagrangian which we used in the remainder of this report and which can be found in appendix A.

The masses of the wino, the photino, and the zino (along with the heavy wino and zino masses) are determined in terms of μ_2 and m_1 . Alternatively, we may choose $m_{\tilde{\gamma}}$ instead of μ_2 [see eq. (1.20c)] and $m_{\tilde{W}}$ as free parameters in terms of which the other masses and couplings are all determined. The only remaining arbitrariness is in the masses of the scalar partners of the fermions. As has already been discussed, these are obtained in terms of a common mass m_0 at the unification scale. For three effective generations in the β -function and for $\sin^2 \theta_w = 0.22$, we have [1.35],

$$\begin{aligned}
m_{\tilde{d}_L}^2 &= m_0^2 + 0.43m_Z^2 \cos 2\alpha + 30.2m_{\tilde{\gamma}}^2 \\
m_{\tilde{d}_R}^2 &= m_0^2 + 0.07m_Z^2 \cos 2\alpha + 28.4m_{\tilde{\gamma}}^2 \\
m_{\tilde{u}_L}^2 &= m_0^2 - 0.36m_Z^2 \cos 2\alpha + 30.2m_{\tilde{\gamma}}^2 \\
m_{\tilde{u}_R}^2 &= m_0^2 - 0.14m_Z^2 \cos 2\alpha + 28.4m_{\tilde{\gamma}}^2 \\
m_{\tilde{e}_L}^2 &= m_0^2 + 0.28m_Z^2 \cos 2\alpha + 2.2m_{\tilde{\gamma}}^2 \\
m_{\tilde{e}_R}^2 &= m_0^2 + 0.22m_Z^2 \cos 2\alpha + 0.6m_{\tilde{\gamma}}^2 \\
m_{\tilde{\nu}}^2 &= m_0^2 - 0.5m_Z^2 \cos 2\alpha + 2.2m_{\tilde{\gamma}}^2
\end{aligned} \tag{1.21}$$

The first term is the common sfermion mass at unification scale, the second term arises from the D-term contribution [notice that it is zero for $v = v'$, implying that the squarks (sleptons) are degenerate for small $m_{\tilde{\gamma}}$], and the last term comes from the radiative corrections. Not surprisingly, the strongly interacting squarks are more affected by these than the sleptons.

We note that for small values of α , the $T_{3L} = +1/2$ components of the scalar doublet tend to be lighter than the $T_{3L} = -1/2$ components. It is, therefore, possible to get the sneutrino considerably lighter than the selectron. Care must be taken to ensure $\tilde{u}_L$ does not become too light since it could not have escaped detection if it had a mass smaller than the present limit. We will concern ourselves with the $\alpha \neq 45^\circ$ case when considering the possibility of light sneutrinos in section 2.4.4.

Finally, since we have diagonalized the neutralino sector only for small values of μ_1 and μ_2 , our analysis is confined to $m_{\tilde{\gamma}} \leq 10$ GeV. Thus almost all of this report will be confined to the case of light photinos. The possibility of a heavier photino and the validity of the approximations used in the calculation will be studied in section 2.2.2.1, where we discuss selectron pair production for this case.

1.6 Experimental strategy

As discussed in section 1.4, sparticles are always produced in pairs. Thus, unless a sparticle is produced in association with another very light one, such as the photino or sneutrino, the mass of any sparticle that can be produced at an e^+e^- collider is obviously limited to the beam energy. Before proceeding to discuss the rates and signatures for the various processes, we briefly outline the strategy which we have adopted for the search for weakly interacting sparticles. We first concentrate on pair-production processes which have the largest cross-sections if they are kinematically accessible. If sleptons were lighter than the gauginos ($\tilde{W}$, $\tilde{Z}$), they would be copiously produced [1.36] if $E_{\text{beam}} > m_{\tilde{\tau}}$. These would then decay 100% of the time via $\tilde{\ell} \rightarrow \ell \tilde{\gamma}$. On the other hand, if the

winos and zinos were lighter than the sleptons, the threshold for $e^+e^- \rightarrow \tilde{W}\tilde{W}$ [1.37] would be crossed first. Winos of such a mass would decay via the three-body modes $\tilde{W} \rightarrow q\bar{q}\tilde{\gamma}$ or $\tilde{W} \rightarrow \ell\bar{\nu}\tilde{\gamma}$ [1.28]. Another possibility is that $\tilde{\nu}$ is light with charged sfermions being heavy, in which case the dominant decay would be $\tilde{W} \rightarrow \ell\tilde{\nu}$. In this case, the zino would decay invisibly via $\tilde{Z} \rightarrow \nu\bar{\nu}$ or $\bar{\nu}\nu$. This possibility can be realized within the class of models we are considering if $v'/v \neq 1$. It could also be that both slepton and wino are so heavy that they cannot be pair-produced. In that case one allowed process could well be $e^+e^- \rightarrow \tilde{Z}\tilde{\gamma}$ [1.38,1.39], since the photino has been assumed to be light. Recall that for light photinos $m_{\tilde{Z}} < m_{Z^0}$ [1.34]. A zino lighter than the slepton would decay via the three-body modes [1.29,1.37], $\tilde{Z} \rightarrow \bar{q}q\tilde{\gamma}$ or $\tilde{Z} \rightarrow \ell\bar{\ell}\tilde{\gamma}$ ^{*)}. Thus, in our computations for slepton pair production, we have assumed that the sleptons decay 100% via $\tilde{\ell} \rightarrow \ell\tilde{\gamma}$, whereas in those for gaugino pair production we have assumed that the gauginos decay via the three-body mode.

Motivated by the possibility of 'neutrino counting' experiments [1.40], we have considered the reactions

$$e^+e^- \rightarrow \tilde{\gamma}\tilde{\gamma}\gamma \quad [1.41]$$

and

$$e^+e^- \rightarrow \tilde{\nu}\tilde{\nu}\gamma \quad [1.42]$$

in chapter 3, where we assume that the photinos and sneutrinos escape detection [1.43]. Even though the rates of these processes are suppressed by an extra power of the fine structure constant, they lead to the characteristic signature of a single photon recoiling against nothing.

In addition to the single-photon processes, in chapter 3 we have also considered other three-body processes in which a heavy sparticle is produced in association with a light photino or sneutrino and an ordinary particle. Such reactions are of interest since a range of sparticle masses larger than the beam energy can be explored just as in the case of $e^+e^- \rightarrow \tilde{Z}\tilde{\gamma}$. The examples of these associated production processes that we have considered are

$$e^+e^- \rightarrow \tilde{e}\tilde{e}\tilde{\gamma} \quad [1.44]$$

and

$$e^+e^- \rightarrow \tilde{W}\tilde{\nu}\tilde{e} \quad [1.45].$$

The event topology we look for in the case of the sparticle pair-production processes is a pair of charged leptons acoplanar with the beam, a substantial amount of missing energy and missing transverse momentum carried off by the photinos, and an absence of hadronic activity. Obviously, in the case of gaugino pair production there will also be events with jets + $\cancel{p}_T$ and jets + leptons + $\cancel{p}_T$ corresponding to the hadronic decay modes of the zino and the wino.

1.7 SUSY signatures

First we briefly comment on SUSY signatures and backgrounds. As shown later, each SUSY reaction has its particular topological signature; moreover the characteristic feature of any SUSY

*) It could be that $m_{Z^0}/2 < m_{\tilde{e}} < m_{\tilde{Z}}$, in which case $\tilde{Z}\tilde{\gamma}$ production could take place at LEP I without slepton pair production. The zino would then decay via the two-body modes $\tilde{Z} \rightarrow q\bar{q}$, $\ell\bar{\ell}$, or $\nu\bar{\nu}$. These branching fractions would depend on $\tilde{q}$ and $\tilde{\ell}$ masses, the leptonic one being somewhat larger in view of the UA1 limit on the squark mass discussed in section 1.8. Some details of the distribution would also be altered.

production is the large missing momentum and energy of undetected photinos or sneutrinos. The detectors to be sensitive to such processes must obviously be hermetic as much as possible. A second very strong constraint on the detectors comes from the need to reject the QED background. In fact at our energies the Standard Model background to SUSY processes comes from:

- i) $e^+e^- \rightarrow \tau^+\tau^-; \tau^+\tau^-\gamma$, with τ 's decaying leptonically;
- ii) $e^+e^- \rightarrow e^+e^-\gamma$ or $q\bar{q}\gamma$, with the γ going down the beam pipe;
- iii) $e^+e^- \rightarrow \ell^+\ell^- e^+e^-$, where the e^+e^- goes down the beam pipe and $\ell = e, \mu, \tau$.

In order to eliminate these backgrounds completely, the $\not{p}_T$ must be larger than

$$\sqrt{s} [\sin \theta_{\min}/1 + \sin \theta_{\min})],$$

where $\theta_{\min}$ is the minimum angle above which electrons, muons or photons can be detected efficiently. Therefore in order to tag the electron/photon from QED events and to distinguish them from SUSY-particle production we need to extend the detector coverage for photons and charged particles down to a few degrees in the polar angle.

At $\sqrt{s} = 190$ GeV, W^+W^- , Z^0Z^0 , and $t\bar{t}$ pairs if $m_t < 70$ GeV, are the main sources of background. The QED processes have been studied using the Berends-Daverveldt-Kleiss code [1.46]; the W^+W^- , Z^0Z^0 reactions using the Monte Carlo generators developed by Kleiss, Blondel and Raimondi [1.47] and the $t\bar{t}$ channel using the LUND Monte Carlo [1.48]. All the calculations are made with $m_W = 82$ GeV, $m_{Z^0} = 93.8$ GeV, and $\sin^2 \theta_w = 0.23$.

The resulting $\not{p}_T$ spectrum for the standard QED processes $e^+e^- \rightarrow \ell^+\ell^-\gamma$ at $\sqrt{s} = m_{Z^0}$ is shown in fig. 1.2, along with a typical spectrum for a two-body SUSY process. As expected, a $\not{p}_T > 0.05\sqrt{s}$ essentially eliminates these events without substantially reducing the SUSY signal. Thus, such a $\not{p}_T$ cut, in conjunction with acoplanarity and acollinearity angle cuts, is sufficient to remove the Standard Model backgrounds for all the supersymmetry processes leading to leptonic final states. Above the $e^+e^- \rightarrow W^+W^-$ threshold there are also $\cancel{E}$ and $\cancel{E}_T$ backgrounds from $W^\pm \rightarrow \ell^\pm \nu$ decays. Although $\sigma(e^+e^- \rightarrow W^+W^-)$ is large (~ 17 pb) these backgrounds can also be beaten down, as will be shown in the next chapter.

In order to get quantitative results for observable cross-sections, a fast detector simulation has been set up with the energy and angular resolutions and acceptances for electrons, photons, hadrons and muons given in table 1.2. For the calorimetry we have checked the impact on our results of going from the 'good' to the 'average' performances quantified in the table.

1.8 Experimental scenario

Electron-positron colliders are ideal for new particle searches because the production mechanisms only involve the electroweak interactions, the initial state is rather simple, and cross-sections can be reliably calculated. Because of this, in the last few years an extensive experimental effort in searching for SUSY particles has been carried out at PEP and PETRA up to a centre-of-mass energy of $\sqrt{s} = 46.8$ GeV.

The processes studied can be divided into three groups:

- pair production of sparticles which have the largest cross-section but where the SUSY masses are limited to the beam energy;
- reactions, like $\tilde{W}e\tilde{\nu}$ or $\tilde{e}e\tilde{\gamma}$, where a sparticle is produced in association with another very light one, such as the photino or the sneutrino, improving the mass limits beyond the beam energy;
- 'single-photon' reactions such as $\tilde{\gamma}\tilde{\gamma}\gamma$ or $\tilde{\nu}\tilde{\nu}\gamma$.

No SUSY particle has been found up to now and these negative results have been interpreted as lower limits on the masses. Table 1.3 is a compilation of these limits at the time this report was prepared [1.49].

On the other hand, hadron-hadron collisions are more complicated, and the interpretation of sparticle mass limits coming from these interactions is more difficult. Nevertheless, squarks and gluinos, if light enough, can be copiously produced at hadron colliders. They rapidly decay into ordinary quarks and a photino, which escapes detection and so gives rise to $\cancel{p}_T$ in hadronic events. Very recently [1.50], the UA1 Collaboration, by an analysis of the $\cancel{p}_T$ data sample obtained at the CERN $p\bar{p}$ Collider, have obtained $m_{\tilde{q}} \gtrsim 70$ GeV, $m_{\tilde{g}} \gtrsim 60$ GeV for $m_{\tilde{\gamma}} = 0$. In a recent paper, Baer, Karatas and Tata [1.51] show that these limits hold for $m_{\tilde{\gamma}} \leq 30$ –40 GeV ($\tilde{q}$) and 20–30 GeV ($\tilde{g}$). Within the framework of the supergravity model, these limits imply restrictions on slepton and photino masses [1.18].

In this report, however, we will concern ourselves with the direct limits on sparticle masses that can be independently obtained at LEP, and hence we will not use the squarks and gluino limits just discussed as input for our considerations. It should be mentioned that the UA1 Collaboration has also announced [1.52] a limit of 41 GeV on the mass of a new charged sequential heavy lepton. Theoretical analyses [1.53] have suggested that this translates into a lower limit of about 40 GeV on the mass of the $\tilde{W}$ and 45 GeV on that of the $\tilde{Z}$. Once again we have not incorporated these limits into our calculations. The cleanliness of e^+e^- collisions should enable us to deduce the nature of the couplings of any sparticles observed, allowing us to test whether they fit into any one of the theoretical schemes proposed.

We have not considered the signals for the production of Higgs bosons in this report, since, as already mentioned, the Higgs sector is the most model-dependent. Unlike as in the standard model, supersymmetric theories always contain more than one Higgs scalar. In the case of the minimal two doublet model discussed in this report, there is one charged Higgs boson, two scalar neutral bosons and one pseudoscalar neutral boson. Further, in the minimal model, the charged Higgs is always heavier than the W-boson whereas the lighter of the two scalars always weighs less than m_Z . Also, their masses and couplings are all fixed in terms of one additional parameter which may be taken to be the mass of the charged Higgs. In models with a more complicated scalar sector, obviously there are no tree-level flavour changing neutral currents (these are absent in the minimal model). Since we do not discuss the detection of Higgs bosons in this report, we will refer the reader to the literature [1.54] on this subject.

We present here the results of a detailed study of the characteristic signatures of the production of some supersymmetric particles and of their use to distinguish these particles from the background. The calculations of the production of these particles, when they are not readily available in the literature, have been carried out by us for this report and are described in detail along with some methods of analysis for separating the supersymmetric particles from the background. The requirement of eliminating the background puts important constraints on the design of the detectors to be used in the search for these particles.

REFERENCES

- [1.1] G. Barbiellini et al., *in* Physics at LEP (CERN 86-02, Geneva, 1986), vol. 2, p. 1.
- [1.2] Y.A. Golfand and E.P. Likhtman, JETP Lett. **13** (1971) 323;
D.V. Volkov and V.P. Akulov, Phys. Lett. **46B** (1973) 109;
J. Wess and B. Zumino, Nucl. Phys. **B70** (1974) 39.
- [1.3] For early reviews, see
P. Fayet and S. Ferrara, Phys. Rep. **32** (1977) 249;
A. Salam and J. Strathdee, Fortschr. Phys. **26** (1978) 57.
- [1.4] R. Haag, J.T. Łopuszanski and M. Sohnius, Nucl. Phys. **B88** (1975) 257.
- [1.5] J. Iliopoulos and B. Zumino, Nucl. Phys. **B76** (1974) 310;
S. Ferrara and O. Piguet, Nucl. Phys. **B93** (1975) 261;
M.T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. **B159** (1979) 429.
- [1.6] S. Mandelstam, Nucl. Phys. **B213** (1983) 149;
L. Brink, O. Lindgren and B. Nilsson, Phys. Lett. **123B** (1983) 323.
- [1.7] S. Deser and B. Zumino, Phys. Lett. **62B** (1976) 113;
D.Z. Freedman, S. Ferrara and P. van Nieuwenhuizen, Phys. Rev. **D13** (1976) 3214.
- [1.8] J.H. Schwarz, Phys. Rep. **89C** (1982) 223;
M.B. Green, Surveys in High Energy Physics **3** (1983) 127;
P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. **B258** (1985) 46;
E. Witten, Nucl. Phys. **B258** (1985) 75.
- [1.9] S.L. Glashow, Nucl. Phys. **22** (1961) 579;
S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264;
A. Salam, Proc. 8th Nobel Symposium, Aspenäsgråden, 1968, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.
- [1.10] For a review, see: W. Marciano and H. Pagels, Phys. Rep. **36** (1978) 137.
- [1.11] UA1 Collab., G. Arnison et al., Phys. Lett. **122B** (1983) 103;
UA2 Collab., M. Banner et al., Phys. Lett. **122B** (1983) 476.
- [1.12] UA1 Collab., G. Arnison et al., Phys. Lett. **126B** (1983) 398;
UA2 Collab., P. Bagnaia et al., Phys. Lett. **129B** (1983) 130.
- [1.13] E. Gildner, Phys. Rev. **D14** (1976) 1667.
- [1.14] For a review see: M. Peskin, Proc. Int. Symposium on Lepton and Photon Interactions at High Energies, Bonn, 1981, ed. W. Pfeil (Univ. Bonn, 1981), p. 880.
- [1.15] For reviews, see:
R. Kaul, Rev. Mod. Phys. **55** (1981) 449;
E. Farhi and L. Susskind, Phys. Rep. **74** (1981) 277.
- [1.16] E. Witten, Nucl. Phys. **B185** (1981) 513;
S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981) 150;
N. Sakai, Z. Phys. **C11** (1981) 153.
- [1.17] G. 't Hooft, Recent developments in field theories, eds. G. 't Hooft et al. (Plenum, New York, 1980), p. 135.
- [1.18] P. Nath, R. Arnowitt and A. Chamseddine, Applied N = 1 supergravity, ICTP Series in Theoretical Physics (World Scientific, Singapore, 1984), Vol. 1, p. ;
H.P. Nilles, Phys. Rep. **110** (1984) 1;
H.E. Haber and G.L. Kane, Phys. Rep. **117** (1985) 75;
J. Ellis, preprint CERN TH.4255 (1985), Lectures given at the 28th Scottish Universities Summer School in Physics, Edinburgh, Scotland, 1985.

- [1.19] P. Fayet, Nucl. Phys. **B90** (1975) 104;
A. Salam and J. Strathdee, Nucl. Phys. **B87** (1975) 85.
- [1.20] S. Wolfram, Phys. Lett. **82B** (1979) 65;
C.B. Dover, T.K. Gaisser and G. Steigman, Phys. Rev. Lett. **42** (1979) 1117.
- [1.21] P.F. Smith and J.R.J. Bennett, Nucl. Phys. **B149** (1979) 525;
P.F. Smith et al., Nucl. Phys. **B206** (1982) 333, and references cited therein.
E. Norman, S. Gazes and D. Bennett, Phys. Rev. Lett. **58** (1987) 1403.
- [1.22] See, for example: P. Fayet, Proc. Europhysics Study Conference on Unification of the
Fundamental Particle Interaction, Erice, Italy, eds. S. Ferrara et al. (Plenum, New
York, 1980), p. 583.
- [1.23] L. Alvarez-Gaumé, M. Claudson and M. Wise, Nucl. Phys. **B207** (1982) 96;
L. Hall and I. Hinchliffe, Phys. Lett. **112B** (1982) 351;
G. Farrar and S. Weinberg, Phys. Rev. **D27** (1983) 2732.
- [1.24] Such models were discussed by many authors, among whom are:
E. Witten, Phys. Lett. **105B** (1981) 267;
S. Dimopoulos and S. Raby, Nucl. Phys. **B192** (1981) 353;
J. Polchinski and L. Susskind, Phys. Rev. **D26** (1982) 3661;
for a more complete list, see L. Hall et al. in ref. [1.25].
- [1.25] R. Arnowitt, A. Chamseddine and P. Nath, Phys. Rev. Lett. **49** (1982) 970 and **50** (1983)
232;
R. Barbieri, S. Ferrara and C. Savoy, Phys. Lett. **119B** (1982) 343;
L. Ibáñez, Phys. Lett. **118B** (1982) 73;
H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. **120B** (1983) 346;
L. Hall, J. Lykken and S. Weinberg, Phys. Rev. **D27** (1983) 2359, and references therein;
see also references cited in H.P. Nilles in ref. [1.18].
- [1.26] J. Ellis, C. Kounnas and D. Nanopoulos, Phys. Lett. **143B** (1984) 410.
- [1.27] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. **67** (1982) 1889 and
68 (1982) 927.
- [1.28] J. Ellis and S. Rudaz, Phys. Lett. **128B** (1983) 248.
- [1.29] J. Ellis, L. Ibáñez and G.G. Ross, Nucl. Phys. **B221** (1983) 29;
R. Arnowitt et al. in ref. [1.25];
P. Nath, R. Arnowitt and A. Chamseddine, Harvard preprint HUTP83/A077 (1983);
J. Ellis et al., Phys. Lett. **132B** (1983) 436;
B. Grinstein, J. Polchinski and M. Wise, Phys. Lett. **130B** (1983) 285;
S. Dawson, E. Eichten and C. Quigg, Phys. Rev. **D31** (1985) 1581;
D. Dicus, S. Nandi and X. Tata, Phys. Lett. **129B** (1983) 451; Erratum Phys. Lett. **145B**
(1984) 448.
H.E. Haber and G.L. Kane in ref. [1.18];
A. Bartl, H. Fraas and W. Majerotto, Z. Phys. **C30** (1986) 441;
H. Komatsu and J. Kubo, Nucl. Phys. **B263** (1986) 265.
- [1.30] X. Tata and D. Dicus, Phys. Rev. **D35** (1987) 2110.
- [1.31] C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quiros, Phys. Lett. **132B** (1983) 95
and Nucl. Phys. **B236** (1984) 438;
S. Jones and G.G. Ross, Phys. Lett. **135B** (1984) 69;
L. Ibáñez and C. Lopéz, Nucl. Phys. **B233** (1984) 511.
- [1.32] R. Arnowitt et al. in ref. [1.25].
- [1.33] S. Ferrara, D.V. Nanopoulos and C. Savoy, Phys. Lett. **123B** (1983) 214;
P. Nath, R. Arnowitt and A. Chamseddine, Northeastern Univ. preprint NUB 2680 (1983).

- [1.34] S. Weinberg, Phys. Rev. Lett. **50** (1983) 387;
R. Arnowitt et al. in ref. [1.25].
- [1.35] C. Kounnas et al. in ref. [1.31];
L.J. Hall and J. Polchinski, Phys. Lett. **152B** (1985) 335.
- [1.36] G. Farrar and P. Fayet, Phys. Lett. **89B** (1980) 191;
D. Dicus, S. Nandi, W.W. Repko and X. Tata, Phys. Rev. Lett. **51** (1983) 1030;
M. Gluck and E. Reya, Phys. Lett. **130B** (1983) 423;
T. Kobayashi and M. Kuroda, Phys. Lett. **134B** (1984) 271;
T. Schimert and X. Tata, Phys. Rev. **D32** (1985) 721;
S. Dawson et al. in ref. [1.29];
A.R. Allan, N. Brown and A.D. Martin, Z. Phys. **C31** (1986) 479;
X. Tata and D. Dicus in ref. [1.30].
- [1.37] D. Dicus et al., in ref. [1.36];
P. Nath et al. in ref. [1.29];
T. Schimert, C. Burgess and X. Tata, Phys. Rev. **D32** (1985) 721;
A. Bartl, H. Frans and W. Majerotto, Z. Phys. **C30** (1986) 441;
T. Fukai, N. Oshimo, Y. Otake and N. Sugiyama, Waseda preprint WU-HEP-85-3 (1985);
X. Tata and D. Dicus in ref. [1.30].
- [1.38] D.A. Dicus, S. Nandi, W.W. Repko and X. Tata, Phys. Rev. **D29** (1984) 1317 and **D30** (1984) 1112.
- [1.39] E. Reya, Phys. Lett. **133B** (1983) 245.
D. Dicus et al. in ref. [1.36];
B. Campbell et al., Phys. Lett. **131B** (1983) 213;
V. Barger et al., Phys. Lett. **131B** (1983) 372.
- [1.40] E. Ma and J. Okada, Phys. Rev. Lett. **41** (1978) 287;
K.J.F. Gaemers, R. Gastmans and F.M. Renard, Phys. Rev. **D19** (1979) 1605.
- [1.41] P. Fayet, Phys. Lett. **117B** (1982) 460;
J. Ellis and J. Hagelin, Phys. Lett. **122B** (1983) 303;
T. Kobayashi and M. Kuroda, Phys. Lett. **139B** (1984) 206;
K. Grassid and P.N. Paneita, Phys. Rev. **D30** (1984) 22;
J. Ware and H. Machacek, Phys. Lett. **142B** (1984) 300;
M. Drees, C.S. Kim and X. Tata, Univ. of Wisconsin preprint MAD/PH/355 (1987).
- [1.42] J.A. Grifols, M. Martinez and J. Sola, DESY preprint 85-112 (1985);
J. Ware and M. Machacek, Phys. Lett. **147B** (1984) 415.
- [1.43] R.M. Barnett, H.E. Haber and K. Lackner, Phys. Lett. **126B** (1983) 64.
- [1.44] M.K. Gaillard et al., Phys. Lett. **116B** (1982) 279 [Erratum **119B** (1982) 471];
M. Kuroda et al., Phys. Lett. **127B** (1983) 467;
J.E. Johnson and S. Rudaz, Phys. Rev. **D30** (1984) 1590.
- [1.45] P. Salati and J.C. Wallet, Phys. Lett. **122B** (1983) 397;
G. Eilam and E. Reya, Phys. Lett. **145B** (1984) 425 and **148B** (1984) 502;
M. Martinez, J. Grifols and R. Pascual, Z. Phys. **C29** (1985) 309.
- [1.46] F. Berends, P.H. Daverveldt and R. Kleiss, Comput. Phys. Commun. **40** (1986) 285.
F.A. Berends and R. Kleiss, Nucl. Phys. **B177** (1981) 237, **B178** (1981) 141, **B228** (1983) 537.
F.A. Berends, R. Kleiss and S. Jadach, Nucl. Phys. **B202** (1982) 63.
- [1.47] R. Kleiss, A. Blondel and P. Raimondi, private communication during the ECFA Workshop on LEP 200, Aachen, 1987.
- [1.48] T. Sjöstrand, Lund Univ. preprint LUTP 85-10 (1985).

- [1.49] These limits are from
- a) PETRA:
 CELLO Collab., H.-J. Behrend et al., Phys. Lett. **176B** (1986) 247 and preprint DESY 87-013 (1987);
 JADE Collab., W. Bartel et al., Phys. Lett. **139B** (1984) 327, **146B** (1984) 126, **152B** (1985) 385 and 392; Z. Phys. **C29** (1985) 505;
 JADE Collab., B. Nasaka DESY 86-113 (1986);
 MARK J. Collab., B. Adeva et al., Phys. Rev. Lett. **53** (1984) 1806, Phys. Lett. **152B** (1985) 439; to be published in Phys. Lett. B (1987); B. Zhou, MIT Thesis (1986) unpublished;
 TASSO Collab., M. Althoff et al., Z. Phys. **C26** (1984) 337; R. Brandelik et al., Phys. Lett. **117B** (1982) 365.
- b) PEP:
 ASP Collab., G. Bartha et al., Phys. Rev. Lett. **56** (1986) 685;
 ASP Collab., C. Hearty et al., Phys. Rev. Lett. **58** (1987) 1711;
 MAC Collab., E. Fernandez et al., Phys. Rev. Lett. **52** (1984) 22 and **54** (1985) 1118;
 MAC Collab., W.T. Ford et al., SLAC-PUB-4003 (1986);
 MARK II Collab., L. Gladney et al., Phys. Rev. Lett. **51** (1983) 225.
- c) $p\bar{p}$ Collider:
 A. Honma, Proc. 23rd Int. Conf. on High-Energy Physics, Berkeley, 1986 (World Scientific, Singapore, 1987), vol. II, p. 1005.
- d) For recent updates, see:
 W. de Boer, Proc. 17th Int. Symp. on Multiparticle Dynamics, Seewinkel (Austria), 1986, and Munich preprint MPI-PAE/Exp. El. 167;
 M. Davier, same Proc. as A. Honma in (c), vol. I, p. 25, and Orsay preprint LAL 86-31 (1986);
 J.F. Grivaz, same Proc., vol. I, p. 614, and Orsay preprint LAL 86-28 (1986);
 S. Whitaker, same Proc., vol. I, p. 602, and Bonn preprint 86-10 (1986).
- [1.50] A. Honma in ref. [1.49c].
- [1.51] H. Baer, D. Karatas and X. Tata, Phys. Lett. **183B** (1987) 220.
- [1.52] H. Baer, K. Hagiwara and X. Tata, Phys. Rev. Lett. **57** (1986) 294 and Phys. Rev. **D35** (1987) 1598;
 A.H. Chamseddine, P. Nath and R. Arnowitt, Phys. Lett. **174B** (1986) 399;
 R. Arnowitt and P. Nath, Phys. Rev. **D35** (1987) 108.
- [1.53] H. Baer and E.L. Berger, Phys. Rev. **D34** (1986) 1361;
 E. Reya and D.P. Roy, Z. Phys. **C32** (1986) 615.
- [1.54] J.F. Gunion and H.E. Haber, Nucl. Phys. **B272** (1986) 1 and Nucl. Phys. **B278** (1986) 449;
 K. Inoue et al., Ref. 1.27:
 P. Majumdar and P. Roy, Phys. Rev. **D30** (1984) 2432;
 M. Drees, M. Gluck and K. Grassie, Phys. Lett. **159B** (1985) 32.

Table 1.1

Minimal particle content

Spin	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
Matter	$\tilde{\ell}_L, \tilde{\ell}_R$	ℓ			
Multipl.	$\tilde{q}_L, \tilde{q}_R$	q			
Field Multipl.		$\tilde{g}$	g		
	$H^\pm$	$\tilde{W}^\pm, \tilde{W}_h^\pm$	$W^\pm$		
		$\tilde{\gamma}$	γ		
	Z	$\tilde{Z}, \tilde{Z}_h$	Z^0		
	H_1^0, H_2^0	$\tilde{H}$			
				$\tilde{G}$	G

Table 1.2

Resolution and acceptances in detector simulations

Particle type	$\sigma(E)/\sqrt{E}$ $\sigma(\%)$		$\sigma(p)/p^2$ $(\% \cdot \text{GeV}^{-1})$	Angular resolution	Polar angle acceptance ($^\circ$)
$e^\pm$	Good	Aver.	0.2	$\sigma_\theta \sigma_\phi$	
γ	2	18		0.01	$5 \leq \theta \leq 175$
Had. cal.	2	18		0.01	$5 \leq \theta \leq 175$
μ	50	120		0.02	$10 \leq \theta \leq 170$
Veto				0.01	$10 \leq \theta \leq 170$
					$2 \leq \theta \leq 178$

Table 1.3

Lower limits on sparticle masses in GeV (95% confidence) from e^+e^- collisions

Experiment	MARK J	CELLO	JADE	TASSO	MARK II	MAC	ASP	Model assumptions
Sparticle								
photino $\tilde{\gamma}$	20.5	20	18	6				Unstable $\tilde{\gamma}$, e.g. $\tilde{\gamma} \rightarrow \gamma \tilde{H}_{\pm}^0$ Goldstino; $m_{\tilde{e}} = 50$ GeV in $o(e^+e^- \rightarrow \gamma\tilde{\gamma})$
selectron $\tilde{e}^{(a)}$								
$\tilde{e}\tilde{e}$	22	22	22	16.6	22.2			$\tilde{e} \rightarrow e\tilde{\gamma}$ decay; $m_{\tilde{\gamma}} = 0$
$\tilde{e}e\tilde{\gamma}$	25.5	30	25.2					
$\tilde{\gamma}\tilde{\gamma}$	34	34	16			50 ^{d)}	58 ^{d)}	
smuon $\tilde{\mu}$	21	20	20.3	16.4				$\tilde{\mu} \rightarrow \mu\tilde{\gamma}$ decay
stau $\tilde{\tau}$	18	20						$\tilde{\tau} \rightarrow \tau\tilde{\gamma}$ decay
charged shiggs $\tilde{H}^{\pm}$	22.5		22.5					$m_{\tilde{\gamma}} = 4$ GeV
zino ^{b)} $\tilde{Z}$	35	32	30					$m_{\tilde{\gamma}} < 2$ GeV; $m_{\tilde{e}} < 40$ GeV
charged wino ^{c)} $\tilde{W}^{\pm}$								
$\tilde{W}\tilde{W}$	22.5	22.5	22.5					$m_{\tilde{\gamma}} = 0$ $\tilde{W}^{\pm}$ decays leptonically
$\tilde{W}e\tilde{\nu}$	26.5	26						
$\tilde{\gamma}\tilde{\nu}\tilde{\nu}$	39	37.5				51 ^{d)}	61 ^{d)}	
squarks $\tilde{q}$ (charge 2/3)			20 17.8 21.4 21.0					$\left. \begin{array}{l} m_{\tilde{q}_1} = m_{\tilde{q}_2} \\ m_{\tilde{q}_1} < m_{\tilde{q}_2} \end{array} \right\} \tilde{q} \rightarrow q\tilde{q}: m_{\tilde{g}} = 3 \text{ GeV}, m_{\tilde{\gamma}} = 0.5 \text{ GeV}$ $\left. \begin{array}{l} m_{\tilde{q}_1} = m_{\tilde{q}_2} \\ m_{\tilde{q}_1} < m_{\tilde{q}_2} \end{array} \right\} \tilde{q} \rightarrow q\tilde{\gamma}: m_{\tilde{\gamma}} < 10 \text{ GeV}$

Commentsa) Here and elsewhere, the reaction used has been indicated where appropriate. We assume $m_{\tilde{e}_L} = m_{\tilde{e}_R}$.

b) Neutral shiggs-zino mixing can reduce the production rate and hence the lower mass limit.

c) Charged shiggs-wino mixing changes the pair-production cross-section only slightly. The decay of a charged shiggs is expected either i) to proceed via W -exchange, giving a signal identical to that of $\tilde{W}^+\tilde{W}^-$ pair-production, or ii) if the sneutrino is light, to give predominantly $\tau + \tilde{\nu}$ or $\tilde{\tau} + \nu$, which would yield acoplanar τ pairs. These final states were looked for in the $\tilde{H}^+\tilde{H}^-$ search. In either case, there is a solid lower limit of 22.5 GeV on the masses of both the charged wino $\tilde{W}^{\pm}$ and the charged shiggs $\tilde{H}^{\pm}$.

d) 90 % c.l.

Figure captions

- Fig. 1.1** Some examples of Standard Model vertices along with the corresponding supersymmetric vertices. In the last example, the factor f_+ denotes the amount of admixture of the gaugino in the mass eigenstate $\tilde{W}$. Such a factor does not occur in the wino coupling to the photon since the photon couples to the electric charge.
- Fig. 1.2** A comparison of the p_T distributions of lepton pairs from selectron pair production and from radiative corrections to pair production in the Standard Model, where the photon either goes down the beam pipe or has an energy less than 1 GeV so that it escapes detection. The backgrounds from two-photon processes are even smaller. Other supersymmetric processes have similar p_T distributions, as may be expected from the fact that the photinos are a genuine source of missing p_T .

Fig. 1.1

Fig. 1.2

2. TWO-BODY FINAL STATES

2.1 Introduction

In this chapter we concentrate on signals from pair production of SUSY particles which have the largest cross-section when the reactions are kinematically accessible. Various two-body reactions have been studied in the literature [2.1–2.8] and, on the experimental side, the current mass lower limits from the non-observation of sparticle pair production or radiative $\tilde{\nu}$ and $\tilde{\gamma}$ pair production at PEP and PETRA are summarized in table 1.3.

Here we will study how far beyond these current limits LEP I and LEP II can reach. Our results have already been presented at the ‘LEP Physics Jamboree’ [2.9] and at the ‘LEP 200 ECFA Workshop’ [2.10]. In what follows, we supplement the previous papers with a more complete analysis, we present the details of our calculations and we give the relevant formulae.

The SUSY search is divided into two parts: i) scalar lepton search and ii) search for charginos and neutralinos. In each case the results are then split in ‘below’ and ‘above’ the WW threshold since W pair production presents a formidable background to most ‘new physics’ search.

2.2 Scalar SUSY search: slepton, squark pair production

Sfermion pair production occurs via annihilation graphs involving the photon and the Z^0 boson and leads to production of $\tilde{f}_L\tilde{f}_L$ and $\tilde{f}_R\tilde{f}_R$ pairs, but no $\tilde{f}_L\tilde{f}_R$ or $\tilde{f}_R\tilde{f}_L$ pairs ($f = e, \mu, \tau, q$), see fig. 2.1a.

For the production of scalar electrons in addition to the annihilation graphs, we also have contributions from the t-channel exchanges of the $\tilde{\gamma}$, the $\tilde{Z}$ and the $\tilde{Z}_h$ (see fig. 2.1b). These exchanges also lead to $\tilde{e}_R\tilde{e}_L$ $\tilde{e}_L\tilde{e}_R$ production. Therefore only $\sigma(\tilde{e}\tilde{e})$ depends on the photino mass while, as shown later, there is a much weaker dependence on the zino mass.

As discussed in section 1.6, we assume in our analysis that the sfermions subsequently decay via the two-body mode, $\tilde{f} \rightarrow f\tilde{\gamma}$, with 100% branching ratio. The production and decay processes are uncorrelated; thus the only dynamically non-trivial cross-section is that for sfermion production, while the constant matrix element for the two-body decay of the sfermion can be trivially folded into the Monte Carlo generator.

The total and differential cross-sections formulae for the above processes are given in appendix B, and we now turn to the rates and distributions for the final states.

2.2.1 From 70 GeV up to below the WW threshold

2.2.1.1 Scalar electrons and scalar muons [2.1–2.4, 2.9, 2.10]: The signature for pair production of $\tilde{e}$ ’s, where $\tilde{e}$ decays via $\tilde{e} \rightarrow e\tilde{\gamma}$, is two acoplanar electrons with large E_T and large $\not{p}_T$. In what follows we assume that events from initial-state bremsstrahlung with the γ energy larger than 1 GeV and the γ inside the acceptance will be rejected. The events from $e^+e^- (\gamma)$, $e^+e^- (e^+e^-)$ and $\tau^+\tau^- (\gamma)$ with $\tau \rightarrow e\nu\nu$, where the particles between brackets are either in the beam pipe or, if gammas, have less than 1 GeV in the detectors, are rejected with 100% efficiency, at any energy, by appropriate cuts in $\not{p}_T$ and acoplanarity, defined as the supplement of the opening angle between the final electrons projected in the transverse plane.

Shown in fig. 2.2a is the accepted cross-section as a function of $\sqrt{s}$ after a p_T cut ($p_T > 0.05\sqrt{s}$) and incorporating the acceptance cuts and resolutions discussed in section 1.7. Also shown separately is the cross-section for opposite-handed selectron pairs. A few comments regarding these cross-sections are in order:

- i) For $m_{\tilde{\gamma}} \leq 10$ GeV the $\tilde{e}_L\tilde{e}_L + \tilde{e}_R\tilde{e}_R$ cross-sections are almost independent of the zino and photino masses. Here we take $|\mu_2| = 10$ GeV corresponding to $m_{\tilde{\gamma}} = 6$ GeV.

- ii) The t-channel exchanges give a rise in the cross-section above the Z pole. Since smuons are produced only via the s channel there will be no such rise for smuon pair production (see fig. 2.2b).
- iii) The production of opposite handed selectrons occurs only via t-channel exchanges of the photino and the zinos, the corresponding amplitude being proportional to the mass of the exchanged particle. Thus this cross-section is quite sensitive to the photino and zino masses. For $0 \leq m_{\tilde{\gamma}} \leq 10$ GeV and for a wide range of zino masses, it is always $\leq 25\%$ of that of the $\tilde{e}_L\tilde{e}_L + \tilde{e}_R\tilde{e}_R$ cross-section and is negligible at LEP I.

The obvious signature for smuon pair production is again two acoplanar muons with large $\cancel{E}$ and $\cancel{p}_T$. Moreover, because the only contribution is from the s channel, the angular distributions for muons are symmetric. As we will see, this is useful for discriminating WW events at LEP II. Below the WW threshold, $\cancel{p}_T$ and acoplanarity cuts are enough to separate smuons from the QED backgrounds.

The cross-section for smuon pair production (in this case, only $\tilde{\mu}_L\tilde{\mu}_L$ and $\tilde{\mu}_R\tilde{\mu}_R$, since only s-channel graphs contribute) is shown in fig. 2.2b. This cross-section is obviously independent of the gaugino masses, except for a small dependence of $m_{\tilde{\gamma}}$ due to the effect of our cuts on the altered phase space in the decay of the smuon.

It is clear from figs. 2.2a and 2.2b that slepton pair production, if kinematically allowed, would lead to several hundreds to several tens of thousands of events per 100 pb^{-1} at LEP I, the precise number depending on the mass of the slepton and the beam energy. The only Standard Model background we have not removed in these graphs is from τ pairs with both τ 's decaying leptonically. The total cross-section for this reaction with both τ 's decaying into the same flavour of lepton at LEP I is between $\sim 5\%$ (for $m_{\tilde{e}} \approx 25$ GeV) to $\sim 20\%$ (for $m_{\tilde{e}} = 40$ GeV) of the slepton pair-production cross-section. Moreover a cut on the acoplanarity, not shown, completely eliminates the τ 's, reducing the slepton cross-section by only $\sim 5\%$ at LEP I.

We now turn to a study of the various distributions of the leptons. We first concentrate on the single-particle distributions. It goes without saying that consideration of distributions makes sense only if a sufficiently large number of events can be accumulated.

The angular distributions for the electrons (muons) coming from the selectron (smuon) decays are shown in fig. 2.3. We see that the electrons are sharply peaked in the forward direction (except at $\sqrt{s} = m_Z$), whereas the muon distribution is comparatively flat and symmetric. This is just a reflection of the t-channel exchange of a (light) photino that is present in the first case alone. This sharp peak obviously emphasizes the need for detection capability close to the beam pipe. The energy distributions of the leptons are absolutely flat [2.2] with

$$(m_{\tilde{\ell}}^2 - m_{\tilde{\gamma}}^2)/2(E + p) \leq E_{\ell} \leq (m_{\tilde{\ell}}^2 - m_{\tilde{\gamma}}^2)/2(E - p) \quad , \quad (2.1)$$

reflecting the two-body decay of a scalar particle. A measurement of the upper and lower end points could thus yield a measure of the slepton and photino masses.

We now turn to a study of the correlations between the leptons. The $\cancel{E}$ distributions (not shown), unlike in the $\tilde{Z}\tilde{\gamma}$ case (see fig. 2.35), are quite symmetric about their peak and do not show a sharp lower edge. Moreover they peak only slightly above $\cancel{E}_T = \sqrt{s}/2$, reflecting the smallness of the photino mass.

Shown in fig. 2.4 is the distribution in acollinearity angle, defined to be the supplement of the opening angle between the leptons for a) $\sqrt{s} = 70$ GeV, b) $\sqrt{s} = m_{Z^0}$, and c) $\sqrt{s} = 180$ GeV, and for different slepton masses. For electrons, off the Z pole the distributions tend to peak at smaller values than at the pole; this is again a reflection of the forward peaking of the selectron production due to the photino exchange, and hence no such trend is seen in the muon case. The position of the peak in

the electron case of course depends on $m_{\tilde{e}}/\sqrt{s}$ moving to smaller values as the ratio decreases. It is interesting to contrast this with the corresponding distribution for the zino-photino case in fig. 2.32.

The invariant-mass distribution of the lepton pairs is shown in fig. 2.5 and should be compared with the corresponding distribution for the zino-photino channel shown in fig. 2.36. Shown in fig. 2.6 is the distribution in missing mass squared: $mm^2 \equiv \not{p}^2$. The lower edge of this distribution is rather sharp and the measurement of this end point could yield a direct measure of the photino mass since kinematically

$$mm^2 \geq 4m_{\tilde{\gamma}}^2.$$

Our simulation, after cuts and resolution smearing, shows that the distribution indeed starts at that point. We underline here the importance of having a sensitive electromagnetic calorimeter made of BGO. Moreover, we note that this distribution is broader than in the $\tilde{Z}\tilde{\gamma}$ case (see fig. 2.37), reflecting the fact that the two photinos have, on average, equal energies and are in opposite hemispheres. The distributions in acoplanarity angle and in $\not{p}_T$ are shown in figs. 2.7 and 2.8, respectively.

To summarize, the flatness of the lepton energy distributions, the correlation in the rates and distributions for e^+e^- and $\mu^+\mu^-$ pairs, the absence of $e^+\mu^-$ or $e^-\mu^+$ pairs and the presence of large amounts of missing energy could simultaneously be used to separate Standard Model backgrounds and also to distinguish the slepton signal from other exotic processes. We emphasize once again that such a detailed study is due to the cleanliness of e^+e^- interactions and the simplicity of the initial state. Thus, once a signal has been found, it may be possible to identify the nature of the new physics.

2.2.1.2 Stau pairs: Stau pair production occurs by the same processes that lead to smuon pairs. Therefore it has the same cross-section. As for smuons, staus decay via $\tilde{\tau} \rightarrow \tau\tilde{\gamma}$. However, the signature for staus is quite different. As the τ decays into at least one neutrino, there is missing energy already involved in standard τ^- production. Whereas the magnitude of the energy from the neutrinos in τ decays is similar to that of the $\tilde{\gamma}$ in $\tilde{\mu}$ decays, it has a different angular distribution. At LEP energies the visible decay products from the τ have nearly the same direction as the τ itself, whilst the visible decay products from a stau have quite a different direction from that of the original stau. Thus standard τ pair production leads to collinear τ -decay products but the visible remnants from stau production are rather acollinear. The τ pairs are not collinear either in processes like $e^+e^- \rightarrow \tau^+\tau^-\gamma$ and $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$. However, all LEP detectors are able to tag the γ or the e^+ and e^- over a large range of solid angle. The particles along the beam pipe which evade detection can lead to pairs which are acollinear in space, but do not influence the momenta in the directions perpendicular to the beam.

These arguments suggest that one uses the acollinearity in the plane transverse to the beam to separate staus from the conventional τ pair production. For the present study the following properties were required:

- i) Two well-separated clusters of particles had to be present. Summing up the momenta $\vec{p}_i$ of charged and neutral particles in each cluster leads to $\vec{p}_{\alpha,c}$ ($\alpha = 1, 2$). The opening angle between the two momenta should be larger than 45° .
- ii) To ensure that the clusters are coming from taus, the two clusters had to consist of either one or three charged particles. In the case that both clusters only contained one particle, the two particles had to be of different kinds, e.g. one an electron, the other a muon, or at least one of them a hadron. In this way, events in which only e^+e^- or $\mu^+\mu^-$ pairs are detected were rejected.

- iii) To allow for an appropriate determination of the τ direction, a minimum visible energy of 2 GeV was required for each τ .
- iv) The difference in azimuthal angle of the two clusters $\Delta\phi = |\phi_1 - \phi_2|$ had to be $\Delta\phi < \pi - 0.5$. This criterion is equivalent to the acoplanarity cut used to set limits on stau pairs at PETRA.

Applying the above cuts and using the detector parametrization of table 1.2 reduces the acceptance of stau pairs to about 30%, the precise value depending on the centre-of-mass energy and on the assumed mass of the stau. For a fixed mass the visible energy cut (iii) becomes slightly less important with increasing c.m. energy, but the $\Delta\phi$ cut (i) becomes more significant^{*)}. Increasing the mass of the stau reduces the overall cross-section, but the acceptance increases because the typical values of $\Delta\phi$ are larger. The accepted cross-section as a function of energy for stau masses of 30 and 60 GeV can be seen in fig. 2.9. Obviously the accepted rates should allow detection even if the $\tilde{\tau}$ is heavier than 75% of the beam energy.

The background within these cuts from radiative processes and from two-photon collisions is negligible. The $\tau\tau\gamma$ background is more than two orders of magnitude smaller than the accepted $\tilde{\tau}\tilde{\tau}$ production. The two-photon τ pair production process has a high cross-section, but its contribution can be further suppressed by demanding that neither the electron nor the positron is detected in the typical 2γ taggers of the LEP detectors ($45 < \theta < 115$ mrad), and in particular that the invariant mass of the detected $\tau^+\tau^-$ decay products exceed $m_{\min} = 0.075\sqrt{s}$. These additional cuts hardly affect the stau pair acceptance, but reduce the background from $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ to less than 10^{-2} of the possible $\tilde{\tau}\tilde{\tau}$ signal.

2.2.2 $\sqrt{s} = 190$ GeV: above the WW threshold

At $\sqrt{s} = 190$ GeV the W^+W^- production has its maximum, $\sigma_{\text{tot}}(W^+W^-) \simeq 17$ pb [2.11], with large $\cancel{E}$ and $\cancel{p}_T$. For the SUSY search to be successful we have then to find out 'selection criteria' good enough to get rid of the WW events that have more than a factor of 10 higher cross-section with similar signatures.

As will be clear in chapter 3, three-body reactions have cross-sections that are too low to be detectable for $\sqrt{s} \geq 160$ GeV, up to integrated luminosities of 500 pb^{-1} .

2.2.2.1 Scalar electrons: At $\sqrt{s} = 190$ GeV, selecting electron pairs with $\cancel{p}_T > 10$ GeV and $\theta_{\text{acopl}} > 34^\circ$, the only background events left are from $WW \rightarrow e\nu e\nu$ and $e\nu\tau\nu$ where $\tau \rightarrow e\nu\nu$. In table 2.1 the rates before and after these cuts are summarized for $m_{\tilde{e}} = 85$ and 90 GeV and $m_{\tilde{\gamma}}$ from 0 to 40 GeV. The dependence of the accepted electron-pair cross-section on the photino mass, for $m_{\tilde{e}} = 75, 85$ and 90 GeV, is shown in fig. 2.10, where $m_{\tilde{Z}} = 90$ GeV. It is clear from this figure that selectrons can be well detected above the WW background up to masses of 90 GeV, with $m_{\tilde{\gamma}} = 40$ GeV, if the integrated luminosity will be as big as 500 pb^{-1} , see again table 2.1. With 100 pb^{-1} for a five standard deviations signal we have to go down to $m_{\tilde{e}} = 85$ GeV, but still with $m_{\tilde{\gamma}} = 40$ GeV. It is worth mentioning here that for $m_{\tilde{\gamma}} = 30$ GeV and $m_{\tilde{e}_R} = 85$ and 90 GeV, only $\tilde{e}_R\tilde{e}_R$ can be produced. For $m_{\tilde{\gamma}} = 40$ GeV this is also true with $m_{\tilde{e}_R} = 80$ GeV. This explains the corresponding drops in the accepted cross-sections of fig. 2.10.

We note here that our use of the neutralino couplings derived in chapter 1 is, strictly speaking, invalid since the gaugino masses are no longer small compared to m_W . We have, however, checked by an exact numerical diagonalization [2.12] of the neutralino mass matrix that the $e\tilde{e}\tilde{\gamma}$ couplings are substantially unaltered from the approximate analytic form given in chapter 1. For example for $m_{\tilde{\gamma}} = 34$ GeV and $m_{\tilde{Z}} = 90$ GeV, the exact results for the coupling differ from our approximation

^{*)} Note that the $\Delta\phi$ distributions of the background processes are rather independent of the c.m. energy.

by less than 5% and from the no-mixing case by less than 10%. The $\tilde{Z}$ and $\tilde{Z}_h$ couplings differ by as much as 30–40%, but since the $\tilde{Z}$ exchanges contribute only a small amount to the cross-section, the results in fig. 2.10 are a good approximation to the exact situation.

Because of the t-channel contribution, the end caps play a relevant role in the $\tilde{e}$ -pair detection. In fact the accepted cross-section for electron pairs in the barrel + end caps regions (defined as $12^\circ < \theta < 168^\circ$) is 3 times bigger than in the barrel region ($42^\circ < \theta < 138^\circ$) for $m_{\tilde{e}} = 60$ GeV and 1.7 times for $m_{\tilde{e}} = 90$ GeV.

2.2.2.2 Scalar muon and tau: Again the obvious signature for smuon (stau) pair production is two acoplanar muons (taus) with large E and p_T . Moreover, because the only contribution is from the s channel, the angular distributions for muons (taus) are symmetric.

For extracting a $\tilde{\mu}$ signal we require two μ 's which are well inside the detector with $|\cos \theta| < 0.9$ and an energy of more than 5 GeV each. To eliminate the background from QED processes we require that the opening angle ϕ of the two μ 's in the plane transverse to the beam fulfils $\cos \phi > -0.9$.

Similar criteria can be used for the selection of $\tilde{\tau}$ events. One has to keep in mind that also in the standard τ events there is missing energy due to the ν from the τ decay. However, because of the relative small τ mass, the decay products of a high-energy τ are close to the direction of the original τ and the acoplanarity cut can still be applied.

In the τ selection we use the topology 1-1 or 1-3 charged particles. From the detected charged and neutral particles we form clusters which have to be well inside the detector, $|\cos \theta| < 0.9$, and are consistent with τ decay products (mass of clusters smaller than m_τ). Besides each cluster has to have an energy greater than 2 GeV with the total energy of the two clusters greater than 10 GeV. To reject hard initial-state bremsstrahlung we keep only events which have no isolated gammas. The most important cut against $\tau\tau$ production is again the acoplanarity angle ϕ in the plane transverse to the beam $-0.8 < \cos \phi < 0.87$.

With these criteria one reaches efficiencies of $\approx 70\%$ for $\tilde{\mu}$'s and $\approx 30\%$ for $\tilde{\tau}$'s. With 100 pb^{-1} integrated luminosity one gets a typical sensitivity up to masses of $0.8\text{--}0.9 \times E_{\text{beam}}$.

At $\sqrt{s} = 190$ GeV, as already stated, the $WW \rightarrow \mu\nu\mu\nu$ events have signatures similar to SUSY ones. The cross-section times the branching ratios is $17 \times 0.10 \times 0.10 = 0.17$ pb. However while the $\cos \theta_\mu$ distribution of the μ 's from W decay shows a very strong forward-backward asymmetry (see ref. [2.11]), the ones from $\tilde{\mu}$ decays, as already mentioned, have a symmetric distribution. Selecting in a given hemisphere the faster muon with a charge opposite to the one expected for a μ from a W decay, the WW background can be reduced by a factor of 5 while the signal is only reduced by a factor of 2. In table 2.2 we summarize the rates before and after these cuts. As can be seen from fig. 2.11 a 5 standard-deviation signal from $\tilde{\mu}$'s can be obtained up to masses of about 80 GeV for 100 pb^{-1} integrated luminosity, and 85–90 GeV for 500 pb^{-1} .

For $\tilde{\tau}$'s the situation is more complex because in principle all events where both W's decay to leptons (e, μ , τ) can fake the SUSY signature. Therefore we suggest using only the hadronic τ decay modes in order that there be no confusion with W decay to e, μ events. This would reduce the signal by a factor of 3 but the WW background by more than a factor of 20! A 5 standard-deviation signal can be obtained up to masses of the order of 70–75 GeV.

It is important to underline once more that the cleanliness of sparticle production in e^+e^- collisions allows one to verify the supersymmetric nature of the observed new phenomena, and also to investigate in detail the properties of the new particles. For example, the spin and the mass of the $\tilde{\mu}$ could be determined [2.13].

Because they have spin 0, the $\tilde{\mu}$ are produced with $\sin^2\theta$ angular distribution, and there is a significant correlation between the angles of the parent $\tilde{\mu}$ and the daughter μ if $m_{\tilde{\mu}}/E_{\text{beam}}$ is small

enough, which can be used to verify the spin of the $\tilde{\mu}$. Figure 2.12 shows the distribution in $\langle \cos \theta_{\mu} \rangle = \frac{1}{2} (\cos \theta_{\mu 1} - \cos \theta_{\mu 2})$ for $m_{\tilde{\mu}} = 70$ GeV, $m_{\tilde{\gamma}} = 20$ GeV, $\sqrt{s} = 190$ for 500 pb^{-1} and the corresponding distribution that would be given by two-particle decay of a comparison particle of spin $\frac{1}{2}$.

As seen from fig. 2.11 the cross-section for $\tilde{\mu}$ is strongly mass-dependent and this can be used to measure the $\tilde{\mu}$ mass. Figure 2.13 shows the total cross-section for $m_{\tilde{\mu}} = 70$ GeV as a function of the E_{beam} . At $\sqrt{s} = 190$ GeV with 100 pb^{-1} one would reach an accuracy of $\sim 10\%$. If one could then take $\sim 50 \text{ pb}^{-1}$ at $\sqrt{s} = 160$ GeV an accuracy of about 2% can be obtained. Once the $\tilde{\mu}$ mass is known one can then use the selectron cross-section, fig. 2.10, to determine the $\tilde{\gamma}$ mass. Accuracies of the order of $5\text{--}10\%$ can be reached.

2.3 Gaugino pair production

We have seen that if the photino is relatively light, we expect mass eigenstates $\tilde{W}^{\pm}$ and $\tilde{Z}$ in the gaugino-higgsino sector that are lighter than the $W^{\pm}$ and Z^0 bosons, respectively [2.14]. Besides it may well be the case that the gauginos ($\tilde{W}, \tilde{Z}$) are lighter than all the corresponding sfermions and the threshold for $\tilde{W}$ pair production would then be crossed first (with the exception of the $\tilde{Z}\tilde{\gamma}$ reaction if $m_{\tilde{\gamma}}$ is very light, see section 2.6). Winos of such a mass would decay via the three-body modes $q\bar{q}\tilde{\gamma}$ or $\ell\bar{\nu}\tilde{\gamma}$. Zino decays would also be dominated by the three-body decays $\tilde{Z} \rightarrow q\bar{q}\tilde{\gamma}$ or $\ell\bar{\ell}\tilde{\gamma}$.

Another possible scenario is that the $\tilde{\nu}$ is light [2.15] and the charged sfermions are heavy—see formula (1.21) with $v'/v < 1$ —in which case the dominant decay would be $\tilde{W} \rightarrow \ell\bar{\nu}$, ($\tilde{\nu} \rightarrow \nu\tilde{\gamma}$), and the zino would decay invisibly via $\tilde{Z} \rightarrow \nu\bar{\nu}$ or $\tilde{\nu}\bar{\nu}$. In the following subsections we discuss $\tilde{W}\tilde{W}$ and $\tilde{Z}\tilde{\gamma}$ production at LEP I and II for both the heavy- $\tilde{\nu}$ and the light- $\tilde{\nu}$ cases.

There is one important difference between the discussion of squark and slepton pair production considered previously and gaugino pair production. The sfermion mass eigenstates $\tilde{\ell}_i, \tilde{q}_i$ ($i = L, R$) were supersymmetric partners of the leptons and quarks and so, their couplings were completely fixed by SUSY in a model-independent way. In contrast, the states $\tilde{W}$ and $\tilde{Z}$ are, in general, complicated mixtures of the gauginos and the higgsinos, so that their couplings, in general, depend on model-dependent mixing angles. We have, however, seen in section 1.5.2 that in a general class of models these mixings can be parametrized in terms of a few parameters (v'/v , $\mu_{1/2}$, and $2m_1$). As a result the masses and couplings of all the winos and zinos are determined by $m_{\tilde{\gamma}}$, $m_{\tilde{W}}$, and v'/v . This feature makes it possible to present a fairly general analysis without considering each model on an individual basis and thus enables us to compare between theory and experiment. We now turn to the consideration of each of the gaugino pair-production processes at LEP.

2.4 Wino pair production [2.4–2.6, 2.9, 2.10]

2.4.1 Generalities

We first consider the pair production of winos which takes place via s-channel γ and Z^0 exchanges and via the t-channel $\tilde{\nu}$ exchange (see fig. 2.14). As anticipated in the previous section we have studied two possible scenarios:

- i) The sfermions are all heavier than the wino so that the wino decays via:

$$\tilde{W} \rightarrow \ell\bar{\nu}\tilde{\gamma} \quad \text{or} \quad \tilde{W} \rightarrow q\bar{q}\tilde{\gamma},$$

giving as experimental signatures two leptons + $\cancel{p}_T$ or jet(s) + lepton + $\cancel{p}_T$ or jet(s) + $\cancel{p}_T$. This is the signal expected in a wide class of SUSY models with $v'/v \simeq 1$ and heavy sfermions.

- ii) The $\tilde{\nu}$ is much lighter than the charged sfermions (such a sneutrino would have escaped experimental detection), in which case the dominant decay would be

$$\tilde{W} \rightarrow \ell \tilde{\nu} \quad (\ell = e, \mu, \tau),$$

leading to two acoplanar leptons with large $\cancel{E}$ and $\cancel{p}_T$ as experimental signatures. Unlike the slepton case discussed in section 2.2.1, $\tilde{W}$ pairs give rise to equal numbers of $e\bar{e} + \mu\bar{\mu}$ and $e\bar{\mu} + \mu\bar{e}$ pairs.

The results for the first scenario are presented in sections 2.4.2 ($\tilde{W}\tilde{W} \rightarrow \ell\bar{\nu}\tilde{\gamma} + \bar{\ell}\nu\tilde{\gamma}$) and 2.4.3 ($\tilde{W}\tilde{W} \rightarrow \ell\bar{\nu}\tilde{\gamma} + \text{jet jet } \tilde{\gamma}$) while, the light sneutrino case is discussed in section 2.4.4.

In the case of wino pair production the decay process is not independent of the production since the wino has spin, i.e. in the calculation of the final-state distributions, one must take into account correlations that occur in the amplitude due to the matching of the spin of the produced and the decaying particle [2.16]. For the case of the leptonic decay of the winos, for which we have studied the various distributions in detail, we have kept these spin correlations in our Monte Carlo. When one or both of the winos decay hadronically, our interest has been limited to a study of the total cross-section for the various topologies that may result. In this case, the inclusion of these spin correlations is comparatively less important, and so we have assumed a factorizable form for the production and decay processes. Likewise, for the light sneutrinos study we have once again confined ourselves to a discussion of the total cross-sections after the cuts, and so have ignored the correlations coming from the spin of the $\tilde{W}$.

Because we have incorporated these spin correlations for the process

$$e^+e^- \rightarrow \tilde{W}\tilde{W} \rightarrow e\bar{\nu}\tilde{\gamma} + \bar{e}\nu\tilde{\gamma}, \quad (2.2)$$

the expressions for the differential cross-sections are rather lengthy. These are presented in complete detail in appendix C. The angular distribution of the wino and the decay matrix element squared for the three-body wino decay—needed for the computation of the uncorrelated multijet + $\cancel{p}_T$ and jet(s) + lepton + $\cancel{p}_T$ signals—are also presented there. In appendix C we also give the formulae we used to compute the total cross-sections.

We now turn to the decay of the wino. Its leptonic decay proceeds via a virtual W boson or via a virtual $\tilde{e}_L$, whereas for the hadronic decay virtual W, virtual $\tilde{d}_L$, and virtual $\tilde{u}_L$ graphs can contribute. Here, we neglect the small $\tilde{\nu}\nu\tilde{\gamma}$ coupling X_ν (see appendix C) induced via $m_{\tilde{\nu}} \neq 0$.

In the case $m_{\tilde{e}} \simeq m_{\tilde{q}}$, which is well satisfied for heavy sfermions and a light photino and $v'/v \simeq 1$ [see eq. (1.21)], we find that the leptonic branching ratio for the wino decay (assuming two-body decays are suppressed) is [2.6]:

$$\Gamma(\tilde{W} \rightarrow e\bar{\nu}\tilde{\gamma})/\Gamma(\tilde{W} \rightarrow \text{everything}) = [9 - (8/3)r]^{-1} \quad (2.3a)$$

where

$$r = \Gamma_{\tilde{e}}(\tilde{W} \rightarrow \ell\bar{\nu}\tilde{\gamma})/\Gamma(\tilde{W} \rightarrow \ell\bar{\nu}\tilde{\gamma}) \quad (2.3b)$$

$$\Gamma_{\tilde{e}}(\tilde{W} \rightarrow \ell\bar{\nu}\tilde{\gamma}) = (e^4 f_+^2 m_{\tilde{W}}/256\pi^3 \sin^2 \theta_w) \left[3q^2 - (5/2) - (3q^4 - 4q^2 + 1) \ln \left(\frac{q^2}{q^2 - 1} \right) \right] \quad (2.4)$$

and

$$\Gamma(\tilde{W} \rightarrow \ell \bar{\nu} \tilde{\gamma}) = \frac{e^4 f_+^2 m_{\tilde{W}}}{256 \pi^3 \sin^2 \theta_w} \left[5\varrho^2 - 4 - \frac{1}{3}\beta^2 + \frac{1}{\beta^2} - (3\varrho^4 - 4\varrho^2 + 1) \ln \frac{\varrho^2}{\varrho^2 - 1} + \frac{1 - \beta^2}{\beta^2} \left(\frac{1 - \beta^2}{\beta^2} + 2\varrho^2 \right) \times \right. \\ \left. \times \ln(1 - \beta^2) - 2\varrho^2(1 - \varrho^2) \int_{\beta^2(\varrho^2 - 1)}^{\beta^2 \varrho^2} \frac{dx}{x} \ln(1 + \varrho^2 \beta^2 - \beta^2 - x) \right] \quad (2.5)$$

with $\varrho = m_{\tilde{e}}/m_{\tilde{W}}$ and $\beta = m_{\tilde{W}}/m_W$. In deriving (2.3) we have assumed that $m_{\tilde{e}} > m_{\tilde{W}}$ so that the decay $\tilde{W} \rightarrow q \bar{q} \tilde{g}$ is kinematically suppressed.

For $m_{\tilde{q}} \approx m_{\tilde{e}} \gg m_W$, the dominant decay is via the virtual W in which case the leptonic branching fraction is 1/9 per lepton family. From eqs. (2.3a) and (2.3b) we see that since $r > 0$, the leptonic branching ratio is increased if the decays via sfermions are important (the photino couples more strongly to sleptons than to squarks because the coupling is like the electric charge). Over a wide range of masses this branching fraction is between 1/7 and 1/9 and approaches 1/7 only when $m_{\tilde{W}} \approx m_{\tilde{e}} \approx m_{\tilde{q}}$.

Finally, we consider the branching fraction for the case $\tilde{W} \rightarrow \tilde{\nu} \ell$ which, as we have discussed, can yield potentially interesting signatures at LEP. We have, for three down- and two up-type families:

$$\frac{\Gamma(\tilde{W} \rightarrow \tilde{\nu} e)}{\Gamma(\tilde{W} \rightarrow \text{all})} = (m_{\tilde{W}}^2 - m_{\tilde{\nu}}^2) \sin \gamma_R / [3 \sin^2 \gamma_R (m_{\tilde{W}}^2 - m_{\tilde{\nu}}^2) + 3 \sin^2 \gamma_L (m_{\tilde{W}}^2 - m_{\tilde{e}_L}^2) \theta(m_{\tilde{W}} - m_{\tilde{e}_L}) + \\ + 6 \sin^2 \gamma_R (m_{\tilde{W}}^2 - m_{\tilde{u}_L}^2) \theta(m_{\tilde{W}} - m_{\tilde{u}_L}^2) + 9 \sin^2 \gamma_L (m_{\tilde{W}}^2 - m_{\tilde{d}_L}^2) \theta(m_{\tilde{W}} - m_{\tilde{d}_L})] \quad (2.6)$$

where the θ functions in the denominator indicate the channels which are open [$\theta(x) = 0$ if $x \leq 0$ and equal to 1 otherwise]. Note that only the left-handed sparticles couple to the wino. Assuming that the sfermion masses are as given in eq. (1.21), and that α is such that the sneutrino is light, for a wide range of wino masses the only other allowed channel is $\tilde{W} \rightarrow \tilde{u}_L d$, since $\tilde{d}_L$ and $\tilde{e}_L$ are both expected to be relatively heavy. For a heavier photino, even the $\tilde{u}_L d$ channel may well be closed.

We are now in a position to discuss the various signatures at LEP I and II resulting from wino pair production.

2.4.2 $m_{\tilde{\nu}} \approx m_{\tilde{e}}$ case. Acoplanar lepton signal from three-body decays of the wino.

We first consider wino pair production and subsequent leptonic decay of both the winos via the three-body mode:

$$\tilde{W} \tilde{W} \rightarrow \ell \bar{\nu} \tilde{\gamma} + \bar{\ell}' \nu \tilde{\gamma} \quad (\ell, \ell' = e \text{ or } \mu). \quad (2.7)$$

The experimental signature is again a pair of acoplanar leptons plus large $\not{p}_T$ and $\not{E}$.

An estimate of the total cross-section without any cut can be obtained using eq. (C.11) of appendix C: see fig. 2.15a. We see that at the Z^0 peak the rate is very large. This is mostly due to the fact that the wino contains (for $\nu' = \nu$) a substantial gaugino ($I = 1$) component which has an enhanced coupling to the Z^0 . The largeness of the wino cross-section allows us to focus here on the clean (though suppressed) pure leptonic channel. Above the WW threshold, this channel is flooded by the WW with $W \rightarrow \ell \bar{\nu}$ leptonic events and, as shown in the next subsection, we have to look for a better signature in order to identify wino pair production.

The accepted cross-section for electron pairs after acceptance and p_T cuts ($p_T > 0.05\sqrt{s}$ removes most of the Standard Model backgrounds up to below the WW threshold) is shown in fig. 2.15b for

different values of $m_{\tilde{W}}$. One can also replace the dielectron final states by dimuon or $e\text{-}\mu$ final states with a similar rate, up to a correction factor which is close to one because of the slight differences in the muon and electron acceptances. The following features are worth noting:

- i) If the decay $Z^0 \rightarrow \tilde{W}\tilde{W}$ is kinematically accessible, there is a very large signal from wino production even after taking into account the branching fraction ($\approx 11\%$) for the $\tilde{W}$ to decay leptonically. Off the Z^0 pole, the cross-section is still rather large and, more importantly, free of Standard Model background.
- ii) The resulting event topologies from the leptonic decays of winos are $e^+e^- + \cancel{p}_T$, $\mu^+\mu^- + \cancel{p}_T$ and $e^-\mu^+ + e^+\mu^- + \cancel{p}_T$ in the ratio 1:1:2 before acceptance cuts. The fact that $e\text{-}\mu$ events are present in this case serves to distinguish winos from sleptons or zinos merely on the basis of event topology. As has already been discussed, Standard Model backgrounds from $\tau^+\tau^-$, $\tau^+\tau^-\tilde{\gamma}$ can be removed via the acoplanarity cut.
- iii) The resulting cross-sections are not very sensitive to the photino mass; at $\sqrt{s} = 180$ GeV, varying $m_{\tilde{\gamma}}$ between 0 and 10 GeV changes the cross-section by about 5%.
- iv) Except at the Z^0 pole, the wino cross-section does depend on the sneutrino mass because of the t -channel $\tilde{\nu}$ exchange graph. Below the Z^0 pole the interference between the s and t channels is positive and it is negative above it.

We now turn to a discussion of the lepton distribution from $\tilde{W}$ decay. The energy distribution of the outgoing lepton is shown in fig. 2.16 for different wino masses and beam energies. We see that unlike the corresponding distribution for the case of slepton decay, this distribution is not flat. The angular distribution of the outgoing lepton is shown in fig. 2.17. The distribution is almost backward-forward symmetric at LEP I energies; this corresponds to the fact that the winos from Z^0 decay are produced with backward-forward symmetry. The small asymmetry is due to the spin correlation [2.6] [see appendix C, eq. (C.3)]. At 180 GeV, the electron distribution shows a forward peaking whereas the reverse is the case for $\sqrt{s} < m_Z$. We note here that the distributions from selectron decay were forward peaking both above and below the Z^0 pole (as discussed, this was due to the small $m_{\tilde{\gamma}}$) which is not the case for wino decay.

The acollinearity angle between the outgoing leptons is shown in fig. 2.18. We see that typically the opening angle is around 60° . Thus unless $m_{\tilde{W}}/\sqrt{s}$ is very small, so that the decay products are more or less back to back, acollinear leptons (with as many $e^+e^- + \mu^+\mu^-$ as $e^+\mu^- + e^-\mu^+$ pairs) would serve to signal wino production. We note that although this distribution is not very different from that for $\tilde{Z}\tilde{\gamma}$ production (see fig. 2.30), for relatively heavy zinos, the event rates and topologies together with other distributions can easily serve to distinguish between the two.

The $\cancel{E}$ distribution is shown in fig. 2.19. The distributions are all rather broad, but we see that for all values of energy they peak at $\approx 2/3 \sqrt{s}$. This reflects the three-body decay of the wino with the $\tilde{\gamma}$ and ν escaping detection. These distributions also reflect the equipartition of energy (on an average) between the three decay products of the wino, which, in turn, reflects the lightness of the photino mass. It is clear that if this was not the case, then the photino (on an average) would carry an energy different from the (massless) electron and neutrino, shifting the peak of the missing energy spectrum from $2/3 \sqrt{s}$. The $\cancel{p}_T$ distributions for these events are shown in fig. 2.20. These distributions, as expected, peak at much smaller values than the $\cancel{E}$ distributions since the transverse momentum vectors of the two neutrinos and two photinos tend to cancel. We also see from these that the $\cancel{p}_T$ cut imposed to remove the Standard Model backgrounds cuts out only a very small part of the $\tilde{W}\tilde{W}$ cross-section.

The invariant-mass distribution of the outgoing leptons is shown in fig. 2.21. We see that it is a broad distribution, extending from zero up to a maximum which, as in the slepton case, is about $2/3 \sqrt{s}$. This is to be contrasted with the corresponding distribution for the $e^+e^- \rightarrow \tilde{Z}\tilde{\gamma}$ case (see fig. 2.36), which cuts off at $m_{\tilde{Z}} - m_{\tilde{\gamma}}$, as we will discuss in section 2.6.1.

This completes our discussion of the lepton distributions from the three-body decay of the wino that is expected to be dominant when the squarks and sleptons are heavier than the gauginos.

2.4.3 $m_{\tilde{g}} = m_t$ case. Lepton + jets signal from three-body decays of the wino

2.4.3.1 Generalities

As already underlined, in addition to the clean, but suppressed, leptonic decays discussed in the previous section, there are expected to be multijet and lepton + jet + $\cancel{p}_T$ signals from the hadronic decay of the $\tilde{W}$. The signals occur at a rate that is about 20 times larger than the clean leptonic decay of both $\tilde{W}$'s discussed in section 2.4.2. Here we use these channels to look for winos, above the WW threshold [2.11], at $\sqrt{s} = 190$ GeV and also at $\sqrt{s} = 160$ GeV.

Figure 2.22 shows the total cross-sections of wino pair production as a function of the wino mass for three values of $\sqrt{s}$ and $m_{\tilde{g}} = 100$ GeV. At $\sqrt{s} = 190$ GeV for $m_W = 80$ GeV, $\sigma(\tilde{W}\tilde{W}) \simeq 1$ pb, to be compared with $\sigma(WW) \simeq 17$ pb. However, because of the interference between the s-channel and the t-channel amplitudes, the total cross-section strongly depends on the $m_{\tilde{g}}$ mass value, see fig. 2.23. For our study we consider the most pessimistic scenario: $m_{\tilde{W}} \leq 80$ GeV, $m_{\tilde{g}} = 100$ GeV and $m_{\tilde{\gamma}} = 20$ GeV.

To get a clean signal we propose the case where one wino (or W) decays leptonically into $e\nu\tilde{\gamma} + \mu\nu\tilde{\gamma}$ ($e\nu + \mu\nu$) and the other hadronically (see fig. 2.24). Since this signature can come only from W or $\tilde{W}$ production (we will comment later on new heavy leptons), all other background can in principle be removed! The branching ratios for these decays are still quite high $\sigma_{acc} = 2 \times 0.2 \times 0.7 \times \sigma_{WW}(\sigma_{WW}) \simeq 30\%$ σ_{tot} . If one looks into these events there are a few important differences, listed in table 2.3, that will allow the rejection of WW events.

Because we have confined our attention only to the total cross-sections, we have neglected the correlations coming from the spin of the wino, i.e. we have assumed that the production and the decay of the wino are completely independent processes. The relevant cross-sections can be readily obtained using eqs. (C.8)–(C.10) and eq. (3.2) for the wino branching fraction.

2.4.3.2 Selection of wino events [1.10]

For our analysis we used WW and $\tilde{W}\tilde{W}$ generators which simulate W's and $\tilde{W}$'s only at the parton level; in this case neither gluon radiation nor fragmentation are taken into account. Since the number of jets is not crucial to our analysis, we believe that the parton level calculation is a reasonable representation of the actual situations. In the following we have used both the good and average detector performances seen in table 1.2.

To select the wino sample we used the following criteria:

- i) Each parton has to have an energy of more than 10 GeV and should be well inside the detector. To assure this the polar angle θ has to fulfil $\cos \theta < 0.94$ (20°) for the lepton and $\cos \theta < 0.87$ (30°) for quarks. We require the bigger angle for quarks in order to make sure that no energy is missing along the beam pipe, remembering that the final size of a hadronic jet has $\sim 10^\circ$ of semiaperture and, typically, LEP detectors have acceptance for hadrons down to 10° .
- ii) *Isolation criteria*: each parton (e, μ , q, $\bar{q}$) should be isolated, i.e. no other parton should be in a cone of 30° . This criterion is also used for the two quarks, in order to make sure that no jet is missing along the beam pipe. This may not however be necessary if one checks that there is no strong hadronic activity near this pipe. Since this strongly depends on specific detectors, background conditions and on a more realistic fragmentation, we kept this isolation criterion in the following.
- iii) *Mass of the hadronic system*: if we compare the WW and $\tilde{W}\tilde{W}$ mass distributions of the hadronic system, see fig. 2.25, we realize that the two samples of events are well separated. In the following we ask $m_{had} < 60$ GeV. With $50\%/\sqrt{E}$ hadronic resolution this cut alone gives

already a clean signal for winos. It is worth mentioning that in our simulation we have also included effects from $W \rightarrow tb \rightarrow \nu + \text{hadrons}$. The signal can also be isolated by looking at the total energy and the total momentum of the hadronic system where we have seen that the WW and $\tilde{W}\tilde{W}$ events are well separated.

- iv) *The lepton spectrum:* for a further separation we have used the lepton spectrum. The two W 's are produced back to back; therefore one can boost the lepton to the W rest frame by using the average W momentum $\langle p_W \rangle = \sqrt{E_{\text{beam}}^2 - m_W^2}$ and taking as W direction the one reconstructed from the hadronic system. Since the momentum of the lepton in the W rest frame is exactly $m_W/2$ one gets a very strong separation between $\tilde{W}$'s and W 's. This can be seen in fig. 2.26, where we show the lepton momentum in the W rest frame versus the hadronic mass distribution a) for $\tilde{W}\tilde{W}$ and b) for WW .

Table 2.4 summarizes the results of our selection criteria. From there we can see that the WW events can be eliminated, from 4.8 pb down to 0.002 pb with $\sigma_{\text{had}} = 50\%/\sqrt{E}$ and down to 0.01 pb for $\sigma_{\text{had}} = 120\%/\sqrt{E}$. This has to be compared with a final $\tilde{W}\tilde{W}$ signal of ~ 0.11 pb. Therefore a wino signal can be well separated from the WW events, with an integrated luminosity of 100 pb^{-1} , up to $m_{\tilde{W}} \approx 80 \text{ GeV}$.

The separation will be more difficult if $t\bar{t}$ events can be produced in the same energy region of 190 GeV. We have applied our selection criteria to LUND hadronic events with a top mass varying between 40 and 70 GeV; it turned out that some contaminations survive from $t\bar{t} \gamma$ events. Such events contain b -quarks and hence may be removed by tagging these with the vertex detector.

The accepted cross-section for these events varies from 0.24 pb ($m_t = 40 \text{ GeV}$) to 0.06 pb ($m_t = 55 \text{ GeV}$) and 0.04 pb ($m_t = 70 \text{ GeV}$). Most of these events are of the type where both top quarks decay semileptonically one to $e(\mu) + b + \text{jet}$ and the other to $\tau + b + \text{jet}$ and a high-momentum ($\sim 70 \text{ GeV}$) gamma along the beam pipe. If we increase the minimum energy cut for the partons from 10 GeV to 20 GeV the $t\bar{t}$ background could be reduced drastically by more than a factor of 3 but also the wino cross-section would drop by a factor of ~ 1.5 .

Below the WW threshold, $\sqrt{s} = 160 \text{ GeV}$, we no longer need to cut on the 'lepton momentum in the W rest frame', whereas all other cuts remain. The only background we are left with comes from $t\bar{t}(\gamma)$. In table 2.5 we summarize our results for $m_{t\bar{t}} = 40, 55, 70 \text{ GeV}$ and with two different cuts on the lepton momentum.

For $p_{e,\mu} > 10 \text{ GeV}$ (20 GeV) the $\sigma_{\tilde{W}\tilde{W}}$ accepted is 0.19 pb (0.13 pb) with $m_{\tilde{W}} = 75 \text{ GeV}$ and $m_{\tilde{\nu}} = 100 \text{ GeV}$. Again the $t\bar{t}(\gamma)$ events are of the same type as the ones discussed at $\sqrt{s} = 190 \text{ GeV}$ with a high-momentum gamma along the beam pipe.

But in a scenario with a top mass from 40 to 70 GeV at the time of LEP II the top mass will be well known and we are sure that we can further optimize the cuts to get rid completely of $t\bar{t}$ events, for instance by using the known b -tagging techniques and methods to identify the isolated tracks from the tau decays.

We underline here that a new heavy lepton would decay to a ν plus a virtual W , giving exactly the same signature as the one studied in this and in the previous sections. Of course our selection of winos could also be applied for a new heavy lepton and, once a signal has been found, one should look more carefully into differences between the two candidates (such as lepton spectrum, angular distributions, and cross-section dependence). It has been emphasized that the rate for sequential heavy lepton production is quite different from that for $\tilde{W}$ pair production in e^+e^- collisions [2.6].

2.4.4 $m_{\tilde{\nu}} < m_t$ case. Wino signals for the light sneutrino scenario.

Up to this point, our analysis has been confined to the case $v'/v = 1$, with all the sfermions approximately degenerate.

We see from eq. (1.21), however, that even if the photino is relatively light, the degeneracy between the various sfermions may be lifted if v'/v is substantially different from unity ($\alpha \neq 45^\circ$). Of particular interest is the case when the $\tilde{\nu}$ is light [2.14], with all charged sfermions relatively heavy. Such a sneutrino would dominantly decay invisibly via $\tilde{\nu} \rightarrow \nu \tilde{\gamma}$ (if $m_{\tilde{\gamma}} < m_{\tilde{\nu}}$) and so would give $\cancel{p}_T$ in the detector. If $m_{\tilde{\gamma}} > m_{\tilde{\nu}}$, the sneutrino may well be the lightest SUSY particle and hence very long lived or stable. As discussed in section 1.6 such sneutrinos would have escaped experimental detection.

For fixed photino mass, one cannot vary v'/v in an arbitrary fashion since for either very small or very large values of this ratio, $m_{\tilde{W}}$ becomes smaller than the PETRA lower limit [2.3] [see eq. (1.13)]. This severely limits the values of sfermion masses, at least within the present framework.

Changing v'/v from unity has two effects:

- i) The $\tilde{W} \tilde{W}$ pair production cross-section [see eq. (C.10)] is altered because the mixings, and hence the wino couplings, are different. In particular, for the case $v'/v < 1$ which gives lighter sneutrinos, this cross-section is reduced if the average sfermion mass is large.
- ii) The mass degeneracy between the sfermions is lifted. This results in a complete change in the wino branching ratios from the $v'/v = 1$ case discussed in sections 2.4.2 and 2.4.3, if one or more of the sfermion masses falls below $m_{\tilde{W}}$ [see eq. (2.6)]. For the case of the light sneutrino that we are considering, the next lightest sfermion is $\tilde{u}_L$ which is considerably heavier than $\tilde{\nu}$. In fact, there is a substantial region of parameter space when $\tilde{\nu}$ is the only sfermion lighter than the wino, so that the leptonic decay $\tilde{W} \rightarrow \ell \tilde{\nu}$ is the only allowed two-body decay of the wino.

The fact that the leptonic branching ratio is enhanced to as much as 100% ($\ell = e, \mu, \tau$) greatly increases the number of lepton pairs from

$$e^+e^- \rightarrow \tilde{W} \tilde{W} \rightarrow \tilde{\nu} \bar{\ell} + \tilde{\nu} \ell.$$

If $m_{\tilde{\nu}} < m_{\tilde{W}}$, $m_{\tilde{W}} < m_{\tilde{q}}$ and $m_{\tilde{\ell}}$, the signature would then be two acoplanar electrons or muons or electron-muon with large $\cancel{p}_T$ and $\cancel{E}$. The wino would decay 100% into $\tilde{\nu} \ell$ and the cross-section for such a signature would be 44% of the total cross-section. The background is the same as in the selectron or smuon case, except that we have $e\mu$ pairs from $\tilde{W} \tilde{W}$ decays, and at $\sqrt{s} = 160$ GeV (190 GeV) we select the signal by asking $p_T > 8$ GeV (10 GeV) and $\theta_{\text{acopl}} > 30^\circ$ (34°). After those cuts we are left with no background at $\sqrt{s} = 160$ GeV and only with WW events contamination at $\sqrt{s} = 190$ GeV. The cut cross-sections for production of $e\bar{e}$, $\mu\bar{\mu}$, $e\bar{\mu} + \mu\bar{e}$ pairs from the decays of the wino are shown in fig. 2.27 for $\sqrt{s} = 160$ GeV (just below the W^+W^- threshold) and in fig. 2.28 for $\sqrt{s} = 190$ GeV as a function of the scalar neutrino mass. For definiteness, we have chosen a value of $v'/v = 0.57$, for which case it is possible to obtain the sfermion masses so that only the leptonic two-body mode is accessible for wino decay. We note, however, that to obtain large values of $m_{\tilde{W}}$ (see section 1.5.2) for this value of v'/v , we are forced to choose large values of $|\mu_2|$ which would correspond to larger values of $m_{\tilde{\gamma}}$.

We see from the figures that even for wino masses almost all the way up to the kinematic limit, the cross-section is detectable and background-free provided $m_{\tilde{\nu}} < m_{\tilde{W}}$. Below the W^+W^- threshold, winos with masses as large as 75 GeV lead to ≥ 15 events per 100 pb^{-1} after cuts are made to remove QED backgrounds. At $\sqrt{s} = 190$ GeV, imposing acoplanar and $\cancel{p}_T$ cuts severely reduces the W^+W^- background, and signal-to-background ratios exceeding unity can be obtained for wino masses as large as 90 GeV. In reaching this conclusion, we have assumed that $m_{\tilde{\nu}} \geq 20$ GeV, so that the kinematics of the wino and W -pair events are sufficiently different so that we can separate one from the other. It is also clear that if $m_{\tilde{\nu}} < m_{\tilde{W}}$ and $m_{\tilde{W}} < m_Z/2$, wino pairs with the winos always decaying leptonically could also be seen at LEP I, although the $\tilde{W}$ production cross-section may be reduced owing to the change in the $Z^0 \tilde{W} \tilde{W}$ coupling [2.3].

It seems therefore that for a wide class of SUSY models, it would be possible to identify the wino at LEP II energies for a broad range of $m_{\tilde{W}}$. Moreover, it seems possible to separate the wino signal from the Standard Model physics signals including those from W^+W^- pair production, although this latter complication can be avoided by running the machine at $\sqrt{s} = 160$ GeV.

To conclude we have seen in the case $m_{\tilde{Z}} = m_{\tilde{W}} > m_{\tilde{W}}$ that by selecting the mixed leptonic and hadronic decays of $\tilde{W}\tilde{W}$, a clean signal up to wino masses of 80 GeV with an integrated luminosity of 100 pb^{-1} can be obtained in the region of WW production with accepted cross-sections $\sigma_{\text{acc}}(\tilde{W}\tilde{W}) \approx 15 \times \sigma_{\text{acc}}(WW)$. If $m_{\tilde{Z}} < m_{\tilde{W}}$, a similar mass range can be reached; therefore, as in the case of sfermions, the best place to look for wino production will be the highest available energy of LEP II. Also for this reaction the end caps + barrel, see subsection 2.2.2.1, detect two times more cross-section than the barrel alone for $m_{\tilde{W}} = 80$ GeV.

2.5 Zino-photino production [2.3, 2.7-2.9]

We now turn to a discussion of the final state resulting from the decay of the zino which is produced via the process

$$e^+e^- \rightarrow \tilde{Z}\tilde{\gamma}. \quad (2.8)$$

From the couplings in appendix A we can see that this occurs only via the t-channel exchange of the selectron ($\tilde{t}_L$ or $\tilde{t}_R$), see fig. 2.29, but that there are no s channel Z^0 or γ contributions.

The absence of the photon contribution follows from the neutrality of the zino and the photino, whereas the absence of the Z^0 contribution perhaps merits a little more discussion. The gaugino components ($\tilde{\lambda}_3$ and $\tilde{\lambda}_0$) do not couple to the Z^0 because of the $SU(2)_L \times U(1)_Y$ structure of the gauge interactions. Thus the photino, which has higgsino components only because of the small mixing induced by $m_{\tilde{\gamma}} \neq 0$, has only very small direct coupling to the Z^0 . In this paper we have neglected this coupling which could only arise from the coupling of the Z^0 to the two neutral higgsinos because for $v' = v$ it identically vanishes even for large soft SUSY-breaking gaugino masses.

Notice that while our argument shows that the $Z^0\tilde{Z}\tilde{\gamma}$ coupling may be very small it does not rule out the possibility of the coupling of Z^0 to other neutralinos. For the minimal model discussed in this paper [2.12], these always vanish for v'/v except when one of the neutralinos is a pure higgsino (recall that for $v' = v$ there is always a higgsino state of mass $-2m_1$) irrespective of gaugino masses. The decay of Z^0 into this state has been studied in ref. [2.17]. In general, the couplings of Z^0 to neutralinos depend on the Higgs sector and will not be discussed here.

2.6 Zino search

If $m_{\tilde{t}}$ and $m_{\tilde{W}}$ are both greater than E_{beam} the zino production via reaction (2.8) may be either the first two-body supersymmetric process to be accessible at LEP I or the only one accessible at LEP II if $m_{\tilde{t}}$ and $m_{\tilde{W}}$ are heavier than 100 GeV. This is of particular interest since $m_{\tilde{Z}} < m_{Z^0}$ is predicted in SUSY models with a light photino.

On the other hand the total cross-section is very small (from few picobarns down to ~ 0.3 pb depending on $m_{\tilde{Z}}$ and $m_{\tilde{g}}$), owing to the suppression coming from the exchange of heavy selectrons. Thus although this reaction may well be the most accessible kinematically, the smallness of the rate will require a very careful analysis of the final state for its identification. It is to this that we now turn.

In deriving the Monte Carlo distributions for the final state, we have assumed a factorization of the production and decay processes, i.e. we have ignored correlations due to the zino spin. The total

and differential cross-section formulae for the $\tilde{Z}\tilde{\gamma}$ production are given in appendix D together with the squared and spin-averaged matrix element for $\tilde{Z} \rightarrow \ell\bar{\ell}\tilde{\gamma}$ decay.

Finally, for $m_{\tilde{q}} = m_{\tilde{l}}$, the branching fraction for the decay $\tilde{Z} \rightarrow \ell\bar{\ell}\tilde{\gamma}$ is given by [2.9, 2.16],

$$\frac{\Gamma(\tilde{Z} \rightarrow \ell\bar{\ell}\tilde{\gamma})}{\Gamma(\tilde{Z} \rightarrow \text{everything})} = \frac{(X_\ell^2 + P_\ell^2 + X_\ell'^2 P_\ell'^2)}{\{n_\ell(X_\ell^2 P_\ell^2 + X_\ell'^2 P_\ell'^2) + 3n_u(X_u^2 P_u^2 + X_u'^2 P_u'^2) + 3n_d(X_d^2 P_d^2 + X_d'^2 P_d'^2)\}} \quad (2.9)$$

where n_ℓ , n_u , and n_d are the number of types of leptons, up quarks, and down quarks into which the zino can decay. For $m_{\tilde{\gamma}} = 0$, and $n_\ell = n_d = 3$ and $n_u = 2$, the leptonic branching fraction is 13.2%. Increasing $m_{\tilde{\gamma}}$ up to 10 GeV slightly increases the branching fraction, with the increase being quite significant for light zinos ($m_{\tilde{Z}} \leq 30$ GeV) [2.16].

Under our hypotheses the signature for such a reaction will be either two acoplanar leptons + $\cancel{E} + \cancel{p}_T$ from $Z \rightarrow \ell\bar{\ell}\tilde{\gamma}$, or two or more jets + $\cancel{E} + \cancel{p}_T$ from $Z \rightarrow q\bar{q}\tilde{\gamma}$. Moreover from the leptonic decay we can have, depending on the ratio $m_{\tilde{Z}}/\sqrt{s}$, the two leptons of the same flavour in one side of the detector and nothing in the opposite one. In the case of the hadronic decay the event is 'less one-sided' and the signature is essentially given by $\cancel{E} + \cancel{p}_T$ [2.7-2.9].

2.6.1 The leptonic decay signature of the zino

The QED background to the zino leptonic reactions is the same as the one treated for the sleptons. Moreover, above the WW threshold, we have contaminations from:

- 1) $Z^0 Z^0 \rightarrow \ell\nu\bar{\nu}$;
- 2) WW if both W's decay into electrons or muons;
- 3) WW when both W's give taus that give a pair of electrons or muons.

The results for the accepted cross-section of electron pairs, after p_T , acoplanarity and acceptance cuts, are shown in fig. 2.30 from 70 GeV up to energies below the WW threshold. We emphasize that although the cross-sections are very small the events are completely free from Standard Model backgrounds contamination. As before, we have fixed $m_{\tilde{\gamma}} \approx 6$ GeV and have also fixed $m_{\tilde{e}} = 70$ GeV. It is clear that increasing the beam energy is of no advantage since the cross-section is limited by $m_{\tilde{l}}$ rather than by the available phase space.

We now move to the single-particle distributions. The angular distribution of the leptons with respect to the beam, not shown, is almost uniform, reflecting the flatness of the zino distributions.

The energy distributions of the leptons, for a given zino mass, are shown for the energies of TRISTAN, LEP I (SLC) and LEP II in fig. 2.31. These are quite unlike those for sleptons, reflecting the three-body decay of the zino. Moreover the mean energy of the leptons slightly exceeds a third of that of the beam, which is merely a reflection of the assumed lightness of the photino.

We now turn to study the correlations between the observed leptons. Dicus et al. [2.7] have emphasized that if the zino is light enough, the leptons produced in the decay $\tilde{Z} \rightarrow \ell\bar{\ell}\tilde{\gamma}$ will tend to have a small opening angle $\theta_{\ell+\bar{\ell}}$ between them, leading to a characteristic signature of two leptons in a single hemisphere recoiling against $\cancel{p}_T$. Clearly this is crucially dependent on $m_{\tilde{Z}}/\sqrt{s}$. The distribution in acollinearity angle, which is the supplement of $\theta_{\ell+\bar{\ell}}$, for a fixed zino mass is shown for TRISTAN, LEP I (SLC) and LEP II energies in fig. 2.32. We have checked for other values of $m_{\tilde{Z}}$ and $\sqrt{s}$ that the shape of the distributions is indeed governed by $m_{\tilde{Z}}/\sqrt{s}$. The evolution of the shape of the distribution with increased beam energy could provide a striking check of the existence of zinos. Again, these have to be compared with the corresponding distributions of the selectron case, fig. 2.4, and of the wino case, fig. 2.16. Shown in fig. 2.33 is the distribution in acoplanarity angle at the same energies as in fig. 2.32. These may be compared with the corresponding slepton distributions of fig. 2.7.

The $\not{p}_T$ and $\not{E}$ distributions are shown in figs. 2.34 and 2.35 respectively. The former clearly show that the cut on $\not{p}_T$ needed to eliminate the Standard Model background does not greatly reduce the SUSY cross-section.

The missing-energy distributions are of interest in that $\not{E}$ is bounded below by

$$\not{E} \geq \frac{(\sqrt{s} + m_{\tilde{\gamma}})^2 - m_{\tilde{Z}}^2}{2\sqrt{s}}$$

and that the lower edge of the distribution is rather sharp. A measurement of this end point in conjunction with the end point of fig. 2.36 below could lead to a measurement of the zino and photino masses [2.9].

In fig. 2.34 is shown the distribution of the invariant mass of the lepton pair $m_{\ell+\ell-}$. Kinematics requires that

$$m_{\ell+\ell-} < m_{\tilde{Z}} - m_{\tilde{\gamma}},$$

independently of $\sqrt{s}$. Again we note that the sharpness of the upper end point could lead to a determination of the zino and photino masses.

Finally the width of the distribution in missing mass squared $mm^2 \equiv \not{p}^2$, see fig. 2.37, indicates as in the slepton case that the missing energy is carried by more than one particle. The narrowness of this distribution compared with that in the slepton case (see fig. 2.6) is a reflection of the fact that one of the photinos carries a much larger energy than the other one.

Above the WW threshold [2.10], $\sqrt{s} = 190$ GeV, we have used in our analysis the following set of values: $m_{\tilde{Z}} = 100$ GeV, $m_{\tilde{g}} = 120$ GeV, $m_{\tilde{\gamma}} = 10$ –40 GeV. We note here that the value of $m_{\tilde{\gamma}}$ does not influence the results significantly. Table 2.6 shows the cross-sections at $\sqrt{s} = 190$ GeV. The QED events are easily eliminated with the cuts $\theta_{\text{acopl}} < 34^\circ$, $\not{p}_T > 10$ GeV. Moreover $\sigma(ZZ \rightarrow \nu\bar{\nu}\ell\ell) \simeq 0.006$ pb, 12 times smaller than for $\tilde{Z}\tilde{\gamma}$. However these cuts plus cuts in $\not{E}$ and in $\cos \theta_{\text{prod}}$ distributions do not get rid of the WW $\rightarrow \ell\bar{\nu}\ell\nu$ events. Notice that $\sigma(\text{WW} \rightarrow \ell\bar{\nu}\ell\nu)/\sigma(\tilde{Z}\tilde{\gamma} \rightarrow \ell\bar{\ell}\tilde{\gamma}\tilde{\gamma}) = 6$. Our conclusion is that it will be very hard to detect a $\tilde{Z}\tilde{\gamma}$ signal at $\sqrt{s} = 190$ GeV via the leptonic decay. The WW is the main obstacle against this.

On the other hand below the WW threshold at $\sqrt{s} = 160$ GeV, taking $m_{\tilde{Z}} = 90$ GeV, $m_{\tilde{g}} = 100$ GeV, $m_{\tilde{\gamma}} = 10$ –40 GeV and selecting $\theta_{\text{acopl}} > 30^\circ$ and $\not{p}_T > 8$ GeV, QED processes are 100% suppressed, while 93% of the signal survives with an accepted cross-section of 0.19 pb for the leptonic decay (electron or muon). These results are obtained with the ‘good’ detector of table 1.2.

Thus, running the machine just below the WW threshold should lead to clearly identifiable signatures of the zino, since this is predicted to have a mass smaller than m_{Z^0} , provided only that $m_{\tilde{\gamma}}$ is small [2.14].

2.6.2 Zino hadronic decay [2.10]

The background for the zino hadronic reactions is given at any energy by:

- 1) $q\bar{q}\gamma$ when the γ is not detected
and above the corresponding thresholds by:
- 2) $Z^0 Z^0 \rightarrow q\bar{q}\nu\bar{\nu}$;
- 3) $\text{WW} \rightarrow q\bar{q}\ell\bar{\nu}$ with $\ell \neq \tau$ and ℓ is either not detected or lost in one of the two jets;
- 4) $\text{WW} \rightarrow q\bar{q}\tau\bar{\nu}$ with $\tau \rightarrow \ell\nu\bar{\nu}$ ($\ell = e$ or μ) and again the lepton goes undetected or lost into the jets;
- 5) $\text{WW} \rightarrow q\bar{q}\tau\bar{\nu}$ with $\tau \rightarrow \text{hadrons} + \nu$ and the τ hadrons are confused within the jets.

Table 2.7 shows the cross-sections at $\sqrt{s} = 190$ GeV.

In our analysis we have used again the following set of values at $\sqrt{s} = 190$ GeV: $m_{\tilde{Z}} = 100$ GeV, $m_{\tilde{e}} = 120$ GeV, $m_{\tilde{\gamma}} = 10\text{--}40$ GeV. Again $m_{\tilde{\gamma}}$ does not influence the results significantly.

For the hadronic case selecting the events with $\cancel{p}_T > 35$ GeV and $\cancel{E} > 80$ GeV we can get rid of (1), (2), and (3), but not of (4) and (5). To eliminate the events from the leptonic tau decay, we have rejected the events with an isolated lepton, defined either as

- i) $10 \text{ GeV} < E_\ell < 20 \text{ GeV}$ and, within a 20° cone around the lepton, $E_h < 1$ GeV, or
- ii) $E_\ell > 20 \text{ GeV}$ and $E_h < 5 \text{ GeV}$.

Similarly, in order to eliminate the events from the hadronic tau decay, we have thrown away the events with a leading isolated particle, defined as above exchanging the lepton with the fastest particle of the event (lepton or hadron).

The results of our analysis are summarized in table 2.7. We conclude that $WW \rightarrow \tau\nu + q\bar{q}$ constitutes the main obstacle to detect the $\tilde{Z}\tilde{\gamma}$ events when $\tilde{Z} \rightarrow q\bar{q}\tilde{\gamma}$.

Our conclusion is that it will be very hard to detect the $\tilde{Z}\tilde{\gamma}$ signal above the WW threshold via both the leptonic *and* the hadronic decay of $\tilde{Z}^0$. The WW is the main obstacle against this.

Although it appears that some of the supersymmetry signals are visible above the W pair background, running LEP just below the W-pair threshold may yield unexpected surprises.

REFERENCES

- [2.1] Slepton pair production has been considered by
 G.R. Farrar and P. Fayet, Phys. Lett. **89B** (1980) 191.
 M. Glück and E. Reya, Phys. Lett. **130B** (1983) 423.
 D. Dicus, S. Nandi, W. Repko and X. Tata, Phys. Rev. Lett. **51** (1983) 1030.
 S. Dawson, E. Eichten and C. Quigg, Phys. Rev. **D31** (1985) 1581.
 M.E. Haber and G.L. Kane, Phys. Rep. **117** (1985) 75.
 M. Glück and E. Reya, Phys. Rev. **D31** (1985) 620.
- [2.2] T. Schimert and X. Tata, Phys. Rev. **D32** (1985) 721.
- [2.3] X. Tata and D. Dicus, Phys. Rev. **D35** (1987) 2110.
- [2.4] A.R. Allan, N. Brown and A.D. Martin, Z. Phys. **C31** (1986) 479. There, the effects of including off-mass-shell selectron contributions to the reaction $e^+e^- \rightarrow \tilde{e}\tilde{e} \rightarrow e^+\tilde{\gamma}e^-\tilde{\gamma}$ are discussed.
- [2.5] Wino pair production has been considered by
 P. Nath, R. Arnowitt and A. Chamseddine, Harvard report HUTP-83/A077 (1983).
 V. Barger, R. Robinett, W.-Y. Keung and R.J.N. Phillips, Phys. Lett. **131B** (1983) 372.
 D. Dicus et al., ref. 2.1.
 J. Ellis et al., Phys. Lett. **127B** (1983) 233 and **132B** (1983) 463.
 S. Dawson et al., ref. 2.1.
 H.E. Haber and G.L. Kane, ref. 2.1.
 A. Bartl, M. Fraas and W. Majerotto, Z. Phys. **C30** (1986) 441.
 T. Fukai et al., Waseda Univ. preprint, WU-HEP-85-3 (1986).
 B.F.L. Ward, Phys. Rev. **D35** (1987) 2092.
- [2.6] T. Schimert, C. Burgess and X. Tata, Phys. Rev. **D32** (1985) 707.
- [2.7] D. Dicus, S. Nandi, W. Repko and X. Tata, Phys. Rev. **D29** (1984) 1317 and **D30** (1984) 1112.
- [2.8] See also:
 E. Reya, Phys. Lett. **133B** (1983) 245.
 D. Dicus et al., ref. 2.1.
 S. Dawson et al., ref. 2.1.
 H.E. Haber and G.L. Kane, ref. 2.1.
 D. Campbell et al., Phys. Lett. **131B** (1983) 213.
 V. Barger et al., ref. 2.5.
- [2.9] H. Baer et al., in Physics at LEP (CERN 86-02, Geneva, 1986), Vol. I, p. 297.
- [2.10] C. Dionisi et al., ECFA Workshop on LEP 200, Aachen, 1987 (CERN 87-08/ECFA 87-108, Geneva, 1987), Vol. II, p. 380.
- [2.11] G. Barbiellini et al., in Physics at LEP (CERN 86-02, Geneva, 1986), Vol. 2, p. 1.
 K. Hagiwara, R.D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. **B282** (1987) 253.
- [2.12] This has been done by several authors. Formulae for the exact couplings in terms of numerically calculated coefficients may be found in:
 H.E. Haber and G.L. Kane, ref. 2.1.
 S. Dawson et al., ref. 2.1.
 H. Baer, V. Barger, D. Karatas and X. Tata, Phys. Rev. D (in press).
 H. Baer, D. Dicus, M. Drees and X. Tata, Wisconsin preprint MAD/PH/329 (1987), Phys. Rev. D (in press).
- [2.13] C. Dionisi and M. Dittmar, in Proc. of Workshop on Physics at Future Accelerators, La Thuile and Geneva, 1987, Vol. II, in preparation.

- [2.14] S. Weinberg, Phys. Rev. Lett. **50** (1983) 387,
R. Arnowitt, A. Chamseddine and P. Nath, Phys. Rev. Lett. **49** (1982) 970 and **50** (1983) 232.
- [2.15] J.S. Hagelin, G.L. Kane and S. Raby, Nucl. Phys. **B247** (1984) 638.
L. Ibañez, Phys. Lett. **137B** (1984) 160.
R.M. Barnett, M.E. Haber and K.S. Lackner, Phys. Lett. **126B** (1983) 64; Phys. Rev. **D29** (1984) 1990.
- [2.16] H. Baer, K. Hagiwara and X. Tata, Phys. Rev. **D35** (1987) 1598; Phys. Rev. Lett. **57** (1986) 294.
- [2.17] H. Komatsu, Phys. Lett. **177B** (1986) 201.

Table 2.1

$\tilde{e}\tilde{e}$ and WW rates at $\sqrt{s} = 190$ GeV before and after the acoplanarity and p_T cuts for $m_{\tilde{e}} = 85$ and 90 GeV. The last column shows the statistical significance in terms of standard deviations for integrated luminosities of 100 and 500 pb^{-1}

$M_{\tilde{e}}$ (GeV)	$M_{\tilde{\gamma}}$	$\sigma_{\text{tot}} \times BR_{ee}$ (pb)	σ_{acc} (pb)	100 (pb^{-1})	500 (pb^{-1})	# σ 's
85	0	0.52	0.41	41	205	12
	10	0.49	0.38	38	190	11
	20	0.39	0.31	31	155	9
	30	0.22	0.17	17	85	5
	40	0.19	0.16	16	80	5 11
90	0	0.17	0.14	14	70	4 9
	10	0.16	0.13	13	65	4 9
	20	0.15	0.12	12	60	3 8
	30	0.09	0.07	7	35	2 5
	40	0.08	0.06	6	30	2 4
WW $\rightarrow e\nu e\nu$		0.17	0.10	11	55	
WW $\rightarrow \tau\nu e\nu$ $\downarrow$ $\tau e\nu\nu$		0.06	0.01			

Table 2.2

$\tilde{\mu}\tilde{\mu}$ and WW rates at $\sqrt{s} = 190$ GeV before and after the selection cuts for $m_{\tilde{\mu}} = 75, 80$ and 85 GeV

	σ_{tot} (pb)	σ_{acc} (pb)	σ_{acc} (pb) after μ charge asymmetry cut
$\mu\mu(\gamma)$	7.8		
$WW \rightarrow \mu\nu\mu\nu$	0.17	0.09	0.03
$+ \tau\nu\mu\nu$	0.06	0.03	
$\downarrow \rightarrow \mu\nu$			
$(\tilde{\mu}\tilde{\mu}) \ M_{\tilde{\mu}} = 75\text{GeV}$	0.31	0.20	0.10
$M_{\tilde{\mu}} = 80\text{GeV}$	0.22	0.15	0.075
$M_{\tilde{\mu}} = 85\text{GeV}$	0.11	0.07	0.035

Table 2.3

$\tilde{W}$ and W production characteristics

	WW	$\tilde{W}\tilde{W}$
Mass of hadronic system	M_W	$\sim \frac{2}{3} M_{\tilde{W}}$
$E_{hadrons}$	E_{beam}	$\sim \frac{2}{3} E_{beam}$
Lepton momentum in the W reference frame	$\frac{M_W}{2}$	$\sim \frac{1}{3} M_{\tilde{W}}$

Table 2.4

$\tilde{W}\tilde{W}$ and WW rates at $\sqrt{s} = 190$ GeV before and after the selection.
For WW the results using an average hadron calorimeter are also shown for comparison.

$\tilde{W}\tilde{W}$ $M_{\tilde{W}}=80\text{GeV}$ $M_{\tilde{Z}}=100\text{GeV}$	WW		Remarks on cuts
	50%/ $\sqrt{E}$	120%/ $\sqrt{E}$	
0.36pb	4.76pb	4.76pb	$\sigma_{\text{tot}} \cdot BR_{e\mu} \cdot BR_{q\bar{q}} \cdot 2$ without cuts
0.17pb	2.8pb	2.8pb	all event selection criteria
0.16pb	0.02pb	0.1pb	additional cut: $M_{Had.} < 60\text{GeV}$
0.11pb	$7 \cdot 10^{-4}\text{pb}$	$3.5 \cdot 10^{-3}\text{pb}$	further cut: $P_{Lept.CMS} < 35\text{GeV}$
	0.002pb	$\sim 0.01\text{pb}$	background from $WW \rightarrow \tau\nu q\bar{q}$ $\hookrightarrow e, \mu, \nu\nu$

Table 2.5

$\tilde{W}\tilde{W}$ and $t\bar{t}$ rates at $\sqrt{s} = 160$ GeV for different
cuts on the lepton momentum

m_t (GeV)	$\sigma_{t\bar{t}}$ (pb)	$\sigma_{t\bar{t}}$ accepted (pb)		ϵ % of $\sigma_{\tilde{W}\tilde{W}}$ accepted	
		$p_{e,\mu} > 10\text{ GeV}$	$p_{e,\mu} > 20\text{ GeV}$	$p_{e,\mu} > 10\text{ GeV}$	$p_{e,\mu} > 20\text{ GeV}$
40	13	0.62	0.35		
55	7	0.20	0.13	0.40	0.28
70	4	0.10	0.08		

Table 2.6

Leptonic zino decay rate and its backgrounds
before any cuts

	$\tilde{Z}\gamma$	WW		$\ell\ell\gamma$	$Z^0 Z^0$
$\sigma_{tot}(pb)$	0.3	17		20	0.9
		$e(\mu)\nu + e(\mu)\nu$	$\tau\nu + \tau\nu$ $\tau \rightarrow e\nu\nu$		
$\sigma_{tot} \times BR(pb)$	0.07	0.42	0.09		0.006

Table 2.7

Hadronic zino decay rate and its backgrounds.
The last row gives the rates after the selection.

	$\tilde{Z}\tilde{\gamma}$	WW			$q\bar{q}\gamma$	$Z^0 Z^0$
$\sigma_{tot}(pb)$	0.3	17			82.2	0.9
		$W \rightarrow e(\mu)\nu$	$W \rightarrow \tau\nu$ $e(\mu)\nu\nu$	$W \rightarrow \tau\nu$ $\nu had's$		
		$W \rightarrow q\bar{q}$	$W \rightarrow q\bar{q}$	$W \rightarrow q\bar{q}$		
$\sigma_{tot} \times BR(pb)$	0.2	5.04	0.86	1.64	82.2	0.19
$\sigma_{acc} \times BR(pb)$	0.11	0.06	0.1	0.26	0.	0.

Figure captions

- Fig. 2.1** Feynman diagrams for the production of: a) scalar fermions, b) scalar electron.
- Fig. 2.2** The total cross-sections for $e^+e^- \rightarrow \tilde{\ell}\tilde{\ell}^* \rightarrow \ell\bar{\ell}\tilde{\gamma}\tilde{\gamma}$ for (a) $\ell = e$ and (b) $\ell = \mu$, with the acceptance and p_T cut incorporated, as functions of the centre-of-mass energy $\sqrt{s}$. The hollow points refer to the total cross-section, whereas the solid points in (a) refer to the cross-section for opposite-handed selectron pairs as discussed in the text. The τ -cut will reduce the σ 's by about 5% at the Z^0 pole. These graphs have been calculated with $m_\gamma \approx 6$ GeV, $m_{\tilde{Z}} = 70$ GeV.
- Fig. 2.3** The angular distributions of the electrons (muons) coming from selectron (smuon) pair production. Here θ is the angle between the incoming electron and the outgoing negative lepton. The acceptances and cuts are the same as in fig. 2.2.
- Fig. 2.4** The acollinearity distributions for electrons (muons) from $\tilde{e}\tilde{e}$ ($\tilde{\mu}\tilde{\mu}$) production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV full curve, $m_{\tilde{\mu}} = 30$ GeV dashed curve;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{\mu}} = 40$ GeV dashed curve;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{e}} = 60$ GeV dashed curve, $m_{\tilde{\mu}} = 40$ GeV dash-dotted curve, $m_{\tilde{\mu}} = 60$ GeV dotted curve.
- Fig. 2.5** The invariant mass distributions of the electron (muon) pairs from $\tilde{e}\tilde{e}$ ($\tilde{\mu}\tilde{\mu}$) production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV full curve, $m_{\tilde{\mu}} = 30$ GeV dashed curve;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{\mu}} = 40$ GeV dashed curve;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{e}} = 60$ GeV dash-dotted curve, $m_{\tilde{\mu}} = 40$ GeV dashed curve, $m_{\tilde{\mu}} = 60$ GeV dotted curve.
- Fig. 2.6** The missing mass squared distributions from $\tilde{e}\tilde{e}$ and $\tilde{\mu}\tilde{\mu}$ production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV full curve, $m_{\tilde{\mu}} = 30$ GeV dashed curve;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 40$ GeV dashed curve, $m_{\tilde{\mu}} = 40$ GeV full curve;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{e}} = 60$ GeV dotted curve, $m_{\tilde{\mu}} = 40$ GeV dashed curve and $m_{\tilde{\mu}} = 60$ GeV dash-dotted curve.
- Fig. 2.7** The acoplanarity angle distributions for electrons from $\tilde{e}\tilde{e}$ production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 40$ GeV;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{e}} = 60$ GeV dashed curve.
- Fig. 2.8** The p_T distributions from $\tilde{e}\tilde{e}$ production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 40$ GeV;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{e}} = 60$ GeV dashed curve.
- Fig. 2.9** Accepted cross-section for $e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$ after applying the cuts described in the text. Here a $\Gamma_{Z^0} = 2$ GeV instead of 3 GeV was used.
- Fig. 2.10** The accepted acoplanar di-electron cross-sections from $\tilde{e}\tilde{e}$ production above the WW threshold.
- Fig. 2.11** The accepted acoplanar di-muon cross-sections from $\tilde{\mu}\tilde{\mu}$ production above the WW threshold.
- Fig. 2.12** Average angular distributions of muons at $\sqrt{s} = 190$ GeV. The solid line corresponds to a spin-0 smuon of mass 70 GeV, the dashed line to a spin- $1/2$ particle.
- Fig. 2.13** Total cross-section as a function of beam energy. The solid (dashed) (dash-dotted) lines correspond to smuons of mass 70 (± 2) (± 5) GeV. The bars represent the statistical errors in cross-section measurements with the luminosities indicated in the text.

- Fig. 2.14** Feynman diagrams for the $\tilde{W}\tilde{W}$ production.
- Fig. 2.15** a) The total cross-section for wino pair production as a function of $\sqrt{s}$ without any acceptance corrections or cuts, for $m_{\tilde{\gamma}} = 10$ GeV and various wino masses. b) The cross-section times (branching ratio)² for $e^+e^- \rightarrow \tilde{W}\tilde{W} \rightarrow \ell\bar{\ell}\nu\bar{\nu}\tilde{\gamma}\tilde{\gamma}$ for $\ell = e$ with the acceptance and p_T cut incorporated, plotted as a function of the centre-of-mass energy, taking $m_{\tilde{\gamma}} \approx 6$ GeV, $m_{\tilde{e}} = 70$ GeV, $m_{\tilde{\nu}} = 70$ GeV and various wino masses. The τ cut has not been included.
- Fig. 2.16** The energy distribution of the charged lepton arising from the three-body decay of the wino for various wino masses.
- Fig. 2.17** The angular distributions of the electrons coming from wino pair production. Here θ is the angle between the incoming electron and the outgoing negative lepton. The acceptance and cuts are the same as in fig. 2.15b.
- Fig. 2.18** The acollinearity distributions for leptons from $\tilde{W}\tilde{W}$ production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 40$ GeV;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{e}} = 60$ GeV dashed curve.
- Fig. 2.19** The missing energy distribution from $\tilde{W}\tilde{W}$ production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 30$ GeV full line, $m_{\tilde{e}} = 40$ GeV dashed line;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 30$ GeV full line, $m_{\tilde{e}} = 40$ GeV dashed line, $m_{\tilde{e}} = 60$ GeV dash-dotted line.
- Fig. 2.20** The p_T distributions from $\tilde{W}\tilde{W}$ production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 40$ GeV;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 40$ GeV full-line, $m_{\tilde{e}} = 60$ GeV dashed line.
- Fig. 2.21** The invariant mass distributions of the lepton pairs from $\tilde{W}\tilde{W}$ production for:
a) $\sqrt{s} = 70$ GeV, $m_{\tilde{e}} = 30$ GeV;
b) $\sqrt{s} = m_Z$, $m_{\tilde{e}} = 40$ GeV;
c) $\sqrt{s} = 180$ GeV, $m_{\tilde{e}} = 40$ GeV full curve, $m_{\tilde{e}} = 60$ GeV dashed line.
- Fig. 2.22** Total cross-section for $\tilde{W}\tilde{W}$ production as a function of $m_{\tilde{W}}$.
- Fig. 2.23** Total cross-section for $\tilde{W}\tilde{W}$ production as a function of $m_{\tilde{\nu}}$.
- Fig. 2.24** Signature for: a) $\tilde{W}\tilde{W}$ and b) WW events in our analysis.
- Fig. 2.25** Mass of the $q\bar{q}$ system for $\tilde{W}\tilde{W}$ (full line) and for WW (dashed line).
- Fig. 2.26** Lepton momentum in the W reference frame versus the $q\bar{q}$ mass combination for a) $\tilde{W}\tilde{W}$ events, b) WW events.
- Fig. 2.27** Results from $\tilde{W}\tilde{W}$ with $m_{\tilde{\nu}} < m_{\tilde{e}}$ at $\sqrt{s} = 160$ GeV for several values of $\tilde{\gamma}$ mass.
- Fig. 2.28** Results from $\tilde{W}\tilde{W}$ with $m_{\tilde{\nu}} < m_{\tilde{e}}$ at $\sqrt{s} = 190$ GeV for several values of $\tilde{\gamma}$ mass. The WW background is also shown as the dashed lines.
- Fig. 2.29** Feynman diagram for the $\tilde{Z}\tilde{\gamma}$ production.
- Fig. 2.30** The cross-section for $e^+e^- \rightarrow \tilde{Z}\tilde{\gamma}$ followed by $\tilde{Z} \rightarrow \ell\bar{\ell}\tilde{\gamma}$ as a function of $\sqrt{s}$ for several zino masses. The cross-sections shown have been corrected for the acceptance and has both the p_T and the τ cuts discussed in the text. We have taken $m_{\tilde{e}} \approx 40$ GeV, $m_{\tilde{\gamma}} \approx 6$ GeV. The expected cross-section from $\tilde{Z} \rightarrow \mu^+\mu^-\tilde{\gamma}$ is, except for the slight difference in the acceptance for muons, identical to this and so the total event rate expected (neglecting taus) is about twice that shown. All the curves were calculated with a leptonic branching ratio of 11% for each of the e , μ and τ .

Fig. 2.31 The energy distributions of the charged leptons from $\tilde{Z}\tilde{\gamma}$ production for:

- a) $\sqrt{s} = 70 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$;
- b) $\sqrt{s} = m_Z$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line;
- c) $\sqrt{s} = 180 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line.

Fig. 2.32 The acollinearity angle distributions from $\tilde{Z}\tilde{\gamma}$ production for:

- a) $\sqrt{s} = 70 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$;
- b) $\sqrt{s} = m_Z$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line;
- c) $\sqrt{s} = 180 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line.

Fig. 2.33 The acollinearity angle distributions for leptons from $\tilde{Z}\tilde{\gamma}$ production for:

- a) $\sqrt{s} = 70 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$;
- b) $\sqrt{s} = m_Z$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line;
- c) $\sqrt{s} = 180 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line.

Fig. 2.34 The $\not{p}_T$ distributions from $\tilde{Z}\tilde{\gamma}$ production for:

- a) $\sqrt{s} = 70 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$;
- b) $\sqrt{s} = m_Z$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line;
- c) $\sqrt{s} = 180 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line.

Fig. 2.35 The missing energy distributions from $\tilde{Z}\tilde{\gamma}$ production for:

- a) $\sqrt{s} = 70 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$;
- b) $\sqrt{s} = m_Z$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line;
- c) $\sqrt{s} = 180 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line.

Fig. 2.36 The invariant mass distributions of the lepton pairs from $\tilde{Z}\tilde{\gamma}$ production for:

- a) $\sqrt{s} = 70 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$;
- b) $\sqrt{s} = m_Z$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line;
- c) $\sqrt{s} = 180 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line.

Fig. 2.37 The missing mass squared distributions from $\tilde{Z}\tilde{\gamma}$ production for:

- a) $\sqrt{s} = 70 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$;
- b) $\sqrt{s} = m_Z$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line;
- c) $\sqrt{s} = 180 \text{ GeV}$, $m_{\tilde{l}} = 40 \text{ GeV}$ full line, $m_{\tilde{l}} = 60 \text{ GeV}$ dashed line.

Fig. 2.1

Fig. 2.2

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Fig. 2.36

Fig. 2.37

3. THREE-BODY FINAL STATES

3.1 Introduction

As shown in the previous chapter, two-body reactions, owing to their relatively large cross-sections, can be used to search for SUSY particles, with the exception of $\tilde{\nu}$ and $\tilde{\gamma}$, with masses up to the beam energies. In this chapter we will look into the production of three-body final states to further improve the sensitivity in searching for SUSY particles. The two techniques which we will explore here are:

- i) the radiative production of photino or sneutrino pairs, and
- ii) the production of a heavy SUSY particle in association with a light one (e.g. $\tilde{\nu}$ or $\tilde{\gamma}$).

3.2 Radiative processes: photino and sneutrino searches

If the sneutrino is heavier than the photino and has a mass smaller than about m_W , it will decay via $\tilde{\nu} \rightarrow \nu + \tilde{\gamma}$ [3.1]. If the photino is indeed the lightest SUSY particle (LSP), this decay is invisible. On the other hand, if the LSP is a higgsino, the photino decays via $\tilde{\gamma} \rightarrow \gamma + \text{higgsino}$ [3.2] so that its presence is signalled by a photon + $\cancel{E}$. In global SUSY models, $\cancel{E}$ could also be carried away by the goldstino resulting from the breaking of SUSY. Photino pairs can be created at e^+e^- colliders by t-channel selectron exchange. The production of unstable photino pairs would be signalled by events with two acoplanar, energetic photons. The cross-section for this process, as for zino-photino production, discussed in section 2.6, is governed by the selectron mass. For $m_{\tilde{e}} < 100$ GeV, PETRA experiments already rule out unstable photinos with masses below about 20 GeV. At LEP, the mass limit on unstable photinos could be pushed up to 50 GeV provided the selectron is not too heavy. A similar limit could be obtained on the sneutrino mass (independent of the selectron mass, since these can be produced from Z decays) if the sneutrino decays into an unstable photino which then decays to photon + $\cancel{E}$. This is, however, an unlikely possibility in the class of models being considered.

In the cases where the photino is the LSP, we can still look for sneutrino and photino by looking for an excess of a single electron, a single muon or a single photon over the QED background. This is the technique which we will describe in this chapter.

At present, the experimental limit on the mass of such a photino or sneutrino is just a few GeV even with the assumption $m_{\tilde{e}} \leq 50$ GeV. Thus they should be searched for even below the LEP energy region. These particles can be pair-produced, and the events tagged by a bremsstrahlung photon in the final state:

$$e^+ + e^- \rightarrow \gamma + \tilde{\gamma} + \tilde{\gamma} \quad (3.1)$$

$$\rightarrow \gamma + \tilde{\nu} + \tilde{\nu} \quad (3.2)$$

They are similar to the radiative production of neutrino pairs. Thus the photon accompanied by no other detectable particles in the final state is used to identify the production of invisible photinos and sneutrinos in e^+e^- annihilation. The Feynman diagrams for the production of the $\tilde{\gamma}$ are shown in fig. 3.1. The details of the calculation can be found in ref. [3.3] and we will not repeat them here. The relevant formulae for the cross-sections are listed in appendix E.

Only incomplete calculations of sneutrino pair production were available in the literature [3.4]. Thus we made our own calculation and present here details for this process, including radiative corrections [3.5]. The Feynman diagrams used for our calculations are also shown in fig. 3.1. The amplitudes are listed in appendix F and have been calculated numerically for several sets of parameter values. In fig. 3.2a we show the cross-sections as functions of the sneutrino mass for several values of the wino mass and the centre-of-mass energy. The cross-section with vanishing sneutrino mass is shown in fig. 3.2b as a function of the wino mass.

The main sources of background are single-photon events from the QED processes in which all other final-state particles, produced at small angles, escape detection. There are two methods to identify the signals from $\tilde{\nu}$, $\tilde{\gamma}$, or ν cleanly. The first is to select high- p_T photon events above the kinematic limits of the QED events. This method can be used at all energies. When $\sqrt{s} \approx m_Z$, however, the production cross-sections of ν and $\tilde{\nu}$ are greatly enhanced owing to the large contribution of the Z^0 pole. Thus the second method is to detect low-energy ($E_\gamma \ll \Gamma_Z$) photons (using some high-resolution photon devices such as BGO, NaI, etc.) in order to see a significant excess of events above the huge QED background. We will describe each of them separately in the following sections.

3.2.1 *The use of high- p_T photon events above the kinematic limits of QED processes*

QED-type events, for example $e-e-\gamma$, $e-e-\mu-\mu$, and $\mu-\mu-\gamma$ final states, conserve energy and momentum, and thus, in principle, can be identified by using energy-momentum conservation. However, it happens frequently that electrons, photons, and muons produced at small angles can escape detection and thus apparently spoil the conservation of energy and momentum parallel to the beams. When a single photon from QED processes is detected, the maximum p_T is $\sin \theta_{\min} \sqrt{s} / (1 + \sin \theta_{\min})$, where $\theta_{\min}$ is the minimum angle above which electrons or photons or muons can be detected efficiently. With a large acceptance and a good energy resolution for the μ , e , and γ , one can detect the QED final particles for events when the transverse momentum of the particles at large angles is greater than the kinematic limit, i.e. $p_T > \sin \theta_{\min} \sqrt{s} / (1 + \sin \theta_{\min})$. Therefore, a large unbalanced p_T is the signature of SUSY-particle production. It follows also that a detector searching for SUSY particles must cover a large solid angle in order not to miss large- p_T particles and be able to measure energy-momentum imbalance accurately.

To study the background from QED processes, we have made our own Monte Carlo calculation [3.6]. This efficiently generates events characterized by the small phase space allowed by our selection criterion that $\theta_{\min}$ be small for all final-state particles besides the photon, which results in a large variation in the values of the matrix element squared. The results, as shown in fig. 3.3, agree well with the single γ , e , and μ data at 40 GeV from Mark J [3.7], which was taken with a veto angle $\theta_{\min} = 12^\circ$.

The expected rates for the production of photino pairs (for $m_{\tilde{\gamma}} = 1$ and 10 GeV), sneutrino pairs (for $m_{\tilde{\nu}} = 9$ and 20 GeV), and three types of massless neutrino pairs with the experimental photon acceptance $= |\cos \theta_\gamma| < 0.87$ and $p_T(\gamma) > 0.05\sqrt{s}$ described in subsection 3.3.1 are shown in fig. 3.4a as functions of the centre-of-mass energy. To demonstrate the sensitivity of the rates to the acceptance, we show in fig. 3.4b the rates with slightly different cuts $= p_T(\gamma) > 0.1\sqrt{s}$ and $20^\circ < \theta_\gamma < 160^\circ$. One sees that even the peaks of the rates depend critically on the acceptance. In general, the smaller the p_T cut is, the closer the peaks of the cross-sections will be to m_Z . For example for a p_T cut as low as 100 MeV, the peak cross-sections occur at $\sqrt{s} \approx m_Z$. One can also see from fig. 3.4 that to search for sneutrinos one would like to run the machine energy below the Z^0 peak, say around the 70 to 80 GeV region, where the background from neutrinos is not too large. To search for photinos, the region around 50 GeV is the most favourable. The experimental signature for all three processes is an excess of events with a single high- p_T photon. Since photino, sneutrino, and neutrino production cross-sections have very different energy dependence, once one observes an excess of single-photon events above QED background one can use the energy dependence of the cross-section to pin down the origin of the signal.

3.2.2 Excess of low-energy photon events at $\sqrt{s} \approx m_Z$

In the above, we have described how to use the kinematic cut on p_T to eliminate all the QED background to search for $\tilde{\nu}$, $\tilde{\gamma}$, and new types of ν 's. When $\sqrt{s}$ is equal to or slightly above m_Z , there is an additional signal, due to an excess of single- γ events at low p_T , significantly above the QED background. To demonstrate this, we compute the expected rates for $\gamma\nu\nu$ and QED background from $ee\gamma$ final states. In order to reduce the relative rate of $ee\gamma$, with $\sqrt{s} \approx m_Z^0$, we assume that the γ are detected in the angular region $40^\circ < \theta < 140^\circ$, and that any events with additional $e^\pm$ or photon are vetoed for $3^\circ < \theta < 177^\circ$. The expected rate from QED is almost independent of the c.m. energy. The expected rates for $\gamma\nu\bar{\nu}$ with $\sqrt{s} = 90, 93$, and 94.5 GeV are shown in fig. 3.5a as functions of $p_T(\gamma)$. Figure 3.5b shows the $p_T(\gamma)$ distributions for ν , $\tilde{\nu}$, $\tilde{\gamma}$ radiative pair production and $ee\gamma$. We see that for $\sqrt{s} = m_Z^0$, there is a significant excess of single- γ events at $0.2 < p_T < 1.0$ GeV due to ν and, to a lesser degree, also to $\tilde{\nu}$ pair production above the QED background. This shows once more the importance of having a highly sensitive electromagnetic detector such as BGO.

3.3 Associated production

If the masses of the supersymmetric particles are larger than the beam energy, one can still look for them via associated-production processes, where one heavy sparticle is produced together with a lighter neutral sparticle such as a photino or sneutrino. These reactions involve three final-state particles in order to conserve quantum numbers. Two reactions which are particularly interesting are the $\tilde{e}$ [3.8, 3.9] and $\tilde{W}$ [3.10, 3.11] associated productions:

$$e^+ + e^- \rightarrow e + \tilde{e} + \tilde{\gamma}, \quad (3.3)$$

and

$$e^+ + e^- \rightarrow e + \tilde{W} + \tilde{\nu}, \quad (3.4)$$

where each reaction involves a dozen or so diagrams with both s- and t-channel contributions.

For the decays of the $\tilde{W}$ and the $\tilde{e}$ we make the following assumptions:

- i) $m_{\tilde{q}} > m_{\tilde{W}}$ and $m_{\tilde{l}} > m_{\tilde{W}} > m_{\tilde{\nu}}$, hence winos undergo only leptonic decay;
- ii) the wino obeys universality in its e , μ , or τ decay;
- iii) for reaction (3.3), since we are mainly interested in the kinematic region $m_{\tilde{W}} > E_{beam} > m_{\tilde{\nu}}$, the two-body decay $m_{\tilde{W}} \rightarrow e + \tilde{\nu}$ is dominant;
- iv) for reaction (3.4), the left- and the right-handed selectrons have equal masses, and the main decay channel for selectrons is to an electron and a photino.

The experimental signals of these processes depend on the centre-of-mass energies as described below.

- i) At centre-of-mass energies far away from the Z^0 peak, the reaction is dominated by the t-channel contribution. The final-state direct electron often escapes detection because it is predominantly produced in the small-angle region. Therefore the most important signature in a typical detector is the production of a single high- p_T lepton from the decay of the $\tilde{W}$ or $\tilde{e}$ without other detectable particles.
- ii) Near the Z^0 peak, these processes are dominated by virtual-pair production intermediate states in the s channel. As a result, the rate of acoplanar lepton pairs, as described in chapter 2 on pair production, becomes larger than that of single leptons.

This explains why the relative rates of the single-lepton channel and the acoplanar dilepton channel (as shown later in figs. 3.7 to 3.11) depend on whether the centre-of-mass energy is near the Z^0 .

3.3.1 $e\tilde{e}\tilde{\gamma}$ [3.12]

Reaction (3.3) is important mainly in the case where the photino mass is much smaller than the selectron mass. In these circumstances the main decay channel of the selectron is the $e\tilde{\gamma}$ final state. In order for it to be valid for the case $m_{\tilde{\gamma}} \neq 0$, we include the two diagrams with Z^0 intermediate state and twelve QED diagrams including two diagrams with amplitudes proportional to $m_{\tilde{\gamma}}$. The Feynman diagrams for this reaction are shown in fig. 3.6, and the amplitudes are given in appendix G. We present the rates for two separate classes of events distinguished by the final observable particles, i.e. i) the class of events with a single electron only, and ii) the class of events with a pair of acoplanar electrons. Figure 3.7a shows the expected rates for the associated production of $e\tilde{e}\tilde{\gamma}$ with $m_{\tilde{\gamma}} = 0$ at the Z^0 peak. Similarly, figs. 3.7b and 3.7c show the rates at $\sqrt{s} = 44$ and 160 GeV. To illustrate the dependence on the photino mass, we show in fig. 3.8 the rate at the Z^0 peak for a photino mass of 10 GeV. As can be seen from figs. 3.7 and 3.8, with an integrated luminosity of 100 pb^{-1} at the Z^0 peak, one can search for the selectron up to a mass of 57 (51) GeV for the case of a photino mass of 0 (10 GeV), respectively.

3.3.2 $e\tilde{W}\tilde{\nu}$

The complete set of Feynman diagrams included in our calculation of reaction (3.4) is shown in fig. 3.9. All the 12 diagrams and a total of 78 terms are computed, and the cross-section formulae are given in appendix H. We do not take into account spin correlations. This introduces a negligible correction in the total cross-section after acceptance cuts, but may induce small errors in differential final-state distributions. The calculations are first carried out under the idealized assumption that there is no mixing between the higgsino and the wino. The contribution from $\tilde{W}_h$ is also neglected [3.10, 3.11]. We use this simplification as a guide in discussing the cross-section for associated production of $\tilde{W}$ and $\tilde{\nu}$ for a light sneutrino. We will consider the general case with mixing and with $v'/v \neq 1$ in the next subsection. The expected rates for events with one muon only and for the $e-\mu$ type of events are shown in fig. 3.10a, as a function of the wino mass at a centre-of-mass energy of 93 GeV. The selection criteria for these events are shown in the caption to fig. 3.10. One can also replace the muon by an electron and thus obtain the rate of single-electron and dielectron events up to a correction factor close to one, owing to the slight difference between the muon and electron acceptances. Similarly, figs. 3.10b, 3.10c and 3.10d show the expected rates as functions of the wino mass for centre-of-mass energies 44, 110, and 160 GeV, respectively. We see that the single- μ rate is higher than the $e-\mu$ rate outside the Z^0 region. At the Z^0 peak, however, the s channel diagrams dominate and the $e-\mu$ rate becomes significantly higher than the single- μ rate. To study the dependence on the sneutrino mass, we show the rate of events at the Z^0 peak in fig. 3.10 for a sneutrino mass of 10 GeV. As seen from figs. 3.10 and 3.11, at the Z^0 peak with 100 pb^{-1} integrated luminosity, one can reach a wino mass of up to 70 GeV if $m_{\tilde{\nu}} \sim 0$, significantly above the beam energy. This result shows the importance of the associated production of supersymmetric particles.

3.3.3 $e\tilde{W}\tilde{\nu}$: general case of mixing and unequal Higgs vacuum expectation values

The Lagrangian for the relevant interactions for this general case is described in appendix I [3.13]. We note the following important modifications to the couplings.

- i) There is now a new axial-vector coupling constant in the $Z\tilde{W}\tilde{W}$ vertex, proportional to

$$A_c \equiv \frac{1}{4} \sec^2 \theta_w (\cos^2 \gamma_L - \cos^2 \gamma_R) \quad , \quad (3.5)$$

whilst the vector coupling constant is changed from

$$e \cot \theta_w \left(1 - \frac{1}{4} f^2 \sec^2 \theta_w \right) \quad (3.6a)$$

to

$$e \cot \theta_w (1 - V_c), \quad (3.6b)$$

where

$$V_c \equiv \frac{1}{4} \sec^2 \theta_w (\cos^2 \gamma_L + \cos^2 \gamma_R).$$

Note that for $v'/v = 1$, $\gamma_L = \gamma_R = \gamma$, and $f_+ = \sin \gamma$ and $f_- = -\cos \gamma$.

ii) The $e\tilde{W}\tilde{\nu}$ vertex is modified by the factor

$$F \equiv \sin \gamma_R / f_+; \quad (3.7)$$

F is obviously $= 1$ for $v' = v$.

We work within the framework of the model described in section 1.5.2. The selectron, sneutrino, wino, and photino masses are given by equations (1.13) (1.20c) and (1.21). We ignore the small contribution from the high-mass wino. The parameters $r = (v'^2 - v^2)/(v'^2 + v^2)$ and $m_0, \mu_{1,2}$ are all defined in subsection 1.5.2 and in appendix G. The explicit expressions for all the parameters in terms of physical masses can be found in appendix G.

For $r > 0$, the $\tilde{\nu}$ can be much lighter than the $\tilde{e}$, so that $\tilde{W} \rightarrow \ell\tilde{\nu}$ dominates and the associated-production process (3.4) becomes a useful channel to search for SUSY particles such as the wino.

With a given set of masses for the sneutrino, wino, photino, and selectron as shown in the four leftmost columns of table 3.1, we can calculate the following quantities up to a quadratic ambiguity discussed in ref. [3.4]:

- i) all the parameters r, α, m_1 , the mass of the high-mass wino, γ_1 , and γ_r , which are shown in the next six columns of table 3.1;
- ii) the coupling constants f_+, A_c , and $1 - V_c$, which are shown in the columns on the right-hand side of table 3.1;
- iii) the ratio of the cross-section with respect to that of the no-mixing results, where $V_c = A_c = 0$ and $\sin \gamma_R = 1$. This ratio can be approximated by the following formula:

$$\sigma(v' \neq v)/\sigma(\text{no-mixing}) \approx \sin^2 \gamma_R (1 - V_c - A_c)^2 \quad (3.8)$$

which is shown in the rightmost column of table 3.1. This ratio can be compared with the results of a rigorous calculation for several important cases as described below.

If A_c and V_c are small and f_+ is close to one, the calculation simplifies to the previous special case $v' = v$, as can be seen in the simplified ratio shown in the rightmost column of table 3.1. For a few cases when A_c and V_c are large and f_+ is far away from 1, we have carried out a rigorous calculation, and we show the results in table 3.2. We see from this table that the simple ratios are in most of the cases within 20% of the values obtained with the rigorous calculation.

We conclude that this associated-production mechanism is indeed useful for searching for a wino with mass above the beam energy, in the case where the sneutrino is not too heavy.

REFERENCES

- [3.1] R.M. Barnett, M.E. Haber and K.S. Lackner, Phys. Lett. **126B** (1983) 64; Phys. Rev. **D29** (1984) 1990.
- [3.2] H. Komatsu and J. Kubo, Phys. Lett. **157B** (1985) 90.
- [3.3] Photino pair calculations:
 P. Fayet, Phys. Lett. **177B** (1982) 460;
 J. Ellis and J. Hagelin, Phys. Lett. **122B** (1982) 303;
 T. Kobayashi and M. Kuroda, Phys. Lett. **139B** (1984) 206;
 K. Grassie and P.N. Pandita, Phys. Rev. **D30** (1984) 22;
 J. Ware and M. Machacek, Phys. Lett. **142B** (1984) 300;
 M. Martinez, Mark-J Internal Rep. No. 4, unpublished (1985);
 Electronic Yellow Book, CERN IBM file T3.EYB.PHOTINOS.
- [3.4] J.A. Grifols, X. Mor-mur and J. Sola, Phys. Lett. **114B** (1982) 35;
 J. Ware and M. Machacek, Phys. Lett. **147B** (1984) 415;
- [3.5] Sneutrino pair calculations:
 J. Berdugo, Y. Chang, M. Chen, M. Martinez and H.G. Wu, MIT LNS Rep. No. 145 (June 1985);
 Electronic Yellow Book, CERN IBM file T3.EYB.SNUS;
 J.A. Grifols, M. Martinez and J. Sola, DESY preprint 112 (1985);
 J. Ellis and R. Peccei (eds.), Physics at LEP (CERN 86-02, Geneva, 1986), vol. 1.
- [3.6] QED $e\bar{e}\gamma$ calculation:
 M. Martinez, Univ. of Barcelona thesis, unpublished (1986).
- [3.7] B. Adeva et al., to be published in Phys. Lett. B (1987).
 B. Zhou, MIT Thesis ('86).
- [3.8] Electron-selectron-photino:
 1. First calculation: (WWA + zero photino mass)
 M.K. Gaillard, L. Hall and I. Hinchliffe, Phys. Lett. **116B** (1982) 279;
 M. Kuroda, K. Ishikawa, T. Kobayashi and S. Yamada, Phys. Lett. **127B** (1983) 467;
 2. Generalized massive photino:
 T. Kobayashi and M. Kuroda, Phys. Lett. **134B** (1984) 271;
- [3.9] Complete calculation:
 J. Berdugo, Y. Chang, M. Chen, M. Martinez and H.G. Wu in ref. 3.5;
 M. Martinez, Mark-J Internal Rep. No. 5, unpublished (1985).
- [3.10] Photino-wino-sneutrino
 J. Berdugo, Y. Chang, M. Chen, M. Martinez and H.G. Wu in ref. 3.5;
 G. Eilam and E. Reya, Phys. Lett. **145B** (1984) 425 and **148B** (1984) 502;
 P. Salati and J.C. Wallet, Phys. Lett. **122B** (1983) 379.
- [3.11] M. Martinez, J.A. Grifols and R. Pascual, DESY preprint 045 (1985).
- [3.12] Electronic Yellow Book, CERN IBM file T3. EYB. ESEPHINO
- [3.13] Electronic Yellow Book, CERN IBM file T3. EYB. EWINOSNU

Table 3.1

Coupling constants and parameters in terms of physical masses

Input masses (GeV)				Output parameters						Couplings ^{*)}			$\sigma_{V+V'}/\sigma_{(no\ mix.)}$
m_V	m_W	m_Y	m_e	x	α	m (GeV)	m_{high} (GeV)	γ_L (deg.)	γ_R (deg.)	f_+	A_C	$(1-V_C)$	Ratio = $f_+^2(1-V_C-A_C)^2$
		0	60	0.59	27.05	69.2	177	124	104	0.970	0.798×10^{-1}	0.882	0.606
		10	30	0.15	40.79	252	516	99.3	97.9	0.991	0.229×10^{-2}	0.986	0.949
		10	45	0.33	35.37	239	491	101	97.2	0.992	0.570×10^{-2}	0.984	0.942
		10	60	0.59	27.05	200	415	104	96.4	0.994	0.142×10^{-1}	0.978	0.917
		0	60	0.59	27.05	39.2	133	138	105	0.965	0.152	0.804	0.397
		10	30	0.15	40.79	126	275	107	104	0.970	0.822×10^{-12}	0.955	0.844
		10	45	0.33	35.37	118	261	110	103	0.976	0.207×10^{-1}	0.956	0.822
		10	60	0.59	27.05	93.5	216	117	100	0.984	0.538×10^{-1}	0.926	0.737
0	50	0	60	0.59	27.05	13.0	106	161	99.0	0.988	0.277	0.708	0.182
0	50			0.15	40.79	75.5	184	114	109	0.944	0.195×10^{-1}	0.911	0.708
0	50	10	45	0.33	35.37	69.0	173	119	106	0.959	0.498×10^{-1}	0.899	0.663
0	50	10	60	0.59	27.05	45.8	139	134	100	0.984	0.145	0.835	0.462
0	60			0.15	40.79	23.3	108	134	120	0.868	0.751×10^{-1}	0.769	0.362
0	60			0.33	35.37	16.8	103	148	110	0.942	0.194	0.734	0.259
0	60			0.15	40.79	46.3	136	122	113	0.919	0.410×10^{-1}	0.860	0.566
0	60	10	45	0.33	35.37	39.4	127	131	107	0.954	0.110	0.833	0.476
0	70			0.15	40.79	9.67	92.7	147	116	0.900	0.165	0.716	0.246
0	70			0.15	40.79	25.9	106	133	114	0.916	0.963×10^{-1}	0.802	0.417
20	30			0.81×10^{-1}	42.66	93.9	218	111	109	0.943	0.725×10^{-2}	0.923	0.747
20	30			0.26	37.33	89.8	211	114	107	0.954	0.252×10^{-1}	0.918	0.725
20	30	0	60	0.52	29.30	75.2	187	121	105	0.966	0.638×10^{-1}	0.894	0.644
20	30			0.81×10^{-1}	42.66	254	521	98.9	98.2	0.990	0.125×10^{-2}	0.986	0.950
20	30	10	45	0.26	37.33	245	502	100	97.5	0.992	0.437×10^{-2}	0.985	0.945
20	40			0.81×10^{-1}	42.66	61.7	163	118	115	0.908	0.135×10^{-1}	0.875	0.612
20	40	0	45	0.26	37.33	58.1	158	122	112	0.929	0.469×10^{-1}	0.866	0.578
20	40			0.52	29.30	44.9	140	133	107	0.956	0.120	0.826	0.455
20	40			0.81×10^{-1}	42.66	128	277	106	105	0.968	0.449×10^{-2}	0.956	0.848
20	40	10	45	0.26	37.33	122	267	108	103	0.974	0.158×10^{-1}	0.952	0.832
20	40	10	60	0.52	29.30	102	231	114	101	0.982	0.418×10^{-1}	0.935	0.770
20	50			0.81×10^{-1}	42.66	40.3	131	124	119	0.871	0.232×10^{-1}	0.824	0.486
20	50			0.26	37.33	36.7	127	131	115	0.910	0.813×10^{-1}	0.810	0.439
20	50	0	60	0.52	29.30	21.9	112	150	105	0.968	0.215	0.744	0.262
20	50			0.81×10^{-1}	42.66	76.6	185	113	110	0.938	0.106×10^{-1}	0.913	0.716
20	50	10	45	0.26	37.33	72.0	178	117	108	0.954	0.377×10^{-1}	0.905	0.684
20	50	10	60	0.52	29.30	54.3	151	128	103	0.976	0.107	0.863	0.544
20	60			0.81×10^{-1}	42.66	24.3	109	131	123	0.841	0.408×10^{-1}	0.774	0.380
20	60			0.26	37.33	20.0	105	142	114	0.915	0.146	0.751	0.306
20	60			0.81×10^{-1}	42.66	47.4	137	120	115	0.905	0.222×10^{-1}	0.864	0.580
20	60	10	45	0.26	37.33	42.7	131	128	110	0.941	0.813×10^{-1}	0.846	0.519
20	70			0.81×10^{-1}	42.66	11.2	93.4	139	123	0.840	0.884×10^{-1}	0.725	0.286
20	70			0.81×10^{-1}	42.66	27.3	107	128	118	0.884	0.510×10^{-1}	0.811	0.451
20	70	10	45	0.26	37.33	19.6	100	148	103	0.976	0.212	0.758	0.284

*) Note that corresponding to each set of masses, in general there are two sets of coupling constants, of which we only show one here.

Table 3.2

Cross-section for $e^+e^- \rightarrow e^+\tilde{W}^+\tilde{\nu}$ ($\sqrt{s} = 93$ GeV)

$m_{\tilde{\nu}}$	$m_{\tilde{W}}$	$m_{\tilde{\chi}}$	$m_{\tilde{e}}$	γ_L (deg.)	γ_R (deg.)	σ_{tot} (pb)	σ_{cut} (pb)	Ratio _{tot} ^{b)}	Estimated _{tot} ^{c)} [$\equiv f_+^2 (\tilde{V}_C - A_C)^2$]	f_+	A_C	$\tilde{V}_C$ ($\equiv 1 - V_C$)
0	70	-	- ^{d)}	90	90	0.301(2)	0.296(1)	1	1	1	0	1
		10	30	133.0	113.7	0.110(2)	0.101(1)	0.37	0.41	0.92	0.10	0.80
		30 ^{e)}	30	23.57	42.36	0.0170(2)	0.0138(1)	0.056	0.10	0.67	0.09	0.56
		40 ^{e)}	60	173.6	58.6	0.0202(1)	0.0148(1)	0.067	0.09	0.85	0.23	0.60
10	60	-	- ^{d)}	90	90	1.54(1)	1.50(1)	1	1	1	0	1
		10	30	121.7	113.8	0.856(4)	0.828(4)	0.55	0.56	0.91	0.04	0.86
		30 ^{e)}	30	29.37	41.25	0.152(1)	0.139(1)	0.099	0.12	0.66	0.06	0.58
		40 ^{e)}	60	4.13	52.87	0.156(1)	0.139(1)	0.10	0.09	0.80	0.20	0.57

a) The cut cross-section has $5^\circ \leq \theta_+ \leq 175^\circ$.

b) Result of rigorous calculation.

c) Simple ratio $\sigma_{\tilde{\nu} \neq \tilde{\nu}} / \sigma(\text{no mix.})$.d) No mixing between $\tilde{W}_1$ and Higgsino has been included in this case.e) Note that the diagonalization of the neutral supersymmetric fermion mass matrix which has been used here is strictly valid only for $m_{\tilde{\chi}} \ll m_W$.

Figure captions

- Fig. 3.1** The Feynman diagrams for the production of photino and sneutrino pairs in association with a radiated photon of reactions (3.1) and (3.2).
- Fig. 3.2a** The production cross-sections of sneutrino pairs together with a photon, as functions of the sneutrino mass, for three different sets of the centre-of-mass energies and wino masses.
- Fig. 3.2b** The production cross-sections of sneutrino pairs together with a photon, as functions of the wino mass for vanishing sneutrino mass at centre-of-mass energies $\sqrt{s} = 44, 93$, and 160 GeV.
- Fig. 3.3** Comparison of a) single- μ events, b) single-e events and c) single- γ events as measured by Mark J at 44 GeV (solid points) with the predictions of a QED calculation (histograms) with $\theta_{\min} = 12^\circ$.
- Fig. 3.4a** The expected numbers of single high- p_T photon events per 100 pb^{-1} of running, as functions of the centre-of-mass energy, for the pair production of photinos (with $m_{\tilde{\gamma}} = 1$ and 10 GeV), sneutrinos (with $m_{\tilde{\nu}} = 9$ and 20 GeV) and three types of massless neutrinos in association with a photon, for the cuts $30^\circ < \theta_\gamma < 150^\circ$ and $p_T(\gamma) > 0.05\sqrt{s}$.
- Fig. 3.4b** The expected numbers of single high- p_T photon events per 100 pb^{-1} of running, as functions of the centre-of-mass energy, for the pair production of photinos (with $m = 1$ and 10 GeV), sneutrinos (with $m = 9$ and 20 GeV) and three types of massless ν , $\tilde{\nu}$, and $\tilde{\gamma}$ with cuts $20^\circ < \theta_\gamma < 160^\circ$ and $p_T(\gamma) > 0.1\sqrt{s}$.
- Fig. 3.5a** The differential cross-sections in p_T of the single photon for $\gamma\nu\nu$ production at $\sqrt{s} = m_Z = 93, 90$, and 94.5 GeV. The background from $ee\gamma$ (dashed curve) is also shown.
- Fig. 3.5b** The differential cross-section in p_T of the single photon for ν , $\tilde{\nu}$, $\tilde{\gamma}$ radiative pair production and $ee\gamma$.
- Fig. 3.6** Fourteen Feynman diagrams used for the calculation of the process $e^+ + e^- \rightarrow e + \tilde{e} + \tilde{\gamma}$.
- Fig. 3.7** The expected rates for both the single-electron and the acoplanar dielectron final states from reaction (3.3), calculated with zero photino mass as functions of the selectron mass at a centre-of-mass energy of (a) 93 GeV, (b) 44 GeV, and (c) 160 GeV.
- Fig. 3.8** Similar to fig. 3.7a, but with a photino mass of 10 GeV.
- Fig. 3.9** Feynman diagrams for the reaction $e^+ + e^- \rightarrow e + \tilde{W} + \tilde{\nu}$.
- Fig. 3.10** The expected number of single- μ events and of acoplanar $e\mu$ events through reaction (3.4), the associated production of a wino together with a sneutrino and an electron, as functions of the wino mass at a centre-of-mass energy of 93 GeV. The points are for the particular case with no-mixing, as described in the text. One can replace the muon with an electron and obtain the event rates of single-electron and acoplanar dielectron events, after a minor correction factor due to the slight difference in detector acceptances for muons and electrons. The graphs are for $\sqrt{s} =$ (a) 93 GeV, (b) 44 GeV, (c) 110 GeV, and (d) 160 GeV.
- Fig. 3.11** Same as fig. 3.10a but with a sneutrino mass of 10 GeV.

Fig. 3.1

Fig. 3.2

Fig. 3.3

Fig. 3.4

Fig. 3.5

Fig. 3.6

Fig. 3.7

Fig. 3.8

Fig. 3.9

Fig. 3.10

Fig. 3.11

4. FINAL CONSIDERATIONS

4.1 Summary of LEP mass limits

Table 4.1 summarizes the improvements from the experimental side which we expect to achieve after LEP I and LEP II on the lower limits on SUSY masses.

With 500 pb^{-1} of integrated luminosity at LEP II, even in the environment of WW pair production, sleptons ($\tilde{e}$, $\tilde{\mu}$, $\tilde{\tau}$) can be detected up to masses of about 80–90% of the beam energy. We can set limits $m_{\tilde{e}} \geq 90 \text{ GeV}$ with $m_{\tilde{\gamma}} = 40 \text{ GeV}$, $m_{\tilde{\mu}} \geq 85 \text{ GeV}$, and $m_{\tilde{\tau}} \geq 75 \text{ GeV}$. As quoted in table 4.1 the present limits for $\tilde{e}$, $\tilde{\mu}$, and $\tilde{\tau}$ are $m_{\tilde{\gamma}} \geq 22 \text{ GeV}$, $m_{\tilde{\mu}} \geq 20 \text{ GeV}$, and $m_{\tilde{\tau}} \geq 17 \text{ GeV}$; from LEP I they are already $m_{\tilde{e}} \geq 57 \text{ GeV}$, $m_{\tilde{\mu}} \geq 40 \text{ GeV}$, $m_{\tilde{\tau}} \geq 40 \text{ GeV}$.

We emphasize once again that it is the cleanliness of e^+e^- annihilation that allows a mass determination up to a few per cent for the sleptons once a signal has been found. The situation for the selectron needs some more comments, because of its dependence on the photino mass. The experimentally excluded region in the $m_{\tilde{e}}-m_{\tilde{\gamma}}$ plane is shown in fig. 4.1. At present, the most stringent limits which come from the study of the process $e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$ by the ASP experiment [4.1] are $m_{\tilde{e}} \geq 58 \text{ GeV}$ and $m_{\tilde{\gamma}} = 0$ at 90% c.l. However, as soon as we move the photino mass above 10–15 GeV the selectron limit goes down to 50–23 GeV. On the other hand fig. 4.1 shows the spectacular improvement that LEP II will give to such a knowledge, pushing the mass limits to $m_{\tilde{e}} \geq 90 \text{ GeV}$ even with $m_{\tilde{\gamma}}$ as big as 40–50 GeV.

Wino masses up to 40 GeV can be probed at $\sqrt{s} = m_{\tilde{Z}_0}$, LEP I, from the study of wino pair production followed by the three-body decay of the wino. If the sneutrino is relatively light, higher wino masses, up to 60 GeV, can be probed via $e^+e^- \rightarrow \tilde{W}\tilde{\nu}e$. At $\sqrt{s} = 190 \text{ GeV}$, we have found that a clean wino signal, for wino masses up to about 90 GeV, can be obtained by studying the final state from wino pair production, where one of the winos decays leptonically and the other hadronically. In the case of a light sneutrino, a similar mass range can be probed.

For the zino mass at LEP I, we can also reach $m_{\tilde{Z}} \geq 60 \text{ GeV}$ with $m_{\tilde{\gamma}} = 10 \text{ GeV}$ and $m_{\tilde{e}} > 70 \text{ GeV}$. If we go above the WW threshold, then WW seems to be an unsurmountable background to detect $\tilde{Z}\tilde{\gamma}$ events in the leptonic as well as in the hadronic final states.

On the contrary, just below the WW threshold, at $\sqrt{s} = 160 \text{ GeV}$, we can reach $m_{\tilde{Z}} \geq 90 \text{ GeV}$ with $m_{\tilde{e}}$ up to 100 GeV and $m_{\tilde{\gamma}}$ up to 40 GeV.

The experimentally excluded region in the $m_{\tilde{Z}}-m_{\tilde{e}}$ plane is shown in fig. 4.2. Again we underline the big improvement on the mass limits from LEP II.

We conclude that the best place to look for SUSY would be the highest available energy of LEP II! We also underline that these results should be achievable by any of the LEP experiments as they are described in the technical proposals. The requirements on detector constraints are good tracking for isolated charged particles and muons over a large range of the polar angle, have good electromagnetic calorimetric measurement and coverage, and electron/photon tagging down to $1-2^\circ$ in the polar angle.

Although the W-pair background can be removed in most cases, running the collider just below its threshold would probably be a simpler way of looking for signals of new physics.

4.2 SUSY mass limits from $p\bar{p}$ colliders and HERA

From the UA1 mass limits on the gluino and squark discussed in section 1.8 [4.2] ($m_{\tilde{g}} \geq 60 \text{ GeV}$, $m_{\tilde{q}} \geq 70 \text{ GeV}$), one deduces a lower bound for the photino mass using $m_{\tilde{\gamma}} \approx m_{\tilde{g}}/6$, a relation which holds in the simplest $N = 1$ supergravity models, with canonical gauge kinetic terms

$$m_{\tilde{\gamma}} > 10 \text{ GeV} . \quad (4.1)$$

A recent analysis [4.3] has shown that the UA1 squark (gluino) mass limit is insensitive to increases in photino mass up to $m_{\tilde{\gamma}} \sim 30\text{--}40$ GeV (20–30 GeV). The Tevatron is sensitive to squark and gluino masses of up to about 200 GeV [4.4].

The masses of the weak gauginos ($\tilde{W}, \tilde{Z}$), on general grounds are expected to be smaller than m_W and m_Z . It is, therefore, possible that these can be copiously produced at hadron colliders via the decays of the gauge bosons $W^\pm$ and Z^0

$$W^\pm \rightarrow \tilde{W}^\pm \tilde{\gamma}, \tilde{W}^\pm \tilde{Z}$$

$$Z^0 \rightarrow \tilde{W}^\pm \tilde{W}^\mp$$

The subsequent decays of the gauginos, $\tilde{W} \rightarrow \ell \bar{\nu} \tilde{\gamma}$ or $q \bar{q}' \tilde{\gamma}$, $\tilde{Z} \rightarrow \ell \bar{\ell} \tilde{\gamma}$ or $q \bar{q} \tilde{\gamma}$, give rise to monojet and dijet + $\cancel{p}_T$ events as well as clean acollinear dilepton and trilepton signals free from hadronic activity. These signals have been analysed (for $m_{\tilde{\gamma}} \lesssim 10$ GeV) in detail for the CERN $p\bar{p}$ Collider and Fermilab Tevatron energies [4.5]. It has been shown that the recent UA1 bound [4.6] on the mass of a sequential fourth-generation lepton, $m_L > 41$ GeV, translates to a limit of 35–40 GeV on the $\tilde{W}$ mass. The $\cancel{p}_T$ reduces sharply with increasing $m_{\tilde{\gamma}}$, as has been studied in ref. [4.7]. Off-resonance production of the gauginos is very small so that LEP II is the optimal machine for detecting winos heavier than $m_Z/2$.

Supersymmetric particles can also be searched for at the HERA ep collider. The pair production cross-section for a typical subprocess such as $e q \rightarrow \tilde{e} \tilde{q}$ is too small to be observable, unless $m_{\tilde{q}} + m_{\tilde{e}} \lesssim 180$ GeV [4.8]. Since this is relatively close to the known lower limits on the mass of these particles, the prospects for discovering SUSY at HERA are not very bright.

4.3 Recent theoretical developments

Even since the discovery of the ten-dimensional anomaly-free superstring theory by Green and Schwarz [4.9] much attention has been paid to the emerging low-energy theory in four dimensions. This is generally believed to be a supergravity theory, based on the gauge group E_6 [4.10], with the chiral fields transforming as the 27 or $\bar{27}$ representation of the gauge group; E_6 is broken by the Hosotani mechanism [4.11] to a gauge group with rank ≥ 5 . These models, therefore, contain at least one neutral gauge boson Z^0 that may be relatively light. This, of course, opens up the possibility of building a Z^0 factory. In addition to the extra gauge bosons, these models necessarily contain other exotic fermions (and their supersymmetric partners). Among these are the charge $-1/3$ isosinglet quarks and new $SU(2)$ doublet fermions with the same quantum numbers as the Higgs doublets. In this report, we have confined ourselves to minimal supergravity models and have not studied the feasibility of looking for these new particles at LEP.

Since the E_6 is broken down to $SU(3) \times SU(2) \times U(1)$, many of the general features discussed in this report continue to hold at low energies although details may be altered. If the extra $Z^{0'}$ boson is heavy compared to the W , the gaugino-higgsino sector reduces to that of minimal supergravity discussed in section 1. Otherwise, the neutralino contains admixtures of the $Z^{0'}$ gaugino and the Higgsino corresponding to the breaking of the extra gauge group. For models with a light right-handed neutrino, cosmological considerations indicate $m_{Z'} \geq 300\text{--}400$ GeV [4.12] so that the additional mixings are small. The supergravity mass relations (1.21) are, however, altered [4.13], but note that we have not used them in a crucial way, in our discussion of the masses of SUSY particles that can be probed at LEP. We should mention that in some of the superstring-inspired models, the SUSY particles tend to be heavier than in minimal supergravity models. For a review of superstring-inspired phenomenology, we refer the reader to [4.14].

4.4 Conclusions

In this report we have examined in detail various non-strongly-interacting two-body and three-body SUSY reactions. We have presented appropriate techniques to clearly isolate the SUSY signals from QED, $t\bar{t}$, ZZ and WW events.

We have seen that mass limits as high as 90 GeV for both sfermions and gauginos can be reached at LEP II. These limits are well above the ones from the Tevatron and HERA, and we conclude that LEP is a *unique* machine for SUSY search. Moreover the cleanliness of e^+e^- interactions, once a signal of new physics has been found, allows to identify the nature of this new physics. For instance, a detailed study of the final-state correlations, together with the energy dependence of the cross-section, can distinguish slepton from wino or zino production, or wino from heavy-lepton production, and also from other new physics signals.

Our studies indicate that the best place to look for SUSY particles will be the highest available energy at LEP II with the exception of the zino case which can be better studied below the WW threshold. The very low SUSY cross-sections call for the highest possible luminosity. We have shown that 100 pb^{-1} of integrated luminosity is already good enough to get a five standard deviation signal from most of the reactions studied.

We underline again that our results can be achieved by all the four approved LEP experiments without any major modifications. Our recommendations here are the following:

- i) the missing energy carried by photinos or sneutrinos calls for hermetic detectors to be sensitive to such event signature;
- ii) the tagging of electrons and photons from QED events demands the extensions of the detector to cover photons and charged particles down to a few degrees in the polar angle;
- iii) the extensions of regions with clean charge assignment and good momentum measurement for isolated tracks in the forward regions down to $\sim 20^\circ$;
- iv) the forward regions should be properly equipped, since the end caps at LEP II detect a big fraction of the SUSY total cross-sections;
- v) some statistics should be taken also just below the WW threshold for the new physics search.

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REFERENCES

- [4.1] G. Bartha et al., Phys. Rev. Lett. **56** (1986) 685.
- [4.2] A. Honma, Proc. 23rd Int. Conf. on High-Energy Physics, Berkeley, 1986, ed. S. Loken (World Scientific, Singapore, 1987), vol. II, p. 1005.
- [4.3] H. Baer, D. Karatas and X. Tata, Phys. Lett. **183B** (1987) 220.
- [4.4] H. Baer and E.L. Berger, Phys. Rev. **D34** (1986) 1361;
E. Reya and D.P. Roy, Z. Phys. **C32** (1986) 615.
- [4.5] H. Baer, K. Hagiwara and X. Tata, Phys. Rev. Lett. **57** (1986) 294; Phys. Rev. **D35** (1987) 1598.
A.H. Chamseddine, P. Nath and R. Arnowitt, Phys. Lett. **174B** (1986) 399;
R. Arnowitt and P. Nath, Phys. Rev. **D35** (1987) 108.
- [4.6] C. Albajar et al., Phys. Lett. **185B** (1987) 241.
- [4.7] M. Glück, M. Godbele and E. Reya, Phys. Lett. **186B** (1987) 421.
- [4.8] R.J. Cashmore et al., Phys. Rep. **122** (1985) 275.
S.K. Jones and C.H. Llewellyn Smith, Nucl. Phys. **B217** (1983) 145.
G. Altarelli et al., Nucl. Phys. **B262** (1985) 2042.
G. Wolf, preprint DESY 86-089 (1986).
- [4.9] M. Green and J. Schwarz, Phys. Lett. **143B** (1984) 117.
- [4.10] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. **B258** (1985) 46;
E. Witten, Nucl. Phys. **B258** (1985) 75.
- [4.11] Y. Hosotani, Phys. Lett. **129B** (1983) 193.
- [4.12] J. Ellis et al., Phys. Lett. **167B** (1986) 452;
G. Steigman et al., Phys. Lett. **176B** (1986) 33.
- [4.13] M. Drees, Wisconsin preprint MAD/PH/328 (1986).
- [4.14] J. Ellis, CERN-TH 4225 (1985); CERN-TH 4474 (1986).

Table 4.1

Improvements on SUSY mass limits from LEP

SUSY particles	Current mass limits (GeV)	Mass limits at LEP I (GeV)	Mass limits at LEP II (GeV)
$\tilde{e}$ $\tilde{e}\tilde{e}^{\text{a)}$ $\tilde{e}e\tilde{\gamma}$	22 30	40 57	90
$\tilde{\mu}$	21	40	85
$\tilde{\tau}$	20	40	80
$\tilde{W}$ $\tilde{W}\tilde{W}$ $\tilde{W}e\tilde{\nu}$	22.5 35	40 60	80
$\tilde{Z}$	35	60	90

a) Here and elsewhere, the reaction used has been indicated where appropriate.

Figure captions

Fig. 4.1 Excluded $m_{\tilde{\gamma}} - m_{\tilde{e}}$ domain for stable photino.

Fig. 4.2 Excluded $m_{\tilde{Z}} - m_{\tilde{e}}$ plane.

APPENDIX A
Supersymmetric Lagrangian

The Lagrangian we used is given by

$$\mathcal{L} = \mathcal{L}_{\text{Standard}} + \mathcal{L}_{\text{gaugino}} + \mathcal{L}_{\text{sfermions}} + \mathcal{L}_{\text{gaugino-sfermions}} \quad (\text{A.1})$$

with

$$\begin{aligned} \mathcal{L}_{\text{Standard}} = & -e \sum_f q_f \bar{f} \gamma_\mu f A_\mu + e \sum_f \bar{f} \gamma_\mu (\alpha_f + \beta_f \gamma_5) f Z_\mu \\ & - \frac{g}{\sqrt{2}} \left[\bar{u} \gamma^\mu \frac{(1-\gamma_5)}{2} d W_\mu^+ + \bar{\nu} \gamma^\mu \frac{(1-\gamma_5)}{2} e W_\mu^+ + \text{h.c.} \right], \end{aligned} \quad (\text{A.2a})$$

where the summation over f runs over the quarks and leptons. The values of the parameters are listed in table A.1

Table A.1 where $c = \cot\theta_w$; $t = \tan\theta_w$

f	q_f	α_f	β_f
l	-1	$+\frac{1}{4}(3t-c)$	$+\frac{1}{4}(c+t)$
ν	0	$\frac{1}{4}(c+t)$	$-\frac{1}{4}(c+t)$
u	2/3	$-\frac{5}{12}t + \frac{1}{4}c$	$-\frac{1}{4}(c+t)$
d	-1/3	$-\frac{c}{4} + \frac{t}{12}$	$+\frac{1}{4}(c+t)$

$$\begin{aligned} \mathcal{L}_{\text{gaugino}} = & e \bar{\tilde{W}} \gamma^\mu \tilde{W} A_\mu - e \cot\theta_w \bar{\tilde{W}} \gamma_\mu (x_c - y_c \gamma_5) \tilde{W} Z_\mu \\ & + \left[\bar{\tilde{W}} \gamma^\mu (iA + B\gamma_5) \tilde{W} + \bar{\tilde{W}} \gamma^\mu (iC + D\gamma_5) \tilde{W} + \text{h.c.} \right], \end{aligned} \quad (\text{A.2b})$$

where

$$x_c = 1 - \frac{1}{2} \sec^2 \theta_w \left(\frac{\cos^2 \gamma_L + \cos^2 \gamma_R}{2} \right),$$

$$y_c = \frac{1}{2} \sec^2 \theta_w \left(\frac{\cos^2 \gamma_R - \cos^2 \gamma_L}{2} \right),$$

$$\begin{aligned}
A &= g f_+ \sin \theta_w + \left[\frac{g N_1 f_-}{\sqrt{2}} + \frac{g \sqrt{2} N_1 f_+ M_Z \cos \theta_w}{\mu_+} \right] \varepsilon_1, \\
B &= \left[\frac{g f_- N_2}{\sqrt{2}} - \frac{g f_+ \sqrt{2} M_Z \cos \theta_w}{\mu_+} \right] \varepsilon_2, \\
C &= \left[\frac{g N_1 f_-}{\sqrt{2}} + \frac{g \sqrt{2} N_1 f_+ M_Z \cos \theta_w}{\mu_+} \right] - g f_+ \sin \theta_w \varepsilon_1, \\
D &= \left[\frac{g f_- N_2}{\sqrt{2}} - \frac{\sqrt{2} g f_+ N_2 M_Z \cos \theta_w}{\mu_+} \right] \delta,
\end{aligned}$$

where $\gamma_R, \gamma_L, f_+, \varepsilon_{1,2}, \delta$, and $N_{1,2}$ are defined in section 1.5.2.

It should be mentioned that only the $\tilde{Z}\tilde{W}\tilde{W}$ couplings have been written for $v \neq v'$; the $\tilde{Z}\tilde{W}\tilde{W}$ coupling becomes purely vector, with x_c reducing to $(1 - \frac{1}{2} f_-^2 \sec^2 \theta_w)$. Continuing with the Lagrangian,

$$\begin{aligned}
\mathcal{L}_{\text{sfermion}} &= -e \sum_f q_f (i \tilde{f}_L^\dagger \partial_\mu \tilde{f}_L + i \tilde{f}_R^\dagger \partial_\mu \tilde{f}_R) A^\mu + \text{h.c.} \\
&+ i e \sum_f \left[(\alpha_f - \beta_f) \tilde{f}_L^\dagger \partial_\mu \tilde{f}_L + (\alpha_f + \beta_f) \tilde{f}_R^\dagger \partial_\mu \tilde{f}_R \right] Z^\mu + \text{h.c.} \\
&- i \frac{g}{\sqrt{2}} \left[\tilde{l}_L^\dagger \partial_\mu \tilde{\nu} - (\partial_\mu \tilde{l}_L)^\dagger \tilde{\nu} \right] W^\mu + \text{h.c.} \\
&- i \frac{g}{\sqrt{2}} \left[\tilde{d}_L^\dagger \partial_\mu \tilde{u}_L - (\partial_\mu \tilde{d}_L)^\dagger \tilde{u}_L \right] W^\mu + \text{h.c.} \tag{A.2c}
\end{aligned}$$

Equation (A.2c) can readily be seen to be the SUSY transform of the standard Lagrangian (A.2a) with the fermions being replaced by the scalars. We have not written the higher order four-point interaction. Finally,

$$\begin{aligned}
\mathcal{L}_{\text{gaugino-sfermion}} &= e \tilde{f}_L^\dagger \tilde{\tilde{Y}} (i \tilde{X}_f + \tilde{Y}_f) \frac{1 - \gamma_5}{2} f + \text{h.c.} \\
&+ e \tilde{f}_R^\dagger \tilde{\tilde{Y}} (i \tilde{X}'_f + \tilde{Y}'_f) \frac{1 - \gamma_5}{2} f + \text{h.c.} \\
&+ e \tilde{f}_L^\dagger \tilde{\tilde{Z}} (i \tilde{P}_f + \tilde{Q}_f) \frac{1 - \gamma_5}{2} f + \text{h.c.} \\
&+ e \tilde{f}_R^\dagger \tilde{\tilde{Z}} (i \tilde{P}'_f + \tilde{Q}'_f) \frac{1 + \gamma_5}{2} f + \text{h.c.} \\
&+ e R_f \tilde{f}_L^\dagger \tilde{\tilde{h}} \frac{1 - \gamma_5}{2} f + \text{h.c.}
\end{aligned}$$

$$+ e R'_f \tilde{f}^\dagger_R \tilde{Z} \frac{1+\gamma_5}{2} f + \text{h.c.} \quad (\text{A.2d})$$

$$- g \sin \gamma_R \tilde{d}^\dagger_L \tilde{W}^C \frac{1-\gamma_5}{2} u + g \sin \gamma_R \tilde{U}^\dagger_L \tilde{W} \frac{1-\gamma_5}{2} d + \text{h.c.}$$

$$- g \sin \gamma_R \tilde{l}^\dagger_L \tilde{W}^C \frac{1-\gamma_5}{2} \nu + g \sin \gamma_R \tilde{\nu}^\dagger_L \tilde{W} \frac{1-\gamma_5}{2} l + \text{h.c.}$$

The parameters, X , Y , X' , Y' , P , Q , P' , Q' , R , and R' are listed in table A.2. This completes the list of couplings that will be relevant for our purposes.

Table A.2

	l	v	u	d
X	$-\sqrt{2} + \frac{1}{\mu_-} (t-c) \varepsilon_1$	$\frac{N_1 m_z}{\mu_-} \frac{\varepsilon_1}{\sin \theta_w \cos \theta_w}$	$\frac{2\sqrt{2}}{3} + \frac{N_1 m_z}{\mu_-} \left(c - \frac{t}{3}\right) \varepsilon_1$	$-\frac{\sqrt{2}}{3} - \frac{N_1 m_z}{\mu_-} \left(c + \frac{t}{3}\right) \varepsilon_1$
Y	$-\frac{N_2 m_z}{\mu_+} (t-c) \varepsilon_2$	$-\frac{N_2 m_z}{\mu_+} \frac{\varepsilon_2}{\sin \theta_w \cos \theta_w}$	$-\frac{N_2 m_z}{\mu_+} \left(c - \frac{t}{3}\right) \varepsilon_2$	$\frac{N_2 m_z}{\mu_+} \left(c + \frac{t}{3}\right) \varepsilon_2$
X'	$-\sqrt{2} + \frac{2N_1 m_z}{\mu_-} t \varepsilon_1$	0	$\frac{2\sqrt{2}}{3} - \frac{4}{3} \frac{N_1 m_z}{\mu_-} t \varepsilon_1$	$-\frac{\sqrt{2}}{3} + \frac{2}{3} \frac{N_1 m_z}{\mu_-} t \varepsilon_1$
Y'	$\frac{2N_2 m_z}{\mu_+} t \varepsilon_2$	0	$-\frac{4}{3} \frac{N_2 m_z}{\mu_+} t \varepsilon_2$	$\frac{2}{3} \frac{N_2 m_z}{\mu_+} \varepsilon_2$
P	$\frac{N_1 m_z}{\mu_-} (t-c) + \sqrt{2} \varepsilon_1$	$\frac{N_1 m_z}{\mu_-} \frac{1}{\sin \theta_w \cos \theta_w}$	$\frac{N_1 m_z}{\mu_-} \left(c - \frac{t}{3}\right) - \frac{2}{3} \sqrt{2} \varepsilon_1$	$-\frac{N_1 m_z}{\mu_-} \left(c + \frac{t}{3}\right) + \frac{\sqrt{2}}{3} \varepsilon_1$
Q	$\frac{N_2 m_z}{\mu_+} (t-c) \delta$	$\frac{N_2 m_z}{\mu_+} \frac{\delta}{\sin \theta_w \cos \theta_w}$	$\frac{N_2 m_z}{\mu_+} \left(c - \frac{t}{3}\right) \delta$	$-\frac{N_2 m_z}{\mu_+} \left(c + \frac{t}{3}\right) \delta$
P'	$\frac{2N_1 m_z}{\mu_-} t + \sqrt{2} \varepsilon_1$	0	$-\frac{4}{3} \frac{N_1 m_z}{\mu_-} t - \frac{2\sqrt{2}}{3} \varepsilon_1$	$\frac{2}{3} \frac{N_1 m_z}{\mu_-} t + \frac{\sqrt{2}}{3} \varepsilon_1$
Q'	$-\frac{2N_2 m_z}{\mu_+} t \delta$	0	$\frac{4}{3} \frac{N_2 m_z}{\mu_+} t \delta$	$-\frac{2}{3} \frac{N_2 m_z}{\mu_+} t \delta$
R	$-\frac{N_2 m_z}{\mu_+} (t-c)$	$-\frac{N_2 m_z}{\mu_+} \frac{1}{\sin \theta_w \cos \theta_w}$	$-\frac{N_2 m_z}{\mu_+} \left(c - \frac{t}{3}\right)$	$-\frac{4}{3} \frac{N_2 m_z}{\mu_+} t$
R'	$\frac{2N_2 m_z}{\mu_+} t$	0	$\frac{N_2 m_z}{\mu_+} \left(c + \frac{t}{3}\right)$	$\frac{2}{3} \frac{N_2 m_z}{\mu_+} t$

APPENDIX B
Slepton and squark pair production

For $\tilde{f} = \tilde{\mu}, \tilde{\tau}$ or $\tilde{q}$, the differential cross-section at a centre-of-mass energy $\sqrt{s}$ is given by

$$\frac{d\sigma}{dz} (e^+ e^- \rightarrow \tilde{f}_i \tilde{f}_i^-) = \frac{e^4 N_f \beta^3 \Phi_{\tilde{f}_i}(z)}{512\pi} \quad (i = L, R) \quad (B.1a)$$

with

$$\Phi_{\tilde{f}_i}(z) = (1-z^2) \left[\frac{8q_f^2}{5} + \frac{2A_f^2(\alpha_e^2 + \beta_e^2)s - 8\alpha_e q_f A_f(s-m_Z^2)}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right] \quad (B.1b)$$

In eq. B.1a, $N_f = 1(3)$ for sleptons (squarks), $z = \cos \theta$ where θ is the angle between the incoming electron and the outgoing sfermion and β is the speed of the produced sfermion: $\beta \equiv \sqrt{(1-4m_{\tilde{f}}^2/s)}$. The constant A_f that appears in eq. B.1b is given by

$$A_f = \begin{cases} 2(\alpha_f - \beta_f) & \text{for left-handed sfermions} \\ 2(\alpha_f + \beta_f) & \text{for right-handed sfermions} \end{cases}$$

The other parameters are as defined in table A.1.

As mentioned in section 2.2, the production of $\tilde{e}_R \tilde{e}_R$ and $\tilde{e}_L \tilde{e}_L$ pairs occurs via exchanges of Z^0 and γ in the s-channel and via $\tilde{\gamma}, \tilde{Z},$ and $\tilde{Z}_h$ exchanges in the t-channel. In this case, the cross-section is given by

$$\frac{d\sigma}{dz} (e^+ e^- \rightarrow \tilde{f}_i \tilde{f}_i^-) = \frac{e^4}{512\pi} \beta^3 \Phi_{\tilde{e}_i}(z),$$

with

$$\Phi_{\tilde{e}_L}(z) = \Phi_{\tilde{\mu}_i}(z)$$

$$\begin{aligned} & + \frac{X_e^4 s(2E\zeta - \zeta^2 - m_{\tilde{e}_L}^2)}{p^2 (2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{\gamma}}^2)^2} + \frac{P_e^4 s(2E\zeta - \zeta^2 - m_{\tilde{e}_L}^2)}{p^2 (2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}}^2)^2} \\ & + \frac{R_e^4 s(2E\zeta - \zeta^2 - m_{\tilde{e}_L}^2)}{p^2 (2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}_h}^2)^2} - 4e^2(1-z^2) \times \left[\frac{X_e^2}{2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{\gamma}}^2} \right] \end{aligned}$$

$$\begin{aligned}
& + \left[\frac{P_e^2}{2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}}^2} + \frac{R_e^2}{2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}_h}^2} \right] \\
& - \frac{2A_e(\alpha_e - \beta_e)s(s - m_Z^2)(1 - z^2)}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2} \times \left[\frac{X_e^2}{2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Y}}^2} \right. \\
& + \left. \frac{P_e^2}{2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}}^2} + \frac{R_e^2}{2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}_h}^2} \right] \\
& + \frac{X_e^2 P_e^2 s(1 - z^2)}{(2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Y}}^2)(2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}}^2)} + \frac{X_e^2 R_e^2 s(1 - z^2)}{(2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Y}}^2)(2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}_h}^2)} \\
& + \frac{P_e^2 R_e^2 s(1 - z^2)}{(2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}_h}^2)(2E\zeta - m_{\tilde{e}_L}^2 + m_{\tilde{Z}}^2)} \quad (B.2b)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\Phi}_{\tilde{e}_L}(z) &= \tilde{\Phi}_{\tilde{e}_L}(z) \\
&\times \left[X_e \rightarrow X'_e, P_e \rightarrow P'_e, R_e \rightarrow R'_e, m_{\tilde{e}_L} \rightarrow m_{\tilde{e}_R} \text{ and } (\alpha_e - \beta_e) \rightarrow (\alpha_e + \beta_e) \right].
\end{aligned}$$

In eq. B.2, the constants X, Y , etc, are as defined in table A.2, where p is the selectron momentum ($p = (E^2 - m_{\tilde{e}}^2)^{1/2}$) and $\zeta = E - p_Z$. Finally, we turn to $\tilde{e}_L \tilde{e}_R$ and $\tilde{e}_R \tilde{e}_L$ pair production. The differential cross-section for this is given by

$$\begin{aligned}
\frac{d\sigma}{dz} (e^+ e^- \rightarrow \tilde{e}_L \tilde{e}_R) &= \frac{d\sigma}{dz} (e^+ e^- \rightarrow \tilde{e}_R \tilde{e}_L) = \\
& \frac{e^4 \beta}{128\pi} \left[\frac{X_e^2 X_e'^2 m_{\tilde{Y}}^2}{(2E\zeta - m_{\tilde{e}}^2 + m_{\tilde{Y}}^2)^2} + \frac{P_e^2 P_e'^2 m_{\tilde{Z}}^2}{(2E\zeta - m_{\tilde{e}}^2 + m_{\tilde{Z}}^2)^2} \right. \\
& + \frac{R_e^2 R_e'^2 m_{\tilde{Z}_h}^2}{(2E\zeta - m_{\tilde{e}}^2 + m_{\tilde{Z}_h}^2)^2} + \frac{2X_e X_e' P_e P_e' m_{\tilde{Z}} m_{\tilde{Y}}}{(2E\zeta - m_{\tilde{e}}^2 + m_{\tilde{Y}}^2)(2E\zeta - m_{\tilde{e}}^2 + m_{\tilde{Z}}^2)} \quad (B.3)
\end{aligned}$$

$$+ \frac{2X_e X'_e R_e R'_e m_{\tilde{Z}_h} m_{\tilde{\gamma}}}{(2E\zeta - m_e^2 + m_{\tilde{\gamma}}^2)(2E\zeta - m_e^2 + m_{\tilde{Z}_h}^2)} + \frac{2P_e P'_e R_e R'_e m_{\tilde{Z}} m_{\tilde{Z}_h}}{(2E\zeta - m_e^2 + m_{\tilde{Z}}^2)(2E\zeta - m_e^2 + m_{\tilde{Z}_h}^2)} \Bigg].$$

In deriving eqs B.2 and B.3, we have neglected the terms proportional to Z and Q in the Lagrangian (see appendix A), i.e. the terms that arise owing to the mixing of $\tilde{Z}_h(0)$ with $\tilde{\gamma}_h(0)$ and $\tilde{Z}(0)$. Numerically, this makes no difference since the mixing is suppressed by the large mass of $\tilde{Z}_h(0)$ (unless $m_{\tilde{Z}}(0)$ is very close to m_Z). In any case, as we shall see, the zino contribution to the cross-section is rather effectively negligible.

We now turn to the total cross-section formulae which for the production of smuons, staus or squarks are given by

$$\sigma(e^+e^- \rightarrow \tilde{f}_i \tilde{f}_i^-) = \frac{N_f e^4 \beta^3}{192\pi} \left[\frac{4q_f^2}{s} + \frac{A_f^2(\alpha_e^2 + \beta_e^2)s - 4\alpha_e q_f A_f(s - m_Z^2)}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right]. \quad (B.4)$$

The cross-section for the production of left-(right-) handed selectron pairs is given by ($i = L, R$)

$$\begin{aligned} \sigma(e^+e^- \rightarrow \tilde{e}_i \tilde{e}_i^-) = & \frac{e^4}{384\pi} \beta^3 \left[\frac{8}{s} + \frac{2A_e^2(\alpha_e^2 + \beta_e^2)s + 8\alpha_e A_e(s - m_Z^2)}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right. \\ & - C_i^4 F(a_{\tilde{Z}}) + B_i^4 F(a_{\tilde{\gamma}}) - D_i^4 F(a_{\tilde{Z}_h}) - 2C_i^2 G(a_{\tilde{Z}}) - 2B_i^2 G(a_{\tilde{\gamma}}) \\ & - 2D_i^2 G(a_{\tilde{Z}_h}) - \frac{A_e(\alpha_e^2 + \beta_e^2)s(s - m_Z^2)}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \left[C_i^2 G(a_{\tilde{Z}}) + B_i^2 G(a_{\tilde{\gamma}}) + D_i^2 G(a_{\tilde{Z}_h}) \right] \\ & + \frac{2B_i^2 C_i^2 E^2}{m_{\tilde{Z}}^2 - m_{\tilde{Z}_h}^2} \left[G(a_{\tilde{\gamma}}) - G(a_{\tilde{Z}}) \right] + \frac{2B_i^2 D_i^2 E^2}{m_{\tilde{Z}}^2 - m_{\tilde{Z}_h}^2} \left[G(a_{\tilde{\gamma}}) - G(a_{\tilde{Z}_h}) \right] \\ & \left. + \frac{2C_i^2 D_i^2 E^2}{m_{\tilde{Z}_h}^2 - m_{\tilde{Z}}^2} \left[G(a_{\tilde{Z}}) - G(a_{\tilde{Z}_h}) \right] \right]. \quad (B.5) \end{aligned}$$

with

$$F(x) = \frac{3}{p^2} - \frac{3}{2} \frac{E+x}{p^3} \ln \frac{E+p+x}{E-p+x}, \quad (B.6a)$$

$$G(x) = \frac{3}{4E} \left[\frac{2}{p^2} (E+x) + \frac{1}{p} \ln \left(\frac{E+p+x}{E-p+x} \right) - \frac{(E+x)^2}{p^3} \ln \left(\frac{E+p+x}{E-p+x} \right) \right] \quad (B.6b)$$

and

$$a_X = (m_X^2 - m_{e_1}^2)/2E . \quad (\text{B.6c})$$

In eqs. (B.5) and (B.6), E is the beam energy and p is the momentum of the sfermion produced. The minus (plus) sign in eq. (B.5) refers to production of left- (right-) handed selectrons. Finally, the cross-section for the production of $\tilde{e}_L \tilde{e}_R$ or $\tilde{e}_R \tilde{e}_L$ pairs is given by

$$\begin{aligned} \sigma(e^+ e^- \rightarrow \tilde{e}_L \tilde{e}_R) &= \sigma(e^+ e^- \rightarrow \tilde{e}_L \tilde{e}_R) \\ &= \frac{e^4}{128\pi} \frac{p}{sE} \left[\frac{2B_L^2 B_R^2 m_{\tilde{\gamma}}^2}{m_{\tilde{\gamma}}^2 + a_{\tilde{\gamma}}^2} + \frac{2C_L^2 C_R^2 m_{\tilde{Z}}^2}{m_{\tilde{Z}}^2 + a_{\tilde{Z}}^2} \right. \\ &\quad + \frac{2D_L^2 D_R^2 m_{\tilde{Z}_h}^2}{m_{\tilde{Z}_h}^2 + a_{\tilde{Z}_h}^2} + \frac{2B_L B_R C_L C_R m_{\tilde{Z}} m_{\tilde{\gamma}}}{p(a_{\tilde{\gamma}} - a_{\tilde{Z}})} \ln \frac{(E+a_{\tilde{\gamma}}-p)(E+a_{\tilde{Z}}+p)}{(E+a_{\tilde{Z}}-p)(E+a_{\tilde{\gamma}}+p)} \\ &\quad + \frac{2B_L B_R D_L D_R m_{\tilde{\gamma}} m_{\tilde{Z}_h}}{p(a_{\tilde{\gamma}} - a_{\tilde{Z}_h})} \ln \frac{(E+a_{\tilde{\gamma}}-p)(E+a_{\tilde{Z}_h}+p)}{(E+a_{\tilde{Z}_h}-p)(E+a_{\tilde{\gamma}}+p)} \\ &\quad \left. + \frac{2C_L C_R D_L D_R m_{\tilde{Z}} m_{\tilde{Z}_h}}{p(a_{\tilde{Z}} - a_{\tilde{Z}_h})} \ln \frac{(E+a_{\tilde{Z}}-p)(E+a_{\tilde{Z}_h}+p)}{(E+a_{\tilde{Z}_h}-p)(E+a_{\tilde{Z}}+p)} \right] . \quad (\text{B.7}) \end{aligned}$$

APPENDIX C
Wino pair production

In this appendix, we present the differential cross-section for the process

$$e^+e^- \rightarrow \tilde{W} \tilde{W}^* \rightarrow l \bar{\nu} \tilde{\gamma} \rightarrow \bar{l} \nu \tilde{\gamma} , \quad (C.1)$$

including all the spin correlations discussed in section 2.4.1.

We also present here the squared matrix elements for the decays $\tilde{W} \rightarrow q' \bar{q} \tilde{\gamma}$ and $l \bar{\nu} \tilde{\gamma}$ and the angular distribution for wino pair production that are required for the calculation of multijet + $\cancel{p}_T$ and jet(s) + lepton + $\cancel{p}_T$ signatures neglecting spin correlations. The angular distribution of winos has also been used in the computation of the cross-sections for $e^+e^- \rightarrow \tilde{W} \tilde{W}^* \rightarrow l \bar{\nu} \rightarrow \bar{l}' \nu'$ for the case of the light sneutrinos. We first consider the reaction (C.1).

In terms of the kinematics shown in fig. C.1 we have

$$\begin{aligned} d\sigma = & \frac{(2\pi)^4}{(2s)^4} \delta^4(p_1 + p_2 - \Sigma q_i - \Sigma q'_i) \frac{1}{|1^2 - m_{\tilde{W}}^2 + i\Gamma_{\tilde{W}} m_{\tilde{W}}|^2} \frac{1}{|1'^2 - m_{\tilde{W}}^2 + i\Gamma_{\tilde{W}} m_{\tilde{W}}|^2} \\ & \times \left[W(1, 1') R(1, q_i) R'(1', q'_i) + X_V(1, 1') S^V(1, q_i) R'(1', q'_i) \right. \\ & \left. + Y_V(1, 1') S'^V(1', q'_i) R(1, q_i) + Z_{\mu\nu}(1, 1') S^\mu(1, q_i) S^\nu(1', q'_i) \right] \\ & \times \prod_i \frac{d^3 q_i}{(2\pi)^3 2E q_i} \prod_j \frac{d^3 q'_j}{(2\pi)^3 2E q'_j} . \end{aligned} \quad (C.2)$$

Before proceeding to the definitions of the various quantities that occur above, we briefly describe the structure of eq. (C.2). The functions W , X_μ , Y_ν , and $Z_{\mu\nu}$ describe the production part of the process, whereas the functions $R(R')$ and $S_\mu(S'_\mu)$ describe the decay of the wino (antiwino). Had we ignored all spin correlations, i.e. had we factorized the cross-section into production and decay parts as if the wino were spinless, we would have only the first term, with $R(R')$ describing the spin averaged decay of a wino (antiwino). The second and third terms are due to spin correlations arising from only one of the winos and the last term from these correlations involving both wino spins. Some useful formulae for the derivation of the squared matrix element may be found in the appendix of Schimert et al. [2.6].

We now present expressions for each of the four terms in eq. (C.2):

$$W(1,1')R(1,q_1)R'(1',q'_1) =$$

$$= \frac{64m_W^4 e^2 f_+^4}{\sin^4 \theta_W} \left[A1 \cdot q_2 + B1 \cdot q_1 + Cq_1 \cdot q_2 \right] \left[A'1' \cdot q'_2 + B'1' \cdot q'_1 + C'q'_1 \cdot q'_2 \right] \\ \times \left[8\Sigma_1 [(1 \cdot p_2)(1' \cdot p_1) + (1' \cdot p_2)(1 \cdot p_1) + m_W^2(p_1 \cdot p_2)] \right. \\ \left. + \frac{2e^2 f_+^2}{\sin^2 \theta_W} \Sigma_2 \frac{[2(p_1 \cdot 1)(p_2 \cdot 1') + m_W^2(p_1 \cdot p_2)]}{(p_1 - 1)^2 - m_V^2} + \frac{e^4 f_+^4}{\sin^4 \theta_W} \frac{(p_1 \cdot 1)(p_2 \cdot 1')}{[(p_1 - 1)^2 - m_V^2]^2} \right] \quad (C.3a)$$

$$x_V(1,1')S^\mu(1,q'_1)R'(1',q'_1) = \frac{64m_W^4 e^8 f_+^4}{\sin^4 \theta_W} \\ \times [A'1' \cdot q'_2 + B'1' \cdot q'_1] \times \left[8\Sigma_3 \left[\tilde{A}(m_W^2[(p_1 \cdot q_2)p_2 \cdot (1+1') - (p_2 \cdot q_2)p_1 \cdot (1+1')]) \right. \right. \\ \left. \left. + (p_2 \cdot 1)(q_2 \cdot 1)p_1(1+1') - (p_1 \cdot 1)(q_2 \cdot 1)p_2 \cdot (1+1') \right] - \tilde{B}(m_W^2[(p_1 \cdot q_1)p_2 \cdot (1+1') \right. \\ \left. - (p_2 \cdot q_1)p_1 \cdot (1+1')]) + (p_2 \cdot 1)(q_1 \cdot 1)p_1 \cdot (1+1') - (p_1 \cdot 1)(q_1 \cdot 1)p_2(1+1') \right) \\ \left. - \frac{2e^2 f_+^2}{\sin^2 \theta_W} \frac{1}{(p_1 - 1)^2 - m_V^2} \left[\tilde{A}(m_W^2[(p_2 \cdot q_2)(1 \cdot p_1) - (p_1 \cdot q_2)(1 \cdot p_2) - 2(p_1 \cdot q_2)(1' \cdot p_2)] \right. \right. \\ \left. \left. + 2(p_1 \cdot 1)(q_2 \cdot 1)(p_2 \cdot 1') - \tilde{B}(m_W^2[(p_2 \cdot q_1)(1 \cdot p_1) - (p_1 \cdot q_1)(1 \cdot p_2) - 2(p_1 \cdot q_1)(1' \cdot p_2)] \right. \right. \\ \left. \left. + 2(p_1 \cdot 1)(q_1 \cdot 1)(p_2 \cdot 1') \right] - \frac{e^4 f_+^4}{\sin^4 \theta} \frac{1}{[(p_1 - 1)^2 - m_V^2]^2} \left[\tilde{A}(1' \cdot p_2)((q_2 \cdot 1)(p_1 \cdot 1) \right. \right. \\ \left. \left. - m_W^2(p_1 \cdot q_2)) - \tilde{B}(1' \cdot p_2)((p_1 \cdot 1)(q_1 \cdot 1) - m_W^2(p_1 \cdot q_1)) \right] \right] \quad (C.3b)$$

The third term, $Y_V(1,1') S'^V(1',q'_1) R(1,q_1)$, is obtained from eq. (C.3b) by the following replacements and interchange:

$$\Sigma_3 \rightarrow \Sigma_3^* \quad (\text{irrelevant because } \Sigma_3 \text{ is real})$$

$$(\tilde{A}, \tilde{B}) \rightarrow (\tilde{A}', \tilde{B}'),$$

$$(A, B, C) \rightarrow (A', B', C'),$$

$$q_1 \rightarrow q'_1,$$

$$p_1 \rightarrow p_2,$$

$$1 \rightarrow 1'. \quad (C.3c)$$

Finally, the last term in eq. (C.2) is given by

$$\begin{aligned} Z_{\mu\nu}(1,1') S^\mu(1,q_1) S'^\nu(1',q'_1) &= \frac{64 m_W^2 e^8 f_+^4}{\sin^4 \theta_w} \left[\left[8\Sigma_1 (1 \cdot 1' + m_W^2) \right. \right. \\ &+ \frac{2e^2 f_+^4}{\sin^2 \theta_w} \Sigma_2 \frac{1 \cdot 1'}{(p_1 - 1)^2 - m_V^2} \left. \right] \times \left[\tilde{A}\tilde{A}'(F(q_2, p_1, q'_2, p_2) + F(q_2, p_2, q'_2, p_1)) \right. \\ &- \tilde{A}\tilde{B}'(F(q_2, p_1, q'_1, p_2) + F(q_2, p_2, q'_1, p_1)) \\ &- \tilde{B}\tilde{A}'(F(q_1, p_1, q'_2, p_2) + F(q_1, p_2, q'_2, p_1)) \\ &+ \tilde{B}\tilde{B}'(F(q_1, p_1, q'_1, p_2) + F(q_1, p_2, q'_1, p_1)) \left. \right] + \left[8\Sigma_1 + \frac{2e^2 f_+^2}{\sin^2 \theta_w} \Sigma_2 \frac{1}{(p_1 - 1)^2 - m_V^2} \right] \\ &\times \left[(\tilde{A}\tilde{A}'F(q_2, 1', q'_2, 1) - \tilde{A}\tilde{B}'F(q_2, 1', q'_1, 1) \right. \\ &- \tilde{B}\tilde{A}'F(q_1, 1', q'_2, 1) + \tilde{B}\tilde{B}'F(q_1, 1', q'_1, 1))(p_1, p_2) \end{aligned}$$

$$\begin{aligned}
& - (\tilde{A}\tilde{A}'F(q_2, p_2, q_2', 1) - \tilde{A}\tilde{B}'F(q_2, p_2, q_1', 1)) \\
& - \tilde{B}\tilde{A}'F(q_1, p_2, q_2', 1) + \tilde{B}\tilde{B}'F(q_1, p_2, q_1', 1))(1' \cdot p_1) \\
& - (\tilde{A}\tilde{A}'F(q_2, 1', q_2', p_1) - \tilde{A}\tilde{B}'F(q_2, 1', q_1', p_1)) \\
& - \tilde{B}\tilde{A}'F(q_1, 1', q_2', p_1) + \tilde{B}\tilde{B}'F(q_1, 1', q_1', p_1))(1 \cdot p_2) \\
& - (\tilde{A}\tilde{A}'F(q_2, p_1, q_2', 1) - \tilde{A}\tilde{B}'F(q_2, p_1, q_1', 1)) \\
& - \tilde{B}\tilde{A}'F(q_1, p_1, q_2', 1) + \tilde{B}\tilde{B}'F(q_1, p_1, q_1', 1))(1' \cdot p_2) \\
& - (\tilde{A}\tilde{A}'F(q_2, 1', q_2', p_2) - \tilde{A}\tilde{B}'F(q_2, 1', q_1', p_2)) \\
& - \tilde{B}\tilde{A}'F(q_1, 1', q_2', p_2) + \tilde{B}\tilde{B}'F(q_1, 1', q_1', p_2))(1 \cdot p_1) \\
& + (\tilde{A}\tilde{A}'G(q_2, q_2') - \tilde{A}\tilde{B}'G(q_2, q_1') - \tilde{B}\tilde{A}'G(q_1, q_2') + \tilde{B}\tilde{B}'G(q_1, q_1')) \left[(1 \cdot p_1)(1' \cdot p_2) \right. \\
& + (1 \cdot p_2)(1' \cdot p_1) - (p_1 \cdot p_2)(1 \cdot 1') \left. \right] + \left[\frac{e^2 f_+^2}{\sin^2 \theta_w} 4\epsilon_2 \frac{m_{\tilde{W}}^2}{(p_1 - 1)^2 - m_{\tilde{V}}^2} \right. \\
& + \left. \frac{e^4 f_+^4}{\sin^2 \theta_w} \frac{m_{\tilde{W}}^2}{[(p_1 - 1)^2 - m_{\tilde{V}}^2]^2} \right] \times \left[\tilde{A}\tilde{A}'F(q_1, p_1, q_2', p_2) - \tilde{A}\tilde{B}'F(q_2, p_1, q_1', p_2) \right. \\
& - \tilde{B}\tilde{A}'F(q_1, p_1, q_2', p_2) + \tilde{B}\tilde{B}'F(q_1, p_1, q_1', p_2) \left. \right] \quad (C.3d)
\end{aligned}$$

The factors A, B, C (A', B', C') are given by

$$A = \frac{q_1 \cdot q_3}{m_W^2} \left[\frac{1}{\left[(1-q_3)^2 - m_W^2 \right]^2} + \frac{1}{(1-q_2)^2 - m_e^2} + 2 \frac{1}{\left[(1-q_3)^2 - m_W^2 \right] \left[(1-q_2)^2 - m_e^2 \right]} \right] \quad (C.4a)$$

$$B = \frac{1}{\left[(1-q_3)^2 - m_W^2 \right]^2} \frac{q_2 \cdot q_3}{m_W^2} \quad (C.4b)$$

$$C = - \frac{m_Y}{m_W} \left[\frac{1}{\left[(1-q_3)^2 - m_W^2 \right]^2} + \frac{1}{\left[(1-q_2)^2 - m_W^2 \right] \left[(1-q_2)^2 - m_e^2 \right]} \right]. \quad (C.4c)$$

With the quantities A', B', C' being obtained from A, B, C by replacing the momenta (l, q_i) by (l', q'_i) , $\tilde{A}$ and $\tilde{B}$ are given by

$$\tilde{A} = A - \frac{m_Y}{m_W} \frac{(q_1 \cdot l)}{m_W^2} \left[\frac{1}{\left[(1-q_3)^2 - m_W^2 \right]^2} + \frac{1}{\left[(1-q_3)^2 - m_W^2 \right] \left[(1-q_2)^2 - m_e^2 \right]} \right] \quad (C.5a)$$

and

$$\tilde{B} = B - \frac{m_Y}{m_W} \frac{q_2 \cdot l}{m_W^2} \left[\frac{1}{\left[(1-q_3)^2 - m_W^2 \right]^2} + \frac{1}{\left[(1-q_3)^2 - m_W^2 \right] \left[(1-q_2)^2 - m_e^2 \right]} \right]. \quad (C.5b)$$

Again $\tilde{A}'$ and $\tilde{B}'$ are obtained from $\tilde{A}$ and $\tilde{B}$ by replacing the momenta by the primed ones. The constants Σ_1, Σ_2 , and Σ_3 are given by

$$\begin{aligned} \Sigma_1 = e^4 & \left[\frac{1}{s^2} + \left(1 - \frac{1}{2} f_-^2 \sec^2 \theta_W \right)^2 \frac{\cot^2 \theta_W (\alpha_e^2 + \beta_e^2)}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right. \\ & \left. - 2 \left(1 - \frac{1}{2} f_-^2 \sec^2 \theta_W \right) \cot \theta_W \alpha_1 \frac{1}{s} \frac{s - m_Z^2}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right] \end{aligned} \quad (C.6a)$$

$$\Sigma_2 = e^2 \left[\frac{1}{s} - \left(1 - \frac{1}{2} f_-^2 \sec^2 \theta_W \right) \cot \theta_W (\alpha_1 - \beta_1) \frac{s - m_Z^2}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right] \quad (C.6b)$$

$$\Sigma_3 = \frac{e^4}{2 \sin^2 \theta_W} \left[\left(1 - \frac{1}{2} f_-^2 \sec^2 \theta_W \right) \frac{1}{s} \frac{s - m_Z^2}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right]$$

$$- \frac{\alpha_1 \cot \theta_w (1 - \frac{1}{2} f_-^2 \sec^2 \theta_w)^2}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \Bigg] . \quad (C.6c)$$

The functions F and G that appear in eq. (C.3d) are given by

$$F(q, p, q', p') = \left[m_W^2 q p - (l \cdot q)(l \cdot p) \right] \left[m_W^2 q' p' - (l' \cdot q')(l' \cdot p') \right] \quad (C.7a)$$

and

$$G(q, q') = m_W^4 (q q') - m_W^2 \left[(l \cdot q')(l \cdot q) + (l' \cdot q)(l' \cdot q') \right] + (l \cdot q)(l' \cdot q')(l \cdot l') . \quad (C.7b)$$

The other constants are as defined in section 1.5.2.

At this point, we note that the production amplitudes have been calculated exactly retaining the effects of $m_{\tilde{Y}} = 0$ in the mixing angle. In the decay amplitude, however, we have neglected the photino mass-induced couplings and so, in effect, we are producing the state $\tilde{\gamma}(0)$, but with a non-zero mass. Further, we have used a leptonic branching fraction for the wino that was calculated for $m_{\tilde{Y}} = 0$. We do not expect this approximation to significantly alter our results. In fact, it has been shown in ref. [2.6] that for $0 < m_{\tilde{Y}} < 10$ GeV, the branching fraction is almost independent of $m_{\tilde{Y}}$ for $m_{\tilde{W}} < m_{\tilde{e}}$ and $m_{\tilde{e}} \geq 100$ GeV. The main effect of the mixings induced because $\mu_2 \neq 0$ is in the production rate, and this has been treated exactly.

Before closing this appendix on the wino, we recall our discussion on the hadronic decays of the wino, which we have calculated using a factorizable form for the cross-section. This, therefore, entails a knowledge of the decay matrix element for

$$\tilde{W}(q) \rightarrow \tilde{\gamma}(l) + \bar{u}(p) + d(k) .$$

This is given by (using the same approximation as discussed above):

$$\begin{aligned} |M(\tilde{W} \rightarrow \bar{u} d \tilde{\gamma})|^2 &= 4g^4 f_+^2 \sin^2 \theta_w \left[\frac{2(q \cdot p)(k \cdot l) + 2(p \cdot l)(k \cdot q)}{[(q-l)^2 - m_W^2]^2} \right. \\ &\quad \left. + \frac{2}{9} \frac{(p \cdot q)(k \cdot l)}{[(q-p)^2 - m_Q^2]^2} + \frac{4}{3} \frac{(p \cdot q)(k \cdot l)}{[(q-l)^2 - m_W^2][(q-p)^2 - m_Q^2]} \right] \end{aligned}$$

$$+ \frac{8}{9} \frac{(q \cdot k)(p \cdot l)}{[(q-k)^2 - m_{\tilde{q}}^2]^2} + \frac{8}{3} \frac{(q \cdot k)(p \cdot l)}{[(q-l)^2 - m_w^2][(q-k)^2 - m_{\tilde{q}}^2]} \Bigg] . \quad (C.8a)$$

The colour sum is not included in eq. (C.8a). This may be compared with the leptonic decay matrix element for

$$\tilde{W}(q) \rightarrow \tilde{\gamma}(l) + \bar{\nu}(p) + l(k)$$

$$|M(\tilde{W} \rightarrow l \bar{\nu} \tilde{\gamma})|^2 = 8g^4 f_+^2 \sin^2 \theta_w \left[\frac{(p \cdot q)(k \cdot l) + (q \cdot k)(p \cdot l)}{[(q-l)^2 - m_w^2]^2} + \frac{(p \cdot q)(k \cdot l)}{[(q-p)^2 - m_e^2]^2} + \frac{2(p \cdot q)(k \cdot l)}{[(q-p)^2 - m_e^2][(q-l)^2 - m_w^2]} \right] . \quad (C.8b)$$

Equations (C.8a) and (C.8b) are useful when computing the total cross-sections for jet(s) + p_T and jet(s) + lepton(s) + p_T that result from wino decay. The only other distribution needed for this computation is the angular distribution of the produced winos. Because these are also used in the computation of the acollinear lepton pair cross-sections in the light sneutrino case, we present this for an arbitrary value of v'/v . We emphasize that all other equations presented in this appendix are valid only for the case $v'/v = 1$.

If θ is the angle between the incoming electron and the outgoing $\tilde{W}$, we have,

$$\frac{d\sigma}{d\cos\theta} (e^+ e^- \rightarrow \tilde{W}\tilde{W}) = \frac{1}{128\pi s} \frac{p}{E} |M|^2 \quad (C.9a)$$

with

$$|M|^2 = |M_Y|^2 + |M_Z|^2 + |M_{\tilde{V}}|^2 + M_{YZ} + M_{Y\tilde{V}} + M_{Z\tilde{V}} , \quad (C.9b)$$

where

$$|M_Y|^2 = \frac{16e^4}{s} [E^2(1+\cos^2\theta) + m_w^2 \sin^2\theta] \quad (C.10a)$$

and

$$|M_Z|^2 = \frac{16e^4 \cot^2 \theta_w s}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \left\{ (x_C^2 + y_C^2)(\alpha_1^2 + \beta_1^2) \left[E^2 (1 + \cos^2 \theta) + m_W^2 \sin^2 \theta \right] - 2y_C(\alpha_1^2 + \beta_1^2) \right. \\ \left. \times m_W^2 - 8x_C y_C \alpha_1 \beta_1 E p \cos \theta \right\} \quad (C.10b)$$

$$|M_{\tilde{V}}|^2 = \frac{g^4 \sin^4 \theta_R s}{\left[E^2 + p^2 - 2Ep \cos \theta + m_{\tilde{V}}^2 \right]} (E - p \cos \theta)^2 \quad (C.10c)$$

$$M_{YZ} = \frac{-32e^4 \cot \theta_w (s-m_Z^2)}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \left\{ \alpha_1 x_C \left[E^2 (1 + \cos^2 \theta) + m_W^2 \sin^2 \theta \right] - 2\beta_1 y_C E p \cos \theta \right\} \quad (C.10d)$$

$$M_{Y\tilde{V}} = -4e^2 g^2 \sin^2 \theta_R \frac{\left[(E - p \cos \theta)^2 + m_W^2 \right]}{\left[E^2 + p^2 - 2Ep \cos \theta + m_{\tilde{V}}^2 \right]} \quad (C.10e)$$

$$M_{Z\tilde{V}} = \frac{4e^2 g^2 \cot \theta_w \sin^2 \theta_R (s-m_Z^2)(\alpha_1 - \beta_1)s}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \\ \times \frac{\left[(x_C - y_C) \left[(E - p \cos \theta)^2 + m_W^2 \right] + 2y_C m_W^2 \right]}{\left[E^2 + p^2 - 2Ep \cos \theta + m_{\tilde{V}}^2 \right]} \quad (C.10f)$$

Here $\sqrt{s}$ is the centre-of-mass energy, $E = \sqrt{s}/2$, and p , the momentum of the wino, is $\sqrt{E^2 - m_W^2}$. The various constants that enter into eq. (C.10) have been defined in section 1.5.2, equation (C.10) takes a much simpler form for $v' = v$ since, in that case, $y_C = 0$. In the form given above, we note that eqs. (C.9) and (C.10) for wino pair production are valid for all values of v'/v and $m_{\tilde{V}}$ since we have diagonalized the charged gaugino sector exactly.

The cross-section for wino pair production is given by

$$\sigma(e^+ e^- \rightarrow \tilde{W}\tilde{W}) = \frac{e^4}{128\pi s} \frac{p}{E} \left[\frac{32}{3} \left(1 + \frac{2m_W^2}{s} \right) \right. \\ \left. + \frac{32 \cot^2 \theta_w s}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \left[(x_C^2 + y_C^2)(\alpha_1^2 + \beta_1^2) \frac{(s+2m_W^2)}{3} - 2y_C^2(\alpha_1^2 + \beta_1^2)m_W^2 \right] \right]$$

$$\begin{aligned}
& + \frac{\sin^4 \gamma_R}{\sin^4 \theta_w} \left[2 - \frac{m_V^2 - m_W^2}{Ep} \ln A + \frac{2(m_V^2 - m_W^2)^2}{(m_V^2 - m_W^2)^2 + m_V^2 s} \right] \\
& - \frac{64}{3} \cot^2 \theta_w (s - m_Z^2) \alpha_1 x_c \frac{(s + 2m_W^2)}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \\
& + \frac{4 \cot \theta_w}{\sin^2 \theta_w} \sin^2 \gamma_R \frac{(s - m_Z^2)(\alpha_1 - \beta_1)s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \\
& \times \left[(x_c - y_c) \left(1 - \frac{2(m_V^2 - m_W^2)}{s} + \frac{(m_V^2 - m_W^2)^2 + m_W^2 s}{2Eps} \ln A \right) + y_c \frac{m_W^2}{Eps} \ln A \right] \\
& - \frac{4 \sin^2 \gamma_R}{\sin^2 \theta_w} \left[1 - \frac{2(m_V^2 - m_W^2)}{s} + \frac{(m_V^2 - m_W^2)^2 + m_W^2 s}{2Eps} \ln A \right] , \quad (C.11)
\end{aligned}$$

where p is the momentum of the wino ($p = \sqrt{E^2 - m_W^2}$), and $A = [2E(E+p) + m_V^2 - m_W^2]/[2E(E-p) + m_V^2 - m_W^2]$.

Figure caption

Fig. C.1 Sketch of the kinematics for the reaction C.1.

Fig. C.1

APPENDIX D
 $\tilde{Z}\tilde{Y}$ production

The total cross-section for the process is given by

$$\begin{aligned} \sigma = & \frac{1}{32\pi s} \frac{k}{E} (X_1^2 P_1^2 + X_1'^2 P_1'^2) \\ & \times \left[1 + \frac{m_{\tilde{Z}}^2 + m_{\tilde{Y}}^2 + 2m_1^2}{2k\sqrt{s}} \ln \left[\frac{(s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) + 2m_1^2 + 2k\sqrt{s}}{(s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) + 2m_1^2 - 2k\sqrt{s}} \right] \right. \\ & - \frac{[m_1^2(m_{\tilde{Z}}^2 + m_{\tilde{Y}}^2) - m_1^4 - m_{\tilde{Z}}^2 m_{\tilde{Y}}^2]}{\left[\frac{1}{2} (s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) + m_1^2 \right]^2 - k^2 s} \\ & \left. - \frac{m_{\tilde{Z}} m_{\tilde{Y}} \sqrt{s}}{k} \frac{1}{(s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) + 2m_1^2} \ln \left[\frac{(s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) + 2m_1^2 + 2k\sqrt{s}}{(s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) + 2m_1^2 - 2k\sqrt{s}} \right] \right], \end{aligned} \quad (D.1)$$

whereas the angular distribution is given by

$$\begin{aligned} \frac{d\sigma}{d \cos \theta} (e^+ e^- \rightarrow \tilde{Z}\tilde{Y}) = & \frac{1}{512\pi s} \frac{k}{E} (X_1^2 P_1^2 + X_1'^2 P_1'^2) \\ & \times \left[\frac{s^2 - (m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2)^2 + 4sk^2 \cos^2 \theta + 4s^{3/2} k \cos \theta}{\left[\frac{1}{2} (s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) + \sqrt{s} k \cos \theta + m_e^2 \right]^2} \right. \\ & + \frac{s^2 - (m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2)^2 + 4sk^2 \cos^2 \theta - 4s^{3/2} k \cos \theta}{\left[\frac{1}{2} (s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) - \sqrt{s} k \cos \theta + m_e^2 \right]^2} \\ & \left. - \frac{8m_{\tilde{Z}} m_{\tilde{Y}} s}{\left[\frac{1}{2} (s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) - \sqrt{s} k \cos \theta + m_e^2 \right] \left[\frac{1}{2} (s - m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2) + \sqrt{s} k \cos \theta + m_e^2 \right]} \right] \end{aligned} \quad (D.2)$$

with k , the zino momentum, being

$$k = \frac{1}{2\sqrt{s}} \left[s^2 + (m_{\tilde{Z}}^2 - m_{\tilde{Y}}^2)^2 - 2s (m_{\tilde{Z}}^2 + m_{\tilde{Y}}^2) \right]^{1/2}.$$

The last term in eq. (D.2) results from the interference between the t- and u-channel graphs for the process. Note that it vanishes in the limit of zero photino mass.

The relevant squared and spin-averaged matrix element for the decay is given by

$$\frac{1}{2} |M(\tilde{Z} \rightarrow l\bar{l}\tilde{\gamma})|^2 \approx 2(X_f^2 P_f^2 + X_f'^2 P_f'^2)$$

$$\times \left[\frac{(P \cdot p_-)(p_{\tilde{\gamma}} \cdot p_+)}{[(P - p_-)^2 - m_{\tilde{e}}^2]^2} + (p_+ \leftrightarrow p_-) + \frac{m_{\tilde{Z}} m_{\tilde{\gamma}} P_+ \cdot p_-}{[(P - p_-)^2 - m_{\tilde{e}}^2][(P - p_+)^2 - m_{\tilde{e}}^2]} \right], \quad (D.3)$$

Here, P is the four-momentum of the zino, p_+ and p_- , the four-momentum of f and $\bar{f}$, and $p_{\tilde{\gamma}}$ that of the photino. The colour factor for $f = q$ has not been included in eq. (D.3). We note here that the decay $\tilde{Z} \rightarrow v\bar{v}\tilde{\gamma}$ is highly suppressed relative to the decay into charged leptons or quarks since the $v\bar{v}\tilde{\gamma}$ coupling which arises is only due to the tiny Higgsino component of the $\tilde{\gamma}$ (see table A.2).

APPENDIX E

Matrix Element and Cross-section for $e^+e^- \rightarrow \tilde{\gamma}\tilde{\gamma}$

The Feynman diagrams contributing to this process in leading order are shown in fig. E.1. Let p_1 stand for the four-momentum of the electron and p_2 for that of the positron; q for that of the photon; and p'_1, p'_2 for each of the two photinos respectively. All four-momentum squares are defined to be mass squared > 0 . The matrix element due to either $\tilde{e}_R$ or $\tilde{e}_L$ exchange can be written [3.3] in the form

$$M = (M_1 + M_2 + M_3) - (\text{exchange contribution}), \quad (\text{E.1})$$

where

$$\begin{aligned} M_1 &= 2e^3 \left[\bar{v}(p_2) \frac{1 - \gamma_5}{2} v(p'_2) \right] \frac{1}{2N_{22} q \cdot p_1} \left[\bar{u}(p'_1) \frac{1 + \gamma_5}{2} (p_1 - q) \varepsilon u(p_1) \right], \\ M_2 &= 2e^3 \left[\bar{v}(p_2) \varepsilon (q - p_2) \frac{1 - \gamma_5}{2} v(p'_2) \right] \frac{1}{2N_{11} q \cdot p_2} \left[\bar{u}(p'_1) \frac{1 + \gamma_5}{2} u(p_1) \right], \\ M_3 &= 2e^3 \left[\bar{v}(p_2) \frac{1 + \gamma_5}{2} v(p'_2) \right] \frac{\varepsilon \cdot (2p_1 - 2p'_1 - q)}{N_{11} N_{22}} \left[\bar{u}(p'_1) \frac{1 + \gamma_5}{2} u(p_1) \right], \end{aligned} \quad (\text{E.2})$$

are the contributions from fig. E.1, and the exchange contribution has the same form with p'_1 and p'_2 interchanged (same three diagrams with photino exchange). The dot products are the four-vector inner products. In eq. (E.2),

$$N_{11} \equiv \Delta^2 + 2p \cdot p'_1, \quad N_{22} \equiv \Delta^2 + 2p_2 \cdot p'_2, \quad \Delta^2 \equiv m_{\tilde{e}_{R \text{ or } L}}^2 - m_{\tilde{\gamma}}^2. \quad (\text{E.3})$$

The differential cross-section can be obtained from eqs. (E.1) TO (E.3). It takes the following form in the e^+e^- centre-of-mass frame:

$$\begin{aligned} \frac{d\sigma}{dx d \cos \theta} &= \frac{\alpha^3}{16\pi} \times \int \left[\left[(T_1 + T_2 + T_3 + T_{13} + T_{23} + T_{16} + T_{26}) + (p'_1 \leftrightarrow p'_2) \right] \right. \\ &\quad \left. + \left[T_{14} + (p_1 \leftrightarrow p_2) \right] + T_{36} \right] \left(1 - \frac{s'}{s} \right) \left(1 - \frac{4m_{\tilde{\gamma}}^2}{s'} \right)^{1/2} d \cos \theta^* d\phi^*, \end{aligned} \quad (\text{E.4})$$

where $s' = (p_1' + p_2')^2$ is the invariant mass of the photino pair, $0 < x \equiv (E_Y/E) < 1 - m_Y^2/E^2$, where E being the beam energy, E_Y , the energy of the detected γ , and θ the angle it makes with the beam direction. The angles θ^* and ϕ^* are the polar and azimuthal angles of one $\tilde{\gamma}$ in the $\tilde{\gamma} - \tilde{\gamma}$ centre-of-mass frame, with the z -axis defined opposite to the γ direction. The $\tilde{\gamma}$ momenta in the e^+e^- centre-of-mass frame can be written in terms of these angles as

$$\begin{aligned}
p_1' &= \left[(\gamma E_1'^* + \beta p_1'^* \cos \theta^*), p_1'^* \sin \theta^* \cos \phi^*, \right. \\
&\quad (p_1'^* \sin \theta^* \sin \phi^*) \cos \theta + (\beta E_1'^* + \gamma p_1'^* \cos \theta^*) \sin \theta, \\
&\quad \left. - \sin \theta (p_1'^* \sin \theta^* \sin \phi^*) + \cos \theta (\beta E_1'^* + \gamma p_1'^* \cos \theta^*) \right], \\
p_2' &= \left[(\gamma E_1'^* - \beta p_1'^* \cos \theta^*), -p_1'^* \sin \theta^* \cos \phi^*, \right. \\
&\quad (-p_1'^* \sin \theta^* \sin \phi^*) \cos \theta + (\beta E_1'^* - \gamma p_1'^* \cos \theta^*) \sin \theta, \\
&\quad \left. + \sin \theta (p_1'^* \sin \theta^* \sin \phi^*) + \cos \theta (\beta E_1'^* - \gamma p_1'^* \cos \theta^*) \right], \quad (E.5)
\end{aligned}$$

where $E_1'^*$ and $p_1'^*$ are the energy and momentum of one of the $\tilde{\gamma}$ in the $\tilde{\gamma} - \tilde{\gamma}$ centre-of-mass frame and,

$$\beta \equiv \frac{s - s'}{2\sqrt{ss'}}, \quad \gamma \equiv \frac{s + s'}{s\sqrt{ss'}}, \quad p_1'^* \equiv |p_1'^*| = \frac{1}{2} \sqrt{s' - m_Y^2}, \quad E_1'^* = \frac{\sqrt{s'}}{2}. \quad (E.6)$$

The quantities T_i and T_{ij} in eq. (E.4) are

$$\begin{aligned}
T_1 &= \frac{p_2 \cdot p_2' \cdot q \cdot p_1'}{q \cdot p_1 N_{22}^2}, \\
T_2 &= \frac{p_1 \cdot p_1' \cdot q \cdot p_2'}{q \cdot p_2 N_{11}^2}, \\
T_3 &= \frac{-2p_1 \cdot p_1' p_2 \cdot p_2'}{N_{11} N_{22}} \left[\frac{2(m_{\tilde{\gamma}}^2 - 2p_1 \cdot p_1' - q \cdot p_1 + q \cdot p_1')}{N_{11} N_{22}} - \frac{(2p_1 \cdot p_1' + q \cdot p_1)}{q \cdot p_1 N_{22}} - \frac{p_1 \cdot p_2}{q \cdot p_1 \cdot q \cdot p_2} \right. \\
&\quad \left. - \frac{(2p_1 \cdot p_2 - 2p_1' \cdot p_2 - q \cdot p_2)}{q \cdot p_2 N_{11}} \right],
\end{aligned}$$

$$\begin{aligned}
T_{13} &= \frac{2p_2 \cdot p'_2}{q \cdot p_1 N_{22}} \left[\frac{m_\gamma^2 q \cdot p_1 - 2p_1 \cdot p'_1 q \cdot p'_1}{N_{11} N_{22}} - \frac{p_1 \cdot p'_1 q \cdot p_2 + p_1 \cdot p_2 q \cdot p'_1 - q \cdot p_1 p'_1 \cdot p_2}{2q \cdot p_2 N_{11}} \right], \\
T_{23} &= \frac{-2p_1 \cdot p'_1}{q \cdot p_2 N_{11}} \left[\frac{q \cdot p'_2 (p_1 \cdot p_2 - p'_1 \cdot p_2) + p_2 \cdot p'_2 (q \cdot p_1 - q \cdot p'_1) - q \cdot p_2 (p_1 \cdot p'_2 - p'_1 \cdot p'_2)}{N_{11} N_{22}} \right. \\
&\quad \left. + \frac{p_1 \cdot p_2 q \cdot p'_2 + p_2 \cdot p'_2 q \cdot p_1 - q \cdot p_2 p_1 \cdot p'_2}{2q \cdot p_1 N_{22}} \right], \\
T_{16} &= \frac{m_\gamma^2}{q \cdot p_1 N_{22}} \left[\frac{q \cdot p_2 p_1 \cdot p'_2 + p_1 \cdot p_2 q \cdot p'_2 - q \cdot p_1 p_2 \cdot p'_2}{N_{21} N_{12}} + \frac{p_1 \cdot p_2}{N_{12}} \right], \\
T_{26} &= \frac{m_\gamma^2}{q \cdot p_2 N_{11}} \left[\frac{(q \cdot p_1 - q \cdot p'_1) p_1 \cdot p_2 + (p_1 \cdot p_2 - p_2 \cdot p'_2) q \cdot p_1 + q \cdot p_2 p_1 \cdot p'_2}{N_{12} N_{21}} + \frac{p_1 \cdot p_2}{N_{21}} \right], \\
T_{14} &= \frac{-m_\gamma^2 q \cdot p_2}{q \cdot p_1 N_{21} N_{22}}, \\
T_{36} &= m_\gamma^2 p_1 \cdot p_2 \left\{ \frac{2}{N_{11} N_{22}} \left[\frac{-2p_1 \cdot p_2 + q \cdot p'_1 + q \cdot p'_2 + 2p'_1 \cdot p'_2}{N_{21} N_{12}} \right] \right. \\
&\quad - \left[\frac{1}{N_{11} N_{22}} \left(\frac{2p_1 \cdot p'_1 + q \cdot p_1}{q \cdot p_1 N_{21}} + \frac{2p_1 \cdot p_2 - 2p'_1 \cdot p_2 - q \cdot p_2}{q \cdot p_2 N_{12}} \right) + (p'_1 \leftrightarrow p'_2) \right] \\
&\quad \left. - \frac{p_1 \cdot p_2}{q \cdot p_1 q \cdot p_2} \left[\frac{1}{N_{12} N_{22}} + \frac{1}{N_{11} N_{21}} \right] \right\}, \tag{E.7}
\end{aligned}$$

where $T_i \equiv |M_i|^2$ and $T_{ij} \equiv 2\text{Re}(M_i M_j^*)$.

Figure caption

Fig. E.1 The Feynman diagrams contributing to the process $e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$.

Fig. E.1

APPENDIX F
Amplitudes for $e^+e^- \rightarrow \tilde{\nu}\tilde{\nu}\gamma$

In general, mass eigenstates of SUSY particles are not weak eigenstates. The scalar partners of left-handed and right-handed leptons are mixed together, as are the fermionic partners of gauge bosons and Higgs bosons. Indeed the mass eigenstates for supersymmetric fermions are mixtures of higgsinos, winos, zinos, with their mixing angles which depend on the relative strengths of the different supersymmetry and $[SU(2) \times U(1)]$ breaking parameters.

In models without right-handed neutrinos, the partner of the left-handed neutrino, the sneutrino $\tilde{\nu}_1$, is also a mass eigenstate. Furthermore, in models where the sneutrino is the lightest supersymmetric particle, one ends up with a light chargino state (charged gauge fermion) which is almost a weak eigenstate, a wino, and a charged heavy chargino which is an almost pure Higgsino state. The latter decouples for any practical purpose from electron interactions, so that only the light wino plays a role in the process discussed here.

The Feynman diagrams [3.1, 3.2, 3.5] that participate in the process $e^+e^- \rightarrow \gamma\tilde{\nu}\tilde{\nu}$ are shown in fig. F.1. The first three amplitudes involve, apart from the electromagnetic vertex, the interaction

$$L = ig \cos \varphi \bar{e}_L \tilde{W}^C \tilde{\nu}_L + \text{h.c.} , \quad (\text{F.1})$$

where $\tilde{W}$ is the (light) wino field and φ is the mixing angle between the $\tilde{W}$ and the $\tilde{H}$. In the following we will first assume that $\cos \varphi = 1$ (no mixing).

The last two amplitudes in fig. F.1 involve the $Z\tilde{\nu}\tilde{\nu}$ vertex

$$L' = \frac{ig}{2 \cos \theta} \tilde{\nu}_L + \partial \tilde{\nu}_L Z^\mu , \quad (\text{F.2})$$

W

and the familiar Standard Model couplings to electrons.

It is now straightforward to write down the explicit formulae for the five amplitudes. They read

$$T_1 = \frac{ieg^2 \left[\bar{e}^+(p_1) \not{\epsilon} (\not{p}_3 - \not{p}_1) \not{p}_4 P_L e^-(p_2) \right]}{[(p_3 - p_1)^2 - m_e^2][p_2 - p_4]^2 - m_W^2} ,$$

$$\begin{aligned}
T_2 &= \frac{-ieg^2 \left[\bar{e}^+(p_1) \not{p}_5 (\not{p}_2 - \not{p}_3) \not{p}_L e^-(p_2) \right]}{[(p_5 - p_1)^2 - m_{\tilde{W}}^2] [(p_2 - p_3)^2 - m_e^2]}, \\
T_3 &= \frac{-ieg^2 \left[\bar{e}^+(p_1) P_R (\not{p}_5 + m_{\tilde{W}}) \not{p}_L (-\not{p}_4 + m_{\tilde{W}}) P_L e^-(p_2) \right]}{[(p_5 - p_1)^2 - m_{\tilde{W}}^2] [(p_2 - p_4)^2 - m_{\tilde{W}}^2]}, \\
T_4 &= \frac{ieg^2}{2 \cos^2 \theta_W} \frac{\left[\bar{e}^+(p_1) \not{p}_3 (\not{p}_3 - \not{p}_1) (\not{p}_4 - \not{p}_5) (C_e^L P_L + C_e^R P_R) e^-(p_2) \right]}{[(p_3 - p_1)^2 - m_e^2] [(p_4 + p_5)^2 - m_Z^2 + i\varepsilon]}, \\
T_5 &= \frac{ieg^2}{2 \cos^2 \theta_W} \frac{\left[\bar{e}^+(p_1) (\not{p}_4 - \not{p}_5) (C_e^L P_L + C_e^R P_R) (\not{p}_3 - \not{p}_2) \not{p}_L e^-(p_2) \right]}{[(p_2 - p_3)^2 - m_e^2] [(p_4 + p_5)^2 - m_Z^2 + i\varepsilon]}, \quad (F.3)
\end{aligned}$$

where $p_{1,2}$ are the four-momenta of the positron and electron; p_3 is the four-momentum of the photon; and $p_{4,5}$ are the four-momenta of the sneutrino and anti-sneutrino respectively;

$$P_{R,L} = \frac{1}{2} (1 \pm \gamma_5), \quad C_e^L = \sin^2 \theta_W - \frac{1}{2}, \quad C_e^R = \sin^2 \theta_W,$$

and ε^μ is the polarization vector of the photon, and $\varepsilon \equiv m_Z \Gamma_Z$.

The generalization to significant mixing between the $\tilde{W}^\pm$ and $\tilde{H}^\pm$ is straightforward, and the general result is given in the Electronic Yellow Book [3.5]. The contribution of the higher mass wino has been ignored here under the assumption that its mass is much higher than that of the wino.

For completeness, we also include here the lowest order cross-section for $e^+e^- \rightarrow \gamma \nu \bar{\nu}$ based on five diagrams:

$$\begin{aligned}
\frac{d\sigma}{dx d \cos \theta} &= \frac{G_F^2 \alpha}{6\pi^2} \left(\frac{m_Z \{ N_\nu (g_V^2 + g_A^2) + 2(g_V + g_A) [1 - s(1-x)/m_Z^2] \}}{[s(1-x) - m_Z^2]^2 + m_Z^2 \Gamma_Z^2} + 2 \right) \\
&\times \left[\frac{s(1-x)}{x \sin^2 \theta} \left[\left(1 - \frac{x}{2}\right)^2 + \frac{1}{4} x^2 \cos^2 \theta \right] \right], \quad (F.4)
\end{aligned}$$

where

$$g_V = -\frac{1}{2} + 2 \sin^2 \theta_W, \quad g_A = -\frac{1}{2}. \quad (F.5)$$

The higher-order radiative corrections on these processes tend to shift the single photon energy spectra toward lower values. As an example, we show in fig. F.2 the photon energy spectra for the reaction $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ using the lowest order calculations and using higher order radiative corrections at $\sqrt{s} = 100$ GeV, with $\mu_Z = 93$ GeV, $\Gamma_Z = 2.6$ GeV, $R_{\min} = 0.005$, $E_\gamma > 2$ GeV, $20^\circ < \theta_\gamma < 160^\circ$. We see that the peak has shifted by half a GeV mainly due to the emission of a second hard photon (dashed histogram).

Figure captions

Fig. F.1 The Feynman diagrams contributing to the process $e^+e^- \rightarrow \gamma\tilde{\nu}\tilde{\nu}$.

Fig. F.2 The photon energy spectra for the reaction $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ using the lowest and the higher order radiative corrections.

Fig. F.1

Fig. F.2

APPENDIX G

Matrix Elements for $e^+e^- \rightarrow e\tilde{e}\tilde{\gamma}$

This associative reaction is important mainly in the cases when the photino is light while the zino is heavy. Also the current limit on the zino mass from PETRA is much larger than that of the photino mass. Assuming that the zino mass is much larger than the photino mass, the contribution due to the exchange of a zino thus is negligible compared to that of a photino. We also ignore the contributions of the Z^0 in the t channel. Under these assumption, there are fourteen diagrams contributing to this process. Out of them, twelve do not involve the Z^0 and are shown in fig. G.1a, b. The other two with direct channel Z^0 poles are shown in fig. G.2. Let p_1 (k_1) be the four-momentum of the positron (electron) in the initial state; p_2 that of the positron in the final state; and k_3 and k_4 for the photinos and the selectron respectively. The matrix elements are as follows:

$$\begin{aligned}
 M^{(1)} &= e^2 g [\bar{v}(p_1) \gamma_\mu v(p_2)] (p_1 - p_2)^{-2} \left[\bar{u}(k_3) \frac{(1 + \gamma_5)}{2} \frac{k_4 + k_3 + m_e}{(k_4 + k_3)^2 - m_e^2} \gamma^\mu u(k_1) \right], \\
 M^{(2)} &= e^2 g [\bar{v}(p_1) \frac{k_4 - k_3 + k_1}{(k_1 - k_3)^2 - m_{\tilde{e}_R}^2} v(p_2)] (p_1 - p_2)^{-2} \left[\bar{u}(k_3) \frac{(1 + \gamma_5)}{2} u(k_1) \right], \\
 M^{(3)} &= -e^2 g [\bar{v}(p_1) \gamma_\mu u(k_1)] (p_1 + k_1)^{-2} \left[\bar{u}(k_3) \frac{(1 + \gamma_5)}{2} \frac{k_4 + k_3 + m_e}{(k_4 + k_3)^2 - m_e^2} \gamma^\mu v(p_2) \right], \\
 M^{(4)} &= -e^2 g [\bar{v}(p_1) \frac{k_4 - k_3 - p_2}{(p_2 + k_3)^2 - m_{\tilde{e}_R}^2} u(k_1)] (p_1 + k_1)^{-2} \left[\bar{u}(k_3) \frac{(1 + \gamma_5)}{2} v(p_2) \right], \\
 M^{(5)} &= g^3 \left[\bar{v}(p_1) \frac{(1 - \gamma_5)}{2} \frac{k_1 - k_4}{(k_1 - k_4)^2 - m_{\tilde{\gamma}}^2} u(k_1) \right] \left[(k_3 + p_2)^2 - m_{\tilde{e}_R}^2 \right]^{-1} \\
 &\quad \left[\bar{u}(k_3) \frac{(1 + \gamma_5)}{2} v(p_2) \right], \\
 M^{(6)} &= g^3 \left[\bar{v}(p_1) \frac{(1 - \gamma_5)}{2} \frac{p_2 + k_4}{(p_2 + k_4)^2 - m_{\tilde{\gamma}}^2} v(p_2) \right] \left[(k_3 - k_1)^2 - m_{\tilde{e}_R}^2 \right]^{-1} \\
 &\quad \left[\bar{u}(k_3) \frac{(1 + \gamma_5)}{2} u(k_1) \right], \\
 M^{(7)} &= g^3 \left[\bar{v}(p_1) \frac{(1 - \gamma_5)}{2} v(k_3) \right] \left[(p_1 - k_3)^2 - m_{\tilde{e}_R}^2 \right]^{-1} \\
 &\quad \left[\bar{u}(p_2) \frac{(1 + \gamma_5)}{2} \frac{m_{\tilde{\gamma}}}{(k_1 - k_4)^2 - m_{\tilde{\gamma}}^2} u(k_1) \right], \\
 M^{(8)} &= g^3 \left[\bar{v}(p_1) \frac{(1 - \gamma_5)}{2} v(k_3) \right] \left[(p_1 - k_3)^2 - m_{\tilde{e}_R}^2 \right]^{-1} \\
 &\quad \left[\bar{u}(p_2) \frac{(1 + \gamma_5)}{2} \frac{m_{\tilde{\gamma}}}{(p_2 + k_4)^2 - m_{\tilde{\gamma}}^2} u(k_1) \right], \\
 M^{(9)} &= g^3 \left[\bar{v}(p_1) \frac{(1 + \gamma_5)}{2} \frac{m_{\tilde{\gamma}}}{(k_1 - k_4)^2 - m_{\tilde{\gamma}}^2} u(k_1) \right] \left[(k_3 + p_2)^2 - m_{\tilde{e}_L}^2 \right]^{-1} \\
 &\quad \left[\bar{u}(k_3) \frac{(1 - \gamma_5)}{2} v(p_2) \right],
 \end{aligned}$$

$$\begin{aligned}
M^{(10)} &= -g^3 \left[\bar{v}(p_1) \frac{(1+\gamma_5)}{2} \frac{m_{\tilde{\gamma}}}{(p_2+k_4)^2 - m_{\tilde{\gamma}}^2} v(p_2) \right] \left[(k_3-k_1)^2 - m_{\tilde{e}_L}^2 \right]^{-1} \\
&\quad \left[\bar{u}(k_3) \frac{(1-\gamma_5)}{2} u(k_1) \right], \\
M^{(11)} &= g^3 \left[\bar{v}(p_1) \frac{(1+\gamma_5)}{2} v(k_3) \right] \left[(p_1-k_3)^2 - m_{\tilde{e}_L}^2 \right]^{-1} \\
&\quad \left[\bar{u}(p_2) \frac{(1-\gamma_5)}{2} \frac{k_1-k_4}{(k_1-k_4)^2 - m_{\tilde{\gamma}}^2} u(k_1) \right], \\
M^{(12)} &= g^3 \left[\bar{v}(p_1) \frac{(1+\gamma_5)}{2} v(k_3) \right] \left[(p_1-k_3)^2 - m_{\tilde{e}_L}^2 \right]^{-1} \\
&\quad \left[\bar{u}(p_2) \frac{(1+\gamma_5)}{2} \frac{k_2+k_4}{(p_2+k_4)^2 - m_{\tilde{\gamma}}^2} u(k_1) \right], \\
M^{(13)} &= \frac{-e^2 g}{(\sin\theta_W \cos\theta_W)^2} \left[\bar{v}(p_1) \gamma_\mu (C_L P_L + C_R P_R) u(k_1) \right] \left[(p_1+k_1)^2 - m_z^2 + im_z \Gamma_z \right]^{-1} \\
&\quad \left[\bar{u}(k_3) \frac{(k_3+k_4) \gamma_\mu (C_L P_L + C_R P_R)}{(k_3+k_4)^2 - m_e^2} v(p_2) \right], \\
M^{(14)} &= \frac{-e^2 g}{(\sin\theta_W \cos\theta_W)^2} \left[\bar{v}(p_1) (k_4-k_3-k_2) (C_L P_L + C_R P_R) u(k_1) \right] \\
&\quad \left[(p_1+k_1)^2 - m_z^2 + im_z \Gamma_z \right]^{-1} \left[\bar{u}(k_3) \frac{P_R}{(k_3+p_2)^2 - m_{\tilde{e}_R}^2} v(p_2) \right], \tag{G.1}
\end{aligned}$$

with

$$P_{R,L} = \frac{(1 \pm \gamma_5)}{2}, \quad C_L = \sin^2 \theta_W - \frac{1}{2}, \quad C_R = \sin^2 \theta_W,$$

where $q = p_1 - p_2$ and $g \equiv \sqrt{2}e$. The matrix elements of the four diagrams (9) to (12) are equal to (5) to (8) by just replacing $\tilde{e}_R$ by $\tilde{e}_L$ in the internal line. Matrix elements for producing $\tilde{e}_L$ can be obtained from eq. G.1 by replacing γ_5 by $-\gamma_5$.

Figure caption

- Fig. G.1** The fourteen diagrams contributing to the process $e^+e^- \rightarrow \tilde{e}e\tilde{\gamma}$
- a) without Z^0 exchange (1–12)
 - b) with Z^0 exchange (13–14).

Fig. G.1

APPENDIX H

Matrix Element for $e^+e^- \rightarrow e\tilde{W}\tilde{\nu}$ with no mixing

The twelve Feynman diagrams for this process are shown in fig. H.1, H.2. The corresponding amplitudes are listed below:

$$A_{1a} = \frac{ie^3}{\sin \theta_w} \frac{[\bar{e}^+(p_1)\gamma^\mu e^+(p_3)][\tilde{W}^-(p_4)P_L(\not{p}_4 + \not{p}_5 + m_e)\gamma_\mu e^-(p_2)]}{[(p_1 - p_3)^2][(p_4 + p_5)^2 - m_e^2]}$$

$$A_{1b} = \frac{ie^3}{\sin \theta_w} \frac{[\bar{e}^+(p_1)\gamma^\mu e^+(p_3)][\tilde{W}^-(p_4)\gamma_\mu(\not{p}_2 - \not{p}_5 + m_{\tilde{W}})P_L e^-(p_2)]}{[(p_1 - p_3)^2][(p_2 - p_5)^2 - m_{\tilde{W}}^2]}$$

$$A_{1c} = \frac{-ie^3}{\sin \theta_w} \frac{[\bar{e}^+(p_1)\gamma^\mu e^-(p_2)][\tilde{W}^-(p_4)P_L(\not{p}_4 + \not{p}_5 + m_e)\gamma_\mu e^+(p_3)]}{[(p_1 + p_2)^2][(p_4 + p_5)^2 - m_e^2]}$$

$$A_{1d} = \frac{-ie^3}{\sin \theta_w} \frac{[\bar{e}^+(p_1)\gamma^\mu e^-(p_2)][\tilde{W}^-(p_4)\gamma_\mu(-\not{p}_3 - \not{p}_5 + m_{\tilde{W}})P_L e^+(p_3)]}{[(p_1 + p_2)^2][(p_3 + p_5)^2 - m_{\tilde{W}}^2]}$$

$$A_{1e} = \frac{-ie^3}{\sin \theta_w} \frac{[\bar{e}^+(p_1)(-\not{p}_3 - \not{p}_5)P_L e^+(p_3)][\tilde{W}^-(p_4)P_L e^-(p_2)]}{[(p_2 - p_4)^2 - m_{\tilde{W}}^2][(p_3 + p_5)^2 - m_{\tilde{W}}^2]}$$

$$A_{1f} = \frac{ie^3}{\sin \theta_w} \frac{[\bar{e}^+(p_1)(\not{p}_2 - \not{p}_5)P_L e^-(p_2)][\tilde{W}^-(p_4)P_L e^+(p_3)]}{[(p_2 - p_5)^2 - m_{\tilde{W}}^2][(p_3 + p_4)^2 - m_{\tilde{W}}^2]}$$

$$A_{2a} = ie^2 g\alpha \frac{[\bar{e}^+(p_1)\gamma^\nu(\alpha P_L + \beta P_R)e^+(p_3)][g_{\mu\nu} - k_\mu k_\nu / m_Z^2][\tilde{W}^-(p_4)(\not{p}_4 + \not{p}_5)\gamma^\mu P_L e^-(p_3)]}{[(p_4 + p_5)^2][k^2 - m_V^2 + im_Z\Gamma_Z]}$$

$$A_{2b} = -ie^2 g \cot \theta_w$$

$$\frac{[\bar{e}^+(p_1)\gamma^\nu(\alpha P_L + \beta P_R)e^+(p_3)][g_{\mu\nu} - k_\mu k_\nu / m_Z^2][\tilde{W}^-(p_4)\gamma^\mu(\not{p}_2 - \not{p}_5 + m_{\tilde{W}})P_L e^-(p_2)]}{[(p_2 - p_5)^2 - m_{\tilde{W}}^2][k^2 - m_V^2 + im_Z\Gamma_Z]}$$

$$A_{2c} = -ie^2 g\alpha \frac{[\bar{e}^+(p_1)\gamma^\nu(\alpha P_L + \beta P_R)e^-(p_2)][g_{\mu\nu} + Q_\mu Q_\nu / m_Z^2][\tilde{W}^-(p_4)(\not{p}_4 + \not{p}_5)\gamma^\mu P_L e^+(p_3)]}{[(p_4 + p_5)^2][Q^2 - m_V^2 + im_Z\Gamma_Z]}$$

$$A_{2d} = ie^2 g \cot \theta_w$$

$$\frac{[\bar{e}^+(p_1)\gamma^\nu(\alpha P_L + \beta P_R)e^-(p_2)][g_{\mu\nu} - Q_\mu Q_\nu / m_Z^2][\tilde{W}^-(p_4)\gamma^\mu(-\not{p}_3 - \not{p}_5 + m_{\tilde{W}})P_L e^+(p_2)]}{[(p_3 + p_5)^2 - m_{\tilde{W}}^2][Q^2 - m_V^2 + im_Z\Gamma_Z]}$$

$$A_{2e} = +ie^2 g\Omega$$

$$\frac{[\bar{e}^+(p_1)\gamma^\nu(\alpha P_L + \beta P_R)e^+(p_3)][g_{\mu\nu} - k_\mu k_\nu/m_Z^2][\tilde{W}^-(p_4)(p_5 + p_2 - p_4)^\mu P_L e^-(p_2)]}{[(p_2 - p_4)^2 - m_V^2][k^2 - m_Z^2 + im_Z\Gamma_Z]}$$

$$A_{2f} = -ie^2 g\Omega$$

$$\frac{[\bar{e}^+(p_1)\gamma^\nu(\alpha P_L + \beta P_R)e^-(p_2)][g_{\mu\nu} - Q_\mu Q_\nu/m_Z^2][\tilde{W}^-(p_4)(p_5 - p_3 - p_4)^\mu P_L e^+(p_3)]}{[(p_3 + p_4)^2 - m_V^2][Q^2 - m_Z^2 + im_Z\Gamma_Z]}$$

where $P_{1,2}$ are the four-momenta of the positron and the electron and p_3 is that of the final state positron; p_4 is that of the wino, and p_5 is that of the sneutrino, and

$$k \equiv p_1 - p_3 ; \quad Q \equiv p_1 + p_2 ; \quad P_{R,L} \equiv \frac{1}{2} (1 \pm \gamma_5)$$

$$\alpha \equiv \frac{2 \sin^2 \theta_w - 1}{2 \sin \theta_w \cos \theta_w} ; \quad \beta \equiv \tan \theta_w ; \quad \Omega \equiv \frac{1}{2 \sin \theta_w \cos \theta_w} ;$$

$$g \equiv e/\sin \theta_w .$$

In order to compute the $e^+e^- \rightarrow e^+\tilde{W}^-\tilde{\nu}$ cross-section we use subsets of the 12 terms which contribute to the total cross-section for different energy regions. One subset is appropriate for energies much below the Z^0 pole and the other one can be used for energies near or beyond the Z^0 . We show here the output of the REDUCE computation of these subsets (fig. H.1, H.2), which was used in the calculation [3.11, 3.13]. Denoting by S_1 , S_2 , and S_3 the signs +1, 0, and -1, respectively, we define

$$P_{ij} \equiv P_i \cdot P_j$$

$$(P_i S_1 S_2 S_3) \equiv P_{ii} (S_1 P_{3i} P_{12} + S_2 P_{2i} P_{13} + S_3 P_{1i} P_{23})$$

$$(S_1 S_2 S_3) \equiv P_{15} (S_1 P_{45} P_{23} + S_2 P_{36} P_{24} + S_3 P_{34} P_{25}) \quad (H.2)$$

to obtain the matrix elements for the two following cases.

Low-energy region (fig. H.1a-f)

$$\begin{aligned}
A(1a, 1a) &= 16C_1 \{ m_W^2 [P_{12}(2P_{35} + P_{14}) + P_{23}(2P_{15} + P_{34})] - m_P^2 (P_{34}P_{12} + P_{23}P_{14}) \\
&\quad + 2P_{45}(P_{35}P_{12} + P_{23}P_{15}) \} \\
A(1b, 1b) &= 16C_1 \{ m_W^2 (P_{34}P_{12} + 2P_{25}P_{13} + P_{23}P_{14}) - m_P^2 (P_{34}P_{12} + P_{23}P_{14}) \\
&\quad + 2P_{25}P_{14} + P_{34}P_{15} \} \\
A(1c, 1c) &= A(1a, 1a; P_2 \leftrightarrow -P_3) \\
A(1d, 1d) &= A(1b, 1b; P_2 \leftrightarrow -P_3) \\
A(1e, 1e) &= -4C_2 \{ m_P^2 P_{24}P_{13} - 2P_{35}P_{24}P_{15} \} \\
A(1f, 1f) &= A(1e, 1e; P_2 \leftrightarrow -P_3) \\
A(1a, 1b) &= 32C_1 \{ m_W^2 [P_{12}(P_{35} + P_{34}) + P_{23}(P_{15} + P_{14}) + P_{23}P_{13}] + (P_5 - + -) \\
&\quad + P_{45}(P_{35}P_{12} + P_{23}P_{15}) + P_{35}(P_{25}P_{14} - 2P_{24}P_{15}) + P_{34}P_{25}P_{15} \} \\
A(1a, 1c) &= 32C_1 \{ m_W^2 P_{23}(-2P_{15} - P_{14}) + m_P^2 P_{23}P_{14} - 2P_{45}P_{23}P_{15} \} \\
A(1a, 1d) &= 16C_1 \{ m_W^2 [-P_{35}P_{12} + P_{25}P_{13} - P_{23}(3P_{15} + 2P_{14})] + (P_5 - + +) + 2(- - +) \} \\
A(1a, 1e) &= 8C_3 \{ (P_4 - + -) + (P_5 - + +) + 2(- - +) \} \\
A(1a, 1f) &= 8C_3 \{ (P_4 - + +) + (P_5 - + -) + 2(- - +) \} \\
A(1b, 1c) &= 16C_1 \{ m_W^2 [P_{35}P_{12} - P_{25}P_{13} - P_{23}(3P_{15} + 2P_{14})] + (P_5 + - +) + 2(- + -) \} \\
A(1b, 1d) &= 16C_1 \{ -2m_W^2 P_{23}(P_{14} + P_{15}) + (P_5 - - +) + 2(- + +) \} \\
A(1b, 1e) &= 8C_3 \{ (P_4 - + -) + (P_5 - - +) + 2(- + +) \} \\
A(1b, 1f) &= 8C_3 \{ (P_4 - + +) - 2m_P^2 P_{34}P_{21} + 4(00+) \} \\
A(1c, 1d) &= 32C_1 \{ m_W^2 [P_{35}P_{12} + P_{13}(P_{25} + P_{24}) + P_{23}(P_{15} + P_{14})] + (P_5 + - -) \\
&\quad + P_{45}(P_{25}P_{13} + P_{23}P_{15}) + P_{35}(P_{25}P_{14} + P_{24}P_{15}) - 2P_{34}P_{25}P_{15} \} \\
A(1c, 1e) &= 8C_3 \{ (P_4 + - +) + (P_5 + - -) + 2(+ + -) \} \\
A(1c, 1f) &= 8C_3 \{ (P_4 + - -) + (P_5 + - +) + 2(- + -) \} \\
A(1d, 1e) &= 8C_3 \{ (P_4 + - +) - 2m_P^2 P_{24}P_{13} + 4(0 + 0) \} \\
A(1d, 1f) &= 8C_3 \{ (P_4 + - -) + (P_5 - - +) + 2(- + +) \} \\
A(1e, 1f) &= 4C_2 \{ (P_5 - - +) + 2(- + +) \} ,
\end{aligned} \tag{H.3}$$

where we have used the following definitions:

$$C_1 \equiv K_1^2, \quad C_2 \equiv K_2^2, \quad C_3 \equiv K_1 K_2, \tag{H.4}$$

with

$$K_1 \equiv (e)^{3/2} / \sin \theta_w ,$$

$$K_2 \equiv K_1 / \sin^2 \theta_w . \quad (H.5)$$

Notice that propagators are defined including the global sign in front of the 'i' of each amplitude: this means that they include the global sign of the diagram.

LEP energies (fig. H.2a-f)

Defining now,

$$(LS_1 S_2 S_3 S_4) \equiv m_W^2 (S_1 p_{35} p_{12} + S_2 p_{25} p_{13} + S_3 p_{23} p_{15} + S_4 p_{23} p_{14}) ,$$

$$(RS_1 S_2 S_3 S_4) \equiv m_W^2 (S_1 p_{35} p_{12} + S_2 p_{25} p_{13} + S_3 p_{24} p_{13} + S_4 p_{23} p_{15}) ,$$

$$(LS_1 S_2 S_3) \equiv S_1 p_{45} p_{23} p_{15} + S_2 p_{35} p_{24} p_{15} + S_3 p_{34} p_{25} p_{15} ,$$

$$(RS_1 S_2 S_3) \equiv S_1 p_{45} p_{25} p_{13} + S_2 p_{35} p_{25} p_{14} + S_3 p_{34} p_{25} p_{15} , \quad (H.6)$$

for $S_k = -, 0, +, 2, 3$ ($k = 1, 2, 3, 4$) we have

$$A(2c, 2c) = -16C_{2c}^2 \{ \alpha^2 [(p_5^{00+}) - (L002+) + 2(L-00)] + \beta^2 [(p_5^{0+0}) - (R02+0) + 2(R-00)] \} ,$$

$$A(2d, 2d) = -16C_{2d}^2 \{ \alpha^2 [(p_5^{0+0}) + (L-+-) + 2(L0-0)] + \beta^2 [(p_5^{00+}) + (R---) + 2(R0-0)] \} ,$$

$$A(2f, 2f) = -16C_{2f}^2 \{ (\alpha^2 + \beta^2) p_{34} (m_V^2 p_{12} - 2p_{25} p_{15}) \} ,$$

$$A(2c, 2d) = 16C_{2c} C_{2d} \{ \alpha^2 [(p_5^{+-}) + (L+-32) + 2(L+-)] + \beta^2 [(p_5^{+-}) + (R+32-) + 2(R+-)] \} ,$$

$$A(2d, 2f) = -16C_{2d}C_{2f} \{ \alpha^2 [(p_5^{++}) + (L^{+-}0) + 2(L^{+-})] + \beta^2 [(p_5^{+-}) + (R^{+}0-) + 2(R^{+-})] \} ,$$

$$A(2c, 2f) = 16C_{2c}C_{2f} \{ \alpha^2 [(p_5^{+-}) + (L^{++}0) + 2(L^{++})] + \beta^2 [(p_5^{++}) + (R^{+}0+) + 2(R^{++})] \} ,$$

$$A(1a, 2c) = 32\alpha C_{2c}K_1 \{ R_Z p_{23} [m_V^2 p_{14} + m_W^2 (-p_{14} - 2p_{15}) - 2p_{45} p_{15}] \} / P_Z ,$$

$$A(1a, 2d) = -16\alpha C_{2d}K_1 \{ G_Z + R_Z [(p_5^{+-}) + (L^{+}32) + 2(L^{+-})] \} / P_Z ,$$

$$A(1a, 2f) = 16\alpha C_{2f}K_1 \{ G_Z + R_Z [(p_5^{++}) + (L^{+-}0) + 2(L^{+-})] \} / P_Z ,$$

$$A(1b, 2c) = -16\alpha C_{2c}K_1 \{ G_Z + R_Z [(p_5^{+-}) + (L^{+}32) + 2(L^{+-})] \} / P_Z ,$$

$$A(1b, 2d) = -16\alpha C_{2d}K_1 \{ G_Z + R_Z [(p_5^{+-}) + (L^{00}22) + 2(L^{+-})] \} / P_Z ,$$

$$A(1b, 2f) = 16\alpha C_{2f}K_1 \{ \hat{G}_Z + R_Z [(p_5^{00}) + (L^{+-}0) + 4(L^{00})] \} / P_Z ,$$

$$A(1e, 2c) = -8\alpha C_{2c}K_2 \{ G_Z + R_Z [(p_5^{++}) + (L^{+-}0) + 2(L^{+-})] \} / P_Z ,$$

$$A(1e, 2d) = -8\alpha C_{2d}K_2 \{ \hat{G}_Z + R_Z [(p_5^{00}) + (L^{+-}0) + 4(L^{00})] \} / P_Z ,$$

$$A(1e, 2f) = -8\alpha C_{2f}K_2 \{ G_Z - \hat{G}_Z + R_Z [(p_5^{--}) + 2(L^{+-})] \} / P_Z , \quad (H.7)$$

where

$$C_{2c} \equiv \alpha K_1 , \quad C_{2d} \equiv K_1 / \beta , \quad C_{2f} \equiv K_1 / \sin 2\theta_W ,$$

$$R_Z \equiv (p_1 + p_2)^2 - m_Z^2 , \quad P_Z \equiv (R_Z^2 + I_Z^2)^{1/2} , \quad I_Z \equiv \Gamma_Z m_Z ,$$

$$\hat{G}_Z \equiv -m_W^2 G_e , \quad G_Z \equiv -(m_W^2 - m_V^2 + 2p_{15}) G_e .$$

$$G_e \equiv \gamma_Z \varepsilon_{\mu\nu\sigma} p_1^\mu p_2^\nu p_3^\sigma p_4^\sigma , \quad (H.8)$$

Here the propagators are again defined including the global sign in front of the 'i' of each amplitude. In the Z^0 propagators, only the modulus must be included, because the phase has already been included in the trace calculation.

Figure captions

Fig. H.1 The six Feynman diagrams for the process $e^+e^- \rightarrow \widetilde{W}e\bar{\nu}$ at low energy.

Fig. H.2 The six Feynman diagrams for the process $e^+e^- \rightarrow \widetilde{W}e\bar{\nu}$ at LEP energies.

Fig. H.1

Fig. H.2

APPENDIX I

Matrix Elements for $e^+e^- \rightarrow e^+\tilde{W}\tilde{v}$ in the case where $v' \neq v$

For the general case with mixing, most of the interactions remain the same as in the case with no mixing described in Appendix H [3.13]. The only ones that are different are as follows:

i) The vertex $Z\tilde{W}\tilde{W}$ is now

$$L_{Z\tilde{W}\tilde{W}} = -e \cot \theta_w \tilde{W}^\mu \gamma^\mu (1 - \frac{1}{2} \sec^2 \theta_w V_C - \gamma_5 \frac{1}{2} \sec^2 \theta_w A_C) \tilde{W} Z_\mu, \quad (I.1)$$

where

$$V_C = (\cos^2 \gamma_L + \cos^2 \gamma_R)/2,$$

and

$$A_C = (\cos^2 \gamma_R - \cos^2 \gamma_L)/2. \quad (I.2)$$

ii) The vertex $\tilde{l}\tilde{l}\tilde{W}$ is now

$$L_{\tilde{l}\tilde{l}\tilde{W}} = g \sin \gamma_R \tilde{v}^+ \tilde{W} \frac{1 - \gamma_5}{2} e - g \sin \gamma_R \tilde{e}^+ \tilde{W}^C \frac{1 - \gamma_5}{2} v, \quad (I.3)$$

where $\sin \gamma_R = f_+$. These couplings can be expressed in terms of the parameters m_1 and μ_2 (with $m_1 > 0$ and $\mu_2 < 0$), the mixing angle α ($\tan \alpha = v/v'$), and $\tan \gamma_L$ and $\tan \gamma_R$.

We can express these parameters in terms of the physical masses of the $\tilde{\gamma}$, $\tilde{v}$, $\tilde{e}$ and $\tilde{W}$ as follows:

$$\mu_2 = - \frac{3}{8 \sin^2 \theta_w} m_{\tilde{\gamma}}^2, \quad (I.4)$$

$$m_0^2 = m_{\tilde{V}}^2 + \frac{1}{2} \frac{(m_{\tilde{e}}^2 - m_{\tilde{V}}^2)}{\left(\sin^2 \theta_w + \frac{1}{2}\right)}, \quad (I.5)$$

$$r = \frac{m_{\tilde{e}}^2 - m_{\tilde{V}}^2}{m_{\tilde{Z}}^2 \left(\sin^2 \theta_w + \frac{1}{2}\right)}, \quad (I.6)$$

and

$$m_1 = \frac{\mu_2^2 m_W^2 \sin 2\alpha + m_{\tilde{W}}^2 \Delta^{1/2}}{2(\mu_2^2 - m_{\tilde{W}}^2)}, \quad (I.7)$$

with

$$\Delta = (m_W^2 - m_{\tilde{W}}^2)^2 + 2\mu_2^2 (m_W^2 - m_{\tilde{W}}^2) \mu_2^4 - m_W^4 \cos^2 2\alpha. \quad (I.8)$$

With given sets of the $\tilde{\gamma}$, $\tilde{e}$, $\tilde{\nu}$, and $\tilde{W}$ masses, we have calculated the parameters μ_2 , m_0 , γ , α , and m_1 and the vertex factors $\sin \gamma_R$, V_C and A_C , which are given in table 3.1. We see that in most cases $\sin \gamma_R \approx 1$, $V_C \approx A_C \approx 0$, and the interactions reduce to the previous special case described in appendix H. For the other cases, the cross-sections can be very different. We have calculated them for the cases shown in table 3.2. Also shown for comparison are the cross-sections that would have been obtained using the results of appendix H, although in these cases the full mass relations of broken supersymmetry have not been used.