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BRST INVARIANCE AND GAUGE INDEPENDENCE OF STRING PATH-INTEGRALS

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A B S T R A C T

The covariant quantization of relativistic strings is performed in the BRST formulation of path-integrals. BRST-invariant measures are constructed in the critical number of dimensions and the gauge independence of the resulting path-integrals is proved.

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1. - INTRODUCTION

The quantum theory of massless relativistic strings can be cast into the form of a conformally-invariant relativistic field theory on two-dimensional Riemann surfaces¹⁾⁻³⁾. Bosonic fields X^M ($M=1, \dots, D$), scalars on the world surface, describe the embedding of the world surface in external space-time: fermionic fields λ^i , spinors on the world surface, may be introduced to describe compactified dimensions of the string⁴⁾⁻⁶⁾. The geometry of the Riemann surface is described by the zweibein field e^a_μ ⁷⁾⁻⁹⁾; in this formulation, the reparametrization invariance of the string dynamics is manifest. In the special case in which the numbers of bosonic and fermionic degrees of freedom of the string are equal, the reparametrization invariance on the world-sheet can be extended to a local supersymmetry by including a two-dimensional gravitino field ψ_μ ⁷⁾⁻⁹⁾. The resulting theory is actually invariant under the larger set of local superconformal transformations^{1),9)-11)}.

The large number of gauge symmetries of string theories allow one to remove the dependence of string dynamics on local fluctuations in the geometry of the (super) world surface completely; there remain a finite number of parameters (moduli), the number of which depends on the genus of the world surface^{2),3),12)-14)}. It is this feature which allows the construction of consistent string theories and singles out strings among theories of extended objects: for higher-dimensional extended objects, like membranes, the geometrical degrees of freedom connected with the world volume become dynamical. The corresponding quantum theories are ill-defined, quantum gravity not being renormalizable in three or more dimensions^{*)}. As is well known, the complete decoupling of world-surface geometry from string dynamics does not in general carry over to quantum string theories, because of the possible presence of conformal anomalies^{1),16)}. The requirement of the absence of such an anomaly leads to the introduction of the concept of a critical dimensionality for the space-time in which the string moves¹⁷⁾.

A convenient and covariant description of quantum strings is obtained by choosing a specific gauge for the zweibein field, leaving essentially only the string variables (X^M, λ^i) as dynamical degrees of freedom. However, in doing so

*) A possible exception is the theory of supermembranes in eleven dimensions, due to extra local symmetries of $d = 11$ supergravity¹⁵⁾.

one must show that the physical content of the theory is independent of the particular choice of gauge. Most effort has been directed to showing the invariance of string path-integrals under conformal changes of gauge^{16),2),3),17)}. Although this is an important step in proving the consistency of quantum string theories, by investigating the absence of conformal anomalies, an equally important requirement is the invariance of the path-integral under traceless deformations of the metric (reparametrizations^{1),3),18)-20)}). This requirement is especially relevant, because reparametrization invariance is necessary for the decoupling of indefinite metric states, associated with longitudinal and time-like polarizations of the string, from the physical Hilbert space^{17),21),22)}.

Using canonical quantization methods, the requirement of nilpotency of the BRST charge has been shown to lead to the correct values for the intercept and central charge in the Virasoro algebra^{21),23)}. Hence one expects that gauge independence of the path integral is a direct consequence of BRST invariance. However, previous work on this subject suggested^{16),24),25)} that the BRST transformations for the co-ordinate ghosts had to be modified in order to obtain the correct critical dimensions of string theories, effectively treating these ghosts as a linearly transforming (anticommuting) vector field^{16),26)-28)}. This is in apparent disparity with the results from canonical quantization, in which the standard non-linear transformation rules for the ghosts were used²⁹⁾.

In a recent paper, two of us showed³⁰⁾ that full BRST invariance and independence of the conformal gauge parameter of the path integral for the bosonic string follow indeed from standard, non-linear BRST transformations, without the modifications of Refs. 16), 24) and 25). In this paper these results are generalized in two ways. First, we present the proof of the connection of gauge independence and BRST invariance in the context of string path-integrals, including non-conformal gauges. Second, we extend the proof of BRST invariance and the existence of BRST-invariant measures to other string models, including supersymmetric ones, in the appropriate critical number of dimensions. As in Ref. 30), we find that more general BRST-invariant measures exist than the one proposed in 16) and 24).

Other approaches to the construction of string path-integrals exist, using powerful geometrical and complex analytical methods^{2),3),10)-14),31)}. Our treatment is closer to the usual covariant quantization procedures for Yang-Mills theories [see for example Ref. 32)], and supergravity³³⁾, taking BRST symmetry as

the starting point for the construction of gauge invariant quantum theories. Indeed, BRST invariance is a useful tool in quantum field theory and may become so in related areas of mathematics, such as the theory of determinant bundles.

In our construction of BRST-invariant functional measures, we use the Fujikawa regularization procedure for the computation of determinants. In Appendix A, we explain the relation of this procedure with well-known heat-kernel methods [34), and references therein]. Appendix B contains some remarks on our notation and its translation to that of Ref. 35).

2. - BRST INVARIANCE AND GAUGE INDEPENDENCE

In this section we present a compact and general proof that, within any given class of allowed gauges, BRST invariance implies gauge independence of path-integrals. Although our proof applies in principle to any field theory with local symmetries, such as Yang-Mills theory or gravity, it is cast here in the language of string models in covariant gauges for unity of presentation.

We first consider strings with ordinary local Poincaré $\times$ Weyl invariance on the world-sheet. Afterwards we generalize to the case of supersymmetric strings. The string is described by co-ordinates X^M ($M=1, \dots, D$), and possibly an additional set of 2-d spinor degrees of freedom λ^i ($i=1, \dots, N$)^{*}). As explained in Section 1, the zweibein e_μ^a allows the theory to be written in a manifestly co-ordinate invariant form, starting with a classical action

$$S_d = \int d^2z \, e \, \mathcal{L}(X^M, \lambda^i, e_a^\mu). \quad (2.1)$$

Here $\mathcal{L}$ is a scalar function of the string variables, e is the determinant of the zweibein and z and $\bar{z}$ are the complex-conjugate world-sheet co-ordinates, with the world-sheet a compact Riemann surface with some fixed topology. We take $S_{d\mathcal{L}}$ to be invariant under local Poincaré $\times$ Weyl transformations, acting on the string co-ordinates as

*) We take the λ^i to be two-component Majorana spinors.

$$\begin{aligned}\delta X^M &= \xi^\lambda \partial_\lambda X^M \\ \delta \lambda_\pm^i &= \xi^\lambda \partial_\lambda \lambda_\pm^i + \frac{1}{2} (\xi_D \mp \xi_M) \lambda_\pm^i.\end{aligned}\tag{2.2}$$

Here ξ^λ , ξ_D and ξ_M are the parameters of infinitesimal general co-ordinate, Weyl- and Lorentz-transformations respectively, whilst $\lambda_\pm^i$ are the upper (lower) components of λ^i . We have chosen the inverse zweibein e_a^μ as the independent variable in $\mathcal{L}$, Eq. (2.1), rather than e_μ^a , for later convenience. A convenient representation of the inverse zweibein is in terms of two orthogonal world vectors

$$e_a^\mu = (e^\mu, \bar{e}^\mu),\tag{2.3}$$

with inverse:

$$e_\mu = -\varepsilon_{\mu\nu} e \bar{e}^\nu, \quad \bar{e}_\mu = \varepsilon_{\mu\nu} e e^\nu,\tag{2.4}$$

where

$$e^{-1} = \det e_a^\mu = \varepsilon_{\mu\nu} \bar{e}^\mu e^\nu; \quad \varepsilon_{zz} = -\varepsilon_{\bar{z}\bar{z}} = 1.\tag{2.5}$$

Then

$$\begin{aligned}e_\mu e^\mu &= 1 = \bar{e}_\mu \bar{e}^\mu, & e_\mu \bar{e}^\mu &= 0 = \bar{e}_\mu e^\mu, \\ e_\mu e^\nu + \bar{e}_\mu \bar{e}^\nu &= \delta_\mu^\nu.\end{aligned}\tag{2.6}$$

For further details, see Appendix B.

Under the infinitesimal transformations (2.2), these fields transform as

$$\begin{aligned}\delta e^\mu &= \xi^\nu \partial_\nu e^\mu - e^\nu \partial_\nu \xi^\mu + (\xi_D + \xi_M) e^\mu, \\ \delta \bar{e}^\mu &= \xi^\nu \partial_\nu \bar{e}^\mu - \bar{e}^\nu \partial_\nu \xi^\mu + (\xi_D - \xi_M) \bar{e}^\mu.\end{aligned}\tag{2.7}$$

Then e transforms as a density and S_{cl} is invariant under (2.2) and (2.7) modulo boundary terms, which vanish if we impose appropriate boundary conditions on the string. The transformations (2.7) allow for an almost complete elimination of the

components $(e^\mu, \bar{e}^\mu)$, leaving only a finite number of parameters (the moduli); thus we may choose

$$e^\mu = h^\mu(z, \bar{z}; m_i), \quad \bar{e}^\mu = \bar{h}^\mu(z, \bar{z}; m_i), \quad (2.8)$$

with $(h^\mu, \bar{h}^\mu)$ arbitrary but fixed functions of $(z, \bar{z})$ for each value of the moduli $(m_i)^{(2), (3), (12)-(14)}$. In the path-integral this gauge choice is implemented by restricting functional integration over $(e^\mu, \bar{e}^\mu)$ to the hypersurface (2.8). The full BRST-invariant path-integral is

$$Z[h, \bar{h}] = \int d\mu e^{-S}, \quad (2.9)$$

where

$$S = S_{cl} + S_{FP},$$

and

$$S_{FP} = \int d^2z \left\{ b_\mu (e^\mu - h^\mu) + \bar{b}_\mu (\bar{e}^\mu - \bar{h}^\mu) - C_\mu^* (C^\nu \partial_\nu e^\mu - e^\nu \partial_\nu C^\mu + (C_D + C_M) e^\mu) - \bar{C}_\mu^* (C^\nu \partial_\nu \bar{e}^\mu - \bar{e}^\nu \partial_\nu C^\mu + (C_D - C_M) \bar{e}^\mu) \right\}. \quad (2.10)$$

In this expression $(b_\mu, \bar{b}_\mu)$ are a set of Lagrange multipliers imposing the constraints (2.8), and $(C^\mu, C_D, C_M; C_\mu^*, \bar{C}_\mu^*)$ are the Faddeev-Popov ghosts and anti-ghosts corresponding to the local symmetries (2.7) and (2.2). The action S has been constructed in such a way as to imply the constraints (2.8) and be invariant under the anti-commuting BRST variations

$$\begin{aligned} \delta_\Lambda X^M &= C^\nu \partial_\nu X^M, \\ \delta_\Lambda \lambda_\pm^i &= C^\nu \partial_\nu \lambda_\pm^i + \frac{1}{2} (C_D \mp C_M) \lambda_\pm^i, \\ \delta_\Lambda e^\mu &= C^\nu \partial_\nu e^\mu - e^\nu \partial_\nu C^\mu + (C_D + C_M) e^\mu, \\ \delta_\Lambda \bar{e}^\mu &= C^\nu \partial_\nu \bar{e}^\mu - \bar{e}^\nu \partial_\nu C^\mu + (C_D - C_M) \bar{e}^\mu. \end{aligned}$$

$$\begin{aligned}
\delta_\Lambda C^\mu &= C^\nu \partial_\nu C^\mu, \\
\delta_\Lambda C_D &= C^\nu \partial_\nu C_D, \\
\delta_\Lambda C_M &= C^\nu \partial_\nu C_M, \\
\delta_\Lambda C_\mu^* &= b_\mu, & \delta_\Lambda \bar{C}_\mu^* &= \bar{b}_\mu, \\
\delta_\Lambda b_\mu &= 0, & \delta_\Lambda \bar{b}_\mu &= 0.
\end{aligned}
\tag{2.11}$$

These transformations are nilpotent counterparts of (2.2) and (2.7):

$$\delta_\Lambda^2 = 0. \tag{2.12}$$

The measure $d\mu$ in Eq. (2.9) has not yet been specified. It must be chosen so as to be BRST invariant as well:

$$\left\| \frac{\partial \mu'}{\partial \mu} \right\|_{\text{BRST}} = 1, \tag{2.13}$$

where the generalized Jacobian on the left-hand side symbolizes the transformation factor of $d\mu$. We can now prove our assertion about gauge independence of $Z[h, \bar{h}]$:

$$\frac{\delta Z}{\delta h^\mu} = 0, \quad \frac{\delta Z}{\delta \bar{h}^\mu} = 0. \tag{2.14}$$

First, observe that

$$\begin{aligned}
\frac{\delta Z}{\delta h^\mu} &= \langle b_\mu \rangle = \langle \delta_\Lambda C_\mu^* \rangle, \\
\frac{\delta Z}{\delta \bar{h}^\mu} &= \langle \bar{b}_\mu \rangle = \langle \delta_\Lambda \bar{C}_\mu^* \rangle,
\end{aligned}
\tag{2.15}$$

where the brackets denote functional averages:

$$\langle \Theta \rangle = \int d\mu \, \Theta e^{-S}. \tag{2.16}$$

We prove the vanishing of the right-hand side of Eqs. (2.15). Introduce arbitrary sources $(\eta^\mu, \bar{\eta}^\mu)$ for the anti-ghosts:

$$Z[\eta, \bar{\eta}; h, \bar{h}] = \int d\mu e^{-(S + \int d^2z [C_\mu^* \eta^\mu + \bar{C}_\mu^* \bar{\eta}^\mu])} \quad (2.17)$$

Next, perform a BRST transformation on (2.17); since both $d\mu$ and S are invariant, only the source terms contribute to the variation of the path-integral. However, the transformation only redefines (dummy) integration variables; thus the variation must vanish:

$$\delta Z = \int d^2z \langle (\delta_\lambda C_\mu^*) \eta^\mu + (\delta_\lambda \bar{C}_\mu^*) \bar{\eta}^\mu \rangle = 0. \quad (2.18)$$

Because $(\eta^\mu, \bar{\eta}^\mu)$ are arbitrary, we immediately obtain

$$\langle \delta_\lambda C_\mu^* \rangle = 0, \quad \langle \delta_\lambda \bar{C}_\mu^* \rangle = 0. \quad (2.19)$$

This completes the proof of (2.14).

We see that the definition of a path-integral by a BRST-invariant action as in (2.10) in combination with a BRST-invariant measure automatically guarantees the gauge independence of the functional integral. This would also hold, had we added a quadratic term in the fields $(b_\mu, \bar{b}_\mu)$; thus it is not peculiar to Landau-type gauges.

It is now obvious how to generalize the proof to the case of the supersymmetric string. We just have to introduce extra gauge choices with corresponding Lagrange multipliers for the local supersymmetries. These symmetries have their own ghosts and antighosts which in this case commute. The proof of gauge independence then follows the steps (2.15)-(2.19) exactly, provided the BRST invariance is extended to include the supersymmetry ghosts and Lagrange multipliers. We will not spell out the details here, but only present a few relevant formulae.

The supersymmetric string is a two-dimensional field theory with superconformal invariance. The gauge fields of local superconformal transformations are the zweibein $(e^\mu, \bar{e}^\mu)$, as before, and the gravitino $\psi_\mu^\pm$, with the $(\pm)$ signs again denoting the chirality of the spinors. The local superconformal transformations consist of local Weyl- and Lorentz transformations, general co-ordinate transformations and local Q- and S-supersymmetry transformations³⁵⁾⁻³⁷⁾ with parameters $(\xi_D, \xi_M, \xi^\mu; \epsilon^\pm, \eta^\pm)$:

$$\begin{aligned}
\delta e^\mu &= \xi^\nu \partial_\nu e^\mu - \partial_\nu \xi^\mu e^\nu + (\xi_D + \xi_M) e^\mu - 2i e^\mu e^\nu \varepsilon^+ \psi_\nu^+ \\
&\quad + 2i \bar{e}^\mu e^\nu \varepsilon^- \bar{\psi}_\nu, \\
\delta \bar{e}^\mu &= \xi^\nu \partial_\nu \bar{e}^\mu - \partial_\nu \xi^\mu \bar{e}^\nu + (\xi_D - \xi_M) \bar{e}^\mu - 2i e^\mu \bar{e}^\nu \varepsilon^+ \psi_\nu^+ \\
&\quad + 2i \bar{e}^\mu e^\nu \varepsilon^- \bar{\psi}_\nu, \\
\delta \psi_\mu^+ &= \xi^\nu \partial_\nu \psi_\mu^+ + \partial_\mu \xi^\nu \psi_\nu^+ + D_\mu \varepsilon^+ - e_\mu \eta^- - \frac{1}{2} (\xi_D + \xi_M) \psi_\mu^+, \\
\delta \bar{\psi}_\mu^- &= \xi^\nu \partial_\nu \bar{\psi}_\mu^- + \partial_\mu \xi^\nu \bar{\psi}_\nu^- + D_\mu \varepsilon^- - \bar{e}_\mu \eta^+ - \frac{1}{2} (\xi_D - \xi_M) \bar{\psi}_\mu^-, \tag{2.20}
\end{aligned}$$

where

$$D_\mu \varepsilon^\pm = (\partial_\mu \pm \frac{1}{2} \omega_\mu) \varepsilon^\pm,$$

ω_μ being the spin-connection. To define the path-integral, one chooses gauge conditions

$$\begin{aligned}
e^\mu &= h^\mu(z, \bar{z}; m_i), \quad \bar{e}^\mu = \bar{h}^\mu(z, \bar{z}; m_i), \\
\psi_\mu^\pm &= \theta_\mu^\pm(z, \bar{z}; m_i). \tag{2.21}
\end{aligned}$$

The Faddeev-Popov ghost action can be written

$$\begin{aligned}
S_{FP} &= \delta_\Lambda \int d^2 z \left\{ C_\mu^* (e^\mu - h^\mu) + \bar{C}_\mu^* (\bar{e}^\mu - \bar{h}^\mu) \right. \\
&\quad \left. + s_+^* (\psi_\mu^+ - \theta_\mu^+) + s_-^* (\bar{\psi}_\mu^- - \bar{\theta}_\mu^-) \right\}. \tag{2.22}
\end{aligned}$$

It is invariant by construction under the nilpotent BRST transformations which are obtained by replacing the parameters $(\xi^\mu, \xi_D, \xi_M; \varepsilon^\pm, \eta^\pm)$ in Eqs. (2.20) by the Faddeev-Popov ghosts $(C^\mu, C_D, C_M; s^\pm, f^\pm)$, and which act on the ghosts, antighosts and Lagrange multipliers as follows:

$$\begin{aligned}
\delta_\Lambda C^\mu &= C^\nu \partial_\nu C^\mu - i (s_+^2 e^\mu - s_-^2 \bar{e}^\mu), \\
\delta_\Lambda C_D &= C^\nu \partial_\nu C_D + i (s_- f^+ - s_+ f^-),
\end{aligned}$$

$$\begin{aligned}
\delta_\Lambda C_M &= C^\nu \partial_\nu C_M - i(s_- f^+ + s_+ f^-) + i(s_+^\pm e^\nu - s_-^\pm \bar{e}^\nu) \omega_\nu, \\
\delta_\Lambda s^\pm &= C^\nu \partial_\nu s^\pm - \frac{1}{2} (C_D \pm C_M) s^\pm + i(s_+^\pm e^\nu - s_-^\pm \bar{e}^\nu) \psi_\nu^\pm, \\
\delta_\Lambda f^+ &= C^\nu \partial_\nu f^+ + \frac{1}{2} (C_D - C_M) f^+ - s_- e^\nu \partial_\nu C_D + i s_- \bar{e}^\nu (f^+ \psi_\nu^- - f^- \psi_\nu^+) \\
&\quad + i s_+^\pm (\frac{1}{e} \epsilon^{\lambda\nu} D_\lambda \psi_\nu^-) + i s_+ s_- (\frac{1}{e} \epsilon^{\lambda\nu} D_\lambda \psi_\nu^+), \\
\delta_\Lambda f^- &= C^\nu \partial_\nu f^- + \frac{1}{2} (C_D + C_M) f^- - s_+ \bar{e}^\nu \partial_\nu C_D + i s_+ e^\nu (f^+ \psi_\nu^- - f^- \psi_\nu^+) \\
&\quad + i s_-^\pm (\frac{1}{e} \epsilon^{\lambda\nu} D_\lambda \psi_\nu^+) + i s_+ s_- (\frac{1}{e} \epsilon^{\lambda\nu} D_\lambda \psi_\nu^-), \\
\delta_\Lambda C_\mu^* &= b_\mu, & \delta_\Lambda \bar{C}_\mu^* &= \bar{b}_\mu, \\
\delta_\Lambda b_\mu &= 0, & \delta_\Lambda \bar{b}_\mu &= 0, \\
\delta_\Lambda s_\pm^{*\mu} &= \chi_\pm^\mu, \\
\delta_\Lambda \chi_\pm^\mu &= 0.
\end{aligned} \tag{2.23}$$

In order to prove the nilpotence of the BRST transformations, which guarantees the invariance of (2.22), one uses the explicit expression for the spin-connection ω_μ :

$$\omega_\mu = -\frac{1}{e} \epsilon^{\lambda\nu} [e_\mu (\partial_\lambda \bar{e}_\nu + i \psi_\lambda^- \psi_\nu^-) + \bar{e}_\mu (\partial_\lambda e_\nu - i \psi_\lambda^+ \psi_\nu^+)].$$

Clearly, the full dependence of S_{FP} in Eq. (2.22) on the gauge functions $(h, \bar{h}, \theta^\pm)$ is contained in the terms

$$\int d^4z \left\{ b_\mu (e^\mu - h^\mu) + \bar{b}_\mu (\bar{e}^\mu - \bar{h}^\mu) + \chi_+^\mu (\psi_\mu^+ - \theta_\mu^+) + \chi_-^\mu (\psi_\mu^- - \theta_\mu^-) \right\}. \tag{2.24}$$

Defining the path-integral

$$Z[h, \bar{h}; \theta^\pm] = \int d\mu e^{-(S_d + S_{FP})}, \tag{2.25}$$

we immediately derive

$$\begin{aligned}\frac{\delta Z}{\delta h^\mu} &= \langle b_\mu \rangle = \langle \delta_\Lambda C_\mu^* \rangle, & \frac{\delta Z}{\delta \bar{h}^\mu} &= \langle \bar{b}_\mu \rangle = \langle \delta_\Lambda \bar{C}_\mu^* \rangle, \\ \frac{\delta Z}{\delta \theta_\mu^\pm} &= - \langle \chi_\pm^\mu \rangle = - \langle \delta_\Lambda s_\pm^{*\mu} \rangle.\end{aligned}\tag{2.26}$$

The vanishing of these expressions is proved exactly as in Eqs. (2.17)-(2.19), provided the measure $d\mu$ is itself BRST-invariant.

3. - BRST INVARIANCE OF GENERALIZED BOSONIC STRINGS

Having demonstrated the importance of BRST-invariant measures in path-integrals, we now turn to the actual construction of such measures for string theories. In this section we consider strings with standard local Poincaré $\times$ Weyl symmetry on the world sheet. Supersymmetric strings will be studied in a later section.

Consider a string described by co-ordinates X^M ($M=1, \dots, D$), mapping the world surface into D-dimensional space. We assume the D-dimensional space to be flat, but we do allow extra fermionic degrees of freedom λ^i ($i=1, \dots, N$) on the world surface of the string, which may be interpreted as representing certain compactified extra dimensions⁴⁾⁻⁶⁾. The reparametrization-invariant classical action for the string is

$$S_{cl} = \int d^2z \left\{ e^\mu \bar{e}^\nu \partial_\mu X \cdot \partial_\nu X + i \lambda_+^i e^\mu \partial_\mu \lambda_+^i - i \lambda_-^i \bar{e}^\mu \partial_\mu \lambda_-^i \right\}.\tag{3.1}$$

There are no Lorentz-covariant derivatives in (3.1) because

$$\lambda_+^{i\,2} = \lambda_-^{i\,2} = 0.\tag{3.2}$$

The action (3.1) is invariant under the local Poincaré $\times$ Weyl transformations given in Eqs. (2.2) and (2.7). Substitution of S_{cl} into Eqs. (2.9) and (2.10) defines the starting point for our construction of the string path-integral.

However, before proceeding, it is convenient to simplify this path-integral by eliminating the Lagrange multipliers b_μ and $\bar{b}_\mu$, in favour of imposing the

constraints (2.8). The ghost fields C_D and C_M also act as Lagrange multipliers and can be eliminated in like manner, implying the constraints

$$e^\mu C_\mu^* = 0, \quad \bar{e}^\mu \bar{C}_\mu^* = 0. \quad (3.3)$$

The remaining degrees of freedom are $\{X^M, \lambda_\pm^i, C^\mu\}$, the moduli $\{m_i\}$, and two anti-ghost components:

$$C^* = \bar{e}^\nu C_\nu^*, \quad \bar{C}^* = e^\nu \bar{C}_\nu^*. \quad (3.4)$$

Using reparametrization invariance, h^μ and $\bar{h}^\mu$ can be chosen such that

$$h^z = 0, \quad \bar{h}^{\bar{z}} = 0. \quad (3.5)$$

Then the non-vanishing components (3.4) are $(C_z^*, \bar{C}_{\bar{z}}^*)$. Moreover, through a local Lorentz transformation we can reach the standard conformal gauge

$$h^{\bar{z}} = \rho(z, \bar{z}; m_i), \quad \bar{h}^z = \rho(z, \bar{z}; m_i). \quad (3.6)$$

In this gauge, the action S of Eq. (2.10) becomes

$$S = \int d^2 z \left\{ \partial_{\bar{z}} X \cdot \partial_z X + i \left(\frac{1}{\sqrt{\rho}} \lambda_+^i \right) \partial_{\bar{z}} \left(\frac{1}{\sqrt{\rho}} \lambda_+^i \right) - i \left(\frac{1}{\sqrt{\rho}} \lambda_-^i \right) \partial_z \left(\frac{1}{\sqrt{\rho}} \lambda_-^i \right) \right. \\ \left. + \rho C_z^* \partial_{\bar{z}} C^z + \rho \bar{C}_{\bar{z}}^* \partial_z \bar{C}^{\bar{z}} \right\}. \quad (3.7)$$

This action is invariant under the residual BRST transformations

$$\begin{aligned} \delta_\Lambda X &= C^\nu \partial_\nu X, \\ \delta_\Lambda \left(\frac{1}{\sqrt{\rho}} \lambda_+^i \right) &= C^\nu \partial_\nu \left(\frac{1}{\sqrt{\rho}} \lambda_+^i \right) + \frac{1}{2} (\partial_z C^z) \frac{1}{\sqrt{\rho}} \lambda_+^i, \\ \delta_\Lambda \left(\frac{1}{\sqrt{\rho}} \lambda_-^i \right) &= C^\nu \partial_\nu \left(\frac{1}{\sqrt{\rho}} \lambda_-^i \right) + \frac{1}{2} (\partial_{\bar{z}} \bar{C}^{\bar{z}}) \frac{1}{\sqrt{\rho}} \lambda_-^i, \\ \delta_\Lambda C^\mu &= C^\nu \partial_\nu C^\mu, \\ \delta_\Lambda C_z^* &= -T_z + C^\nu \partial_\nu C_z^* + (2\partial_z C^z + C^\nu \partial_\nu \ln \rho) C_z^*, \\ \delta_\Lambda \bar{C}_{\bar{z}}^* &= -\bar{T}_{\bar{z}} + C^\nu \partial_\nu \bar{C}_{\bar{z}}^* + (2\partial_{\bar{z}} \bar{C}^{\bar{z}} + C^\nu \partial_\nu \ln \rho) \bar{C}_{\bar{z}}^*, \end{aligned} \quad (3.8)$$

where

$$\begin{aligned} T_z &= \frac{1}{e} \left\{ \partial_z \underline{X} \cdot \partial_z \underline{X} + i \left(\frac{1}{\sqrt{e}} \lambda_+^i \right) \partial_z \left(\frac{1}{\sqrt{e}} \lambda_+^i \right) \right\}, \\ \overline{T}_{\bar{z}} &= \frac{1}{e} \left\{ \partial_{\bar{z}} \underline{X} \cdot \partial_{\bar{z}} \underline{X} - i \left(\frac{1}{\sqrt{e}} \lambda_-^i \right) \partial_{\bar{z}} \left(\frac{1}{\sqrt{e}} \lambda_-^i \right) \right\}. \end{aligned} \quad (3.9)$$

Note that these transformations are only nilpotent after using the equation of motion following from S. However, S [Eq. (3.7)] is still fully invariant under the transformations (3.8). We now write the path-integral as

$$Z[e] = \int dm_i \int D\underline{X} D\lambda_{\pm}^i DC^{\mu} DC_{\mu}^* e^{-S}. \quad (3.10)$$

Our objective is to construct a functional integration measure which is invariant under (3.8). In the conformal gauge, we only have the function $\rho(z, \bar{z})$ at our disposal. Thus we consider measures of the form

$$\begin{aligned} D\underline{X} &= \prod_{(z, \bar{z})} [d\tilde{\underline{X}}(z, \bar{z})], \quad D\lambda_{\pm}^i = \prod_{(z, \bar{z})} [d\tilde{\lambda}_{\pm}^i(z, \bar{z})], \\ DC^{\mu} &= \prod_{(z, \bar{z})} [d\tilde{C}^{\mu}(z, \bar{z})], \quad DC_{\mu}^* = \prod_{(z, \bar{z})} [d\tilde{C}_{\mu}^*(z, \bar{z})], \end{aligned} \quad (3.11)$$

with

$$\tilde{\underline{X}} = e^m \underline{X}, \quad \tilde{\lambda}_{\pm}^i = e^s \lambda_{\pm}^i, \quad \tilde{C}^{\mu} = e^k C^{\mu}, \quad \tilde{C}_{\mu}^* = e^n C_{\mu}^*. \quad (3.12)$$

For these variables the BRST-transformations (3.8) become:

$$\begin{aligned} \bar{\delta}_{\Lambda} \tilde{\underline{X}} &= \Lambda e^{-k} [\tilde{C}^{\nu} \partial_{\nu} \tilde{\underline{X}} - m \tilde{C}^{\nu} \partial_{\nu} \ln e \tilde{\underline{X}}], \\ \bar{\delta}_{\Lambda} \tilde{\lambda}_+^i &= \Lambda e^{-k} [\tilde{C}^{\nu} \partial_{\nu} \tilde{\lambda}_+^i + (\frac{1}{2} \partial_z \tilde{C}^z - (s + \frac{1}{2}) \tilde{C}^{\bar{z}} \partial_{\bar{z}} \ln e \\ &\quad - (s + \frac{k+1}{2}) \tilde{C}^z \partial_z \ln e) \tilde{\lambda}_+^i], \end{aligned}$$

$$\begin{aligned}
\bar{\delta}_\Lambda \tilde{\lambda}_-^i &= \Lambda e^{-k} [\tilde{C}^\nu \partial_\nu \tilde{\lambda}_-^i + (\frac{1}{2} \partial_{\bar{z}} \tilde{C}^{\bar{z}} - (s + \frac{k+1}{2}) \tilde{C}^{\bar{z}} \partial_{\bar{z}} \ln \rho \\
&\quad - (s + \frac{1}{2}) \tilde{C}^z \partial_z \ln \rho) \tilde{\lambda}_-^i], \\
\bar{\delta}_\Lambda \tilde{C}^\mu &= \Lambda e^{-k} [\tilde{C}^\nu \partial_\nu \tilde{C}^\mu - k \tilde{C}^\nu \tilde{C}^\mu \partial_\nu \ln \rho], \\
\bar{\delta}_\Lambda \tilde{C}_z^* &= \Lambda e^{-k} [- e^{k+n} \overline{T}_z + \tilde{C}^\lambda \partial_\lambda \tilde{C}_z^* + \{ (1-n-2k) \tilde{C}^z \partial_z \ln \rho \\
&\quad + (1-n) \tilde{C}^{\bar{z}} \partial_{\bar{z}} \ln \rho + 2 \partial_z \tilde{C}^z \} \tilde{C}_z^*], \\
\bar{\delta}_\Lambda \tilde{C}_{\bar{z}}^* &= \Lambda e^{-k} [- e^{k+n} \overline{T}_{\bar{z}} + \tilde{C}^\lambda \partial_\lambda \tilde{C}_{\bar{z}}^* + \{ (1-n) \tilde{C}^z \partial_z \ln \rho \\
&\quad + (1-n-2k) \tilde{C}^{\bar{z}} \partial_{\bar{z}} \ln \rho + 2 \partial_{\bar{z}} \tilde{C}^{\bar{z}} \} \tilde{C}_{\bar{z}}^*].
\end{aligned}
\tag{3.13}$$

We have introduced the constant anticommuting parameter Λ in (3.13) to keep track of minus signs, so that $\bar{\delta}_\Lambda = \Lambda \delta_\Lambda$ is an even (commuting) operator. The condition for the measure (3.11) to be invariant under (3.13) is a unit graded Jacobian. The Jacobian is:

$$J = 1 + \Lambda (J_X + J_\lambda + J_C + J_{C^*}), \tag{3.14}$$

with

$$\begin{aligned}
J_X &= D \mathcal{T}_{(z,z')} [C^\nu \partial_\nu \delta_X^2(z-z') - m \delta_X^2(z-z') C^\nu \partial_\nu \ln \rho], \\
J_\lambda &= -2 N \mathcal{T}_{(z,z')} [C^\nu \partial_\nu \delta_\lambda^2(z-z') + \delta_\lambda^2(z-z') (\frac{1}{4} \partial_\nu C^\nu - (s + \frac{1}{2}) C^\nu \partial_\nu \ln \rho)], \\
J_C &= -2 \mathcal{T}_{(z,z')} [C^\nu \partial_\nu \delta_C^2(z-z') - \delta_C^2(z-z') (\frac{1}{2} \partial_\nu C^\nu + k C^\nu \partial_\nu \ln \rho)], \\
J_{C^*} &= -2 \mathcal{T}_{(z,z')} [C^\nu \partial_\nu \delta_{C^*}^2(z-z') + \delta_{C^*}^2(z-z') (\partial_\nu C^\nu + (1-n) C^\nu \partial_\nu \ln \rho)]
\end{aligned}
\tag{3.15}$$

For notational convenience we have written the expressions for the J_i in terms of C^λ rather than $\tilde{C}^\lambda$. The delicate point in Eqs. (3.15) is the interpretation of the

δ -functions. This can be traced back to the problem of regularizing the measures (3.11). We follow the procedure of Fujikawa^{16),24)}, which is really the heat-kernel method in disguise. Thus for a given δ -function we write

$$\delta^2(z-z') = \lim_{t \downarrow 0} K(z, z' | t), \quad (3.16)$$

where

$$K(z, z' | t) = \sum_n \varphi_n(z') e^{t\Delta} \varphi_n(z) \quad (3.17)$$

is the heat kernel for the Laplacian Δ , which is appropriate for the particular field being considered. For details, see Appendix A. The Laplacians are all of the form

$$\Delta = e^{-\alpha} \partial_{\bar{z}} e^{2\beta} \partial_z e^{-\alpha}. \quad (3.18)$$

The values of (α, β) have been collected for the various fields in Table 1. Equation (3.14) now reads

$$\begin{aligned} J-1 = \lim_{t \downarrow 0} \int d^2z \left\{ -\frac{D}{2} K_X(z, z | t) (\partial_{\bar{z}} C^{\nu} + 2m C^{\nu} \partial_{\bar{z}} \ln \rho) \right. \\ \left. + \frac{N}{2} K_{\lambda}(z, z | t) (\partial_{\bar{z}} C^{\nu} + 2(2s+1) C^{\nu} \partial_{\bar{z}} \ln \rho) \right. \\ \left. + 2 K_C(z, z | t) (\partial_{\bar{z}} C^{\nu} + k C^{\nu} \partial_{\bar{z}} \ln \rho) \right. \\ \left. - K_{C^*}(z, z | t) (\partial_{\bar{z}} C^{\nu} + 2(1-n) C^{\nu} \partial_{\bar{z}} \ln \rho) \right\}. \quad (3.19) \end{aligned}$$

In order for the right-hand side to cancel, we must first of all require

$$m = 2s+1 = \frac{1}{2}k = 1-n. \quad (3.20)$$

In Appendix A it is shown that for Δ as in (3.18)

$$K_{(\alpha, \beta)}(z, z | t) = \frac{1}{4\pi} \left[\frac{1}{t} e^{\frac{2(\alpha-\beta)}{t}} + \frac{1}{3}(\alpha+2\beta)(-\square \ln \rho) + O(t) \right]. \quad (3.21)$$

See also Refs. 15) and 25). The final result for (3.19) is:

$$\mathcal{J}_{-1} = \frac{\Lambda}{4\pi} \lim_{t \downarrow 0} \int d^2 z \left[\frac{1}{2t} (2+N-D) + \frac{m}{12} (D + \frac{N}{2} - 26) e^{-2m} \square \ln e^{2m} + O(t) \right] \partial_\nu (e^{2m} C^\nu)$$

(3.22)

The first term vanishes, since it is a total derivative. Only the (finite) second term remains. It vanishes as well if and only if

$$D = 26 - \frac{N}{2}, \quad (3.23)$$

which defines the critical dimension of the string. As expected, we find $D = 26$ for the ordinary bosonic string. A string in $D = 4$ exists for $N = 44$. As we will see below, an interesting case is also $N = 16$, $D = 18$, a critical dimension intermediate between the bosonic string and the superstring³⁸⁾.

4. - GAUGE INDEPENDENCE OF BOSONIC STRINGS

In Section 3, we have proved the existence of a functional measure (3.11) invariant under the BRST-transformations (3.13). However, these transformations are a modified version of the transformations (2.11); therefore it may be questioned to what extent the argument presented in Eqs. (2.15)-(2.19) also proves the gauge independence of the path-integral (3.10). This question will now be addressed.

The main issue is that by eliminating the ghosts (C_D, C_M) we have also eliminated the two antighost components (3.3). In fact, only the antighosts connected with the co-ordinate ghosts C^μ remain, and we may therefore expect the proof of gauge independence in Section 2 to apply only to traceless symmetric changes of the zweibein gauge conditions (related to reparametrizations). This is indeed the case. It is straightforward to show that a change of gauge

$$\delta h^z \neq 0, \quad \delta \bar{h}^{\bar{z}} \neq 0, \quad (4.1)$$

changes the path-integral (3.10) by

$$\delta Z = \int d^2 z \left(\delta h^z \langle \delta_\lambda C_z^* \rangle + \delta \bar{h}^{\bar{z}} \langle \delta_\lambda \bar{C}_{\bar{z}}^* \rangle \right), \quad (4.2)$$

with the antighost variations given by (3.8). Thus the proof in Section 2 can be applied to the non-conformal changes of gauge (4.1).

It remains to check the invariance of $Z[\rho]$ under local Lorentz and conformal changes of gauge parametrized by functions σ and ϕ . These take the form

$$\begin{aligned} \delta h^z &= \delta \bar{h}^{\bar{z}} = 0, \\ \delta h^{\bar{z}} &= (\sigma + \phi) h^{\bar{z}}, \\ \delta \bar{h}^z &= (\sigma - \phi) \bar{h}^z. \end{aligned} \quad (4.3)$$

Defining ρ in this case as

$$\rho = h^{-\frac{1}{2}} = (h^z \bar{h}^{\bar{z}})^{\frac{1}{2}}, \quad (4.4)$$

we find

$$\delta \rho = \sigma \rho. \quad (4.5)$$

A careful analysis of the derivation of Eq. (3.7) shows that such a change of gauge may be absorbed by a redefinition of the integration variables

$$\begin{aligned} \delta \tilde{X} &= m \sigma \tilde{X}, \\ \delta \tilde{\lambda}_\pm^i &= \left[\frac{m}{2} \sigma \mp \frac{1}{2} \phi \right] \tilde{\lambda}_\pm^i, \\ \delta \tilde{C}^\mu &= 2m \sigma \tilde{C}^\mu, \\ \delta \tilde{C}_z^* &= -(m \sigma + \phi) \tilde{C}_z^*, \quad \delta \tilde{C}_{\bar{z}}^* = -(m \sigma - \phi) \tilde{C}_{\bar{z}}^*. \end{aligned} \quad (4.6)$$

Hence the path integral (3.10) with ρ defined by (4.4) is invariant under the changes of gauge (4.3), provided the measure (3.11) is invariant under (4.6). To first order in the parameters (σ, ϕ) the graded Jacobian of the transformations (4.6) is

$$\mathcal{J} = 1 + m \mathcal{I}_{(z, z')} \left[\sigma (D \delta_\lambda^z(z-z') - N \delta_\lambda^z(z-z') - 4 \delta_C^z(z-z') + 2 \delta_{C^*}^z(z-z')) \right]. \quad (4.7)$$

Here the δ -functions are interpreted as before. Applying Eq. (3.21) we obtain

$$\mathcal{J}_{-1} = \frac{m}{4\pi} \lim_{t \downarrow 0} \int d^2z \, \sigma \left[(D-N-2) \frac{1}{t} e^{2m} + \frac{m}{3} (16 - \frac{N}{2} - D) \ln e + O(t) \right]. \quad (4.8)$$

In the critical dimension (3.23), the second term vanishes. The ultra-violet divergent first term can in fact be absorbed by introducing a cut-off $1/M^2$ for t and adding a BRST-invariant field-independent counter term to the action (3.7) of the form

$$\Delta S = \frac{M^2}{8\pi} (2+N-D) \int d^2z \, e^{2m}. \quad (4.9)$$

This is just a renormalization of the free energy of the string^{1),16)}. We may therefore summarize our results as follows. Taking S as in (3.7), ΔS as in (4.9) and the measure as in (3.11), the path-integral

$$Z[e] = \int dm_i \int D\tilde{\chi} D\lambda_{\pm}^i D\zeta^{\mu} D\zeta_{\mu}^* e^{-[S+\Delta S]} \quad (4.10)$$

is invariant under changes of gauge of the type (4.1) and of the type (4.3), corresponding to reparametrizations, local Lorentz- and Weyl-transformations of the zweibein. This is true for all values of m ³⁰⁾. We close this section by remarking that the counter term (4.9) vanishes (hence the string free energy is not renormalized) if $N = 16$, $D = 18$. This special case of bosonic string theory was studied in Ref. 38).

5. - SUPERSYMMETRIC STRINGS

We now turn to the generalization of the results obtained in Sections 3 and 4 to strings with local world-sheet supersymmetry. The classical action of such a string is^{36),37),10),11),35)}:

$$S_a = \int d^2z \, e \left\{ \bar{e}^\mu e^\nu \partial_\mu X \cdot \partial_\nu X + i \lambda_\mu^+ e^\mu \partial_\mu \lambda_\mu^+ - i \lambda_\mu^- \bar{e}^\mu \partial_\mu \lambda_\mu^- \right. \\ \left. + 2i \bar{e}^\mu e^\nu (\partial_\mu X \cdot \lambda_\mu^+ \psi_\nu^- - \partial_\nu X \cdot \lambda_\mu^- \psi_\mu^+) \right. \\ \left. + 2 \bar{e}^\mu e^\nu \lambda_\mu^+ \lambda_\mu^- \psi_\mu^+ \psi_\nu^- \right\}. \quad (5.1)$$

Both $\underline{X}$ and $\lambda_\mu^\pm$ are vectors in D-dimensional space. This action is invariant under the transformations (2.20), supplemented by

$$\delta \underline{X} = \xi^\nu \partial_\nu \underline{X} + i (\varepsilon^- \lambda_\mu^+ - \varepsilon^+ \lambda_\mu^-), \\ \delta \lambda_\mu^+ = \xi^\nu \partial_\nu \lambda_\mu^+ + \frac{1}{2} (\xi_D - \xi_M) \lambda_\mu^+ - (\bar{e}^\mu D_\mu X) \varepsilon^-, \\ \delta \lambda_\mu^- = \xi^\nu \partial_\nu \lambda_\mu^- + \frac{1}{2} (\xi_D + \xi_M) \lambda_\mu^- - (e^\mu D_\mu X) \varepsilon^+, \quad (5.2)$$

with

$$D_\mu X = \partial_\mu X - i (\psi_\mu^- \lambda_\mu^+ - \psi_\mu^+ \lambda_\mu^-). \quad (5.3)$$

The Faddeev-Popov terms may be computed from (2.22) and (2.23). Just as for the bosonic string, the path integral may be simplified by elimination of the Lagrange multipliers $b_\mu, \bar{b}_\mu$ and $\chi_\pm^\mu$, which remove $e^\mu, \bar{e}^\mu$ and $\psi_\mu^\pm$ as independent degrees of freedom (with the exception of moduli). In addition, the ghosts C_D, C_M and $f^\pm$ act as Lagrange multipliers imposing the constraints (3.3) and

$$e_\mu s_+^{*\mu} = 0, \quad \bar{e}_\mu s_-^{*\mu} = 0. \quad (5.4)$$

Therefore the only surviving antighost components are

$$C^* = \bar{e}^\mu C_\mu^*, \quad \bar{C}^* = e^\mu \bar{C}_\mu^*, \\ s_+^* = \bar{e}_\mu s_+^{*\mu}, \quad s_-^* = e_\mu s_-^{*\mu}, \quad (5.5)$$

balancing the ghost degrees of freedom of C^μ and $s^\pm$. As in Eqs. (3.4) and (3.6), it is convenient to choose a conformal gauge

$$\begin{aligned} h^{\bar{z}} &= e(z, \bar{z}; m_i), & \bar{h}^z &= e(z, \bar{z}; m_i), \\ \theta_{\bar{z}}^+ &= \frac{1}{e} \chi^-(z, \bar{z}; m_i), & \theta_z^- &= \frac{1}{e} \chi^+(z, \bar{z}; m_i), \end{aligned} \quad (5.6)$$

and

$$h^z = \bar{h}^{\bar{z}} = 0, \quad \theta_z^+ = \theta_{\bar{z}}^- = 0.$$

Inserting this into (5.1) and (2.22), one finds that all $\chi^\pm$ dependence cancels if one redefines the antighosts as follows:

$$\begin{aligned} C_z^* + \frac{1}{e} s_-^{*\bar{z}} \chi^+ &\rightarrow C_z^*, \\ \bar{C}_{\bar{z}}^* + \frac{1}{e} s_+^{*z} \chi^- &\rightarrow \bar{C}_{\bar{z}}^*. \end{aligned} \quad (5.7)$$

The resulting path-integral is

$$Z[e] = \int dm_i \int D\underline{X} D\underline{\lambda}^\pm DC^\mu DC_\mu^* Ds^\pm Ds_\pm^* e^{-S}, \quad (5.8)$$

with

$$\begin{aligned} S = \int d^2z \left\{ \partial_{\bar{z}} \underline{X} \cdot \partial_z \underline{X} + i \left(\frac{1}{\sqrt{e}} \underline{\lambda}^+ \right) \cdot \partial_{\bar{z}} \left(\frac{1}{\sqrt{e}} \underline{\lambda}^+ \right) - i \left(\frac{1}{\sqrt{e}} \underline{\lambda}^- \right) \cdot \partial_z \left(\frac{1}{\sqrt{e}} \underline{\lambda}^- \right) \right. \\ \left. + e C_z^* \partial_{\bar{z}} C^z + e \bar{C}_{\bar{z}}^* \partial_z C^{\bar{z}} \right. \\ \left. + \frac{1}{\sqrt{e}} s_+^{*z} \partial_z (\sqrt{e} s_+) + \frac{1}{\sqrt{e}} s_-^{*\bar{z}} \partial_{\bar{z}} (\sqrt{e} s_-) \right\}. \end{aligned} \quad (5.9)$$

The measure in (5.8) is specified below. The action is invariant under the residual BRST transformations

$$\begin{aligned} \delta_\lambda \underline{X} &= C^\nu \partial_\nu \underline{X} + i (s_- \underline{\lambda}^+ - s_+ \underline{\lambda}^-), \\ \delta_\lambda \left(\frac{1}{\sqrt{e}} \underline{\lambda}^+ \right) &= C^\nu \partial_\nu \left(\frac{1}{\sqrt{e}} \underline{\lambda}^+ \right) + \frac{1}{2} (\partial_z C^z) \frac{1}{\sqrt{e}} \underline{\lambda}^+ - \sqrt{e} s_- \partial_z \underline{X}, \end{aligned}$$

$$\begin{aligned}
\delta_\lambda \left(\frac{1}{\sqrt{e}} \lambda^- \right) &= C^\nu \partial_\nu \left(\frac{1}{\sqrt{e}} \lambda^- \right) + \frac{1}{2} (\partial_z C^{\bar{z}}) \frac{1}{\sqrt{e}} \lambda^- - \sqrt{e} s_+ \partial_z X, \\
\delta_\lambda C^z &= C^\nu \partial_\nu C^z + i e s_-^z, \\
\delta_\lambda C^{\bar{z}} &= C^\nu \partial_\nu C^{\bar{z}} - i e s_+^z, \\
\delta_\lambda (s_+ \sqrt{e}) &= C^\nu \partial_\nu (s_+ \sqrt{e}) - \frac{1}{2} (\partial_z C^{\bar{z}}) s_+ \sqrt{e}, \\
\delta_\lambda (s_- \sqrt{e}) &= C^\nu \partial_\nu (s_- \sqrt{e}) - \frac{1}{2} (\partial_z C^z) s_- \sqrt{e}, \\
\delta_\lambda (e C_z^*) &= -e T_z + C^\nu \partial_\nu (e C_z^*) + 2 (\partial_z C^z) e C_z^* - \frac{1}{2} \partial_z (s_-^{*z} s_-) \\
&\quad - \frac{1}{\sqrt{e}} s_-^{*z} \partial_z (s_- \sqrt{e}), \\
\delta_\lambda (e \bar{C}_{\bar{z}}^*) &= -e \bar{T}_{\bar{z}} + C^\nu \partial_\nu (e \bar{C}_{\bar{z}}^*) + 2 (\partial_z C^{\bar{z}}) e \bar{C}_{\bar{z}}^* - \frac{1}{2} \partial_z (s_+^{*z} s_+) \\
&\quad - \frac{1}{\sqrt{e}} s_+^{*z} \partial_z (s_+ \sqrt{e}), \\
\delta_\lambda \left(\frac{1}{\sqrt{e}} s_+^{*z} \right) &= -\frac{1}{\sqrt{e}} S^z + C^\nu \partial_\nu \left(\frac{1}{\sqrt{e}} s_+^{*z} \right) + \frac{3}{2} (\partial_z C^{\bar{z}}) \frac{1}{\sqrt{e}} s_+^{*z} - 2i e^{3/2} \bar{C}_{\bar{z}}^* s_+, \\
\delta_\lambda \left(\frac{1}{\sqrt{e}} s_-^{*z} \right) &= -\frac{1}{\sqrt{e}} \bar{S}^{\bar{z}} + C^\nu \partial_\nu \left(\frac{1}{\sqrt{e}} s_-^{*z} \right) + \frac{3}{2} (\partial_z C^z) \frac{1}{\sqrt{e}} s_-^{*z} + 2i e^{3/2} C_z^* s_-,
\end{aligned}
\tag{5.10}$$

where

$$\begin{aligned}
T_z &= \frac{1}{e} \left[\partial_z X \cdot \partial_z X + i \left(\frac{1}{\sqrt{e}} \lambda^+ \right) \cdot \partial_z \left(\frac{1}{\sqrt{e}} \lambda^+ \right) \right], \\
\bar{T}_{\bar{z}} &= \frac{1}{e} \left[\partial_z X \cdot \partial_z X - i \left(\frac{1}{\sqrt{e}} \lambda^- \right) \cdot \partial_z \left(\frac{1}{\sqrt{e}} \lambda^- \right) \right], \\
S^z &= -2i \lambda^- \cdot \partial_z X, \quad \bar{S}^{\bar{z}} = 2i \lambda^+ \cdot \partial_z X.
\end{aligned}
\tag{5.11}$$

As for the bosonic string, we wish to construct a local measure of the type (3.11) for our functional integral which is invariant under the transformations (5.10). Thus we define

$$\begin{aligned}
 D\tilde{X} &= \prod_{(z,\bar{z})} [d\tilde{X}(z,\bar{z})], & D\tilde{\lambda}^\pm &= \prod_{(z,\bar{z})} [d\tilde{\lambda}^\pm(z,\bar{z})], \\
 DC^\mu &= \prod_{(z,\bar{z})} [d\tilde{C}^\mu(z,\bar{z})], & DC_\mu^* &= \prod_{(z,\bar{z})} [d\tilde{C}_\mu^*(z,\bar{z})], \\
 Ds^\pm &= \prod_{(z,\bar{z})} [d\tilde{s}^\pm(z,\bar{z})], & Ds_\pm^* &= \prod_{(z,\bar{z})} [d\tilde{s}_\pm^*(z,\bar{z})],
 \end{aligned} \tag{5.12}$$

with

$$\begin{aligned}
 \tilde{X} &= e^m X, & \tilde{\lambda}^\pm &= e^s \lambda^\pm, \\
 \tilde{C}^\mu &= e^k C^\mu, & \tilde{s}^\pm &= e^l s^\pm, \\
 \tilde{C}_\mu^* &= e^n C_\mu^*, & \tilde{s}_\pm^* &= e^p s_\pm^*.
 \end{aligned} \tag{5.13}$$

Like in (3.13), the transformations (5.10) must now be rewritten in terms of the fields (5.13); then the graded Jacobian can be computed by a long but straightforward calculation. The result is

$$J = 1 + \Lambda(J_X + J_\lambda + J_C + J_{C^*} + J_s + J_{s^*}), \tag{5.14}$$

with

$$\begin{aligned}
 J_X &= D \mathcal{T}_{(z,z')} [C^\nu \partial_\nu \delta_X^2(z-z') - m \delta_X^2(z-z') C^\nu \partial_\nu \ln \varrho], \\
 J_\lambda &= -2 D \mathcal{T}_{(z,z')} [C^\nu \partial_\nu \delta_\lambda^2(z-z') + \delta_\lambda^2(z-z') (\frac{1}{4} \partial_\nu C^\nu - (s+\frac{1}{2}) C^\nu \partial_\nu \ln \varrho)], \\
 J_C &= -2 \mathcal{T}_{(z,z')} [C^\nu \partial_\nu \delta_C^2(z-z') - \delta_C^2(z-z') (\frac{1}{2} \partial_\nu C^\nu + k C^\nu \partial_\nu \ln \varrho)], \\
 J_{C^*} &= -2 \mathcal{T}_{(z,z')} [C^\nu \partial_\nu \delta_{C^*}^2(z-z') + \delta_{C^*}^2(z-z') (\partial_\nu C^\nu + (1-n) C^\nu \partial_\nu \ln \varrho)],
 \end{aligned}$$

$$\begin{aligned} \mathcal{J}_S &= 2 \int_{(z,z')} \mathcal{K} [C^\nu \partial_\nu \delta_S^2(z-z') - \delta_S^2(z-z') (\frac{1}{4} \partial_\nu C^\nu + (1-\frac{1}{2}) C^\nu \partial_\nu \ln \rho)], \\ \mathcal{J}_{S^*} &= 2 \int_{(z,z')} \mathcal{K} [C^\nu \partial_\nu \delta_{S^*}^2(z-z') + \delta_{S^*}^2(z-z') (\frac{3}{4} \partial_\nu C^\nu - (p+\frac{1}{2}) C^\nu \partial_\nu \ln \rho)]. \end{aligned} \quad (5.15)$$

Contd.

Inserting the heat-kernel expression for the δ -functions, and using the result (3.21) as well as the values of α and β collected in Table 2, we find first of all that we should take

$$m = 2S+1 = \frac{k}{2} = 1-n = \frac{2}{3} (1-\frac{1}{2}) = -(2p+1); \quad (5.16)$$

then we obtain

$$\mathcal{J}_{-1} = \frac{\Delta m}{16\pi} (D-10) \int d^2z (\bar{e}^{-2m} \square \ln e^{2m}) \partial_\nu (e^{2m} C^\nu). \quad (5.17)$$

Thus the BRST invariance (5.10) is preserved in the quantum theory for $D = 10$, a result well known from many earlier calculations^{1),17)}.

It remains to discuss to what extent gauge independence of the path-integral is implied by the vanishing of (5.17). First, note that by our simplification of the path-integral and the choice of conformal gauge (5.6) we have lost the ghosts and antighosts corresponding to local Weyl-, Lorentz- and S-supersymmetry. Therefore the residual BRST invariance (5.10) only implies invariance of the path-integral under changes of gauge generated by general co-ordinate and Q-supersymmetry transformations:

$$(\delta h^z, \delta \bar{h}^{\bar{z}}) \neq 0, \quad (\delta \theta_z^+, \delta \theta_{\bar{z}}^-) \neq 0. \quad (5.18)$$

To prove the independence of the path-integral under local Weyl-, Lorentz- and S-supersymmetry we must investigate separately the change of $Z[\rho]$, Eq. (5.8), under

$$\begin{aligned}\delta h^{\bar{z}} &= (\sigma + \varphi) h^{\bar{z}}, & \delta \bar{h}^z &= (\sigma - \varphi) \bar{h}^z, \\ \delta \theta_{\bar{z}}^+ &= -\frac{1}{\rho} \eta^- - \frac{1}{2}(\sigma + \varphi) \theta_{\bar{z}}^+, & \delta \theta_z^- &= -\frac{1}{\rho} \eta^+ - \frac{1}{2}(\sigma - \varphi) \theta_z^-. \end{aligned} \quad (5.19)$$

Again, whenever we are not strictly in the conformal gauge, we define $\rho = 1/\sqrt{h}$ [cf. (4.4)]. Thus (5.19) induces the changes

$$\delta \rho = \sigma \rho, \quad \delta \chi^\pm = -\eta^\pm + \frac{1}{2}(\sigma \mp \varphi) \chi^\pm. \quad (5.20)$$

The first change in $\chi^\pm$ can be absorbed in a redefinition of C_z^* , $\bar{C}_{\bar{z}}^*$:

$$\begin{aligned}\delta C_z^* &= -\frac{1}{\rho^2} S_-^{*\bar{z}} \eta^+, \\ \delta \bar{C}_{\bar{z}}^* &= -\frac{1}{\rho^2} S_+^{*z} \eta^-. \end{aligned} \quad (5.21)$$

Since this redefinition is off-diagonal, it has a unit Jacobian: the measure (5.12) is preserved and the path-integral is invariant. It remains to consider the variations induced by local Lorentz and Weyl transformations. They can be absorbed by redefinitions of the fields as follows:

$$\begin{aligned}\delta \tilde{\chi} &= m \sigma \tilde{\chi}, & \delta \tilde{\chi}^\pm &= \frac{1}{2}(m \sigma \mp \varphi) \tilde{\chi}^\pm, \\ \delta \tilde{C}^\mu &= 2m \sigma \tilde{C}^\mu, & \delta \tilde{S}^\pm &= \frac{1}{2}(3m \sigma \pm \varphi) \tilde{S}^\pm, \\ \delta \tilde{C}_z^* &= -(m \sigma + \varphi) \tilde{C}_z^*, & \delta \tilde{C}_{\bar{z}}^* &= -(m \sigma - \varphi) \tilde{C}_{\bar{z}}^*, \\ \delta \tilde{S}_+^{*z} &= -\frac{1}{2}(m \sigma - \varphi) \tilde{S}_+^{*z}, & \delta \tilde{S}_-^{*\bar{z}} &= -\frac{1}{2}(m \sigma + \varphi) \tilde{S}_-^{*\bar{z}}. \end{aligned} \quad (5.22)$$

Computing the Jacobian of this transformation is straightforward. To first order in (σ, φ) one finds:

$$\begin{aligned} \mathcal{J} = 1 + m \mathcal{T}_{(z, z')} & \left[\sigma \left(D \delta_\chi^2(z-z') - D \delta_\lambda^2(z-z') - 4 \delta_c^2(z-z') \right. \right. \\ & \left. \left. + 2 \delta_{c*}^2(z-z') + 3 \delta_s^2(z-z') - \delta_{s*}^2(z-z') \right) \right]. \end{aligned} \quad (5.23)$$

After inserting the appropriate heat-kernel traces, one gets:

$$\mathcal{I}_{-1} = -\frac{m^2}{8\pi} (D-10) \int d^2z \, \sigma \square \ln \varrho. \quad (5.24)$$

There is no quadratic divergence because of supersymmetry. Thus we have verified the complete invariance of the supersymmetric string path-integral in the conformal gauge (5.6) for $D = 10$ under arbitrary changes of gauge.

6. - CONCLUSIONS

The simple argument presented in Section 2 shows the direct relation between gauge independence and BRST invariance of path-integrals for theories with local gauge invariance. However, the argument cannot be used if certain (anti-) ghost degrees of freedom are eliminated. In our construction of path integrals for strings this happened to be the case. Thus, BRST invariance could still be used to prove the invariance of the path integrals under changes of gauge generated by general co-ordinate transformations and, if applicable, local Q-supersymmetry. In contradistinction, changes of gauge generated by other (super-) conformal transformations had to be checked separately. As expected, the results were mutually consistent: whenever the path-integral is invariant under the residual BRST transformations, it is also invariant under local Weyl-, Lorentz- and S-supersymmetry transformations of the gauge.

In this paper we have only considered left-right symmetric strings. It is well known that, due to conformal invariance in the critical dimension, string Green functions factorize with respect to left- and right-moving fields. This holds also for the cancellation of anomalies: they cancel separately in the left- and right-handed sectors. Therefore all results obtained here can be extended to heterotic types of strings. In particular, the critical dimensions remain the same as obtained here, though they may be different for left- and right-movers.

A point which we have not addressed in detail in this paper is the exact form of the integration measure over modular space. This is a finite-dimensional integral (to finite order in the string-loop expansion), the domain of which is not generally known. This integration may require some finite-dimensional Jacobian factor, to define an invariant volume on modular space. A discussion of BRST invariance of the bosonic string, keeping track explicitly of the moduli, can be found in Refs. 12) and 19).

Finally we emphasize that we have only discussed strings in a flat background. It is desirable to extend our methods to the case of strings in background fields. In particular, effective field equations for the background fields should be obtainable by requiring BRST invariance³⁹⁾ as an alternative to requiring the vanishing of β functions. These ideas are presently being studied.

Field	α	β
$\tilde{X}$	m	0
$\tilde{\lambda}$	$s + \frac{1}{2} = m/2$	$-(s + \frac{1}{2}) = -m/2$
$\tilde{C}$	$k = 2m$	$l - n = m$
$\tilde{C}^*$	$n - l = -m$	$-k = -2m$

Table 1 - Exponents in bosonic-string
Laplacians.

Field	α	β
$\tilde{X}$	m	0
$\tilde{\lambda}$	$s + \frac{1}{2} = m/2$	$-(s + \frac{1}{2}) = -m/2$
$\tilde{C}$	$k = 2m$	$l - n = m$
$\tilde{C}^*$	$n - l = -m$	$-k = -2m$
$\tilde{s}$	$\lambda - \frac{1}{2} = \frac{3}{2} m$	$-(p + \frac{1}{2}) = m/2$
$\tilde{s}^*$	$p + \frac{1}{2} = -m/2$	$-(\lambda - \frac{1}{2}) = -\frac{3}{2} m$

Table 2 - Exponents in supersymmetric-string
Laplacians.

APPENDIX A - HEAT KERNELS AND FUJIKAWA REPRESENTATION

In this Appendix we review some aspects of the heat-kernel formalism^{*)}. We will derive Eq. (3.21) and show how to use the Fujikawa method for computing the coefficients in the short-distance expansion of the heat kernel³⁴⁾.

Let $-\Delta$ be an elliptic operator with eigenvalues $\lambda_n \geq 0$. The heat kernel is the solution of the equation

$$(\partial_t - \Delta) K(x, x' | t) = 0, \quad (\text{A.1})$$

satisfying the boundary condition

$$\lim_{t \downarrow 0} K(x, x' | t) = \delta^d(x - x'). \quad (\text{A.2})$$

If $\{\phi_n\}$ is a complete orthonormal set of eigenfunctions of Δ

$$-\Delta \phi_n = \lambda_n \phi_n, \quad (\text{A.3})$$

then $K(x, x' | t)$ has a representation

$$\begin{aligned} K(x, x' | t) &= \sum_n e^{-t\lambda_n} \phi_n^*(x') \phi_n(x) \\ &= \sum_n \phi_n^*(x') e^{t\Delta} \phi_n(x), \end{aligned} \quad (\text{A.4})$$

which is the result (3.17). In Eq. (3.19) we are required to compute the diagonal matrix elements

$$K(x, x | t) = \sum_n \phi_n^*(x) e^{t\Delta} \phi_n(x). \quad (\text{A.5})$$

Using the completeness and orthonormality of the $\{\phi_n\}$, Fujikawa assumes a Fourier representation

$$\phi_n(x) = \int \frac{d^d k}{(2\pi)^{d/2}} \tilde{\phi}_n(k) e^{ik \cdot x}. \quad (\text{A.6})$$

*) The heat-kernel formalism is the Euclidean version of the Fock-Schwinger proper-time method.

[For a finite volume the integral must actually be replaced by a discrete sum. This does not affect the small- t behaviour of $K(x, x', |t)$ as used in computing the Jacobians in Sections 3 to 5.] Then

$$K(x, x | t) = \int \frac{d^d k}{(2\pi)^d} e^{-ik \cdot x} e^{t\Delta} e^{ik \cdot x}. \quad (A.7)$$

This is the Fujikawa representation.

For the operator (3.18) (in $d=2$),

$$\Delta = e^{-\alpha} \partial_{\bar{z}} e^{2\beta} \partial_z e^{-\alpha} \quad (A.8)$$

one gets

$$K(z, z | t) = \frac{1}{4\pi^2 t} \int d^2 k e^{\Omega(k, t)}, \quad (A.9)$$

where

$$\Omega(k, t) = -e^{2(\beta-\alpha)} \bar{k} k + t(A + B \partial_z + \bar{B} \partial_{\bar{z}}) + t^2 e^{2(\beta-\alpha)} \partial_{\bar{z}} \partial_z, \quad (A.10)$$

and

$$A = e^{2(\beta-\alpha)} [-i\alpha \bar{k} \partial_{\bar{z}} \ln \rho + i(2\beta-\alpha) k \partial_z \ln \rho - \alpha t \partial_{\bar{z}} \partial_z \ln \rho - \alpha(2\beta-\alpha) t \partial_{\bar{z}} \ln \rho \partial_z \ln \rho],$$

$$B = e^{2(\beta-\alpha)} [ik - \alpha t \partial_{\bar{z}} \ln \rho], \quad \bar{B} = e^{2(\beta-\alpha)} [i\bar{k} + (2\beta-\alpha) t \partial_z \ln \rho]. \quad (A.11)$$

Expanding (A.9) in powers of t , using the Baker-Campbell-Hausdorff identity, one gets after performing the Gaussian integrals^{16), 26)}:

$$K(z, z | t) = \frac{1}{4\pi} \left[\frac{1}{t} e^{2(\beta-\alpha)} + \frac{1}{3} (\alpha + 2\beta) (-\partial_{\bar{z}} \partial_z \ln \rho) + O(t) \right]. \quad (A.12)$$

This is all that is required to derive the result (3.22). However, one can do more. Introducing a regulator m^2 :

$$\begin{aligned} \ln \det(-\Delta+m^2) &= \int_0^{m^2} d\mu^2 \operatorname{tr}(-\Delta+\mu^2) \\ &= \int_0^{m^2} d\mu^2 \int_0^\infty dt \int d^d x e^{-\mu^2 t} K(x,x|t). \end{aligned} \quad (\text{A.13})$$

The heat kernel has a well-known short-distance expansion³⁴⁾:

$$K(x,x'|t) = \frac{1}{(4\pi t)^{d/2}} e^{-\frac{r^2(x,x')}{4t}} \sum_{k \geq 0} t^k C_k(x,x'), \quad (\text{A.14})$$

where $r^2(x,x')$ is the geodesic distance between (x,x') . Inserting this expression into (A.13), one obtains a Laurent series in m for $\ln \det(-\Delta+m^2)$. The Fujikawa representation (A.7) provides a convenient alternative way to derive this result. A simple scaling argument shows that for Laplace-type operators [like (A.8)]:

$$\int \frac{d^d x d^d k}{(2\pi)^d} e^{-ik \cdot x} e^{t\Delta} e^{ik \cdot x} = \sum_{k \geq 0} B_k t^{k - \frac{d}{2}}. \quad (\text{A.15})$$

Then (A.13) becomes:

$$\begin{aligned} \ln \det(-\Delta+m^2) &= \int_0^{m^2} d\mu^2 \int_0^\infty dt e^{-\mu^2 t} \sum_{k \geq 0} B_k t^{k - \frac{d}{2}} \\ &= \sum_{k \geq 0} B_k \Gamma(k - \frac{d}{2}) m^{d-2k} + C, \end{aligned} \quad (\text{A.16})$$

where C is a constant independent of m . The poles of the Γ function represent the ultra-violet divergences in dimensional regularization. The coefficients B_k can be obtained by inverting (A.15):

$$B_k = \frac{1}{k!} \frac{\partial^k}{\partial t^k} \left(t^{\frac{d}{2}} \int d^d x K(x,x|t) \right)_{t=0}. \quad (\text{A.17})$$

Thus the Fujikawa representation may conveniently replace the short-distance expansion (A.14).

APPENDIX B - NOTATION

Throughout this paper we use 2-d complex co-ordinates $(z, \bar{z})$ to parametrize the world surface of the string. Correspondingly, we introduce a complex vector representation of the zweibein, Eqs. (2.3)-(2.6). The relation between this complex notation and the 2-d Euclidean co-ordinate representation $x^\mu = (x^1, x^2)$ is:

$$z = \frac{1}{\sqrt{2}} (x^1 + i x^2), \quad \bar{z} = \frac{1}{\sqrt{2}} (x^1 - i x^2), \quad (\text{B.1})$$

and

$$\begin{aligned} e^z &= \frac{1}{2} (e'' - e^{22} + i (e^{12} + e^{21})), \\ e^{\bar{z}} &= \frac{1}{2} (e'' + e^{22} - i (e^{12} - e^{21})), \\ \bar{e}^z &= \frac{1}{2} (e'' + e^{22} + i (e^{12} - e^{21})), \\ \bar{e}^{\bar{z}} &= \frac{1}{2} (e'' - e^{22} - i (e^{12} + e^{21})). \end{aligned} \quad (\text{B.2})$$

Here $e^{a\mu}$ denotes the component of the real Euclidean inverse zweibein. Similarly, the Euclidean 2-d gamma matrices are

$$\begin{aligned} \gamma^\mu &= e^{a\mu} \gamma_a, \quad \mu = (1, 2); \\ \gamma^\mu &= e^\mu \gamma + \bar{e}^\mu \bar{\gamma}, \quad \mu = (z, \bar{z}); \end{aligned} \quad (\text{B.3})$$

with

$$\gamma = \frac{1}{\sqrt{2}} (\gamma_1 - i \gamma_2), \quad \bar{\gamma} = \frac{1}{\sqrt{2}} (\gamma_1 + i \gamma_2). \quad (\text{B.4})$$

In the conventional Pauli matrix representation

$$\gamma = \sqrt{2} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \bar{\gamma} = \sqrt{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \quad (\text{B.5})$$

Finally, we have decomposed all 2-d (Majorana) spinors into upper and lower components:

$$\lambda = \begin{pmatrix} \lambda_+ \\ \lambda_- \end{pmatrix}. \quad (\text{B.6})$$

These results, together with (2.3)-(2.6) suffice to translate between this paper and the results of Ref. 35).

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