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BOUNDS FOR MULTIPARTICLE DISTRIBUTIONS
IN MINIJET EVENTS FROM SEMI-HARD QCD

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A B S T R A C T

It is shown that, as a consequence of the centrality of the semi-hard amplitude, it is possible to obtain an upper and a lower bound for the ratio $\langle n \rangle_{SH} / \langle n \rangle$ of the minijet average multiplicity over the inelastic average multiplicity and an upper bound for the ratio C_{2SH} / C_2 of the corresponding normalized second moments of the particle distributions. In minijet events multiplicities are higher and the KNO distributions are narrower.

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It is a prediction of perturbative QCD that at higher energies soft physics will be increasingly contaminated by hard physics¹⁾. As the energy grows, the smallness of the typical QCD cross-section tends to be compensated by the fast rise in the number of hard interacting soft gluons. This is the essence of semi-hard physics. The most striking evidence for semi-hard QCD comes from the rapid increase of the minijet cross-section^{2),3)}. Additional evidence comes as well from the violation of several low-energy regularities in particle production and elastic scattering^{4),5)}.

Processes where at least one minijet is detected, semi-hard processes, are more central in the impact parameter than purely soft processes. This is easy to understand. On one hand, hard interactions are of short-range nature and their probability of occurrence, comparing to soft long-range interactions, dies out rapidly as the impact parameter b increases. On the other hand, QCD ladders, involving large virtualities, do not generate the shrinking factor⁶⁾ typical of soft ladders. Both arguments thus lead to the conclusion that semi-hard contributions are more central than soft ones.

The question I would like to address in this note is the question of how centrality of the semi-hard amplitude affects particle distributions, in particular when comparing jet to no-jet events, independently of any specific model for particle production.

As semi-hard interactions do not use much energy, minijets usually occur in a background of energy and particles which has not much to do with perturbative QCD. In fact, the energy and particle density in minijet events, even outside the jet cone, is larger than in no-jet events²⁾. This happens because when triggering on minijets one is triggering on small impact parameters⁷⁾ (centrality of semi-hard interactions). The physics of particle production, except for the few particles in the minijets, is the familiar particle production physics known in soft processes. Here I shall then make the assumption that, in first approximation, the particle production mechanism in minijet and no-jet events is the same. This can only be acceptable to the extent that most of minijets are of low transverse energy E_T . Clearly, for large E_T jets ($E_T \gtrsim 80$ GeV) the QCD event configurations and multiplicities dominate⁸⁾.

At high energy the impact parameter b is a good quantum number. In general, the probability $P(n,s)$ of producing n particles at a given energy $\sqrt{s}$ can be written in the impact parameter approximation as

$$P(n,s) = \int P(b^2,s) P(b^2,n,s) db^2 \quad (1)$$

where

$$P(b^2, s) \equiv \frac{1}{\sigma} \frac{d\sigma}{db^2} \equiv \frac{\pi}{\sigma} G(b^2, s) \quad (2)$$

is the inelastic collision probability at a given b^2 and s , σ is the inelastic (or the non-diffractive) cross-section, G is the inelastic overlap function with

$$0 \leq G(b^2, s) \leq 1, \quad (3)$$

and $P(b^2, n, s)$ is the multiparticle distribution at fixed b^2 .

Instead of working with the particle distribution $P(n, s)$ it is equivalent to use the moments $\langle n^q \rangle$, $\langle n^q \rangle \equiv \sum n^q P(n, s)$, $q = 1, 2, \dots$. From (1) and (2),

$$\langle n^q(s) \rangle = \frac{\pi}{\sigma} \int G(b^2, s) \bar{n}^q(b^2, s) db^2, \quad (4)$$

where $\bar{n}^q(b^2, s)$ are the fixed b distribution moments, $\bar{n}^q(b^2, s) \equiv \sum n^q P(b^2, n, s)$. Without loss of generality one can write⁹⁾

$$\bar{n}^q(b^2, s) \equiv \langle n(s) \rangle^q f_q(b^2, s) \quad (5)$$

with the functions $f_q(b^2, s)$ satisfying the constraints

$$f_q(b^2, s) \geq f_{q-1}(b^2, s) f_1(b^2, s) \geq f_1^q(b^2, s) \geq 0. \quad (6)$$

For convenience I shall use the variable

$$\beta \equiv \frac{\pi b^2}{\sigma} \quad (7)$$

It follows, from (2), that

$$\int G(\beta, s) d\beta = 1 \quad (8)$$

and, from (4) and (5),

$$C_q(s) \equiv \langle n^q \rangle / \langle n \rangle^q = \int f_q(\beta, s) G(\beta, s) d\beta, \quad (9)$$

with

$$C_1 = \int f_1(\beta, s) G(\beta, s) d\beta = 1, \quad (10)$$

where the C_q are the normalized KNO moments.

Up to this point all the formulae are entirely general. Physics, and experiment in the case of the overlap function, requires, at a given s , $G(\beta, s)$, $f_1(\beta, s)$ and in general the $f_q(\beta, s)$ to be decreasing functions of β , as more interactions occur and more particles are produced at small impact parameters.

The same formalism can be extended to semi-hard processes by making the changes

$$\begin{aligned} G &\longrightarrow G_{SH} \\ f_q &\longrightarrow f_{qSH} = f_q, \end{aligned} \quad (11)$$

where G_{SH} is the minijet overlap function satisfying

$$\sigma_{SH}(s) = \pi \int G_{SH}(b^2, s) db^2, \quad (12)$$

σ_{SH} being the minijet total cross section, and the equality of the functions f_q in inelastic and minijet events simply expresses our basic assumption that the dominant particle production mechanism is the same. At a given impact parameter the particle distribution is, approximately at least, the same in events with and without minijets.

Introducing now the function

$$\tilde{G}_{SH}(\beta, s) \equiv \frac{\sigma(s)}{\sigma_{SH}(s)} G_{SH}(\beta, s), \quad (13)$$

Eq. (12), similarly to (8), becomes

$$\int \tilde{G}_{SH}(\beta, s) d\beta = 1 \quad (14)$$

As the semi-hard amplitude is more central and is then expected to decrease faster with β , from (8) and (14) it follows that (see Fig. 1)

$$\begin{aligned} \tilde{G}_{SH}(\beta, s) &> G(\beta, s), \quad 0 \leq \beta < c, \\ \tilde{G}_{SH}(\beta, s) &< G(\beta, s), \quad \beta > c, \end{aligned} \quad (15)$$

with $c > 0$.

The moments of the particle distribution for minijet events can be written as

$$\langle n_{SH}^q \rangle = \langle n \rangle^q \int f_q(\beta, s) \tilde{G}_{SH}(\beta, s) d\beta \quad (16)$$

and in particular for $q = 1$ one has, see (10),

$$\langle n_{SH} \rangle / \langle n \rangle = \frac{\int f_1(\beta, s) \tilde{G}_{SH}(\beta, s) d\beta}{\int f_1(\beta, s) G(\beta, s) d\beta} \equiv \frac{\int f_1 \tilde{G}}{\int f_1 G} \quad (17)$$

A simplified notation for integrals was introduced in (17). It is now straightforward to prove the following:

$$1) \quad \langle n_{SH} \rangle / \langle n \rangle > 1 \quad (18)$$

Let $f_1(c)$ be the value of f_1 at $\beta = c$ (see Fig. 1). Then

$$\int_0^c f_1 (\tilde{G} - G) > f_1(c) \int_0^c (\tilde{G} - G)$$

and, because of (8) and (14)

$$\int_0^\infty f_1 (\tilde{G} - G) + \int_c^\infty (f_1(c) - f_1) (\tilde{G} - G) > 0$$

with the result, as the second integral is negative, that the ratio (17) is larger than 1.

$$2) \quad \langle n \rangle_{SH} / \langle n \rangle < \tilde{G}_{SH}^{(0)} / G(0). \quad (19)$$

The ratio (17) can be written in the form

$$(17) = \frac{\tilde{G}_{SH}^{(0)}}{G(0)} \frac{\int f_1 \tilde{H}}{\int f_1 H}$$

with $\tilde{H}(\beta) = \tilde{G}_{SH}(\beta) / \tilde{G}_{SH}(0)$ and $H(\beta) \equiv G(\beta) / G(0)$ and as

$$\tilde{H} < H, \quad \beta > 0$$

then

$$\int f_1 \tilde{H} / \int f_1 H < 1$$

and (17) < $\tilde{G}_{SH}^{(0)} / G(0)$.

Combining (18) and (19) our result is:

$$1 < \langle n \rangle_{SH} / \langle n \rangle < \frac{\sigma}{\sigma_{SH}} \frac{G_{SH}^{(0)}}{G(0)} \quad (20)$$

Note that these bounds are not related to quantities from multiplicity measurements. They exclusively depend on unitarity and centrality of semi-hard interactions [Eqs. (8), (14) and (15)].

In order to test the bounds (20) we use the model of Ref. 5). As this model fits the minijets cross-section, the inelastic cross-section and the differential elastic cross-section at the CERN Collider energies it probably gives a good

parametrization of the overlap function. The quantities needed are⁵⁾:

$$i) \quad G_{SH}(0) = 1 - \exp(-2\chi_{SH}(0)) \quad (21)$$

where χ_{SH} is the semi-hard eikonal with

$$\chi_{SH}(0, s) = \frac{1}{2} \frac{0.71}{12\pi} \frac{1}{0.389} \sigma_{SH}^{(0)}(s) \quad (22)$$

being given by the product of the Fourier transform of the square of the dipole form factor by the bare semi-hard cross-section $\sigma_{SH}^{(0)}(s)$ given, in the $0.1 \lesssim \sqrt{s} \lesssim 40$ TeV region, by

$$\sigma_{SH}^{(0)}(s) = 0.15 s^{0.35} \text{ mb} \quad (23)$$

$$ii) \quad G(0) = 1 - \exp(-2\chi(0)) \quad (24)$$

with

$$\chi(0, s) = \chi_S(0, s) + \chi_{SH}(0, s), \quad (25)$$

where χ_S is the soft eikonal and

$$\chi_S(0, s) = 1.4. \quad (26)$$

iii) the ratio σ_{SH}/σ can be taken from Ref. 5) or directly from experimental data.

The data on $\langle n \rangle_{SH}/\langle n \rangle$ were obtained from UA1 preliminary results¹⁰⁾. It was assumed that the multiplicities from minimum bias events and multiplicities from inelastic events were the same.

The test of (20) is shown in Fig. 2.

The bounds (20), in a Gaussian approximation for the overlap functions,

$$G(b^2, s) \simeq \frac{\sigma(s)}{\pi} \frac{1}{B(s)} \exp(-b^2/B(s)) \quad (27)$$

and

$$G_{SH}(b^2, s) \approx \frac{\sigma_{SH}(s)}{\pi} \frac{1}{B_{SH}(s)} \exp(-b^2/B_{SH}(s)) \quad (28)$$

can be written as

$$1 < \langle n \rangle_{SH} / \langle n \rangle < B(s)/B_{SH}(s) \quad (29)$$

making very transparent that the origin of the upper bound is in the centrality of the semi-hard interactions. In a rough (unabsorbed) approximation for the ratios $B(s)/B_{SH}(s)$ one can still obtain

$$\langle n \rangle_{SH} / \langle n \rangle < (1 - \sigma_{SH}/\sigma)(B_s/B_{SH} - 1) + 1 \quad (30)$$

where B_s is the soft amplitude slope parameter. The ratio σ_{SH}/σ increases rapidly with energy due to the large value of the semi-hard Pomeron intercept. The ratio B_s/B_{SH} also increases because of shrinking of the soft amplitude. The fact that at low energy the upper bound is approximately constant (see Fig. 2) means that the two effects tend to compensate, the shrinking part of the slope of the soft amplitude being of the same order of the constant part of it or of the semi-hard slope. As energy increases $\sigma_{SH}/\sigma \rightarrow 1$ and $\langle n \rangle_{SH}/\langle n \rangle \rightarrow 1$.

Next I compute the ratio of the normalized second moments of the multiparticle distributions of the minijet and inelastic events. From (16) one has, in a simplified notation,

$$C_{2SH}/C_2 = \frac{\int f_2 \tilde{G}}{\int f_2 G} \left(\frac{\int f_1 G}{\int f_1 \tilde{G}} \right)^2, \quad (31)$$

C_{2SH} and C_2 being the semi-hard and inelastic KNO second moments, respectively.

As the function $\tilde{G}_{SH}$ is narrower than the function G , one has for the dispersions of the distributions $\tilde{G}_{SH}$ and G , considered as functions f_1 , the inequality

$$\int f_1^2 \tilde{G} - \left(\int f_1 \tilde{G} \right)^2 < \int f_1^2 G - \left(\int f_1 G \right)^2 \quad (32)$$

and, from (18) and (31),

$$C_{2SH}/C_2 < \frac{\int f_2 \tilde{G}}{\int f_2 G} \frac{\int f_1^2 G}{\int f_1^2 \tilde{G}} \quad (33)$$

or

$$C_{2SH}/C_2 < \frac{\int c_2 \tilde{\rho}}{\int c_2 \rho} \quad (34)$$

with

$$\rho(\beta, s) \equiv f_1^2(\beta, s) G(\beta, s) / \int f_1^2 G \quad (35)$$

and

$$\tilde{\rho}(\beta, s) \equiv f_1^2(\beta, s) \tilde{G}_{SH}(\beta, s) / \int f_1^2 \tilde{G}$$

being normalized functions and

$$c_2(\beta, s) \equiv f_2(\beta, s) / f_1^2(\beta, s) \quad (36)$$

being the KNO second moment at a fixed impact parameter, see (5).

Finally, as physically more particles are produced at small values of the impact parameter, general statistical arguments tell us that $c_2(\beta, s)$ should be, at a given s , a function increasing with b . The larger the number of particle emitting sources is, the narrower is the KNO distribution: smaller b values correspond to smaller c_2 .

Using now arguments similar to the ones used to obtain (18), from (34) I arrive at the result:

$$C_{2SH}/C_2 < 1, \quad (37)$$

i.e., KNO multiplicity distributions in minijet events are narrower than in elastic ones. From the UA1 Collaboration¹⁰⁾ results, one finds for the ratio (37) the value ≈ 0.9 .

In conclusion, perturbative QCD plays an important but indirect role in determining the differences between minijet and no-jet multiplicity distributions. Centrality of the semi-hard interactions and general statistical arguments naturally lead to the observed differences and to the bounds (20) and (37).

I would like to mention two papers that have recently called the attention respectively to the importance of the impact parameter summation¹¹⁾ and of statistical arguments¹²⁾ in understanding multiparticle distributions.

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FIGURE CAPTIONS

Fig. 1: The inelastic overlap function G , (2), and the modified minijet overlap function $\tilde{G}_{SH}$, (12) and (13), as functions of the scaled variable $\beta \equiv \pi b^2/\sigma$, (7). The semi-hard interaction is more central. For both cases $\int G = \int \tilde{G} = 1$.

Fig. 2: Bounds for the ratio $\langle n \rangle_{SH}/\langle n \rangle$ of the minijet event multiplicities over the inelastic event multiplicities, Eq. (20), as function of energy. Data points are from the UA1 Collaboration.

Fig. 1

Fig. 2