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Experimental Test of the PCAC-Hypothesis in Charged Current Neutrino and Antineutrino Interactions on Protons

*Birmingham-CERN-Imperial College-München(MPI)-Oxford-
University College Collaboration (WA21)*

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ABSTRACT

Data obtained with the bubble chamber BEBC at CERN are used for the first significant test of Adler's prediction for the neutrino and antineutrino-proton scattering cross sections at vanishing four-momentum transfer squared Q^2 . An Extended Vector Meson Dominance Model (EVDM) is applied to extrapolate Adler's prediction to experimentally accessible values of Q^2 . The data show good agreement with Adler's prediction for $Q^2 \rightarrow 0$ thus confirming the PCAC hypothesis in the kinematical region of high leptonic energy transfer $\nu > 2$ GeV. The good agreement of the data with the theoretical predictions also at higher Q^2 , where the EVDM terms are dominant, also supports this model. However, an EVDM calculation without PCAC is clearly ruled out by the data.

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1. Introduction

The hypothesis of a “Partially Conserved Axial Current” (PCAC) was put forward by *Gell-Mann* and *Levy* in 1960 [1]. It connects the axial part of the weak hadronic current with the charged pion field. The most notable success of the PCAC-hypothesis was in the explanation of the *Goldberger-Treiman*-relation [2].

In 1964 *Adler* [3] proposed a test of PCAC in high energy neutrino experiments: using PCAC he showed that the cross section for neutrino-nucleon scattering in the forward direction can be expressed in terms of the pion-nucleon cross section. First tests of *Adler*’s prediction at energies of the CERN PS [4, 5] suffered from low statistics and a poor experimental resolution. A big step forward was possible due to the BEBC experiments with a high energy neutrino beam from the SPS. Two pieces of evidence for the PCAC hypothesis have been found so far: The reasonable agreement of the data with *Adler*’s model [6] for neutrino-induced $\Delta(1232)$ production in hydrogen [7], and the success of the PCAC based model of *Rein* and *Sehgal* [8] in predicting the differential cross section for coherent pion production off neon nuclei by antineutrinos [9].

The subject of the present analysis is a test of *Adler*’s prediction using data from the neutrino and antineutrino-proton scattering experiment, WA21, with BEBC. A similar analysis with (anti)neutrino-Neon scattering data can be found in Ref. 10. These are the first two significant tests of PCAC in neutrino scattering in the kinematical regime of leptonic energy transfer ν above 2 GeV and thus complement previous tests which all have been performed at comparatively low ν .

The paper is organized as follows: After an outline of the theoretical concepts (Section 2), the experimental details are presented (Section 3). In Section 4 the methods and results of the experimental data analysis are described; the evaluation of the theoretical predictions is presented in Section 5. The comparison of experimental data and theoretical results and the conclusions follow in Sections 6 and 7.

2. Theoretical Concepts

This paper deals with the charged current (CC) scattering of muon (anti)neutrinos off protons, i.e. with the reactions

$$\nu_{\mu}p \rightarrow \mu^{-}X \quad \text{and} \quad \bar{\nu}_{\mu}p \rightarrow \mu^{+}X, \quad (1)$$

where X stands for the final hadronic system. Apart from the degrees of freedom of the particles in X, the kinematics of the reactions (1) is fully described by the three four-momenta k_{α} (neutrino), k'_{α} (muon) and P_{α} (proton); the energies of the (anti)neutrino and the muon in the laboratory system are denoted by E_{ν} and E_{μ} . Derived from these quantities are the four-momentum transfer $q_{\alpha} = k_{\alpha} - k'_{\alpha}$, the energy transfer $\nu = q_0 = E_{\nu} - E_{\mu}$ and the Lorentz-invariants $Q^2 = -q_{\alpha}q^{\alpha} > 0$ and $W^2 = (P_{\alpha} + q_{\alpha})(P^{\alpha} + q^{\alpha})$ (squared invariant mass of X).

a) The cross section for forward scattering

Consider now the case of vanishing muon scattering angle ϑ in the laboratory, the so called "parallel configuration", which corresponds to minimal Q^2 . Adler demonstrated, that as a consequence of the Conserved Vector Current (CVC) hypothesis the cross section for neutrino^{†)} proton scattering at $\vartheta = 0$ (and neglecting the muon mass m_{μ}) depends only on the divergence $\partial_{\alpha}H^{\alpha(A)}$ of the axial part $H^{\alpha(A)}$ of the weak hadronic current. The PCAC hypothesis (valid for the strangeness S and charm C conserving part of $H^{\alpha(A)}$) reads

$$\partial_{\alpha}H^{\alpha(A)}\big|_{\Delta S=\Delta C=0} = f_{\pi}M_{\pi}^2\Phi_{\pi^{+}}, \quad (2)$$

where $f_{\pi} = 0.932 M_{\pi}$ [11] is the pion decay constant, M_{π} the pion mass and $\Phi_{\pi^{+}}$ denotes the interpolating pion field operator.^{‡)} Using eq. (2) one can express $\sigma(\nu_{\mu}p \rightarrow \mu^{-}X)$ for reactions with $\Delta S = \Delta C = 0^{*})$ in terms of the $\pi^{+}p$ cross section. The result is Adler's formula for $m_{\mu} = 0$ [3]:

$$\left. \frac{d^2\sigma(\nu_{\mu}p \rightarrow \mu^{-}X)}{dQ^2 d\nu} \right|_{\substack{Q^2=0 \\ m_{\mu}=0 \\ \Delta S=\Delta C=0}} = \left(\frac{G_F f_{\pi}}{2\pi} \right)^2 \frac{2E_{\mu}}{\nu E_{\nu}} \sigma(\pi^{+}p \rightarrow X; Q^2) \bigg|_{Q^2=0}, \quad (3)$$

^{†)} Only the case of neutrino scattering is described here. The results for antineutrino scattering follow by the replacements $\nu_{\mu} \rightarrow \bar{\nu}_{\mu}$, $\mu^{-} \rightarrow \mu^{+}$, $W^{+} \rightarrow W^{-}$ and $\pi^{+}, \rho^{+}, A_1^{+} \rightarrow \pi^{-}, \rho^{-}, A_1^{-}$.

^{‡)} Field theoretical details can be found in Ref. 12.

^{*}) All the rest of this paper will be restricted to reactions without change of strangeness S and charm C , i.e. X carries no open strangeness or charm.

where G_F is the weak *Fermi* coupling constant and $\sigma(\pi^+ p \rightarrow X; Q^2)$ is the cross section of virtual pions which carry the four-momentum q_α of the W-boson exchanged in the νp -reaction.

Corrections to eq. (3) due to the finite muon mass and to small, but nonzero scattering angles ϑ can be calculated in the one-pion-exchange approximation. The result is an additional multiplicative term on the right-hand side of eq. (3):

$$\left. \frac{d^2\sigma(\nu_\mu p \rightarrow \mu^- X)}{dQ^2 d\nu} \right|_{\substack{Q^2 \lesssim M_\pi^2 \\ \Delta S = \Delta C = 0}} = \left(\frac{G_F f_\pi}{2\pi} \right)^2 \frac{2E_\mu}{\nu E_\nu} \sigma(\pi^+ p \rightarrow X; Q^2) \cdot \left[1 - \frac{\nu}{E_\mu} \frac{m_\mu^2}{Q^2 + M_\pi^2} + \frac{\nu^2}{4E_\nu E_\mu} \frac{m_\mu^2(Q^2 + m_\mu^2)}{(Q^2 + M_\pi^2)^2} \right]. \quad (4)$$

This formula is valid for small Q^2 , where the scale is given by $M_\pi^2 \approx 0.02 \text{ GeV}^2$.

b) Extrapolation to higher Q^2

In order to obtain predictions which can be compared to the data, *Adler's* relation (4) has to be extrapolated to a region of Q^2 substantially higher than M_π^2 .^{†)} For this purpose an approach is used based on an Extended Vector Meson Dominance Model (EVDM) put forward by *Piketty* and *Stodolsky* [13]. According to this model the intermediate W-boson virtually dissociates into a quark-antiquark-pair which strongly interacts with the target nucleon yielding the main contribution to the W-nucleon cross section. The EVDM assumes that this quark-antiquark-pair predominantly behaves like the pion or the lightest vector or axial vector mesons, the ρ and its chiral partner A_1 (c.f. the *Feynman*-diagram in Fig. 1). In this picture the ρ couples to the vector part and the pion and the A_1 couple to the axial part of the weak hadronic current.

Without additional assumptions and neglecting small terms of the order m_μ^2 originating from the pion-exchange, the EVDM yields a cross section $\sigma(\nu_\mu p \rightarrow \mu^- X)$ which

^{†)} The experimental resolution allows the determination of Q^2 with an accuracy of about 0.1 GeV^2 (see Section 4b).

is completely given by the ρ and A_1 -exchange:

$$\begin{aligned}
\left. \frac{d^2\sigma(\nu_\mu p \rightarrow \mu^- X)}{dQ^2 d\nu} \right|_{\substack{m_\mu=0 \\ \Delta S=\Delta C=0 \\ \text{EVDM}}} &= \left(\frac{G_F}{2\pi} \right)^2 \frac{Q^2 |\vec{q}|}{E_\nu^2} \frac{1}{1-\epsilon} \\
&\cdot \sum_{V=\rho, A_1} \left(\frac{f_{V^\pm}}{Q^2 + M_V^2} \right)^2 \left[\sigma_T(V^+ p \rightarrow X; Q^2) + \epsilon \sigma_L(V^+ p \rightarrow X; Q^2) \right] \\
&+ C \cdot \left(\frac{G_F}{\pi} \right)^2 Q^2 \frac{E_\nu + E_\mu}{2E_\nu^2} \frac{f_{\rho^\pm} f_{A_1^\pm}}{(Q^2 + M_\rho^2)(Q^2 + M_{A_1}^2)} \\
&\cdot \sqrt{\sigma_T(\rho^+ p \rightarrow X; Q^2) \cdot \sigma_T(A_1^+ p \rightarrow X; Q^2)};
\end{aligned} \tag{5}$$

M_V are the masses of the ρ and A_1 , the indices T and L indicate cross sections of transversely and longitudinally polarized mesons, the $f_{V^\pm}$ are the lepton- $\rho^\pm$ and lepton- $A_1^\pm$ coupling constants, C is a real number between -1 and 1 with opposite signs for neutrino and antineutrino scattering, and

$$\epsilon = \left(1 + \frac{2|\vec{q}|^2}{4E_\nu E_\mu - Q^2} \right)^{-1}, \quad 0 < \epsilon < 1 \tag{6}$$

($|\vec{q}|$ being the modulus of the three-momentum transfer). The sum in eq. (5) gives the “diagonal” $\rho \times \rho$ and $A_1 \times A_1$ -contributions to $\sigma(\nu_\mu p \rightarrow \mu^- X)$, both of which resemble the familiar expression for electron-proton scattering [14, 15]. The last term in eq. (5) is (for $|C| = 1$) an upper limit for the contribution of the ρ - A_1 -interference[†]). The range in Q^2 where eq. (5) is expected to be a good approximation is $Q^2 \lesssim M_\rho^2 \approx 0.6 \text{ GeV}^2$.

Apart from m_μ^2 -terms the cross section (5) vanishes linearly with Q^2 as $Q^2 \rightarrow 0$, in contrast to Adler’s prediction (4). Hence, EVDM does not contain PCAC. In order to combine these two models and to restore Adler’s formula in eq. (5), an assumption has to be made which goes beyond the PCAC hypothesis and the EVDM. The finite PCAC contribution to the neutrino cross section at $Q^2 = 0$ is due to a nonconserved part of the weak hadronic current. In the framework of the EVDM such a term can only be contained in the contribution proportional to $\sigma_L(A_1^+ p \rightarrow X; Q^2)$ in eq. (5). Thus Adler’s prediction (3) is reproduced if the limiting behaviour of this term is postulated to be [13]

$$\frac{f_{A_1}^2}{M_{A_1}^4} Q^2 \sigma_L(A_1^+ p \rightarrow X; Q^2) \xrightarrow{Q^2 \rightarrow 0} f_\pi^2 \sigma(\pi^+ p \rightarrow X; Q^2). \tag{7}$$

Henceforth $\sigma_L(A_1^+ p \rightarrow X; Q^2)$ will be split into a part $\sigma_L^{(0)}$ obeying eq. (7) (interpreted as an equation) and a part $\sigma_L^{(1)}$ vanishing as $Q^2 \rightarrow 0$. Thus an extra term containing

[†]) This term differs from the corresponding expression in Ref. 13 by the additional factor $(E_\nu + E_\mu)/2E_\nu$.

$\sigma_L^{(0)}$ is added to eq. (5) which is motivated solely by PCAC and turns out to be dominant at $Q^2 \lesssim 0.1 \text{ GeV}^2$. A comparison of the theoretical prediction with the data in this kinematical regime represents a test of the PCAC hypothesis.

After including the pion exchange terms from eq. (4) (where the EVDM yields an additional A_1 -propagator for the π - A_1 -interference term), the final expression for the neutrino proton cross section reads

$$\begin{aligned}
\left. \frac{d^2\sigma(\nu_\mu p \rightarrow \mu^- X)}{dQ^2 d\nu} \right|_{\Delta S = \Delta C = 0} &= \left(\frac{G_F}{2\pi} \right)^2 f_\pi^2 \frac{|\vec{q}|}{E_\nu^2} \frac{\epsilon}{1-\epsilon} \sigma(\pi^+ p \rightarrow X; Q^2) \\
&\cdot \left[\left(\frac{M_{A_1}^2}{Q^2 + M_{A_1}^2} \right)^2 - \frac{\nu}{E_\mu} \frac{M_{A_1}^2}{Q^2 + M_{A_1}^2} \frac{m_\mu^2}{Q^2 + M_\pi^2} + \frac{\nu^2}{4E_\nu E_\mu} \frac{m_\mu^2(Q^2 + m_\mu^2)}{(Q^2 + M_\pi^2)^2} \right] \\
&+ \left(\frac{G_F}{2\pi} \right)^2 \frac{Q^2 |\vec{q}|}{E_\nu^2} \frac{1}{1-\epsilon} \cdot \left(\frac{f_{\rho^\pm}}{Q^2 + M_\rho^2} \right)^2 \left[\sigma_T(\rho^+ p \rightarrow X; Q^2) + \epsilon \sigma_L(\rho^+ p \rightarrow X; Q^2) \right] \\
&+ \left(\frac{G_F}{2\pi} \right)^2 \frac{Q^2 |\vec{q}|}{E_\nu^2} \frac{1}{1-\epsilon} \cdot \left(\frac{f_{A_1^\pm}}{Q^2 + M_{A_1}^2} \right)^2 \left[\sigma_T(A_1^+ p \rightarrow X; Q^2) + \epsilon \sigma_L^{(1)}(A_1^+ p \rightarrow X; Q^2) \right] \\
&+ C \cdot \left(\frac{G_F}{\pi} \right)^2 Q^2 \frac{E_\nu + E_\mu}{2E_\nu^2} \frac{f_{\rho^\pm} f_{A_1^\pm}}{(Q^2 + M_\rho^2)(Q^2 + M_{A_1}^2)} \\
&\cdot \sqrt{\sigma_T(\rho^+ p \rightarrow X; Q^2) \cdot \sigma_T(A_1^+ p \rightarrow X; Q^2)}.
\end{aligned} \tag{8}$$

c) Calculation of virtual π , ρ and A_1 scattering cross sections

The theoretical prediction (8) is not suitable for a comparison with the experimental data, because scattering cross sections of charged ρ and A_1 -mesons as well as those of virtual pions are not accessible to experimental measurements. In order to replace these cross sections by well measured ones a series of further assumptions and approximations will be made:

1. Chiral symmetry implies that the ρ and A_1 behave alike in hadronic interactions; this is used to set

$$f_{A_1^\pm} = f_{\rho^\pm} \tag{9a}$$

$$\sigma_T(A_1^+ p \rightarrow X; Q^2) = \sigma_T(\rho^+ p \rightarrow X; Q^2). \tag{9b}$$

The longitudinal cross sections cannot be related that easily; in fact the finiteness of $\sigma_L^{(0)}(A_1^+ p \rightarrow X; Q^2)$ at $Q^2 = 0$ is a manifestation of chiral symmetry breaking. It is plausible though to put

$$\sigma_L^{(1)}(A_1^+ p \rightarrow X; Q^2) = \sigma_L(\rho^+ p \rightarrow X; Q^2). \tag{9c}$$

2. In the next step the scattering cross section of charged ρ -mesons is replaced by that of the ρ^0 . This is achieved by the total cross section relation^{†)}

$$\sigma(\rho^+ p; Q^2) \approx \frac{\sigma(\pi^+ p; Q^2)}{\sigma(\pi^0 p; Q^2)} \cdot \sigma(\rho^0 p; Q^2) \quad (10)$$

which is motivated by quark counting and is expected to hold for $W \gtrsim 2 \text{ GeV}$. The present analysis will be restricted to the case of total cross sections.

3. Using isospin symmetry the total cross section $\sigma(\pi^0 p; Q^2)$ can be calculated to be

$$\sigma(\pi^0 p; Q^2) = \frac{\sigma(\pi^+ p; Q^2) + \sigma(\pi^- p; Q^2)}{2}. \quad (11)$$

4. It is assumed that cross sections of virtual and real pions on protons at a given centre of mass energy only differ by their flux factors. The energy ν_π of the on-shell pion is related to the energy ν of the off-shell pion by

$$\nu_\pi = \nu - \frac{Q^2 + M_\pi^2}{2M_p}, \quad (12)$$

and the above approximation should hold if $(\nu - \nu_\pi)/\nu \ll 1$, i.e. for $\nu \gg Q^2/M_p$; in the data analysis $\nu > 2 \text{ GeV}$ is required. The virtual-pion proton cross section is then given by

$$\sigma(\pi^+ p; Q^2) \approx \sqrt{\frac{\nu_\pi^2 - M_\pi^2}{\nu^2 + Q^2}} \cdot \sigma(\pi^+ p) = \xi \cdot \sigma(\pi^+ p). \quad (13)$$

The ratio ξ of the flux factors has values near 1 for small Q^2 ; $Q^2 = 0.1 \text{ GeV}^2$ and $\nu > 2 \text{ GeV}$ yield $0.95 < \xi < 1$.

5. The Vector Meson Dominance Model (VDM)^{‡)} is used to express the virtual cross sections $\sigma_{T,L}(\rho^0 p; Q^2)$ in terms of photo- and electroproduction cross sections. For electron-proton scattering the VDM gives [13]:

$$\frac{d^2\sigma(e^- p \rightarrow e^- X)}{dQ^2 d\nu} = 2\alpha^2 \frac{1}{1 - \epsilon} \frac{|\vec{q}|}{E_e^2 Q^2} \cdot \sum_{V=\rho,\omega,\phi} \left(\frac{f_{V^0}}{Q^2 + M_V^2} \right)^2 \left[\sigma_T(V^0 p \rightarrow X; Q^2) + \epsilon \sigma_L(V^0 p \rightarrow X; Q^2) \right] \quad (14)$$

^{†)}Total cross sections will be denoted by $\sigma(\pi^+ p; Q^2)$, $\sigma(\rho^+ p; Q^2)$ etc. The absence of the argument Q^2 indicates the cross section of on-shell particles.

^{‡)}A detailed review of experimental and theoretical aspects of the VDM can be found in Ref. 16.

where $\alpha = e^2/4\pi$ and the f_{V0} are the photon–vector meson coupling constants. The experimentally inferred f_{V0} values are compiled in Ref. 16, a consistent set can be found in Ref. 17 ($f_{\rho^0}^2 = 0.015 \text{ GeV}^4$, $f_\omega^2 = 0.0022 \text{ GeV}^4$). The ϕ –contribution in eq. (14) turns out to be negligible in photoproduction experiments [13] and will not be taken into account. Using $M_\rho = 0.770 \text{ GeV} \approx M_\omega = 0.783 \text{ GeV}$ [18] (thus also neglecting effects due to the finite widths of the ρ and the ω) and the assumption $\sigma(\rho^0 p \rightarrow X) \approx \sigma(\omega p \rightarrow X)$, which is motivated by quark counting, one can eliminate the isoscalar ω –contribution to the electron proton cross section to good approximation by simply scaling eq. (14) with the factor

$$s^{IV} = \frac{f_{\rho^0}^2/M_\rho^4}{f_{\rho^0}^2/M_\rho^4 + f_{\omega^0}^2/M_\omega^4}. \quad (15)$$

The label IV stands for “isovector”, indicating that s^{IV} projects out the contribution of the isovector part of the electromagnetic current to the cross section (14). The numerical value for s^{IV} calculated from the coupling constants given above is $s^{IV} = 0.883$; using other values quoted in Ref. 16 results in s^{IV} values between 0.870 and 0.893, from which an error $\delta s^{IV} = 0.01$ is deduced. Using eqs. (14) and (15) the combination $\sigma_T(\rho^0 p; Q^2) + \epsilon \sigma_L(\rho^0 p; Q^2)$ can now be expressed by the measured electron–proton cross section. The error caused by the different lepton masses in electron and neutrino scattering is of the order $(m_\mu/E_\mu)^2$ and is negligible for the data used in this analysis ($E_\mu > 5 \text{ GeV}$).

6. Finally $\sigma_T(\rho^0 p; Q^2)$ in the interference term of eq. (8) is derived from the photoproduction cross section. The VDM yields [14]

$$\sigma_T(\rho^0 p \rightarrow X; Q^2) = \frac{(Q^2 + M_\rho^2)^2}{4\pi\alpha f_{\rho^0}^2} s^{IV} \sigma_T(\gamma p \rightarrow X; Q^2), \quad (16)$$

where the ρ –contribution has been projected out of the total photon–proton cross section by s^{IV} in the same manner as in the previous point 5. In the VDM $\sigma_T(\rho^0 p; Q^2)$ is assumed to depend only weakly on Q^2 , which is confirmed by the success of the VDM in predicting photoproduction cross sections [19] and the Q^2 –dependence of electroproduction cross sections (see Section 5). Therefore the right hand side of eq. (16) can be evaluated at $Q^2 = 0$ using the photoproduction cross section which is well measured as a function of the photon energy $E_\gamma = \nu$.

Inserting the relations (9–16) into eq. (8) finally yields the theoretical prediction for

the inclusive νp cross section which can directly be compared to the data [20]:

$$\begin{aligned}
\left. \frac{d^2\sigma(\nu_\mu p)}{dQ^2 d\nu} \right|_{\Delta S=\Delta C=0} &= \left(\frac{G_F}{2\pi} \right)^2 f_\pi^2 \frac{|\vec{q}|}{E_\nu^2} \frac{\epsilon}{1-\epsilon} \sqrt{\frac{\nu_\pi^2 - M_\pi^2}{\nu^2 + Q^2}} \sigma(\pi^+ p) \\
&\cdot \left[\left(\frac{M_{A_1}^2}{Q^2 + M_{A_1}^2} \right)^2 - \frac{\nu}{E_\mu} \frac{M_{A_1}^2}{Q^2 + M_{A_1}^2} \frac{m_\mu^2}{Q^2 + M_\pi^2} + \frac{\nu^2}{4E_\nu E_\mu} \frac{m_\mu^2(Q^2 + m_\mu^2)}{(Q^2 + M_\pi^2)^2} \right] \\
&+ 2 \left(\frac{G_F}{4\pi\alpha} \right)^2 Q^4 \frac{f_{\rho^\pm}^2}{f_{\rho^0}^2} \frac{2\sigma(\pi^+ p)}{\sigma(\pi^- p) + \sigma(\pi^+ p)} \left[1 + \left(\frac{Q^2 + M_\rho^2}{Q^2 + M_{A_1}^2} \right)^2 \right] s^{IV} \frac{d^2\sigma(ep)}{dQ^2 d\nu} \\
&+ C \left(\frac{G_F}{2\pi} \right)^2 \frac{Q^2}{2\pi\alpha} \frac{f_{\rho^\pm}^2}{f_{\rho^0}^2} \frac{2\sigma(\pi^+ p)}{\sigma(\pi^- p) + \sigma(\pi^+ p)} \frac{E_\nu + E_\mu}{E_\nu^2} \frac{M_\rho^4}{(Q^2 + M_\rho^2)(Q^2 + M_{A_1}^2)} s^{IV} \sigma(\gamma p) .
\end{aligned} \tag{17}$$

The ratio of the coupling constants $f_{\rho^\pm}$ and f_{ρ^0} is derived from an application of the EVDM to nuclear beta decay [13] yielding $f_{\rho^\pm}/f_{\rho^0} = \sqrt{2}$.

3. Experimental Details

a) The Detector

The data analysed in this paper has been taken at CERN in the years 1977 to 1983, where the BEBC hybrid detector has been exposed in several runs to the SPS neutrino and antineutrino wide band beams. The three main components of the detector are

1. The Big European Bubble Chamber BEBC (see Ref. 21 for technical details). It was filled with approx. 30 m^3 of liquid hydrogen and kept in a vertical magnetic field of 3.5 Tesla. Momenta of charged particles are determined with high accuracy (average error below 2%) from the direction and curvature of their tracks. Neutral particles can only be detected if they produce charged secondaries by interaction or decay.
2. The External Muon Identifier (EMI) [22]. This device is divided into three planes of multiwire proportional counters, one upstream of BEBC and two downstream. The two downstream planes are separated from BEBC by the magnetic coils and from each other by an additional iron shielding of about one meter thickness. Almost all hadrons, photons and electrons are stopped before they are able to reach the two downstream planes of the EMI; tracks extrapolating to hits in these planes can thus be identified as muons. The upstream EMI plane serves as a veto counter in order to eliminate muons not originating from events inside BEBC.
3. The Internal Picket Fence (IPF) [23], which was operational in the 1981 and 1983 runs. It consists of an array of vertical proportional tubes surrounding BEBC like a fence in the immediate vicinity of the BEBC vessel's outer surface. The IPF gives information about the timing of events in BEBC and thus improves the linking of neutral particles to primary vertices and the muon identification. Its upstream part supports the function of the EMI veto plane.

b) Selection of Events

In total 458400 pictures in the neutrino and 307400 pictures in the antineutrino beam have been taken, each picture consisting of 4 stereoscopic photographs of the BEBC volume. Events found in these pictures have been accepted for this analysis if they fulfilled the following criteria:

- The primary vertex lies in a fiducial volume of about 19 m^3 . This condition ensures sufficient track lengths and accuracy of geometrical track reconstruction.
- The event has exactly one EMI-identified muon with an energy greater than 5 GeV. By this requirement reactions of the type (1) are separated from a background of neutral

current and hadron induced events.

- The number N_{ch} of charged particles in the final state (“prongs”) has to be greater or equal to 3. Since N_{ch} is always odd, this cut only excludes the one-prong events, i.e. events where the muon is the only charged particle in the final state. One-prong events (which occur only in antineutrino scattering) are rejected because they have a low and not well known scanning efficiency (Section 4) and the event energy is poorly constrained.
- The estimated neutrino energy (see below) has a value between 10 GeV (neutrino) or 20 GeV (antineutrino) and 200 GeV. These cuts are to avoid regions with uncertain neutrino flux and to minimize the influence of one-prong events in the antineutrino case.
- For theoretical reasons ν is required to be greater than 2 GeV (see Section 2); this cut also implies, that for low Q^2 the hadronic invariant mass is restricted to $W \gtrsim 2$ GeV.
- The crucial kinematical variable is $Q^2 \approx 4E_\nu E_\mu \sin^2(\vartheta/2)$. In order keep the uncertainty in Q^2 low, events with a relative muon momentum error $\delta|\vec{k}'|/|\vec{k}'| > 0.2$ are rejected.

In total 14772 events in neutrino and 8970 events in antineutrino scattering fulfil all selection criteria.

c) The Monte Carlo Program

The raw data have to be corrected for systematic errors resulting from experimental insufficiencies and from the cuts applied. The corrections are performed by means of a Monte Carlo (MC) program in which the experiment is simulated.

The generation of events in the MC (“initial” events) is based on a parametrisation of the neutrino flux density as a function of the neutrino energy, on the quark distribution functions for the proton [24] and on the weak couplings as given by the standard model. By knowing the flavours of the quarks involved in the weak reaction this procedure allows to estimate the contribution from events not conserving strangeness or charm.^{†)} The hadronic final state of the “initial” events is generated according to a longitudinal phase space (LPS) model, where longitudinal and transverse momenta of the hadrons relative to the current direction are assumed to be independent.

By a further MC simulation of the experimental situation the “initial” events are transformed to “modified” events which correspond to the experimental data. The comparison of

^{†)}In the limit of low Q^2 this approach corresponds to using the model for deep inelastic scattering, extrapolated to $x_{Bj} = Q^2/2M_p\nu \rightarrow 0$.

the latter two samples has been used to tune the MC carefully, whereas the comparison of “initial” and “modified” events yields the correction factors to be applied to the data.

Some selected comparisons of experimental data to MC results are shown in Fig. 2; detailed information on the MC and its applications for this analysis can be found in Ref. 20.

d) Estimation of the Neutrino Energy

In more than 80% of the selected events one or several neutral particles escape detection; this is mainly due to the poor detection efficiency of about 7% for photons which originate mainly from π^0 -decays. In these events with unseen neutral particles, the neutrino energy E_ν is estimated on an event-by-event basis using the *Heilmann* method [25]. This method is based on the assumption that charged and neutral hadrons have on average the same direction; the neutrino energy is calculated according to

$$E_\nu^{(H)} = k'_L + P_L^s \left(1 + \frac{P_T^u}{\sum_s p_T^s} \right), \quad (18)$$

where k'_L and P_L^s are the components of the muon and the total seen hadronic momentum in the direction of the neutrino beam, P_T^u (determined from transverse momentum conservation) is the modulus of the unseen momentum transverse to the beam direction and the sum contains the moduli of the transverse momenta of all seen hadrons.

Systematic over- or under-estimates of E_ν by the *Heilmann* method were investigated with MC events, for which the “true” value of E_ν is known. It turned out that on average $E_\nu^{(H)}$ is too small by about 2 GeV. In order to correct for this shift, the average difference of the true and estimated values ν and $\tilde{\nu}$ of the leptonic energy transfer was determined as a function of $\tilde{\nu}$ for MC events and fitted by a polynomial in $\tilde{\nu}$. The final result $\tilde{\tilde{E}}_\nu$ for the estimated (anti)neutrino energy of an event was obtained by evaluating the polynomial at the appropriate $\tilde{\nu}$ and adding its value to $E_\nu^{(H)}$. Again by MC simulation it was shown that the mean difference $\langle \tilde{\tilde{E}}_\nu - E_\nu \rangle$ is negligible, and that the average error of $\tilde{\tilde{E}}_\nu$ is below 20%.

4. Experimental Results

The comparison between experimental results and theoretical predictions is carried out on the basis of event numbers and not on the basis of cross sections. This approach is chosen since it provides the highest possible significance in the comparison of data and theory. The theoretically calculated inclusive differential cross section $d^2\sigma(\nu p)/dQ^2 d\nu$ is converted into expected differential event distributions by convolution with the neutrino flux spectrum $\Phi(E_\nu)$:

$$\frac{\Delta^3 N}{\Delta Q^2 \Delta \nu \Delta E_\nu} = \frac{1}{\Delta E_\nu} \int_{\Delta E_\nu} dE_\nu \Phi(E_\nu) \frac{1}{\Delta Q^2} \int_{\Delta Q^2} dQ^2 \frac{1}{\Delta \nu} \int_{\Delta \nu} d\nu \frac{d^2\sigma(\nu_\mu p)}{dQ^2 d\nu}, \quad (19)$$

where ΔE_ν , ΔQ^2 and $\Delta \nu$ are arbitrary intervals which also can comprise the whole range allowed kinematically.

The flux spectrum $\Phi(E_\nu)$ is determined by

$$\Phi(E_\nu) = \frac{1}{\sigma_0 E_\nu} \cdot \frac{dN(E_\nu)}{dE_\nu}, \quad (20)$$

where $dN(E_\nu)/dE_\nu = n(E_\nu)$ is the energy spectrum of all (anti)neutrino CC events and $\sigma_0 E_\nu$ is the known total cross section for CC reactions. The constant σ_0 has been measured in BEBC experiments [26] to be

$$\sigma_0 = \begin{cases} (0.474 \pm 0.029) \cdot 10^{-38} \text{ cm}^2 / \text{GeV} & \text{for } \nu p \text{ scattering} \\ (0.500 \pm 0.032) \cdot 10^{-38} \text{ cm}^2 / \text{GeV} & \text{for } \bar{\nu} p \text{ scattering.} \end{cases} \quad (21)$$

a) Measurement of the Event Energy Spectrum

The uncorrected event energy spectrum $\tilde{n}(E_\nu) = d\tilde{N}(E_\nu)/dE_\nu$ is obtained with the selection criteria of Section 3, except that the two cuts $\delta|\vec{k}'|/|\vec{k}'| < 0.2$ and $\nu > 2 \text{ GeV}$ are omitted. The spectrum $\tilde{n}(E_\nu)$ is shown in Fig. 3a, b for neutrino and antineutrino scattering, respectively; also shown is the result obtained by parametrising $\tilde{n}(E_\nu)$ with smoothed cubic spline functions (solid lines).

Corrections have to be applied in order to convert the raw distribution $\tilde{n}(E_\nu)$ to the “true” distribution $n(E_\nu)$. The following effects have been taken into account:

1. For several reasons the true distribution includes events which are not contained in $\tilde{n}(E_\nu)$. The major contributions to this difference are due to:
 - The muon energy cut $E_\mu > 5 \text{ GeV}$.
 - The one-prong events in antineutrino scattering.
 - Those events where the muon misses the EMI and is thus not identified. The geometrical EMI acceptance ϵ_g has been determined as a function of $|\vec{k}'|$ and

ϑ using a program which extrapolates tracks to the EMI. An average value of $\epsilon_g \approx 98\%$ has been found for $E_\mu > 5 \text{ GeV}$.

- A certain fraction of events is not found in the film scan. The scanning efficiencies for different event topologies have been estimated from independent double scans of the same film and are given in Table 1.

2. On the other hand $\tilde{n}(E_\nu)$ contains events where a hadron was falsely identified as a muon. The rate r_μ of fake muons has been determined as a function of modulus and transverse component of the muon momentum relative to the beam direction by combining track measurements and EMI information of different pictures and submitting them to the normal muon identification algorithm. The average of r_μ over the selected events is about 1%, however for low muon energies ($E_\mu < 10 \text{ GeV}$) r_μ increases considerably up to about 30%.

Another source of fake muons is the unobserved decay of pions or kaons in flight. Since the average hadron momenta are much lower than $5 \text{ GeV}/c$ this effect is negligible for $E_\mu > 5 \text{ GeV}$.

3. The smearing of E_ν causes a variation of both the shape and the integral of $n(E_\nu)$.

An E_ν dependent correction function has been calculated by MC simulations and has been suitably parametrised. The true energy distribution $n(E_\nu)$ is obtained by multiplying $\tilde{n}(E_\nu)$ with this correction function. The results for $\nu_\mu p$ and $\bar{\nu}_\mu p$ scattering are shown in Fig. 3 as dashed lines. Only for small E_ν , where most events with low muon energy and one-prong events occur, does the correction function considerably exceed an average value of about 1.1 to 1.2.

In order to estimate the contributions from the different effects to the total correction, the ratio of the corrected and raw total event numbers N^{tot} and $\tilde{N}^{\text{tot}}$ has been calculated for each single effect without considering the others. The resulting ratios are given in Table 2. Also shown is the ratio $N^{\text{tot}}/\tilde{N}^{\text{tot}}$ for the total correction; it is equal to the correction function averaged over the observed event energy spectrum $\tilde{n}(E_\nu)$. The errors quoted are purely statistical, i.e. they vanish as the number of generated MC events goes to infinity.

b) Measurement of differential distributions for the variables Q^2 , ν and E_ν at low Q^2

The most important kinematic variable for this analysis besides E_ν and ν is Q^2 . From a MC simulation it was found that the mean deviation of the measured value $\tilde{Q}^2$ from the true value Q^2 , $\langle |\tilde{Q}^2 - Q^2| \rangle$, is $\lesssim 0.1 \text{ GeV}^2$ for all Q^2 of interest (Fig. 4) and thus justifies a binning of Q^2 in intervals of 0.1 GeV^2 . In the most interesting region below 0.1 GeV^2 , Q^2 is determined with even better accuracy, so that the smearing of Q^2 in this interval is a minor

effect.

The theoretical result (17) together with eq. (19) predicts event numbers in a three-dimensional array of intervals in the kinematic variables Q^2 , ν and E_ν . Since the event numbers at Q^2 -values below some 0.1 GeV^2 are not sufficient to produce a detailed three-dimensional distribution one has to integrate in eq. (19) over the full range of at least one of the variables ν or E_ν . Consequently for each Q^2 -interval of width 0.1 GeV^2 those event numbers are determined which correspond to the following integrations in eq. (19):

- integration over the full ranges of ν and E_ν ,
- integration over the full range of E_ν in single ν -intervals chosen to be equidistant on a logarithmic scale,
- the same with interchanged rôles of ν and E_ν .

The full integration area for a fixed value of Q^2 is shown in Fig. 5. The boundaries of this area are given by the following conditions:

1. The energy transfer ν is restricted to $\nu > \nu^{\min} = 2 \text{ GeV}$ and $\nu < \nu^{\max} = 50 \text{ GeV}$. The first cut is introduced in Section 3, whereas the second limitation is used in order to exclude ν regions with very little statistics and kinematically badly smeared events.
2. For the (anti)neutrino energy E_ν ($E_{\bar{\nu}}$) the required limits are $E_\nu > E_\nu^{\min} = 10 \text{ GeV}$, $E_{\bar{\nu}} > E_{\bar{\nu}}^{\min} = 20 \text{ GeV}$, and $E_\nu, E_{\bar{\nu}} < E_\nu^{\max} = 150 \text{ GeV}$. The lower limit is motivated in Section 3, whereas the upper one is introduced for similar reasons as the $\nu^{\max}$ cut.
3. The muon energy cut $E_\mu > E_\mu^{\min} = 5 \text{ GeV}$ yields

$$\nu < E_\nu - E_\mu^{\min}. \quad (22)$$

4. For very small Q^2 there is another boundary resulting from the lower limit $Q^2 > m_\mu^2 \nu / E_\mu + O(m_\mu^4 / E_\mu^2)$:

$$\nu < E_\nu \frac{Q^2}{Q^2 + m_\mu^2} + O(m_\mu^4 / E_\mu^2); \quad (23)$$

for $Q^2 \gg m_\mu^2$ this is always guaranteed by eq. (22).

The raw event numbers determined according to the above scheme have to be corrected for reasons which differ from those for the correction of the event energy spectrum in the following points:

1. The muon energy cut is taken into account by choosing the appropriate integration limits both in the data and in the MC (see eq. (22) and Fig. 5).
2. The theory predicts a cross section for reactions without a change of strangeness and charm. Since in most cases it is not possible to identify strange or charmed secondaries

(e.g. an unseen neutral particle can be a pion or a kaon), all events are accepted and the correction for the strangeness or charm non-conserving reactions is determined by MC calculations.

3. There is an additional correction due to the muon momentum error cut $\delta|\vec{k}'|/|\vec{k}'| < 0.2$.
4. Not only E_ν is smeared, but also the kinematic variables Q^2 and ν .

The raw event numbers $\Delta\tilde{N} \pm \sqrt{\Delta\tilde{N}}$, correction factors K and corrected event numbers ΔN for the case of full integration over E_ν and ν are listed in Table 3a, b for neutrino and antineutrino scattering, respectively. Also given are the correction factors K' obtained if only one of the corrections is taken into account.

The errors quoted for the the “single correction” factors K' are purely statistical and would vanish if infinitely many MC events were generated (an exception is the fake muon background correction where the error is set equal to the difference $(1 - K')$). The error of K is obtained by quadratically adding the statistical error and a systematic uncertainty due to the imperfections of the MC program, which can be estimated by varying parameters of the MC such as the quark distribution functions, the shape of the neutrino energy distribution, the cross sections for different exclusive resonance production channels or details of the production of hadrons. One concludes that the systematic error of $(1 - K)$ is of the order of 20%. The relative error of ΔN is determined by quadratically adding the relative errors of $\Delta\tilde{N}$ and K .

In Table 4 the raw and corrected event numbers $\Delta\tilde{N}$ and ΔN as well as the corresponding correction factors K are presented for the distributions of ν and E_ν in the lowest Q^2 -interval; the results for the remaining intervals of Q^2 can be found in Ref. 20. The errors are treated in the same manner as described above.

5. Evaluation of the Theoretical Prediction

The theoretical prediction (17) contains the cross sections of pion-proton scattering and of photo- and electroproduction off protons. These cross sections have been measured in various experiments. In order to use formula (17) in the integration (19), the measured experimental values are parametrised as analytic functions of the energy of the incident particle or of Q^2 and ν , respectively.

a) The pion-proton total cross sections

Most measurements of $\sigma(\pi^\pm p)$ available until the end of 1982 are compiled in Ref. 27. When parametrising these data points one has to take into account the rather strong variations of $\sigma(\pi^\pm p)$ with the pion energy in the resonance region. To cope with this problem a smoothed cubic spline parametrisation was chosen. In the region $E_\pi = \nu_\pi \gtrsim 5$ GeV and for the energy range considered here $\sigma(\pi^+ p)$ and $\sigma(\pi^- p)$ flatten out to plateaus of about 24 mb in both cases.

The error of the resulting parametrisation can be estimated by examining the increase of χ^2 when varying the spline parameters. The error is found to fall below 2% and will be neglected henceforth.

b) The total cross section for photoproduction off protons

Data of photoproduction off protons with $E_\gamma \lesssim 16$ GeV is collected in Ref. 28, and for incident photon energies E_γ up to 185 GeV in Ref. 19. The data points for $E_\gamma > 2$ GeV were fitted by an ansatz motivated by the *Regge*-theory [19]

$$\sigma(\gamma p) = a_1 + a_2/\sqrt{E_\gamma} + a_3 E_\gamma. \quad (24)$$

The values determined for the parameters are $a_1 = (0.1007 \pm 0.0007)$ mb, $a_2 = (0.0609 \pm 0.0025)$ mb GeV^{1/2} and $a_3 = (0.8944 \pm 0.0648) \cdot 10^{-4}$ mb GeV⁻¹. The cross section $\sigma(\gamma p)$ varies in the range from 0.11 mb to 0.15 mb for $2 \text{ GeV} < E_\gamma < 185 \text{ GeV}$.

The error of $\sigma(\gamma p)$ is given by the errors of the parameters a_i as determined by the fit. Also this error is of the order of 2% and will be neglected.

c) The differential cross section for electroproduction off protons

Experimental results on electron proton scattering at low Q^2 and high ν (as required for this analysis) are published in Ref. 29. In order to find a function of Q^2 , ν , and the energy E_e of the incident electron describing the measured data points one starts with eq. (14), which is rewritten in terms of the ρ cross sections using the scaling factor s^{IV} , eq. (15). The

electroproduction cross section is then

$$\frac{d^2\sigma(ep)}{dQ^2 d\nu} = \kappa \cdot \frac{1}{s^{IV}} \left(\frac{f_{\rho^0}}{Q^2 + M_\rho^2} \right)^2 \left(\sigma_T(\rho^0 p; Q^2) + \epsilon \sigma_L(\rho^0 p; Q^2) \right), \quad (25)$$

where κ contains known constants and kinematic variables. It is assumed that the Q^2 and ν -dependence of $\sigma_T(\rho^0 p; Q^2) + \epsilon \sigma_L(\rho^0 p; Q^2)$ factorizes:

- As mentioned in Section 2, $\sigma_T(\rho^0 p; Q^2)$ is expected to depend only weakly on Q^2 . On the other hand, due to the CVC hypothesis, $\sigma_L(\rho^0 p; Q^2)$ vanishes as $Q^2 \rightarrow 0$ as expected for a zero mass vector particle. The simplest ansatz for its Q^2 -dependence is

$$\sigma_L(\rho^0 p; Q^2) = \lambda \frac{Q^2}{M_\rho^2} \sigma_T(\rho^0 p; Q^2) \quad (26)$$

with a free parameter λ .

- The ν -dependence can be inferred by evaluating eq. (25) at $Q^2 = 0$, where the σ_L -term vanishes and σ_T can be related to the photoproduction cross section using eq. (16), evaluated at $Q^2 = 0$. The parametrisation of $\sigma(\gamma p)$ as a function of ν has been described above.

With these assumptions eq. (25) becomes

$$\frac{d^2\sigma(ep)}{dQ^2 d\nu} = \kappa \cdot \frac{\sigma(\gamma p)}{4\pi\alpha} \left(\frac{M_\rho^2}{Q^2 + M_\rho^2} \right)^2 \cdot \left(1 + \lambda \frac{Q^2}{M_\rho^2} \epsilon \right), \quad (27)$$

where $\sigma(\gamma p)$ is a function of ν , and ϵ depends on Q^2 , ν and E_e . The expression (27) has been fitted to all ep scattering data from Ref. 29 below a certain $(Q^2)^{\max}$ varying the only free parameter λ . The results for λ for different values of $(Q^2)^{\max}$ agree within one standard deviation. For $(Q^2)^{\max} = 1.1 \text{ GeV}^2$ the result is $\lambda = 0.720 \pm 0.024$ with $\chi^2/\text{NDF} = 0.46$. Any other tested assumption on the Q^2 -dependence of $\sigma_L(\rho^0 p; Q^2)$ differing from eq. (26) yielded considerably larger values of χ^2 .

The fact that eq. (27) describes the data well not only justifies the ansatz (26) but also strongly supports the VDM by confirming the predicted Q^2 -dependence.

6. Comparison of Experiment and Theory

In Fig. 6 the experimental distribution of Q^2 (i.e. after integration over the full ranges of ν and E_ν) is compared with the theoretical prediction. The latter was calculated from eq. (17) according to eq. (19), omitting however the averaging over Q^2 . The error on the theoretical result was determined by adding quadratically the three contributions due to

- the error on σ_0 ; this is the dominant contribution (see Section 4),
- the error on s^{IV} (see Section 2c),
- the error on λ as obtained in the fitting described in Section 5c.

The variation of the theoretical prediction within one standard deviation due to the total resulting error is indicated in Fig. 6 by the shaded area.

The theoretical prediction is the sum of three contributions, which are shown separately in Fig. 6:

1. The “PCAC-term” proportional to $\sigma(\pi^\pm p)$, which reproduces *Adler’s* prediction at $Q^2 \rightarrow 0$ (first term of the sum (17)).
2. The EVDM-term proportional to the differential electroproduction cross section off protons (second part of eq. (17)).
3. The $V \times A$ interference term, whose maximum absolute size (i.e. $C = 1$ in eq. (17)) is displayed in Fig. 6. Since this contribution is of the same order as the error on the theoretically predicted event distributions, the $(V \times A)$ -term is neglected henceforth (i.e. $C = 0$); it is also not included in the overall theoretical result (solid line) in Fig. 6.

The comparisons of experimental and theoretically predicted ν and E_ν distributions are shown in Figs. 7 and 8 for the lowest Q^2 interval ($Q^2 < 0.1 \text{ GeV}^2$). The single contributions to the theoretical result are indicated in the same manner as in Fig. 6.

All comparisons show good agreement between data and theory, in shape as well as in absolute values. This is especially striking for low Q^2 where the PCAC-term is dominant; the excellent agreement in the Q^2 -distributions below 0.1 GeV^2 confirms *Adler’s* prediction for $\sigma(\nu_\mu p \rightarrow \mu^- X)$ in the limit $Q^2 \rightarrow 0$.

In order to quantify the agreement between data and theory, χ^2 -values have been calculated for all distributions which were evaluated in this analysis. The hypotheses tested are (1) the theoretical prediction (17) (with $C = 0$) and (2) the same prediction, but without the PCAC-term. In computing the χ^2 -values the correlation between the errors on the averaged theoretical predictions in different kinematical bins has been taken into account.

The results are shown in Table 5. They indicate, that hypothesis (1) is compatible with

the experiment in (nearly) all kinematical intervals, whereas hypothesis (2) without PCAC can be discarded with an extremely high confidence level.

7. Conclusions

In 1964 *Adler* proposed a test of the PCAC hypothesis in high energy neutrino interactions at $Q^2 = 0$ [3]. Using an Extended Vector Meson Dominance Model (EVDM) [13] and various additional assumptions concerning the cross sections of virtual π , ρ and A_1 -mesons, *Adler's* prediction is extrapolated to higher, experimentally accessible values of Q^2 .

The data of this experiment is used to test the theoretical prediction in the regime of high energy transfer $\nu > 2 \text{ GeV}$. Good agreement is found, both in absolute magnitude and in the shapes of the distributions. This applies in particular for $Q^2 \lesssim 0.2 \text{ GeV}^2$, where the main contribution to the theoretically predicted cross section is due to the PCAC hypothesis. Hence this analysis, which constitutes a significant test of *Adler's* theorem, confirms the PCAC hypothesis for $\nu > 2 \text{ GeV}$.

The good agreement of the data and theory also for $Q^2 \gtrsim 0.3 \text{ GeV}^2$, where the EVDM gives the dominant contributions to the cross section, supports also this model. However, a calculation exclusively based on the EVDM, without PCAC, is clearly ruled out by the data.

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Table 1: Scanning efficiencies for different event topologies.

Topology	Scanning Efficiency ϵ_s
1 prong	$\lesssim 0.3$
3 prongs	0.95
≥ 5 prongs	0.99

Table 2: Ratio of total event numbers for corrected and raw data, for each correction performed separately and for the total correction.

Corrected Effect	$N^{\text{tot}}/\tilde{N}^{\text{tot}}$	
	ν	$\bar{\nu}$
$E_\mu > 5 \text{ GeV}$	1.2028 ± 0.0014	1.0897 ± 0.0008
One-Prong Events	—	1.1812 ± 0.0012
Scanning Efficiency (≥ 3 Prongs)	1.0204 ± 0.0004	1.0235 ± 0.0004
EMI Acceptance for $E_\mu > 5 \text{ GeV}$	1.0242 ± 0.0004	1.0177 ± 0.0003
Fake Muons	0.9990 ± 0.0001	0.9991 ± 0.0001
Smearing of E_ν	1.0028 ± 0.0002	1.0038 ± 0.0002
Total	1.2723 ± 0.0017	1.3752 ± 0.0021

Table 3a: Raw event numbers, correction factors and corrected event numbers in different intervals of Q^2 for neutrino scattering (full integration over ν and E_ν). Also shown are the contributions of the single corrections to the total correction.

νp	$Q^2 \text{ (GeV}^2\text{)}$					
	0 – 0.1	0.1 – 0.2	0.2 – 0.3	0.3 – 0.4	0.4 – 0.5	0.5 – 0.6
Raw Number of Events	109 ± 10.4	133 ± 11.5	151 ± 12.3	201 ± 14.2	177 ± 13.3	184 ± 13.6
Total Monte Carlo Correction Factor	0.796 ± 0.046	0.770 ± 0.045	0.803 ± 0.041	0.858 ± 0.039	0.836 ± 0.039	0.826 ± 0.039
Corrected Number of Events	86.8 ± 9.7	102.4 ± 10.7	121.3 ± 11.6	172.5 ± 14.5	148.0 ± 13.1	152.0 ± 13.3
$\Delta S = \Delta C = 0$	0.868 ± 0.023	0.866 ± 0.019	0.875 ± 0.018	0.872 ± 0.018	0.878 ± 0.018	0.878 ± 0.018
Smearing of Q^2	0.906 ± 0.021	0.882 ± 0.022	0.898 ± 0.022	0.956 ± 0.023	0.925 ± 0.023	0.912 ± 0.024
Smearing of E_ν	0.965 ± 0.008	0.974 ± 0.011	0.963 ± 0.013	1.028 ± 0.014	0.986 ± 0.015	0.971 ± 0.017
Smearing of ν	0.882 ± 0.020	0.862 ± 0.020	0.886 ± 0.020	0.949 ± 0.021	0.906 ± 0.021	0.903 ± 0.022
$\delta \vec{k}' / \vec{k}' < 0.2$	1.049 ± 0.016	1.038 ± 0.010	1.037 ± 0.008	1.034 ± 0.006	1.035 ± 0.006	1.033 ± 0.006
EMI Acceptance	1.004 ± 0.005	1.005 ± 0.004	1.006 ± 0.003	1.008 ± 0.004	1.007 ± 0.003	1.008 ± 0.003
Scanning Efficiency	1.043 ± 0.014	1.040 ± 0.010	1.040 ± 0.009	1.036 ± 0.007	1.036 ± 0.007	1.033 ± 0.007
Fake Muons	0.962 ± 0.038	0.962 ± 0.038	0.964 ± 0.036	0.970 ± 0.030	0.970 ± 0.030	0.970 ± 0.030

Table 3b: Raw event numbers, correction factors and corrected event numbers in different intervals of Q^2 for antineutrino scattering (full integration over ν and E_ν). Also shown are the contributions of the single corrections to the total correction.

$\bar{\nu}p$	Q^2 (GeV ²)					
	0 – 0.1	0.1 – 0.2	0.2 – 0.3	0.3 – 0.4	0.4 – 0.5	0.5 – 0.6
Raw Number of Events	38 ± 6.2	79 ± 8.9	75 ± 8.7	80 ± 8.9	84 ± 9.2	86 ± 9.3
Total Monte Carlo Correction Factor	0.949 ± 0.033	0.917 ± 0.029	0.940 ± 0.029	0.956 ± 0.030	0.900 ± 0.029	0.934 ± 0.030
Corrected Number of Events	36.1 ± 6.0	72.4 ± 8.3	70.5 ± 8.4	76.5 ± 8.9	75.6 ± 8.6	80.3 ± 9.0
$\Delta S = \Delta C = 0$	0.870 ± 0.029	0.873 ± 0.023	0.877 ± 0.022	0.868 ± 0.021	0.875 ± 0.020	0.876 ± 0.020
One-Prong Events	1.075 ± 0.010	1.079 ± 0.009	1.083 ± 0.008	1.073 ± 0.008	1.083 ± 0.008	1.067 ± 0.007
Smearing of Q^2	0.992 ± 0.027	0.969 ± 0.026	0.978 ± 0.026	1.002 ± 0.027	0.944 ± 0.027	0.961 ± 0.027
Smearing of E_ν	1.007 ± 0.010	1.001 ± 0.013	1.014 ± 0.015	1.020 ± 0.017	0.990 ± 0.018	0.997 ± 0.019
Smearing of ν	0.911 ± 0.019	0.935 ± 0.018	0.964 ± 0.019	0.950 ± 0.019	0.939 ± 0.019	0.961 ± 0.019
$\delta \vec{k}' / \vec{k}' < 0.2$	1.049 ± 0.032	1.043 ± 0.019	1.039 ± 0.015	1.039 ± 0.013	1.036 ± 0.012	1.036 ± 0.011
EMI Acceptance	1.003 ± 0.008	1.005 ± 0.006	1.006 ± 0.006	1.007 ± 0.006	1.008 ± 0.005	1.009 ± 0.005
Scanning Efficiency	1.018 ± 0.020	1.020 ± 0.014	1.021 ± 0.012	1.023 ± 0.011	1.024 ± 0.010	1.025 ± 0.010
Fake Muons	0.995 ± 0.005	0.994 ± 0.006	0.994 ± 0.006	0.994 ± 0.006	0.994 ± 0.006	0.993 ± 0.007

Table 4: Raw event numbers, correction factors and corrected event numbers for the E_ν and ν -distributions in the lowest Q^2 -interval.

	ν -distribution in νp -scattering ($Q^2 < 0.1 \text{ GeV}^2$)						
	$\nu \text{ (GeV)}$						
	2.00 – 3.42	3.42 – 5.85	5.85 – 10.00	10.00 – 17.10	17.10 – 29.24	29.24 – 50.00	
Raw Event Numbers	32 ± 5.7	36 ± 6.0	19 ± 4.4	10 ± 3.2	10 ± 3.2	2 ± 1.4	
MC Correction Factor	0.845 ± 0.070	0.743 ± 0.074	0.842 ± 0.087	0.713 ± 0.099	0.869 ± 0.153	0.729 ± 0.194	
Corrected Event Numbers	27.0 ± 5.3	26.7 ± 5.2	16.0 ± 4.0	7.1 ± 2.5	8.7 ± 3.1	1.5 ± 1.1	
	ν -distribution in $\bar{\nu} p$ -scattering ($Q^2 < 0.1 \text{ GeV}^2$)						
Raw Event Numbers	7 ± 2.6	10 ± 3.2	7 ± 2.6	10 ± 3.2	4 ± 2.0	0 ± 0.0	
MC Correction Factor	1.038 ± 0.085	0.986 ± 0.086	0.890 ± 0.079	0.880 ± 0.094	0.844 ± 0.141	1.122 ± 0.264	
Corrected Event Numbers	7.3 ± 2.8	9.9 ± 3.2	6.2 ± 2.4	8.8 ± 2.9	3.4 ± 1.8	0.0 ± 0.0	
	E_ν -distribution in νp -scattering ($Q^2 < 0.1 \text{ GeV}^2$)						
	$E_\nu \text{ (GeV)}$						
	10.00 – 15.70	15.70 – 24.66	24.66 – 38.73	38.73 – 60.82	60.82 – 95.52	95.52 – 150.00	
Raw Event Numbers	19 ± 4.4	29 ± 5.4	32 ± 5.7	17 ± 4.1	9 ± 3.0	3 ± 1.7	
MC Correction Factor	0.752 ± 0.148	0.778 ± 0.062	0.821 ± 0.059	0.777 ± 0.064	0.923 ± 0.101	0.781 ± 0.163	
Corrected Event Numbers	14.3 ± 4.3	22.6 ± 4.6	26.3 ± 5.0	13.2 ± 3.5	8.3 ± 2.9	2.3 ± 1.4	
	$E_{\bar{\nu}}$ -distribution in $\bar{\nu} p$ -scattering ($Q^2 < 0.1 \text{ GeV}^2$)						
	$E_{\bar{\nu}} \text{ (GeV)}$						
	20.00 – 27.98	27.98 – 39.15	39.15 – 54.77	54.77 – 76.63	76.63 – 107.21	107.21 – 150.00	
Raw Event Numbers	16 ± 4.0	10 ± 3.2	7 ± 2.6	3 ± 1.7	2 ± 1.4	0 ± 0.0	
MC Correction Factor	0.954 ± 0.056	0.951 ± 0.067	0.980 ± 0.087	0.895 ± 0.124	0.803 ± 0.187	1.239 ± 0.387	
Corrected Event Numbers	15.3 ± 3.9	9.3 ± 3.1	6.9 ± 2.7	2.7 ± 1.6	1.6 ± 1.2	0.0 ± 0.0	

Table 5: Values of χ^2 per degree of freedom for all examined distributions for two different hypotheses: (1) the theoretical prediction (17) (with $C = 0$) and (2) the same prediction, but without the PCAC-term.

Q^2 Region	Distribution of	NDF	χ^2 per NDF			
			PCAC and EVDM		EVDM without PCAC	
			ν	$\bar{\nu}$	ν	$\bar{\nu}$
$0 < Q^2 < 0.6 \text{ GeV}^2$	Q^2	6	1.101	1.162	51.819	38.283
$0 < Q^2 < 0.1 \text{ GeV}^2$	ν	6	0.874	0.887	10.486	4.903
	E_ν	6	0.627	0.121	10.333	4.941
$0.1 \text{ GeV}^2 < Q^2 < 0.2 \text{ GeV}^2$	ν	6	0.696	1.794	11.337	10.038
	E_ν	6	1.846	2.137	11.683	10.209
$0.2 \text{ GeV}^2 < Q^2 < 0.3 \text{ GeV}^2$	ν	6	0.990	1.372	13.762	10.059
	E_ν	6	0.697	1.184	13.438	9.623
$0.3 \text{ GeV}^2 < Q^2 < 0.4 \text{ GeV}^2$	ν	6	3.147	1.538	20.800	10.796
	E_ν	6	3.615	1.517	21.740	10.343
$0.4 \text{ GeV}^2 < Q^2 < 0.5 \text{ GeV}^2$	ν	6	0.948	0.898	15.847	9.427
	E_ν	6	0.969	0.847	15.861	9.410
$0.5 \text{ GeV}^2 < Q^2 < 0.6 \text{ GeV}^2$	ν	6	1.469	1.843	16.239	11.422
	E_ν	6	1.034	0.732	15.910	9.890

FIGURE CAPTIONS

1. *Feynman*-diagram for the scattering of neutrinos off protons according to the Extended Vector Meson Dominance Model (EVDM)
2. Comparison of experimental data (points) and “modified” MC events (histograms) for charged current neutrino proton interactions. Distributions of a) $\sqrt{Q^2}$, b) ν and c) pion lab momentum.
3. Spectrum of the (anti-)neutrino energy in the observed events. The points represent the raw data, the solid lines the parametrisation of the raw data and the dashed lines the corrected distribution; a) νp and b) $\bar{\nu} p$ scattering.
4. Smearing of Q^2 (see text). Shown is the result for neutrino scattering which does not significantly differ from the antineutrino result. The error bars indicate the error on the mean.
5. Integration area for the kinematic variables ν and E_ν at a fixed value of Q^2 . The boundaries are labelled according to the points mentioned in the text; boundary (4) is Q^2 -dependent.
6. Q^2 distribution. The data points show the experimental results (Table 3a, b). The solid line is the theoretical prediction (without (3)), the shaded area corresponds to a one standard deviation error. The dashed lines show contributions (1) of the PCAC-term, (2) of the EVDM-term proportional to $\sigma_T + \epsilon\sigma_L$ and (3) the maximum modulus of the $V \times A$ interference term. a) νp and b) $\bar{\nu} p$ scattering.
7. ν distribution for $Q^2 < 0.1 \text{ GeV}^2$. The data points show the experimental results (Table 4), the lines display the theoretical prediction in the same manner as in Fig. 6. a) νp and b) $\bar{\nu} p$ scattering.
8. E_ν distribution for $Q^2 < 0.1 \text{ GeV}^2$. The data points show the experimental results (Table 4), the lines display the theoretical prediction in the same manner as in Fig. 6. a) νp and b) $\bar{\nu} p$ scattering.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8