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ERRATA

NOT THE STANDARD SUPERSTRING REVIEW

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Page 10, line 32 should read: "In fact, explaining where this other small scale in nature comes from remains one of the biggest unsolved problems."

Page 15, line 4: replace "Klein" by "Kaluza".

Page 19, line 7 should read: "The authors of Ref. 27), who used the device of even self-dual Lorentzian lattices of dimension 88, pointed out that there was a lower bound of about 10^{1500} such lattices each giving a different four-dimensional superstring! Not all such lattices give rise to consistent theories, however, so this number does not accurately reflect the number of four-dimensional superstrings²⁷⁾."

Page 19, line 19: replace "non" by "none".

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A B S T R A C T

The boasts of superstring theory are essentially threefold: (1) superstrings provide a consistent (finite) quantum theory of gravity; (2) they are likely to explain the standard model with no free parameters; (3) no other theory (e.g., membranes) would work. We critically examine their current status and conclude that there is no room for complacency.

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The word "orthodoxy" not only no longer means being right; it practically means being wrong.

G.K. Chesterton

1874-1936

1. - SUPERSTRING ORTHODOXY

Each week the CERN preprint library receives about 60 papers on theoretical particle physics of which about 25-30 are devoted to superstrings or related topics. So as I put pen to paper I feel the burden of having to review one half of theoretical physics. To make matters worse, just as the number of superstring publications is mushrooming, so are the number of superstring review talks. There now exist many excellent reviews, some of them by experts who have been studying superstrings for 15 years¹⁾. Well, there are two respects in which this review may differ from previous ones. First, to paraphrase Shelley^{*)}, I never was attached to that great sect whose doctrine was the Dual Resonance Model and who pioneered string theory in the late 60's and early 70's in its original incarnation as a theory of hadrons. I entered the subject comparatively recently via the route of supersymmetry and for me ten-dimensional superstrings were the natural extension of the rather different theoretical developments that had been taking place in supergravity theories and Kaluza-Klein unification via extra space-time dimensions²⁾.

The second, and more important, difference is the degree of scepticism with which I view much of the current superstring orthodoxy. Accordingly, I have chosen the above quote of Chesterton's as my theme. Of course, I doubt that he was referring to superstrings; more probably he had in mind religious belief. Since there is as yet no shred of experimental confirmation of superstrings, however, the comparison is perhaps not altogether inappropriate.

In order not to be misunderstood, let me say straight away that I share the conviction that superstrings are the most exciting development in theoretical physics for many years, and that they offer the best promise to date of achieving the twin goals of a consistent quantum theory of gravity and a unification of all the forces and particles of Nature. Where I differ is the degree of emphasis that I would place on the unresolved problems of superstrings, and the likely time scales involved before superstrings (or something like superstrings) make

*) The poet, not the physicist.

contact with experimental reality. To illustrate what I mean, let us consider the three boasts of superstrings as a Theory of Everything:

1. Superstrings provide a consistent (finite) quantum theory of gravity.
2. They are likely to explain the standard model with no free parameters.
3. No other theory (neither points, nor membranes) would work.

Whereas, the harsh realities are:

1. Finiteness?: Still no proof after 15 years.
2. Uniqueness?: Disappears in going from ten dimensions to four. Now have billions of consistent four-dimensional strings with desired theoretical properties ($N=1$ supersymmetry, chiral, anomaly-free, modular invariant, conformally invariant...) with more or less any gauge group of rank ≤ 22 .
3. No other theory?: Now exists a supermembrane in eleven dimensions which yields a superstring in ten dimensions upon dimensional reduction.

I draw attention to the finiteness problem (1) above and in Section 5, not because I think superstrings are necessarily infinite, but because of an attitude pervading the orthodox string community that finiteness is so obvious that we do not need to prove it! At the recent Nobel Symposium³⁾, for example, almost everybody spoke about strings but almost nobody spoke about finiteness. It was taboo.

Much more serious, in my opinion, is the "vacuum degeneracy" problem (2) which often receives short shrift in superstring review talks. It is particularly ironic that superstrings should founder on the uniqueness problem, since, as discussed in Section 3, it was the apparent uniqueness of ten-dimensional superstrings (following from the Green-Schwarz anomaly-cancellation paper) which triggered the 1984 superstring revolution.

Finally, the maximum space-time dimension permitted by supersymmetry is $D = 11$, whereas superstrings exist in $D = 10$. The recent discovery of a supersymmetric theory of a three-dimensional extended object in $D = 11$ may resolve this old puzzle. We do not yet know whether this "supermembrane" is consistent at the quantum level, but the orthodox claim that only strings can be quantum consistent now looks much less certain.

Before giving a verdict on superstrings' claim to be the Theory of Everything, let us examine what such a theory might be and why we need it.

2. - A THEORY OF EVERYTHING?

Our experimental colleagues tell us that the standard $SU(3) \times SU(2) \times U(1)$ model with three families of quarks and leptons is still in good shape, although there is some uncertainty about the top quark. Moreover, the Higgs bosons necessary for symmetry-breaking have yet to be observed. Nevertheless, the standard model is remarkably robust. There are as yet no signs of proton decay, no signs of monopoles and no signs of supersymmetry. In short, there is no empirical evidence at all that we should go beyond the standard model. So why bother? Although the standard $[SU(3) \times SU(2) \times U(1)]$ model is probably the truth, and although grand unified theories $[SU(5), SO(10), E_6, \dots]$ are possibly the truth, none can claim to be the whole truth. There are still too many unanswered questions. For example:

- * Why chiral fermions?
- * How many families?
- * What dictates the Higgs sector?
- * Why $SU(3) \times SU(2) \times U(1)$?

By "chiral" we mean that we put the left-handed quarks and leptons into doublets of the weak-isospin and the right-handed into singlets. It is true that with these assignments we avoid the otherwise disastrous chiral anomalies in triangle graphs provided we group the quarks and leptons into families, but this logic does not tell us how many families there are; and whereas the gauge principle beautifully pins down the gauge boson sector and its coupling to fermions, the scalar Higgs sector remains quite arbitrary. Nor does the gauge principle alone tell us which gauge group we should start from.

A Theory of Everything must also include the force of gravity whose typical energy scale is given by the Planck mass

$$M_P = \sqrt{\frac{hc}{G}} \sim 10^{-5} \text{ gms} \sim 10^{19} \text{ GeV} \quad (2.1)$$

where G is Newton's constant of gravitation. But if we now worry about gravity, the mystery deepens:

- * How to incorporate gravity?
- * The ultra-violet problem?
- * Why is the cosmological constant $\Lambda \approx 0$?
- * Why is M_W/M_P so small?

It seems unlikely that the fundamental theory will be obtained merely by grafting gravity, as an afterthought, on to one's favourite GUT theory. Indeed, all the current theoretical indications are that gravity is actually at the root of everything. Moreover, the success in formulating the theory of strong, weak and electromagnetic forces relied crucially on the criterion of renormalizability, i.e., the ability to absorb into the masses and coupling constants those infinities which arise from ultra-violet divergent integrals in radiative corrections. Yet Einstein's theory of general relativity, the cornerstone of classical gravity, is at the quantum level non-renormalizable! (just because Newton's constant G has dimensions). Thirdly, there is the problem of the cosmological constant Λ . Originally Einstein suggested that the field equations of gravity should be obtained by equating geometry (the Einstein tensor $G_{\mu\nu}$) to matter (the energy-momentum tensor $T_{\mu\nu}$), i.e.,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (2.2)$$

where $(\mu, \nu = 1, 2, 3, 4)$. But he later realized that the principle of general covariance did not rule out a term proportional to the metric tensor $g_{\mu\nu}$, i.e.,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (2.3)$$

Upper bounds on the constant of proportionality Λ , known as the cosmological constant, can be put by studying the rate of expansion of the Universe, and the result is one of the most accurate of experimental statements:

$$\Lambda < 10^{-120} M_P^2. \quad (2.4)$$

Yet theoretically the most natural value seems to be $\Lambda \sim M_P^2$ or perhaps else $\Lambda \sim M_W^2$ where M_W = W-boson mass $\sim 10^2$ GeV; no one knows why $\Lambda \sim 0$. Fourthly, we have the hierarchy problem: why is $M_W/M_P \sim 10^{-17}$ so small?

There are even more fundamental questions one might ask: questions so fundamental that their answer is usually taken for granted but which have recently been brought into focus by superstrings, namely:

- * Are ultra-violet divergences inevitable?
- * Is field theory right?
- * Are the fundamental objects points? Why not extended objects: strings, membranes...?
- * Why four apparent space-time dimensions?

Ultra-violet divergences have been with us for so long now that we forget the horror with which they were first greeted. Indeed, Dirac maintained to the end that they were only artefacts of our feeble approximations to Nature and that the fundamental theory should be finite. Field theory has been very successful

at low energies but should we expect the field concept to persist beyond the Planck scale? The non-renormalizability of quantum gravity suggests that it does not. Indeed, as we shall see, this is one of the main reasons for introducing extended objects. And finally why does space-time appear to have four dimensions, as opposed to some other number?

Murray Gell-Mann reminds us that a Theory of Everything requires three ingredients: (a) the initial conditions of the Universe; (b) the fundamental laws; (c) the parameters peculiar to our specific Universe. Ingredient (a) is the subject of cosmology which is beyond the scope of this review, but we shall be asking whether superstrings provide not only ingredient (b) but also ingredient (c). For example, no one would expect a Theory of Everything to predict the radius of the earth from its fundamental laws alone, but do we expect it to predict things like the fine-structure constant or the electron-muon mass ratio? We shall consider this problem in Section 8, but let us first understand the excitement of superstrings.

3. - THE SUPERSTRING REVOLUTION

Despite the success of the standard model's electroweak unification, and the possibility of grand electronuclear unification schemes, a Theory of Everything remained only a dream so long as gravity was neglected. Consequently, the arrival of local supersymmetry, a Bose-Fermi symmetry which predicted gravity, completely changed the psychology of theorists. Moreover, the fact that supersymmetry also places an upper limit on the dimension of space-time

$$D \leq 11$$

(3.1)

raised hopes of a superunification involving the higher space-time dimension ideas, originally put forward by Kaluza and Klein in the 1920's. Up until the summer of 1984 various proposals were put forward combining supersymmetry and the Kaluza-Klein idea²⁾ but none with complete success. Those based on conventional field theory (e.g., eleven-dimensional supergravity) suffered from various problems, not least of which was the traditional objection to a non-renormalizable theory of gravity. Those based on string theory seemed better from this point of view but had problems of their own. The realistic-looking strings appeared to suffer from inconsistencies (anomalies akin to the triangle anomalies of the standard model) while the anomaly-free strings did not appear realistic. In particular, they seemed to prefer to live in ten space-time dimensions rather than undergoing a spontaneous compactification to four space-time dimensions as

demanding by the Kaluza-Klein idea. This was the sorry state of affairs until the September 84 superstring revolution:

1. Green and Schwarz⁴⁾ discovered that the gravitational and Yang-Mills anomalies of ten-dimensional superstrings all cancel provided the gauge group is either $SO(32)$ or $E_8 \times E_8$;
2. Gross, Harvey, Martinec and Rohm⁵⁾ discovered the heterotic (hybrid) string with the above gauge groups;
3. Candelas, Horowitz, Strominger and Witten⁶⁾ discovered that the $E_8 \times E_8$ heterotic string admits a "spontaneous compactification" from ten to four dimensions on a six-dimensional Calabi-Yau manifold. The resulting four-dimensional theory resembles a GUT theory based on the group E_6 . In particular, there are chiral families of quarks and leptons.

To understand the hysteria which greeted the Green-Schwarz discovery, one has to realize that theorists love uniqueness. They like to think that the Theory of Everything will one day be singled out on the grounds that it is the only possible theory. In other words, all rival theories will be eliminated not merely because they disagree with experiment but because they are internally inconsistent. Anomalies are just the sort of inconsistency one looks for. Already in the four-dimensional standard model the cancellation of triangle anomalies narrows down the choice of quark and lepton representations. Anomaly cancellation in the ten-dimensional superstring is much more dramatic. Here, and for the first time ever, the Yang-Mills gauge groups were pinned down to either $SO(32)$ or $E_8 \times E_8$. $SO(32)$ is just the rotation group in 32 dimensions and E_8 is the largest of the "exceptional" group in Cartan's classification of Lie groups. The subscript 8 refers to the rank of the group. Both $SO(32)$ and $E_8 \times E_8$ have rank 16, which means that we need 16 quantum numbers to specify a particle multiplet. Both groups have dimension 496, which means that there are 496 massless gauge bosons. [Compare with $SU(2)$ which has rank one (isospin) and dimension 3 or $SU(3)$ which has rank two (isospin and strangeness) and dimension 8.] Of course, $E_8 \times E_8$ and $SO(32)$ are enormous groups but since both contain $SU(3) \times SU(2) \times U(1)$ as a subgroup, one might optimistically hope to find the standard model after some spontaneous symmetry breaking. For example: E_8 contains E_6 , E_6 contains $SO(10)$; $SO(10)$ contains $SU(5)$; and $SU(5)$ contains $SU(3) \times SU(2) \times U(1)$.

Before discussing the heterotic string, its compactification to four dimensions and the subsequent phenomenology let us first recall what strings are.

4. - WHY STRINGS ?

Normally one thinks of fundamental objects as being points in space which sweep out one-dimensional world-lines in space-time as in Fig. 1a. But suppose instead we consider the fundamental objects to be one-dimensional strings which sweep out two-dimensional world-sheets in space-time. Strings can either be "open" so that their ends move freely in space-time as in Fig. 1b or "closed" so that their ends are joined to form a closed-loop as in Fig. 1c. In either case, the string may carry an intrinsic orientation. Two open strings may interact to form a third open string but they may also form a closed string. So all theories with open strings necessarily contain closed strings as well, though the converse is not true.

These strings were originally invented in the 1960's by Nambu⁷⁾ to explain the Veneziano⁸⁾ spectrum of hadrons. The string has vibrational modes and each of these modes represents a particle in the spectrum. So a string describes an infinite number of elementary particles: a finite number of massless particles corresponding to the zero-eigenvalue modes and an infinite tower of massive particles corresponding to the non-zero eigenvalue modes. The mass M of each particle in the spectrum is related to its spin J by

$$J = \alpha_0 + \alpha' M^2 \quad (4.1)$$

where the intercept α_0 is given by 1 for open strings and 2 for closed. So open strings include massless spin-1 particles and closed strings massless spin-2. The slope parameter α' with dimensions of $(\text{mass})^{-2}$ sets the fundamental scale for string theories. In these early attempts to relate strings to hadrons one assumed that $(\alpha')^{-\frac{1}{2}} \sim 1 \text{ GeV}$.

It was not until the 1970's, however, that Yoneya⁹⁾ and Scherk and Schwarz¹⁰⁾ abandoned the hadron interpretation and instead interpreted the massless spin-2 particle as the graviton! In this picture we take $(\alpha')^{-\frac{1}{2}} \sim 10^{19} \text{ GeV}$ i.e., the Planck scale. Similarly, the massless spin-1 particle of the open string was identified with the Yang-Mills gauge boson. At such enormous energy scales, the effects of "stringiness" are important but at much lower energy scales these may be approximated by an ordinary quantum field theory. In particular the low-energy effective Lagrangians will possess general covariance and Yang-Mills gauge invariance and are given by the familiar Einstein and Yang-Mills Lagrangians plus an infinite sequence of corrections suppressed by powers of E/M_p :

$$\mathcal{L}_{\text{GRAVITY}} = \sqrt{-g} R + O(\alpha') \quad (4.2)$$

$$\mathcal{L}_{\text{YANG-MILLS}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + O(\alpha') \quad (4.3)$$

Physically, a string describes a two-dimensional surface embedded in a D-dimensional space-time. Bosonic strings may thus be described by a two-dimensional field theory in which the commuting bosonic fields are the space-time co-ordinates $X^\mu(\tau, \sigma)$ with τ and σ the temporal and spatial co-ordinates on the two-dimensional world-sheet. The index μ ranges over 1 to D. We may also introduce anticommuting world-sheet spinors $\lambda(\tau, \sigma)$. We speak of a "superstring" if there is a supersymmetry which rotates X^μ into λ and vice-versa. These can be classified by $n = 0, 1, 2$ or 4 , where n is the number of world-sheet supercharges. We also demand world-sheet reparametrization invariance under

$$\tau \rightarrow \tau'(\tau, \sigma) \quad \sigma \rightarrow \sigma'(\tau, \sigma) \quad (4.4)$$

so that the laws of physics are the same for all world-sheet "observers". In addition to such local transformations, we also require invariance under "global diffeomorphisms" of the world sheet, i.e., transformations that are not connected to the identity. This invariance, known as "modular invariance" is more difficult to describe but proves to be a crucial constraint on the allowed string theories. Finally we demand Weyl invariance, i.e., invariance under the conformal rescaling

$$\gamma_{ij} \rightarrow \Omega \gamma_{ij} \quad (4.5)$$

where $\gamma_{ij}(\tau, \sigma)$ is the metric that describes the two-dimensional world-sheet geometry and Ω is an arbitrary function of σ and τ . Classically, it is relatively easy to satisfy the above requirements but quantum-mechanically it is a different story.

In 1974 it was shown that there are "Weyl anomalies"¹¹⁾, i.e., the classical conformal invariance (4.5) displayed by various classical matter systems in interaction with gravity, breaks down at the quantum level. This implies, in particular, that the regularized energy-momentum tensor θ^i_j acquires a non-vanishing trace which in the case of two dimensions is proportional to the two-dimensional scalar curvature R ¹²⁾. For a single scalar field¹³⁾

$$\theta^i_i = \frac{1}{24\pi} R \quad (4.6)$$

and, for a Majorana fermion¹³⁾

$$\theta^i_i = \frac{1}{48\pi} R \quad (4.7)$$

In 1981 Polyakov¹⁴⁾ demonstrated that when applied to the two-dimensional string world-sheet, these Weyl anomalies accounted for the previously known "critical dimension" of string theories: the critical dimension D_c being the dimension in which the Weyl anomalies cancel. When a theory with the above symmetries is quantized we must also include a contribution to θ_i^i from the Faddeev-Popov ghosts which Polyakov calculated to be -26 times that of a single scalar in the case of the diffeomorphism ghosts and 11 times in the case of the supersymmetry ghosts. Anomaly-free theories are then obtained by having different numbers of bosons and fermions on the world sheet. The known open strings and their critical dimensions are shown in Table 1.

The $n = 0$ case is just what used to be called the Veneziano model back in the days of dual resonance models. From (4.1) we see that it exhibits an undesirable tachyon ($M^2 < 0$) at $J = 0$. The beauty of the Type I superstring is that it exhibits space-time supersymmetry ($N=1$) which eliminates the tachyon. Moreover, as shown by Green and Schwarz⁴⁾ in 1984, its chiral anomalies vanish only in the case of $SO(32)$. Hence the revolution. The $n = 2$ and $n = 4$ cases are the superstrings which, for obvious reasons, dare not speak their name. (To obtain their critical dimension we also require two scalar ghosts for each world sheet vector.)

In a suitable gauge the equations of motion of the string co-ordinate is just

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2} \right) X^\mu(\tau, \sigma) = 0 \quad (4.8)$$

In the case of the closed string the solution may therefore be written as a sum of left and right moving waves

$$X^\mu(\tau, \sigma) = X_L^\mu(\tau - \sigma) + X_R^\mu(\tau + \sigma) \quad (4.9)$$

and X_L^μ and X_R^μ are then described by the same mathematics as an open string. Symbolically we have

$$\text{CLOSED} = (\text{OPEN})_L \times (\text{OPEN})_R \quad (4.10)$$

The resulting closed strings and their critical dimensions are shown in Table 2 (where we have excluded the unrealistic cases $D_c = +2$ and -2).

The case 26×26 yields what used to be called the Shapiro-Virasoro model in the old days, whereas 10×10 yields the so-called Type IIA or Type IIB strings depending of whether the handedness of the space-time fermions is opposite or

equal. In these cases we can still say with confidence that the space-time dimension is either 26 or 10. Much more mysterious is the heterotic (hybrid) string discovered by Gross, Harvey, Martinec and Rohm⁵⁾ in 1984. This idea came as a rude shock to traditional Kaluza-Klein enthusiasts who were used to treating the extra space-time dimensions as real, albeit compactified. Here we have a theory with 26 left-moving dimensions but only 10 right-moving dimensions. What on earth can this mean?

As we shall see in Section 7, the best way to regard this theory is as a ten-dimensional theory in which the extra 16 left-moving dimensions are interpreted as the internal symmetry of $E_8 \times E_8$ or $SO(32)$. We then still have the luxury of treating the surviving ten dimensions in the usual way. (However, even this luxury will be denied to us when we come to the "asymmetric orbifolds".)

There are thus five consistent ten-dimensional superstrings^{*)}, one open and four closed. The question naturally arises why there should be five consistent superstrings. Should not the Theory of Everything be unique? One intriguing suggestion, which I do not have time to discuss, is that the fundamental theory is actually the closed bosonic string¹⁵⁾, and that the other closed superstrings: heterotic $SO(32)$, heterotic $E_8 \times E_8$, Type IIA and Type IIB are somehow different vacuum states of this one fundamental string. I shall not pursue this idea further here since it has been discussed at length elsewhere¹⁶⁾. Even if it were true, however, we would still be left with the problem of which vacuum to choose, a problem which becomes even more severe in four dimensions. This will be the subject of Section 7.

After compactification to four dimensions, the superstring spectrum looks like Fig. 2. There are infinite towers of bosons and fermions of increasingly high mass and spin and a finite number of massless particles with maximum spin-two. Since the mass gap is 10^{19} GeV, however, these massive particles are way beyond the scope of any current or foreseeable accelerators. If string theory is to explain the real world, we must look for the particles we actually observe amongst the massless sector of the theory. Particles like the electron or W-boson must then acquire their small masses (small compared to the Planck scale $M_W \ll M_P$) through some symmetry breaking effect, probably at the quantum level. In fact, explaining where this other small scale in nature comes from remains one of the biggest unsolved problems. Nevertheless all the right ingredients for a Theory of Everything are present in the massless sector. We have a single massless spin-2 particle, identified with the graviton and its spin 3/2 superpartner, the gravitino (assuming $N=1$ supersymmetry remains unbroken). In addition there are

*) There is also the $SO(16) \times SO(16)$ string which has no space-time supersymmetry but is nevertheless tachyon-free, at least classically.

massless spins 1, $\frac{1}{2}$ and 0 which (if we are lucky) will be the gauge bosons, quarks, leptons and Higgs bosons of the standard model.

At low energies, therefore, these massless string states constitute a gravity-matter system which is in principle capable of describing low-energy particle-physics phenomenology and standard general relativity but which, taken in isolation, would suffer from the usual breakdown of quantum gravity at the Planck scale. But it is just at the Planck scale where the massive string states can no longer be neglected. This phenomenon is at the root of the claims that strings solve the ultra-violet problem. It is to the issue of finiteness that we now turn.

5. - ARE STRINGS FINITE ?

Consider a scattering process in which two incoming strings collide, coalesce, and then split into two outgoing strings as in Fig. 3. Now imagine two world-sheet observers A and B and their corresponding equal time lines. According to observer A, the strings meet at the point marked a whereas according to observer B they meet at the point b. Since the laws of physics should be the same for both observers by virtue of the reparametrization invariance of (4.4), the point at which the strings meet cannot be objectively specified and is without physical significance. In contrast to conventional field theory based on point particles there is no concept of an "interaction-point", i.e., string diagrams have no sharp corners! This has far-reaching consequences when we consider radiative corrections to such scattering processes. Whereas the one-loop field theory diagrams of Fig. 4 would yield ultra-violet divergences owing to the sharp corners (i.e., from products of field operators at the same space-time point), the corresponding string diagram of Fig. 5 (of which there is only one) should be finite; a fact borne out by explicit one-loop calculations.

If we go beyond one loop as in Fig. 6, the same hand-waving argument about no sharp corners still applies. Unfortunately the highly sophisticated mathematical apparatus (involving the theory of Riemann surfaces and its super-extension) necessary to check finiteness beyond one loop has still not been worked out. Why then are orthodox string theorists so confident that superstrings are finite? I put this question to a superstring expert at the recent Nobel Symposium and was told "Experts don't ask that question"!

To be charitable, what he meant (I think) is that those who have worked with superstrings for many years have developed a feeling for the subject which enables them to make educated guesses about the outcome of calculations still too

complicated to carry out. No one could argue with that. Conjecture and refutation is the way that science progresses, after all. But since the claim of a consistent (finite) quantum gravity is the single most important boast of superstrings, this should not be a matter for complacency. Yet there is an attitude pervading the orthodox string community that finiteness is so obvious that we do not need to prove it^{*)}. I suggest that the finiteness question is one that all of us, experts and non-experts alike, should continue to ask since the future of theoretical physics hinges on the outcome.

6. - ORTHODOX PHENOMENOLOGY

Up until recently, superstring phenomenology was based on the Calabi-Yau approach of Ref. 6) and looked like Fig. 7. Aside from the question of why we pick the $E_6 \times E_6$ heterotic string and not one of the others (another question the experts do not like to ask), there are several other questionable features. Must we first take the field-theory limit in ten dimensions before compactifying to four dimensions? Is a Calabi-Yau manifold the only way to go from 10 dimensions to 4? Is the only way to achieve $SU(3) \times (SU(2) \times U(1))$ via E_6 ? Can one prove that the extra $U(1)$'s or $SU(2)$'s are inevitable? Do we understand the mechanism for supersymmetry breaking? The answer to all these questions is "No"!

The main objection to this orthodox phenomenology, however, is that we now understand, much better than we did three years ago, how to arrive at a four-dimensional string theory. In order to appreciate this we shall briefly review the Kaluza-Klein idea, the Frenkel-Kac idea, the Narain idea and the ideas of orbifolds and four-dimensional strings.

7. - FOUR-DIMENSIONAL SUPERSTRINGS

The Kaluza idea

By 1919, Maxwell's theory of electromagnetism had been around for some time and Einstein had recently formulated his general theory of relativity. According to Einstein the laws of physics should be the same to all observers and not merely the same to those in relative uniform motion, as he had earlier proposed in his special relativity. This is the principle of general co-ordinate invariance. In this picture, the gravitational field is described by a symmetric second-rank tensor, $g_{\mu\nu}(x)$ where the indices μ and ν run over the four dimensions

*) Two physicists exempt from this criticism are Stanley Mandelstam¹⁷⁾ and John G. Taylor¹⁸⁾. Indeed, J.G. Taylor even claims to have proved finiteness in a series of papers largely ignored by the orthodox community. He surely deserves a response. By contrast, A. Casher¹⁹⁾ claims that there is no consistent finite string theory.

of space-time ($\mu, \nu = 1, 2, 3, 4$). Moreover, this tensor admits the geometrical interpretation of a metric, i.e., it describes the infinitesimal proper distance ds between points in a curved space-time^{*}):

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu. \quad (7.1)$$

Note that in curved space-time, the metric is a field: it itself depends on the space-time co-ordinates $x^\mu = (ct, x, y, z)$, just like Maxwell's electromagnetic vector-potential $A_\mu(x)$.

Of course, very little was known about the strong and weak nuclear forces at this time so it was natural, in searching for a unified field theory, to try and combine gravity with electromagnetism. This Kaluza²⁰⁾ was able to do by the ingenious device of postulating an extra, fifth dimension for space-time. Kaluza considered Einstein's theory of gravity alone but with a five-dimensional line element

$$ds^2 = \hat{g}_{MN}(z) dz^M dz^N \quad (7.2)$$

where the indices M, N now run over 1 to 5 and z^M are the co-ordinates of a five-dimensional space-time. Thus

$$z^M = (x^\mu, y) \quad (7.3)$$

where x^μ are the usual co-ordinates of space-time and $y = x^5$ is the extra fifth co-ordinate. We put hats on ds and $\hat{g}_{MN}$ to distinguish them from the four-dimensional quantities of (7.1). Kaluza then suggested the following four-dimensional interpretation

$\hat{g}_{\mu\nu} = g_{\mu\nu}(x)$	TENSOR	SPIN-2	GRAVITON	
$\hat{g}_{\mu 5} = A_\mu(x)$	VECTOR	SPIN-1	PHOTON	(7.4)
$\hat{g}_{55} = \phi(x)$	SCALAR	SPIN-0	DILATON	

i.e., the $\mu\nu$ components have the four-dimensional interpretation of a tensor which we identify with the usual gravitational field; the $\mu 5$ components have the four-dimensional interpretation of a vector which (and this is the crucial step) we identify with Maxwell's electromagnetic vector potential; and the 55 components have the four-dimensional interpretation of a scalar which is a new (and in 1919, rather embarrassing) field. All this was before the invention of quantum field theory but nowadays we would make the identification of these

fields with particles, namely a spin 2 graviton, a spin 1 photon, and a spin 0 dilaton (so called because, in Klein's interpretation to be discussed below, the field ϕ measures the size of the fifth dimension).

Of course, it is not enough to call $g_{\mu 5}$ by the name photon, one must also check that it obeys the correct equations and here we see the Kaluza-Klein miracle at work: Einstein's pure gravity equations in $d = 5$ yielded not only the correct gravity equations for $g_{\mu\nu}(x)$ in $d = 4$ but also the correct Maxwell's equations for $A_\mu(x)$ and, as an extra bonus, the Klein-Gordon equation for $\phi(x)$. In other words, Maxwell's theory of electromagnetism was seen to be a consequence of Einstein's theory of gravity, given that one was willing to buy this extra fifth dimension. In particular, conservation of electric charge (gauge invariance) was a consequence of conservation of energy momentum (general co-ordinate invariance).

The Klein idea

Attractive though Kaluza's idea was, it contained a rather unsatisfactory ad hoc assumption. Although the indices were allowed to run over 1 to 5, the metric $\hat{g}_{MN}$ depended on only the four space-time co-ordinates x^μ . For no very good reason, the dependence on the fifth co-ordinate y had been suppressed. An attempt to remedy this defect was made in 1926 by Oskar Klein²¹⁾ who suggested that the fifth dimension should have a different "topology" from the other four. In other words, whereas

$$-\infty < x^\mu < \infty \quad (7.5)$$

the fifth co-ordinate is periodic

$$0 \leq y < 2\pi \quad (7.6)$$

and the fifth dimension has been "compactified" to a circle of radius m^{-1} . Mathematically we say that space-time has topology $R^4 \times S^1$ (the circle S^1 is a special case of a k -dimensional sphere S^k), which means physically that if we set out in the fifth direction we will always return to our starting point.

It is difficult to conceptualize a Universe with $R^4 \times S^1$ topology, but a simpler model with $R^1 \times S^1$ is provided by the hose-pipe analogy. Consider a long, thin hose-pipe as in Figure 8. At large distances it looks like a one-dimensional line (topology R^1), but closer inspection reveals that at each point on the line there is a little circle (topology S^1). So it was that Klein suggested a little circle (S^1) at each point of four-dimensional space-time (R^4).

The point of all this is that we can now retain the dependence on the fifth co-ordinate without giving up the four-dimensional interpretation. All we have to do is perform a Fourier expansion

$$\hat{g}_{MN}(x,y) = \sum_{n=-\infty}^{\infty} \hat{g}_{MN}^{(n)}(x) e^{iny}. \quad (7.7)$$

The $n = 0$ modes in this expansion are just the massless particles that Klein had already discussed, namely the graviton, the photon and the dilaton of (7.4). But now we also have an infinite tower of $n \neq 0$ modes. Substituting this expression (7.7) into the five-dimensional Einstein equations reveals that these extra modes, which satisfy

$$\hat{g}_{MN}^{(n)} = \hat{g}_{MN}^{(-n)*} \quad (7.8)$$

admit the four-dimensional interpretation of charged, massive particles with masses

$$m_n = nm \quad (7.9)$$

and electric charges

$$q_n = n\kappa m \quad (7.10)$$

where

$$\kappa = \sqrt{G}. \quad (7.11)$$

Note that the masses are quantized in units of m , the inverse radius of the circle, and that the charges are quantized in units of a fundamental charge

$$e = \kappa m. \quad (7.12)$$

Although this five-dimensional Kaluza-Klein theory is only a simple model, it illustrates three important points that survive in more realistic models:

(1) The electric charges of the elementary particles are quantized in units of a fundamental charge (a well-known, but hitherto unexplained, empirical fact).

(2) The gravitational constant $\sqrt{G}$ and the electric charge e are proportional to one another, with the constant of proportionality given by the size of the extra compact dimensions, as in (7.2).

(3) This means that with e of order unity, we are forced to take the radius of the extra dimensions to be of the very small; the Planck size

$$m^{-1} \sim 10^{-33} \text{ cm} \quad (7.13)$$

which is satisfactorily consistent with the fact that these extra space-time dimensions are not apparent in our everyday lives. But this in turn means that we are forced to take the fundamental mass m to be Planck size, i.e., 10^{19} GeV.

We may summarize the Kaluza-Klein philosophy as follows: what we perceive to be internal symmetries in $d = 4$ (electric charge, colour, charge conjugation, etc.) are really space-time symmetries in $d > 4$ (general covariance, parity, etc.). For a comprehensive review, see Ref. 2).

The Frenkel-Kac idea

Let us now extend the idea to r extra dimensions. If we choose these extra dimensions to be a torus T^r , i.e., a product of r circles

$$T^r = \underbrace{S^1 \times S^1 \times \dots S^1}_{r \text{ times}}, \quad (7.14)$$

then the four-dimensional gauge group is just Abelian

$$G = \underbrace{U(1) \times U(1) \times \dots U(1)}_{r \text{ times}} \quad (7.15)$$

It might appear at first sight, therefore, that the torus is not a very realistic choice for describing groups containing the $SU(3) \times SU(2) \times U(1)$ of the standard model. However, our Kaluza-Klein discussion has so far dealt only with ordinary field theory. As we shall now describe, the story with string theory has an interesting "twist" to it.

The reason is that the string can wind around the circles of the extra dimensions. For a single circle, one identifies

$$X \equiv X + 2\pi Rn \quad (7.16)$$

where R is the radius of the circle and the integer n is the number of times the string wraps around the circle. When this winding number is non-zero, there are extra topological "soliton" states in the theory in addition to the usual Kaluza-Klein states. For a specific value of the radius R , moreover, some of the

spin-1 states are massless and the Abelian Kaluza-Klein $U(1)$ is enlarged to a non-Abelian $SU(2)$.

One may generalize the argument to r circles and one finds that for specific tori the Kaluza-Klein group $[U(1)]^r$ is enlarged to any simply-laced non-Abelian group of rank r ²²⁾. The simply-laced groups of rank r are the orthogonal groups $SO(2r)$, the unitary groups $SU(r+1)$, and the exceptional groups E_r ($r=6,7,8$). One may also obtain any product of smaller simply-laced groups provided the total rank is r . Thus in string theory, the humble torus (as Abdus Salam calls it) can nevertheless give rich symmetries big enough to contain $SU(3) \times SU(2) \times U(1)$.

In the case of the compactification of the left-moving heterotic string from 26 to 10 dimensions, the requirements of modular invariance force the T^{16} tori to be such as to yield the very special simply-laced rank 16 groups $E_8 \times E_8$ or $SO(32)$. This is how the heterotic string was first constructed⁵⁾.

The Narain idea

We have seen how the heterotic string has 26 left-moving dimensions and 10 right-moving dimensions and how, by compactification of 16 left-movers on T^{16} , we arrive at a string theory with the interpretation of 10 space-time dimensions and internal gauge groups $E_8 \times E_8$ or $SO(32)$.

The next important development in string compactification came from K.S. Narain²³⁾, a young Indian physicist working at the Rutherford Laboratory. Although in detail it is technically complicated, the broad idea is so simple and yet so far-reaching that one wonders why no one had thought of it before. Instead of going from 26 to 10 with the left-movers leaving the right movers alone, why not go straight away from 26 to 4 on the left with T^{22} and go from 10 to 4 on the right with T^6 so as to obtain a four-dimensional string theory directly? This is precisely what Narain did. Of course, one must check that the tori are so chosen as to comply with the requirements of modular invariance. In this way we arrive at a string theory with the interpretation of four space-time dimensions and an internal symmetry of any simply-laced group with rank

$$r = \text{rank} G = 22$$

(7.17)

Some comments are now in order. First, this is a compactification of the string theory itself. There is no need to go first to 10 dimensions, take the field-theory limit, and then compactify the field theory from 10 to 4 as in Section 6. Secondly, by including the right movers in the compactification we have lost the uniqueness of $E_8 \times E_8$ or $SO(32)$. Now we can have a four-dimensional

theory with any simply-laced group of rank 22, of which there are thousands! For example we could have $SO(44)$, a group which neither contains $E_8 \times E_8$ as a subgroup nor is contained in it! Thirdly, the four-dimensional theory so obtained allows $22 \times 6 = 132$ free parameters.

In Narain's original construction, the resulting four-dimensional theory was still far from phenomenology. In particular there was $N = 4$ supersymmetry and no chiral fermions. However, a more realistic class of theories with $N = 1$ supersymmetry and chiral fermions may be obtained by appealing to the idea of "orbifolds" (an idea first applied in the context of going from 10 to 4 in the ten-dimensional heterotic string²⁴⁾). Instead of taking the simple tori T^{22} or T^6 we consider $T^{22}/\mathcal{G}$ and $T^6/\mathcal{G}$ where $\mathcal{G}$ is a discrete group. One may even consider "asymmetric orbifolds"²⁵⁾ where we consider $T^{22}/\mathcal{G}_L$ and $T^6/\mathcal{G}_R$ and where the left-moving and right-moving discrete groups are different: $\mathcal{G}_L \neq \mathcal{G}_R$. Now we must give up the luxury of interpreting the theory as a ten-dimensional string compactified to four dimensions, since there is no such space-time interpretation! The resulting models are then best described by the title "four-dimensional superstrings".

Fermionic Formulations

In Section 4, we saw that the string is described by boson fields $X^\mu(\tau, \sigma)$ and fermion fields $\lambda(\tau, \sigma)$ on the two-dimensional world sheet. It is a peculiar property of two dimensions (where spin is not defined) that we may swap one boson for two Majorana fermions and vice versa. This does not spoil the conformal invariance since from (4.6) one boson contributes to the conformal anomaly by the same amount as two Majorana fermions. This process of "fermionization" is, however, possible only when the bosonic co-ordinate is periodic. We may thus express the compactified X^μ_L as a left-handed Majorana fermion and the compactified X^μ_R as a right-handed Majorana fermion. If so desired, we may dispense completely with the idea of extra space-time dimensions! All we have is a conformally invariant set of bosons and fermions on the world sheet. Such fermionic formulations have proved very popular recently and have led to many new four-dimensional string theories²⁶⁾.

8. - CONCLUSIONS

The result of all this activity is that we may now construct four-dimensional superstrings satisfying all the theoretically desired properties:

conformal invariance, unbroken $N = 1$ supersymmetry, chiral fermions and modular invariance. But the uniqueness of the ten-dimensional strings is nowhere to be seen in four dimensions. We may have any simply-laced (and many non-simply laced) gauge groups with rank ≤ 22 . (Thus, to paraphrase Shelley^{*}), we can in particular incorporate a grand unified theory with the rank 18 group $[SU(4)]^6$.) We may have many difficult numbers of families and many different quark-lepton representations. The authors of Ref. 27), who used the device of even self-dual Lorentzian lattices of dimension 88, pointed out that there was a lower bound of about 10^{1500} such lattices each giving a four-dimensional superstring satisfying the above theoretical requirements! Not all such lattices give rise to different theories, however, so this number does not accurately reflect the number of four-dimensional superstrings. In fact, in the absence of an exhaustive classification, we do not yet know just how many there are but it surely runs into billions. This number becomes even larger when we include the four-dimensional superstrings that come from the Type II construction of Section 4. Contrary to what was originally suspected one may, in fact, construct four-dimensional Type II superstrings with realistic gauge groups and chiral fermions²⁸⁾. Unfortunately, although the possibilities are much more tightly constrained than in the heterotic formulation, none of them seems to give realistic fermion representations²⁹⁾.

For the time being, therefore, the phrase "superstring-inspired phenomenology" can only mean sifting through these billions of heterotic models in the hope of finding one that is realistic. The trouble with this needle-in-a-haystack approach is that even if we found a model with good phenomenology, we would be left wondering in what sense this could be described as a prediction of superstrings.

We have ended on a pessimistic note. Since this review deliberately set out to highlight the unresolved problems of string theory, let me say on the positive side how truly amazing it is that superstrings, motivated as they were by the desire for a consistent quantum theory of gravity, should have come anywhere close to the standard model. The moral I would draw at this stage is that we have seen some remarkable breakthroughs but that this is no time for complacency. We must understand the outstanding problems of finiteness, of supersymmetry breaking, of the cosmological constant and of gauge hierarchy.

Embracing perhaps all of these is the crucial question of vacuum degeneracy. There are good reasons for believing that the billions of four-dimensional superstring models are not billions of different theories but rather billions of different vacua of the same theory. But this will not help the phenomenology of superstrings one iota unless we can find some principle for lifting this

^{*}) The physicist, not the poet. See the talk by Prof. Glashow at this school.

degeneracy. Some string theorists put their faith in non-perturbative effects, others in string field theory, others in both. Some cosmologists, on the other hand, accept vacuum degeneracy as a fact of life. They argue that the Universe has really billions of different vacua and that we just happen to be living in one of them with $SU(3) \times SU(2) \times U(1)$, three families, etc. This view is very difficult, if not impossible, to refute but is certainly rather a pessimistic one for the future of scientific inquiry. As Murray Gell-Mann points out, if it is really true that we have to give not only the fundamental laws, but also hundreds of parameters peculiar to our specific Universe then physics will have been reduced to an environmental science like botany.

Perhaps the vacuum degeneracy problem par excellence is the question of why we appear to live in four space-time dimensions. Superstrings have as yet offered no clue. What about supermembranes?²⁹⁾ This is the subject of my second lecture³⁰⁾.

NAME	n	D _c	COMMENTS
Veneziano	0	26	SO(n), Sp(n) and SU(n) as possible gauge groups. No fermions. Tachyon.
Type I Superstring	1	10	Fermions. Space-time supersymmetry. No tachyons. Chiral. Only SO(32).
?	2	2	Cannot explain D = 4.
?	4	-2	Nonsense.

TABLE 1 - Open strings

NAME	DESCRIPTION	D _c	COMMENTS
Shapiro-Virasoro	26 × 26	26	Gravity but no Yang-Mills. Tachyon. No fermions.
Heterotic	26 × 10 _L	10	N = 1 supergravity plus E ₈ × E ₈ or SO(32) Yang-Mills. Chiral.
Type IIA	10 _R × 10 _L	10	N = 2a supergravity. Non-chiral.
Type IIB	10 _L × 10 _L	10	N = 2b supergravity. Chiral.

TABLE 2 - Closed Strings

REFERENCES

- 1) J.H. Schwarz - "Superstrings - A Progress Report", Preprint CALT-68-1417 (1987).
- 2) M.J. Duff, B.E. W. Nilsson and C.N. Pope - Physics Reports 130 (1986) 1.
- 3) "Unification of the Fundamental Interactions", Proceedings of the Second Nobel Symposium on Elementary Particle Physics, Marstrand, Sweden, June 1986. (Eds. L. Brink et al., Physica Scripta T15, 1987).
- 4) M.B. Green and J.H. Schwarz - Phys.Lett. 149B (1984) 117.
- 5) D. Gross, J. Harvey, E. Martinec and R. Rohm - Phys.Rev.Lett. 54 (1985) 502, Nucl.Phys. B256 (1985) 253.
- 6) P. Candelas, G. Horowitz, A. Strominger and E. Witten - Nucl.Phys. B258 (1985) 46.
- 7) Y. Nambu - in "Proceedings of the International Conference on Symmetries and Quark Models", Wayne State University (Gordon and Breach, 1970).
- 8) G. Veneziano - Nuovo Cimento 57A (1968) 190.
- 9) T. Yoneya - Progr.Theor.Phys. 51 (1974) 1907.
- 10) J. Scherk and J. Schwarz - Nucl.Phys. B81 (1974) 118.
- 11) D.M. Capper and M.J. Duff - Nuovo Cimento 23A (1974) 173.
- 12) S. Deser, M.J. Duff and C.J. Isham - Nucl.Phys. B111 (1976) 45.
- 13) M.J. Duff - Nucl.Phys. B125 (1977) 334.
- 14) A.M. Polyakov - Phys.Lett. 103B (1981) 207, 211.
- 15) P.G.O. Freund - Phys.Lett. 151B (1985) 387;
M.J. Duff, B.E.W. Nilsson and C.N. Pope - Phys.Lett. 163B (1985) 343;
A. Casher, F. Englert, H. Nicolai and A. Taormina - Phys.Lett. 162B (1985) 121;
M.J. Duff, B.E.W. Nilsson, C.N. Pope and N.P. Warner - Phys.Lett. 171B (1986) 170;
F. Englert, H. Nicolai and A. Schellekens - Nucl.Phys. B274 (1986) 315;
W. Lerche, D. Lust and A. Schellekens - Phys.Lett. 181B (1986) 71;
A. Chamseddine, M.J. Duff, B.E.W. Nilsson, C.N. Pope and D.A. Ross - Phys.Lett. 193B (1987) 444.
- 16) M.J. Duff - in Proceedings of the Fourth Capri Symposium, May 1986 (Ed. F. Buccella, World Scientific).
- 17) S. Mandelstam - Proceedings of the Niels Bohr Centennial Conference, Copenhagen (May 1985).
- 18) J.G. Taylor - CERN Preprint TH. 4596 (1986).
- 19) A. Casher - CERN Preprint TH. 4738 (1987).
- 20) Th. Kaluza - Sitzungsber.Preuss.Akad.Wiss. Berlin Math.Phys. K1 (1921) 966.
- 21) O. Klein - Nature 118 (1926) 516.

- 22) I. Frenkel and V.G. Kac - Inv.Math. 62 (1980) 23;
P. Goddard and D. Olive - in "Workshop on Vertex Operators in Mathematics and Physics", Berkeley (1983).
- 23) K.S. Narain - Phys.Lett. 169B (1986) 41.
- 24) L. Dixon, J. Harvey, C. Vafa and E. Witten - Nucl.Phys. B261 (1985) 678 and B274 (1986) 285.
- 25) K.S. Narain, M.H. Sarmadi and C. Vafa - Harvard Preprint HUTP-86/A089 (1986).
- 26) H. Kawai, D.C. Lewellen and S.H.H. Tye - Phys.Rev.Lett. 57 (1986) 1832;
Cornell University Preprint CLNS 86/75 (1986);
I. Antoniadis, C. Bachas and C. Kounnas - LBL Preprint LBL-22709 (1986).
- 27) W. Lerche, D. Lüst and A.N. Schellekens - CERN Preprint TH. 4590 (1986).
- 28) L. Castellani, R. D'Auria, F. Gliozzi and S. Sciuto - Phys.Lett. 168B (1986) 47;
R. Bluhm, L. Dolan and P. Goddard, Rockefeller Preprint RU-B86 (1987);
H. Kawai, D.C. Lewellen and S.H.H. Tye - Cornell Preprint CLNS 87/760 (1987).
- 29) L. Dixon, V. Kaplunovsky and C. Vafa - SLAC Preprint SLAC-PUB-4282 (1987).
- 30) E. Bergshoeff, E. Sezgin and P.K. Townsend - Phys.Lett. 189B (1987) 75.
- 31) M.J. Duff - CERN Preprint TH. 4797 (1987).

FIGURE CAPTIONS

- Fig. 1 a) Point-particle sweeps out world-line in space-time with trajectory $X^\mu(\tau)$.
 b) Open string sweeps out world-sheet in space-time with trajectory $X^\mu(\tau, \sigma)$. The boundary conditions are $X'^\mu = 0$ for $\sigma = 0$ and $\sigma = \pi$.
 c) Closed string sweeps out world-sheet in space-time with trajectory $X^\mu(\tau, \sigma)$. The boundary conditions are $X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \pi)$.
- Fig. 2 The superstring mass spectrum. Bosons are denoted by dots and fermions by crosses.
- Fig. 3 The scattering of strings as seen by two different world-sheet observers A and B.
- Fig. 4 Feynman diagrams for one-loop corrections to four-particle scattering in field theory.
- Fig. 5 The corresponding process in string theory.
- Fig. 6 Higher loop corrections.
- Fig. 7 Orthodox phenomenology.
- Fig. 8 The hose-pipe analogy. At large distances the hose-pipe looks like R ; closer inspection reveals it to be $R \times S^1$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7 - ORTHODOX PHENOMENOLOGY

Fig. 8