Impact picture predictions for $\bar{p}p$ and pp elastic scattering at CERN collider, FNAL collider, LHC and SSC

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Abstract. We present in detail the predictions on $\bar{p}p$ and pp total and elastic differential cross-sections for energies 0.546 to 40 TeV obtained on the basis of the impact picture, including the Coulomb contribution.

1 Introduction

In the 1960's, it was realized that a useful approach to the theoretical understanding of hadronic scattering at high energies is to study the asymptotic behaviour of Feynman diagrams for relativistic quantum gauge field theories. With this approach, the low-order diagrams [1] show features similar to the droplet model of Yang and collaborators [2]. However, these features are greatly modified with the inclusion of higher-order diagrams [3]. In 1970, the unexpected result was found that the total cross sections and the integrated elastic cross sections must increase without bound in the limit of extremely high energies [4]. By incorporating this rising total cross section into the optical model, some phenomenological data analysis has been carried out beginning in the early seventies [5]. The predictions [5, 6] of this impact-picture phenomenology have been repeatedly verified by experiments [7], most recently from the data of the CERN $\bar{p}p$ collider [8] and the Tevatron collider [9].

The purpose of this paper is three-fold:

(1) Since the Tevatron collider at Fermilab is now ready to take more data, the first purpose of this paper is to present detailed predictions of our approach for the center-of-mass energy range of 0.8 to 2.0 TeV;

(2) Because of the recent data at the CERN collider at 0.55 TeV in the Coulomb interference region [8],

our prediction [10] is compared with this data; and

(3) Since it seems quite likely that the large hadron collider (LHC) and/or the superconducting super collider (SSC) will be built, the predictions of our approach is also given in some detail for two selected energies in the multi TeV energy range.

As the experimental data improve in accuracy, the phenomenological parameters becomes better determined. In the high-energy range under consideration, the background due to the exchange of ρ , ω , etc. is negligible. Thus there are six parameters for pp and $\bar{p}p$ elastic scattering, and they are denoted by

c, c', m_1, m_2, a, f .

Of these six, c is related to the strong coupling constant, while m_1 and m_2 are obtained from the electromagnetic form factor. The total cross sections are of course determined through the optical theorem.

Nearly a decade ago, these six parameters were first found accurately [11] on the basis of the ISR data. [Previously, c' was not known, and treated as an input]. Since then, their values have changed little, indicating that our impact-picture phenomenology is reliable when applied to higher energies. Their values at present are given below in (9).

The paper is organized as follows. In Sect. 2 we shall recall the main features of the phenomenological model we use to describe elastic scattering. Section 3 is devoted to the presentation of the numerical results, which will be compared to experimental data in Sect. 4.

2 Description of the model

The $\bar{p}p$ and pp elastic scattering are described by the following amplitudes

$$a(s, t) = a^N(s, t) \pm a^C(t) \quad (1)$$

where upper sign is for $\bar{p}p$ while the lower one is for

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Table 1

$\sqrt{s}$ (TeV)	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00	17.0	40.0
σ_{tot} (mb)	65.5	66.8	67.9	69.0	70.0	70.9	71.8	72.6	73.3	74.1	74.8	75.4	76.1	106.7	121.2
σ_{el}/σ_{tot}	0.216	0.218	0.220	0.222	0.223	0.224	0.225	0.227	0.228	0.229	0.230	0.232	0.233	0.284	0.300
p	0.127	0.128	0.128	0.128	0.128	0.128	0.128	0.128	0.128	0.128	0.128	0.128	0.128	0.119	0.114
$ t $ (GeV) ²	$d\sigma/dt$ mb/(GeV) ²														
0.04	1.14E+02	1.18E+02	1.22E+02	1.25E+02	1.29E+02	1.32E+02	1.35E+02	1.38E+02	1.40E+02	1.43E+02	1.45E+02	1.47E+02	1.50E+02	2.68E+02	3.29E+02
0.05	9.71E+01	1.00E+02	1.04E+02	1.07E+02	1.09E+02	1.12E+02	1.14E+02	1.16E+02	1.19E+02	1.21E+02	1.23E+02	1.25E+02	1.26E+02	2.21E+02	2.67E+02
0.06	8.28E+01	8.56E+01	8.82E+01	9.06E+01	9.28E+01	9.48E+01	9.69E+01	9.88E+01	1.01E+02	1.02E+02	1.04E+02	1.05E+02	1.07E+02	1.82E+02	2.17E+02
0.07	7.07E+01	7.31E+01	7.52E+01	7.72E+01	7.90E+01	8.08E+01	8.24E+01	8.40E+01	8.54E+01	8.68E+01	8.82E+01	8.94E+01	9.06E+01	1.50E+02	1.76E+02
0.08	6.05E+01	6.25E+01	6.43E+01	6.59E+01	6.74E+01	6.89E+01	7.02E+01	7.15E+01	7.27E+01	7.38E+01	7.49E+01	7.60E+01	7.70E+01	1.24E+02	1.43E+02
0.09	5.19E+01	5.35E+01	5.50E+01	5.63E+01	5.76E+01	5.88E+01	5.99E+01	6.09E+01	6.19E+01	6.29E+01	6.38E+01	6.46E+01	6.54E+01	1.02E+02	1.16E+02
0.10	4.46E+01	4.59E+01	4.71E+01	4.83E+01	4.93E+01	5.03E+01	5.12E+01	5.20E+01	5.29E+01	5.36E+01	5.44E+01	5.51E+01	5.57E+01	8.46E+01	9.45E+01
0.11	3.83E+01	3.94E+01	4.05E+01	4.14E+01	4.22E+01	4.30E+01	4.38E+01	4.45E+01	4.52E+01	4.58E+01	4.64E+01	4.70E+01	4.75E+01	6.99E+01	7.66E+01
0.12	3.30E+01	3.39E+01	3.48E+01	3.55E+01	3.62E+01	3.69E+01	3.75E+01	3.81E+01	3.86E+01	3.92E+01	3.96E+01	4.01E+01	4.05E+01	5.77E+01	6.20E+01
0.13	2.85E+01	2.92E+01	2.99E+01	3.06E+01	3.11E+01	3.17E+01	3.22E+01	3.27E+01	3.31E+01	3.35E+01	3.39E+01	3.43E+01	3.46E+01	4.76E+01	5.01E+01
0.14	2.46E+01	2.52E+01	2.58E+01	2.63E+01	2.68E+01	2.72E+01	2.76E+01	2.80E+01	2.84E+01	2.87E+01	2.90E+01	2.93E+01	2.96E+01	3.93E+01	4.04E+01
0.15	2.12E+01	2.18E+01	2.22E+01	2.27E+01	2.31E+01	2.34E+01	2.38E+01	2.41E+01	2.44E+01	2.46E+01	2.49E+01	2.51E+01	2.54E+01	3.24E+01	3.26E+01
0.16	1.84E+01	1.88E+01	1.92E+01	1.96E+01	1.99E+01	2.02E+01	2.04E+01	2.07E+01	2.09E+01	2.11E+01	2.13E+01	2.15E+01	2.17E+01	2.67E+01	2.61E+01
0.17	1.59E+01	1.63E+01	1.66E+01	1.69E+01	1.71E+01	1.74E+01	1.76E+01	1.78E+01	1.80E+01	1.82E+01	1.83E+01	1.85E+01	1.86E+01	2.20E+01	2.09E+01
0.18	1.38E+01	1.41E+01	1.44E+01	1.46E+01	1.48E+01	1.50E+01	1.52E+01	1.53E+01	1.55E+01	1.56E+01	1.57E+01	1.59E+01	1.60E+01	1.80E+01	1.67E+01
0.19	1.20E+01	1.22E+01	1.24E+01	1.26E+01	1.28E+01	1.29E+01	1.31E+01	1.32E+01	1.33E+01	1.34E+01	1.35E+01	1.36E+01	1.37E+01	1.48E+01	1.33E+01
0.20	1.04E+01	1.06E+01	1.08E+01	1.09E+01	1.10E+01	1.11E+01	1.12E+01	1.13E+01	1.14E+01	1.15E+01	1.16E+01	1.17E+01	1.18E+01	1.21E+01	1.03E+01
0.21	9.04E+00	9.20E+00	9.33E+00	9.45E+00	9.56E+00	9.65E+00	9.74E+00	9.81E+00	9.88E+00	9.95E+00	1.00E+01	1.01E+01	1.02E+01	9.89E+00	8.28E+00
0.22	7.86E+00	7.99E+00	8.09E+00	8.19E+00	8.27E+00	8.34E+00	8.41E+00	8.47E+00	8.52E+00	8.57E+00	8.61E+00	8.65E+00	8.68E+00	8.06E+00	6.48E+00
0.23	6.84E+00	6.94E+00	7.03E+00	7.10E+00	7.16E+00	7.22E+00	7.27E+00	7.31E+00	7.35E+00	7.38E+00	7.41E+00	7.44E+00	7.46E+00	6.55E+00	5.03E+00
0.24	5.96E+00	6.04E+00	6.10E+00	6.16E+00	6.20E+00	6.24E+00	6.28E+00	6.31E+00	6.34E+00	6.36E+00	6.38E+00	6.40E+00	6.41E+00	5.31E+00	3.90E+00
0.25	5.19E+00	5.25E+00	5.30E+00	5.34E+00	5.37E+00	5.40E+00	5.43E+00	5.45E+00	5.47E+00	5.48E+00	5.49E+00	5.50E+00	5.51E+00	4.28E+00	2.99E+00
0.26	4.52E+00	4.57E+00	4.60E+00	4.63E+00	4.66E+00	4.68E+00	4.69E+00	4.71E+00	4.72E+00	4.72E+00	4.73E+00	4.73E+00	4.74E+00	3.45E+00	2.28E+00
0.27	3.94E+00	3.98E+00	4.00E+00	4.02E+00	4.04E+00	4.05E+00	4.06E+00	4.07E+00	4.07E+00	4.07E+00	4.07E+00	4.07E+00	4.07E+00	2.76E+00	1.72E+00
0.28	3.44E+00	3.46E+00	3.48E+00	3.49E+00	3.50E+00	3.51E+00	3.51E+00	3.51E+00	3.51E+00	3.51E+00	3.51E+00	3.51E+00	3.50E+00	2.20E+00	1.29E+00
0.29	3.00E+00	3.02E+00	3.03E+00	3.03E+00	3.04E+00	3.04E+00	3.04E+00	3.03E+00	3.03E+00	3.02E+00	3.02E+00	3.01E+00	3.00E+00	1.75E+00	9.54E-01
0.30	2.62E+00	2.63E+00	2.63E+00	2.63E+00	2.63E+00	2.63E+00	2.63E+00	2.62E+00	2.61E+00	2.61E+00	2.60E+00	2.59E+00	2.58E+00	1.38E+00	7.03E-01
0.31	2.29E+00	2.29E+00	2.29E+00	2.29E+00	2.28E+00	2.28E+00	2.28E+00	2.27E+00	2.26E+00	2.25E+00	2.24E+00	2.23E+00	2.22E+00	1.08E+00	5.17E-01
0.32	2.00E+00	1.99E+00	1.99E+00	1.99E+00	1.98E+00	1.97E+00	1.96E+00	1.95E+00	1.94E+00	1.93E+00	1.92E+00	1.91E+00	1.90E+00	8.45E-01	3.82E-01
0.33	1.74E+00	1.73E+00	1.72E+00	1.72E+00	1.72E+00	1.71E+00	1.70E+00	1.69E+00	1.67E+00	1.66E+00	1.65E+00	1.64E+00	1.63E+00	6.56E-01	2.88E-01
0.34	1.52E+00	1.51E+00	1.50E+00	1.50E+00	1.49E+00	1.48E+00	1.47E+00	1.45E+00	1.44E+00	1.43E+00	1.42E+00	1.41E+00	1.39E+00	5.07E-01	2.25E-01
0.35	1.33E+00	1.32E+00	1.31E+00	1.30E+00	1.29E+00	1.28E+00	1.27E+00	1.25E+00	1.24E+00	1.23E+00	1.22E+00	1.21E+00	1.19E+00	3.91E-01	1.86E-01
0.36	1.16E+00	1.15E+00	1.14E+00	1.13E+00	1.12E+00	1.10E+00	1.09E+00	1.08E+00	1.07E+00	1.06E+00	1.04E+00	1.03E+00	1.02E+00	3.01E-01	1.66E-01
0.37	1.01E+00	1.00E+00	9.90E-01	9.78E-01	9.66E-01	9.54E-01	9.42E-01	9.30E-01	9.18E-01	9.07E-01	8.95E-01	8.84E-01	8.73E-01	2.32E-01	1.60E-01
0.38	8.83E-01	8.72E-01	8.60E-01	8.48E-01	8.36E-01	8.24E-01	8.12E-01	8.01E-01	7.89E-01	7.78E-01	7.67E-01	7.56E-01	7.45E-01	1.81E-01	1.63E-01
0.39	7.70E-01	7.59E-01	7.47E-01	7.35E-01	7.23E-01	7.11E-01	7.00E-01	6.88E-01	6.77E-01	6.66E-01	6.56E-01	6.45E-01	6.35E-01	1.44E-01	1.74E-01
0.40	6.72E-01	6.60E-01	6.48E-01	6.37E-01	6.25E-01	6.14E-01	6.02E-01	5.91E-01	5.81E-01	5.70E-01	5.60E-01	5.50E-01	5.41E-01	1.18E-01	1.89E-01
0.41	5.86E-01	5.74E-01	5.62E-01	5.51E-01	5.40E-01	5.29E-01	5.18E-01	5.08E-01	4.98E-01	4.88E-01	4.78E-01	4.69E-01	4.60E-01	1.01E-01	2.08E-01
0.42	5.11E-01	4.99E-01	4.88E-01	4.77E-01	4.66E-01	4.55E-01	4.45E-01	4.35E-01	4.26E-01	4.17E-01	4.08E-01	3.99E-01	3.91E-01	9.09E-02	2.27E-01
0.43	4.45E-01	4.34E-01	4.23E-01	4.12E-01	4.02E-01	3.92E-01	3.82E-01	3.73E-01	3.64E-01	3.55E-01	3.47E-01	3.39E-01	3.31E-01	8.58E-02	2.47E-01
0.44	3.87E-01	3.77E-01	3.66E-01	3.56E-01	3.46E-01	3.37E-01	3.28E-01	3.19E-01	3.11E-01	3.02E-01	2.95E-01	2.87E-01	2.80E-01	8.47E-02	2.67E-01
0.45	2.93E-01	2.83E-01	2.74E-01	2.65E-01	2.56E-01	2.48E-01	2.40E-01	2.33E-01	2.25E-01	2.19E-01	2.12E-01	2.06E-01	2.00E-01	8.66E-02	2.85E-01
0.46	2.35E-01	2.25E-01	2.16E-01	2.08E-01	1.99E-01	1.92E-01	1.85E-01	1.78E-01	1.72E-01	1.66E-01	1.60E-01	1.54E-01	1.48E-01	9.05E-02	3.02E-01
0.47	2.35E-01	2.45E-01	2.36E-01	2.28E-01	2.20E-01	2.12E-01	2.05E-01	1.98E-01	1.92E-01	1.85E-01	1.79E-01	1.74E-01	1.68E-01	9.59E-02	3.17E-01
0.48	2.21E-01	2.12E-01	2.04E-01	1.96E-01	1.89E-01	1.82E-01	1.75E-01	1.69E-01	1.63E-01	1.57E-01	1.52E-01	1.46E-01	1.42E-01	1.02E-01	3.31E-01
0.49	1.92E-01	1.84E-01	1.76E-01	1.69E-01	1.62E-01	1.55E-01	1.49E-01	1.43E-01	1.38E-01	1.33E-01	1.28E-01	1.23E-01	1.19E-01	1.09E-01	3.42E-01
0.50	1.67E-01	1.59E-01	1.51E-01	1.45E-01	1.38E-01	1.32E-01	1.27E-01	1.21E-01	1.17E-01	1.12E-01	1.08E-01	1.03E-01	9.96E-02	1.16E-01	3.51E-01

Table 1 (continued)

$\sqrt{s}$ (TeV)	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00	17.0	40.0
0.52	1.25E-01	1.18E-01	1.12E-01	1.06E-01	1.01E-01	9.59E-02	9.13E-02	8.70E-02	8.31E-02	7.94E-02	7.59E-02	7.27E-02	6.97E-02	1.29E-01	3.64E-01
0.54	9.32E-02	8.75E-02	8.23E-02	7.75E-02	7.31E-02	6.91E-02	6.54E-02	6.20E-02	5.89E-02	5.60E-02	5.34E-02	5.09E-02	4.87E-02	1.40E-01	3.69E-01
0.56	6.93E-02	6.45E-02	6.02E-02	5.63E-02	5.28E-02	4.96E-02	4.67E-02	4.40E-02	4.17E-02	3.95E-02	3.75E-02	3.57E-02	3.41E-02	1.49E-01	3.67E-01
0.58	5.13E-02	4.73E-02	4.38E-02	4.07E-02	3.79E-02	3.55E-02	3.33E-02	3.13E-02	2.95E-02	2.80E-02	2.66E-02	2.54E-02	2.43E-02	1.56E-01	3.60E-01
0.60	3.78E-02	3.46E-02	3.18E-02	2.94E-02	2.73E-02	2.54E-02	2.38E-02	2.24E-02	2.12E-02	2.01E-02	1.92E-02	1.85E-02	1.79E-02	1.59E-01	3.49E-01
0.62	2.77E-02	2.52E-02	2.31E-02	2.12E-02	1.97E-02	1.83E-02	1.72E-02	1.63E-02	1.55E-02	1.49E-02	1.44E-02	1.41E-02	1.38E-02	1.60E-01	3.35E-01
0.64	2.03E-02	1.84E-02	1.68E-02	1.54E-02	1.43E-02	1.35E-02	1.28E-02	1.22E-02	1.18E-02	1.16E-02	1.15E-02	1.14E-02	1.15E-02	1.59E-01	3.18E-01
0.66	1.49E-02	1.35E-02	1.23E-02	1.14E-02	1.07E-02	1.02E-02	9.85E-03	9.65E-03	9.27E-03	9.70E-03	9.74E-03	9.96E-03	1.03E-02	1.57E-01	2.99E-01
0.68	1.10E-02	9.95E-03	9.19E-03	8.64E-03	8.28E-03	8.09E-03	8.03E-03	8.09E-03	8.27E-03	8.53E-03	8.88E-03	9.31E-03	9.80E-03	1.53E-01	2.80E-01
0.70	8.19E-03	7.52E-03	7.08E-03	6.83E-03	6.75E-03	6.80E-03	6.98E-03	7.26E-03	7.63E-03	8.07E-03	8.59E-03	9.17E-03	9.81E-03	1.47E-01	2.60E-01
0.72	6.24E-03	5.87E-03	5.70E-03	5.70E-03	5.84E-03	6.10E-03	6.45E-03	6.90E-03	7.49E-03	8.18E-03	8.92E-03	9.70E-03	1.05E-02	1.34E-01	2.40E-01
0.74	4.92E-03	4.79E-03	4.85E-03	5.04E-03	5.36E-03	5.77E-03	6.28E-03	6.85E-03	7.49E-03	8.18E-03	8.92E-03	9.70E-03	1.05E-02	1.34E-01	2.21E-01
0.76	4.05E-03	4.13E-03	4.36E-03	4.71E-03	5.17E-03	5.71E-03	6.32E-03	6.99E-03	7.71E-03	8.48E-03	9.29E-03	1.01E-02	1.10E-02	1.27E-01	2.02E-01
0.78	3.50E-03	3.75E-03	4.12E-03	4.60E-03	5.16E-03	5.79E-03	6.49E-03	7.23E-03	8.01E-03	8.83E-03	9.67E-03	1.05E-02	1.14E-02	1.19E-01	1.84E-01
0.80	3.19E-03	3.57E-03	4.05E-03	4.62E-03	5.26E-03	5.96E-03	6.71E-03	7.50E-03	8.32E-03	9.17E-03	1.00E-02	1.09E-02	1.18E-02	1.11E-01	1.67E-01
0.82	3.00E-03	3.52E-03	4.09E-03	4.73E-03	5.43E-03	6.17E-03	6.96E-03	7.77E-03	8.61E-03	9.47E-03	1.03E-02	1.12E-02	1.21E-02	1.04E-01	1.51E-01
0.84	3.04E-03	3.55E-03	4.18E-03	4.87E-03	5.61E-03	6.38E-03	7.18E-03	8.01E-03	8.85E-03	9.71E-03	1.06E-02	1.15E-02	1.23E-02	9.63E-02	1.36E-01
0.86	3.02E-03	3.63E-03	4.30E-03	5.02E-03	5.78E-03	6.57E-03	7.38E-03	8.20E-03	9.04E-03	9.89E-03	1.07E-02	1.16E-02	1.25E-02	8.91E-02	1.32E-01
0.88	3.09E-03	3.73E-03	4.43E-03	5.17E-03	5.94E-03	6.72E-03	7.53E-03	8.34E-03	9.17E-03	1.00E-02	1.08E-02	1.17E-02	1.25E-02	8.22E-02	1.10E-01
0.90	3.17E-03	3.84E-03	4.55E-03	5.29E-03	6.06E-03	6.84E-03	7.63E-03	8.43E-03	9.23E-03	1.00E-02	1.08E-02	1.16E-02	1.24E-02	7.55E-02	8.80E-02
0.92	3.25E-03	3.93E-03	4.65E-03	5.39E-03	6.14E-03	6.91E-03	7.68E-03	8.45E-03	9.22E-03	1.00E-02	1.08E-02	1.15E-02	1.23E-02	6.92E-02	8.73E-02
0.94	3.33E-03	4.01E-03	4.72E-03	5.45E-03	6.19E-03	6.93E-03	7.68E-03	8.42E-03	9.16E-03	9.90E-03	1.06E-02	1.14E-02	1.21E-02	6.33E-02	7.76E-02
0.96	3.39E-03	4.07E-03	4.77E-03	5.48E-03	6.20E-03	6.91E-03	7.63E-03	8.34E-03	9.05E-03	9.75E-03	1.05E-02	1.11E-02	1.18E-02	5.78E-02	6.88E-02
0.98	3.44E-03	4.11E-03	4.79E-03	5.48E-03	6.17E-03	6.86E-03	7.54E-03	8.22E-03	8.89E-03	9.56E-03	1.02E-02	1.09E-02	1.15E-02	5.28E-02	6.08E-02
1.00	3.47E-03	4.12E-03	4.78E-03	5.45E-03	6.11E-03	6.76E-03	7.41E-03	8.06E-03	8.70E-03	9.32E-03	9.94E-03	1.06E-02	1.12E-02	4.78E-02	5.37E-02
1.05	3.46E-03	4.06E-03	4.66E-03	5.25E-03	5.83E-03	6.41E-03	6.97E-03	7.53E-03	8.07E-03	8.61E-03	9.13E-03	9.64E-03	1.01E-02	3.72E-02	4.39E-02
1.10	3.36E-03	3.89E-03	4.41E-03	4.92E-03	5.43E-03	5.92E-03	6.40E-03	6.87E-03	7.33E-03	7.78E-03	8.21E-03	8.64E-03	9.05E-03	2.87E-02	2.78E-02
1.15	3.17E-03	3.63E-03	4.08E-03	4.52E-03	4.95E-03	5.37E-03	5.77E-03	6.16E-03	6.54E-03	6.91E-03	7.27E-03	7.62E-03	7.95E-03	2.20E-02	1.97E-02
1.20	2.94E-03	3.33E-03	3.71E-03	4.09E-03	4.44E-03	4.79E-03	5.12E-03	5.44E-03	5.75E-03	6.05E-03	6.35E-03	6.63E-03	6.90E-03	1.67E-02	1.37E-02
1.25	2.67E-03	3.01E-03	3.33E-03	3.64E-03	3.93E-03	4.22E-03	4.49E-03	4.75E-03	5.01E-03	5.25E-03	5.48E-03	5.71E-03	5.92E-03	1.25E-02	9.53E-03
1.30	2.40E-03	2.68E-03	2.95E-03	3.20E-03	3.45E-03	3.68E-03	3.90E-03	4.11E-03	4.32E-03	4.51E-03	4.70E-03	4.87E-03	5.05E-03	9.35E-03	6.56E-03
1.35	2.13E-03	2.37E-03	2.59E-03	2.80E-03	2.99E-03	3.18E-03	3.36E-03	3.53E-03	3.69E-03	3.84E-03	3.99E-03	4.13E-03	4.27E-03	6.93E-03	4.51E-03
1.40	1.88E-03	2.07E-03	2.25E-03	2.42E-03	2.58E-03	2.73E-03	2.87E-03	3.01E-03	3.14E-03	3.26E-03	3.37E-03	3.48E-03	3.59E-03	5.11E-03	3.18E-03
1.45	1.64E-03	1.80E-03	1.95E-03	2.08E-03	2.21E-03	2.33E-03	2.44E-03	2.55E-03	2.65E-03	2.75E-03	2.84E-03	2.92E-03	3.00E-03	3.74E-03	2.18E-03
1.50	1.43E-03	1.55E-03	1.67E-03	1.78E-03	1.89E-03	1.98E-03	2.07E-03	2.15E-03	2.23E-03	2.30E-03	2.37E-03	2.44E-03	2.50E-03	2.73E-03	1.58E-03
1.60	1.06E-03	1.14E-03	1.22E-03	1.29E-03	1.35E-03	1.41E-03	1.47E-03	1.52E-03	1.56E-03	1.61E-03	1.65E-03	1.68E-03	1.72E-03	1.44E-03	9.23E-04
1.70	7.77E-04	8.30E-04	8.78E-04	9.21E-04	9.60E-04	9.95E-04	1.03E-03	1.06E-03	1.08E-03	1.11E-03	1.13E-03	1.15E-03	1.17E-03	7.67E-04	7.53E-04
1.80	5.64E-04	5.97E-04	6.26E-04	6.52E-04	6.75E-04	6.95E-04	7.13E-04	7.30E-04	7.44E-04	7.56E-04	7.68E-04	7.78E-04	7.86E-04	4.26E-04	6.39E-04
1.90	4.06E-04	4.26E-04	4.42E-04	4.59E-04	4.72E-04	4.83E-04	4.93E-04	5.01E-04	5.08E-04	5.14E-04	5.19E-04	5.24E-04	5.27E-04	2.60E-04	5.95E-04
2.00	2.91E-04	3.03E-04	3.13E-04	3.21E-04	3.28E-04	3.34E-04	3.39E-04	3.43E-04	3.46E-04	3.48E-04	3.50E-04	3.51E-04	3.52E-04	1.82E-04	5.49E-04
2.10	2.08E-04	2.15E-04	2.20E-04	2.25E-04	2.28E-04	2.31E-04	2.33E-04	2.34E-04	2.34E-04	2.35E-04	2.35E-04	2.35E-04	2.35E-04	1.44E-04	4.94E-04
2.20	1.48E-04	1.52E-04	1.55E-04	1.57E-04	1.58E-04	1.59E-04	1.59E-04	1.59E-04	1.59E-04	1.58E-04	1.57E-04	1.56E-04	1.55E-04	1.24E-04	4.30E-04
2.30	1.06E-04	1.08E-04	1.09E-04	1.09E-04	1.09E-04	1.09E-04	1.09E-04	1.08E-04	1.08E-04	1.08E-04	1.05E-04	1.04E-04	1.03E-04	1.10E-04	3.64E-04
2.40	7.56E-05	7.62E-05	7.64E-05	7.63E-05	7.59E-05	7.53E-05	7.46E-05	7.38E-05	7.28E-05	7.18E-05	7.07E-05	6.96E-05	6.85E-05	9.79E-05	3.01E-04
2.50	5.41E-05	5.41E-05	5.38E-05	5.34E-05	5.27E-05	5.20E-05	5.12E-05	5.03E-05	4.94E-05	4.85E-05	4.75E-05	4.66E-05	4.56E-05	8.65E-05	2.43E-04
2.60	3.87E-05	3.84E-05	3.80E-05	3.74E-05	3.67E-05	3.60E-05	3.52E-05	3.44E-05	3.36E-05	3.28E-05	3.20E-05	3.12E-05	3.05E-05	7.53E-05	1.93E-04
2.70	2.79E-05	2.74E-05	2.69E-05	2.63E-05	2.57E-05	2.50E-05	2.43E-05	2.36E-05	2.30E-05	2.23E-05	2.17E-05	2.11E-05	2.05E-05	6.43E-05	1.50E-04
2.80	2.01E-05	1.96E-05	1.91E-05	1.86E-05	1.80E-05	1.74E-05	1.69E-05	1.63E-05	1.58E-05	1.53E-05	1.48E-05	1.43E-05	1.39E-05	5.41E-05	1.15E-04
2.90	1.46E-05	1.41E-05	1.37E-05	1.32E-05	1.27E-05	1.22E-05	1.18E-05	1.13E-05	1.09E-05	1.05E-05	1.02E-05	9.81E-06	9.49E-06	4.48E-05	8.76E-05
3.00	1.06E-05	1.02E-05	9.81E-06	9.40E-06	9.01E-06	8.63E-06	8.26E-06	7.93E-06	7.61E-06	7.32E-06	7.05E-06	6.80E-06	6.57E-06	3.67E-05	6.58E-05
3.20	5.76E-06	5.46E-06	5.17E-06	4.90E-06	4.64E-06	4.41E-06	4.20E-06	4.00E-06	3.83E-06	3.67E-06	3.53E-06	3.41E-06	3.31E-06	2.37E-05	6.51E-05
3.40	3.21E-06	3.00E-06	2.81E-06	2.64E-06	2.49E-06	2.35E-06	2.23E-06	2.12E-06	2.03E-06	1.95E-06	1.89E-06	1.83E-06	1.79E-06	1.47E-05	1.92E-05
3.60	1.85E-06	1.71E-06	1.59E-06	1.48E-06	1.39E-06	1.31E-06	1.24E-06	1.18E-06	1.14E-06	1.10E-06	1.07E-06	1.05E-06	1.04E-06	8.88E-06	9.98E-06
3.80	1.10E-06	1.01E-06	9.30E-07	8.63E-07	8.07E-07	7.61E-07	7.23E-07	6.94E-07	6.72E-07	6.56E-07	6.47E-07	6.43E-07	6.44E-07	5.22E-06	5.19E-06
4.00	6.79E-07	6.18E-07	5.67E-07	5.23E-07	4.91E-07	4.63E-07	4.42E-07	4.27E-07	4.16E-07	4.09E-07	4.07E-07	4.09E-07	4.13E-07	2.99E-06	2.74E-06
4.20	4.34E-07	3.92E-07	3.58E-07	3.31E-07	3.09E-07	2.93E-07	2.80E-07	2.72E-07	2.66E-07	2.64E-07	2.64E-07	2.66E-07	2.71E-07	1.69E-06	1.53E-06
4.40	2.88E-07	2.59E-07	2.36E-07	2.17E-07	2.02E-07	1.91E-07	1.83E-07	1.77E-07	1.74E-07	1.73E-07	1.73E-07	1.75E-07	1.79E-07	9.35E-07	9.22E-07
4.60	1.97E-07	1.76E-07	1.60E-07	1.46E-07	1.36E-07	1.28E-07	1.23E-07	1.18E-07	1.16E-07	1.15E-07	1.15E-07	1.16E-07	1.18E-07	5.13E-07	6.10E-07
4.80															

pp , $a^N(s, t)$ is the hadronic amplitude and $a^C(t)$ is the $\bar{p}p$ Coulomb amplitude. The Coulomb amplitude is of course

$$a^C(t) = 2\alpha s \frac{G^2(t)}{|t|} \exp[\mp i\alpha\phi(t)] \quad (2)$$

where α is the fine structure constant, $\phi(t)$ is the West-Yennie phase [12] given by

$$\phi(t) = \ln(t_0/|t|) - \gamma \quad (3)$$

with $t_0 = 0.08 \text{ GeV}^2$ and $\gamma = 0.577$ is the Euler constant. We use

$$G(t) = [(1 - t/m_1^2)(1 - t/m_2^2)]^{-1} \quad (4)$$

as the proton electromagnetic form factor. From the impact picture [4–6] the hadronic amplitude at high energies is given by

$$a^N(s, t) = is \int_0^\infty J_0(b\sqrt{-t})(1 - e^{-\Omega_0(s, b)})b db \quad (5)$$

where

$$\Omega_0(s, b) = S_0(s)F(b^2) + R_0(s, b) \quad (6)$$

with $R_0(s, b)$ being the Regge background which is negligible at the energies under consideration. With Regge background neglected, the $a^N(s, t)$ is the same for pp and $\bar{p}p$. The energy dependence is given by the crossing symmetric function

$$S_0(s) = s^c/(\ln s)^{c'} + u^c/(\ln u)^{c'} \quad (7)$$

which comes from the high energy behaviour of quantum field theory [3.4]. In (7) u is the third Mandelstam variable. The t -dependence of $a^N(s, t)$ is controlled by $F(b^2)$ whose Fourier transform is taken to be [6].

$$\tilde{F}(t) = f[G(t)]^2 [(a^2 + t)/(a^2 - t)]. \quad (8)$$

The values of the parameters are [6]

$$c = 0.167 \quad c' = 0.748$$

Table 2

$\sqrt{s}$ (TeV)	0.546	0.70	0.80	0.90	1.00	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
$ t $ (GeV) ²	Real													
	Imag													
0.001	4.80E+06	7.93E+06	1.04E+07	1.32E+07	1.63E+07	2.35E+07	2.77E+07	3.21E+07	3.69E+07	4.21E+07	4.75E+07	5.33E+07	5.95E+07	6.60E+07
	3.72E+06	6.37E+06	8.50E+06	1.10E+07	1.38E+07	2.04E+07	2.43E+07	2.85E+07	3.31E+07	3.80E+07	4.34E+07	4.91E+07	5.52E+07	6.16E+07
0.002	2.62E+06	4.34E+06	5.70E+06	7.24E+06	8.97E+06	1.30E+07	1.53E+07	1.78E+07	2.05E+07	2.33E+07	2.64E+07	2.96E+07	3.31E+07	3.67E+07
	3.69E+06	6.31E+06	8.42E+06	1.09E+07	1.36E+07	2.02E+07	2.41E+07	2.82E+07	3.28E+07	3.77E+07	4.30E+07	4.86E+07	5.47E+07	6.11E+07
0.003	1.89E+06	3.14E+06	4.13E+06	5.25E+06	6.52E+06	9.46E+06	1.11E+07	1.30E+07	1.49E+07	1.70E+07	1.93E+07	2.17E+07	2.42E+07	2.69E+07
	3.66E+06	6.26E+06	8.35E+06	1.08E+07	1.35E+07	2.01E+07	2.39E+07	2.80E+07	3.25E+07	3.74E+07	4.26E+07	4.82E+07	5.42E+07	6.05E+07
0.004	1.52E+06	2.54E+06	3.34E+06	4.25E+06	5.28E+06	7.68E+06	9.06E+06	1.05E+07	1.21E+07	1.39E+07	1.57E+07	1.77E+07	1.97E+07	2.19E+07
	3.63E+06	6.20E+06	8.28E+06	1.07E+07	1.34E+07	1.99E+07	2.32E+07	2.78E+07	3.22E+07	3.70E+07	4.22E+07	4.78E+07	5.37E+07	6.00E+07
0.005	1.30E+06	2.17E+06	2.86E+06	3.65E+06	4.54E+06	6.61E+06	7.79E+06	9.08E+06	1.05E+07	1.19E+07	1.35E+07	1.52E+07	1.70E+07	1.89E+07
	3.60E+06	6.15E+06	8.21E+06	1.06E+07	1.33E+07	1.97E+07	2.34E+07	2.75E+07	3.19E+07	3.67E+07	4.19E+07	4.74E+07	5.32E+07	5.95E+07
0.006	1.15E+06	1.93E+06	2.54E+06	3.24E+06	4.03E+06	5.88E+06	6.94E+06	8.08E+06	9.32E+06	1.07E+07	1.21E+07	1.36E+07	1.52E+07	1.69E+07
	3.57E+06	6.10E+06	8.14E+06	1.05E+07	1.32E+07	1.95E+07	2.32E+07	2.73E+07	3.17E+07	3.64E+07	4.15E+07	4.69E+07	5.28E+07	5.90E+07
0.007	1.04E+06	1.75E+06	2.31E+06	2.95E+06	3.67E+06	5.35E+06	6.32E+06	7.37E+06	8.50E+06	9.71E+06	1.10E+07	1.24E+07	1.39E+07	1.54E+07
	3.54E+06	6.05E+06	8.07E+06	1.04E+07	1.31E+07	1.94E+07	2.30E+07	2.70E+07	3.14E+07	3.61E+07	4.11E+07	4.65E+07	5.23E+07	5.84E+07
0.008	9.60E+05	1.61E+06	2.13E+06	2.72E+06	3.39E+06	4.95E+06	5.84E+06	6.82E+06	7.87E+06	8.99E+06	1.02E+07	1.15E+07	1.28E+07	1.43E+07
	3.51E+06	6.00E+06	8.00E+06	1.03E+07	1.30E+07	1.92E+07	2.28E+07	2.68E+07	3.11E+07	3.58E+07	4.08E+07	4.61E+07	5.18E+07	5.79E+07
0.009	8.96E+05	1.51E+06	1.99E+06	2.54E+06	3.17E+06	4.63E+06	5.47E+06	6.38E+06	7.37E+06	8.43E+06	9.56E+06	1.08E+07	1.20E+07	1.34E+07
	3.48E+06	5.95E+06	7.94E+06	1.02E+07	1.28E+07	1.90E+07	2.26E+07	2.66E+07	3.08E+07	3.53E+07	4.04E+07	4.57E+07	5.14E+07	5.74E+07
0.010	8.43E+05	1.42E+06	1.87E+06	2.40E+06	2.99E+06	4.37E+06	5.17E+06	6.03E+06	6.96E+06	7.96E+06	9.03E+06	1.02E+07	1.14E+07	1.27E+07
	3.45E+06	5.90E+06	7.87E+06	1.01E+07	1.27E+07	1.89E+07	2.24E+07	2.63E+07	3.06E+07	3.52E+07	4.01E+07	4.53E+07	5.09E+07	5.69E+07
0.012	7.62E+05	1.28E+06	1.70E+06	2.17E+06	2.71E+06	3.97E+06	4.69E+06	5.48E+06	6.33E+06	7.24E+06	8.22E+06	9.26E+06	1.04E+07	1.15E+07
	3.39E+06	5.80E+06	7.74E+06	9.98E+06	1.25E+07	1.86E+07	2.21E+07	2.59E+07	3.01E+07	3.45E+07	3.94E+07	4.45E+07	5.01E+07	5.59E+07
0.014	7.02E+05	1.18E+06	1.57E+06	2.01E+06	2.51E+06	3.67E+06	4.34E+06	5.07E+06	5.86E+06	6.70E+06	7.61E+06	8.58E+06	9.60E+06	1.07E+07
	3.34E+06	5.70E+06	7.61E+06	9.81E+06	1.23E+07	1.82E+07	2.17E+07	2.55E+07	2.95E+07	3.40E+07	3.87E+07	4.38E+07	4.92E+07	5.49E+07
0.016	6.55E+05	1.11E+06	1.47E+06	1.88E+06	2.34E+06	3.44E+06	4.06E+06	4.75E+06	5.48E+06	6.28E+06	7.13E+06	8.03E+06	8.99E+06	1.00E+07
	3.28E+06	5.61E+06	7.48E+06	9.64E+06	1.21E+07	1.79E+07	2.16E+07	2.50E+07	2.90E+07	3.34E+07	3.80E+07	4.30E+07	4.83E+07	5.40E+07
0.018	6.16E+05	1.04E+06	1.38E+06	1.77E+06	2.21E+06	3.24E+06	3.84E+06	4.48E+06	5.18E+06	5.93E+06	6.73E+06	7.59E+06	8.50E+06	9.46E+06
	3.23E+06	5.52E+06	7.36E+06	9.48E+06	1.19E+07	1.76E+07	2.10E+07	2.46E+07	2.85E+07	3.28E+07	3.74E+07	4.23E+07	4.75E+07	5.31E+07
0.020	5.84E+05	9.89E+05	1.31E+06	1.68E+06	2.10E+06	3.08E+06	3.64E+06	4.26E+06	4.92E+06	5.63E+06	6.40E+06	7.21E+06	8.07E+06	8.99E+06
	3.18E+06	5.43E+06	7.30E+06	9.33E+06	1.17E+07	1.73E+07	2.06E+07	2.42E+07	2.81E+07	3.22E+07	3.67E+07	4.16E+07	4.67E+07	5.22E+07
0.022	5.57E+05	9.43E+05	1.25E+06	1.60E+06	2.00E+06	2.94E+06	3.48E+06	4.06E+06	4.70E+06	5.38E+06	6.11E+06	6.88E+06	7.71E+06	8.58E+06
	3.12E+06	5.34E+06	7.12E+06	9.17E+06	1.15E+07	1.71E+07	2.03E+07	2.38E+07	2.76E+07	3.17E+07	3.61E+07	4.09E+07	4.59E+07	5.13E+07
0.024	5.33E+05	9.02E+05	1.20E+06	1.53E+06	1.92E+06	2.81E+06	3.33E+06	3.89E+06	4.50E+06	5.15E+06	5.85E+06	6.59E+06	7.39E+06	8.22E+06
	3.07E+06	5.25E+06	7.00E+06	9.02E+06	1.13E+07	1.68E+07	1.99E+07	2.34E+07	2.71E+07	3.12E+07	3.55E+07	4.02E+07	4.51E+07	5.04E+07
0.026	5.11E+05	8.66E+05	1.15E+06	1.47E+06	1.84E+06	2.70E+06	3.20E+06	3.74E+06	4.32E+06	4.95E+06	5.62E+06	6.34E+06	7.10E+06	7.90E+06
	3.02E+06	5.16E+06	6.88E+06	8.87E+06	1.11E+07	1.65E+07	1.96E+07	2.30E+07	2.67E+07	3.06E+07	3.49E+07	3.95E+07	4.43E+07	4.95E+07
0.028	4.92E+05	8.34E+05	1.11E+06	1.42E+06	1.77E+06	2.60E+06	3.08E+06	3.60E+06	4.16E+06	4.77E+06	5.41E+06	6.10E+06	6.83E+06	7.61E+06
	2.98E+06	5.08E+06	6.77E+06	8.73E+06	1.09E+07	1.62E+07	1.93E+07	2.26E+07	2.62E+07	3.01E+07	3.43E+07	3.88E+07	4.36E+07	4.87E+07
0.030	4.75E+05	8.05E+05	1.07E+06	1.37E+06	1.71E+06	2.51E+06	2.97E+06	3.47E+06	4.02E+06	4.60E+06	5.22E+06	5.89E+06	6.59E+06	7.34E+06
	2.93E+06	5.00E+06	6.66E+06	8.58E+06	1.08E+07	1.59E+07	1.89E+07	2.22E+07	2.58E+07	2.96E+07	3.37E+07	3.81E+07	4.29E+07	4.79E+07

$$\begin{aligned} m_1 &= 0.586 \text{ GeV} & m_2 &= 1.704 \text{ GeV} \\ a &= 1.953 \text{ GeV} & f &= 7.115 \text{ GeV}^{-2}. \end{aligned} \quad (9)$$

The normalization is such that

$$\frac{d\sigma}{dt} = \frac{\pi}{s^2} |a(s, t)|^2 \quad (10)$$

so that the amplitude is dimensionless. This completes the description of the model and we now turn to the numerical results.

3 Results

On the basis of the amplitude described above, theoretical results on pp and $\bar{p}p$ scattering have been calculated for the total cross section σ_{tot} , the integrated elastic cross section σ_{el} , the ratio ρ of the real to the imaginary part of the forward scattering amplitude $a^N(s, 0)$ and the differential cross section $d\sigma/dt$ for various values of $|t|$, the square of the momentum transfer. The results for centre-of-mass energies from 0.8 to 2 TeV and for $|t| \geq 0.04 \text{ GeV}^2$ are presented in Table 1 which have been arranged such that interpolations can be easily carried out. For this range of t the Coulomb amplitude is so small that the predictions are the same for pp and $\bar{p}p$, even in the dip region. Note that the ratio $\sigma_{\text{el}}/\sigma_{\text{tot}}$ increases slowly in

this energy range, its limiting value is $1/2$ for $s = \infty$.

The design value of the centre-of-mass energy of SSC is 40 TeV, but that of LHC is less certain. Since LHC is to be housed in the LEP tunnel at CERN, its centre-of-mass energy is proportional to the strength of the superconducting dipole magnet. For 10 T magnets, the centre-of-mass energy is approximately 17 TeV. Although it is quite feasible to use magnets up to 20 T, we give in Table 1 the pp and $\bar{p}p$ elastic cross sections, total cross sections etc. at these two energies of 17 and 40 TeV, in addition to those for FNAL energies.

In Table 2 we give the values of the real and imaginary parts of $a(s, t)$ in the Coulomb interference region for an energy range between 0.546 TeV and 2.0 TeV. For each value of $|t|$ the upper number is $\text{Re} a(s, t)$ and the lower one is $\text{Im} a(s, t)$. Since this is the energy range for the CERN and FNAL collider we give only the amplitude for $\bar{p}p$, but not pp , elastic scattering. Plots of some of these cross sections have been given before [10].

For LHC and SSC the proposed option is pp scattering due to higher luminosity, but the $\bar{p}p$ alternative is not completely ruled out. We therefore give in Table 3 the amplitudes and differential cross sections in the Coulomb interference region for both pp and $\bar{p}p$ elastic scattering.

Table 3

$\sqrt{s}$ (TeV)	pp			17.0			$\bar{p}p$			17.0			pp			40.0			$\bar{p}p$			40.0		
$ t $ (GeV) ²	Real	Imag	dσ/dt	Real	Imag	dσ/dt	Real	Imag	dσ/dt	Real	Imag	dσ/dt	Real	Imag	dσ/dt	Real	Imag	dσ/dt	Real	Imag	dσ/dt	Real	Imag	dσ/dt
0.0005	-7.67E+09	6.27E+09	1.44E+03	9.16E+09	6.27E+09	1.81E+03	-4.21E+10	3.94E+10	1.59E+03	5.11E+10	3.94E+10	1.99E+03	-4.21E+10	3.94E+10	1.59E+03	5.11E+10	3.94E+10	1.99E+03	5.11E+10	3.94E+10	1.99E+03	5.11E+10	3.94E+10	1.99E+03
0.0010	-3.46E+09	6.24E+09	7.46E+02	4.94E+09	6.24E+09	9.28E+02	-1.88E+10	3.92E+10	9.04E+02	2.77E+10	3.92E+10	1.10E+03	-1.88E+10	3.92E+10	9.04E+02	2.77E+10	3.92E+10	1.10E+03	2.77E+10	3.92E+10	1.10E+03	2.77E+10	3.92E+10	1.10E+03
0.0015	-2.06E+09	6.21E+09	6.27E+02	3.53E+09	6.21E+09	7.47E+02	-1.11E+10	3.90E+10	7.86E+02	1.99E+10	3.90E+10	9.16E+02	-1.11E+10	3.90E+10	7.86E+02	1.99E+10	3.90E+10	9.16E+02	1.99E+10	3.90E+10	9.16E+02	1.99E+10	3.90E+10	9.16E+02
0.0020	-1.36E+09	6.18E+09	5.87E+02	2.82E+09	6.18E+09	6.76E+02	-7.20E+09	3.88E+10	7.44E+02	1.59E+10	3.88E+10	8.41E+02	-7.20E+09	3.88E+10	7.44E+02	1.59E+10	3.88E+10	8.41E+02	1.59E+10	3.88E+10	8.41E+02	1.59E+10	3.88E+10	8.41E+02
0.0025	-9.44E+08	6.15E+09	5.67E+02	2.39E+09	6.15E+09	6.38E+02	-4.90E+09	3.86E+10	7.23E+02	1.36E+10	3.86E+10	8.00E+02	-4.90E+09	3.86E+10	7.23E+02	1.36E+10	3.86E+10	8.00E+02	1.36E+10	3.86E+10	8.00E+02	1.36E+10	3.86E+10	8.00E+02
0.0030	-6.68E+08	6.12E+09	5.55E+02	2.11E+09	6.12E+09	6.13E+02	-3.37E+09	3.84E+10	7.10E+02	1.20E+10	3.84E+10	7.73E+02	-3.37E+09	3.84E+10	7.10E+02	1.20E+10	3.84E+10	7.73E+02	1.20E+10	3.84E+10	7.73E+02	1.20E+10	3.84E+10	7.73E+02
0.0035	-4.72E+08	6.09E+09	5.46E+02	1.90E+09	6.09E+09	5.96E+02	-2.29E+09	3.82E+10	7.00E+02	1.08E+10	3.82E+10	7.53E+02	-2.29E+09	3.82E+10	7.00E+02	1.08E+10	3.82E+10	7.53E+02	1.08E+10	3.82E+10	7.53E+02	1.08E+10	3.82E+10	7.53E+02
0.0040	-3.26E+08	6.06E+09	5.39E+02	1.74E+09	6.06E+09	5.82E+02	-1.49E+09	3.80E+10	6.91E+02	9.97E+09	3.80E+10	7.37E+02	-1.49E+09	3.80E+10	6.91E+02	9.97E+09	3.80E+10	7.37E+02	9.97E+09	3.80E+10	7.37E+02	9.97E+09	3.80E+10	7.37E+02
0.0045	-2.14E+08	6.03E+09	5.33E+02	1.62E+09	6.03E+09	5.71E+02	-8.78E+08	3.78E+10	6.83E+02	9.29E+09	3.78E+10	7.24E+02	-8.78E+08	3.78E+10	6.83E+02	9.29E+09	3.78E+10	7.24E+02	9.29E+09	3.78E+10	7.24E+02	9.29E+09	3.78E+10	7.24E+02
0.0050	-1.25E+08	6.00E+09	5.27E+02	1.52E+09	6.00E+09	5.61E+02	-3.92E+08	3.76E+10	6.76E+02	8.74E+09	3.76E+10	7.12E+02	-3.92E+08	3.76E+10	6.76E+02	8.74E+09	3.76E+10	7.12E+02	8.74E+09	3.76E+10	7.12E+02	8.74E+09	3.76E+10	7.12E+02
0.0055	-5.37E+07	5.97E+09	5.22E+02	1.44E+09	5.97E+09	5.52E+02	1.51E+05	3.74E+10	6.68E+02	8.28E+09	3.74E+10	7.01E+02	1.51E+05	3.74E+10	6.68E+02	8.28E+09	3.74E+10	7.01E+02	8.28E+09	3.74E+10	7.01E+02	8.28E+09	3.74E+10	7.01E+02
0.0060	5.30E+06	5.94E+09	5.17E+02	1.37E+09	5.94E+09	5.44E+02	3.22E+08	3.72E+10	6.62E+02	7.90E+09	3.72E+10	6.91E+02	3.22E+08	3.72E+10	6.62E+02	7.90E+09	3.72E+10	6.91E+02	7.90E+09	3.72E+10	6.91E+02	7.90E+09	3.72E+10	6.91E+02
0.0065	5.45E+07	5.91E+09	5.12E+02	1.31E+09	5.91E+09	5.37E+02	5.89E+08	3.70E+10	6.55E+02	7.56E+09	3.70E+10	6.82E+02	5.89E+08	3.70E+10	6.55E+02	7.56E+09	3.70E+10	6.82E+02	7.56E+09	3.70E+10	6.82E+02	7.56E+09	3.70E+10	6.82E+02
0.0070	9.60E+07	5.88E+09	5.07E+02	1.26E+09	5.88E+09	5.30E+02	8.13E+08	3.68E+10	6.48E+02	7.28E+09	3.68E+10	6.73E+02	8.13E+08	3.68E+10	6.48E+02	7.28E+09	3.68E+10	6.73E+02	7.28E+09	3.68E+10	6.73E+02	7.28E+09	3.68E+10	6.73E+02
0.0075	1.31E+08	5.85E+09	5.02E+02	1.22E+09	5.85E+09	5.23E+02	1.00E+09	3.66E+10	6.42E+02	7.02E+09	3.66E+10	6.65E+02	1.00E+09	3.66E+10	6.42E+02	7.02E+09	3.66E+10	6.65E+02	7.02E+09	3.66E+10	6.65E+02	7.02E+09	3.66E+10	6.65E+02
0.0080	1.62E+08	5.82E+09	4.97E+02	1.18E+09	5.82E+09	5.17E+02	1.17E+09	3.64E+10	6.35E+02	6.80E+09	3.64E+10	6.56E+02	1.17E+09	3.64E+10	6.35E+02	6.80E+09	3.64E+10	6.56E+02	6.80E+09	3.64E+10	6.56E+02	6.80E+09	3.64E+10	6.56E+02
0.0085	1.88E+08	5.79E+09	4.92E+02	1.14E+09	5.79E+09	5.11E+02	1.31E+09	3.62E+10	6.29E+02	6.59E+09	3.62E+10	6.49E+02	1.31E+09	3.62E+10	6.29E+02	6.59E+09	3.62E+10	6.49E+02	6.59E+09	3.62E+10	6.49E+02	6.59E+09	3.62E+10	6.49E+02
0.0090	2.11E+08	5.77E+09	4.87E+02	1.11E+09	5.77E+09	5.05E+02	1.43E+09	3.61E+10	6.22E+02	6.41E+09	3.61E+10	6.41E+02	1.43E+09	3.61E+10	6.22E+02	6.41E+09	3.61E+10	6.41E+02	6.41E+09	3.61E+10	6.41E+02	6.41E+09	3.61E+10	6.41E+02
0.0095	2.31E+08	5.74E+09	4.83E+02	1.08E+09	5.74E+09	4.99E+02	1.53E+09	3.59E+10	6.16E+02	6.24E+09	3.59E+10	6.33E+02	1.53E+09	3.59E+10	6.16E+02	6.24E+09	3.59E+10	6.33E+02	6.24E+09	3.59E+10	6.33E+02	6.24E+09	3.59E+10	6.33E+02
0.0100	2.48E+08	5.71E+09	4.78E+02	1.05E+09	5.71E+09	4.94E+02	1.63E+09	3.57E+10	6.10E+02	6.09E+09	3.57E+10	6.26E+02	1.63E+09	3.57E+10	6.10E+02	6.09E+09	3.57E+10	6.26E+02	6.09E+09	3.57E+10	6.26E+02	6.09E+09	3.57E+10	6.26E+02
0.0110	2.77E+08	5.65E+09	4.69E+02	1.01E+09	5.65E+09	4.83E+02	1.78E+09	3.53E+10	5.97E+02	5.82E+09	3.53E+10	6.12E+02	1.78E+09	3.53E+10	5.97E+02	5.82E+09	3.53E+10	6.12E+02	5.82E+09	3.53E+10	6.12E+02	5.82E+09	3.53E+10	6.12E+02
0.0120	3.00E+08	5.60E+09	4.60E+02	9.66E+08	5.60E+09	4.72E+02	1.89E+09	3.49E+10	5.85E+02	5.58E+09	3.49E+10	5.98E+02	1.89E+09	3.49E+10	5.85E+02	5.58E+09	3.49E+10	5.98E+02	5.58E+09	3.49E+10	5.98E+02	5.58E+09	3.49E+10	5.98E+02
0.0130	3.18E+08	5.54E+09	4.51E+02	9.29E+08	5.54E+09	4.63E+02	1.98E+09	3.46E+10	5.73E+02	5.37E+09	3.46E+10	5.85E+02	1.98E+09	3.46E+10	5.73E+02	5.37E+09	3.46E+10	5.85E+02	5.37E+09	3.46E+10	5.85E+02	5.37E+09	3.46E+10	5.85E+02
0.0140	3.32E+08	5.49E+09	4.43E+02	8.97E+08	5.49E+09	4.53E+02	2.05E+09	3.42E+10	5.62E+02	5.18E+09	3.42E+10	5.72E+02	2.05E+09	3.42E+10	5.62E+02	5.18E+09	3.42E+10	5.72E+02	5.18E+09	3.42E+10	5.72E+02	5.18E+09	3.42E+10	5.72E+02
0.0150	3.43E+08	5.43E+09	4.34E+02	8.69E+08	5.43E+09	4.44E+02	2.11E+09	3.39E+10	5.50E+02	5.02E+09	3.39E+10	5.60E+02	2.11E+09	3.39E+10	5.50E+02	5.02E+09	3.39E+10	5.60E+02	5.02E+09	3.39E+10	5.60E+02	5.02E+09	3.39E+10	5.60E+02
0.0160	3.52E+08	5.38E+09	4.26E+02	8.42E+08	5.38E+09	4.34E+02	2.15E+09	3.35E+10	5.39E+02	4.86E+09	3.35E+10	5.48E+02	2.15E+09	3.35E+10	5.39E+02	4.86E+09	3.35E+10	5.48E+02	4.86E+09	3.35E+10	5.48E+02	4.86E+09	3.35E+10	5.48E+02
0.0170	3.59E+08	5.33E+09	4.18E+02	8.18E+08	5.33E+09	4.26E+02	2.18E+09	3.32E+10	5.28E+02	4.72E+09	3.32E+10	5.36E+02	2.18E+09	3.32E+10	5.28E+02	4.72E+09	3.32E+10	5.36E+02	4.72E+09	3.32E+10	5.36E+02	4.72E+09	3.32E+10	5.36E+02
0.0180	3.64E+08	5.28E+09	4.10E+02	7.96E+08	5.28E+09	4.17E+02	2.20E+09	3.28E+10	5.17E+02	4.59E+09	3.28E+10	5.25E+02	2.20E+09	3.28E+10	5.17E+02	4.59E+09	3.28E+10	5.25E+02	4.59E+09	3.28E+10	5.25E+02	4.59E+09	3.28E+10	5.25E+02
0.0190	3.68E+08	5.22E+09	4.02E+02	7.75E+08	5.22E+09	4.09E+02	2.21E+09	3.25E+10	5.07E+02	4.46E+09	3.25E+10	5.14E+02	2.21E+09	3.25E+10	5.07E+02	4.46E+09	3.25E+10	5.14E+02	4.46E+09	3.25E+10	5.14E+02	4.46E+09	3.25E+10	5.14E+02
0.0200	3.71E+08	5.17E+09	3.94E+02	7.56E+08	5.17E+09	4.00E+02	2.22E+09	3.21E+10	4.96E+02	4.35E+09	3.21E+10	5.03E+02	2.22E+09	3.21E+10	4.96E+02	4.35E+09	3.21E+10	5.03E+02	4.35E+09	3.21E+10	5.03E+02	4.35E+09	3.21E+10	5.03E+02
0.0210	3.73E+08	5.12E+09	3.86E+02	7.38E+08	5.12E+09	3.92E+02	2.22E+09	3.18E+10	4.86E+02	4.24E+09	3.18E+10	4.92E+02	2.22E+09	3.18E+10	4.86E+02	4.24E+09	3.18E+10	4.92E+02	4.24E+09	3.18E+10	4.92E+02	4.24E+09	3.18E+10	4.92E+02
0.0220	3.74E+08	5.07E+09	3.79E+02	7.21E+08	5.07E+09	3.85E+02	2.22E+09	3.15E+10	4.76E+02	4.14E+09	3.15E+10	4.82E+02	2.22E+09	3.15E+10	4.76E+02	4.14E+09	3.15E+10	4.82E+02	4.14E+09	3.15E+10	4.82E+02	4.14E+09	3.15E+10	4.82E+02
0.0230	3.74E+08	5.02E+09	3.72E+02	7.04E+08	5.02E+09	3.77E+02	2.21E+09	3.12E+10	4.66E+02	4.04E+09	3.12E+10	4.72E+02	2.21E+09	3.12E+10	4.66E+02	4.04E+09	3.12E+10	4.72E+02	4.04E+09	3.12E+10	4.72E+02	4.04E+09	3.12E+10	4.72E+02
0.0240	3.74E+08	4.98E+09	3.65E+02	6.89E+08	4.98E+09	3.69E+02	2.20E+09	3.08E+10	4.57E+02	3.95E+09	3.08E+10	4.62E+02	2.20E+09	3.08E+10	4.57E+02	3.95E+09	3.08E+10	4.62E+02	3.95E+09	3.08E+10	4.62E+02	3.95E+09	3.08E+10	4.62E+02
0.0250	3.73E+08	4.93E+09	3.58E+02	6.74E+08	4.93E+09	3.62E+02	2.19E+09	3.05E+10	4.47E+02	3.86E+09	3.05E+10	4.52E+02	2.19E+09	3.05E+10	4.47E+02	3.86E+09	3.05E+10	4.52E+02	3.86E+09	3.05E+10	4.52E+02	3.86E+09	3.05E+10	4.52E+02
0.0260	3.71E+08	4.88E+09	3.51E+02	6.60E+08	4.88E+09	3.55E+02	2.17E+09	3.02E+10	4.38E+02	3.77E+09	3.02E+10	4.43E+02	2.17E+09	3.02E+10	4.38E+02	3.77E+09	3.02E+10	4.43E+02	3.77E+09	3.02E+10	4.43E+02	3.77E+09	3.02E+10	4.43E+02
0.0270	3.70E+08	4.83E+09	3.44E+02	6.46E+08	4.83E+09	3.48E+02	2.16E+09	2.99E+10	4.29E+02	3.69E+09	2.99E+10	4.33E+02	2.16E+09	2.99E+10	4.29E+02	3.69E+09	2.99E+10	4.33E+02	3.69E+09	2.99E+10	4.33E+02	3.69E+09		

4 Comparison with experimental data

Let us first consider the final results obtained at the CERN collider by the UA4 collaboration for the measurement of the differential cross section of $\bar{p}p$ elastic scattering at $\sqrt{s} = 0.546$ TeV and $\sqrt{s} = 0.630$ TeV and which have been extended to large t values [7]. We show in Fig. 1a and b the comparison of the data with our predictions. The agreement is excellent near the forward direction and the dips which occur at the right position are followed by a shoulder. However the theoretical curves overestimate the values of the cross sections in the large t region.

There are preliminary data at $\sqrt{s} = 1.8$ TeV from experiment E710 at FNAL near the forward direction in the t range from 0.03 GeV^2 up to 0.12 GeV^2 . This data is not normalized and provides only the slope of $d\sigma/dt$. We show the comparison of this data with our prediction calculated from the total amplitude in Fig. 2, where the dashed curve represents the pure hadronic cross section. Clearly the quality of the data will improve in the near future. We also hope that the measurements will be extended to smaller values of $|t|$ to reach the Coulomb interference region.

This region has been already investigated at the CERN collider by the UA4 collaboration [8]. The comparison with our prediction is shown in Fig. 3. The agreement is fairly good and it is worth noting that the theoretical prediction corresponds to $\sigma_{\text{tot}} = 62 \text{ mb}$ and $\rho = 0.13$. However the value $\rho = 0.24 \pm 0.04$ was obtained by the UA4 collaboration resulting from the use of an oversimplified form of $a^N(s, t)$ in their data analysis [8]. For $|t| > 0.01 \text{ GeV}^2$ the data points are scattered symmetrically around the theoretical prediction. On the other hand for $|t| < 0.01 \text{ GeV}^2$ the data points lie mostly above the theoretical curve. It is difficult to attribute such an abrupt change of behaviour to the theoretical result and it would be very interesting to ascertain the cause of this phenomena. In our opinion it is unlikely that the so called "odderon" [13] be relevant.

We wish to emphasize that the behaviour of the nuclear part of the amplitude $a^N(s, t)$ is quite complicated in and near the Coulomb region. In Fig. 4 we show, for the real and imaginary parts separately the local slope parameters defined by

$$\begin{aligned} b_1 &= -\frac{\partial}{\partial t} \ln \text{Re } a^N(s, t) \\ b_2 &= -\frac{\partial}{\partial t} \ln \text{Im } a^N(s, t). \end{aligned} \quad (11)$$

The parameter b_1 is fairly constant for $|t| < 0.01 \text{ GeV}^2$ but it increases rapidly for larger values of $|t|$ due to the presence of a zero for $|t|$ around 0.32 GeV^2 . In contrast b_2 decreases monotonically in the range plotted. Note that $\text{Re } a^N(s, t)$ is significantly smaller than $\text{Im } a^N(s, t)$ so that the slope parameter b for the elastic differential cross section is approximately $2b_2$ (see Fig. 5 in the third reference of [6]).

Fig. 1a, b. $d\sigma/dt$ for $\bar{p}p$ elastic scattering at two different energies a $\sqrt{s} = 0.546$ TeV b $\sqrt{s} = 0.630$ TeV. Data from [7]

Fig. 2. $d\sigma/dt$ for $\bar{p}p$ elastic scattering at $\sqrt{s} = 1.8$ TeV in the small t region. Unnormalized data from [9]

Fig. 3. $d\sigma/dt$ for $\bar{p}p$ elastic scattering at $\sqrt{s} = 0.546$ TeV. Data from [8]

Fig. 4. Local slopes of $\text{Re} a^N(s, t)$ (b_1 upper curve) and $\text{Im} a^N(s, t)$ (b_2 lower curve) for $\sqrt{s} = 0.546$ TeV

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References

1. H. Cheng, T.T. Wu: Phys. Rev. Lett. 22 (1969) 666; Phys. Rev. 182 (1969) 1852, 1868, 1873, 1899
2. T.T. Wu, C.N. Yang: Phys. Rev. B137 (1965) 708; N. Byers, C.N. Yang: Phys. Rev. 142 (1966) 976; T.T. Chou, C.N. Yang: Phys. Rev. Lett. 20 (1968) 1213; Phys. Rev. 170 (1968) 1591; ibid 175 (1968) 1832
3. H. Cheng, T.T. Wu: Phys. Rev. D1 (1970) 2775
4. H. Cheng, T.T. Wu: Phys. Rev. Lett. 24 (1970) 1456
5. T.T. Wu, H. Cheng: A.I.P. Conf. Proc. n° 15 54 (Stony Brook 1973) C. Quigg (ed.); H. Cheng, J.K. Walker, T.T. Wu: Phys. Lett. 44B (1973) 97
6. C. Bourrely, J. Soffer, T.T. Wu: Phys. Rev. D19 (1979) 3249; Phys. Lett. 121B (1983) 284; Nucl. Phys. B247 (1984) 15; Phys. Rev. Lett. 54 (1985) 757
7. U. Amaldi et al.: Phys. Lett. 44B (1973) 112; S.R. Amendolia et al.: Phys. Lett. 44B (1973) 119; UA1 Collaboration, G. Arnison et al.: Phys. Lett. 121B (1983) 472; UA4 Collaboration, R. Battiston et al.: Phys. Lett. 127B (1983) 472; M. Bozzo et al.: Phys. Lett. 147B (1984) 392; ibid 155B (1985) 197; ibid 171B (1986) 142
8. UA4 Collab., D. Bernard et al.: Preprint CERN/EP 87-147
9. J. Peoples: Status report on the Tevatron I, presented at the EPS International Conference on High Energy Physics, Uppsala, 1987
10. C. Bourrely, J. Soffer, T.T. Wu: Phys. Lett. 196B (1987) 237. J. Soffer: Marseille preprint CPT-87/P.2022. Proceedings of the EPS International Conference on High Energy Physics, Uppsala, 1987. (to appear)
11. C. Bourrely, J. Soffer, T.T. Wu: Proceedings of the Conference Hadron Physics at High Energies. Marseille June 1978, Marseille preprint CPT-78/P.1032, p II. 41
12. G.B. West, D.R. Yennie: Phys. Rev. 172 (1968) 1413
13. K. Kang, B. Nicolescu: Phys. Rev. D11 (1975) 2461