CERN-TH.4823/87

BI-TP 87/11

ON $W^\pm$, Z_0 , γ SELF-INTERACTIONS; HIGH ENERGY BEHAVIOUR AND TREE UNITARITY BOUNDS^{*)}

C. Bilchak, M. Kuroda⁺⁾

University of Bielefeld
D-4800 Bielefeld, FRG

and

D. Schildknecht^{")}

CERN - Geneva

A B S T R A C T

We analyze the high-energy behaviour of vector boson scattering amplitudes within the framework of a recently suggested Lagrangian model based on global weak isospin symmetry broken by electromagnetism. Requiring vanishing of the most strongly (as s^2) rising contribution to vector boson scattering amplitudes leads to vector boson self-interactions dependent on a single parameter, for which the anomalous $W^\pm$ magnetic moment, κ , can be chosen. Tree unitarity is violated at about 2 TeV for arbitrary κ as in the $SU(2)_L \times U(1)_Y$ theory for $m_H \rightarrow \infty$. The model is well suited for significant tests of the vector boson sector of the $SU(2)_L \times U(1)_Y$ electroweak theory in processes such as $e^+e^- \rightarrow W^+W^-$.

^{*)} Supported by Bundesministerium für Forschung und Technologie, Bonn, FRG.

⁺⁾ Permanent address: Dept. of Physics, Meiji-Gakuin University, Yokohama 244, Japan.

^{")} Permanent address: Dept. of Theoretical Physics, University of Bielefeld, Bielefeld, FRG.

CERN-TH.4823/87

BI-TP 87/11

August 1987

1. Introduction

The discovery¹⁾ of the intermediate vector bosons has brilliantly confirmed the predictions of the $SU(2)_L \times U(1)_Y$ spontaneously broken electroweak theory²⁾. A detailed experimental investigation of the properties of these particles will be a most important task for the near and more distant future. A particularly interesting and moreover well-defined problem concerns the structure and magnitude of the self-couplings³⁾ of the weak vector bosons which appear as a strict consequence^{*} of the $SU(2)_L \times U(1)_Y$ local gauge principle.

Is it at all conceivable that the true self-couplings of the vector bosons differ from the $SU(2)_L \times U(1)_Y$ predictions, or are experimentally known facts on electroweak interactions sufficiently restrictive already to exclude any alternatives? In examining this question we have recently suggested an effective Lagrangian⁵⁾ for electroweak interactions which accounts for all observed electroweak phenomena while at the same time allowing for deviations in the structure and magnitude of the vector boson self-interactions from their $SU(2)_L \times U(1)_Y$ values. With two free parameters in the trilinear and two free parameters in the quadrilinear interaction terms, this Lagrangian is general enough to allow for significant deviations from and tests^{5,6)} of the $SU(2)_L \times U(1)_Y$ predictions which are recovered for a special choice of the free parameters. On the other hand, the suggested framework excludes radical departures from theoretical assumptions which have been proven empirically successful in electroweak interactions so far. This latter feature of the model is important, if it is to provide stringent tests of the $SU(2)_L \times U(1)_Y$ theory.

The KMSS Lagrangian model^{5,6)} for vector boson self-interactions rests on global $SU(2)_{WI}$ weak isospin symmetry broken by electromagnetism as a basic assumption. Electromagnetic interactions of the $W^\pm$ bosons are assumed to be P and C conserving and, for simplicity, only dimensionless couplings are allowed, thus suppressing possible dimension six quadrupole terms^{**}. The resulting four parameter Lagrangian is the most general one under the assumptions mentioned.

In the present paper we study the high energy behaviour of the tree graphs for

* Compare, however, ref. 4) for an alternative derivation of the standard vector boson self-couplings based on global $SU(2)_{WI}$ symmetry, the W-dominance substitution rule $W_\mu^3 \rightarrow W_\mu^3 + (e/g_W) A_\mu$ and electromagnetic gauge invariance.

** For a different approach including quadrupole terms compare ref. 4).

vector boson scattering in the KMSS Lagrangian model. We are thus supplementing a recent note⁷⁾ in which it was shown that requiring the absence of the most strongly divergent vector boson loop contributions to the NC to CC ratio (the empirically well known ρ -parameter) actually leads to relations among the free parameters of the model reducing the number of free trilinear couplings to a single one.

In Section 2 we will show how a reduction of the number of four free parameters to a single one, which may be chosen to be κ , the anomalous magnetic moment of the $W^\pm$, is achieved by reviving the analysis of tree diagrams for vector boson scattering actually undertaken^{8,9,10)} a long time ago. Requiring the vanishing of those contributions to the vector boson scattering amplitudes which are increasing most strongly (as s^2), reduces the number of free parameters to a single one, κ . The other trilinear and quadrilinear couplings are then related to κ by simple algebraic relations*. Even though the one parameter model still violates unitarity, as certain vector boson scattering amplitudes increase as s^1 , it is noteworthy that the power of s does not depend on whether κ is equal to the $SU(2)_L \times U(1)_Y$ value of $\kappa = 1$ or rather assumes any other value of $\kappa \neq 1$.

The question of the critical energy, at which the partial wave unitarity bounds will be saturated as a function of κ will be investigated in Section 3. We find that the critical energy is given by $\sqrt{s}_{\text{crit.}} \cong 1.8 \text{ TeV to } 2.5 \text{ TeV}$, slightly dependent on what value of κ is chosen in the interval $-0.5 \lesssim \kappa \lesssim 2$. The unitarity bounds thus turn out to be of roughly the same magnitude (for any κ in the above interval) as in the $SU(2)_L \times U(1)_Y$ theory with $\kappa = 1$ in the limit of $m_H \rightarrow \infty$, m_H denoting the mass of the Higgs scalar.

2. Restrictions on Vector Boson Self-Couplings

In the notation of⁵⁾ the basic Lagrangian for the vector boson self-interactions is given by

* A further restriction to $\kappa = 1$, i.e., to the $SU(2)_L \times U(1)_Y$ gauge theory couplings, is not achieved, unless one desires^{8,9,10)} to cancel the remaining s^1 increase by postulating the existence of a neutral scalar particle (the Higgs scalar).

$$\begin{aligned}
& L_{\tilde{A}} + L_W + L_{\text{MIX}} + L_{\text{SI}} + L_{\text{AN}} = \\
& = -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu} + \frac{1}{2} m_W^2 \vec{W}_\mu \cdot \vec{W}^\mu \\
& - \frac{1}{4} \lambda (\tilde{F}_{\mu\nu} W^{\mu\nu} + W_{\mu\nu}^3 \tilde{F}^{\mu\nu}) \\
& - \frac{1}{2} \hat{g} \vec{W}_{\mu\nu} \cdot (\vec{W}^\mu \times \vec{W}^\nu) - \frac{1}{4} \hat{g} (\vec{W}_\mu \times \vec{W}_\nu) \cdot (\vec{W}^\mu \times \vec{W}^\nu) \\
& - \frac{1}{4} \tilde{g} (\vec{W}_\mu \cdot \vec{W}^\mu) (\vec{W}_\nu \cdot \vec{W}^\nu) + i e \kappa \tilde{F}_{\mu\nu} W^{+\mu} W^{-\nu} . \quad (2.1)
\end{aligned}$$

The strength of the γW^3 current mixing term,

$$\lambda = \frac{e}{g_W} = \sin\theta_W , \quad (2.2)$$

where g_W denotes the $W^\pm$ fermion coupling, guarantees the empirically correct form of the neutral current interaction as well as the empirically verified magnitude of the vector boson masses. The electromagnetic field in (2.1) has been denoted by $\tilde{F}_{\mu\nu}$ in order to discriminate this (unmixed or primordial) field from the physical one obtained via diagonalization of the mixing term in (2.1). Electromagnetic gauge invariance requires the electromagnetic interactions of the vector bosons to be introduced in (2.1) via the gauge derivative substitution rule $\partial_\mu \rightarrow \partial_\mu - i e Q \tilde{A}_\mu$, thus breaking the underlying global $SU(2)_W$ symmetry of the kinetic energy and the trilinear interaction terms. The arbitrary vector boson self-couplings in (2.1) have been denoted by $\hat{g}$, $\tilde{g}$, $\tilde{g}$, and the anomalous magnetic moment of the $W^\pm$ by κ .

Upon diagonalization one obtains⁵⁾ for the part of the Lagrangian which describes the vector boson self-interactions, L_{SI} ,

$$\begin{aligned}
L_{\text{SI}} = & i e A_\mu (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) + i e \kappa F_{\mu\nu} W^{+\mu} W^{-\nu} \\
& - i(e \tan\theta_W - \frac{\hat{g}}{\cos\theta_W}) Z_\mu (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) \\
& - i(e \kappa \tan\theta_W - \frac{\tilde{g}}{\cos\theta_W}) Z_{\mu\nu} W^{+\mu} W^{-\nu} \\
& - e^2 (A_\mu A^\mu W_\nu^+ W^{-\nu} - A_\mu A_\nu W^{+\mu} W^{-\nu}) \\
& + 2e(e \tan\theta_W - \frac{\hat{g}}{\cos\theta_W}) (A_\mu Z^\mu W_\nu^+ W^{-\nu} - \frac{1}{2} A_\mu Z_\nu (W^{+\mu} W^{-\nu} + W^{-\mu} W^{+\nu})) \\
& - [e(e \tan^2\theta_W - \frac{2\hat{g}}{\cos\theta_W} \tan\theta_W) + \frac{\tilde{g}}{\cos^2\theta_W}] (Z_\mu Z^\mu W_\nu^+ W^{-\nu} - Z_\mu Z_\nu W^{+\mu} W^{-\nu}) \\
& - \frac{\tilde{g}}{\cos^2\theta_W} Z_\mu Z^\mu W_\nu^+ W^{-\nu} - \frac{\tilde{g}}{4\cos^4\theta_W} Z_\mu Z^\mu Z_\nu Z^\nu \\
& + \frac{1}{2} \hat{g} (W_\mu^- W_\nu^+ W^{-\mu} W^{+\nu} - W_\mu^- W_\nu^+ W^{+\mu} W^{-\nu}) - \tilde{g} W_\mu^- W_\nu^+ W^{+\mu} W^{-\nu} . \quad (2.3)
\end{aligned}$$

We note that this four parameter Lagrangian constitutes the most general^{5,6)} vector boson self-interaction consistent with the above principles of dimensionless couplings, no intrinsic violation of global $SU(2)_{WI}$ weak isospin symmetry, electromagnetic gauge invariance and finally no P or C violations by electromagnetic interactions of the vector bosons. The $SU(2)_L \times U(1)_Y$ self-interactions are obtained as a special case in (2.3) for $\hat{g} = e/\sin\theta_W$, $\kappa = 1$, $\hat{g} = \hat{g}^2$ and $\tilde{g} = 0$.

Let us now turn to the derivation of the restrictions on the couplings in (2.3) which are obtained by requiring vanishing of the most strongly growing and unitarity violating terms of order s^2 in the amplitudes for charged and neutral vector boson scattering, $WW \rightarrow WW$ and $ZZ \rightarrow WW$. The Feynman diagrams are given in Fig. 1, the corresponding Feynman rules in ref. 5). One easily convinces oneself that terms proportional to s^2 only appear when all vector bosons are in the longitudinal polarization state. For the purely longitudinal c.m.s. helicity amplitude, for $Z_L Z_L \rightarrow W_L^+ W_L^-$ one finds

$$A(Z_L Z_L \rightarrow W_L^+ W_L^-) = \frac{i s^2}{16M_W^4} (6(\hat{g}^2 - \hat{g}) - 8\tilde{g} - 2(\hat{g}^2 - \hat{g}) \cos^2\vartheta) + O(s) \quad , \quad (2.4)$$

where ϑ denotes the c.m.s. scattering angle. Requiring vanishing s^2 contributions thus implies

$$\hat{g} = \hat{g}^2 \quad (2.5)$$

and

$$\tilde{g} = 0 \quad . \quad (2.6)$$

Likewise one obtains for $W^+ W^- \rightarrow W^+ W^-$

$$\begin{aligned} A(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \frac{i s^2}{16M_W^4} [& 3(e^2 \kappa^2 + g_{ZWW}^2 \kappa_Z^2 - \hat{g} - 10\tilde{g}) \\ & - (6(e^2 \kappa^2 + g_{ZWW}^2 \kappa_Z^2 - \hat{g}) - 4\tilde{g}) \cos\vartheta \\ & - (e^2 \kappa^2 + g_{ZWW}^2 \kappa_Z^2 - \hat{g} + 2\tilde{g}) \cos^2\vartheta] + O(s) \quad , \quad (2.7) \end{aligned}$$

where the notation

$$g_{ZWW} \equiv -e \tan\theta_W + \frac{\hat{g}}{\cos\theta_W} \quad (2.8)$$

and

$$\kappa_Z g_{ZWW} \equiv \frac{\hat{g}}{\cos\theta_W} - e \kappa \tan\theta_W \quad (2.9)$$

has been introduced for the ZWW coupling, g_{ZWW} , and the anomalous weak magnetic moment κ_Z of the $W^\pm$, respectively. Requiring vanishing of the term proportional to s^2 in (2.7) again implies (2.6) as well as

$$\hat{g} = e^2 \kappa^2 + g_{ZWW}^2 \kappa_Z^2 \quad . \quad (2.10)$$

With (2.5) and (2.9) one deduces from (2.10) that

$$\hat{g} = \frac{e}{\sin\theta_W} \kappa \quad . \quad (2.11)$$

Altogether we thus have obtained three conditions

$$\hat{g} = \frac{e}{\sin\theta_W} \kappa \quad , \quad (2.12a)$$

$$\hat{\hat{g}} = \hat{g}^2 = \frac{e^2}{\sin^2\theta_W} \kappa^2 \quad , \quad (2.12b)$$

$$\tilde{g} = 0 \quad , \quad (2.12c)$$

which are necessary and sufficient for vanishing s^2 terms in vector boson scattering and reduce the four arbitrary parameters to a single one, κ . (Due to the transverse nature of the photon no s^2 terms are present in $\gamma\gamma \rightarrow W^+W^-$ and $\gamma Z \rightarrow W^+W^-$.)

It may be appropriate to add a remark at this point on the physical meaning⁷⁾ of the essential relation (2.12a), which connects the value of the anomalous magnetic moment, κ , with the strength, $\hat{g}$, of the trilinear coupling of the members of the $SU(2)_{WI}$ triplet, $W^\pm, W^3$. (Relation (2.12b) simply generalizes the corresponding $SU(2)_L \times U(1)_Y$ relation.) One may understand (2.12a) in terms of the notion of W-dominance¹¹⁾ which, within the γW^3 mixing approach, is essential for the fermion sector of the theory, as it implies $\lambda = e/g_W = \sin\theta_W$ and thus assures the correct neutral current structure and vector boson masses. It is well known that the requirement of vector boson dominance also for vector boson photon interactions implies^{4,12)} the $SU(2)_L \times U(1)_Y$ trilinear and quadrilinear self-couplings, i.e. $\kappa = 1$ as well as $\hat{g} = g_W = e/\sin\theta_W$ and $\kappa_Z = 1$. Condition (2.12a) is a weaker one. It is necessary and sufficient for the validity of W-dominance for the magnetic moment interaction of the $W^\pm$ with the photon. Indeed, applying the vector dominance substitution rule $W_\mu^3 \rightarrow W_\mu^3 + \lambda \tilde{A}_\mu = W_\mu^3 + \sin\theta_W \tilde{A}_\mu$ to the trilinear interaction in (2.1) generates the correct magnetic moment interaction (but it does not lead to the correct charge interaction, unless $\kappa = 1$), as with (2.12a)

$$\begin{aligned} & - \frac{\hat{g}}{2} \vec{W}_{\mu\nu} \cdot (\vec{W}^\mu \times \vec{W}^\nu) = \\ & = i \hat{g} W_\mu^3 (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) + i \hat{g} W_{\mu\nu}^3 W^{+\mu} W^{-\nu} \\ & \rightarrow i e \kappa A_\mu (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) + i e \kappa F_{\mu\nu} W^{+\mu} W^{-\nu} \quad . \end{aligned} \quad (2.13)$$

Requiring (2.12a) thus means validity of W-dominance for the magnetic moment interaction.

For completeness we now write down the single parameter Lagrangian following from (2.1) upon substituting (2.12):

$$\begin{aligned}
 L_{SI} = & i e A_{\mu} (W^{-\mu\nu} W_{\nu}^{+} - W^{+\mu\nu} W_{\nu}^{-}) + i e \kappa F_{\mu\nu} W^{+\mu} W^{-\nu} \\
 & + i e \frac{\kappa - \sin^2 \theta_W}{\sin \theta_W \cos \theta_W} (Z_{\mu} (W^{-\mu\nu} W_{\nu}^{+} - W^{+\mu\nu} W_{\nu}^{-}) + \frac{\kappa \cos^2 \theta_W}{\kappa - \sin^2 \theta_W} Z_{\mu\nu} W^{+\mu} W^{-\nu}) \\
 & - e^2 (A_{\mu} A^{\mu} W_{\nu}^{+} W^{-\nu} - A_{\mu} A_{\nu} W^{+\mu} W^{-\nu}) \\
 & - 2e^2 \frac{\kappa - \sin^2 \theta_W}{\sin \theta_W \cos \theta_W} (A_{\mu} Z^{\mu} W_{\nu}^{+} W^{-\nu} - \frac{1}{2} A_{\mu} Z_{\nu} (W^{+\mu} W^{-\nu} - W^{-\mu} W^{+\nu})) \\
 & - e^2 \frac{(\kappa - \sin^2 \theta_W)^2}{\sin^2 \theta_W \cos^2 \theta_W} (Z_{\mu} Z^{\mu} W_{\nu}^{+} W^{-\nu} - Z_{\mu} Z_{\nu} W^{+\mu} W^{-\nu}) \\
 & + \frac{1}{2} \frac{e^2}{\sin^2 \theta_W} \kappa^2 (W_{\mu}^{-} W_{\nu}^{+} W^{-\mu} W^{+\nu} - W_{\mu}^{-} W_{\nu}^{+} W^{+\mu} W^{-\nu}) \quad . \quad (2.14)
 \end{aligned}$$

The $SU(2)_L \times U(1)_Y$ self-interactions are (obviously) obtained by substituting $\kappa = 1$ in (2.14). According to the above derivation, the couplings in (2.14) are such that for any value of κ one obtains vector boson scattering amplitudes which increase at most as s^1 .

Besides $\kappa = 1$, another distinct case is obtained for $\kappa = 0$ in (2.14). This case of $\kappa = 0$ according to (2.12) corresponds to vanishing intrinsic self-interactions of the vector bosons. It may be generated⁵⁾ by applying the electromagnetic minimal substitution principle to the kinetic energy term in (2.1) with $\hat{g} = \hat{\tilde{g}} = \tilde{g} = \kappa = 0$. The self-interactions in (2.14) in this case are due entirely to λW^3 mixing.

3. Energy Bounds from Partial Wave Unitarity

As the interaction Lagrangian (2.14) does not contain additional particles apart from the vector bosons, in particular no scalars, we expect that helicity amplitudes for vector boson scattering calculated in tree approximation will grow as s^1 and eventually violate unitarity. In this section we discuss the energy dependence of the helicity amplitudes as a function of κ and subsequently give the critical energies at which partial wave amplitudes reach the unitarity bounds.

The Feynman diagrams for the processes $W^+ W^- \rightarrow W^+ W^-$, $ZZ \rightarrow W^+ W^-$, $\gamma Z \rightarrow W^+ W^-$ and

$\gamma\gamma \rightarrow W^+W^-$ are shown in Fig. 1. Using the Feynman rules explicitly given in ref. 5), the helicity amplitudes following from (2.14) for these processes have been calculated. The leading terms of order s^1 are listed in Tables 1a and 1b. In addition to the full amplitudes, the partial wave amplitudes, which will be used subsequently, are also listed in Tables 1a,b. In Table 1, Θ denotes the c.m.s. scattering angle and η stands for

$$\eta = 16M_W^2 / \alpha \cong (3.89 \text{ TeV})^2, \quad (3.1)$$

while the constants A_J , B_J and C_J are given by

$$A_J = \frac{1}{2} \int_{-1}^1 d_{1-1}^J(Z) \frac{1}{2}(1+Z) dZ = \begin{cases} 1/6 & , J = 1 \\ 1/J(J+1) & , J \geq 2 \end{cases}, \quad (3.2a)$$

$$B_J = \frac{1}{2} \int_{-1}^1 d_{20}^J(Z) \frac{1}{2}(1+Z) dZ = 1 / \sqrt{(J+2)(J+1)J(J-1)}, \quad J \geq 1, \quad (3.2b)$$

$$C_J = \frac{1}{2} \int_{-1}^1 d_{20}^J(Z) dZ = \begin{cases} 0 & , J = \text{odd} \\ 2 / \sqrt{(J+2)(J+1)J(J-1)} & , J = 2, 4, \dots \end{cases}, \quad (3.2c)$$

As regards the dependence on κ , there are two values of κ , namely $\kappa = 1$ and $\kappa = 0$, which lead to a reduction of the number of amplitudes growing as s^1 . For the $SU(2)_L \times U(1)_Y$ value of $\kappa = 1$, Tables 1a,b reduce to the amplitudes of purely longitudinal scattering processes of $W_L W_L \rightarrow W_L W_L$ and $Z_L Z_L \rightarrow W_L W_L$. This s^1 behaviour is cancelled within the $SU(2)_L \times U(1)_Y$ electroweak theory by introducing the Higgs scalar with sufficiently low mass, m_H . For the case of $\kappa = 0$ the amplitudes $W_L W_L \rightarrow W_L^+ W_L^-$ and $Z_L Z_L \rightarrow W_L^+ W_L^-$ are excluded from the Table 1, but all other amplitudes with two longitudinal vector bosons, in particular $\gamma\gamma \rightarrow W_L W_L$, rise as s^1 . It is not known whether a relatively simple mechanism exists which could cancel the s^1 high energy behaviour in this case. For $\kappa \neq 0$ and $\kappa \neq 1$, all amplitudes listed in Tables 1a,b lead to eventual violation of partial wave unitarity.

The partial wave amplitudes $a_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}^J(s)$ in Tables 1a,b are defined as usual by expanding the helicity amplitudes in partial waves in the c.m. system,

$$f_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}(s, \cos\Theta) = 16\pi \sqrt{\frac{s}{p_i p_f}} \sum_{J=M}^{\infty} (2J+1) a_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}^J(s) e^{i(\lambda-\mu)\varphi} d_{\lambda\mu}^J(\Theta), \quad (3.3)$$

where $\lambda = \lambda_1 - \lambda_2$ and $\mu = \lambda_3 - \lambda_4$ and the lowest total angular momentum J is given by $M = \max(|\lambda|, |\mu|)$. The most stringent constraints on the energy $\sqrt{s}$ from the unitarity condition

$$|a_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}^J(s)| \leq 1 \quad (3.4)$$

when applied to the partial waves in Tables 1a,b, are listed in Table 2 and are shown in Fig. 2. In the range of κ shown in Fig. 2, the most stringent bound is due to relations (D), (F) and (B) in their respective intervals. As seen in Fig. 2, a constraint for $\kappa = 1$ can only come from the purely longitudinal process $Z_L Z_L \rightarrow W_L W_L$, while for $\kappa = 0$ the only constraints are due to processes involving two transversely and two longitudinally polarized particles in the initial and final states.

More stringent bounds from partial wave unitarity are obtained by diagonalizing the T matrix in particle as well as in helicity space. The unitarity requirement for the 24×24 matrix $a^J(s)$ corresponding to the following two body channels*

$$\begin{aligned} & W_{\pm}^+ W_{\pm}^-, \quad W_{\pm}^+ W_{\mp}^-, \quad W_{\pm}^+ W_L^-, \quad W_L^+ W_{\pm}^-, \quad W_L^+ W_L^-, \\ & \frac{1}{\sqrt{2}} Z_{\pm} Z_{\pm}, \quad Z_{+} Z_{-}, \quad Z_{\pm} Z_L, \quad \frac{1}{\sqrt{2}} Z_L Z_L, \\ & Z_{\pm} \gamma_{\pm}, \quad Z_{\pm} \gamma_{\mp}, \quad Z_L \gamma_{\pm}, \quad \frac{1}{\sqrt{2}} \gamma_{\pm} \gamma_{\pm}, \quad \gamma_{+} \gamma_{-}, \end{aligned} \quad (3.5)$$

implies that the eigenvalues $\lambda^J(s)$ have to fulfil the constraint

$$|\lambda^J(s)| \leq 1. \quad (3.6)$$

It is straightforward to obtain all of the 24 eigenvalues λ^J for each partial wave amplitudes. Most of the eigenvalues are zero, and therefore no constraint on $\sqrt{s}$ is obtained from them. The strongest constraints are given by

$$s \{ \kappa^2 + \sqrt{9\kappa^4 + 96(1-\kappa)^4 \tan^4 \theta_W + 128(1-\kappa)^2 \kappa^2 \tan^2 \theta_W} \} \leq 4\eta \sin^2 \theta_W, \quad (3.7)$$

which is obtained from $|\lambda^0| \leq 1$, and by

$$s \{ 5(1-\kappa)^2 + \sqrt{25(1-\kappa)^4 + 32\kappa^2(1-\kappa)^2 \cot^2 \theta_W} \} \leq 12\eta \cos^2 \theta_W, \quad (3.8)$$

originating from $|\lambda^1| \leq 1$. For $\eta = (3.89 \text{ TeV})^2$ and a value of the weak mixing angle of $\sin^2 \theta_W \approx 0.22$, it happens that the condition (3.8) is much weaker than (3.7) for the range of κ we are interested in. The most stringent bound is therefore given by (3.7).

* In principle also lepton and quark channels should have been included (for $\kappa \neq 1$) in the diagonalization procedure. We believe that our results will not be drastically affected by including these channels.

As seen in Fig. 2 (curve (G)), the bound (3.7) nicely interpolates and strengthens the individual bounds. For $\kappa = 1$ we recover the well-known restriction¹³⁾ of the $SU(2)_L \times U(1)_Y$ gauge theory in the limit of $m_H \rightarrow \infty$, which is given by

$$\sqrt{s} \leq 4M_W \sqrt{\frac{\sin^2 \theta_W}{\alpha}} \cong 1.8 \text{ TeV} \quad . \quad (3.9)$$

For $\kappa = 0$ the allowed energy range is restricted by

$$\sqrt{s} \leq 4M_W \sqrt{\frac{\cos^2 \theta_W}{\sqrt{6} \alpha}} \cong 2.2 \text{ TeV} \quad , \quad (3.10)$$

while for $0.2 \lesssim \kappa \lesssim 0.6$, the bound moves upward to

$$\sqrt{s} \lesssim 2.5 \text{ TeV} \quad . \quad (3.11)$$

As demonstrated in Fig. 2, unitarity restricts the energy range for which a tree level evaluation of the effective Lagrangian (2.14) makes sense to energies below approximately $\sqrt{s} \cong 2 \text{ TeV}$, an energy value not strongly dependent on κ .

4. Conclusions

Let us end with the following summarizing and concluding remarks:

i) We have given an effective Lagrangian for vector boson self-interactions depending on a single free parameter, for which we have chosen κ , the anomalous magnetic moment of the $W^\pm$. If one restricts oneself to dimensionless couplings, this Lagrangian for vector boson self-interactions is the most general one based on global $SU(2)_{WI}$ weak isospin symmetry broken by electromagnetism and fulfilling the additional constraint of implying vanishing s^2 terms in vector boson scattering amplitudes. The content of the Lagrangian may be summarized by noting that it interpolates between self-interactions generated by minimal electromagnetic substitution ($\kappa = 0$ as well as $\hat{g} = \hat{\tilde{g}} = \tilde{g} = 0$) and γW^3 mixing and the $SU(2)_L \times U(1)_Y$ self-interactions ($\kappa = 1$, $\hat{g} = e/\sin\theta_W$, $\hat{\tilde{g}} = \hat{g}^2$, $\tilde{g} = 0$) in such a manner that s^2 terms are absent in vector boson scattering.

ii) Within our framework, even for $\kappa \neq 1$ (i.e., $-0.5 \lesssim \kappa \lesssim 2$) the critical energy at which tree-level amplitudes violate unitarity bounds is of the order of 2 TeV as in the $SU(2)_L \times U(1)_Y$ spontaneously broken gauge theory for $m_H \rightarrow \infty$. This energy sets the scale below which new phenomena, not contained in the effec-

tive Lagrangian, should appear in the self-interactions of vector bosons.

iii) While for $\kappa = 1$ tree unitarity may be saved in the conventional way of introducing the Higgs scalar, for $\kappa \neq 1$ no such simple mechanism is known which would lead to well-behaved high energy amplitudes for, e.g., $\gamma\gamma \rightarrow W_L W_L$ or $\gamma Z \rightarrow W_L W_L$. A richer structure would certainly be necessary for $\kappa \neq 1$, in particular for the case of $\kappa = 0$ corresponding to vanishing genuine vector boson self-interactions. If one entertains the point of view that the massive vector bosons are of composite nature, one might be tempted to be guided by the example¹⁴⁾ of reactions explored in $\gamma\gamma$ physics, such as $\gamma\gamma \rightarrow \rho^+ \rho^-$, $\rho^0 \rho^0$, $\rho\omega$ etc. to stretch one's imagination on possible richer structures.

iv) As stressed⁷⁾ recently, measurements of $e^+e^- \rightarrow W^+W^-$ at LEP 200 will be well suited for determining the remaining free parameter κ in (2.14). In particular the cases of $\kappa = 0$ and $\kappa = 1$ can be discriminated⁷⁾ fairly easily. If future measurements will find $\kappa = 1$, the case for a (composite or elementary?) scalar particle will be strongly supported. On the other hand, a significantly different value of κ , e.g. $\kappa \simeq 0$ (and $\hat{g} = \tilde{g} \simeq 0$), would clearly imply a spectacular and radically new situation as regards electroweak interaction.* The simple solution of adding a Higgs scalar would be excluded with certainty. This shows the importance of the LEP 200 experiments on $e^+e^- \rightarrow W^+W^-$.

v) Let us stress that the single parameter Lagrangian (2.14) (as well as the more general one (2.3) containing two arbitrary trilinear and two arbitrary quadrilinear couplings) is particularly well suited for significant empirical tests of the $SU(2)_L \times U(1)_Y$ theory. This is due to the fact that a variation of the $W^\pm$ anomalous magnetic moment, κ , evidently requires that all coupling constants in (2.14) vary in such a manner that the underlying principles, which to a large extent are well supported by empirical evidence on electroweak interactions, remain valid. This is not the case, if the large number of nine free parameters allowed¹⁶⁾ on kinematical grounds is reduced by ad hoc assumptions, such as, e.g., allowing for $\kappa \neq 1$ while adopting the $SU(2)_L \times U(1)_Y$ values for all the other parameters. Empirical tests of the $SU(2)_L \times U(1)_Y$ theory are then unduly restricted to making a comparison with very specific and radical alternatives which are not motivated by theoretical principles.

* It is amusing to note in this connection that arguments have recently been advanced¹⁵⁾ that the $SU(2)_L \times U(1)_Y$ theory if taken literally (cut-off $\rightarrow \infty$) might lead to vanishing genuine vector boson self-interactions, thus to a situation effectively described by $\hat{g} = \tilde{g} = \tilde{\tilde{g}} = 0$.

References

- 1) UA1 Collab., G. Arnison et al., Phys. Lett. B166 (1986) 484 and references therein;
UA2 Collab., J.A. Appel et al., Z. Phys. C30 (1986) 1 and references therein.
- 2) S. L. Glashow, Nucl. Phys. 22 (1961) 579;
S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A. Salam, Proc. 8th Nobel Symposium, ed., N. Svartholm (Almqvist and Wiksell, Stockholm 1968) p. 367.
- 3) e.g., D. Schildknecht, in Proc. of the VIth Topical Workshop on Proton-Antiproton Collider Physics, Aachen, June 1986, ed. K. Eggert, H. Faissner and E. Radermacher (World Scientific 1987), p. 125.
- 4) M. Kuroda, F.M. Renard and D. Schildknecht, Phys. Lett. 183B (1987) 366.
- 5) J. Maalampi, D. Schildknecht and K.-H. Schwarzer, Phys. Lett. B166 (1986) 361;
M. Kuroda, J. Maalampi, K.-H. Schwarzer and D. Schildknecht, Nucl. Phys. B284 (1987) 271.
- 6) M. Kuroda, J. Maalampi, D. Schildknecht and K.-H. Schwarzer, Phys. Lett. 190B (1987) 217.
- 7) H. Neufeld, J. Stroughair and D. Schildknecht, CERN preprint, CERN-TH.4720/87, Phys. Lett. B, to appear.
- 8) C.H. Llewellyn Smith, Phys. Lett. 46B (1973) 233.
- 9) J.M. Cornwall, D.N. Levin and G. Tiktopoulos, Phys. Rev. Lett. 30 (1973) 1268, Phys. Rev. D10 (1974) 1145.
- 10) S.D. Joglekar, Ann. Phys. 83 (1974) 427.
- 11) R. Kögerler and D. Schildknecht, CERN preprint, CERN-TH.3231 (1982) (unpublished);
D. Schildknecht, in Electroweak Effects at High Energies, ed. N.B. Newman (Plenum, New York 1985) p. 551;
M. Kuroda, D. Schildknecht and K.-H. Schwarzer, Nucl. Phys. B261 (1985) 432.
- 12) B. Schrempp and F. Schrempp, DESY 84-055 (1984) (unpublished).
- 13) B.W. Lee, C. Quigg and H.B. Thacker, Phys. Rev. D16 (1977) 1519.
- 14) M. Poppe, Int. Journal of Mod. Phys., A1 (1986) 545.

- 15) A. Hasenfratz and P. Hasenfratz, FSU-SCRI-86-30 (1986);
P. Hasenfratz and J. Nager, BUTP-86/20 (1986).
- 16) K. Gaemers and G. Gounaris, Z. Phys. C1 (1979) 259

Table 1a: The leading contributions (of order s^1) to the helicity amplitudes for the process $W^+W^- \rightarrow W^+W^-$ and the corresponding partial wave amplitudes. The constants η , A_J and B_J are defined in (3.1) and (3.2) in the text.

$W^+W^- \rightarrow W^+W^-$					
$\lambda_1\lambda_2 \rightarrow \lambda_3\lambda_4$	λ	μ	amplitudes	J	$a_{\lambda_1\lambda_2\lambda_3\lambda_4}^J(s)$
LL $\rightarrow$ LL	0	0	$(1 + \cos\theta) s \kappa^2 \frac{8\pi i}{\eta \sin^2\theta_W}$	0	$s \kappa^2 \frac{i}{2\eta \sin^2\theta_W}$
		0		1	$s \kappa^2 \frac{i}{6\eta \sin^2\theta_W}$
$\pm\pm \rightarrow$ LL	0	0	$s(1 - \kappa)^2 \frac{16\pi i}{\eta \cos^2\theta_W}$	0	$s(1 - \kappa)^2 \frac{i}{\eta \cos^2\theta_W}$
LL $\rightarrow$ $\pm\pm$	0	0			
L $\pm \rightarrow$ L $\pm$	∓ 1	∓ 1	$(1 + \cos\theta) s(1 - \kappa)^2 \frac{-8\pi i}{\eta \cos^2\theta_W}$	1	$s(1 - \kappa)^2 \frac{-i}{3\eta \cos^2\theta_W}$
$\pm L \rightarrow$ $\pm L$	± 1	± 1		≥ 1	$s(1 - \kappa)^2 \frac{-i A_J}{\eta \cos^2\theta_W}$
L $\pm \rightarrow$ $\pm L$	∓ 1	± 1		≥ 2	$s(1 - \kappa)^2 \frac{-i B_J}{\eta \cos^2\theta_W}$
$\pm L \rightarrow$ L $\pm$	± 1	∓ 1			
LL $\rightarrow$ $\pm\mp$	0	± 2	$(1 + \cos\theta) s(1 - \kappa)^2 \frac{-8\pi i}{\eta \cos^2\theta_W}$	≥ 2	$s(1 - \kappa)^2 \frac{-i B_J}{\eta \cos^2\theta_W}$
$\pm\mp \rightarrow$ LL	± 2	0			
L $\pm \rightarrow$ L $\mp$	∓ 1	± 1	$(1 - \cos\theta) s(1 - \kappa)^2 \frac{-8\pi i}{\eta \cos^2\theta_W}$	1	$s(1 - \kappa)^2 \frac{-i}{3\eta \cos^2\theta_W}$
$\pm L \rightarrow$ $\mp L$	± 1	∓ 1			
L $\pm \rightarrow$ $\mp L$	∓ 1	∓ 1	$(1 + \cos\theta) s(1 - \kappa)^2 \frac{4\pi i}{\eta \cos^2\theta_W}$	1	$s(1 - \kappa)^2 \frac{2i}{3\eta \cos^2\theta_W}$
$\pm L \rightarrow$ L $\mp$	± 1	± 1			

Fig. 2

(A)

Table 1b: The leading contributions (of order s^1) to the helicity amplitudes for the processes $ZZ \rightarrow W^+W^-$, $\gamma Z \rightarrow W^+W^-$ and $\gamma\gamma \rightarrow W^+W^-$ and the corresponding partial wave amplitudes. The constants η and C_J are defined in (3.1) and (3.2) in the text.

$ZZ \rightarrow W^+W^-$						
$\lambda_1\lambda_2 \rightarrow \lambda_3\lambda_4$	λ	μ	amplitudes	J	$a_{\lambda_1\lambda_2\lambda_3\lambda_4}^J(s)$	Fig. 2
LL $\rightarrow$ LL	0	0	$s \kappa^2 \frac{16\pi i}{\eta \sin^2\theta_W}$	0	$s \kappa^2 \frac{i}{\eta \sin^2\theta_W}$	(B)
$\pm\pm \rightarrow$ LL	0	0	$s(1-\kappa) [1+\kappa - \frac{1-\kappa}{\cos^2\theta_W}] \frac{32\pi i}{\eta}$	0	$s(1-\kappa) [1+\kappa - \frac{1-\kappa}{\cos^2\theta_W}] \frac{2i}{\eta}$	(C)
$+- \rightarrow$ LL	2	0	$s(1-\kappa)^2 \frac{-i \tan^2\theta_W}{\eta}$	≥ 2	$s(1-\kappa)^2 \frac{-i \tan^2\theta_W C_J}{\eta}$	
$\pm L \rightarrow \mp L$	± 1	∓ 1	$(1 - \cos\theta) s \kappa(1-\kappa) \frac{8\pi i}{\eta \cos\theta_W}$	1	$s \kappa(1-\kappa) \frac{i}{3\eta \cos\theta_W}$	
$L\pm \rightarrow L\mp$	∓ 1	± 1	$(1 + \cos\theta) s \kappa(1-\kappa) \frac{8\pi i}{\eta \cos\theta_W}$			
$\pm L \rightarrow L\mp$	± 1	± 1	$(1 + \cos\theta) s \kappa(1-\kappa) \frac{8\pi i}{\eta \cos\theta_W}$			
$L\pm \rightarrow \mp L$	∓ 1	∓ 1	$(1 + \cos\theta) s \kappa(1-\kappa) \frac{8\pi i}{\eta \cos\theta_W}$			

Table 1b continued

$\gamma Z \rightarrow W^+W^-$						
$\lambda_1\lambda_2 \rightarrow \lambda_3\lambda_4$	λ	μ	amplitudes	J	$a_{\lambda_1\lambda_2\lambda_3\lambda_4}^J(s)$	Fig. 2
$\pm\mp \rightarrow LL$	± 2	0	$s(1-\kappa)^2 \frac{16\pi i \tan\theta_W}{\eta}$	≥ 2	$s(1-\kappa)^2 \frac{i \tan\theta_W}{\eta} C_J$	
$\pm\pm \rightarrow LL$	0	0	$s(1-\kappa)(1-\kappa \cot^2\theta_W) \frac{32\pi i \tan\theta_W}{\eta}$	0	$s(1-\kappa)(1-\kappa \cot^2\theta_W) \frac{2i \tan\theta_W}{\eta}$	(D)
$\pm L \rightarrow \mp L$	± 1	∓ 1	$(1-\cos\theta) s \kappa(1-\kappa) \frac{-8\pi i}{\eta \sin\theta_W}$	1	$s \kappa(1-\kappa) \frac{-i}{3\eta \sin\theta_W}$	(E)
$\pm L \rightarrow L\mp$	± 1	± 1	$(1+\cos\theta) s \kappa(1-\kappa) \frac{-8\pi i}{\eta \sin\theta_W}$			
$\gamma\gamma \rightarrow W^+W^-$						
$\pm\pm \rightarrow LL$	0	0	$s(1-\kappa^2) \frac{-32\pi i}{\eta}$	0	$s(1-\kappa^2) \frac{-2i}{\eta}$	(F)
$+- \rightarrow LL$	2	0	$s(1-\kappa)^2 \frac{-16\pi i}{\eta}$	≥ 2	$s(1-\kappa)^2 \frac{-iC_J}{\eta}$	

Table 2: The most stringent constraints obtained by requiring partial wave unitarity for different vector boson scattering amplitudes.

processes		constraints
$W_{\pm}^{+} W_{\pm}^{-} \rightarrow W_L^{+} W_L^{-}$	(A)	$s (1 - \kappa)^2 \leq \eta \cos^2 \theta_W$
$Z_L Z_L \rightarrow W_L^{+} W_L^{-}$	(B)	$s \kappa^2 \leq \eta \sin^2 \theta_W$
$Z_{\pm} Z_{\pm} \rightarrow W_L^{+} W_L^{-}$	(C)	$s (1 - \kappa)\{1 + \kappa - (1 - \kappa) \operatorname{cosec}^2 \theta_W\} \leq \frac{1}{2} \eta$
$\gamma_{\pm} Z_{\pm} \rightarrow W_L^{+} W_L^{-}$	(D)	$s (1 - \kappa)(\tan \theta_W - \kappa \cot \theta_W) \leq \frac{1}{2} \eta$
$\gamma_{\pm} Z_L \rightarrow W_T^{+} W_L^{-}$	(E)	$s (1 - \kappa) \kappa \leq 3 \eta \sin \theta_W$
$\gamma_{\pm} Z_L \rightarrow W_L^{+} W_T^{-}$		
$\gamma_{\pm} \gamma_{\pm} \rightarrow W_L^{+} W_L^{-}$	(F)	$s 1 - \kappa^2 \leq \frac{1}{2} \eta$

Figure Captions

Figure 1: Feynman diagrams for the processes $W^+W^- \rightarrow W^+W^-$, $ZZ \rightarrow W^+W^-$, $\gamma Z \rightarrow W^+W^-$ and $\gamma\gamma \rightarrow W^+W^-$, which contribute at tree level according to our effective Lagrangian.

Figure 2: The critical energy $\sqrt{s_{\text{crit}}}$ for which vector boson scattering amplitudes violate tree unitarity as a function of κ . The curves (A) to (F) correspond to the constraint equations given in Table 2, the curve (G) corresponds to the constraint (3.7) which is obtained by diagonalization of the T matrix.

Fig.1

Fig. 2