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KÄHLER INVARIANCE OF N=1 QUANTUM SUPERGRAVITY

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A B S T R A C T

The Kähler invariance of the N=1 supergravity theory is investigated beyond the classical level. In the popular gauge fixing condition $e_{m\mu} - e_{\mu m} = 0$, $\gamma^\mu \phi_\mu = 0$, $\partial_\mu (\sqrt{g} g^{\mu\nu}) = 0$ one has a manifestly Kähler invariant theory at the quantum level and for that the form of the path integral measure is important. We show that Fujikawa's measure is consistent with quantum Kähler invariance. The integration of the auxiliary fields of the closed algebra theory brings down local determinants $(\det g_{zz})^{-1}$ necessary for maintaining the Kähler invariance at the quantum level.

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$N=1$ supergravities in four dimensions though not complete as physical theories are of great interest since they are believed to be the effective theories at scales $E \lesssim M_{\text{Planck}}$ of a superstring theory. To many people this may be the ultimate theory encompassing all known forces including gravity.

The effective supergravity theory though not renormalizable can consistently represent the low-energy phenomena by resumming in the string theory the effect of the heavy degrees of freedom in the string loop amplitudes. This consistency will be guaranteed if there are no anomalies related to the classical symmetries of the effective Lagrangian at the tree level. In four dimensions gravitational and supersymmetry anomalies are generally believed to be absent in the perturbation theory and so one has to look at other classical symmetries of the theory. The Kähler symmetry which is strikingly similar to the conformal invariance of the string theory is a possible candidate. Therefore it is very important to pay attention to the correct quantization procedure with a cut-off which preserves this symmetry. In the case of the effective supergravity theory describing the zero mass sector of the four-dimensional superstrings, the Kähler symmetry is of geometrical origin and it is expected that string perturbation theory will preserve it.

$N=1$ supergravity theories have been widely studied in the past and their Lagrangians have been explicitly constructed^{1),2)*)}. A special feature of an $N=1$ supergravity theory is the fact that it is completely specified in terms of two arbitrary functions namely $\mathcal{G}$ a real function of the scalars Z_i, Z_i^* involved, known as Kähler potential, and a chiral function $f_{\alpha\beta}(Z_i)$. $f_{\alpha\beta}$ depends only on Z_i and α, β are group indices [for details see Refs. 1) and 2)]. One of the main issues of the superstring theory is to determine the form of these two functions since in this way the low-energy theory is completely determined and unique physical predictions can be made which will be of relevance to the experimentalists.

The aim of this letter is the study of some quantum aspects of the $N=1$ supergravity and in particular to investigate the Kähler structure of the quantized theory.

We shall assume a simple $N=1$ supergravity involving only one matter multiplet (Z, X_L) and no gauge structure but our results are expected to hold in the more general case as well. The Kähler potential $\mathcal{G}$ is therefore a real function $\mathcal{G}(Z, Z^*)$ depending on Z and its conjugate Z^* which is the notation of Refs. 1) and 2) can be split into.

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*) More general higher derivative $N=1$ supergravities have been also constructed. See Refs. 3) and 4).

$$g = J - \ln \frac{|g(Z)|^2}{4} \equiv 3 \ln \left(-\frac{\phi}{3} \right) - \ln \frac{|g(Z)|^2}{4} \quad (1)$$

$g(Z)$ bearing the name superpotential. In constructing the N=1 supergravity theory, one starts from the N=1 locally supersymmetric action [see Refs. 1) and 2)]

$$S \equiv \int d^4x \mathcal{L} = \sum_{\eta, \eta'} a_{\eta\eta'} \int d^4x \left(\mathcal{L}(\Sigma^\eta T(\Sigma^{\eta'})) d^4x + \sum_n b_n \left(\mathcal{L}(\Sigma^\eta) d^4x + (h.c.) \right) \right)_{(2)}$$

where Σ denotes the multiplet ($Z = A+iB, X_L, h' = F'+iG'$) and $T(\Sigma)$ the kinetic multiplet of Σ (non-analytic forms of the action can also be treated but will not be discussed here). $\int d^4x \mathcal{L}(\Sigma)$ in Eq. (1) stands for the N=1 invariant action^{*})

$$\int d^4x \mathcal{L}(\Sigma) = \int d^4x e \{ h' + u \bar{z} + \bar{\Psi}_L \cdot \gamma X_L + \bar{\Psi}_L^\mu \sigma_{\mu\nu} \Psi_R^\nu \bar{z} \} \quad (2')$$

Eliminating the auxiliary fields A_m , $u \equiv S-iP$ as well as h' and properly redefining the vierbein and the fermion fields one arrives at a Lagrangian depending only on g as defined in Eq. (1) where ϕ and g are given by

$$\phi = \sum_{\eta, \eta'} a_{\eta\eta'} z^\eta \bar{z}^{\eta'}, \quad g = \sum_m b_m z^m$$

Upon quantizing the theory one integrates functionally over all fields which we denote collectively by f_i and defines the functional

$$W = \int [D_\mu(f_i)] e^{-S_{cl}} \quad (3)$$

where S_{cl} is the action given by Eq. (2). We work with a closed algebra theory so that the auxiliary fields appear within S_{clas} and $D_\mu(f_i)$ ⁺). The integration measure $D_\mu(f_i)$ has been extensively discussed in the literature^{7), 8)} [see also Ref. 9) and references therein] and more recently by Fujikawa¹⁰⁾. At the quantum level the elimination of the auxiliary fields does not proceed as classically since the Faddeev-Popov ghosts associated with the fixing of the gauge symmetries of the theory will modify the equations of motion for the auxiliary fields.

*) Throughout we prefer to write $\xi_L X_R$, $\xi_L \gamma^\mu X_L$ etc. instead of $\xi_R X_R$, $\xi_R \gamma^\mu X_L$ used in Refs. 1) and 2).

+) The open algebra gauge theory, its BRST transformation and the derivation of the Ward identities has been considered by B. de Wit and J.W. van Holten⁵⁾. Also R.E. Kallosh⁶⁾ has considered the open algebra pure supergravity.

In fact for the auxiliary fields a^{aux} one has the equations of motion

$$a^{\text{aux}} = (\text{class.}) + (\bar{c}c) \quad (4)$$

where the first bracket denotes the classical expression and the second the ghost contribution. It is known that the effect of Eq. (4) is to modify the Lagrangian and to include among other terms a four-ghost interaction^{9),11)-15)} $\Delta L^{\text{quartic}}$. In the classical Lagrangian L_{class} the Kähler structure becomes manifest once the following field redefinitions are performed after the elimination of the auxiliary fields

$$e_{m\mu} = e^{\sigma} e_{m\mu}^N \quad (5a)$$

$$\psi_{\mu L} = e^{i\theta} e^{\sigma/2} \left(\psi_{\mu L}^N - \gamma_{\mu}^N \frac{\phi_{\bar{z}^*}}{\phi} \chi_R^N \right) \quad (5b)$$

$$\chi_L = e^{-i\theta} e^{-\sigma/2} \chi_L^N \quad (5c)$$

In Eqs. (5) $e_{m\mu}$ is the vierbein field and ψ_{μ} the gravitino and the superscript "N" denotes the new fields. The angle θ is defined as $e^{i\theta} \equiv (g/g^*)^{1/4}$ while the real function σ is

$$e^{2\sigma} = -3/\phi = e^{-J/3} \quad (5d)$$

For the quantized theory one has to perform the same field redefinitions after the gauge fixing conditions are inserted within the functional of Eq. (3) and properly taking care of the form of the integration measure $D\mu(f_i)$. The quantized theory is not a priori known to have a Kähler structure. The gauge fixing conditions we choose should respect the Kähler symmetry so that Kähler invariance is manifest as far as gauge fixing conditions are concerned. There are 14 gauge symmetries, four for the general co-ordinate transformations (g.c.t), four for the local supersymmetry (S) and six for the local Lorentz (L) and the gauge fixing conditions we employ are¹⁶⁾

$$G_N \equiv \partial_\mu (\sqrt{g_N} g_N^{\mu\nu}) = 0 \quad (\text{g.c.t.}) \quad (6a)$$

$$S_N^{L,R} \equiv \gamma_N^\mu \psi_{\mu L}^N = \gamma_N^\mu \psi_{\mu R}^N = 0 \quad (S) \quad (6b)$$

$$L_N \equiv \delta_n^\mu e_{\mu\mu}^N - (m \neq n) = 0 \quad (L) \quad (6c)$$

This is a set of popular gauge fixing conditions and they refer to the new fields $e_{\mu\mu}^N$ ψ_μ^N not having any dependence on the matter fields so that the requirement that they have a Kähler structure is obviously satisfied. In terms of the initial field variables these become proportional to the gauge conditions

$$G = \partial_\mu (\sqrt{g} g^{\mu\nu} e^{-2\sigma}) = 0 \quad (7a)$$

$$S^{L(R)} = \gamma^\mu \psi_{\mu L(R)} + 4 \frac{\phi_{z^*}(z)}{\phi} \chi_{R(L)} = 0 \quad (7b)$$

$$L = e^{-\sigma} (\delta_n^\mu e_{\mu\mu} - m \neq n) = 0 \quad (7c)$$

In an elegant quantization method proposed by Fujikawa¹⁰⁾ the BRST algebra is unitarily represented on the fields of the theory and there is a well-defined prescription to determine the Faddeev-Popov part of the Lagrangian for any legitimate gauge fixing as well as the path integral variables. To this end this method forces invariance of the measure for each integration variable under the general coordinate transformation. It happens that this guarantees also invariance under supergravity transformations.

Following Fujikawa¹⁰⁾ and to implement the gauge conditions (7) we use superfields of the BRST symmetry

$$\tilde{\zeta}^i(x, \theta) = \tilde{\zeta}^i(x) + \theta B^i(x) \quad (\text{antighosts})$$

$$\eta^i(x, \theta) = \eta^i(x) + \theta K^i(x) \quad (\text{ghosts})$$

with K^i bilinear in the ghosts η^i ($\delta \eta^i = K^i$). The index "i" labels the various gauge symmetries. For any gauge fixing condition F^i , say, we similarly form the superfields

$$F^i(x, \theta) = F^i(x) + \theta \Delta F^i(x)$$

where ΔF^i is the variation of F^i under the gauge symmetry with all parameters replaced by n^i . The part of the action containing the ghost fields is then assumed to be of the form

$$\begin{aligned} S_{g.f.} &= \int d^4x \int d\theta \bar{z}^i(x, \theta) F^i(x, \theta) \\ &= \int d^4x [\bar{z}^i(x) \Delta F^i(x) + B^i(x) F^i(x)] \end{aligned}$$

The first term is the well-known Faddeev-Popov term and the second term will yield delta functions $\delta(F^i)$ when the integrations with respect to the multiplier fields B^i are performed. Therefore the functional of Eq. (3) is

$$W = \int [D\mu(f_i)] \delta(G) \delta(s_{L,R}) \delta(L) e^{-S_{cl} - S_{F.P.}} \quad (3')$$

where S_{cl} denotes the classical part of the action and $S_{F.P.}$ the Faddeev-Popov part. The measure $D\mu(f_i)$ is given by¹⁰⁾

$$\begin{aligned} [D\mu(f_i)] &= D(e^{1/4} e_r^m) D(e^{1/4} \psi_r) D(e^{1/2} S) D(e^{1/2} P) D(e^{1/2} A_m) \\ &\times D(e^{3/4} C^r) D(e^{1/2} C) D(e^{1/2} C^{mn}) D(e^{1/2} \text{matter fields}) \\ &\times D(\text{multiplier fields}) \end{aligned}$$

In this measure the integration variables are completely specified and the measure itself is BRST general co-ordinate and supersymmetry invariant. Note that all matter fields Z, X_L and among these the auxiliary h' appear with weight $e^{1/2}$. The B^i fields do not appear within the measure since they have already integrated out to give the delta functions appearing in Eq. (3'). The Faddeev-Popov action $S_{F.P.}$ involves all ghost fields, associated with local Lorentz ($C^{mn}, \bar{C}^{mn}$ anticommuting) general co-ordinate transformations ($C^\mu, \bar{C}^\mu$ anticommuting) and supersymmetry transformations ($C, \bar{C}$ Dirac spinor commuting). Note that we prefer to use in the following the commonly used notation $C, \bar{C}$ for the ghosts-antighosts instead of the η, ξ used by Fujikawa. In the measure $[D\mu]$ defined previously the "multiplier fields" are all antighost fields and appear without any weight in the integrations.

Taking the variations ΔG , ΔL , $\Delta S_{L,R}$ of the gauge condition (7) and taking into account the appearance of the delta functions within the functional in Eq. (3') we have for the Faddeev-Popov Lagrangian

$$\begin{aligned} L_{F.P.}^{g.c.t.} = & 4 \bar{C}_\nu \partial_\mu [e^{-2\sigma} (\sigma_\mu \bar{X}_R C_L + h.c.) \sqrt{g} g^{\mu\nu}] \\ & + 2 \bar{C}_\nu \partial_\mu [e^{-2\sigma} C^\lambda \partial_\lambda \sigma \sqrt{g} g^{\mu\nu}] \\ & - \bar{C}_\nu \partial_\mu \{ -e^{-2\sigma} \sqrt{g} (\partial_\lambda C^\mu g^{\lambda\nu} + \nu \rightleftharpoons \mu) + \partial_\lambda (C^\lambda \sqrt{g} g^{\mu\nu}) \\ & - \sqrt{g} (\bar{C} \gamma^\mu \psi + \nu \rightleftharpoons \mu) + \bar{C} \gamma_\mu \psi \sqrt{g} g^{\mu\nu} \} \end{aligned} \quad (8a)$$

$$L_{F.P.}^L = - \sum_{b>a} \bar{C}^{ab} \{ \delta_b^\mu [e_{a\lambda} \partial_\mu C^\lambda + C^\lambda \partial_\lambda e_{a\mu} + C_a e_{\mu}^c - \bar{\psi}_\mu \gamma_a C] e^{-\sigma} - (a \rightleftharpoons b) \} \quad (8b)$$

$$\begin{aligned} L_{F.P.}^S = & -2 \bar{C} \not{D}(e, \psi) C + \frac{i}{3} A^\mu \bar{C} \gamma_\mu \gamma_5 C - \frac{4}{3} [(u^* + J_{2*} h') \bar{C}_L C_R + h.c.] \\ & - \frac{4}{3} [J_{2*} (\partial_\mu \bar{Z}^* - \bar{\psi}_\mu X_R) \bar{C}_L \gamma^\mu C_L + J_2 (\partial_\mu \bar{Z} - \bar{\psi}_\mu X_L) \bar{C}_R \gamma^\mu C_R] \\ & - \frac{8}{3} [J_{22*} (\bar{C}_L X_R) (\bar{X}_R C_L) + J_{22} (\bar{C}_R X_L) (\bar{X}_L C_R) \\ & + J_{22*} (\bar{C}_R X_L) (\bar{X}_L C_R) + J_{22} (\bar{C}_L X_R) (\bar{X}_R C_R)] - (\bar{C} \gamma^\mu \psi_\mu) (\bar{\psi}_\mu \gamma^\mu C) \end{aligned} \quad (8c)$$

Note that there is no ghost mixing term in $L_{F.P.}^S$ owing to the fact that we are in the gauge $\gamma_{\mu N}^{N\mu} = 0$. We shall come back to this point later. The ghost term $L_{F.P.}^S$ is the only place where the auxiliary fields interfere with the ghosts and the auxiliary field part of the Lagrangian is given by

$$\begin{aligned} L^{aux} = & - \frac{e\phi}{9} A_\mu^2 + \frac{ie}{3} A_\mu (F_A^\mu + \Delta F_A^\mu) + \frac{e\phi}{9} |u + J_2 h'|^2 \\ & + [e(u + J_2 h') (F_B^* + \Delta F_B^*) + (h.c.)] \\ & + \frac{e\phi}{3} J_{22*} |h'|^2 + [e h' (F_C^* + \Delta F_C^*) + (h.c.)] \end{aligned} \quad (9)$$

The expressions for $F_{A,B,C}$ can be found in Ref. 1) while the bilinears in the ghosts $\Delta F_{A,B,C}$ are given by

$$\Delta F_A^m = \frac{1}{e} \bar{c} \gamma^m \gamma_5 c, \quad \Delta F_B = -\frac{4}{3e} \bar{c}_R c_L, \quad \Delta F_C = 0$$

Performing the integrations with respect A_m , S , P and h' , h'^* in the functional (3') L^{aux} becomes

$$L^{\text{aux}} = L_0^{\text{aux}} + L_{\text{F.P.}}^{\text{aux}}$$

where L_0^{aux} is the contribution already taken into account in Ref. 1) and $L_{\text{F.P.}}^{\text{aux}}$ is the part which stemmed from the $\Delta F_{A,B,C}$ terms above and which involves quartic ghost terms. The explicit form of $L_{\text{F.P.}}^{\text{aux}}$ is given by

$$\begin{aligned} L_{\text{F.P.}}^{\text{aux}} = & -\frac{1}{2\phi} \left[\phi_{\bar{z}} \partial_{\bar{z}} z - \phi_{\bar{z}^*} \partial_{\bar{z}^*} z^* + \frac{1}{2} \phi_{\bar{z}z^*} \bar{\chi}_r \gamma_5 \chi - \bar{\psi}_r \gamma_5 \hat{\phi}_{\bar{z}} \chi \right] \bar{c} \gamma^r \gamma_5 c \\ & + \frac{12}{\phi} \left[\left(\frac{3}{2} - \frac{\phi}{3} \left(\frac{1}{3} J_{\bar{z}z^*} + \frac{1}{9} J_{\bar{z}^*}^2 \right) \bar{\chi}_L \chi_R \right) \bar{c}_L c_R + (\text{H.C.}) \right] \\ & - \frac{1}{e\phi} \left[\frac{1}{4} (\bar{c} \gamma^r \gamma_5 c)^2 + 16 (\bar{c}_L c_R) (\bar{c}_R c_L) \right] \end{aligned} \quad (10)$$

The quartic ghost terms are exactly the same as those found in Ref. 9) apart from the $1/\phi$ factor. Therefore from Eq. (3') we have, after integrating with respect the auxiliary fields

$$\begin{aligned} W = & \left[D\mu \right]_{\text{no aux.}}^{\text{no}} (\det e)^4 (\det e\phi)^{-3} (\det e\phi J_{\bar{z}z^*})^{-1} \delta(G) \delta(L) \delta(S_{L,R}) \\ & \times \exp[-\bar{S}_c - S_{\text{F.P.}}^{\text{aux}} - \Delta S_{\text{F.P.}}] \end{aligned} \quad (11)$$

In Eq. (11) $\bar{S}_{c1}$ is the part of the classical action containing no ghosts after the elimination of the auxiliary fields and it is identical to the Lagrangian of Ref. 1) while $S_{\text{F.P.}}^{\text{aux}}$ is the action of the Lagrangian (10). $\Delta S_{\text{F.P.}}$ stands for all remaining ghost terms. In Eq. (11) the subscript "no aux" in $[D\mu]_{\text{no aux.}}^{\text{no}}$ denotes that the auxiliary fields do not appear any longer in the measure. The determinants appearing within Eq. (11) arose from the A_m , S , P integrations $((\det e\phi)^{-3})$ as well as the h' , h'^* integrations $((\det e\phi J_{\bar{z}z^*})^{-1})^*$. The additional $(\det e)^4$ stems from the weight $e^{1/2}$ accompanying each of the auxiliary fields S , P , A_m , h' , h'^* [see $D\mu(f_i)$ following Eq. (3')].

The kinetic ghost term in $L_{\text{F.P.}}^S$ [Eq. (8c)] and the ghost "mass" term in $L_{\text{F.P.}}^{\text{aux}}$ [Eq. (10)] yield after redefining the vierbein field e_m^μ and using new ghost fields

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*) $\{\det e\phi\}$ stands for $\det\{e(x)\phi(x)\delta(x-y)\}$ and similarly for the remaining determinants.

$$C_R = e^{-i\theta} e^{\sigma/2} C_R', \quad C_L = e^{i\theta} e^{\sigma/2} C_L' \quad (12)$$

$$\begin{aligned} & -2\bar{C}\mathcal{D}(e,\psi)C - 6\left(\frac{g}{\Phi}\bar{C}_L C_R + \text{H.c.}\right) \\ & = -2\bar{C}'[\mathcal{D}(e_r,\psi') + 2e^{-\frac{g}{2}}]C' - 4\gamma_5 \bar{C}'\gamma_\mu^r C' + \frac{1}{2}(\gamma_5 \ln \frac{g}{g^*})\bar{C}'\gamma_\mu^r \gamma_5 C' \end{aligned} \quad (13)$$

The angle θ in (12) is $\theta \equiv 1/2 \arg g$. The connection $D_\mu(e_N, \psi')$ in Eq. (13) depends on the new vierbein field $e_{N\mu}^m$ and a new gravitino which is related to ψ_μ by

$$\psi_\mu \equiv e^{\sigma/2} \psi_\mu' \quad (14)$$

Performing the same redefinitions in the $\partial_\mu Z$ dependent parts of $L_{F.P.}^S + L_{F.P.}^{\text{aux}}$, the last two terms of Eq. (13) cancel and the rest has the known Kähler structure, that is, it only depends on Z and its derivatives, except from a part in which two ghosts and two fermions are coupled. Going to the final fermionic variables X_N, ψ_N^μ [see Eqs. (5b), (5c) and (14)] one has

$$\begin{aligned} L_{F.P.}^S + L_{F.P.}^{\text{aux}} &= -2\bar{C}'[\mathcal{D}(e_r,\psi') + 2e^{-\frac{g}{2}}]C' + \frac{1}{2}(\gamma_5 \gamma_\mu \gamma^\mu - \text{h.c.})\bar{C}'\gamma_\mu^r \gamma_5 C' \\ &\quad - \frac{4}{3}\gamma_5 \gamma_\mu \gamma^\mu (\bar{C}'\gamma_\mu^r \gamma_5 C')(\bar{\chi}_\mu \gamma_\mu^r \gamma_5 \chi_\mu) \\ &\quad + [(\bar{C}'C' \bar{\psi}_\mu^r \psi_\mu^r + \text{h.c.}) - \frac{1}{2}(\bar{C}'\gamma_\mu^r \gamma_5 C')(\bar{\psi}_\mu^r \gamma_\mu^r \gamma_5 \psi_\mu^r)] \\ &\quad + \frac{1}{3e_r} [16(\bar{C}'C')(\bar{C}'C') + \frac{1}{4}(\bar{C}'\gamma_\mu^r \gamma_5 C')(\bar{C}'\gamma_\mu^r \gamma_5 C')] \end{aligned} \quad (15)$$

The last quartic ghost term has no dependence on the matter fields so that the Kähler structure is not spoiled by these terms, nor by the rest of the terms in Eq. (15). The quartic ghost term is identical to that of Ref. 9). For the ghost term $L_{F.P.}^{\text{gct}}$ it is found that after the field redefinitions [Eqs. (5) and (12)] it receives the form

$$\begin{aligned} L_{F.P.}^{\text{gct}} &= \bar{C}_\nu \gamma_\mu \{ e_\nu [\gamma_\lambda c^\lambda g_\mu^{\lambda\nu} + \gamma_\lambda c^\lambda g_\mu^{\lambda\nu} - \frac{1}{e_\nu} \gamma_\lambda (e_\nu c^\lambda g_\mu^{\lambda\nu}) \\ &\quad + (\bar{C}'\gamma_\mu^r \psi_\nu^r + \mu \delta^\mu_\nu)] \} \end{aligned} \quad (16)$$

and so it has a Kähler structure. For the Lorentz ghost term [Eq. (8b)] things are not so straightforward. By redefining the vierbein field $e_{m\mu} = e^\sigma e_{m\mu}^N$ and defining

the graviton field $h_{m\mu}^\mu \equiv \delta_{m\mu} - e_{m\mu}^\mu$ one gets a bilinear term [see Ref. 8a)]

$$-\bar{C}^{ab} (\partial_b C_a - \partial_a C_b + 2C_{ab})$$

which suggests using a new Lorentz ghost $C'_{ab} = C_{ab} + 1/2(\partial_b C_a - \partial_a C_b)$. All terms in Eq. (8b), but the one in which a gravitino is involved, have the desired Kähler structure. However, the ghost C'_{ab} couples only to the antighost $\bar{C}^{ab}$ and not to the other ghost fields due to the fact that we are in the gauge $\gamma^\mu \psi_\mu = 0$ [see discussion following Eq. (8c)]. This, in addition to the fact that the Lorentz ghost kinetic term is $\bar{C}^{ab} C'_{ab}$ it makes impossible to draw a ghost graph in which a $\bar{C}^{ab} [\bar{\psi}_\mu \gamma_a C \delta_b^\mu - (b \rightleftharpoons a)]$ vertex participates (see the Figure).

Therefore the disturbing $\bar{C}^{ab} \psi C$ terms in $L_{F.P.}^L$ do not participate and the rest has the Kähler structure. In fact using the gauge fixing condition $e_{a\mu} \delta_b^\mu - a \rightleftharpoons b = 0$ (8b) becomes

$$L_{F.P.}^L = - \sum_{b>a} \bar{C}^{ab} \{ \delta_b^m [e_{a\lambda}^\lambda \partial_\lambda C^m + C^\lambda \partial_\lambda e_{a\lambda}^m + C_{am} e_{\lambda\lambda}^m] - (a \rightleftharpoons b) \} \quad (17)$$

where in Eq. (17) we preferred to write $L_{F.P.}^L$ in terms of the old Lorentz ghost C_{ab} instead of C'_{ab} . Therefore one concludes that the quantum action $S_{\text{quant}} \equiv \bar{S}_{cl} + S_{F.P.}^{\text{aux}} + \Delta S_{F.P.}^{\text{aux}}$ has a Kähler structure and it remains to examine the integration measure and the rest of the terms appearing within the measure in Eq. (11) to check the Kähler invariance of the quantum theory.

The gauge fixing delta functions inside the path integral of Eq. (11) are such that when expressed in terms of the new variables give

$$\delta(L) = \delta(L_N), \quad \delta(G) = \delta(G_N), \quad \delta(S_{L,R}) = \delta(e^{-\frac{\sigma}{2}} \gamma_r^\mu \psi_{r,L,R}^\mu) \quad (18)$$

[see Eqs. (6) and (7)] so that one has from Eq. (11)

$$\begin{aligned} [D\mu]_{aux}^{no} (\det e)^4 (\det e\phi)^{-3} (\det e\phi J_{zz}^*)^{-1} \delta(G) \delta(L) \delta(S_{L,R}) \\ = [D\mu]_{aux}^{no} (\det G_{zz}^*) (\det \phi)^{-3} \delta(G_N) \delta(L_N) \delta(S_{L,R}^N) \end{aligned} \quad (19)$$

where we replaced J_{zz}^* by G_{zz}^* . The measure $[D\mu]_{aux}^{no}$ is of the form

$$[D_\mu]_{\text{no aux}} = \prod_i [De^{K_i} f_i] \quad (e = \text{vierbein determinant}) \quad (20)$$

the index "i" running over all fields (except the auxiliary) and all ghosts and antighosts. K_i is the proper weight factor [see the measure following Eq. (3')]. Under the field redefinitions of Eqs. (5) and (12) one has

$$\prod_i [De^{K_i} f_i] = \prod_i (\det e^\sigma)^{\lambda_i} [De_N^{K_i} f_{iN}] \quad (21)$$

the subscript N indicating that new field variables are now integrated and $(\det e^\sigma)^{\lambda_i}$ is a Jacobian factor. Therefore

$$[D_\mu]_{\text{no aux}} = (\det e^\sigma)^R [D_\mu^N]_{\text{no aux}} \quad (22)$$

$[D_\mu^N]_{\text{no aux}}$ defining the new measure which in form is identical to $[D_\mu]_{\text{no aux}}$ and R is given by

$$R = \sum_i s_i \lambda_i m_i \quad (23)$$

In Eq. (23) m_i is the number of degrees of freedom and s_i is either +1 (for boson or commuting ghost) or -1 (for fermion or anticommuting ghost). The λ_i , m_i and s_i are given in the Table. Therefore R of Eq. (23) is

$$R = -6$$

so that there is a cancellation between $(\det e^\sigma)^R$ and $(\det \phi)^{-3}$ in Eq. (19) and the final form of the functional W is given by

$$W = \int [D_\mu^N]_{\text{no aux}} (\det g_{zz^*})^{-1} \delta(G_N) \delta(L_N) \delta(S_N^{LR}) e^{-S_{\text{quantum}}} \quad (24)$$

One should note the appearance of $(\det g_{zz^*})^{-1}$ in the functional of Eq. (24). Such a determinant is necessary if one has to have Kähler invariance at the quantum theory. In fact Kähler invariance requires invariance under the field redefinitions $Z \rightarrow f(Z)$, $X_L \rightarrow (df/dZ)X_L$ and the measure $[D_\mu^N]_{\text{no aux}} (\det g_{zz^*})^{-1}$ is

indeed invariant owing to the appearance of the determinant. Such determinants of the Kähler metric are also known to occur within the partition function of supersymmetric σ -models¹⁷⁾ and are expected to appear in other quantization schemes [see Refs. 15a) and 15b)]. Since the $N=1$ supergravity theories can represent the low-energy effective theory for superstring interactions the non-renormalizability of these theories can be treated consistently with a momentum space cut off at the Planck scale M_{Planck} . In this case the determinant factors appearing within the functional integral are important terms of the effective Lagrangian and should not be omitted using an all-scale regularization scheme as for instance dimensional regularization. In such schemes it is very difficult to formulate the notion of the effective theory. It is an interesting fact which the present work proves that the elimination of the auxiliary degrees of freedom and the redefinitions of the classical fields do not produce only a Kähler invariant classical Lagrangian but also take care of the factors in the functional measure to promote this invariance at the quantum level. It should be noticed that this Kähler invariance appears in the zero-mass sector Lagrangian of some four-dimensional string theories and its origin is connected with conformal invariance of the string. This invariance is implemented in the string spectrum and in particular in the ground state of the string.

In this note we have investigated the Kähler properties of the quantized $N=1$ supergravity theories coupled to matter and found that all the classical level properties are preserved. This symmetry has been used to construct the simplest supergravity models which may be of relevance to the real world and its validity should be guaranteed also at the quantum level. For this the importance of the BRST symmetry implemented by Fujikawa's approach was crucial. The specification of integration variables of the measure proposed by Fujikawa led to a Kähler invariant path integral preserving therefore at the quantum level all the implications of the classical symmetries such as Ward-identities as well as restricting the possible form of local counterterms of this effective theory. The cut-off dependence of the supergravity theory hides all the physics of the underlying superstring dynamics for energies higher than the cut-off. The elimination of the heavy degrees of freedom is a big present day problem of the quantized string theory because it is related to the locality problem of this theory. Due to the extended structure of the string, the string interactions are believed to produce non-local interactions for the particles when the string tension is finite. It remains to be seen if the removal of the cut-off of the effective supergravity theory will produce a non-local interacting field theory for the massless sector of the string consistent with all basic principles of Quantum Field Theory.

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FIGURE CAPTION

Figure 1: Ghost graph involving a $\bar{C}^{ab}(\phi_\mu C)$ vertex. The ghost loop does not close since C'_{ab} does not couple to ghosts other than $\bar{C}^{ab}$.