CERN-TH.4822/87

MESON-EXCHANGE CURRENTS AND ENERGETIC PARTICLE EMISSION FROM μ^- CAPTURE

M. Lifshitz^{*)}

Technion-Israel Institute of Technology, Haifa

and

P. Singer⁺⁾

CERN - Geneva

A B S T R A C T

We consider corrections to the impulse approximation calculation of the excitation function generated by muon capture in complex nuclei, in the form of captures on pairs of nucleons induced via meson-exchange currents. Within the Fermi gas model for the nucleus, these multi-nucleon captures are found to account for 4-8% of the experimental muon capture rates and to produce high excitation energies. The emission of pre-equilibrium protons with energies between 40 and 90 MeV is calculated for several nuclei using the hybrid exciton model. The agreement with experiment on emission spectra and rates is generally good. The emission of neutrons at these energies is also discussed.

^{*)} Present address: IMI, P.O.Box 1044, Ramat Hasharon, Israel.

⁺⁾ On leave from Technion - Israel Institute of Technology, Haifa, Israel.

CERN-TH.4822/87

August 1987

1. - INTRODUCTION

The emission of energetic neutrons [1-4] and protons [5-7] following muon capture in medium-heavy nuclei has received increased attention recently, especially since these processes are sensitive to the poorly known high-energy tail of the nuclear response function [8,9], which is of interest in a variety of nuclear processes, like (μ, ν_μ) , (π, γ) , (p, p') and (e, e') . While the low-energy emission of particles after muon capture can be explained satisfactorily in the impulse approximation (see Refs. [10,11] for detailed reviews of the experimental and theoretical situation), this picture turns out to be inappropriate for the high energy part of the spectra, beyond ~ 30 MeV.

The apparent failure of the impulse picture indicates the possibly important rôle of multinucleon processes [9] in those captures in which a large energy transfer to the final nuclear state occurs. In fact, it has been suggested already at an early stage [12,13] that the high energy emission could be due to the effect of short-range pair correlations in the nucleus. Muon capture by a correlated nucleon pair with large relative momentum, $\mu^- + (NN) \rightarrow N' + N'' + \nu_\mu$, would thus lead to the emission of correlated high energy nucleons. In the most recent experiments, direct evidence for the emission of correlated high energy nucleons has indeed been obtained by coincidence measurements [1,3] of two neutrons at very large opening angles (135° - 180°) following muon capture in O^{16} and Ca^{40} . The observed angular distribution pattern is similar to that obtained in pion absorption which is known to occur mostly on a nucleon pair. While in the above quoted experiments the muon capture would happen in the more frequently available (n-p) pairs, preliminary evidence for captures on (p-p) pairs has also been put forward [14] from measurements of (p,n) coincident pair emission following muon capture in C^{12} .

On the theoretical side, the first calculation of muon capture by nucleon pairs was performed [12] for AgBr by considering only the pseudodeuterons available in the nuclear surface region, thus avoiding the strong suppression of proton emission due to total internal reflection. With a reasonable amount of correlations, the previously unaccounted strength of the proton emission could be explained semi-quantitatively by this picture. Bernabéu, Ericson and Jarlskog [13] have used the formalism [15] which explicitly connects the nuclear axial current to the nuclear pion field. In this approach, which exploits PCAC and the assumption that the pole free part of the axial current is determined by the local pion source, the matrix elements of the axial current are related [15] to the local s and p wave interaction of pions in the nucleus. Thus, at maximal energy transfer in μ -capture, the inclusive capture rate on nucleon pairs is related to the off-shell pion absorption on these pairs. Quantitatively, this approach leads [13] to $\sim 1\%$ of captures with energy transfer of more than 80 MeV

in ^{12}C , which translates to a first approximation into emission of similar percentage of correlated pairs of nucleons carrying more than ~ 30 MeV of kinetic energy per nucleon. Using this picture for an estimate of the inclusive spectrum of high energy neutrons emitted after muon capture in ^{40}Ca , the authors of Ref. [4] are able to reproduce fairly well their experimental results.

The effect of the short-range nuclear correlations has also been taken into account in more phenomenological treatments [8,11,16] by the use of effective nuclear momentum distributions. A more sophisticated approach along these lines has been developed by Ohta and Fujita [17].

Since we are interested in the two-body correlations in the capture process, one may proceed by inquiring what are the most likely additional nuclear processes which become possible in the regime of large energy transfer to the final nuclear state, beyond the impulse approximation. Naturally, we start by considering the role of the two-body current in the process of ejection of nucleons following the muon capture. The obvious candidate for this current is the two-body meson exchange current (MEC), which is known to be relevant in a multitude of electromagnetic as well as weak processes (for recent reviews, see Refs. [18-22]).

The contribution of the meson exchange currents to the muon capture in deuterons was found [23] to increase the total rate by about 6%. Recently, Goulard, Lorazo and Primakoff [24] have considered the segment of this process involving outgoing neutrinos with very small momentum and have shown that in this restricted region (the same as the one considered in Ref. [13]), the exchange currents become the dominant mechanism for capture. This prediction was confirmed recently [25] by the measurement (with a time-of-flight method) of the neutron spectrum from 20 to 52 MeV resulting from μ^- capture in liquid deuterium. The energetic neutron part of the spectrum exhibits a sizeable increase, which can be accounted for by the inclusion of the contribution of MEC [13,23,24].

In this paper, we propose to calculate the contribution of the two-nucleon current due to MEC to the nuclear excitation function in medium-heavy nuclei following muon capture. We then calculate the absolute rate of high-energy nucleon emission due to this additional effect, with emphasis on proton emission, and find it to be consistent with existing measurements. Our calculation of the two-nucleon excitation via two-body MEC is carried out within the Fermi gas model, using to some extent the approach of Van Orden and Donnelly [26] who described successfully two-nucleon ejection in deep inelastic electron scattering in nuclei.

In Section 2 we present a concise background discussion of meson exchange effects in μ -capture and then proceed to obtain the two-body hadronic MEC of relevance for our problem. To this end, we use Feynman diagrams derived from an effective phenomenological Lagrangian. In Section 3 we use the Fermi gas model to calculate the nuclear excitation function and the capture probability via the MEC mechanism for some 15 selected isotopes from Mg^{24} to Bi^{209} . In Section 4, we describe our formalism for particle ejection which is based on the hybrid exciton model approach to pre-equilibrium emission [27]. Section 5 contains our results and discussion.

2. - MUON CAPTURE WITH EXCHANGE CURRENT

During recent years, it has become evident that exchange currents play an important role in electromagnetic and weak nuclear processes. We do not intend to review this here especially as good review articles are available [18-22], but in order to give the reader some background we briefly survey some works dealing with exchange currents in μ -capture. The pioneering work in our context is due to Dautry, Rho and Riska [23] who have shown that the contribution of MEC increases the muon capture rate in deuteron by 6.5%, bringing it to the range of the experimental figure. A slightly higher contribution, of 8.5%, is found when the MEC are calculated [28] by using hard pion methods. Additional work [29-32] has dealt with the effect of MEC on partial capture rates of muons in nuclei, with particular emphasis on nuclei with $A = 12, 14, 16$. Processes originating from the contribution of the time component of the axial current, like $0^+ \rightarrow 0^-$ transitions, reveal sizeable contributions from MEC [20, 21, 29]. Of special interest, since it relates to the effect of MEC on the nuclear excitation distribution with consequences for the spectrum of the final state neutrons, is the paper of Goulard, Lorazo and Primakoff [24]. The near tripling of the highest energy neutron production from $\mu^- + d \rightarrow n + n + \nu_\mu$ obtained by these authors, as compared to the expectation [33] without MEC has been verified for the region $35 \text{ MeV} \lesssim E_n \lesssim 52 \text{ MeV}$ in a recent experiment [25]. Qualitatively, such an appreciable change is implied also by the work of Bernabéu, Ericson and Jarlskog [13].

Motivated by these results and by the argumentation presented in the Introduction, we attempt here to estimate the changes induced by exchange currents in the nuclear excitation distribution following muon capture in complex nuclei. It will be shown that these changes seem indeed to be responsible for the emission of highly energetic particles, as recently observed [1-7].

Several approaches are available in the literature for the derivation of the MEC terms [18, 20]. We choose here the simplest and well-tested diagrammatic approach of expressing the exchange operator in terms of a sum of Feynman

diagrams [23,26,34,35], which are calculated by using a phenomenological effective Hamiltonian [36].

The Feynman diagrams contributing to the two-body exchange currents of vector (V_μ) and axial-vector (A_μ) nature are exhibited in Fig. 1. We limit ourselves mainly to diagrams of pionic range, although we have also included the contribution of Fig. 1d when a ρ is exchanged, which we found to be of some importance. In Fig. 1 we exhibit only those exchange contributions not included [34] in the nuclear wave function, which already contains meson exchanges through the nuclear potential. The weak current is denoted by I_μ ; the fact that the weak current is first class [37] restricts the contribution in Fig. 1a to the vector current and the contributions to diagrams 1e and 1f to the axial current. In Fig. 1 only one typical diagram for each case is depicted, the various permutations for the vertex location being understood.

The contribution of Fig. 1a has been invoked already long ago [38] as a possible source for higher excitation in muon capture. However, as a result of Siegert's theorem [39] which, although rigorous for zero momentum transfer, is known to be effective also for transfer values as high as the pion mass, the contribution on Fig. 1a turns out to be negligible [40]. Thus, it is to be expected that the main exchange contributions will come from the axial current. This agrees with the findings of Ref. [13] who found the vector part to be under 10%. Turning to diagram 1b, the so-called "recoil diagram", we remark that to leading order in a non-relativistic expression this recoil contribution is cancelled by the effects of wave-function renormalization [41]. Also, using considerations developed by Cheng and Goulard [31], one finds that the contribution of diagram 1f is negligible, the time component vanishing and the space component being of high order in k/m in the non-relativistic regime. Hence, we need to calculate only the contributions due to (1c)-(1e). However, since we found that the contribution of (1c) turns out to be smaller than the rest, in agreement with previous estimates [31], we deal in the following with diagrams 1d and 1e only.

The two-particle meson exchange operators are written in the momentum space from the Feynman diagrams of Figs. 1d and 1e. We follow the notation and conventions of Ref. [34]. For the diagrams involving Δ we use the definitions of Ref. [42]. In the Appendix we list the interaction terms and the values of the coupling constants used. Then the basic term for muon capture by a pair of nucleons via MEC is

$$S_{fi}^{(MEC)} = \frac{G \cos \theta_c}{\sqrt{2}} \delta^{(4)}(p^{(\mu)} + p_1 + p_2 - p^{(\nu)} - p'_1 - p'_2) \times \sqrt{\frac{m_\mu M^4}{2 E_1 E_2 E'_1 E'_2 E_\mu E_\nu}} L_\lambda J_\lambda^{(MEC)} \tau(1,2) \quad (1)$$

where $p^{(\mu)}$, $p^{(\nu)}$, p_i , $i = 1, 2$ are the four-momenta of the muon, neutrino and the nucleons, respectively, a prime indicating the final state. The leptonic current is

$$L_\lambda = \bar{u}(p^{(\nu)}) \gamma_\lambda (1 + \gamma_5) u(p^{(\mu)}) \quad (2)$$

and the isotopic spin factor $\tau(1,2)$ is

$$\tau(1,2) = \tau_-(1) \tau_3(2) - \tau_3(1) \tau_-(2) \quad (3)$$

with τ_- being the lowering operator. The hadronic current is

$$J_\lambda^{(MEC)} = J_\lambda^{(\pi S)} + J_\lambda^{(\pi \Delta)} + J_\lambda^{(S \Delta)} \quad (4)$$

We obtain

$$J_\lambda^{(\pi S)} = N_{\pi S} \frac{\bar{u}(p'_1) \gamma_\lambda u(p_1) \cdot \bar{u}(p'_2) \gamma_5 u(p_2)}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)} - (1 \leftrightarrow 2), \quad (5)$$

$$J_\lambda^{(\pi \Delta)} = N_{\pi \Delta} \left\{ \frac{q_{1,\mu} \bar{u}(p'_1) \gamma_\lambda u(p_1) \cdot u(p'_2) (\gamma \cdot k + i m_\Delta)}{(q_1^2 + m_\pi^2)(k^2 + m_\Delta^2)} \times \left[(\delta_{\mu\lambda} - \frac{1}{3} \gamma_\mu \gamma_\lambda) + (\delta_{\lambda\mu} - \frac{1}{3} \gamma_\lambda \gamma_\mu) \right] u(p_2) \right\} - (1 \leftrightarrow 2), \quad (6)$$

$$J_\lambda^{(S \Delta)} = N_{S \Delta} \left\{ \bar{u}(p'_1) [(\gamma \cdot k + i m_\Delta) (\delta_{\lambda\mu} - \frac{1}{3} \gamma_\lambda \gamma_\mu) \gamma_5 - \gamma_5 (\gamma \cdot k + i m_\Delta) (\delta_{\lambda\mu} - \frac{1}{3} \gamma_\lambda \gamma_\mu)] u(p_1) \frac{\bar{u}(p'_2) \gamma_\mu u(p_2)}{(q_1^2 + m_\pi^2)(k^2 + m_\Delta^2)} \right\} - (1 \leftrightarrow 2), \quad (7)$$

where

$$q_{1,2} = p'_{1,2} - p_{1,2}$$

$$k = p'_1 + p'_2 - p_2$$

and we have lumped numerical coefficients into $N_{\pi\rho}$, $N_{\pi\Delta}$, $N_{\rho\Delta}$.

In the non-relativistic regime which we use in our calculation and to first order in the inverse nucleon mass, $1/M$, the matrix elements of these operators for capture by a nucleon pair become proportional to the following expressions:

$$M_{fi}^{(\pi\rho)} \propto \langle \varphi_{(\nu)} | (1 - \vec{\sigma} \cdot \hat{k}_{(\nu)}) | \varphi_{(\mu)} \rangle \left[\frac{\langle \varphi'_1, \varphi'_2 | \vec{\sigma}(2) \cdot \vec{q}_2 | \varphi_1, \varphi_2 \rangle}{(\vec{q}_1^2 + m_\rho^2)(\vec{q}_2^2 + m_\pi^2)} - (1 \leftrightarrow 2) \right], \quad (5a)$$

$$M_{fi}^{(\Delta\pi)} \propto \langle \varphi_{(\nu)} | (1 - \vec{\sigma} \cdot \hat{k}_{(\nu)}) \sigma_j | \varphi_{(\mu)} \rangle \times \left[\frac{\langle \varphi'_1, \varphi'_2 | \vec{\sigma}(1) \cdot \vec{q}_1 \sigma_{2,j} | \varphi_1, \varphi_2 \rangle}{\vec{q}_2^2 + m_\pi^2} - (1 \leftrightarrow 2) \right], \quad (6a)$$

$$M_{fi}^{(\rho\Delta)} \propto \langle \varphi_{(\nu)} | 1 - \vec{\sigma} \cdot \hat{k}_{(\nu)} \sigma_j | \varphi_{(\mu)} \rangle \left[\frac{\langle \varphi'_1, \varphi'_2 | \sigma_j(1) | \varphi_1, \varphi_2 \rangle}{\vec{q}_2^2 + m_\rho^2} - (1 \leftrightarrow 2) \right], \quad (7a)$$

where $\phi_{(\mu)}$, $\phi_{(\nu)}$, ϕ_i , ϕ'_i are the wave functions of the muon, neutrino, initial nucleon i and final nucleon i , respectively. $\hat{k}_{(\nu)}$ is a unit vector in the direction of the neutrino momentum.

Hence, in the non-relativistic limit we adopted here, it is the matrix elements of the time component of $A_\mu^{(\pi\rho)}$ and the space components of $A_\mu^{(\pi\Delta)}$ and $A_\mu^{(\rho\Delta)}$ which survive.

3. - THE NUCLEAR EXCITATION

In order to compute the nuclear excitation produced by muon capture via MEC, we need the nuclear wave function for the initial and final states. In this work, which is a first attempt to estimate the contribution of MEC to the nuclear excitation in complex nuclei, we adopt the simplest nuclear model, the non-interacting Fermi gas model. Hence, the nuclear wave functions are approximated by plane waves and the ground state is constructed by filling the levels up to the Fermi momentum k_F . Such a model should provide a good first approximation for our purpose, as it did, for example, in inelastic electron scattering [26]. Following now the approach of Van Orden and Donnelly [26] we use

the matrix elements calculated in the previous section to define a two-body quantized MEC operator $\hat{J}$ [43]

$$\hat{J} = \frac{1}{2} \sum_{\substack{k_1, k_2 \\ k'_1, k'_2}} J(k'_1 - k_1; k'_2 - k_2) a_{k_1}^\dagger a_{k_2}^\dagger a_{k_2} a_{k_1} \quad (8)$$

where a_k ($a_k^\dagger$) are annihilation (creation) operators of a nucleon with momentum k and J is the two-particle MEC matrix element defined in [4]. The spin and isospin indices are omitted in the formulae presented, for simplicity.

Let us denote the ground state of the nucleus by $|G\rangle$. The operator $\hat{J}$ can connect between $|G\rangle$ and one particle - one hole $|1ph\rangle$ and two particle-two holes $|2p2h\rangle$ states. The $|1ph\rangle$ states can be realized in two ways: (A) one nucleon is raised above the Fermi sea creating a hole in the Fermi occupation region, while the second nucleon is scattered elastically; to this, the operator $J(k'_1 - k_1; 0)$ contributes. (B) One nucleon is raised from k_1 to k'_1 above the Fermi sea, while the second nucleon moves from k_2 to k'_2 ; to such a situation corresponds $J(k'_1 - k_1; k'_2 - k_2)$. However, the $|1ph\rangle$ excited states thus achieved do not differ appreciably from those produced in the impulse approximation. In fact, for such processes we find $\langle 1ph | \hat{J} | G \rangle$ to provide an excitation energy distribution with maximum around 25 MeV and peaking around 15 MeV. Moreover, the single nucleon thus formed must be a neutron and as such, it will contribute very little to the appearance of high energy protons. Accordingly, we shall consider here only processes induced by $|2p2h\rangle \hat{J} | G \rangle$.

The two particle-two holes states are constructed as follows:

$$|2p2h\rangle \equiv |p'_1 p'_2 p_1 p_2\rangle = \theta(p'_1 - k_F) \theta(p'_2 - k_F) \times \theta(k_F - p_1) \theta(k_F - p_2) a_{p'_1}^\dagger a_{p'_2}^\dagger a_{p_2} a_{p_1} |G\rangle \quad (9)$$

where $\theta(p)$ is the step function. Our calculation involves therefore matrix elements of the form

$$\langle p'_1 p'_2 p_1 p_2 | \hat{J} | G \rangle = \theta(p'_1 - k_F) \theta(p'_2 - k_F) \times \theta(k_F - p_1) \theta(k_F - p_2) \frac{1}{2} \sum_{\substack{k_1, k_2 \\ k'_1, k'_2}} J(k'_1 - k_1; k'_2 - k_2) \langle G | \Omega | G \rangle \quad (10)$$

where $J \equiv J^{(MEC)}$ of Eqs. (4)-(7a) and

$$\Omega = a_{p_1}^\dagger a_{p_2}^\dagger a_{p_2} a_{p_1} a_{k'_1}^\dagger a_{k'_2}^\dagger a_{k_2} a_{k_1} \quad (11)$$

Using the commutation relations among the a_k , $a_k^\dagger$ operators and the properties of

$|G\rangle$, namely

$$a_{p'_1}|G\rangle = a_{p'_2}|G\rangle = a_{p_1}^\dagger|G\rangle = a_{p_2}^\dagger|G\rangle = 0 \quad (12)$$

one easily finds

$$\begin{aligned} \langle p'_1 p'_2 p_1 p_2 | \hat{J} | G \rangle &= \theta(p'_1 - k_F) \theta(p'_2 - k_F) \theta(k_F - p_1) \theta(k_F - p_2) \\ &\times \left[J(p'_1 - p_1; p'_2 - p_2) - J(p'_1 - p_2; p'_2 - p_1) \right], \end{aligned} \quad (13)$$

having thus a direct and an exchange contribution to the matrix element.

We are ready now to calculate the excitation of the nucleus caused by capture via MEC. This capture probability per unit time is given by

$$\omega_{MEC} = N \langle \varphi_{(\mu)} \rangle^2 V I_{MEC} \quad (14)$$

where V is the nuclear volume and N is a numerical coefficient. Since in any case we neglect the muon's momentum, we may use an average muon wave function [44]

$$\langle \varphi_{(\mu)} \rangle^2 = \frac{Z_{eff}^4}{\pi Z} (\alpha \bar{m}_\mu)^3 \quad (15)$$

where Z_{eff} is a known tabulated value for each isotope, $\bar{m}_\mu$ is the muon reduced mass and α the fine structure constant. Also,

$$\begin{aligned} I_{MEC} &= \int d^3 p_1 d^3 p_2 d^3 p'_1 d^3 p'_2 d^3 k_{(\nu)} \delta(\vec{k}_{(\nu)} + \vec{p}'_1 + \vec{p}'_2 - \vec{p}_1 - \vec{p}_2) \\ &\times \delta(E_0 - k_{(\nu)} - E) |M_{fi}^{(MEC)}|^2 \theta(p'_1 - k_F) \theta(p'_2 - k_F) \theta(k_F - p_1) \theta(k_F - p_2) \end{aligned} \quad (16)$$

where E_0 is the energy available to the process

$$E_0 = M(A, Z)c^2 - M(A, Z-1)c^2 + m_\mu c^2 - B_\mu \quad (17)$$

with B_μ the muon binding energy in the capturing nucleus. The excitation energy E of the nucleus is defined in terms of particle momenta by

$$E = (\vec{p}'_1{}^2 + \vec{p}'_2{}^2 - \vec{p}_1{}^2 - \vec{p}_2{}^2) / 2M. \quad (18)$$

Substituting in (16) the matrix elements calculated from (5a)-(7a) and (13) we are left after using the δ -function with a ten-dimensional integration in order to obtain the excitation distribution function $I(E)dE$. We have performed the

multidimensional numerical integration by a Monte Carlo method based on an algorithm developed by Lepage [45].

In Fig. 2 we give the nuclear excitation function obtained from (16) for muon capture by MEC in ^{207}Pb . This function is typical for the isotopes we calculated.

In order to obtain the contribution of the exchange currents to the particle emission, we firstly calculate from Eq. (14) the absolute capture rates due to MEC. The results are given in column 3 of Table 1 in which we list the percentage contribution $F(\%)$ of ω_{MEC} to the μ -capture process in 13 isotopes; this is determined by using for the total capture rate the experimental figure [46], so that $F = \omega_{\text{MEC}}/\omega_{\text{exp}}$. In column 2 we give the average excitation energies $\bar{E}$ determined from the respective $I_{\text{MEC}}(E)$.

We find that the main contribution to the capture by MEC is due to the Δ diagram of Fig. 1d with pion exchange. The Δ diagram with ρ exchange contributes approximately one-quarter of the previous one, while the $(\rho\pi)$ diagram of Fig. 1e contributes only one-tenth of the Δ diagram with pion exchange. Since phases of the couplings involved are not established, we have not included interference terms in the calculation of $I_{\text{MEC}}(E)$. However, since one diagram is clearly dominant, this procedure is acceptable. It should be remarked here that our results for the smallness of the contribution of Fig. 1e compared to Fig. 1d agrees quantitatively with the findings of Refs. [28] and [29]. In Ref. [23], although the $(\pi\rho)$ exchange is also found to contribute less than the Δ diagram, their relative contribution is about one third.

4. - THE PARTICLE EMISSION PROCESS

The excitation energy induced by μ -capture via MEC in complex nuclei described by a Fermi gas was found in the previous section to span a range between ~ 30 MeV and ~ 80 MeV. It is well known that for this range of energies, pre-equilibrium emission is the major mechanism for nucleus de-excitation. The hybrid exciton model has been widely used in recent years in this context and has been shown [27] to describe well the emission of neutrons and protons in nuclear reactions at excitation energies of several tens of MeV and up to nearly 100 MeV. The model has been applied [47] successfully also to the calculation of exclusive and inclusive neutron spectra up to ≈ 90 MeV produced by pion capture in ^{165}Ho . Recently, Blann has demonstrated [48] that a modified version of the model, which includes effects of the diffuse nuclear surface, is suitable to account for the emission of protons and neutrons from nuclear reactions and pion

capture also for energies of the emitted particles well beyond 100 MeV. The predictions of the new version [48] coincide with those of the usual formulation [11,27,47] up to energies of 70-80 MeV.

Our approach here to the emission of protons will be to use the excitation functions calculated from (16), of which Fig. 2 is an example, in order to follow with the aid of the hybrid exciton model the equilibration process in the nucleus, during which time emission of energetic particles occurs. The equilibration process is achieved by a cascade of nucleon collisions with increasing complexity, each intermediate stage being characterized by n , the number of excitons (particles+holes, $n=p+h$), with transitions occurring always towards more complicated states ($n \rightarrow n+2$). The probability of emitting a particle x with energy ϵ after μ capture during the pre-equilibrium stage is given by (we refer to [11,27,47,48] for more details of the model):

$$P_x(\epsilon) d\epsilon = \int_0^{E_0} I(E) dE \sum_{\substack{n=n_0 \\ (\Delta n=2)}} f_x \left[\frac{\rho_{n-1}(u) g}{\rho_n(E)} \right] D_n \left[\frac{\lambda_c(E) d\epsilon}{\lambda_c(E) + \lambda_{n+2}(E)} \right] \\ = \int_0^{E_0} I(E) dE \sum_n P_x(\epsilon, E) d\epsilon \quad (20)$$

The term in the second bracket in (20) is the branching ratio for emission, λ_c being the transition rate for emission into continuum and λ_{n+2} giving the transition rate to a higher exciton state. The other factors express the probability of populating the particle emitting state: f_x is the number of nucleons of type x in an n -exciton state, D_n is the depletion factor accounting for emission from lower exciton states and the expression in the first bracket is the statistical factor; it uses densities of states to give the probability that in state n there is a particle in an unbound level with energy ϵ in the continuum. u is the residual excitation energy after particle emission, so that $\epsilon_x = E - u - B_x$, with B_x the binding energy of a particle of type x . For ρ_n one uses the Ericson expression

$$\rho_n(E) = \frac{g(gE)^{n-1}}{(n-k)! k! (n-1)!} \quad , \quad (21)$$

g being the single particle level density proportional to the number of nucleons in the nucleus. Using well-tested expressions for the quantities involved, one arrives [27] at Blann's parametric expression for $P_x(\epsilon_x, E)$, the probability of emitting a particle of type x with energy ϵ_x from the exciton state n at excitation energy E

$${}_n P_x(\epsilon, E) = \frac{(n-1) u^{n-2} f_x D_n}{E^{n-1}} \times \frac{G_x(\epsilon_x) \epsilon_x / g_x}{[G_x(\epsilon_x) \epsilon_x / g_x] + [1890(\epsilon_x + B_x) - 8(\epsilon_x + B_x)^2]} \quad (22)$$

where $g_x = g/2$, σ is in mb and ϵ_x , B_x are in MeV. Using $I_{\text{MEC}}(E)$ of (16) in (20) we calculate from (20) and (22) the proton emission spectra and probabilities. The lowest exciton state from MEC capture which is relevant for our purpose is $|2p2h\rangle$, as it was explained in Section 3. Therefore, the summation in (20) starts with $n_0 = 4$.

5. - DISCUSSION

In this work we have examined the contribution of μ captures via meson-exchange currents to the excitation of a complex nucleus. Our aim was to investigate whether these captures by correlated nucleons could lead to the emission of the energetic part of the emission spectrum, which is unaccounted for in the impulse approximation [10,11]. Our calculations were performed for the emission of protons with energies beyond ~ 35 MeV, for which no previous detailed treatments of the spectra are available. After presenting our results for the proton spectra, we shall discuss shortly the energetic neutron emission.

Firstly, in order to underline the problematics of the present situation, we reproduce in Fig. 3 the result of a previous calculation [11] in which the emission spectrum of protons following μ capture in ${}^{75}_{33}\text{As}$ is calculated in the impulse approximation, using "realistic" momentum distributions for the nucleons. The low energy evaporation protons (designated as statistical in the figure) die out beyond 25 MeV, while the spectrum of those emitted during the equilibration process (the "precompound" protons) extends up to ~ 40 MeV. Beyond 20 MeV, the spectrum is dominated by the latter. However, there is practically no component beyond 40 MeV, while the experiments indicate [5,6] proton emission up to ~ 80 MeV with an integrated probability of 10^{-4} - 10^{-3} beyond 40 MeV for the several isotopes measured so far. Obviously, the impulse approximation fails badly for this region.

Turning to our calculation, we see from Table 1 that two-nucleon captures via MEC account for ~ 4 -9% of the experimental rate, and one observes that on the average this percentage increases mildly with A . This may be compared with the 6-8% contribution [23,28] of MEC to the total capture rate in deuteron. The percentage contribution of MEC to partial transition rates in light nuclei, $A = 6$ -14, has been found [30] to vary between 2 and 20%. The figures we obtain

are indeed of same order of magnitude. A comparison with the "1% probability at 80 MeV" of Ref. [13] is not straightforward but an estimate may be made by taking the part of our excitation curve (Fig. 2) between 70 and 90 MeV. This part constitutes 1/6 of the calculated MEC capture rate in Pb^{207} , leading indeed to 1.3% capture probability from this region.

In Table 2 we compare the calculation of integrated proton emission with energies $E > E_{\text{min}} = 40$ MeV to the available data [5,6,49,50]. Since the contribution from the impulse approximation is essentially nil in this region, the emission calculated from MEC should ideally account for all the observed emission. For Si, Mg and Pb our results are indeed very close to the experimental ones, but we are a factor of 4 too low for Al and Cu. Our result for AgBr is an order of magnitude higher than the old emulsion measurement [49], which appears suspiciously low. The general trend of our results is gratifying. We could have inverted this analysis and calculate from the experiment the percentage of MEC captures needed to account for the observed proton emission above 40 MeV. One would conclude that our simplified approach accounts quite well for these captures, with the exception of Al and Cu where something is still missing to explain the sizeable high energy proton emission.

In Figs. 4-6, we compare the calculated spectra with the experimental ones [6]. For this comparison, we have normalized the theoretical spectrum to the total experimental proton emission above 40 MeV. The spectra we obtain fit nicely the experimental shape, except possibly in the highest region above 60 MeV, where it tends to fall below it.

Recently, spectra of high energy neutrons with $40 \text{ MeV} < E_n < 90 \text{ MeV}$ have also been measured, from μ capture in ^{40}Ca [4] and ^{165}Ho [2]. These spectra are quite similar in shape to the proton ones, which is understandable if they have a common origin. This would indeed be so, if the highest energy particle emission comes from the $n = 4$ exciton state formed in MEC captures. Nevertheless, one would expect the neutron spectrum to have a certain component from the lower exciton states, even in this high energy region.

We have calculated the neutron emission for these isotopes with the formalism described here, and we find that we do not obtain the observed amount of high energy neutrons. Thus, for ^{165}Ho our model gives 0.5% neutron emission for $E_n > 40$ MeV, while the observed [2] figure is 0.019 ± 0.004 neutrons per μ stop. With an average of neutron emission per capture [10] of $\langle n \rangle = 1.64$ this implies 1.2% high energy neutron production. For ^{40}Ca , with a similar theoretical result, one can read from Fig. 2 of Ref. [4] that there are 0.015 neutrons per μ stop above 40 MeV. With $\langle n \rangle = 0.75$ for Ca [10] one has 2% of high energy neutron production. It thus appears that a pre-equilibrium analysis of the

neutron spectra might have to include [48] the geometry dependent aspects related to the diffuse nuclear surface and to the neutron-richness of the nuclear skin. Moreover, one should consider the possibility of a higher likelihood of a p-n pair vs. a p-p pair to act as a pseudodeuteron in the capture process.

To summarize, μ capture by meson-exchange currents appears to be a viable candidate for producing the higher excitation energies needed for the emission of energetic nucleons in the range between 40 and 90 MeV. From among the exchange currents considered (Fig. 1), the π - Δ turns out to be the dominant contribution. A detailed calculation of proton emission using the hybrid exciton model accounts well for most existing data on rates and spectra. To account for the neutron emission by the same mechanism, some refinements appear necessary. The availability of more data will greatly facilitate progress in the understanding of the mechanism responsible for the energetic particle emission following μ capture.

ACKNOWLEDGEMENTS

We are thankful to T. Ericson and N. Mukhopadhyay for helpful remarks. P.S. acknowledges with thanks the hospitality of the Theory Division at CERN.

APPENDIX

We list here the effective Lagrangian terms [36,42] and the values of the coupling constants [36] we used:

$$\mathcal{L}_{\pi NN} = i g_{\pi} \bar{\Psi} \gamma_5 \vec{\tau} \Psi \cdot \vec{\Phi}_{\pi} , \quad (A1)$$

$$\mathcal{L}_{\rho NN} = i g_{\rho} \bar{\Psi} [\gamma_{\mu} + i \frac{K_V}{2M} \sigma_{\mu\nu} \partial_{\nu}] \vec{\tau} \cdot \vec{\rho}_{\mu} \Psi , \quad (A2)$$

$$\mathcal{L}_{\pi N \Delta} = g_{\pi \Delta} \vec{\chi}_{\mu} \partial_{\mu} \cdot \vec{\Phi}_{\pi} \Psi + h.c. , \quad (A3)$$

$$\mathcal{L}_{\rho N \Delta} = g_{\rho \Delta} \left[\vec{\chi}_{\mu} \gamma_5 \cdot \vec{\rho}_{\mu} + \frac{1}{m_{\Delta} + M} \vec{\chi}_{\nu} \cdot \partial_{\nu} \vec{\rho}_{\mu} \gamma_5 \delta_{\mu} \right] \Psi + h.c. \quad (A4)$$

$$\mathcal{L}_{\pi \rho}^{(W)} = \frac{G \cos \theta_c}{\sqrt{2}} f_{\pi \rho} L_{\lambda} (\epsilon_{1jk} - i \epsilon_{2jk}) \rho_{\lambda}^j \varphi_{\pi}^k \quad (A5)$$

$$\mathcal{L}_{N \Delta}^{(W)} = \frac{G \cos \theta_c}{\sqrt{2}} f_{N \Delta} L_{\lambda} (\epsilon_{1jk} - i \epsilon_{2jk}) \bar{\chi}_{\lambda}^j \tau^k \Psi + h.c. \quad (A6)$$

where we denoted by Ψ the nucleon spinor-isospinor and by $\vec{\chi}_{\mu}$ the vector-spinor and isospin vector-spinor of Δ in the Rarita-Schwinger formalism.

$$\frac{g_{\pi}^2}{4\pi} = 14.5 ; \quad \frac{g_{\rho}^2}{4\pi} = 0.64 ; \quad K_V = 3.7 ; \quad (A7)$$

$$\frac{g_{\pi \Delta}^2}{4\pi m_{\pi}^2} = 0.21 ; \quad g_{\rho \Delta} = \frac{6\sqrt{2}}{5} g_{\rho} (1 + K_V) ; \quad (A8)$$

$$f_{\pi \rho} = \sqrt{2} g_{\rho \pi \pi} f_{\pi} = \sqrt{2} m_{\rho} ; f_{N \Delta} = g_{\pi \Delta} \frac{M_{\Delta}}{g_{\pi}} . \quad (A9)$$

g_{ρ} is determined with vector-meson dominance, $g_{\pi \Delta}$ from the decay width of Δ and for the $g_{\rho \Delta}$ we use the value of the quark model. To express $f_{\pi \rho}$, $f_{N \Delta}$ one has made use of the KSUF and Goldberger-Treiman relations respectively.

Capturing Nucleus	$\bar{E}$ (MeV)	$F(\%) = \omega_{\text{MEC}}/\omega_{\text{exp}}$
Mg ²⁴	54.8	4.2
Al ²⁷	56.6	3.6
S ³²	56.5	4.2
Ca ⁴⁰	56.4	4.5
Cu ⁶³	57.9	5.1
Br ⁷⁹	58.1	6.0
Ag ¹⁰⁷	57.5	6.2
Ho ¹⁶⁵	56.4	5.7
Os ¹⁸¹	57.5	8.3
Ir ¹⁸⁶	57.7	8.6
Tl ¹⁹⁴	57.9	9.8
Pb ²⁰⁷	56.7	7.9
Bi ²⁰⁹	55.5	5.2

Table 1

The average excitation energy and the capture probability due to meson-exchange currents in various isotopes from the present calculation.

Capturing Nucleus	Exp $\times 10^3$	Present Model $\times 10^3$
Al	$1.38 \pm 0.09^{\text{a)}}$	0.3
Si	$0.87 \pm 0.14^{\text{b)}}$	0.5
Mg	$0.17 \pm 0.05^{\text{c)}}$	0.2
Cu	$1.96 \pm 0.12^{\text{a)}}$	0.5
AgBr	$(4.7 \pm 1.1) \times 10^{-2 \text{d)}}$	0.4
Pb	$0.17 \pm 0.03^{\text{a)}}$	0.3

a) Ref. [6]; b) Ref. [50]; c) Ref. [5]; d) Ref. [49]

Table 2

Probability of inclusive proton emission with $E_p > 40$ MeV following μ capture.

REFERENCES

- 1) A. van der Schaaf et al. - Nucl.Phys. A408 (1983) 573.
- 2) E.K. McIntyre et al. - Phys.Lett. 137B (1984) 339.
- 3) T. Kozlowski et al. - Nucl.Phys. A436 (1985) 717.
- 4) J. van der Pluym et al. - Phys.Lett. 177B (1986) 21.
- 5) M.P. Balandin et al. - Yad.Fiz. 28 (1978) 582 [Sov.J.Nucl.Phys. 28 (1978) 297].
- 6) K.S. Krane et al. - Phys.Rev. C20 (1979) 1873.
- 7) G.E. Belovitskii et al. - Yad.Fiz. 43 (1986) 1057 [Sov.J.Nucl.Phys. 43 (1986) 673].
- 8) P. Singer, N.C. Mukhopadhyay and R.D. Amado - Phys.Rev.Lett. 42 (1979) 162.
- 9) N.C. Mukhopadhyay, J. Hadermann and P. Singer - Phys.Rev.Lett. 44 (1980) 1653.
- 10) P. Singer - in Springer Tracts in Modern Physics, ed. G. Höhler (Springer, Berlin, 1974), Vol. 71, p. 39;
N.C. Mukhopadhyay - Physics Reports 30C (1977) 1;
Yu.A. Batusov and R.A. Eramzhan - Fiz.Elem.Chastits At.Yadra 8 (1977) 229 [Sov.J.Part.Nucl. 8 (1977) 95].
- 11) M. Lifshitz and P. Singer - Phys.Rev. C22 (1980) 2135.
- 12) P. Singer - Phys.Rev. 124 (1961) 1602.
- 13) J. Bernabéu, T.E.O. Ericson and C. Jarlskog - Phys.Lett. 69B (1977) 161.
- 14) C.J. Martoff et al. - Czech.J.Phys. B36 (1986) 378.
- 15) T.E.O. Ericson - Progr.Part.Nucl.Phys. 1 (1978) 173;
T.E.O. Ericson and J. Bernabéu - Phys.Lett. 70B (1977) 170.
- 16) T. Fujita and K. Kubodera - Phys.Lett. 149B (1984) 451.
- 17) N. Ohta and T. Fujita - to be published.
- 18) R.J. Blin-Stoyle - in Progress in Particle and Nuclear Physics, ed. D. Wilkinson (Pergamon Press, Oxford, 1978), p. 5.
- 19) E.A. Ivanov and E. Truhlik - Fiz.Elem.Chastits At.Yadra 12 (1981) 492 [Sov.J.Part.Nucl. 12 (1981) 198].
- 20) M. Kirschbach and E. Truhlik - Fiz.Elem.Chastits At.Yadra 17 (1986) 224 [Sov.J.Part.Nucl. 17 (1986) 93].
- 21) I.S. Towner - Comments Nucl.Part.Phys. 15 (1986) 145.
- 22) D.O. Riska - Progr.Part.Nucl.Phys. 11 (1984) 199.
- 23) F. Dautry, M. Rho and D.O. Riska - Nucl.Phys. A264 (1976) 507.
- 24) B. Goulard, B. Lorazo and H. Primakoff - Phys.Rev. C26 (1982) 1237.
- 25) Y.K. Lee et al. - Phys.Lett. 188B (1987) 33.
- 26) J.W. Van Orden and T.W. Donnelly - Ann.Phys. 131 (1981) 451.

- 27) M. Blann - Ann.Rev.Nucl.Sci. 25 (1975) 123.
- 28) E. Ivanov and E. Truhlik - Nucl.Phys. A316 (1979) 451.
- 29) S. Nozama, Y. Kohyama and K. Kubodera - Progr.Theor.Phys. 70 (1983) 613;
Phys.Lett. 140B (1984) 11.
- 30) P.A.M. Guichon et al. - Z.Phys. A285 (1978) 183; Phys.Rev. C19 (1979) 987;
P.A.M. Guichon, M. Giffon and C. Samour - Phys.Lett 74B (1978) 15;
P. Desgrolard and P.A.M. Guichon - Phys.Rev. C19 (1979) 120.
- 31) W.K. Cheng, B. Lorazo and B. Goulard - Phys.Rev. C21 (1980) 374;
W.K. Cheng and B. Goulard - Phys.Rev. C23 (1981) 869.
- 32) P. Parthasarathy and V.N. Sridhar - Phys.Lett. 106B (1981) 363;
M. Fukui et al. - Progr.Theor.Phys. 70 (1983) 827;
I.S. Towner and F.C. Khanna - Nucl.Phys. A372 (1981) 331;
H.-U. Jäger, M. Kirkbach and E. Truhlik - Nucl.Phys. A404 (1983) 456.
- 33) S.L. Mintz - Phys.Rev. C28 (1983) 556.
- 34) M. Chemtob and M. Rho - Nucl.Phys. A163 (1971) 1.
- 35) P. Herczeg and P. Singer - Phys.Rev. D11 (1975) 611.
- 36) M. Chemtob - in "Mesons in Nuclei", Vol. II, eds. M. Rho and D. Wilkinson
(North Holland, 1979), p. 495.
- 37) S. Weinberg - Phys.Rev. 112 (1958) 1375.
- 38) M. Bertero, G. Passatore and G.A. Viano - Nuovo Cimento 38 (1965) 1669.
- 39) A.J.F. Siegert - Phys.Rev. 52 (1937) 787.
- 40) M. Bertero, G. Passatore and G.A. Viano - Nuovo Cimento 52 (1967) 1379.
- 41) J.L. Friar - Phys.Lett. 69B (1977) 51;
M. Gari and H. Hyuga - Z.Phys. A277 (1976) 291.
- 42) G.N. Epstein and D.O. Riska - Z.Phys. A283 (1977) 193.
- 43) A.L. Fetter and J.D. Walecka - "Quantum Theory of Many-Particle Systems"
(McGraw Hill, New York, 1971).
- 44) Y. Kohyama and A. Fujii - Progr.Theor.Phys.Suppl. 60 (1976) 171.
- 45) G.P. Lepage - J.Comp.Phys. 27 (1978) 192.
- 46) M. Eckhause et al. - Nucl.Phys. 81 (1966) 575.
- 47) Y.K. Lee - Phys.Rev. C33 (1986) 1707;
E.K. McIntyre et al. - Phys.Rev. C34 (1986) 2223.
- 48) M. Blann - Phys.Rev. C28 (1983) 1648;
M. Blann and H.K. Vonach - Phys.Rev. C28 (1983) 1475.
- 49) A.O. Vaisenberg, E.D. Kolganova and N.V. Rabin - Yad.Fiz. 11 (1970) 830
[Sov.J.Nucl.Phys. 11 (1970) 464].
- 50) Yu.G. Budyashov et al. - Zh.Eksp.Teor.Fiz. 60 (1971) 10 [Sov.Phys. JETP 33
(1971) 11].

FIGURE CAPTIONS

- Fig. 1 Meson-exchange diagrams for the two-nucleon muon capture process. The weak current is $I_{\mu} = V_{\mu} + A_{\mu}$.
- Fig. 2 Nuclear excitation function following muon capture by meson-exchange currents in ^{207}Pb .
- Fig. 3 Precompound (PC) and statistical (ST) spectra of protons from muon capture in $^{75}_{33}\text{As}$, calculated in the impulse approximation (Ref. [11]).
- Fig. 4 Theoretical (present calculation) vs. experimental (Ref. [6]) spectrum of protons following muon capture by Pb.
- Fig. 5 Theoretical (present calculation) vs. experimental (Ref. [6]) spectrum of protons following muon capture by Al.
- Fig. 6 Theoretical (present calculation) vs. experimental (Ref. [6]) spectrum of protons following muon capture by Cu.

- Fig. 1 -

- Fig. 2 -

- Fig. 3 -

- Fig. 4 -

- Fig. 5 -

- Fig. 6 -