CERN-TH.4857/87

MUTUAL FOCUSING OF GRAVITON BEAMS

G. Veneziano
CERN -- Geneva

ABSTRACT

Using recent results on high-energy superstring scattering, we discuss the mutual focusing of two colliding graviton beams. The phenomenon, which could be of some astrophysical interest, bears amusing similarities to the one leading to singularities in the classical treatment of gravitational wave collisions.

It is well known that singularities appear in classical general relativity, while considering collisions of infinitely extended gravitational plane waves [1-3]. Much less is known on the more realistic case of finite (and possibly curved) fronted wave collisions.

In this short note we wish to point out that some recently derived [4,5] properties of superstring scattering at Planckian energies have some amusing implications on this problem.

In the idealized case we analyze, we consider two, finite-size graviton beams B_1 and B_2 colliding in a (gedanken) graviton-graviton collider, as sketched in the figure.

In a convenient reference system, the gravitons in $B_{1,2}$ move in opposite directions along the z-axis with momenta :

$$k_1^i = (|k_1^i|, 0, 0, |k_1^i|); \quad k_2^j = (|k_2^j|, 0, 0, -|k_2^j|) \quad (1)$$

where i, j label the different gravitons in each beam.

Let us denote by R the radius of the (circular) transverse section of either beam, assumed to consist of a short burst with constant energy density along its profile. Besides R , one can construct two lengths:

$$r_{s,1} = 2G_N E_1, \quad r_{s,2} = 2G_N E_2 \quad (2)$$

corresponding to the Schwarzschild radii associated with the energies of each beam. Out of these we can form two more lengths, whose meaning will become clear in the following:

$$f_1 = R^2 / 4 r_{s,2} = (8\pi G_N \rho_2)^{-1} \text{ and symmetrically for } f_2. \quad (3)$$

where ρ_i is the energy density (per unit area) in B_i . We shall work in the limit in which $r_{s,i} \ll R \ll f_i$, $i=1,2$, corresponding, as we shall see, to small deflection angles. This is the opposite limit to the one considered in refs.[1-3] in which $R \rightarrow \infty$ at fixed ρ_i .

Consider now a generic graviton g_1 belonging to B_1 and travelling at a distance b from the (common) beam axis (cf. figure). Such a graviton will be deflected by the gravitational pull exerted

on it by the gravitons in B_2 . It is straightforward to compute this deflection angle, using the results of ref.[4], in the limit of small angles and when string size (α') effects are neglected. Each infinitesimal section $d\Sigma_2$ of B_2 induces on g_1 a transverse momentum given by:

$$d\mathbf{K}_1/|k_1| = 8 G_N \rho_2 r dr d\phi \ \underline{\Delta} / \Delta^2 \quad (4)$$

where r and ϕ are the polar coordinates of $d\Sigma_2$, $\underline{\Delta}$ is the relative impact parameter (a function of b , r , and ϕ) and underlining means a transverse vector (see again the picture). Eq. (4) also follows if one considers the deflection in the field produced by a massless particle [6] as implicitly done recently in ref.[7].

Straightforward integration of (4) shows that, due to a sort of Gauss law, the global effect on g_1 is the same as if B_2 was replaced by a single graviton travelling on the beam axis and carrying the energy contained in that part of B_2 lying within a distance b from the axis (shaded region in the figure). In other words, one finds :

$$\mathbf{K}_1/|k_1| = 8 G_N E_2(r < b) \ \underline{b}/b^2 = 8\pi G_N \rho_2 \ \underline{b} \quad (5)$$

i.e. a scattering angle proportional to the impact parameter. This implies that, for small deflection angles, the gravitons in B_1 are all focused towards a point located on the beam axis at a distance from the interaction plane given by f_1 of eq. (3). Indeed:

$$b/\theta_1 = (8\pi G_N \rho_2)^{-1} = f_1 \ ; \ \theta_1 = |\mathbf{K}_1|/|k_1| \quad (6)$$

Also, evidently, the gravitons converge at f_1 a time f_1/c after the collision. Of course, a symmetric focusing occurs at f_2 for B_2 .

Focusing also occurs for any D : indeed eq.(6) continues to hold^(*) after the simple replacement:

$$\rho_2 \rightarrow \rho_2 (D/2-1)^{-1} \quad (7)$$

(*) I thank M. Ciafaloni for this remark.

Before discussing corrections, let us compare the result (6) with the location of the singularity arising [1] from the collision of two infinite gravitational plane waves (we are unaware of a generalization to any D to compare with (7)). In this case, unlike in ours, there is translational invariance along the x - y plane. In the z - t plane, instead, the tip of the singularity occurs precisely at the point:

$$t = z = u/\sqrt{2} = (\sqrt{2} \sigma^2 u_0)^{-1} = (8\pi G_N \rho_2)^{-1} \quad (8)$$

where we have used notations and formulae from ref. [2], in the limit of short bursts and low energy densities. The position of the singularity in (8) exactly coincides with our result (6)!

This could be a coincidence, of course, since the infinite R limit of our calculation is beyond the range of validity of our approximations. However, the occurrence of the singularity is normally attributed to a focusing effect [8] exerted by gravitational plane waves on null cones, a mechanism very similar to the one we see at work in our beam-beam collision. Furthermore, the energy density in B_1 at a distance d after the scattering plane was found in ref. [2] to behave as:

$$\rho_1(d) = \rho_1(1-d/f_1)^{-2} \quad (9)$$

again in complete agreement with our simple picture's prediction.

We now examine the possible corrections to our eq. (4), which, by itself, would imply the accumulation of a finite amount of energy at the points f_1, f_2 and thus singularities. We can think of various types of corrections:

i) Classical corrections to the deflection formula (2). The results of refs. [4-5] indicate that these corrections are of $O(\theta^3)$. It is easy to see that in such a case they imply a spread in the position of f which is of $O(r_s)$ longitudinally and of $O(r_s^2/R)$ transversally.

ii) Classical reinteractions after the deflection. These corrections become strong when the deflected gravitons get closer to one another than the Schwarzschild radius of their total centre-of-mass energy, ER/f . One finds that the spread just discussed makes the reinteractions small as long as the gravitons do not get at a distance from the focus of the order of the spread itself. What happens afterwards is not completely clear, but it

looks very likely that a gravitational collapse would follow, leading presumably to a black hole. Singularities would remain shielded behind the hole's horizon.

iii) Corrections due to density inhomogeneity of the "target beam" (the focused beam does not need to be homogeneous). These can be found to be under control if $\Delta\rho/\rho < O(R^2/f^2)$. The same is true for corrections due to an imperfect axial symmetry of the beams. Coincidence of the two beam axes, on the other hand, is inessential.

In summary, our results seem to imply that finite front gravitational waves, if sufficiently symmetric and homogeneous, can act as very effective lenses and lead to gravitationally collapsed objects even though, probably, they are unable to generate the naked singularities of the infinite wave collisions. The phenomenon could still be of astrophysical significance, since a typical gravitational wave is expected to collide many times with similar waves before reaching us.

The most interesting question at this point is the one of finite-string-size corrections. Do they alter the picture in our case? Are they capable of eliminating the singularity in the infinite front wave situation?

For graviton-graviton scattering string corrections are under study [5]. They seem to intervene in our process at two different stages:

i) They prevent elastic scattering from occurring with centre-of-mass angles larger than $O(L_p/\sqrt{\alpha'}) = O(g)$, where g is the gauge coupling. At larger angles, elastic scattering is replaced by diffractive excitation [5] and the incident wave is absorbed. Since in our problem we can go to a limit in which the deflections are as small as we like, this effect does not seem to prevent a large energy density from occurring at f_1 .

ii) They modify presumably the final outcome of the focusing process. Indeed, as we said, after going through B_2 , the gravitons in B_1 collide with each other at impact parameters of order of the Schwarzschild radius corresponding to the centre-of-mass energy. If we take this energy to be as small as $O(M_{\text{Planck}}/g)$, the impact parameter becomes $O(\sqrt{\alpha'})$ and string effects presumably intervene to prevent formation of a black hole of an even smaller radius. A similar mechanism should smear out the usual classical singularity of the infinite wave collision over distances $O(\sqrt{\alpha'})$.

Much work is clearly still needed before being able to know if string theory provides the long-sought mechanism by which quantum corrections eliminate the singularities of classical general relativity.

I am grateful to D. Amati, M. Ciafaloni, C. Gomez, N. Sanchez, P. Szekeres and K. Zalewski for discussions. Most of this work was done while I was visiting the Institute of Theoretical Physics, Beijing, China and the Physics Department of the University of Adelaide, Australia. I wish to thank both institutions for the warm hospitality.

REFERENCES

1. P. Szekeres, Nature (London) 228 (1970) 1183;
K. Khan and R. Penrose, *ibid.* 229 (1971) 185.
2. P. Szekeres, J. Math. Phys. 13 (1972) 286.
3. Y. Nutku and M. Halil, Phys. Rev. Lett. 39 (1977) 1379;
S. Chandrasekhar and V. Ferrari, Proc. R. Soc. Lond. A396 (1984) 55.
4. D. Amati, M. Ciafaloni and G. Veneziano, CERN preprint TH. 4782 (1987).
5. D. Amati, M. Ciafaloni and G. Veneziano, in preparation.
6. P.C. Aichelburg and R.U. Sexl, Gen. Rel. Grav. 2 (1971) 303.
7. G. 't Hooft, Utrecht preprint (1987).
8. R. Penrose, Rev. Mod. Phys., 37 (1965) 215.

FIGURE CAPTION

Collision of two graviton beams neglecting string corrections. The shaded region of B_2 is what effectively scatters the graviton g_1 of B_1 focusing it at a distance f_1 from the interaction plane.