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GRAVITATIONAL SUPERVERTICES IN $N = 1$, 4D-SUPERSTRINGS

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A B S T R A C T

The gravitational supermultiplet for target space-time supersymmetry of four-dimensional heterotic strings is obtained. By an explicit construction of supervertex operators it is shown that the underlying superspace geometry corresponds to the "new-minimal" formulation of $N = 1$, 4D-supergravity. The relation between R-symmetry in target space and the $U(1)$ symmetry of the world-sheet $N = 2$ superconformal algebra is outlined.

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Extensive investigation over the last few years of the point-field limit of superstrings has raised the question which is the appropriate supergravity (local supersymmetry) formulation of superstring dynamics.

In his pioneering work Witten^[1] has shown that the point-field limit of a 10D-heterotic string, compactified on a suitable Calabi-Yau manifold, can be approximated by a standard 4D, $N=1$ supergravity lagrangian^[2] describing the effective interaction of the massless supermultiplets. This is true if at least higher order (in α') string corrections are neglected. On the other hand it is known that different off-shell formulations^[3] of supergravity couplings exist which turn out to be equivalent⁽⁴⁾, provided one neglects high derivative interactions.

Because such higher order interactions intrinsically appear in string theories⁽⁵⁾, different off-shell formulations of $N=1$, 4D-supergravity may not be equivalent at the string level.

Indeed, different off-shell formulations of 4D-supergravity are related by Legendre transformations, which become highly non-local when higher derivative interactions are present. As a consequence it is of paramount importance to understand which is the appropriate supergravity formulation of the dynamics of 4D-superstrings.

This issue is related to a similar problem recently investigated by some authors⁽⁶⁾ dealing with the construction of vertex operators for matter F and D auxiliary fields.

It is well known that in space-time supersymmetry particle fields belong to supermultiplets and the latter contain some components which are called "auxiliary fields". For chiral and vector matter multiplets these auxiliary fields have been recently studied in the framework of superstring theory and their vertex operators constructed.

This is not surprising since auxiliary fields are related, by supersymmetry transformations, to physical fields, and the supersymmetry algebra acts linearly on the on-shell vertex operators.

The same strategy can be used for the gravitational multiplet as well and this is the scope of the present paper.

The explicit construction of the gravitational vertex supermultiplet (supervertex) will tell us which is the appropriate

supergravity formulation for superstrings. This is a novel feature with respect to matter multiplets because, as we have already mentioned, the choice of the gravitational multiplet is somewhat ambiguous in supergravity theory.

For instance, if we confine ourselves to a "minimal set" of auxiliary fields⁽³⁾, two different formulations, the old and new minimal, exist.

These formulations have been shown to be inequivalent when higher derivative couplings are introduced⁽⁷⁾.

In the present article we will show that the appropriate gravitational multiplet for N=1, 4D-heterotic superstrings corresponds to the "new minimal set" of auxiliary fields. In this formulation a fundamental role is played by an R-symmetry of the theory. The local U(1) R-symmetry in target space is shown to be deeply related to the U(1) world-sheet symmetry of the N=2 two-dimensional superconformal algebra⁽⁸⁾⁽⁹⁾.

It is because of this microscopic property of the two-dimensional world that the gravitational vertex operator, actually related to the so called Einstein supermultiplet, will be shown to satisfy the Bianchi identities of the superspace geometry corresponding to the new-minimal formulation⁽³⁾. Also the dilaton multiplet will be shown to belong to a "linear multiplet"⁽¹⁰⁾ rather than to a chiral multiplet. Note that in the effective theory, when higher derivative couplings are neglected, these multiplets are equivalent, since they can transform into each other through a duality transformation⁽¹⁾.

VERTICES AND SUPERSPACE BIANCHI IDENTITIES

As discussed in Ref.(11) the SUSY algebra

$$\{Q_\alpha, Q_{\dot{\beta}}\} = 2i(\gamma^\mu)_{\alpha\dot{\beta}} P_\mu$$

$$\{Q_\alpha, Q_\beta\} = [P_\mu, Q_\alpha] = [P_\mu, P_\nu] = 0 \quad (1)$$

is linearly realized on the on-shell vertex operators of a superstring. At first sight, one could think that these vertices belong to on-shell SUSY representations. This is not so, however, since the vertices are related to the corresponding fields by the truncation of the external propagator. Thus they should transform as the corresponding equations of motion. This SUSY representation is dual (or contragradient) to the one for the corresponding fields.

In 4D, for a superstring with $N=1$ space-time supersymmetry, the above argument implies that the vertices should be organized in full superfields, since the equations of motion can be written as superfied equations. More precisely, the vertices should organize themselves into full superfields, up to terms which give a vanishing contribution when inserted in a physical on-shell amplitude*. In the following all our equations are valid up to terms of this type.

The condition for the vertices to belong to a given SUSY representation is then expressed as a set of superspace Bianchi identities for the vertex supermultiplet (SUPERVERTEX in the following).

These super-Bianchi identities contain the usual Ward identities as a subset. This is natural since they just express the invariance of the physical S-matrix not only with respect to the usual bosonic gauge (resp. diffeomorphism) transformations but also under target superspace gauge (resp. diffeomorphism) transformations, where now the gauge parameter is promoted to a full (target) superfield. These super-Ward identities (i.e. the constraints defining the representation) are valid in exactly the same sense of the usual Ward identities, i.e. when all the vertices are put on-shell. For instance, the auxiliary vertex $V_D(z,k)$ of Ref. (6) is the first component of a real-linear multiplet $L^a(z,k)$ **

$$L^a(z,k) = (V_D^a(z,k), V_\alpha^a(z,k), V_\mu^a(z,k)) . \quad (2)$$

* And up to some picture-changing operation⁽¹¹⁾, which can be shown to be safe.

** Here V_α is the gaugino vertex, V_μ the gauge-vector vertex. All components of L^a are supergauge covariant.

The algebraic constraint telling that L^a is a real linear multiplet ($D^\alpha D_\alpha L^a = 0$) implies for its vector component $V_\mu(z, k)$ (which is the gauge vertex) the Ward identity

$$k^\mu V_\mu^a(z, k) \sim 0 \quad (3)$$

(when inserted in a on-shell physical amplitude)

i.e. the usual Ward identity expressing gauge-invariance of S-matrix elements in the Wess-Zumino gauge.

The simplest way to construct the auxiliary vertices needed to complete the gravitational supervertex, is to start from the Bianchi identities they should obey, much as Eq. (2) can be deduced from Eq. (3).

In the gravitational case, however, there is an additional difficulty since the super-Bianchi identities are not uniquely fixed by the request of SUSY invariance. Different possibilities exist, which correspond to the different auxiliary field formulations⁽³⁾.

It turns out that the superspace geometry chosen by the heterotic string corresponds to the so-called "new minimal" formulation⁽³⁾. The reason why it should be so is deeply rooted in the 2d superconformal algebra. It has been shown⁽⁹⁾ that, in order to have space-time N=1 supersymmetry, we need an N=2 superconformal symmetry⁽⁹⁾ at the world-sheet level. The

corresponding U(1) superconformal current⁽⁹⁾

$$J(z) \quad (4)$$

can be used to construct a generator

$$R = \frac{2}{3} \oint J(z) \quad (5a)$$

which when acting on physical vertices generates an R-symmetry transformation. It is natural to think that this R-symmetry is in fact a local symmetry since in the theory there exists a vertex with the correct structure to correspond to a connection field for the R-symmetry

$$C_{(0)}^\mu(z, k) = \frac{2}{3} J(z) \bar{\partial} X^\mu(\bar{z}) e^{i k \cdot X(z, \bar{z})} \quad (5b)$$

Now, in the new-minimal formulation, we have exactly one auxiliary field A_μ , which is the connection for the local R-symmetry. So it is tempting to identify Eq. (5b) with the vertex for the auxiliary field A_μ of the new-minimal formulation. This will be explicitly proved below using the Bianchi identities.

Notice that the vertex in Eq. (5b) is not BRS-invariant. This is correct since it does not correspond to a physical propagating particle but to an auxiliary field. However, it can be shown that this BRS-variance does not lead to any ambiguity for reasonable amplitudes, in particular, the picture-rearrangement can be used safely. Moreover, there exists an explicitly BRS-invariant approach, which will be briefly discussed at the end of the paper.

THE EINSTEIN SUPERVERTEX

In the new-minimal formulation the gravitational equations have the same SUSY transformation properties of the Einstein multiplet E^μ . From the gravitational Bianchi identities we learn that E^μ satisfies the following two conditions

$$\mathcal{D}_\mu E^\mu = 0 \quad (6)$$

$$\bar{\mathcal{D}}^{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\alpha}} E^\mu = \mathcal{D}_\alpha \mathcal{D}^\alpha E^\mu = 0 \quad (7)$$

as well as other identities relating its derivatives to those of the chiral gravitational curvatures $W_{\alpha\beta\gamma}$ and W_α . In particular the scalar curvature multiplet W_α is given by

$$\mathcal{D}^\alpha E_{\alpha\dot{\alpha}} = -2 \bar{W}_{\dot{\alpha}} \quad (8)$$

Eqs. (6) and (7) state that the Einstein multiplet E^μ transforms according to the SUSY representation which would be described as "a transverse Lorentz-vector real-linear superfield".

E^μ contains in its θ -component the equation of motion for the gravitino, and in the $\theta\sigma^\mu\theta$ -component the equation of motion for the graviton and the 2-form $b_{\mu\nu}$.

Now we shall show that the supervertex which is defined by the first component $C^\mu_{(0)}(z,k)$ - Eq. (5b) - belongs to the same

representation as the Einstein multiplet E^μ (in particular, it satisfies Eqs. (6) and (7)); also it contains the usual vertices for the gravitational fields in its components. This supervertex will be denoted by $E^\mu_{(0)}$ and called the Einstein supervertex. Its components are denoted as

$$E^\mu_{(0)} = (C^\mu_{(0)}(z, k), Z^\mu_{(-1/2)}(z, k), H^\mu_{(-1)}(z, k), \\ K^\mu_{(-1)}(z, k), B^\mu_{(-1)\nu}(z, k), \Lambda^\mu_{(-1/2)\alpha}(z, k), D^\mu_{(0)}(z, k)) \quad (9)$$

(The small numbers in brackets denote our picture-conventions for the various components; their precise value is just a matter of convenience).

The Bianchi identity in Eq. (6) requires that for all components of $E^\mu_{(0)}$,

$$[P_{(0)\mu}, E^\mu_{(0)}] = K_\mu E^\mu_{(0)} \sim 0 \quad (10)$$

where, as always, $\sim$ means equality up to terms which do not contribute to on-shell amplitudes. The second Bianchi identity, Eq. (7), is then equivalent to the following equations for the components in Eq. (9)

$$H^\mu_{(-1)} = K^\mu_{(-1)} = 0 ; \quad K^\nu B^\mu_{(-1)\nu} \sim 0 \quad (11) \\ \Lambda^\mu_{(-1/2)\alpha} \sim -\not{\partial} Z^\mu_{(-1/2)\alpha} ; \quad D^\mu_{(0)} \sim -\square C^\mu_{(0)} .$$

Notice that Eqs. (10), (11) contain all the standard Ward-identities of the gravitational sector.

Now we shall prove Eqs. (10), (11). Eq. (10) is very easy to prove; for instance, for the first component*

$$K_\mu C^\mu_{(0)}(z, k) = \frac{\partial}{\partial \bar{z}} \left[\frac{2}{3} J(z) e^{ik \cdot X} \right] \sim 0 \quad (12)$$

* Notice that Eq. (12) can be interpreted as the Ward identity ensuring Local R-invariance of the on-shell amplitudes.

being a world-sheet total derivative which vanishes upon integration over the compact Riemann surface.

The same argument proves Eq. (10) for all other components, since the SUSY transformation acts only on the right (holomorphic) part of the vertex, so that all vertices in $E^\mu_{(0)}$ have the structure,

$$E^\mu_{(0)} = (\text{holomorphic fields}) \bar{\partial} x^\mu e^{ik \cdot X} \quad (13)$$

so that

$$[\partial^\mu E_\mu]_{(0)} \sim \frac{\partial}{\partial \bar{z}} [(\text{holomorphic}) e^{ik \cdot X}] \sim 0. \quad (14)$$

Before proving Eqs. (11) by an explicit construction of all the vertices in Eq. (9), we recall some well-known facts about the supersymmetry charges in a 4D-heterotic string with an unbroken $N=1$ SUSY.

As we have seen above space-time $N=1$ supersymmetry implies the existence of a superconformal $U(1)$ 2d-current $J(z)$, which can be bosonized in terms of a free scalar $H(z)$ ^(9,6)

$$J(z) = i\sqrt{3} \partial H(z) \quad (15)$$

$$H(z)H(0) \sim -\ln z$$

Using this free field $H(z)$, we can define two "killing spinors" $S^{\pm(9,6)}$

$$S^\pm(z) = e^{\pm i \frac{\sqrt{3}}{2} H(z)} \quad (16)$$

which are spin-fields of conformal dimension $h=3/8$ and conformal $U(1)$ charge $e=\pm 3/2$.

In the $(-1/2)$ -picture the 4D supersymmetry generators (after the GOS projection) are defined as^(9,6)

$$Q_{(-1/2)\alpha}(z) = e^{-\phi(z)/2} S^+(z) S_\alpha(z) \quad (17a)$$

$$Q_{(-1/2)\dot{\alpha}}(\bar{z}) = e^{-\phi(\bar{z})/2} S^{-}(\bar{z}) S_{\dot{\alpha}}(\bar{z}) \quad (17b)$$

where $S_{\alpha}(z)$ (resp. $S_{\dot{\alpha}}(z)$) is a chiral (resp. antichiral) 4D spin-field.

We recall also some useful OPEs needed below, (9,11)

$$T(z) Q_{(-1/2)\alpha}(0) \sim \frac{1}{z} \frac{3}{2} Q_{(-1/2)\alpha}(0) \quad (18a)$$

$$T(z) Q_{(-1/2)\dot{\alpha}}(0) \sim -\frac{1}{z} \frac{3}{2} Q_{(-1/2)\dot{\alpha}}(0) \quad (18b)$$

$$Q_{(-1/2)\alpha}(\bar{z}) Q_{(-1/2)\beta}(0) = \text{FINITE} \quad (18c)$$

$$Q_{(-1/2)\dot{\alpha}}(\bar{z}) Q_{(-1/2)\beta}(0) \sim \frac{1}{z} e^{-\phi} \psi^{\mu} (\gamma^{\mu})_{\dot{\alpha}\beta} \quad (18d)$$

$$(\psi^{\mu} \psi^{\nu})(\bar{z}) Q_{(-1/2)\dot{\alpha}}(0) \sim -\frac{1}{z} \frac{1}{2} (\gamma^{\mu\nu})_{\dot{\alpha}}{}^{\beta} Q_{(-1/2)\beta}(0) \quad (18e)$$

As a piece of notation, we define for any 2d conformal field $A(z)$ its SUSY variation $\delta_{(-1/2)\alpha} A$ and its q-picture derivative $\partial^{\mu}_{(q)} A$ by the equations

$$i Q_{(-1/2)\alpha}(\bar{z}) A(0) \sim \frac{1}{z} \delta_{(-1/2)\alpha} A(0) \quad (19a)$$

$$i P^{\mu}_{(q)}(\bar{z}) A(0) \sim \frac{1}{z} \partial^{\mu}_{(q)} A(0) \quad (19b)$$

where $P^{\mu}_{(0)} = \partial x^{\mu}$, $P^{\mu}_{(-1)} = e^{-\phi} \psi^{\mu}$.

The second component $Z^{\mu}_{(-1/2)}(z, k)$ of the Einstein supervertex is defined by the equation

$$\delta_{(-1/2)\alpha} C^{\mu}_{(0)}(z, k) = i Z^{\mu}_{(-1/2)\alpha}(z, k) \quad (20)$$

Comparing Eqs. (5b), (18a) and (19a) we get

$$Z^{\mu}_{(-1/2)\alpha}(z, k) = Q_{(-1/2)\alpha}(\bar{z}) \bar{\partial} X^{\mu}(\bar{z}) e^{i k \cdot X} \quad (21)$$

i.e. the gravitino vertex (in the $(-1/2)$ -picture) as advertized. Notice that for this component, Eqs. (10) and (14) correspond to the usual Ward identity of local supersymmetry.

Next, the two vertices $H_{(-1)}^\mu(z,k)$ and $K_{(-1)}^\mu(z,k)$ are defined by

$$\begin{aligned} \delta_{(-1/2)\alpha} \bar{Z}_{(-1/2)\beta}^\mu(z,k) &= \\ &= -k_{(-1)}^\mu(z,k) \epsilon_{\alpha\beta} + i H_{(-1)}^\mu(z,k) \epsilon_{\alpha\beta} \end{aligned} \quad (22)$$

But from Eqs. (21) and (18c) it follows that the L.H.S. of Eq. (22) just vanishes. Then

$$H_{(-1)}^\mu(z,k) = K_{(-1)}^\mu(z,k) = 0 \quad (23)$$

which is just the first of Eq. (11). It is well-known that Eq. (23) implies (by supersymmetry) all the other conditions in Eq. (11); (however, we shall check them directly, below). Thus we have proved that the Einstein supervertex $E_{(0)}^\mu$, defined by the first component $C_{(0)}^\mu(z,k)$ of Eq. (6), satisfies all the Bianchi identities, Eq. (6) and Eq. (7) of the new-minimal formulation.

The vector component $B_{(-1)\nu}^\mu(z,k)$ is now defined by

$$\begin{aligned} -i \delta_{(-1/2)\alpha} \bar{Z}_{(-1/2)\beta}^\mu(z,k) &= \\ &= -B_{(-1)\nu}^\mu(z,k) (\gamma^\nu)_{\dot{\alpha}\beta} - i (\gamma^\nu)_{\dot{\alpha}\beta} [\partial_\nu C^\mu]_{(-1)} \end{aligned} \quad (24)$$

(The last term in the R.H.S. of Eq. (24) can be safely ignored since, $P_{(-1)}^\nu(z)C_{(0)}^\mu(z,k) \approx \text{FINITE}$).

Then, from Eq. (18d) we get

$$\begin{aligned} B_{(-1)\nu}^\mu(z,k) &= \\ &= e^{-\phi(z)} \psi_\nu(z) \bar{\partial} X^\mu(\bar{z}) e^{i k \cdot X} \end{aligned} \quad (25)$$

As is well known, the symmetric part $B^{(\mu\nu)}_{(-1)}(z,k)$ is the vertex of the graviton (in the (-1) -picture) and the antisymmetric part $B^{[\mu\nu]}_{(-1)}(z,k)$ is the vertex for the antisymmetric field $b^{\mu\nu}$ in the same picture. Now we see how the gravitational Ward identities follow from the SUSY representation constraints: we already know that the vertex in Eq. (25) is conserved with respect to the index μ . Eq. (11) says also that the conservation with

respect to the index ν follows from the linear character of $E^\mu_{(0)}$, that is from Eq. (23). This is most easily seen by changing the picture

$$\begin{aligned} [Q_{BRS}, 2 \sum_{\nu} i k^\nu B^\mu_{(-1)\nu}(z, k)] &\sim \\ &\sim \frac{\partial}{\partial z} (\bar{\partial} X^\mu e^{i k \cdot X}) \sim 0 \end{aligned} \quad (26)$$

A simple computation shows that

$$\begin{aligned} \partial^\nu_{(-1)} Z^\mu_{(-1/2)\dot{\beta}}(z, k) &= \\ &= \frac{1}{\sqrt{2}} (\gamma^\nu)_{\dot{\beta}}{}^\gamma (e^{-\frac{3}{2}\phi} s^+ s_\gamma) \bar{\partial} X^\mu e^{i k \cdot X} \end{aligned} \quad (27)$$

Extracting as above $\Lambda^\mu_{(-3/2)\alpha}(z, k)$ from the variation $\delta_{(-1/2)\alpha} B^\mu_{(-1)\nu}(z, k)$, it is easy to see that

$$\Lambda^\mu_{(-3/2)}(z, k) = -(\gamma_\nu)_\alpha{}^{\dot{\beta}} \partial^\nu_{(-1)} Z^\mu_{(-1/2)\dot{\beta}}(z, k) \quad (28)$$

with the derivative in the R.H.S. as in Eq. (27).

Returning to our conventions for the pictures (see Eq. (9)), Eq. (28) becomes

$$\Lambda^\mu_{(-1/2)\alpha}(z, k) = -(\gamma_\nu)_\alpha{}^{\dot{\beta}} \partial^\nu_{(-1)} Z^\mu_{(-1/2)\dot{\beta}}(z, k) \quad (29)$$

as required by Eq. (11). Of course, for an on-shell amplitude the R.H.S. of Eq. (29) is just equivalent to zero.

Finally, computing $D^\mu_{(0)}(z, k)$ from the variation of Eq. (29), we obtain

$$D^\mu_{(0)}(z, k) = -[Q, C^\mu]_{(0)} \sim k^2 C^\mu_{(0)}(z, k) \sim 0 \quad (30)$$

(compare with the comment after Eq. (24)).

This concludes the construction of the Einstein supervertex $E^\mu_{(0)}$. We have seen that it contains all the physical vertices for the gravitational fields, plus a vertex for the auxiliary field A_μ . Since all these fields are gauge fields we have conservation of all

the components of $E^\mu_{(0)}$. $E^\mu_{(0)}$ satisfies all the conditions, Eqs. (6) and (7), of the new-minimal formulation.

$E^\mu_{(0)}$ is not the only supervertex containing the physical gravitational multiplets. We comment briefly on this point below. Full details will be present elsewhere.

THE DILATON SUPERVERTEX

There is another basic supermultiplet to be discussed, the one containing the dilaton, dilatino and axion vertices. Given the physical role of the corresponding particles, it is natural to identify this supervertex with the new-minimal supercurvature W_α defined by Eq. (8). W_α satisfies two Bianchi identities by itself

$$\bar{D}_{\dot{\alpha}} W_\alpha = 0 \quad (31)$$

$$D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \quad (32)$$

Eq. (31) tells that W_α is a chiral-spinor, whereas Eq. (32) tells that W_α is the field-strength supermultiplet of some Abelian gauge vector multiplet*.

We denote the components of the dilaton supervertex as

$$W_{(-1/2)\alpha} = (A_{(-1/2)\alpha}(z, \kappa), \chi_{(-1)\beta\alpha}(z, \kappa), F_{(-1/2)\alpha}(z, \kappa)) \quad (33)$$

The first component of $W_{(-1/2)\alpha}$ is defined by

$$\begin{aligned} W_{(-1/2)\alpha}(z, \kappa) \Big|_{\text{FIRST COMPONENT}} &= \\ &= \frac{1}{2} \left[\delta_{(-1/2)\dot{\alpha}} E^{\alpha\dot{\alpha}}_{(0)}(z, \kappa) \right] \Big|_{\text{FIRST COMPONENT}} . \end{aligned} \quad (34)$$

* This is a good illustration of the dual representation phenomenon. The supervertex of a gauge vector is a linear multiplet⁽⁹⁾, see Eq. (2). Conversely, the supervertex of the dilaton multiplet (which is a linear multiplet) is the field strength of a gauge multiplet⁽¹⁰⁾.

A simple computation gives

$$\begin{aligned}
A_{(-1/2)\alpha}(z, k) &= -\frac{1}{2}(\gamma_\mu)_\alpha{}^{\dot{\alpha}} \bar{z}^\mu_{(-1/2)\dot{\alpha}}(z, k) \\
&= -\frac{1}{2}(\gamma_\mu)_\alpha{}^{\dot{\alpha}} Q_{(-1/2)\dot{\alpha}}(z) \bar{\partial} X^\mu e^{ik \cdot X}
\end{aligned} \tag{35a}$$

$$\begin{aligned}
\chi_{(-1)\beta\alpha}(z, k) &= \\
&= \frac{1}{2} \epsilon_{\beta\alpha} B_{(-1)\mu}^\mu(z, k) + (\sigma_{\mu\nu})_{\beta\alpha} B_{(-1)}^{[\mu\nu]}(z, k)
\end{aligned} \tag{35b}$$

$$\begin{aligned}
F_{(-1/2)\alpha}(z, k) &= \\
&= \frac{1}{2}(\gamma_\mu)_\alpha{}^{\dot{\beta}} \Lambda_{(-1/2)\dot{\beta}}^\mu(z, k) \sim [\not{\partial} A_{(-1/2)}] \sim 0.
\end{aligned} \tag{35c}$$

The Bianchi identity (see Eq. (31)) is equivalent to

$$\delta_{(-1/2)\dot{\alpha}} A_{(-1/2)\alpha}(z, k) \sim 0 \tag{36}$$

which is obvious in view of the Eqs. (35a), (22) and (23). The identity Eq. (32) is also trivial, since it states that $B_{(-1)\mu}^\mu(z, k)$ (the dilaton vertex) is real, as it is manifest in the 0-picture. then the expression $F_{(-1/2)\alpha} \approx \partial A_{(-1/2)\alpha}$ (Eq. (35c)) is a consequence of this Bianchi identity. In the language of vector multiplets, $A_{(-1/2)\alpha}$ is the λ_α -component, $B_{(-1)\mu}^\mu$ is the D component and $\hat{B}^{[\mu\nu]}_{(-1)}$ is the field strength $F_{\mu\nu}$ of the vector component.

(The Abelian Bianchi identities are then a consequence of the identities Eqs. (14) and (26)).

Comparing with the known expressions for the physical vertices, and Eq. (25), we see that the λ -component of $W_{(-1/2)\alpha}$ is the dilatino vertex, the D-term is the dilaton vertex, and the dual of the field strength is the axion (i.e. 2-form) vertex. This

supervertex has no auxiliary component (as it is natural for the supervertex of a linear multiplet, as the dilaton multiplet is).

Thus, we have that also the dilaton vertices organize in a full supermultiplet. Moreover this multiplet gets unified, in the new-minimal formulation of superspace geometry, with the gravitational curvatures, as expected on physical grounds.

OTHER SUPERVERTICES

Besides the supervertex discussed above, and those of Refs. (6), there are other supervertices. For instance, for the gauge-vector sector, besides the linear one L^a of Eq. (2) we have the "magnetic" supervertex $G^{\mu\nu AB}_{(0)}(z,k)$ whose first component is

$$G^{\mu\nu AB}_{(0)}(z,k) = \widetilde{\psi^\mu \psi^\nu}(z) \lambda^A \lambda^B(\bar{z}) e^{ik \cdot X} . \quad (37)$$

Analogously, in the gravitational sector, we have the "torsion" supervertex, $T^{\mu\nu\rho}_{(0)}(z,k)$, whose first component is

$$T^{\mu\nu\rho}_{(0)}(z,k) = \widetilde{\psi^\mu \psi^\nu}(z) \bar{\partial} X^\rho(\bar{z}) e^{ik \cdot X} . \quad (38)$$

Of course, these additional supervertices correspond to the same superspace geometry (the new-minimal formulation) as before. They just give a different parametrization of the same basic geometrical objects. The explanation of their precise role from the point of view of SUSY representation theory is a bit technical and will be discussed elsewhere.

BRS (NON-)INVARIANCE

Since our (0)-picture auxiliary vertices are not BRS invariant (otherwise they would correspond to physically propagating particles), one can wonder the legitimacy of some formal manipulation, and more generally, the uniqueness and non ambiguity of the results for on-shell amplitudes with auxiliary vertex insertions.

One can easily show - by a simple modification of the standard argument⁽¹¹⁾ - that all the usual manipulations are safe,

when used in reasonable amplitudes (in particular when all vertices are physical except an auxiliary one).

Anyhow, it is also possible to work directly in a scheme in which all the auxiliary vertices are BRS invariant. (But this scheme is not suited for practical calculations). Indeed, since the BRS-charge is nilpotent $Q_{\text{BRS}}^2 = 0$, for any operator A the expression

$$A' = [Q_{\text{BRS}}, A]_{\pm} \quad (39)$$

is BRS-invariant. In particular, it is BRS-invariant the auxiliary vertex of the (+1)-picture

$$V_{(+1)}^{\text{AUX}}(z, k) = [Q_{\text{BRS}}, 2 \int V_{(0)}^{\text{AUX}}(z, k)] \quad (40)$$

(here $V_{(0)}^{\text{AUX}}$ denotes any 0-picture auxiliary vertex).

The only thing we have to show, is that $V_{(+1)}^{\text{AUX}}(z, k)$ is indeed the auxiliary vertex for the supervertex in the (+1)-picture. This amounts to show that its $\delta_{(-1/2)}$ -variation is the physical fermion vertex in the (+1/2)-picture. But, since $Q_{(-1/2)\alpha}(z)$ is BRS-invariant, we have

$$\begin{aligned} \delta_{(-1/2)\alpha} V_{(+1)}^{\text{AUX}}(z, k) &= \oint_{C_z} \frac{d\zeta}{2\pi i} Q_{(-1/2)\alpha}(\zeta) V_{(+1)}^{\text{AUX}}(z, k) \\ &= \oint_{C_z} \frac{d\zeta}{2\pi i} Q_{(-1/2)\alpha}(\zeta) [Q_{\text{BRS}}, 2 \int V_{(0)}^{\text{AUX}}(z, k)] = \\ &= \oint_{C_z} \frac{d\zeta}{2\pi i} [Q_{\text{BRS}}, 2 Q_{(-1/2)\alpha}(\zeta) \int V_{(0)}^{\text{AUX}}(z, k)] = \\ &= Q_{(+1/2)\alpha}(z, k). \end{aligned} \quad (41)$$

So in the (+1)-picture all components are BRS-invariant. This explains why the auxiliary amplitudes are well-defined.

CONCLUSION

In the present paper we have shown that the superspace geometry underlying the dynamics of $N=1$, 4D-heterotic superstrings corresponds to the "new-minimal" formulation of space-time supergravity.

This result has immediate applications both in the point-field limit and in the full fledged string dynamics.

For example effective lagrangians and the structure of higher derivative interactions are much more constrained in the "new-minimal" formulation. In particular one can study the modification of the scalar potential due to higher order (in α') string corrections. At the string level the knowledge of the gravitational auxiliary fields may provide an efficient tool for the proof of non-renormalization theorems in string theory.

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