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INTRODUCTION TO THE NEXT GENERATION OF LINEAR COLLIDERS

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1. INTRODUCTION

Since 1965 electron-positron collisions have been producing a continuous stream of first rate physics results. To continue on the same trail it is now necessary to push the centre-of-mass energy to values larger than the $W \approx 200$ GeV achievable with LEP 200, which should be running in 1993-1994. In 1975 it was shown by B. Richter [1] that the cost of e^+e^- storage rings increases as W^2 and it was quite natural to speculate [2] on the advantages of firing, one against the other, electron and positron bunches accelerated to hundreds of GeV by opposite linacs, since in this case there is no radiation and the cost is obviously proportional to W . A few months after the proposal, which prompted a lot of speculations, I was informed that ten years before M. Tigner had suggested a similar approach to produce electron-electron collisions at few GeV [3]; this proposal had been forgotten, since at few GeV's synchrotron radiation was not yet a problem and storage rings work beautifully. During these ten years, related ideas had been floating around but not noticed, so that in preparing a review talk for the 1979 Lepton-Photon Conference, I went through the literature and collected all possible precedents. The interested reader can find them described in Ref. [4].

The crossing over point between the cost of circular and linear e^+e^- colliders depends on many assumptions, but everybody agrees that it is around $W \approx 300$ GeV. This is shown for copper cavity and superconducting cavity linacs in Fig. 1, which is taken from Ref. [4].

Fig. 1 This figure, taken from Ref. [4], compares the cost of electron-positron storage rings with the cost of colliding linacs. The comparison is made both for accelerators based on conventional room temperature cavities (a) and those based on superconducting cavities (b).

For the development of linear colliders the two ICFA workshops, held in 1978 and 1979, were very seminal [5,6]. After the first one, B. Richter and his SLAC collaborators started to work on the SLAC Linear Collider, which is now being commissioned [7]. In the second ICFA workshop, various aspects of the physics of e^+e^- colliders were elucidated and the first $(20 + 20)$ TeV proton-antiproton collider was studied. While waiting to learn from the precious experience which will come from the running-in of SLC, we are now witnessing the flourishing of studies of e^+e^- colliders in the range $0.5 \leq W \leq 2$ TeV. (B. Richter has called this machine the NLC, Next Linear Collider.) CERN, SLAC, Novosibirsk and KEK are involved in these studies. The aim is to have a proposal ready in 2 to 5 years from now. The wild speculations of the first years have thus been replaced by more conservative discussions of the possible acceleration techniques and of the choice of the many parameters. The present review aims at guiding the newcomer through the main issues which are involved in this choice for the 'near future' linear accelerators. As examples I shall use the CERN Linear Collider (CLIC) concept at $(1+1)$ TeV and the SLAC work on a $\sim (0.5+0.5)$ TeV linear collider (to be called SC, for SLAC Collider). For lack of time, I leave aside the acceleration mechanism and the techniques for producing high frequency RF power. These subjects have been reviewed at the Topical Course by R. Palmer and W. Schnell.

2. PHYSICS ISSUES AND LUMINOSITY REQUIREMENTS

2.1 *Beyond the standard model: the questions*

There are very special reasons for which physicists want to reach nowadays about 1 TeV in the electron and positron centre of mass. Before reviewing them, following the lines of Refs. 8 and 9, we must start by stressing that the present theory of all subnuclear phenomena, the so-called *Standard Model*, gives a rationale to the existence of the weak, electromagnetic, and strong forces, but offers *no* explanation for the fact that we have observed only three families of fermions, the matter-particles which act as sources of the known force-particles: the photon (γ), the asthenons (W and Z), and the gluons (g). (As done in Ref. [9] I shall use the term 'asthenon' to indicate a weak intermediate vector boson with a *single* word, which sounds more or less as 'photon' and 'gluon'. I am convinced that, five years after the W and Z discovery, such a symmetry in nomenclature is needed; perhaps a better neologism could be found).

A first very natural series of questions that physicists ask themselves is thus:

Questions 1: Do heavier (sequential) leptons and quarks exist?

What about heavier asthenons both charged (W') and neutral (Z')?

Present data indicate indirectly that the masses of these sequential leptons and quarks cannot be larger than ~ 0.25 TeV.

Whatever the answer to these questions is, there is a fundamental problem related to the force-sector. The Standard Model of the electroweak interaction is very successful in explaining a host of data and, at the same time, very unsatisfactory. This is true not only because it contains about twenty free parameters, but also because it needs an *ad hoc* mechanism, not yet confirmed experimentally, to justify the rest energies of the asthenons. At the fundamental level these bosons are very similar to photons. A photon has zero rest mass and thus, once emitted by a particle in a virtual process, can fly far away from its source, giving rise to a force of infinite range (Fig. 2a). Why is the range R of the weak force $\sim 10^{-16}$ cm instead (Fig. 2b and 2c), i.e. why are the masses of the asthenons ~ 0.1 TeV? (In this case the Heisenberg relation reads: $R \simeq 2 \times 10^{-17}$ cm $\cdot$ TeV/0.1 TeV = 2×10^{-16} cm.)

The answer theorists give is: Because of the *Higgs mechanism*. According to the prevailing point of view, all space is filled by a boson field which behaves as the collection of Cooper pairs in a superconductor. The macroscopic wave function of such a collection repels *out* of the metal the virtual photons of an external magnetic field, so that they do not penetrate in the conductor (Meissner effect). The Higgs field has a similar effect on the asthenons, but the other way around: since it fills all space, it pushes *back* the asthenons radiated by leptons and quarks, so that they can be felt only at less than 10^{-16} cm from their source (Fig. 2b and 2c). The Higgs field fills the physical vacuum because its self-interaction is so strong that the state of minimum energy is reached when the average field is non-zero.

Fig. 2 Leptons (and quarks) radiate virtual photons (γ) and virtual asthenons (W , Z), but according to the Standard Model, the Higgs field pushes back the asthenons and obliges them to remain close to the sources, so that the range of the weak force is $R \approx 10^{-16}$ cm.

As always in quantum theory, there are quanta associated to fields, thus also to the Higgs field. They are called 'Higgs particles' or simply 'Higgses'. Like the Cooper pairs of normal superconductivity, they carry no intrinsic angular momentum - in other words they are bosons of spin $J = 0$. In the simplest model there is only one neutral Higgs field, but it is well possible that charged Higgs fields are also present.

The above arguments do not fix the mass of the neutral Higgs quantum. One of the main arguments in this direction comes from considering $W_L W_L$ scattering at very high energies (Fig. 3a). If the neutral Higgs bosons did not exist, the cross-section computed in perturbation theory would increase as the square of the centre-of-mass energy ($\sigma \sim W^2$) and would violate probability conservation (unitarity, in theoretical parlance) when $W \geq 1800$ GeV. If a scalar ($J = 0$) Higgs particle exists, it will contribute to the virtual processes which take place in the dashed region at the centre of Fig. 3a:

Fig. 3 (a) WW scattering and (b) neutrino quark scattering have cross sections which increase with the centre-of-mass energy, but for different reasons.

perturbative calculations show that in this case there is no unitarity violation provided the Higgs mass is smaller than about 1 TeV. There is also the possibility that perturbation theory does not apply and that new strong forces come into play at this mass scale. Electron-positron collisions are a natural source of $W_L W_L$ collisions, as shown in Fig. 4. This proves that the new accelerators will give us the opportunity of studying directly this fundamental process.

Fig. 4 The luminosity of the 2 TeV CLIC collider is plotted versus the effective centre-of-mass energy $\hat{W}$. The low energy tail in the $e^+ e^-$ luminosity is due to the beamstrahlung phenomenon (Section 4). The dotted and continuous lines are the luminosity for the process $(\gamma + \gamma)$ and $(W_L + W_L)$.

The argument which gives an upper limit for the mass of the Higgs looks simple, but one may wonder if the reasoning is sound enough. We can find some confirmation going back to past experience and recalling that already once unitarity guided us to a correct mass scale. For the neutrino-quark scattering shown in Fig. 3b the old theory of weak interactions, now superseded by the Standard Model, gives a cross section which is proportional to W^2 , so that unitarity would be violated when the energy is larger than about 300 GeV. This was a fundamental problem already thirty years ago, and, while experimentalists were collecting data around 1 GeV, theorists were bold enough to conclude that, to cure this disease, something had to happen at energies smaller than about 300 GeV. They were right and everybody now knows that probability is conserved in weak interactions, because the exchanged W has a finite mass so that the cross section flattens at high energy. Experience is thus telling us that unitarity arguments are very powerful and the present indication of violation in $W_L W_L$ scattering around 2000 GeV, while we are experimenting around 100 GeV, should not be lightly dismissed.

This long argument focuses to *The Problem* of today's physics:

- Questions 2:
- Does the standard neutral Higgs exist?
 - And if yes, what is its mass?
 - Do charged Higgs exist?

Higgs particles could be *bona fide* Cooper pairs, so that they can be broken into two fermions at energies of the order of 1 TeV. For this, and other reasons, some theorists contemplate *composite models*, in which the known 'elementary' particles are made of more fundamental fermions. If a particle is made of two other particles, one expects to find their excited states, in the same way as the $c\bar{c}$ quarks form both the fundamental state J/Ψ and its excited states Ψ' , Ψ'' , etc. Thus the questions arise:

- Questions 3:
- Are quarks and leptons composite systems of other simpler fermions?
 - Are W , Z , and Higgs themselves composite?
 - Do excited leptons (ℓ^*) and quarks (q^*) exist?

Unfortunately a consistent and unique theoretical framework for introducing compositeness does not exist. By necessity, composite models contain various parameters, whose definition has to be closely

scrutinized before they are interpreted in terms of a spatial dimension by applying the Heisenberg uncertainty relation. To increase the confusion, usually all those parameters are indicated by the letter Λ , so that one meets symbols such as Λ_{ee} , Λ_{eq} , Λ_{qq} , etc., which have different meanings in different contexts. In the following I shall use for simplicity a single symbol Λ which has the dimension of energy and can be used to roughly define the linear dimensions d at which the composite nature of the particles would become apparent through the Heisenberg relation $d \simeq 2 \times 10^{-17} \text{ cm TeV}/\Lambda$.

Question 4: Is there a distance $\propto 1/\Lambda$ below which particles appear as composite systems?

Not many theorists are today following the composite route. Many more pursue the idea of a fundamental Higgs, and here they meet with a difficulty: while propagating through space, a Higgs particle H emits and reabsorbs various other particles (Fig. 5) and through their interaction, picks up weight very easily.

Fig. 5 A Higgs particle (H) gets mass by virtual emission and absorption of (a) asthenons, (b) Higgs and (c) fermion-antifermion pairs. The virtual bosons contributions are positive, whilst the fermion ones are negative.

This is a general property of spin-0 particles since there is nothing to protect them from becoming massive. Theorists are then faced with the problem of cancelling the many different contributions to the mass increase. As is shown in Fig. 5, the contributions due to virtual bosons (W , Z , H) are positive, whilst the contributions of virtual fermion pairs ($f\bar{f}$) are negative. It can be shown that a miraculous cancellation occurs if each known boson has a related fermion (and vice versa) and their couplings are in a well-defined relation. *Supersymmetry* gives exactly the needed pattern: for each particle of integer (half-integer) spin there is a sparticle of half-integer (integer) spin (Table 1).

Table 1
Particles and sparticles

Particle	Symbol	Spin	Sparticle	Symbol	Spin
Quark	q	$1/2$	squark	$\tilde{q}$	0
Lepton	ℓ	$1/2$	slepton	$\tilde{\ell}$	0
Photon	γ	1	photino	$\tilde{\gamma}$	$1/2$
Gluon	g	1	gluino	$\tilde{g}$	$1/2$
Charged asthenon	W	1	wino	$\tilde{W}$	$1/2$
Neutral asthenon	Z	1	zino	$\tilde{Z}$	$1/2$
Higgs	H	0	higgsino	$\tilde{H}$	$1/2$
Graviton	G	2	gravitino	$\tilde{G}$	$3/2$

If the cancellation is to be effective, the masses of the sparticles should not be much larger than ≈ 1 TeV, the known limit on the Higgs mass. Other arguments also point to the existence of sparticles, but we cannot discuss them here. We can thus express the following questions:

Questions 5: Do sparticles exist?

If yes, what are their masses?

In the last few years, theorists have become very interested in *superstring theories*, in which fundamental particles are not point-like objects but strings about 10^{-33} cm long. Such theories give rise to a consistent treatment of quantum gravity, fulfilling a long-standing dream, and to supersymmetry; but they are not unique. Some theorists think that in the cold world in which we live it even reduces naturally to the Standard Model, but with some interesting additions. A particularly fashionable model foresees the existence of an additional neutral asthenon Z' , a set of other 'normal' particles (such as a neutrino, a neutral lepton and a quark of charge $-1/3$) and a new kind of particle which carries at the same time the "flavour" of a lepton and a quark. These predictions are very clear but not all experts agree on their necessity and, anyway, the masses of these particles are unknown. The 'leptoquark' D_0 is a particularly interesting object: it is a boson (spin $J=0$) which couples in a well-defined manner to a lepton and a quark. Thus:

Questions 6: Do other quarks and neutral leptons exist?

Do leptoquarks exist?

In summary, theorists indicate to machine builders and experimentalists the following 'discovery targets':

- (i) Higgs particles, both neutral (H^0) (with mass less than ~ 1 TeV) and charged ($H^\pm$);
- (ii) sequential leptons and quarks;
- (iii) sparticles with masses less than about 1 TeV;
- (iv) various new particles, such as a neutral lepton, a new quark and a leptoquark;
- (v) a new neutral asthenon Z' of unknown mass and, possibly, new charged asthenons W' ;
- (vi) compositeness, which reflects either in a compositeness scale Λ , or in
- (vii) the existence of excited states of quarks (q^*) and of leptons (for instance e^*).

2.2 Luminosity requirements

These questions can be (partially) answered either with proton-proton colliders in the range $10 \simeq W \leq 50$ TeV or with electron-positron ones having $1 \leq W \leq 5$ TeV. The comparison of the two types of colliders has been discussed in detail in Jan. '87 at the *La Thuile Workshop* [8]. I concentrate here on the possibility of e^+e^- collisions as discussed in Ref. [9] and first consider the data and predictions of the standard model. The continuous curve of Fig. 6 represents what is known today of the hadronic 'annihilation' cross section, i.e. the sum of the cross sections of all the processes in which the positron and the electron disappear having annihilated. The figure is taken from Ref. [8] and is based on experimental data up to $W \simeq 45$ GeV and on the predictions of the Standard Model above 45 GeV. Since the overall trend is roughly parallel to the dashed curve, which represents the 'point-like' cross section applicable to $\mu^+\mu^-$ and $\tau^+\tau^-$ production, the figure proves that, in the explored energy range, quarks (and leptons) are point-like particles. The four sets of peaks appearing below about 10 GeV are due to the pair-creation of quark-antiquark pairs, which *together* have the same quantum numbers as the electron-positron pairs and, immediately after production, whirl thousands of times one around the other in a resonance state. The four sets are due to the production of unstable bound states of the quark pairs $u\bar{u}$ and $d\bar{d}$ (ρ and ω), $s\bar{s}$ (ϕ), $c\bar{c}$ (J/Ψ) and $b\bar{b}$ (Υ). We would like to complete the figure by plotting the set of resonances due to $t\bar{t}$ bound states, but at present we only know that they must

lie above $W \approx 50$ GeV and below $W \approx 350$ GeV. There are also indications that the lower limit could be as high as ~ 100 GeV.

The Z peak is not only taller than the others but, according to the Standard Model, it is also of a very different nature, because the Z boson is a *single* elementary particle having the quantum numbers of the initial electron-positron pair and not a composite system. After climbing on the Z^0 peak, LEP 200 will open the possibility of exploring the much smaller shoulder due to the production of pairs of charged asthenons: $e^+ + e^- \rightarrow W^+ + W^-$.

Fig. 6 Electron-positron cross section as a function of the centre-of-mass energy. The dashed line is the point-like cross section. The continuous line is measured up to about 45 GeV and computed above. The peak due to a 'superstring inspired' Z' is very high. Compositeness with a form-factor scale $\Lambda \approx 0.5$ TeV would have striking consequences on the total cross section. Sparticles (ee , WW and $\mu\mu$) are expected to be produced with cross sections which are of the order of the point-like cross sections (dashed areas). The thick dotted line represents the cross section for the production of neutral Higgs having a mass of 0.5 TeV. (Figure adapted from Ref. [8].)

The point-like cross section decreases as W^{-2} , so that at CLIC $\sigma_{\mu\mu} \simeq 2.2 \times 10^{-38} \text{ cm}^2$: by running one third of a calendar year ($T = 10^7 \text{ s}$) at a luminosity $L = 10^{33} \text{ cm}^{-2}\text{s}^{-1}$, only 220 μ pairs would be collected. Figure 6 shows that at CLIC the rate of production of W pairs in a 4π detector is 50 times larger than for μ -pairs. It is difficult to say whether such a large production rate will be 'useful' for physics or should only be considered as a source of background. Let us now consider the effect of the 'new' physics.

In electron-positron annihilations the superstring-inspired Z' appears as an enormous peak, which is drawn in Fig. 6 for $m_{Z'} = 1 \text{ TeV}$. Clearly at a 1 TeV collider it would not only be seen, but could also be studied in detail. The cross section for a Higgs of mass $m_H = 0.5 \text{ TeV}$ is large, since it corresponds to $\geq 10^3$ events per year at $W = 2 \text{ TeV}$ and $L = 10^{33} \text{ cm}^{-2}\text{s}^{-1}$. Compositeness would be signalled by a flattening electron-positron total cross section, as indicated by the dash-dotted line of Fig. 6. For a value of the parameter Λ of the order of 0.5 TeV (i.e. for distances $d \simeq 4 \times 10^{-17} \text{ cm}$) the effect shown in the figure is striking, so much so that we are reminded of the surprise of the physicists working in 1969 at Adone when, thanks to the pair-production of point-like quarks, they found a hadron production rate which greatly exceeded the expectations.

In Fig. 6 the shaded areas indicate the cross section ranges of three sparticle channels produced in the annihilation of an electron-positron pair. No unique value can be given because various parameters enter the calculations [9]. In general one can state that selectron pairs are produced about 10 times more abundantly than wino pairs, whose cross section is of the same order as the point-like one (~ 200 events per year for $L = 10^{33} \text{ cm}^{-2}\text{s}^{-1}$). Smuons are expected to be somewhat rarer, but are very interesting because it was shown at the *La Thuile Workshop* [8] that, if found in the reachable mass range, their bosonic nature (spin $J = 0$) can be *proven* by measuring the angular distribution of the decay muons.

Fig. 6 shows that, around $W \simeq 1 \text{ TeV}$, the cross sections of new 'expected' phenomena fall in two broad categories: (i) more or less standard Z' production (with $m_{Z'} \simeq W$) and compositeness (with a scale $\Lambda \simeq W$) have $10^{-34} \leq \sigma \leq 10^{-36} \text{ cm}^2$ and (ii) SUSY particle production (with $m_{\text{SUSY}} \simeq W/2$, and neutral higgs (with $m_H \simeq W/2$) have $10^{-38} \leq \sigma \leq 10^{-36} \text{ cm}^2$. In the first case a luminosity $L =$

$(W/\text{TeV})^2 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ gives between 10^2 and 10^4 events/year, where a running year is taken to be equivalent to 10^7 seconds. In the second case, one needs a luminosity $L = (W/\text{TeV})^2 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ to obtain 10^2 - 10^4 events/year. In short, these are the arguments which brought physicists to require [8-9] the 'large' luminosity

$$L \geq (W/\text{TeV})^2 10^{33} \text{ cm}^{-2} \text{ s}^{-1} \quad (1)$$

when entering in the new energy regime opened by TeV linear colliders.

3. SCALING SLC TO TEV ENERGIES

Figure 7 schematically shows the main components of a linear collider. A few comments are in order: (i) in most schemes the positrons are obtained by using either the photons radiated by ~ 200 GeV electrons in an undulator or the spent bunches, but dedicated sources are not at all excluded (Section 6); (ii) the damping ring(s), which are needed to reduce the invariant emittance ϵ_n of the positron bunches, are not always a small component of the system and in some cases may be a few kilometres long; (iii) high gradients in the main linac are certainly useful, but a viable collider requires much more than an idea on how to produce such high gradients.

Fig. 7 Schematic representation of a linear collider complex. In practice the positron source will make use of either the accelerated or the spent bunches. The electron damping ring system will probably not be needed, because low-emittance sources are now available and, for this reason, is dashed in the figure.

Since the system is very complicated and clearly in its design many different choices are implied, it appears to be didactically useful to start from what is now reasonably well known, the SLAC Linear Collider (SLC), and try to scale it up to higher energies so to gauge the problems one has to face. This is particularly attractive since one of the nice feature of a linear collider is that one can add, at least conceptually, new pieces of accelerator and thus reach higher energies. Of course, this exercise does not imply that it would be economically convenient to build NLC by using the SLAC klystrons and accelerating structures.

3.1 Luminosity and emittance

The luminosity L is determined by the number N of particles contained in the electron and positron bunches, by the repetition frequency of the bunch collisions f_r and by the transverse r.m.s. dimensions σ_x :

$$L = (f_r N^2) / (4\pi \sigma_x^2) \quad (2)$$

(For the moment we suppose that $\sigma_y = \sigma_x$, i.e. we consider only 'round' bunches; in Subsection 4.2 the formulae shall be extended to the case of 'flat' bunches, $\sigma_y < \sigma_x$.)

To get the needed luminosity, $L = 10^{33} \text{ cm}^{-2}\text{s}^{-1}$ at $W = 1 \text{ TeV}$, Eq. (2) shows that the radius of the bunch has to be scaled to $\sigma_x \simeq 0.1 \mu\text{m} = 1000 \text{ \AA}$, if the final SLC parameters collected in Table 2 are used: $N = 7.2 \cdot 10^{10}$ particles/bunch and $f_r = 180 \text{ Hz}$ [7]. In this case the power of each beam, given by

$$P = N f_r E_0, \quad (3)$$

would be $P \simeq 1 \text{ MW}$. With a total transfer efficiency from the plug to the beam $\eta_t \simeq 2 \%$, the power consumption would be $\sim 100 \text{ MW}$, not unreasonable for a 1 TeV collider of such a large luminosity. Is this a reasonable choice of parameters?

Table 2
SLC parameters. (The various quantities are defined in Sections 3 and 4.)

Quantity	Symbol/Unit	SLC 1st stage	SLC 2nd stage
Bunch energy	E_0/GeV	50	50
Luminosity	$L/\text{cm}^{-2}\text{s}^{-1}$	$6.4 \cdot 10^{29} (*)$	$6 \cdot 10^{30} (*)$
Power/bunch	P/MW	0.05	0.10
Bunch average frequency	f_r/kHz	0.12	0.18
Invariant emittance	ϵ_N/m	$4.2 \cdot 10^{-5}$	$4.2 \cdot 10^{-5}$
Disruption parameter	D	0.34	0.76
Pinch factor	Π_D	1.14	2.2
Particles/bunch	N	$5 \cdot 10^{10}$	$7.2 \cdot 10^{10}$
Bunch r.m.s. length	σ_z/mm	1.0	1.0
β -value at IP	β^*/mm	10.	6.0
Bunch r.m.s. radii	$\sigma_x, \sigma_y/\mu\text{m}$	2.07	1.65
Av.critical energy	$\bar{E}_c/\text{MeV}$	60	150
$\bar{E}_c/E_0$	$\bar{T}$	$1.2 \cdot 10^{-3}$	$2.9 \cdot 10^{-3}$
Fluctuation parameter	Γ	$3.9 \cdot 10^{-5}$	$3.9 \cdot 10^{-5}$
Fractional energy loss	$\langle \epsilon \rangle$	$3.3 \cdot 10^{-4}$	$2.1 \cdot 10^{-3}$
Av.number of photons	N_γ	0.6	1.5
Collision energy	W/GeV	100	100
r.m.s. of W	σ_W/MeV	46	190
Fractional r.m.s. of W	σ_W/W	$4.6 \cdot 10^{-4}$	$1.9 \cdot 10^{-3}$

(*) Note that the high energy requirement of Eq. (1) gives at 50 GeV $L \simeq 2.5 \cdot 10^{30} \text{cm}^{-2}\text{s}^{-1}$, in (accidental) agreement with the average of the two SLC project values.

SLC aims at $\sigma_x \simeq 1.65 \mu\text{m}$, which is ~ 15 times larger than $\sigma_x \simeq 0.1 \mu\text{m}$. However, the SLC final beam energy (50 GeV) is 10 times smaller than what we are aiming at, and the transverse emittances of the bunches are reduced by the acceleration as $1/\gamma$. The *invariant* transverse emittances $\epsilon_{nx}, \epsilon_{ny}$ (which are *not* reduced) define the bunch r.m.s. radio σ_x and σ_y at the crossing point when the beam energy is E_0 through the relations:

$$\sigma_x = (\epsilon_{nx} \beta_x^* / \gamma)^{1/2}, \quad (4)$$

$$\sigma_y = (\epsilon_{ny} \beta_y^* / \gamma)^{1/2}, \quad (5)$$

where $\gamma = E_0/mc^2 = W/(2 mc^2)$ and β_x^* and β_y^* are the values of the horizontal and vertical β -functions at the interaction point (IP).

In the absence of wake fields and dissipative phenomenon, the emittance ϵ_n does not vary during acceleration. As discussed in Subsection 7.3, the 'natural' scaling law for the β -function, produced by a final focus system having the same tip field, is $\beta^* \propto \gamma^{1/3}$, so that Eqs. (4) and (5) imply the 'natural' law $\sigma_x, \sigma_y \propto \gamma^{-1/3}$. As a consequence (for fixed values of ϵ_n, f_r and N) $L \propto \sigma_x^{-2} \propto \gamma^{2/3}$ and *not* $L \propto \gamma^2$, as we would like to find in order to follow the requirement of Eq. (1) when passing from SLC to a TeV collider. To compensate for this effect, one has to make $\epsilon_{nx}, \epsilon_{ny} \propto \gamma^{-4/3}$, so that the $4 \cdot 10^{-5}$ m invariant emittance of SLC ($E_0 = 0.05$ TeV), has to become $\epsilon_{nx} = \epsilon_{ny} \simeq 2 \cdot 10^{-6}$ m at $E_0 = 0.5$ TeV, if one wants to keep the same values of f_r and N . As we shall see in Section 5, such invariant emittances are today feasible and, at first glance, the scaling from SLC to a 1 TeV collider is reasonable with $f_r = 0.18$ kHz, $N = 7.2 \cdot 10^{10}$, $\epsilon_{nx} = \epsilon_{ny} \simeq 2 \cdot 10^{-6}$ m, $\beta_x^* = \beta_y^* = 13$ mm, $\sigma_x = \sigma_y \simeq 0.15$ μ m. This conclusion is incorrect because of the focusing and radiation effects that each colliding bunch produces on the opposite one. Let us look into these problems.

3.2 Disruption

Figure 8 defines the relevant quantities.

Fig. 8 Schematic representation of the effects of a moving bunch on a typical particle of the opposite bunch.

A particle at a distance σ_x from the axis sees a magnetic field produced by the opposite bunch which is proportional to the incoming current ($\propto N/\sigma_z$) and inversely proportional to σ_x : $B \propto N/(\sigma_x \sigma_z)$. The radius of curvature ρ in this field, supposed to be uniform, is proportional to γ/B and equal to $\gamma \sigma_x \sigma_z / (N r_e)$, where $r_e = 2.82 \cdot 10^{-13}$ cm is the classical electron radius. The deflection angle is $\Theta = \sigma_z / \rho = \sigma_x / F$, where F is the 'focal distance' defined for little focusing in Fig. 8, and thus $F = \sigma_x \rho / \sigma_z$. Finally, the *disruption parameter* D , defined in the case of weak focusing as $D = \sigma_z / F$, has the form $D = \sigma_z^2 / \sigma_x \rho$, i.e.

$$D = r_e N \sigma_z / (\gamma \sigma_x^2). \quad (6)$$

When $D \ll 1$ the focusing effect of one bunch on the other is clearly negligible, but when $D \geq 0.5$ the bunches pinch and the electron-positron luminosity increases. (Note that it would decrease in electron-electron or positron-positron collisions). The pinch factor H_D has been numerically computed [10] and is plotted in Fig. 9. (*Note added in proof:* the usual picture of disruption [10-17] has to be modified for large D 's as a consequence of more recent computer simulations [18].) The value of H_D appearing for SLC in Table 2 is the result of such a computer simulation.

Fig. 9 Pinch factor H_D versus the disruption parameter D defined in Eq. (6).

By taking into account the pinch effect, Eq. (1) becomes

$$L = f_r N^2 H_D / (4\pi \sigma_x \sigma_y). \quad (7)$$

and the unfavorable scaling laws of a linear collider can be illuminated [4] by combining Eqs. (3), (6) and (7):

$$L = (4\pi)^{-1} DH_D (r_e/\sigma_z) (P/mc^2). \quad (8)$$

Numerically this equation reads

$$L/(10^{33} \text{ cm}^{-2}\text{s}^{-1}) \simeq (DH_D/30) (\text{mm}/\sigma_z) (P/\text{MW}), \quad (9)$$

which implies $P \simeq 1 \text{ MW}$ and $\sigma_z \simeq 0.1 \text{ mm}$ with $DH_D \simeq 1 \times 4 \simeq 4$ to obtain $L = 10^{33} \text{ cm}^{-2}\text{s}^{-1}$.

Even forgetting the problems due to scaling of β^* at the final focus, luminosity is thus *proportional* to beam power; and beam power, for N and f_r constant, increases only *linearly* with γ . The needed extra factor γ can be obtained by making $\sigma_z \propto \gamma^{-1}$, so that the various quantities would scale as summarized in Table 3. (In particular scaling SLC to $W = 1 \text{ TeV}$ would imply $\sigma_z \leq 0.1 \text{ mm}$.)

Table 3
'Natural' scaling laws from SLC to higher energies

Quantity	γ -dependence	Quantity	γ -dependence
N	1	L	γ^2
f_r	1	P	γ
$\sigma_x = \sigma_y$	γ^{-1}	$\epsilon_{nx} = \epsilon_{ny}$	$\gamma^{-4/3}$
σ_z	γ^{-1}	$\beta_x = \beta_y$	$\gamma^{1/3}$
D	1	δ (defined in Eq. 11)	γ^4

Three problems are apparent from inspecting Table 3:

- (i) The beam power is proportional to γ . We see that up to $W \simeq 1 \text{ TeV}$ the extrapolation is still reasonable since $P \leq 1 \text{ MW}$, which for an accelerator efficiency $\eta_t \simeq 2\%$ corresponds to a total consumption of $\sim 100 \text{ MW}$.
- (ii) The invariant emittance decreases on $\gamma^{-4/3}$, which gives reasonable numbers again for $W \leq (1-2) \text{ TeV}$ but not above.
- (iii) The new parameter δ appearing in the Table has a frightening γ -dependence. This is a serious problem already for colliders below $W \simeq 1 \text{ TeV}$, as we shall see in the next Section.

4. BEAMSTRAHLUNG

4.1 Beamstrahlung for round bunches

An electron (positron) deflected by the opposite bunch radiates electromagnetic energy. As shown in the last Subsection, the radius of curvature in the uniform field of a cylindrical bunch is, for a small deflection ($D \ll 1$),

$$\rho = \gamma \sigma_x \sigma_z / (N r_e) . \quad (10)$$

According to classical electrodynamics, the energy radiated per unit length is $P_\ell = 2 r_e mc^2 \gamma^4 / (3\rho^2)$ so that the *fractional energy loss* in a length σ_z can be written in the form

$$\delta = 2/9 (r_e^3 \gamma / \sigma_z) (N^2 / \sigma_x^2) . \quad (11)$$

(The numerical factor, which equals 0.222, has a simple form useful for later simplifications. The exact form computed for gaussian bunches both in the transverse and the longitudinal directions is $8 \pi^{1/2} / 21 \simeq 0.215$ [12].) The quantity δ has been dubbed *beamstrahlung parameter*.

Equation (11) shows that, as anticipated in Table 3, δ increases proportionally to γ^4 , if one decides to scale up a collider with $N = \text{const}$, $f_r = \text{const}$, $P \propto \gamma$, $\sigma_x = \sigma_y \propto \gamma^{-1}$ and $\sigma_z \propto \gamma^{-1}$ so to have $L \propto \gamma^2$. At SLC, with a luminosity corresponding to the requirement of Eq. (1), $\delta \simeq 10^{-3}$ so that at $W = 1$ TeV with the 'natural' scaling of Table 3 one would have $\delta \simeq 10$, clearly unacceptable. This is the third and strongest reason for which, starting from SLC parameters of Table 2, the scaling laws of Table 3 cannot be followed above $E_0 \simeq 300$ GeV.

In 1985 Himel and Siegrist [11] showed that at very large energies linear colliders have more favourable scaling laws than the ones implied by Eq. (11). The issue has been discussed and clarified in many recent papers [12-19] and since Pisin Chen has discussed it at this Course [18], I limit myself to the presentation of the main results.

In a uniform magnetic field the critical energy of the radiated photon is $E_c = (3 \hbar c \gamma^3) / (2\rho)$ so that, using Eq. (10), the *average fractional critical energy* $\bar{\gamma} = \bar{E}_c / E$ has the form

$$\bar{T} \simeq 5/(12\alpha) (r_e^2 \gamma/\sigma_z) (N/\sigma_x), \quad (12)$$

where $\alpha = 1/137$.

Note that Yokoya [13] uses the parameter $\xi_0 = 12 \bar{T}/5$.

(The numerical factor of Eq. (12), computed by Noble [15], has the equivalent form $3^{1/2} (4\alpha)^{-1}$ [17-19], since $5/12 = 0.417$ and $3^{1/2}/4 = 0.433$; Eq. (12) is the one needed for weak round gaussian bunches.)

In general, beamstrahlung has to be described with two parameters, which are often chosen to be δ and $\bar{T}$. However, physically the best ones are the average fractional critical energy $\bar{T}$ of Eq. (12) and the *fluctuation parameter* [15] which I define as

$$\Gamma = (r_e \gamma) / (\alpha \sigma_z). \quad (13)$$

$\Gamma = (\hbar c \gamma) / (mc^2 \sigma_z)$ is proportional to the longitudinal density of each bunch, and is equal, in the rest frame of the radiating particle, to the ratio DE/mc^2 , DE being the uncertainty in the particle energy produced by the finite interaction time with the opposite bunch $\Delta T \propto \sigma_z/\gamma c$. When Γ is large this energy uncertainty is large with respect to mc^2 .

The three parameters $\bar{T}$, Γ and δ are related:

$$\delta = (4/5)^2 2 \alpha \bar{T}^2 / \Gamma \quad (\text{for all } \bar{T}\text{'s}) \quad (14)$$

and all interesting quantities describing the beamstrahlung phenomenon can be expressed as a function of any two of them. I think one should use the *most physical* ones: $\bar{T}$ and Γ .

Before discussing the possible ways of choosing the parameters of a bunch-bunch collision we have to consider flat bunches which are so dense to focus each other.

4.2 Disruption and beamstrahlung for flat and pinching bunches

'Flat' bunches have $\sigma_y < \sigma_x$. By introducing the aspect ratio

$$R = \sigma_x / \sigma_y \geq 1 \quad (15)$$

two disruption parameters determine now the focusing in the two planes [10,19].

$$D_x = (r_e N \sigma_z) (\gamma \sigma_x \sigma_y)^{-1} [2 / (1+R)] , \quad (16)$$

$$D_y = (r_e N \sigma_z) (\gamma \sigma_x \sigma_y)^{-1} [2 R / (1+R)] = D = R D_x > D_x . \quad (17)$$

In the following the largest one, D_y , is still called 'disruption parameter' and is indicated by the symbol D . For flat beams the pinch factor is reduced with respect to the one given as a function of D in Fig. 9 [14,17]:

$$H(R) \simeq H_D (1+R)/2 R , \quad (18)$$

and the luminosity becomes

$$L = f_r N^2 H(R) / (4\pi \sigma_x \sigma_y) . \quad (19)$$

(P. Wilson [19] has the expression $H(R) = R H_D (1 + (R-1) H_D^{1/2})^{-1}$, which agrees numerically with Eq. (18) to better than 10%.)

The important relations (7) and (8) become now

$$L = (4\pi)^{-1} D H(R) [(1+R)/(2R)] (r_e \sigma_z)^{-1} (P/mc^2) , \quad (20a)$$

$$L/(10^{33} \text{ cm}^{-2}\text{s}^{-1}) \simeq [DH(R)/29] [(1+R)/2R] (\text{mm}/\sigma_z) (P/\text{MW}) . \quad (20b)$$

While the fluctuation parameter of Eq. (13), being a longitudinal quantity, does not change when $R > 1$, the fractional average critical energy $\bar{T}$ gets a non-trivial modification [17]:

$$\bar{T} = 5/12\alpha (r_e^2 \gamma / \sigma_z) (N/\sqrt{\sigma_x \sigma_y}) H_b^{1/2}(D,R) , \quad (21)$$

where I have introduced a new factor, which depends on D and R :

$$H_b(D,R) = 4R H_D [1 + R H_D^{(R-1)/2R}]^{-2} . \quad (22)$$

(P. Wilson [19] has the expression $H_b(D,R) = 4R H_D [2 + (R-1) H_D^{1/2}]^{-2}$, which agrees numerically with Eq. (22) to better than 10%.)

For round bunches ($R=1$) with pinch effect $H_b \simeq H_D$ and the parameter $\bar{T}$ is proportional to $H_D^{1/2}$ [14,16], while for very flat bunches ($R \gg 1$) $H_b \simeq 4R^{-1}$ and (with $\sigma_x \sigma_y = \text{const}$) $\bar{T} \propto R^{-1/2}$. Flat bunches can thus be used to reduce $\bar{T}$, while Eq. (20) is modified only by a factor 2.

Once the physical parameters Γ and $\bar{T}$ are known, the energy lost by the particles due to beamstrahlung can be computed. If $\bar{T} \ll 1$ the collider runs in the so-called 'classical regime' [11]. The discussion of Subsection 4.1 applies and the *fractional average energy loss* $\langle \epsilon \rangle$ by a particle due to beamstrahlung, when $R = 1$, is given by Eqs. (11) and (14); when the bunches are flat $\langle \epsilon \rangle$ is the natural extension of Eq. (14):

$$\langle \epsilon \rangle = \delta \simeq (4/5)^2 2 \alpha \bar{T}^2 / \Gamma \quad (\bar{T} \ll 1), \quad (23)$$

where $\bar{T}$ is now given by Eq. (21). Both computer simulations [15] and analytical calculations [13] show that this extension is surprisingly well verified when $\langle \epsilon \rangle$ is small, so that D and R appear only through the factor H_b , which multiplies $\bar{T}$.

Other important quantities are the *average number of radiated photons* N_γ and the *average fractional photon energy* $\langle \hbar\omega \rangle / E_0$. The same computer simulations and analytical calculations give the following remarkably simple approximate expressions [13,15,17]:

$$N_\gamma \simeq 3^{3/2} \alpha \bar{T} / (2 \Gamma) \simeq 2 \langle \epsilon \rangle / \bar{T} \quad (\bar{T} \ll 1) \quad (24)$$

$$\langle \hbar\omega \rangle / E_0 = \langle \epsilon \rangle / N_\gamma \simeq 2^6 3^{-3/2} 5^{-2} \bar{T} \simeq \bar{T} / 2 \quad (\bar{T} \ll 1). \quad (25)$$

By increasing the fractional critical energy $\bar{T}$ one enters in the 'quantum regime', first studied by Himel and Siegrist [11]. The results, obtained for $\bar{T} \gg 1$, are [13,15]

$$\langle \epsilon \rangle \simeq \alpha \bar{T}^{2/3} (2^{1/2} \Gamma)^{-1}, \quad (\bar{T} \gg 1), \quad (26a)$$

$$N_\gamma \simeq 3.7 \langle \epsilon \rangle. \quad (\bar{T} \gg 1). \quad (26b)$$

Eq. (26a) shows that $\langle \epsilon \rangle$ increases with $\bar{T}$ much less rapidly than in the classical regime [35]. The interpolation between formulae (23) and (26a) is obtained through numerical calculations [13] and can be read from the graph of Fig. 10. Following Ref. [14], I introduce the handy approximation, valid to $\sim 5\%$ for $10^{-2} < \bar{T} < \infty$:

$$\langle \epsilon \rangle = \delta F(\bar{T}) = (4/5)^2 2 \alpha \bar{T}^2 F(\bar{T}) / \Gamma, \quad (27)$$

with

$$F(\bar{T}) \simeq (1 + \bar{T}^{-1/2} + 3 \bar{T}/2)^{-4/3}. \quad (28)$$

The function $F(\bar{T})$, called H_Γ by Wilson [19], is plotted as dashed line in Fig. 10.

Fig. 10 The 'quantum' reduction factor $F(\bar{T})$, which multiplies δ (the classical estimate for $\langle \epsilon \rangle$), and the correction factor $G(\bar{T})$, which multiplies the classical average number of photons N_γ , are plotted versus $\bar{T}$. The continuous lines are the results of Noble numerical calculations [15]; the dashed curves are the simple analytic expressions $F(\bar{T}) = (1 + \bar{T}^{-1/2} + 3\bar{T}/2)^{-4/3}$ and $G(\bar{T}) = F(\bar{T}) (1 + \bar{T}^{1/2} + \pi^{1/2}\bar{T})$.

The fractional energy loss $\langle \epsilon \rangle$ of Eq. (27) can be written [19] in a convenient form by expressing $\bar{T}/\Gamma$ of Eqs. (13) and (21) as a function of $(L/f_r)^{1/2}$:

$$\langle \epsilon \rangle = (16 \pi^{1/2} \alpha/15) (H_b/H)^{1/2} (r_e^2 L/f_r)^{1/2} \bar{T} F(\bar{T}). \quad (29)$$

The quantity $\bar{T} F(\bar{T})$ is thus very important because, together with L and f_r , it fixes $\langle \epsilon \rangle$. Noble's results are plotted as a continuous line in Fig. 11, while the dashed line represents the approximation given by the simple formula (28). Following Ref. [19] three regimes can be clearly distinguished, which I define as:

$$\begin{array}{ll} \text{classical regime:} & \bar{T} < 0.2 \\ \text{transition regime:} & 0.2 < \bar{T} < 10^2 \\ \text{quantum regime:} & 10^2 < \bar{T}. \end{array}$$

In the first regime $\langle \epsilon \rangle \propto \bar{T}$, in the second $\langle \epsilon \rangle \approx \text{constant}$ within a factor 2, and in the third

$\langle \epsilon \rangle \propto \bar{T}^{-1/3}$. It is remarked in Ref. [19] that most probably NLC will run in the intermediate regime where $\bar{T} F(\bar{T})$ is about constant. We shall come back to this point in the next Section.

Fig. 11 The function $\bar{T}F(\bar{T})$, which enters in the expression of $\langle \epsilon \rangle$ [Eq. (29)], is plotted versus $\bar{T}$. The continuous curve is due to Noble [15] and the dashed curve represents the simple approximation $\bar{T} (1 + \bar{T}^{1/2} + 3\bar{T}/2)^{-4/3}$.

One can write down simple interpolation formulae also for N_γ and $\langle \hbar\omega \rangle / E_0$:

$$N_\gamma = 3^{3/2} \alpha \bar{T} G(\bar{T}) / (2 \Gamma) \approx 2 \langle \epsilon \rangle (1 + \bar{T}^{1/2} + \pi^{1/2} \bar{T}) / \bar{T} \quad (30)$$

$$\langle \hbar\omega \rangle / E_0 = \langle \epsilon \rangle / N_\gamma \approx (\bar{T}/2) (1 + \bar{T}^{1/2} + \pi^{1/2} \bar{T})^{-1}. \quad (31)$$

Eqs. (30) and (31) reproduce the numerical results of Ref. [13] within about $\pm 5\%$, in the range $10^{-2} < \bar{T} < \infty$, and are better approximations than the expressions given in Ref. [20]. The exact and approximate expressions of $G(\bar{T})$ are plotted in Fig. 10 as continuous and dashed lines. (The 'exact' form has been computed in Ref. [13]; Eq. (30) implies that the approximate expression is $G(\bar{T}) = F(\bar{T}) (1 + \bar{T}^{1/2} + \pi^{1/2} \bar{T})$.)

The r.m.s. fractional fluctuation σ_W/W of the centre-of-mass energy W is [13,15]:

$$\sigma_W/W \approx \langle \epsilon \rangle (1 + 10 / N_\gamma)^{1/2} / 3. \quad (\bar{T} \ll 1) \quad (32a)$$

$$\sigma_W/W \approx 1.9 \langle \epsilon \rangle (1 + 26 / N_\gamma)^{1/2} / 9. \quad (\bar{T} > 10^3) \quad (32b)$$

The first term represents the energy spread due to the change of the energy loss with the particle trajectory. The second describes the effect of the fluctuation in the number of radiated photons. If N_γ is not larger than $\sim 10-20$, the second term is more important than the first. This happens in most cases. For instance, at SLC $N_\gamma \simeq 1$ (Table 2) and the second term dominates. In the quantum regime, $N_\gamma \simeq 3.7$ $\langle \epsilon \rangle \leq 1$, since usually one does not consider values of $\langle \epsilon \rangle$ larger than ~ 0.3 because the energy distribution of the colliding particles would be too wide. It follows that in most practical cases one can neglect the first term in Eqs. (32a) and (32b) and write $\sigma_W/W \simeq \langle \epsilon \rangle / N_\gamma^{1/2}$. For $N_\gamma \simeq 1$, $\langle \epsilon \rangle$ is thus a good measure of the physically more interesting quantity σ_W/W . Also for σ_W/W one can find a suitable, not very elegant, analytical form valid for $10^{-3} \leq \bar{T} \leq 10^3$:

$$\sigma_W/W \simeq (2 + e^{-\bar{T}/3}) [1 + (10 + 3^{1/2} \ln(10\bar{T} + 1)) / N_\gamma]^{1/2} \langle \epsilon \rangle / 9. \quad (33)$$

Note that typically the effective c.m. energy distribution has a peak at the beam energy $W = E_0$, when no photon is radiated by the incoming particles, and a very long tail at smaller energies (the CLIC example is shown in Fig. 4). If $N_\gamma \leq 1$ the r.m.s. spread σ_W has a doubtful meaning, and even if $\sigma_W/W \simeq 0.3$ a large fraction of the electron-positron annihilations take place at the full energy $W = 2 E_0$.

5. RELATIONS CONNECTING COLLISION PARAMETERS

We have introduced a large number of quantities to describe bunch-bunch collisions: E_0 (or γ), L , P , R , D (and $H(R)$), σ_z , f_r , N , σ_x , $\bar{T}$, Γ , $\langle \epsilon \rangle$, N_γ , $\langle \hbar \omega \rangle / E_0$, σ_W/W . These fifteen parameters are linked by nine equations: (3), (13), (17), (19), (21), (27), (30), (31), (33), so that six parameters are enough to define any linear collider. (To help the reader, all the relevant formulae are collected in Table 4.)

How do we orientate ourselves in such a complicated multidimensional space? In this Section we first discuss some relations, which are simple consequences of the equations of Table 4. But to make full use of them we have to review the main components of a linear collider and their scaling laws. In the following Sections we then shall take up in turn: (i) particle sources and emittance forming; (ii) acceleration mechanisms and (iii) final focus.

Table 4
Summary of collider formulae
 (the symbol $\approx$ signifies 5 – 10% accuracy)

Quantity	Formula	Eq.
Energy/beam	$E_0 = \gamma mc^2$	
Power/beam	$P = N f_r E_0$	(3)
Luminosity	$L = f_r N^2 H(R)/(4\pi \sigma_x \sigma_y), (\sigma_x = R\sigma_y)$	(19)
Disruption par.	$D = (r_e N \sigma_z) (\gamma \sigma_x \sigma_y)^{-1} [2R/(1+R)]$	(17)
Pinch factor ($R=1$)	$H_D = \text{graph of Fig. 9}$	
Pinch factor ($R \neq 1$)	$H(R) \approx H_D(1+R)/2R$	(18)
r.m.s. radii	$\sigma_x = (\epsilon_{nx} \beta_x^*/\gamma)^{1/2}; \sigma_y = (\epsilon_{ny} \beta_y^*/\gamma)^{1/2}$	(4),(5)
Fract.av.critical energy	$\bar{T} \approx (5/12\alpha) (r_e^2 \gamma/\sigma_z) (N/\sqrt{\sigma_x \sigma_y}) H_b^{1/2}(D,R)$	(21)
Beamstrahlung factor	$H_b(D,R) \approx 4R H_D [1 + R H_D (R-1)/2R]^{-2}$	(22)
Fluctuation par.	$\Gamma = r_e \gamma/(\alpha \sigma_z)$	(13)
Beamstrahlung par.	$\delta = (4/5)^2 2\alpha \bar{T}^2/\Gamma = 2/9 (r_e^3 \gamma/\sigma_z) (N^2/\sigma_x \sigma_y) H_b(D,R)$	(14)
Fract.av.energy loss ^(*)	$\langle \epsilon \rangle \approx (4/5)^2 2\alpha \bar{T}^2 F(\bar{T})/\Gamma = \delta F(\bar{T})$	(27)
Quantum factor	$F(\bar{T}) \approx (1 + \bar{T}^{1/2} + 3\bar{T}/2)^{-4/3}$	(28)
Av.number of γ 's	$N_\gamma \approx 2 \langle \epsilon \rangle (1 + \bar{T}^{1/2} + \pi^{1/2} \bar{T}) / \bar{T}$	(30)
Av.fract.photon energy	$\langle \hbar\omega \rangle / E_0 \approx (\bar{T}/2) (1 + \bar{T}^{1/2} + \pi^{1/2} \bar{T})$	(31)
Fract.fluctuation of the cm energy	$\sigma_W/W \approx (2 + e^{-\bar{T}/3}) [1 + (10 + 3^{1/2} \ln(10\bar{T} + 1))/N_\gamma]^{1/2} \langle \epsilon \rangle / 9$	(33)

(*) The symbol $\langle \epsilon \rangle$ has been introduced by Yokoya [13]. The notation δ is often used [16-19]. I think that it is better to keep this symbol for the beamstrahlung parameter connected to $\bar{T}$ and Γ for all $\bar{T}$'s by Eq. (14): $\delta = (4/5)^2 2\alpha \bar{T}^2/\Gamma$.

The relations of Table 4 are so complicated and so numerous that each author prefers a different set of derived relations to discuss the interrelations among parameters. Palmer has written a PC computer code which takes all of them into account [36], and this is very useful for planning a collider but does not help in getting a feeling for the freedom of choice and the sources of constraints. In this Subsection I address, on the basis of the equations of Table 4, the questions: (i) Will NLC run with sizeable pinch

effect? (ii) In which beamstrahlung regime NLC will operate? The answers will be: (i) NLC will utilize the pinch effect to increase luminosity; (ii) NLC will either run in the transition regime or very close to it.

Table 5
Relations derived from the equations of Table 4 (*)
(practical units are introduced for convenience in numerical calculations)

Relation	Eq.
$\sigma_z/\text{mm} = [\text{DH}/29] [(1+R)/(2R)] (10^{33}/L) (P/\text{MW})$	(20)
$F(\bar{T}) = (1 + \bar{T}^3/2 + 3\bar{T}/2)^{-4/3}$	(28)
$\langle \epsilon \rangle \approx 3.9 (H_b/H)^{1/2} (L/10^{33})^{1/2} (\text{kHz}/f_r)^{1/2} \bar{T} F(\bar{T})$	(29)
$\bar{T} \approx 0.32 (H_b/H)^{1/2} (E_0/\text{TeV}) (L/10^{33})^{1/2} (\text{kHz}/f_r)^{1/2} (\text{mm}/\sigma_z)$	(34)
$\bar{T} \approx 9.1(\text{DH})^{-1} (H_b/H)^{1/2} [2R/(1+R)] (E_0/\text{TeV}) (L/10^{33})^{3/2} (\text{MW}/P) (\text{kHz}/f_r)^{1/2}$	(35)
$p \approx 3.3 \{ \langle \epsilon \rangle (\text{DH})^{-1} [2R/(1+R)] (E_0/\text{TeV}) (L/10^{33}) (\text{MW}/P) \}^{3/4}$	(40)
$\bar{T} \approx 3^{-1/3} p^{2/3} (1 + 5p/4 + 13p^4/12)$	(39)
$f_r/\text{kHz} \approx 35.5 (\text{DH})^{-1} (H_b/H) [2R/(1+R)] (E_0/\text{TeV}) (L/10^{33})^2 (\text{MW}/P) F(\bar{T}) / \langle \epsilon \rangle$	(41)
$N/10^9 = 6.25 (\text{TeV}/E_0) (P/\text{MW}) (\text{kHz}/f_r)$	(3)
$\sigma_x \sigma_y / \text{nm}^2 = (\epsilon_x \beta_x^* \epsilon_y \beta_y^*)^{1/2} / \gamma \approx 8.75 (\text{DH}^3/H_b) [(1+R)/2R] (\text{TeV}/E_0)^3 (10^{33}/L)^3 (P/\text{MW})^3 \langle \epsilon \rangle / F(\bar{T})$	(42)

(*) In this Table $10^{33} = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ and $H = H(R)$.

We have already met the first three important relations (Eqs. (20), (28) and (29)), which are rewritten in practical units in Table 5. The fourth relation in the Table is obtained by expressing [19] $\bar{T}$ of Eq. (21) in terms of (L/f_r) :

$$\bar{T} \approx (5\pi^{1/2}/6\alpha) (H_b/H)^{1/2} (E_0/mc^2) (r_e^2 L/f_r)^{1/2} (r_e/\sigma_z). \quad (34)$$

By combining Eqs. (20) and (34) I then obtain

$$\bar{T} \approx (10\pi^{3/2}/3\alpha) (\text{DH})^{-1} (H_b/H)^{1/2} [2R/(1+R)] (r_e^2 L E_0/P) (r_e^2 L/f_r)^{1/2}, \quad (35)$$

which relates the parameter $\bar{\Upsilon}$ to the five most important physical parameters (E_0 , L , P , D , R) and the repetition rate f_r . When designing a collider, E_0 and L are given by physics, and the present wisdom was presented in Section 2 and is condensed in Eq. (1). P is fixed by the acceleration technology and the money available, since given a certain energy transfer η_t from wall-plug to the accelerated beam, the power (per linac) is P/η_t . The value of D defines the choice between *pinch effect* ($D > 0.5$) or *no pinch effect* ($D < 0.5$). The aspect ratio R can be varied by using both the transverse emittances ϵ_{ny} and ϵ_{nx} and/or the ratio β_x^*/β_y^* .

Eq. (35) shows that, if one chooses to scale the luminosity according to Eq. (1), $\bar{\Upsilon} \propto E_0^4/(DH)$ and at large enough energy the collider has to run in the quantum regime ($\bar{\Upsilon} > 10^2$). For a 'standard' NLC ($W = 1$ TeV and $L = 10^{33} \text{ cm}^{-2}\text{s}^{-1}$) with $P = 1$ MW (so that the total plug power is ~ 100 MW with $\eta_t \simeq 2\%$) it follows from Eq. (35) that $\bar{\Upsilon} \simeq (4.7/DH) g(H_D, R) (\text{kHz}/f_r)^{1/2}$. For reasonable parameters the function $g(H_D, R)$, tabulated in Table 6, varies in the range $0.2 \leq g(H_D, R) \leq 1$ and for $R \gg 1$ tends to $4 R^{-1/2} H_D^{-1/4}$.

Table 6
Values of the functions $g(H_D, R) = (H_b/H)^{1/2} [2R/(1+R)]$ of Eq. (35)

R	$H_D=1$	$H_D=2$	$H_D=4$	$H_D=6$
1	1.00	1.00	1.00	1.00
3	1.30	1.22	1.14	1.09
10	1.04	0.92	0.80	0.73
30	0.68	0.58	0.50	0.46
100	0.39	0.33	0.28	0.26
300	0.23	0.19	0.16	0.15

A standard NLC, which profits from the pinch effect ($D \geq 0.5$) and runs at the (small) SLC repetition rate ($f_r = 0.2$ kHz), will thus have $0.2 \leq \bar{\Upsilon} \leq 10$ for all reasonable values of R . A related argument prompted Wilson to conclude that a 'standard' NLC will have to run in the transition regime. However, higher rates and beam powers tend to reduce $\bar{\Upsilon}$, and one can get $\bar{\Upsilon} < 10^{-1}$ if the aspect ratio R is large. At higher energy (for instance at $W = 2$ TeV, as for CLIC), the choice is again pushed to the transition regime because of the dependence $\bar{\Upsilon} \propto E^4/DH$.

In conclusion, a low repetition rate NLC profiting from the pinch effect will have to run in the transition regime. At higher repetition rates and for flat bunches one can choose to sit at the higher end of the classical regime. Anyway, by increasing the energy one is pushed towards the quantum regime, where the scaling laws change. We do not dwell on this here because this regime is not interesting for NLC; still it is worth mentioning that, in this case, one would choose D small and no pinch effect because otherwise the required emittances would be unattainably small [14].

The bunch-bunch collision of a NLC running in the pinch mode is described by six independent parameters. The usual choice of five of them has already been indicated: E_0 , L , P , D , R . For physics reasons the most natural choice for the sixth one would be σ_W/W . The form of Eq. (33) is unfortunately very complicated and, anyway, σ_W/W does not fix the *shape* of the luminosity distribution versus the effective collision energy $\hat{W}$, since also N_γ enters. It has thus become customary to use as sixth parameter the *fractional average energy loss* $\langle \epsilon \rangle$, but it has to be clearly understood that two colliders having the same ϵ will in general have different luminosity distributions. It is also generally thought that a value of $\langle \epsilon \rangle \leq 0.3$ can be accepted, but in the case of a search for a narrow resonance, of the type of the Z' -asthenons discussed in Section 2, one has to reduce $\langle \epsilon \rangle$ by a factor of about 10 with a corresponding reduction in luminosity.

Given the 'standard' set of parameters $\{E_0, L, P, D, R, \langle \epsilon \rangle\}$, the advantage offered by the new analytic form of $\langle \epsilon \rangle$ as a function of $\bar{T}$ [Eqs. (27) and (28)], is that the equation can be inverted *analytically*, by solving a simple cubic equation, to get:

$$\bar{T} = \{[R(\kappa) + S(\kappa)]^{1/3} + [R(\kappa) - S(\kappa)]^{1/3} + \kappa/2\}^2, \quad (36)$$

where κ is

$$\kappa = [S^2 \langle \epsilon \rangle \Gamma / (2^5 \alpha)]^{3/4} = \{[5^2 \pi / (2^3 \alpha^2)] (\langle \epsilon \rangle / DH) [2R / (1 + R)] (r_e^2 E_0 L / P)\}^{3/4}, \quad (37)$$

and

$$\begin{aligned} R(\kappa) &= (\kappa^3/4 + \kappa^2/2 + \kappa)/2, \\ S(\kappa) &= (5\kappa^4/48 + 23\kappa^3/108 + \kappa^2/4)^{1/2}. \end{aligned} \quad (38)$$

For $\kappa \ll 1$, one gets $\bar{T} = \kappa^{2/3}$ while, for $\kappa \gg 1$, Eq. (36) simplifies to $\bar{T} = (3\kappa/2)^2$. The complicated general solution is approximated to better than 3% by the simple expression

$$\bar{T} = 3^{-1/3} p^{2/3} (1 + 5p^{1/2}/4 + 13p^{4/3}/12), \quad (39)$$

where the new *regime parameter* p is proportional to κ :

$$p = 3^{1/2} \kappa = \{ [3^2/5^2 \pi / (2^3 \alpha^2)] (\langle \epsilon \rangle / DH) [2R/(1+R)] (r_e^2 E L/P) \}^{3/4}. \quad (40)$$

A practical expression for the parameter p as a function of the 'standard' set of parameters appears in Table 5. Note that the 'regime parameter' p is related to the 'quantum parameter' q introduced in Ref. [14] to achieve the same purpose, i.e. to express the quantum correction $F(\bar{T})$ in terms of $\{E_0, L, P, D, R, \langle \epsilon \rangle\}$: $p = (4q)^{3/2}/3$.

As the name indicates, p defines the beamstrahlung regime:

$$\begin{aligned} \text{classical regime:} & \quad p < 0.1 \\ \text{transition regime:} & \quad 0.1 < p < 10 \\ \text{quantum regime:} & \quad 10 < p. \end{aligned}$$

In the transition regime the function $\bar{T} F(\bar{T})$ varies only by a factor ~ 2 within the range $0.10 - 0.20$, as shown in Fig. 10. CLIC and SC, whose parameters are collected in Table 7, have $p = 0.13$ and $p = 0.69$ respectively, so that both would run in the transition regime.

Table 7
Provisional parameter lists of CLIC [33] and SC [36]

Quantity	Symbol/Unit	CLIC	SC
Bunch energy	E_0/TeV	1.0	0.5
Luminosity	$L/10^{33} \text{cm}^{-2} \text{s}^{-1}$	1.1	1.3
Power/beam	P/MW	5.0	0.14
Bunch average frequency	f_r/kHz	5.8	0.22
Invariant emittances	$\epsilon_{nx}, \epsilon_{ny}/10^{-6} \text{m}$	2.8; 2.8	2.5; 0.025
Disruption parameter	D	0.9	6
Pinch factor	H_D	3.5	2.4
Aspect ratio	R	1.0	180
Particles/bunch	$N/10^9$	5.4	8.0
Bunch r.m.s. length	σ_z/mm	0.5(*)	0.026
β -value at IP	$\beta_x^*, \beta_y^*/\text{mm}$	3; 3	14; 0.04
Bunch r.m.s. radii	$\sigma_x, \sigma_y/\text{nm}$	65; 65	190; 1
Fract.av.crit.en. = $\bar{E}_c/E_0$	$\bar{T}$	0.28	1.5
Av.critical energy	$\bar{E}_c/\text{TeV}$	0.28	0.75
Fluctuation parameter	$\Gamma/10^{-3}$	1.55	0.15
Regime parameter	p	0.13	0.69
Quantum factor	$F(\bar{T})$	0.41	0.13
Fractional energy loss	$\langle \epsilon \rangle$	0.20	0.19
Av.number of photons	N_γ	2.9	1.25
Av.fract.photon energy	$\langle \hbar\omega \rangle / E_0$	0.068	0.15
Collision energy	W/TeV	2	1
Fractional r.m.s. of W	σ_W/W	0.14	0.17
r.m.s. of W	σ_W/GeV	140	85

(*) Later parameter lists foresee $\sigma_z \leq 0.3 \text{ mm}$.

The choice of the set $[E_0, L, P, D, R, <\epsilon>]$ is convenient but arbitrary. At least two other quantities have severe technical implications and are often fixed by external constraints: the repetition rate f_r and the bunch area $\sigma_x \sigma_y = (\epsilon_{nx} \epsilon_{ny} \beta_x^* \beta_y^*)^{1/2} / \gamma$. Each one of them can substitute R , for instance, in the set of independent parameters. Addressing first the inverse problem, given $\{E_0, L, P, D, R, <\epsilon>\}$ the repetition rate can be computed by using Eqs. (13), (21) and (27) and the bunch area obtained by a different combination of the same relations with Eq. (17):

$$\begin{aligned} f_r &= (2^3 \pi^2 / 3^2) (H_b / H) (r_e^3 \gamma L / \sigma_z) F(\bar{T}) / <\epsilon> \\ &= (2^5 \pi^2 / 3^2) (H_b / H) [2R / (1 + R)] [r_e^4 L^2 / (DH)] (E_0 / P) F(\bar{T}) / <\epsilon> \end{aligned} \quad (41)$$

$$\begin{aligned} \sigma_x \sigma_y &= (3^2 / 2^5 \pi^2) (H^2 / H_b) (\gamma r_e^2)^{-1} [P / (E_0 L)]^2 (\sigma_z / r_e) <\epsilon> / F(\bar{T}) \\ &= (3^2 / 2^7 \pi^3) [(1 + R) / 2R] (DH^3 / H_b) (r_e^4 L^3)^{-1} (P / E_0)^3 <\epsilon> / F(\bar{T}). \end{aligned} \quad (42)$$

These relations [20] are written in practical units in Table 5. Unfortunately these expressions are too complicated to be analytically inverted. Even if we cannot give all the interesting quantities explicitly expressed as a function of the sets $\{E_0, L, P, D, f_r, <\epsilon>\}$ and $\{E_0, L, P, D, \sigma_x \sigma_y, <\epsilon>\}$, for every interesting case numerical tables can be easily constructed using R as free parameter and computing f_r and $(\sigma_x \sigma_y)$ from Eqs. (41) and (42). Of course, *a posteriori* checks on the output values have to be made by comparing the values of σ_z , f_r , $\sigma_x \sigma_y$ and N (computed with the relations collected in Table 5) with what is technically feasible.

To go further in the use of all these relations we have now to review the accelerator components and their scaling laws.

6. EMITTANCE DAMPING

6.1 Positron production

At colliders having energy larger than about 20 GeV, positron production does not pose difficult problems since, on average, 20 GeV electrons showering in few radiation lengths produce more than 2 – 3 low-energy positrons. The spent beams, or fraction of them, can thus be used to produce positrons in a showering medium, as done at SLC [7]. Other schemes foresee the radiation of 10 – 50 MeV photons by electrons of a few hundred GeV, which pass through undulators, and the creation of electron-positron pairs in thin heavy targets [21]. Both methods produce positron (and electron) bunches which have initial invariant emittances in the range $\epsilon_0 \simeq 10^{-2} - 10^{-3}$ m, too large to achieve the submicron transverse dimensions, which are needed in the interaction point to obtain high luminosities. As we have seen in the last Section, reasonable extrapolations indicate that for a 1 TeV collider one needs $\epsilon_n \simeq 10^{-6}$ m, and thus the positron invariant emittance has to be reduced by a factor $10^3 - 10^4$.

Recent developments in laser sources promise intense electron bunches having emittances in the range $10^{-6} < \epsilon_n < 10^{-5}$ m, so that for electrons emittance damping may not be required [22]. For positrons, damping rings (DR's) offer at present the only method to produce small emittances, starting from source emittances in the range $\epsilon_0 \simeq 10^{-2} - 10^{-3}$ m. Since the needed damping ring system may be very expensive, other methods of directly producing low emittance positron bunches are worth pursuing [23].

6.2 Emittance of damping rings

Damping rings have been discussed at this Topical Course by Wiedemann [24]. Here the presentation is kept very general.

In a storage ring the transverse emittance decreases with time because the radiated photons carry away a fraction of the momentum which has both transverse and longitudinal components, whilst the RF system restores only the longitudinal momentum. The invariant emittance of a *weak* bunch, which has the initial emittance ϵ_0 , decreases with time towards the equilibrium emittance ϵ_d following the law

$$\epsilon_n = \epsilon_0 e^{-2t/\tau} + \epsilon_d (1 - e^{-2t/\tau}). \quad (43)$$

The (transverse) damping time τ is inversely proportional to the rate at which energy is radiated and, for an isomagnetic lattice of circumference $2\pi R$, is given by the expression [25]

$$\tau = (3/c\epsilon_e) (\rho_m^2/F_m \gamma_d^3), \quad (44)$$

where ρ_m is the bending radius in the n_m (identical) bending magnets of the ring, γ_d is the γ -value of the stored particles and F_m is the fraction of the circumference which is covered by magnets of lengths ℓ ($F_m = n_m \ell / (2\pi R) = \rho_m/R$). (We assume for simplicity that the "damping partition numbers" are $J_x = J_y = 1$ and $J_E = 2$ and, moreover, that the horizontal and vertical oscillations are fully coupled so that the two transverse emittances are equal.)

The equilibrium emittance ϵ_d can be written in the simple form

$$\epsilon_d = (k_e/2) (2\pi/n_m)^3 \gamma_d^3 \quad (45)$$

where the factor $1/2$ comes from the full horizontal - vertical coupling and k_e depends on the particular type of lattice. For any lattice k_e is larger than the absolute minimum given in the first line of Table 4 [26]. The Double Bend Achromat lattice of Chasman and Green [27] has an emittance which is three times larger than the absolute minimum, while for an optimized FODO structure the constant k_e is about fifty times larger than the minimum.

Table 8
Values of the emittance constant k_e in Eq. (45)

Lattice	k_e/m
Fully optimized lattice	$8.25 \cdot 10^{-15}$
Chasman and Green	$2.48 \cdot 10^{-14}$
FODO	$4.75 \cdot 10^{-13}$

If the horizontal and vertical oscillations are not fully coupled, “flat” bunches can be extracted from the ring. The factor $1/2$ of Eq. (45) becomes $(1 + k^2)^{-1}$ for the horizontal emittance and $k^2/(1 + k^2)$ for the vertical one, where k is the coupling factor $0 \leq k \leq 1$. A value $k^2 \approx 10^{-3}$ is the minimum obtained in practice, and $k^2 \approx 10^{-2}$ is considered to be practically achievable in normal rings. In conclusion, one can get $\epsilon_{ny} / \epsilon_{nx} \approx 10^{-2}$; in this case, by multiplying Eq. (45) by 2 one obtains the limiting emittance in the horizontal plane.

The strong γ_d -dependence of Eq. (44) seems to suggest that very low energy positrons should be stored in a damping ring in order to obtain small emittances. However, intrabeam scattering, i.e. multiple Coulomb scattering within one bunch, causes a blow up of the transverse emittance which becomes more important when the energy is lowered. The physics behind this important limiting factor can be described as follows. In the reference system of the bunch, the transverse momentum spread is much larger than the longitudinal one so that any scattering transforms transverse motion into longitudinal motion. This causes fluctuations in the longitudinal momentum and, consequently, jumps in the equilibrium orbit and blow up of the transverse emittance [28,29].

Intrabeam scattering, which is not discussed further here because it has been presented in detail by Piwinsky during this Topical Course [28], tends to increase the emittance where synchrotron radiation alone reduces it with the damping time τ given by Eq. (44) to the equilibrium value. For $N \approx 10^{10}$ and a normal lattice, intrabeam scattering is negligible, with respect to synchrotron radiation damping, for an energy $E_d \geq 2$ GeV. By trying to obtain very small vertical emittances without requiring lifetimes of the order of a millisecond, an optimum value of the energy has been obtained by Palmer [30] in a ring with wigglers [31]; by requiring that the two effects are similar in order of magnitude, he obtained $E_d \approx 1.5$ GeV and $\epsilon_{nx} = 2.5 \cdot 10^{-6}$, $\epsilon_{ny} \approx 2.5 \cdot 10^{-8}$, as needed for SC (Table 7).

6.3 Damping ring parameters

A standard FODO structure characterized by the constant k_e of Table 8, cannot easily produce $\epsilon_{nx} \approx \epsilon_{ny} \approx 10^{-6}$ at $\gamma_d \approx 5 \cdot 10^3$, if one wants a damping time $\tau \leq 5$ ms. With a Chasman and Green lattice one obtains $\epsilon_d \approx 10^{-6}$ if the number of bending magnets is $n_m \approx 60$. With a magnet length $\ell \approx$

0.5 m and $F_m \approx 1/3$, the circumference of such a ring would be $2\pi R \approx 100$ m, the bending radius $\rho \approx 6$ m, the field $B \approx 1.2$ tesla, and the tune $Q_x \approx 30$. The damping time is computed from Eq. (44): $\tau \approx 3$ ms.

As a complement of information with respect to the presentation by Wiedemann [24], Table 9 contains two parameter lists considered in Ref. [20] in connection with CLIC designs. No dynamical apertures have been computed yet, and the second scheme may have problems at this level because the lower the emittance, the stronger are the sextupoles required to correct the chromaticity of the lattice [24]. This, in turn, may limit the aperture of the ring because the dynamics becomes non-linear.

Table 9
Examples of damping rings [18]

Energy, E_d/Gev	2.5	2.5
Number of particles/bunch, N	9.8×10^9	1.10^8
Number of dipoles, n_m	64	180
Dipole length, ℓ/m	.87	.29
Bending field, B/T	1	1
Bending radius, ρ/m	8.3	8.3
Average radius, R/m	25	25
Normalized emittance, $\epsilon_{nx} = \epsilon_{ny}/\text{m}$	5.7×10^{-6}	2.6×10^{-7}
Momentum compaction	5.3×10^{-4}	6.8×10^{-5}
Energy loss/turn, KeV	111	111
Damping time, ms	6	6
r.m.s. energy spread	7.6×10^{-4}	7.6×10^{-4}
RF frequency, MHz	500	500
Voltage/turn, MV	3.5	1.2
r.m.s. bunch length, σ_z/mm	2.2	1.1

The above qualitative arguments show that the lattices made of bending magnets, quadrupoles (and sextupoles) have to be carefully optimized to obtain emittances of the order of 10^{-6} m for bunches containing $N \approx 10^{10}$ particles. Can one improve the situation and obtain even smaller emittances by using wiggler magnets? When wigglers are added the damping time τ decreases and the energy spread σ_E increases, which is good news. The emittance can either increase or decrease depending on the strength of the wigglers and on the value of the lattice dispersion and β -function at their location. In optimal conditions the emittance can be made to decrease as the damping time. This implies a gain on all fronts.

As already mentioned, an interesting variant of a standard lattice has been proposed by Palmer [30] following a suggestion by Steffen [31]: each bending magnet is substituted by a wiggler whose absolute field B is larger than the average field $\bar{B}$, so that the ratio $r = \bar{B}/B < 1$. It can be shown that in this case both the emittance of Eq. (45) and the damping time of Eq. (44) have to be multiplied by r^2 . In the examples of Table 9, $\bar{B} = 1$ T; with $B = 2$ T one gains a factor $r^2 \approx 4$, which shows that average emittances $\sqrt{\epsilon_{nx} \epsilon_{ny}}$ of the order of 10^{-7} m can be obtained. Careful studies of the dynamical aperture and of the alignment tolerances are needed before being sure that such rings will run and perform as predicted. In the SC parameter list of Table 7 the collision frequency is $f_T \approx 0.2$ kHz and the lifetime should not be particularly short. This simplifies the problem.

It follows from Eq. (43) that a bunch which initially has $\epsilon_0 \approx 10^{-3}$ m has to remain in its damping ring a waiting time $T \approx 5 \tau$. If the average repetition frequency at which the (positron) bunches have to be extracted from the system is f_T , the needed number of damping rings is $N_d = f_T T/N_b$, where N_b is the number of bunches stored in each ring at any time [14]. By indicating with the symbol ℓ_b the average distance between two consecutive bunches, the total length ℓ_d of the (positron) damping system is:

$$\ell_d = (T/\tau) f_T \tau \ell_b \approx 5 f_T \tau \ell_b. \quad (46)$$

For $\tau \approx 3$ ms, $f_T \approx 6$ kHz (as in the present list of the CLIC parameters of Table 7 and $\ell_b \approx 10$ m one has to build a positron DR system whose total length is $\ell_d \approx 1000$ m. (At $f_T \approx 0.2$ kHz one needs 60 m, i.e. one DR is enough.) An interbunch distance of 10 m is not too much for the presently available kickers, if they have to extract the positron bunches without disturbing their small emittance. It is clear that the development of *faster* kickers may save a lot of money by entailing a proportional reduction of the total length of the DR system.

It is also possible to reduce ℓ_b by injecting and extracting trains of closely spaced bunches, as required in the multibunch high luminosity scheme described at this Course by Schnell [32]. In any case there will be many bunches in a ring at any time and sophisticated feedback systems have to be used to avoid multibunch instabilities.

In conclusion the development of low emittance electron sources is such that one may avoid the construction of a costly damping system for the electron bunches. For the positrons such a system is needed and may be a few km long, if repetition frequencies in the 10 kHz range are required. At CERN long rings in the ISR or in the LEP tunnels have been considered to be more advantageous than tens of smaller rings [14].

The present understanding of damping rings is such that emittances ten times smaller than SLC (i.e. $\epsilon_n \simeq 10^{-6}$ m) can be expected in the horizontal plane, even if no detail project has been worked out. In the vertical plane, emittances 100 times smaller may be achievable [30].

7. COLLIDERS BASED ON COPPER ACCELERATING STRUCTURES

7.1 "Near future" colliders

It is by now generally accepted that the acceleration techniques of the next generation of linear colliders will be 'reasonable' extrapolations of the ones in use today [19,33]. The most promising candidates are copper cavities running at high frequency ($10 \leq f \leq 30$ GHz), as discussed by W. Schnell at this Course [32]. Superconducting cavities would be an ideal solution, but at present the achievable gradients are too small and imply long and expensive linacs to reach the TeV energy range. This and the next Sections are devoted to these two possibilities, leaving aside the many interesting but more exotic acceleration methods, which have certainly to be pursued for the longer term development of the field, but are not ripe enough for NLC.

7.2 Scaling laws of normal conducting linacs

Following the discussion of Refs. [32-34], let us consider a normal conducting linac made of travelling wave sections of length L at frequency f . Given the stored energy W' per unit length, the power dissipation per unit length P_d' determines the decay time of the energy: W'/P_d' . The structure (unloaded) quality factor Q is defined as the ratio between this decay time and the characteristic time of the RF oscillation: $(2\pi f)^{-1} = 1/\omega$:

$$Q = \omega W' / P_d' . \quad (47)$$

The quantity τ , proportional to the decay time, fixes the time it takes to build up the field:

$$\tau = W'/(2P_d') = Q/2\omega . \quad (48)$$

(The factor 1/2 corresponds to the choice made in Ref. [34]. In general $\tau = Q/\alpha$ where α is the 'attenuation constant' of the structure.)

Each one of the sections is excited in a resonant mode with an electric field component in the direction of the particle motion; the excitation is caused by a square power pulse of frequency f ($\omega = 2\pi f$), duration τ and peak power $\hat{P}_L$. The group velocity v_g , which is the velocity of the energy flow, determines the filling time so that $\tau = L/v_g$. (In the structures to be considered here typically $v_g/c \approx 0.05-0.1$.)

The dissipation per unit length is obviously proportional to the square of the accelerating field E (in volt/m), so that one writes

$$P_d' = E^2 / R' ; \quad (49)$$

the constant R' is the *shunt impedance per unit length* and is measured in Ω/m . By eliminating P_d' between Eqs. (47) and (49) one gets the 'R over Q per unit length'

$$\frac{R'}{Q} = \frac{E^2}{\omega W'} = r' \quad (50)$$

which depends only on the geometry and not on the losses of the structure. The 'R over Q', or the equivalent 'loss parameter' k_0 and the 'elastance' s

$$k_0 = \omega r'/4 , \quad s = \omega r' , \quad (51)$$

determine the peak power needed to obtain the desired accelerating field. A SLAC disc-loaded structure with $v_g \approx 0.07$ ($f \approx 3$ GHz, $Q = 1.3 \cdot 10^4$) has $r' = R'/Q \approx 2.7 \cdot 10^3 \Omega/m$, so that for the present gradient of 17 MV/m one would need $W' \approx 5.7$ J/m. Table 10 shows that, for a fixed geometry, $r' = R'/Q \propto \omega$ so that the linear energy density W' needed to produce the accelerating field E is

$$W' \propto (E/\omega)^2 \propto \omega^{-2} . \quad (52)$$

This equation is simply stating that the energy per meter is proportional to the square of the electric field and to the volume of the structure, itself proportional to the square of the RF wavelength.

Table 10
Frequency dependence of the parameters of a RF accelerating structure
(for fixed geometry and accelerating field)

Quantity	Dependence	Quantity	Dependence
Q	$\omega^{-1/2}$	τ	$\omega^{-3/2}$
R'	$\omega^{1/2}$	P_d'	$\omega^{-1/2}$
$r' = R'/Q$	ω	W'	ω^{-2}
k_0	ω^2	$\hat{P}'$	$\omega^{-1/2}$
s	ω^2	P'_{rf}	ω^{-2}
v_g	1	$\hat{P}_L$	ω^{-2}

SLC ($E_0 = 50$ GeV) runs at ~ 17 MV/m and $f \approx 3$ GHz; Eq. (52) shows that by increasing at the same time the gradient and the frequency $f = \omega/2\pi$ by a factor of about ten, the required energy per meter would not change while the linac would become ten times shorter. What about the peak power and the average power?

Clearly the peak power per meter $\hat{P}' = \hat{P}_L/L$ has to be larger than the dissipated power of Eq. (49).

The dissipation during the filling time can be tuned; a good compromise gives [34]:

$$\hat{P}' \approx 2.5 P_d' = 2.5 E^2/R' \propto \omega^{-1/2} . \quad (53)$$

Such a power is very large. For $f = 30$ GHz the scaling laws of Table 10 applied to the structure with $v_g = 0.07$ give $Q \approx 4 \cdot 10^3$, $r' = 2.7 \cdot 10^4 \Omega/\text{m}$ and $R' = 110 \text{ M}\Omega/\text{m}$, so that to get $E = 170$ MV/m one needs $\hat{P}' \approx 650 \text{ MW/m}$! The filling time of Eq. (48) fixes the energy $\hat{P}'\tau \propto \omega^{-2}$ needed to power once 1 m of structure; the average power per meter to the structure is then obtained multiplying $\hat{P}'\tau$ by the RF repetition rate f_{rf} :

$$P'_{rf} \approx 1.25 E^2 f_{rf} (r'\omega)^{-1} = 1.25 W' f_{rf} \propto \omega^{-2} . \quad (54)$$

Finally, the peak power $\hat{P}_L$ per section of linac is $\hat{P}_L = \hat{P}' L = \hat{P}' \tau v_g$, i.e.

$$\hat{P}_L = 1.25 E^2 v_g (r'\omega)^{-1} \propto \omega^{-2}. \quad (55)$$

In our example at $f = 30$ GHz ($\tau \simeq 10$ ns) with $f_{rf} \simeq 0.2$ kHz (as foreseen for SC in Table 7) Eq. (54) gives the average power $P'_{rf} \simeq 1.3$ kW/m and, with $v_g = 0.07 c$, $L \simeq 0.2$ m and $\hat{P}_L \simeq 130$ MW.

At 170 MV/m such a (0.5+0.5) TeV collider would be 6 km long for a total peak power of about 4 Tevawatts to be given to 30.000 sections (!) and a total average RF power of $1.3 \text{ kW/m} \times 6 \cdot 10^3 \text{ m} \simeq 8$ MW dissipated in the structure. The CLIC choice (Table 7) is $f_{rf} \simeq 6$ kHz and, for the same field, the average power would be 30 times larger. To reduce the plug power P_{ac} one can decrease the accelerating field E since, for a given cm energy $W = 2 E_0$,

$$P_{ac} = \eta_{rf}^{-1} (2 E_0/e E) P'_{rf} = 2.5 \eta_{rf}^{-1} (E_0/e) f_{rf} E (r'\omega)^{-1} \propto \omega^{-2}, \quad (56)$$

where η_{rf} is the efficiency for converting 'wall plug' power to RF power (typically $\eta_{rf} \simeq 0.5$). The present CLIC design foresees $E \simeq 100$ MV/m.

Of course some power is spent to accelerate the electron and positron beams. When a bunch of charge Ne interacts with the structure it induces a field $E_i = 2 k_0 Ne$ which cancels part of the accelerating field. The average particle of the bunch will thus see the field $(E - E_i/2)$ and this causes a momentum spread. If η is the *fraction* of the stored energy extracted by a bunch, when no particular attention is paid to the problem, the momentum spread is of the order of $\eta/2$. One can do better [37] by choosing the phase of the bunch with respect to the RF wave such that, without beam loading, the particles in the tail of the bunch would see a larger accelerating field. If b bunches of N particles each are accelerated during a single RF pulse, and extra power is poored in the structure in between bunches to compensate for the energy extracted, the efficiency of the system can be increased without augmenting the momentum spread. In this case the collision repetition rate f_r is

$$f_r = b f_{rf}, \quad (57)$$

and the fraction of stored energy which is extracted is

$$\eta \simeq b' Ne E/W' = b' Ne r \omega'/E, \quad (58)$$

where $b' < b$ to take into account off-crest acceleration and multibunch compensation. For a single bunch $\eta \leq 10\%$, because of the momentum spread, and one can hope to have $\eta \approx 30\%$ in the multibunch compensated scheme [34]. The total energy transfer efficiency η_t from well-plug to beam is

$$\eta_t = 0.8 \eta_{rf} \eta = 0.8 \eta_{rf} b' N e r \omega' / E, \quad (59)$$

and the power/beam

$$P = \eta_t P_{ac} / 2. \quad (60)$$

We can now make contact with the discussion of Section 4, and in particular with the equations of Table 5. By using, as normalisation, the quoted 'R over Q' value $r' \approx 2.7 \cdot 10^4 \Omega/m$ (for a structure at 30 GHz with $v_g \approx 0.07$) and $\eta_{rf} = 0.5$, in practical units Eq. (56) becomes [19]

$$P_{ac}/MW \approx (880/b') (E_0/TeV) (f_r/kHz) (E/MV m^{-1}) (GHz/f)^2, \quad (61)$$

which clearly displays the gain in AC power implied by high RF frequencies. Another advantage of increasing f is that the fraction of the energy which is extracted from the cavity

$$\eta = 9.1 \cdot 10^{-4} b' (N/10^9) (MV m^{-1}/E) (f/GHz)^2 \quad (62)$$

increases. Since luminosity is proportional to beam power [Eq. (20)] the fraction η has to be as large as possible, at the limit of what is allowed by the momentum spread and other constraints.

By combining Eqs. (29) and (61) one gets [19]:

$$b'^{1/2} f/GHz = 115 (H_b/H)^{1/2} (E_0/TeV)^{1/2} (L/10^{33}) (MW/P_{ac})^{1/2} (E/MeV m^{-1})^{1/2} \bar{T} F(\bar{T}) / \langle \epsilon \rangle. \quad (63)$$

For a 'standard' NLC ($W = 2E_0 = 1$ TeV, $L = 10^{33} \text{cm}^{-2} \text{s}^{-1}$, $P_{ac} = 100$ MW, $D = 2$, $H_D \approx 5.7$, $\langle \epsilon \rangle = 0.3$, $b' = 1$, $E = 100$ MV m^{-1}) which runs in the transition regime, (so that $\bar{T} F(\bar{T}) = 0.15 \pm .05$), Eq. (63) implies $f \approx (40 \pm 13) (H_b/H)^{1/2}$ GHz which, for $R = 1$, is ten times larger than the SLC frequency. The factor $(H_b/H)^{1/2}$, which is equal to 1 for $R = 1$, decreases as $2R^{-1/2} H_D^{-1/4} \approx 1.3 R^{-1/2}$ for $R \geq 3$. Flat bunches are thus useful to decrease either the RF frequency, or the plug power.

As discussed in Section 5, a collider is described by a set of six independent variables, for instance $\{E_0, L, P, D, \sigma_x \sigma_y, <\epsilon>\}$. We have now added two independent relations, Eqs. (60) and (61), since (62) follows from these two. The new variables are η, P_{ac}, E and f . By keeping $\eta_{ff} = 0.5$, the two extra parameters can be chosen to be η and E , so that the collider is determined by the set

$$\{E_0, L, P, D, \sigma_x \sigma_y, <\epsilon>, \eta, b', E\}, \quad (64)$$

where η is (roughly) proportional to the number of bunches per RF pulse b . Given these eight quantities all the others can be computed by using R as auxiliary variable. One can apply the following procedure:

1. Given P compute, with Eq. (40) of Table 5, the regime parameter $p(R)$ as a function of the aspect ratio R . (Eq. (20) helps in checking if the length of the bunch σ_z is 'reasonable'.)
2. Use Eqs. (39) and (28) to get $\bar{T}$ and $F(\bar{T})$ as a function of R .
3. Find by consistency the value of R which satisfies Eq. (42). If no solution is found it means that P is too small for the chosen value of $\sigma_x \sigma_y$. (Note that $\sigma_x \sigma_y \propto P^3$.)
4. Derive f_r from Eq. (29) with the value of R found in step 3.
5. Compute N from Eq. (3).
6. Using $P_{ac} = 5 P/\eta$, obtain $f^2 b'/E$ from Eq. (61).
7. Compute f assuming the value of E appearing in the initial set of data and b' compatible with the input value of η . If f is not in the range of the available technique, E (and b') can be adjusted to get the wanted value of $f^2 b'/E$.
8. Finally, σ_z is obtained from Eq. (20).

A reasonable set of parameters for a 'standard' NLC, with $\sqrt{\epsilon_{nx} \epsilon_{ny}} \simeq 10^{-6}$ m and $\sqrt{\beta_x^* \beta_y^*} \simeq 3$ mm is:

$$\{0.5 \text{ TeV}, 10^{33} \text{ cm}^{-2} \text{s}^{-1}, 1 \text{ MW}, D=2, 3.10^3 \text{ nm}^2, <\epsilon>=0.3, \eta=.05, b'=1, 100 \text{ MV m}^{-1}\}.$$

The above procedure gives: $R=8$, $\sigma_x=155$, $\sigma_y=19$ nm, $p=0.34$, $\bar{T}=0.57$, $F(\bar{T})=0.28$, $f_r=0.88$ kHz, $N=1.4 \cdot 10^{10}$, $f=20$ GHz, $\sigma_z = 0.10$ mm.

7.3 Final focus and alignment

The input value $(\sigma_x \sigma_y)$ is determined by $\sqrt{\epsilon_{nx} \epsilon_{ny}}$ and $\sqrt{\beta_x^* \beta_y^*}$. Emittance forming was discussed in the last Section, where it was concluded that values of the order of 10^{-6} m are certainly obtainable, while a factor ten reduction may be achievable but needs detailed studies of unconventional lattices. For the quantity $\sqrt{\beta_x^* \beta_y^*}$ the minimum achievable value is the subject of considerable debate and progress is continuous. At the Topical Course K. Brown discussed it in detail [38,39] and I limit myself to a summary of the main results.

If the bunches collide head-on the disrupted beams have to pass through the holes of the opposite quadrupoles. When the bunch does not pinch, the typical angles of the deflected particles are equal in the two planes. Following the argument given at the beginning of Subsection 3.2, the angle of a particle at distance σ_x from the axis is

$$\Theta = \sigma_z/F = D_x \sigma_x / \sigma_z = (2 r_e N / \gamma) (\sigma_x + \sigma_y)^{-1}, \quad (65)$$

and the angles in the two planes are equal because $D_x \sigma_x = D_y \sigma_y$ [Eqs. (16) and (17)]. By integrating on the transverse dimensions of the bunch with $\sigma_x = \sigma_y$ it was shown that $\sim 0.9 \Theta$ is the *maximum* angle without pinch [39]. When pinch effects are present, a computer simulation is needed [40]: since the number of betatron oscillations is $\sim \sqrt{D/10}$, for $D \gg 1$ one obtains a maximum angle $\Theta \simeq (3D)^{1/2} (\sigma_y / \sigma_x)$, numerically not very different from Eq. (65) for $D \simeq 5$. In the case of round beams, the β^* which can be achieved with a given momentum spread σ_p/p is [38,41]:

$$\beta_x^* (\min) = \beta_y^* (\min) = 2 \cdot 10^{-7} \text{mm} [N (\sigma_p/p)^2 (T/B_p)]^{2/3} (\gamma m / \epsilon_n)^{1/3} \quad (66)$$

where B_p is the pole tip field in the final focusing quadrupole. For the 'standard' NLC of the last Subsection with $\sigma_p/p \simeq 5 \cdot 10^{-3}$, $B_p = 1$ Tesla and $\epsilon_{nx} = \epsilon_{ny} = \epsilon_n = 10^{-6}$, one would obtain $\beta_x^* (\min) = \beta_y^* (\min) \simeq 9$ mm, about five times larger than what was a priori chosen for $\sqrt{\beta_x^* \beta_y^*}$. Fortunately flat beams help. This can be achieved either having equal transverse emittances ($\epsilon_n = \epsilon_{nx} = \epsilon_{ny}$) but $\beta_x^* = R^2 \beta_y^*$ (as in Ref. [14]), or unequal emittances and unequal β^* 's, as in Ref. [36], where flat beams intersect at an angle so that the disrupted beams do not have to pass through the aperture of the opposite quadrupole. With small momentum spreads ($\sigma_p/p \simeq 10^{-3}$), so that chromatic corrections

are not too large, one can get $\sqrt{\beta_x^* \beta_y^*} \approx 1 \text{ mm}^2$ at 1 TeV and recent ideas [42,43] indicate that, if $\beta_x^* \neq \beta_y^*$, conventional focusing systems can reach this goal even for $\sigma_p/p \approx 5 \cdot 10^{-3}$. Much smaller β 's can be obtained with the high fields available in a plasma lens, as discussed in Refs. [44] and [45], but they need still to be developed and tested.

The magnetic elements of the final focus system have to be aligned *and* be kept aligned to an accuracy which is better than the bunch dimensions. The needed very sophisticated feedback systems work better when the repetition rate f_{rf} is large. As stressed by Panofsky [55], this favours high repetition rate colliders and, in particular, the fully superconducting colliders to be discussed in the next Section. The alignment of the accelerator elements and of the quadrupoles is very delicate, since at frequencies in the range 10–30 GHz micron accuracies are needed. The problem of the alignment tolerances is strongly correlated with the choice of the bunch momentum spread (needed for the Landau damping of the head-tail effects) and with the intensity of the wake fields [46]. The transverse (longitudinal) wake fields are proportional to $f^3(f^2)$, and so put severe limitations on the frequency choice and on the alignment accuracy. These items have been discussed at the Topical Course by R. Ruth [47] and are not repeated here. They enter with a large weight in the overall optimisation of a linear collider [36].

8. FULLY SUPERCONDUCTING LINEAR COLLIDERS

8.1 Superconducting cavities

Linear colliders based on superconducting (SC) cavities were proposed and studied at CERN more than ten years ago [2,48]. In the last years many progresses have been made in the construction and understanding of SC radiofrequency cavities [49,50]. For single cell cavities, gradients as large as $E \approx 23 \text{ MV/m}$ and quality factors up to $Q \approx 10^{10}$ have been obtained. Multicell cavities in the laboratory have reached $E \approx 15 \text{ MV/m}$ for $f = 1.5 \text{ GHz}$, and are routinely constructed by industry with accelerating fields $E \geq 7 \text{ MV/m}$ and $Q \geq 3 \cdot 10^9$ in the frequency range $0.35 \leq f \leq 1.5 \text{ GHz}$. It is believed that few years of technological developments on the preparation of clean and defect free surfaces should allow the reaching of $E \approx 25 \text{ MV/m}$ and $Q = 5 \cdot 10^{10}$. The development of type II supercon-

ductory, like Nb₃Sn and NbN, may be sputtered on a copper substrate [51], offers great promises for reaching even higher gradients and quality factors. For linear colliders economic fabrication and treatment will of course become of paramount importance. New "warm" superconductors have higher critical fields and theoretically could reach even 500 MV/m [52], but the phenomenon is not yet understood and the technology is far into the future.

In the work on fully SC colliders presented in Ref. [53] the following parameters were assumed; $f = 1$ GHz, $E = 25$ MeV/m, $Q = 5 \cdot 10^{10}$, temperature = 1.8° K. In parallel with the work done at CERN, the Cornell group has also studied fully superconducting linear colliders, devoting particular attention to cost optimisation [54]. In particular, Sundelin et al. use $E = 28.8$ MV/m and $Q = 5 \cdot 10^{10}$ (4 K) at a frequency $f = 2.86$ GHz. The independent assumptions and conclusions of these two studies are very similar.

8.2 Superconducting colliders

Superconducting (SC) cavities would offer an ideal solution to the problem posed by linear colliders [33,55], if it was not for the too low gradients presently achievable, which imply long machines. The main advantage of SC linear colliders is that the electromagnetic energy can be kept in the structure from one acceleration cycle to the next and so high efficiencies are achievable. Wall losses (which take place at low temperatures) are given by Eq. (49)

$$P_d' = E^2 (r'Q)^{-1}; \quad (49)$$

r' depends only on the geometry and $Q \propto R_{\text{BCS}}^{-1}$, where R_{BCS} is the electrical resistance of the superconductor, whose behaviour is well described by the Bardeen-Cooper-Schrieffer (BCS) theory. At a temperature $T < T_c/2$, where T_c is the critical temperature, and for frequencies small with respect to the frequency corresponding to the gap Δ of the superconductor (~ 700 GHz for niobium) one has

$$R_{\text{BCS}} \propto T^2 \exp(-\Delta/kT), \quad (67)$$

so that the Q-value is expected to be proportional to f^{-2} . Since in Eq. (49) $r' \propto f$, the RF-losses in the walls, which are the main cause of inefficiency in the transformation of wall-power to beam power, are $P_d' \propto f$. The theoretical limiting values of Q are very large $\sim 10^{11}$, but at present have not been obtained in multicell structures because defects in the SC material give rise to a residual resistance which adds to R_{BCS} .

In the case of SC cavities low frequencies are preferred because: (i) according to Eqs. (49) and (67) RF losses are proportional to f ; (ii) as seen in Subsection 7.3, transverse (longitudinal) wake fields vary as f^3 (f^2) and so are negligibly small when the frequency is ~ 1 GHz instead than 20 GHz, as considered in the last Section; (iii) the ratio between the energy extracted by a bunch and the energy stored is proportional to f^2/E [Eq. (58)]; (iv) alignment of the structure and of the quadrupoles along the linac is not a problem with a hole which is of the order of a few cm's. Moreover, the cost per meter increases with the frequency because the number of cells and feedthroughs increases.

Another advantage of having a relatively large wavelength and negligible longitudinal wake fields is that the momentum spread of the final beams can be very small. (At low energy SC linacs, as the one under construction a CEBAF, the project value is $\sigma_p/p \simeq 10^{-4}$.) A reduction of a factor of 10 in the momentum spread, with respect to what was considered in the last Section, greatly simplifies the design of the final focus system, since Eq. (66) shows that $\beta^* \propto (\sigma_p/p)^{4/3}$. Finally one should also mention that, since a SC linac runs (practically) continuously, the choice of the bunch repetition rate f_r is not as constrained as in normal conducting linac. All these arguments justify the fact that with fully SC colliders one can really aim at luminosities of the order of $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ at $(1+1)$ TeV.

Table 11, very similar to the one appearing in Ref. [53], lists the main parameters of two designs of a high luminosity ($10^{34} \text{ cm}^{-2} \text{ s}^{-1}$) $(1+1)$ TeV fully SC collider, for which some cost optimizations have been performed.

Table 11
Sets of parameters for fully superconducting (1 + 1) TeV colliders
with $L = 10^{34} \text{cm}^{-2} \text{s}^{-1}$ ($D = 2$, $H = 5.7$)

	Energy recovery	No recovery
Beam power, P (MW)	100	10
Recovery efficiency, η	0.90	0.00
Fractional energy loss, $\langle \epsilon \rangle$	0.03	0.3
Fract.av.critical energy, $\bar{T}$	0.025	0.69
Av.number of photons, N_γ	2.9	3.6
Fract. r.m.s. of W, σ_W/W	0.015	0.20
Regime parameter, p	$6.8 \cdot 10^{-3}$	0.22
Invariant emittance, ϵ_n/m	$2.5 \cdot 10^{-6}$	$1.0 \cdot 10^{-6}$
Bunch length, σ_z/mm	3.9	0.39
β -value at crossing, β^*/mm	17	1.2
Bunch radius, $\sigma_x = \sigma_y/\text{nm}$	150	25
Particles/bunch, N	$7.5 \cdot 10^9$	$2.2 \cdot 10^9$
Macroscopic duty factor, C	0.15	0.15
Bunch peak repetition frequency, kHz	550	190
Train peak frequency, kHz	55	19
Average bunch repetition frequency, f_r/kHz	83	28
Average train repetition frequency f_t (kHz)	8.3	2.8
Gradient, MeV/m	25	25
Q-value	$5 \cdot 10^{10}$	$5 \cdot 10^{10}$
Frequency of RF, f/GHz	1	1

The first design uses the principle of *energy recovery* [1,2] to save on the total power, gives a very good energy resolution and has parameters which are less demanding than the ones considered in the last Section for the 'standard' NLC. This is particularly true for the length of the bunch and the β^* -value. (Note that both collider have "round" bunches). The damping ring system is a very important and *expensive* part of the complex since the repetition rate is *very* large (see Eq. (46)) to get a luminosity larger by a factor 10. To reduce the number of rings one can place 10 bunches, for instance, at about 30 cm distance. The 10 pulses would be extracted from their ring and accelerated as a single train. This is possible because wake field effects are very small at low frequencies ($f = 1$ GHz). The second set of parameters is for a collider which has no energy recovery and runs in the transition regime. The required emittance is still reasonable, and the β^* -value is similar to the one considered for the 'standard' NLC in the last Section.

The cost optimisation procedure, which takes into account klystron replacement and electricity consumption over 10 years, leads to the conclusion that the accelerator has to be run with a macroscopic duty factor $C \simeq 10\%$, because the announced $Q = 5 \cdot 10^{10}$ (ten times larger than what is today obtained) is still not large enough to reduce the wall losses. Both designs need a power from the mains of about 350 MW. (Remember that $L \propto P$, so that also here the requirement $L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ has heavy consequences.) By making different hypothesis on the cost components and by assuming that only one bunch is damped at any time in each ring, the Cornell studies concluded that the optimum macroscopic duty factor has to be about ten times smaller [54].

Superconducting accelerating structures, with their capability of running continuously, promise luminosities in the $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ range with parameters which are not very different from the SLC ones. As already underlined, they would be an ideal solution for NLC if it was not for the relatively low value of the accelerating field which is technically feasible today. Since in Nb_3Sn cavities the theoretical limit is 100 MV/m [50], there is space for future improvements. The problem is challenging because Q-factor definitely larger than 10^{10} have to be obtained to reduce the losses at cryogenic temperature. It is comforting that at 1.8° K the theoretical limit is $Q \simeq 10^{11}$ for $\omega = 1 \text{ GHz}$ and $Q \simeq 10^{12}$ at $\omega = 0.35 \text{ GHz}$. We may have technological breakthroughs in the next years, and the use of SC cavities in low energy-high luminosity collider to abundantly produce heavy flavours [56] could help in moving in this direction. If the breakthrough will not take place, one of the approaches presented at the Course by W. Schnell [32] and R. Palmer [57] will be the basis of the construction of a high-frequency and high-field normal conducting Next Linear Collider, which we hope will be running before the turn of the century.

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