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DETAILED NEUTRINO SPECTRA
FROM COLD DARK-MATTER ANNIHILATIONS IN THE SUN

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A B S T R A C T

Cold dark-matter particles captured by the Sun can annihilate and may produce an observable flux of high-energy neutrinos. We discuss the effects of the solar medium upon the neutrino flux. Muons, pions, and kaons lose their energy to the medium before undergoing a weak decay, so that the flux is dominated by prompt neutrinos from τ , c , b , or t decays. We calculate the differential fluxes of neutrinos from annihilations via $\tau\bar{\tau}$, $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ final states using a successful model for final states in e^+e^- collisions augmented by our solar medium considerations. Fragmentation and finite mass effects are important for b and c decays. For sufficiently energetic fermions, stopping and absorption are important. We give the results of our calculations in a form useful for estimating event rates in terrestrial experiments. Finally, we apply our results to the case of a superstring dark-matter candidate.

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1. INTRODUCTION

Observations of the Universe at many different scales suggest that it contains matter which we cannot see [1]. Many candidates for this dark matter have been advanced, a number of which are species of massive elementary particles. Among these, the ones currently favoured are cold dark-matter (CDM) candidates [2]. Whilst CDM could be composed of very light particles (e.g. axions [3]), in most models it consists of heavy particles weighing 1 GeV or more [4]. One would like to test such a hypothesis in a laboratory experiment. One possibility is to look for neutrinos from the annihilations of heavy CDM particles that have been captured in the Sun [5-7]. This possibility is the subject of this paper.

We will not discuss other signals such as gamma rays, positrons, or antiprotons from annihilations in the galactic halo [8]; or the many suggestions [9] for direct detection of CDM via the elastic scattering of CDM particles off nuclei in a low-temperature detector.

Several papers have addressed the possibility of detecting neutrinos from CDM annihilations in the Sun. Only recently has it clearly been stated that the solar medium plays an important role in that muons, pions, and kaons will either stop or be absorbed before they undergo a weak decay [6]. Any observable flux of neutrinos must come from direct annihilation of CDM into neutrinos, or from annihilation into $\tau\bar{\tau}$, $c\bar{c}$, $b\bar{b}$, or $t\bar{t}$ followed by subsequent decay of the heavy fermion. In section 2 we will emphasize this point. Furthermore, we show that the solar medium is potentially important for stopping even the heavy fermions and will play a role in absorbing some of the neutrino flux.

A second shortcoming of recent work is that the neutrino fluxes have been estimated using relatively crude models. We attempt to remedy this situation by using the Lund program [10] for modelling e^+e^- collisions. Our reasoning here is that a particular CDM annihilation

lation event results in a fermion-antifermion pair. Once that pair is produced, its subsequent evolution should be indistinguishable from a similar fermion-antifermion pair produced in an e^+e^- annihilation. Given this, it seems reasonable that the easiest way to handle the multitude of decay possibilities and kinematics is to employ an existing program designed to do just that. All we need do when following the evolution of an event is to keep track of the final-state neutrinos resulting from heavy fermion decay. In section 3 we describe this procedure and its results in more detail.

A third characteristic of existing work is that it tends to concentrate on particular models. In section 4 we try to give our results in a form which is easily adaptable to any CDM model. To illustrate this, we apply our results to a particular CDM candidate arising from a superstring-inspired model [11-13]. This is done in section 5. Finally, in section 6 we summarize our results and comment on their validity.

2. INTERACTIONS WITH THE SOLAR MEDIUM

In this section we outline the basic physical processes and time scales that determine the flux of neutrinos resulting from CDM annihilations in the Sun. In general, an annihilation event results in a shower of ordinary particles (leptons, mesons, baryons) which then decay, scatter electromagnetically or strongly, or otherwise interact with the material of the Sun. The flux and the energy distribution of the neutrinos depend on the importance of weak decays compared with other physical processes. The weak decay rates and branching ratios are well known or may be estimated from theory, but for the other processes we need a knowledge of the environment in which the showers propagate. Therefore, our first task is to determine where in the Sun annihilations take place.

2.1 Annihilation profile

The annihilation rate is given by

$$\Gamma = nn \langle \sigma v \rangle_A . \quad (1)$$

For most dark-matter candidates the thermally averaged annihilation cross-section $\langle \sigma v \rangle_A$ is independent of temperature for temperatures as low as those present in the Sun ($T < 1.5 \times 10^7$ K). Therefore, in determining where annihilations occur, we need only determine the density of annihilating particles n . We model the dark-matter phase-space distribution as being characterized by a temperature T_D , [14, 15], and a chemical potential given by the gravitational potential relative to the core of the Sun,

$$n(r) = n_c e^{-m\phi(r)/T_D} \quad (2)$$

where n_c is the density at the core of the Sun and

$$\phi(r) = \int_0^r \frac{GM(r')dr'}{r'^2}, \quad M(r) = 4\pi \int_0^r \rho(r')r'^2 dr'. \quad (3)$$

The use of a global temperature T_D , independent of position, is justified because for most dark-matter candidates the time between dark matter-nucleus collisions is much longer than an orbital period and much, much less than the age of the Sun.

We determine T_D by demanding that the dark-matter distribution be in equilibrium. Specifically, we demand that the integrated energy exchange between dark-matter particles and nuclear matter be zero. The thermally averaged energy transfer between two ideal gases of particles with masses m_1 and m_2 , temperatures T_1 and T_2 , and densities n_1 and n_2 , is [15]

$$n_1 n_2 \langle \sigma v_{\text{rel}} \Delta E \rangle = 8 \sqrt{\frac{2}{\pi}} n_1 n_2 \sigma (1-Q) \frac{\mu}{M} \left(\frac{m_1 T_2 + m_2 T_1}{m_1 m_2} \right)^{1/2} (T_2 - T_1) \quad (4)$$

where we have assumed that the elastic cross-section σ is independent of the relative velocity v_{rel} , $M = m_1 + m_2$, $\mu = m_1 m_2 / M$, and $Q = (1/\sigma) \int (d\sigma/d\Omega) \cos \theta d\Omega$, with $d\sigma/d\Omega$ and θ given in centre-of-mass coordinates. (Note: $Q = 0$ for isotropic scattering.) We then demand that

$$\int n_o n_n \langle \sigma v_{\text{rel}} \Delta E \rangle dV = 0. \quad (5)$$

In principle, eq. (5) should be summed over nuclear species to take into account different cross-sections of dark matter with different nuclei. In practice, the dark matter is concentrated very close to the core, and the parameters describing the nuclear distributions change very little over the radial range of interest. All the detailed information can then be absorbed into an overall coefficient that does not affect balancing the integral in eq. (5) to zero. We use a solar model of Bahcall et al. [16] to solve

$$\int \rho(r) (T_o - T_N(r)) e^{-m_D \phi(r)/T_o} \left(\frac{m_o T_N + m_p T_o}{m_o m_p} \right)^{1/2} r^2 dr = 0 \quad (6)$$

where m_p is the proton mass. The results of this equation depend only on the mass of the dark-matter particle. We find that for $m_D = 3, 5, 10$, and 20 GeV, $T_D = 1.39, 1.45, 1.50$, and 1.53×10^7 K, respectively. For comparison, the core temperature of the solar model is $T_c = 1.55 \times 10^7$ K. Masses below 3 GeV are uninteresting because such particles are expected to evaporate before they annihilate [7].

Using the dark-matter distributions determined above, we have calculated the annihilation profiles. In Fig. 1 we show the distribu-

tion of dark-matter annihilations as a function of solar density for particles of various masses. As one can see, even for 3 GeV particles almost all annihilations occur in an environment where $\rho > 100 \text{ g/cm}^3$. For more massive particles ($m > 10 \text{ GeV}$) it is a reasonable approximation to take the nuclear density to be $\rho = \rho_c \simeq 150 \text{ g/cm}^3$.

2.2 Absorption and stopping of neutrinos

Now let us see what happens to various particles in the shower. The direct annihilation products may be leptons, quarks, or perhaps gauge bosons. Our attitude is that the dark-matter particle is the lightest particle embodying new physics, so its annihilation products must be within the Standard Model. We do not consider annihilation into Higgs scalars.

The simplest case is annihilation directly into neutrinos. The neutrino energy will be just the mass of the dark-matter particle. Ordinarily we think of the Sun as being transparent to neutrinos, but for sufficiently high energies this is not correct. We must consider absorption of neutrinos due to charge exchange, and energy loss due to neutral-current scattering off nucleons. We will neglect the scattering off electrons, which is smaller by a factor of ~ 1000 . In the energy range in which we are interested ($1 \text{ GeV} < E_\nu < 1 \text{ TeV}$), the charge-exchange cross-section is

$$\sigma = a E \times 10^{-39} \text{ cm}^2 \quad (7a)$$

where E is the neutrino or antineutrino energy in GeV, and $a_{\nu p} = 4.3$, $a_{\bar{\nu} n} = 9.6$, $a_{\nu n} = 4.1$, and $a_{\bar{\nu} p} = 2.3$ were determined using the parton distributions of Eichton et al. [17]. The subscripts (p,n) indicate scattering from protons and neutrons. Similarly, the neutral-current cross-sections are

$$\sigma = b E \times 10^{-39} \text{ cm}^2 \quad (7b)$$

with $b_{\nu p} = 2.1$, $b_{\nu n} = 2.3$, $b_{\bar{\nu} p} = 1.1$, and $b_{\bar{\nu} n} = 1.1$.

To estimate the effects of interactions, we calculate an average stopping power due to neutral currents. This result is then applied to determine the neutrino energy as a function of distance as the neutrino traverses the Sun. Finally, absorption probabilities are calculated for the slowed neutrino. An initial neutrino of energy E_i leaves the Sun with energy E_f and probability P_f .

The average stopping power is given by

$$\frac{dE}{dx} = - \left[(n\sigma y)_p + (n\sigma y)_n \right] E \quad (8)$$

where yE is the average energy lost by the neutrino per collision. We can separate the energy loss into an isoscalar part which depends only on the total density ρ , and a term that is weighted by the hydrogen mass fraction X_H . Recalling that $\sigma \approx E$, we find

$$E(x) = \frac{E_i}{1 + E_i \chi(x)} \quad (9a)$$

where

$$\chi(x) = \chi_o^v I_o(x) + \chi_H^v I_H(x) \quad (9b)$$

$$I_o(x) = \int_0^x \rho dx' \quad (9c)$$

$$I_H(x) = \int_0^x \rho X_H dx' \quad (9d)$$

$$\text{and } \chi_{o/H} = 10^{39} N_A R_o^{\frac{1}{2}} (b_p y_p \mp b_n y_n) \text{ GeV}^{-1} \quad (9e)$$

Here we have expressed ρ in g/cm^3 and used the dimensionless radial coordinate $x = r/R_\odot$. Numerically: $10^{-39} N_A R_\odot = 4.19 \times 10^{-5}$, $I_0(1) = 22.0$, $I_H(1) = 11.6$. Also, for neutrinos $1/2(b_p y_p + b_n y_n) = 1.1$, $1/2(b_p y_p - b_n y_n) = 0.073$; and for antineutrinos $1/2(b_p y_p + b_n y_n) = 0.41$, $1/2(b_p y_p - b_n y_n) = 0.010$. The energy of the neutrino as it leaves the Sun is determined by $\tau_v(1) = 1.01 \times 10^{-3} \text{ GeV}^{-1}$, or for antineutrinos by $\tau_v(1) = 3.8 \times 10^{-4} \text{ GeV}^{-1}$. For the case of very high initial energy, $E_v(f) \rightarrow \tau_v^{-1} = 1 \text{ TeV}$.

If there is a charged-current interaction, the neutrino will change into a charged lepton. In the case of electrons and muons, the charged lepton stops (see subsection 2.3). We therefore treat this as an absorption process. The flux φ decreases according to

$$\frac{d\varphi}{dx} = -[(n\sigma)_p + (n\sigma)_n]\varphi \quad (10)$$

which has the solution

$$\varphi(x) = \varphi_i e^{-I}, \quad (11a)$$

$$I = \int_0^x \frac{E_i}{1 + \chi(x')E_i} (\tilde{\sigma}_0 \rho + \tilde{\sigma}_H \rho X_H) dx' . \quad (11b)$$

We have again separated out the X_H -dependent part and defined

$\tilde{\sigma}_0/H = 10^{-39} N_A R_\odot 1/2(a_p +/ - a_n) \text{ GeV}^{-1}$. In general, it is not possible to evaluate I analytically; however, if we keep X_H fixed we may set $\rho dx = d\tau/(\tau_0 + \tau_H X_H)$ and perform the integral to determine

$$\varphi(x) = \varphi_i \left(\frac{1}{1 + E_i \chi(x)} \right)^\alpha \quad (11c)$$

where $\alpha = (\tilde{\sigma}_0 + \tilde{\sigma}_H X_H)/(\tau_0 + \tau_H X_H)$. In the limit where neutral currents are much weaker than charged currents,

$$\varphi(\nu) \rightarrow \varphi_i e^{-[E_i(\tilde{\sigma}_0 + \sigma_H x_H) I_0(\nu)]} \quad (11d)$$

In fact, if we pick $x_H = 1/2$ as an average value, then $\alpha_\nu = 5.1$ and $\alpha_{\bar{\nu}} = 9.0$. So, neutral currents do not greatly affect the absorption. Neglecting energy loss due to neutral currents, the escape probability is $P_f = \exp(-E_i/E_{\text{Abs}}^i)$, where $E_{\text{Abs}}^v = 198$ GeV and $E_{\text{Abs}}^{\bar{v}} = 296$ GeV.

For the case of a ν_τ , two things change. First, after a charge-exchange interaction the τ lepton is not absorbed (see subsection 2.3). Rather, it decays, reproducing a ν_τ with an average energy ≈ 0.18 times the energy of the original ν_τ . For a $\bar{\nu}_\tau$ the re-emitted energy is ≈ 0.24 times the original. The ν_τ or $\bar{\nu}_\tau$ may of course scatter again. The net effect is to increase the stopping power. For a ν_τ ,

$$\tau_{o/H} = 10^{-39} N_A R_0^{\frac{1}{2}} \left[(b_p y_p + a_p f_p) \pm (b_n y_n + a_n f_n) \right] \quad (12)$$

where $f_p = f_n = 0.82$ for ν_τ and $f_p = f_n = 0.76$ for $\bar{\nu}_\tau$.

The second change for ν_τ is that the τ lepton is massive, which decreases the charge-exchange cross-section at low energies. We will ignore this effect, because for high energies, $E > 100$ GeV, this amounts to $< 10\%$ correction [6, 18], whilst at low energies the Sun is nearly transparent to neutrinos anyway. We note, however, that if one plans to detect ν_τ via charged-current interactions in a detector, this effect must be taken into account.

2.3 Stopping of charged leptons

If the annihilations are into charged leptons then we must compare the decay time with the stopping time in a plasma. The stopping rate for a charged particle is roughly [19]

$$\frac{dE}{dt} \approx 9.2 \times 10^6 \left(\frac{Z\rho}{A} \right) \frac{1}{\beta} \left[\ln \frac{2m_e \gamma \beta^2}{I} - F \right] \text{ GeV/s} \quad (13a)$$

where Z and A refer to the charge and mass of the nuclei in the medium; ρ is the density of the medium in g/cm^3 ; β and γ refer to the incident particle; m_e is the electron mass; F is a fudge factor of order 1 which we will not calculate; and I is a cut-off energy for the photon propagator. For a particle moving through a plasma with a velocity greater than the thermal velocity of electrons in the plasma [20], we use $I = \omega_p = (4\pi\alpha n_e/m_e)^{1/2}$, the plasma frequency. For the core of the Sun, $\omega_p \approx 3 \times 10^{-7} \text{ GeV}$, so we will take the factor in brackets to be ≈ 8 . Adjusting for element abundances at the core, $(Z\rho/A) \approx 105 \rho_{150}$, where $\rho = 150 \rho_{150} \text{ g/cm}^3$, we find

$$\frac{dE}{dt} \approx 8 \times 10^9 \frac{\rho_{150}}{\beta} \text{ GeV/s} \quad (13b)$$

Since the muon lifetime is $2.2 \times 10^{-6} \text{ s}$, we conclude that for muons injected into the core only one muon in 10^4 will decay before it becomes non-relativistic. The resulting neutrino energy spectrum will be for muons decaying at rest. On the other hand, τ leptons only live $3.4 \times 10^{-13} \text{ s}$, so they decay after an average energy loss of only 0.16%. Therefore, in determining the neutrino spectra we will get energetic neutrinos from τ decays but only $\approx 30 \text{ MeV}$ neutrinos from muon decays.

2.4 Stopping and absorption of light hadrons

Next let us consider the case where the annihilation products are quarks. In general, we will get two back-to-back jets of light hadrons (i.e. containing only u , d , or s quarks) plus, possibly, two leading

hadrons containing heavy quarks (c, b, t). First we look at the fate of the light hadrons from the jets. Even if we ignore strong interactions there will be sufficient electromagnetic stopping power to bring charged π 's and K's to rest. Not surprisingly, strong interactions are more important. The interaction time-scale is

$$\tau_{\text{int}} = [n\sigma v]^{-1} = 3.5 \times 10^{-11} [\rho_{150} \sigma_{26} \beta]^{-1} \text{ s} \quad (14)$$

where $\sigma = 10^{-26} \sigma_{26} \text{ cm}^2$ is the cross-section for scattering a light hadron off a nucleon, and β is the hadron velocity. For π 's and K's, $\sigma_{26} \simeq 2$ in the relativistic regime. Non-relativistically, σ_{26} may go as high as 20 if the energy is at a resonance. In the relativistic regime a collision of a light hadron with a nucleon just produces more light hadrons of lower energy and we get a hadronic cascade^{*}). Because of the short interaction time-scale, all energy in light hadrons reduces itself rapidly to non-relativistic hadrons, before any of these hadrons can undergo a weak decay. Therefore, light hadrons cannot produce very many energetic neutrinos. For example, the fastest neutrino-producing decay is $K^+ \rightarrow \mu^+ \nu$ with a rate $5 \times 10^7 \text{ s}^{-1}$. Considering the efficiency of K production, we may conclude from eq. (14) that much less than 10^{-3} of the energy in light hadrons goes into neutrinos of $E > 0.5 \text{ GeV}$.

Even though they do not produce many high-energy neutrinos, we might suspect that light hadronic energy will produce $\approx m_D/m_\pi$ low-energy neutrinos when the stopped π 's and K's finally decay. We argue

^{*}) In fact, for high-energy collisions we expect some secondary charm production, but we argue that this will be unimportant. At asymptotically high energies ($E > 1 \text{ TeV}$), charm production in πp collisions is less than 0.01 times the total cross-section [21]. At lower energies this ratio drops dramatically. Even accounting for many πp collisions during the cascade, we expect less than 0.1 cc pair per injected pion at $E = 1 \text{ TeV}$. In addition, this secondary charm should be soft [22], so the resultant neutrino energies will be low. If there is any substantial branching ratio to heavy quarks or leptons, secondary charm production is unimportant.

that any such low-energy neutrinos will be unobservable. First, we point out that the number of low-energy neutrinos is probably less than $m_D/30 m_\pi$ because of energy lost during the hadronic decay. Energy loss mechanisms include π^0 's produced in the cascade, absorption of π^- s after they stop ($\pi^- p \rightarrow \pi^0 n$), and energy lost via elastic scattering ($\pi p \rightarrow \pi p$ threshold energy is about 600 MeV). Similar considerations apply to injected kaons. Secondly, neutrino-nucleon cross-sections drop dramatically with energy. Thirdly, and most important, the background flux of neutrinos from cosmic-ray air showers [23] increases at low energy roughly as E^{-3} . With any non-zero branching ratio for CDM annihilation into heavy quarks we would see the signal from the prompt neutrinos from heavy-quark decay far more easily than any possible low-energy neutrino signal from stopped light hadrons.

2.5 Stopping of heavy hadrons

The next possibility is that annihilation takes place into heavy quarks. In this case we expect back-to-back jets in which the leading particle is a hadron containing the heavy quark. The only particle of interest in these jets is the heavy hadron which may decay leptonically. The lifetimes for hadrons containing b-quarks and c-quarks are between 10^{-13} and 10^{-12} s, which is considerably shorter than the interaction times in the solar medium. For the most part these hadrons decay before interacting, producing neutrinos with energy $E_\nu \approx 1/3 E_H$, the energy of the hadron, with a branching fraction that depends on the initial hadron.

If the initial heavy hadron has a relativistic gamma factor in excess of $\tau_{\text{int}}/\tau_{\text{decay}}$, then the hadron may lose a considerable fraction of its energy before it decays. To model this, we suppose that after colliding with a nucleon the heavy hadron will have produced a jet with the leading hadron of that jet carrying the incident heavy quark and a momentum fraction Z of the initial hadron. We express this

as $Z = x_i z_j$, where x_i is the fraction of the momentum of the incoming hadron which is carried by the heavy quark, and z_j is the momentum fraction carried by the outgoing heavy hadron within the heavy-quark jet. Thus we model the energy loss as a two-step process. A hadron of momentum p collides with a nucleon producing a jet from a quark with momentum $p_Q = p x_i$. The jet fragments, so that the leading hadron carries the heavy quark and momentum $p_H = z_j p_Q = x_i z_j p$. At high energies we expect x_i and z_j to be uncorrelated so $\langle Z \rangle = \langle x_i \rangle \langle z_j \rangle$.

The average energy loss per collision is then $\langle \Delta E \rangle = \langle 1 - Z \rangle E$, and the energy loss rate is

$$\frac{dE}{dt} = \frac{\langle \Delta E \rangle}{\tau_{int}} \equiv \frac{E}{\tau_{stop}} \quad (15)$$

where $\tau_{stop} \equiv \tau_{int} / \langle 1 - Z \rangle$. After some time t , a heavy hadron of initial energy E_0 will be degraded to

$$E(t) = E_0 e^{-t/\tau_{stop}} \quad (16)$$

Of course this formula only holds in the relativistic regime, but heavy hadrons will decay long before they become non-relativistic. If we calculate the mean energy of the hadron when it decays, we find

$$\langle E \rangle = E_c e^{\gamma_0} E_1(\gamma_0) \quad (17)$$

where $E_c = m\tau_{stop}/\tau_{decay}$, $x_0 = E_c/E_0$, and $E_1(x_0) = \int_{x_0}^{\infty} (e^{-x}/x) dx$. If the γ factor is small, interactions are unimportant. Then $x_0 \gg 1$, $E_1(x_0) \approx (e^{-x_0}/x_0)[1 - (1/x_0)]$, and $\langle E \rangle \simeq E_0$. However, if γ is large then $x_0 \ll 1$ and $E_1(x_0) \simeq -\ln(x_0) - \gamma_E$, where $\gamma_E = 0.577\dots$ is Euler's constant. In this case

$$\langle E \rangle \approx E_c \left[\ln \left(\frac{E_0 \tau_{\text{decay}}}{m \tau_{\text{stop}}} \right) - \gamma_E \right]. \quad (18)$$

As we increase the injected hadron energy above E_c , the mean neutrino energy grows only logarithmically.

With the approximation that only the light quarks in each hadron contribute to scattering, we estimate that for the average b- or c-meson $\sigma_{26} \simeq 1$, and for b- or c-baryons $\sigma_{26} \simeq 2.5$. For charm, we estimate $\langle x_1 \rangle = m_c/m_1$, $\langle z_j \rangle = 0.6$, and average over masses and lifetimes of c-hadrons with probabilities 0.4 for D^0 and D^+ and 0.1 for D_s and Λ_c . Thus we obtain an effective E_c for charm: $E_c^c \approx 250$ GeV. For b-hadrons we estimate $m = 5.3$ GeV, $\langle x_1 \rangle = 0.9$, $\langle z_j \rangle = 0.8$, $\tau_{\text{decay}} \simeq 1.4 \times 10^{-12}$ s, and $E_c^b \approx 470$ GeV. We assume that top-quarks decay so quickly that dE/dx is never important. These estimates may seem rather crude; however, in sections 4 and 5 we will see that they compare fairly well with the more detailed results of our Monte Carlo.

2.6 Gauge bosons

The last direct annihilation products we consider are gauge bosons. Photons or gluons are unlikely to produce neutrinos. Photons are absorbed and gluons act like light quarks. For massive enough dark matter we should consider annihilation into weak bosons; either W^+W^- pairs, Z^0Z^0 , or $Z^0\gamma$. Weak bosons decay long before they interact with the solar medium.

3. NEUTRINO FLUXES FROM INJECTED PARTICLES

In the last section we gave qualitative arguments concerning the effects of the environment at the core of the Sun on the propagation and decay of particles. In this section we combine those results with

a version^{*)} of the Lund [10] Monte Carlo for fragmentation and decay in e^+e^- annihilations to produce neutrino spectra for various injected particles. The justification for using this Monte Carlo is twofold. First, the primary products of a dark-matter annihilation event are quite likely to be a heavy quark-antiquark pair resulting in back-to-back jets. However, once the $Q\bar{Q}$ pair is produced, the resulting jets should look just like the jets resulting from an e^+e^- annihilation. Secondly, although we could in principle use a set of analytic formulae with adjustable parameters, this information is very bulky and is already coded within the Lund program with parameters adjusted to fit the available experimental data.

Although the Lund model describes the fragmentation and decay of initial fermions, it does not take into account the environment in the Sun. Following section 2, we assume that muons and light hadrons do not contribute to the production of high-energy neutrinos.

To account for neutrino absorption and scattering we use a Monte Carlo procedure to model interactions as the neutrino travels through the Sun. If an electron or muon is produced from a charged-current event it is absorbed. For neutral-current events the outgoing neutrino energy is chosen randomly from the distribution predicted by the parton model. We do the same for τ leptons produced in charged-current interactions and then pass them back to the Lund program for decay. In all cases we keep track of the neutrino's position in the Sun for subsequent interactions.

We assume that τ leptons and t -quarks decay with no energy loss; however, we do take into account dE/dt for b - and c -hadrons. We do a Monte Carlo for energy loss on an event-by-event basis. Whenever a hadron interacts before it decays, the outgoing hadron is given momentum $p_j = z_j x_i p_i$, where p_i is the hadron momentum going into the inter-

*) We used version 6.2, modifying the LUDECY subroutine to handle weak semileptonic decays correctly.

action, x_i is taken from a parton distribution for heavy quarks in hadron i , and z_j is taken from a fragmentation function to produce hadron j . For the parton distribution we use

$$P(x_i) = \frac{1}{N} e^{-\frac{(x_i - x_0)^2}{\sigma^2}} x_i (1 - x_i) \quad (19)$$

where $x_0 = m_Q/m_H$, $\sigma = 150 \text{ MeV}/M_Q$, and N is a normalization constant. The rationale for this parametrization is that for a heavy-quark Q the momentum carried by the quark is $x_i \simeq (m_Q + p_{\parallel})/m_{H_i}$, with $p_{\parallel}$ the momentum component of Q within H_i parallel to the motion of H_i . The Gaussian form for $p(x_i)$ thus reflects our guess for the wave function of the heavy quark. The factor $x_i(1 - x_i)$ constrains the momentum to the physical regime $0 < x_i < 1$. We use quark masses; $m_c = 1.6 \text{ GeV}$, $m_b = 5.0 \text{ GeV}$. For the fragmentation function we use the parametrization of Peterson et al. [24],

$$D(z) = N z(1-z)^2 / [(1-z)^2 + \epsilon z]^2 \quad (20)$$

where again N is a normalization constant and we use $\epsilon_c = 0.075$ and $\epsilon_b = 0.008$.

We wish to illustrate the effects of neutrino and hadronic interactions with the solar medium. To do this, in the figures and tables that follow, we give some results including our interaction terms and others for which no interactions were included.

In fig. 2 we show the results of injecting $10^4 \tau^+ \tau^-$ pairs of 60 GeV per τ into the core of the Sun with our interaction terms turned off. We have expressed the neutrino yields per $\tau^+ \tau^-$ pair in terms of the scaled energy $z = E_\nu/E_\tau$. As long as $E_\tau \gg m_\tau$, dN/dz is a function of z alone, i.e. it is independent of E_τ . We can express dN/dz in terms of the energy distribution in the rest frame of the

decaying particle. If we define a scaled energy for the rest frame $y = 2E/m$, then in the limit $E_\tau \gg m_\tau$,

$$\frac{dN}{dz} = \int_z^1 \frac{dN}{dy} \frac{1}{y} dy . \quad (21)$$

We see that dN/dz is a monotonically decreasing function of z , even though dN/dy has a maximum at finite y and vanishes as $y \rightarrow 0$.

The top curve in fig. 2 is the yield of ν_τ , whilst the bottom curve is the yield of ν_μ . For clarity we have not plotted ν_e , $\bar{\nu}_\tau$, $\bar{\nu}_\mu$, or $\bar{\nu}_e$. We have binned our Monte Carlo data in bins of $\Delta z = 0.02$ and adopted a normalization so that integrating under the histograms gives the total yield N of a given neutrino type per pair of initial fermions (i.e. per annihilation event).

In addition to the curves in fig. 2 and in figs. 3 to 7 to follow, in table 1 we have tabulated useful moments of the neutrino distributions. The first three columns give the injection energy of the parent, the parent particle type, and the neutrino flavour. In columns 4 to 8 we give N_{ij} , the total yield of neutrino j from parent i at the given energy; the first two moments $\langle z_{ij} \rangle$ and $\langle z_{ij}^2 \rangle$; and the products $N_{ij} \langle z_{ij} \rangle$, $N_{ij} \langle z_{ij}^2 \rangle$, where

$$N_{ij} = \int \frac{dN_{ij}}{dz} dz , \quad (22a)$$

$$N_{ij} \langle z_{ij} \rangle = \int \frac{dN_{ij}}{dz} z dz , \quad (22b)$$

$$N_{ij} \langle z_{ij}^2 \rangle = \int \frac{dN_{ij}}{dz} z^2 dz . \quad (22c)$$

The purpose of tabulating $N_{ij}\langle z_{ij} \rangle$ and $N_{ij}\langle z_{ij}^2 \rangle$ is that these quantities are directly related to the event rates when observing a neutrino flux at the Earth. We will discuss this in more detail in the next section.

Since the only ν_μ 's in fig. 2 arise from leptonic τ decays, we can calculate their energy distribution and moments from standard weak interaction theory. Neglecting the muon mass, we find $\langle z_{\tau\mu} \rangle = 0.3$ and $\langle z_{\tau\mu}^2 \rangle = 0.133$. The values in table 1 compare favourably. For our sample, the statistical error in $\langle z_{\tau\mu} \rangle$ is approximately 5×10^{-3} . Including the mass of the muon would decrease the theoretical prediction to $\langle z_{\tau\mu} \rangle \simeq 0.299$. Similarly, for leptonic τ decay, neglecting m_μ , we expect $\langle z_{\tau\tau} \rangle = 0.35$ and $\langle z_{\tau\tau}^2 \rangle = 0.178$. Again, we might expect slightly lower values because of the finite mass of final-state particles. However, about 10% of the time the τ undergoes the two-body decay $\tau \rightarrow \nu_\tau \pi^-$. In these cases the ν_τ is considerably more energetic than for leptonic τ decays, and this explains our slightly harder spectrum.

In addition to the sample with no interactions, we performed a similar run with interactions turned on. These results are also reported in table 1. As one can see, at 60 GeV, interactions result in a 10-15% decrease in Nz and Nz^2 . Because of the numerically smaller cross-sections, only about 15% of this effect is due to neutral currents. Most of the decrease in Nz and Nz^2 is due to absorption. For the ν_τ the interactions are overestimated by about a factor of 2 since we have not included the effects of the τ mass in the charge-exchange cross-section.

The τ leptons we injected for fig. 2 were unpolarized. However, under some conditions we may expect polarized τ 's, e.g. the τ^- 's from b decay should have helicity = -1 about three quarters of the time. None the less, we ignore polarization effects since they do not greatly change our results and the Lund program is not designed to incorporate

them. As an example of the effects we omit, consider $\tau \rightarrow \nu_\tau \mu \bar{\nu}_\mu$. In these events the ν_τ momentum is preferentially opposite to the τ polarization. For large E_τ/m_τ , one calculates from standard weak interaction theory that $\langle z \rangle = E_{\nu_\tau}/E_\tau = 0.4, 0.35, 0.3$ for negative helicity, unpolarized, and positive helicity τ 's, respectively. Similarly the $\bar{\nu}_\mu$ is emitted along the τ polarization, and the respective numbers are $z = E_{\bar{\nu}_\mu}/E_\tau = 0.2, 0.3, 0.4$. Ignoring polarizations leads to at most a 50% error in neutrino energy. Furthermore, for most dark-matter candidates, τ 's resulting directly from annihilation will not be polarized. The lone exception we know of is the pedagogical case of a massive Dirac neutrino. Otherwise, although τ 's from b or t decays are polarized, the secondary neutrinos produced do not dominate the flux. With these warnings we will ignore polarization effects for τ 's. It is even easier to justify ignoring polarization effects for b - and c -quarks, which usually find themselves in spin-0 mesons; however, this may not be the case for t -quarks which may decay weakly inside the spin-1 mesons.

In figs. 3 to 5 we show the results of injecting 10^4 $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ pairs at 60 GeV. From c -quarks we plot just ν_μ , and from b - or t -quarks just ν_τ and ν_μ . Other flavours are related in a trivial way. The effect of fragmenting the quarks, including hadron masses etc., is most clearly seen in the c -quark data. Naïvely, we might expect a mean neutrino z of 0.35 from simple weak decay of an unfragmented quark. However, the results of our Monte Carlo yield $\langle z \rangle = 0.14$. Some of this decrease is due to the fact that the mean energy of the hadron containing the c -quark is only about 0.6 of the mass of the dark-matter particle and some comes from sharing the mass of the hadron among more than three final-state particles. The situation is a little more complicated for b -quarks. If we tag just the primary neutrinos from $b \rightarrow c \ell^- \nu$ we might expect $\langle z \rangle_\mu = 0.30$, where we observed $\langle z \rangle_\mu = 0.18$. About one half of this reduction is due to the charm mass and the rest

is due to fragmentation. When we include secondary neutrinos from c or τ decay, $\langle z \rangle_\mu$ drops to around 0.125.

In fig. 3 we compare the results of running with interactions turned on and off. Two effects are comparable. First, some of the neutrinos are absorbed. Although this number is small, they are preferentially the higher-energy ν 's, so the effect on Nz and Nz^2 is noticeable. For comparison, see the effects on ν_μ 's from injected τ 's. Secondly, on the average, c -hadrons are slowed down by the medium. This results in depleting the high-energy end of the spectrum and piling up neutrino flux at low z .

In discussing our results for injected τ leptons we stated that dN/dz is independent of injection energy as long as $E_\tau \gg m_\tau$. The same remark holds here for injected b 's and c 's. However, for the t -quarks we used $m_t = 40$ GeV and we are not in the relativistic regime. The effect is only obvious if we tag the primary neutrinos arising directly from top decay. We show the tagged ν_μ 's as the cross symbols in fig. 5. There is a pronounced maximum in their distribution at $z \approx 0.25$ which is not apparent in the full muon sample. Having tagged the primary ν_μ , something else becomes apparent. Comparing the tagged with the full ν_μ sample, we see that the primary neutrinos will account for most of the signal in an experiment, i.e. the tagged ν_μ 's dominate $\langle N \rangle \langle z \rangle$ and $\langle N \rangle \langle z^2 \rangle$.

In figs. 6 and 7 we illustrate the effects of dE/dx and neutrino absorption on the neutrino distribution for high injection energies. Figures 6a and 6b give the results of injecting 10^4 $\tau^+ \tau^-$ pairs at 1000 GeV per fermion. Here we have plotted both neutrino and anti-neutrino distributions, along with the average flux $(\nu + \bar{\nu})/2$ with interactions turned off. The effect of absorption is apparent in the reduced ν_μ yield at large z (fig. 6a). For ν_τ (fig. 6b) the neutrinos are not absorbed, but stopped. As a result, the flux depleted from the large- z region is piled up at small z , but the total number of neu-

trinos is held fixed. Curiously, the ν_μ yield at small z is also enhanced. This is because the ν_μ yield is being fed by the decay of τ 's subsequent to $\bar{\nu}_\tau$ 'absorption' events. Here, however, there is a net change in number because the ν and $\bar{\nu}$ charge-exchange cross-sections are not the same.

Figure 7 compares the ν_μ yield for 10^4 $c\bar{c}$ pairs injected at 60 and 1000 GeV. We see the effect of stopping power on c -quarks in that the 1000 GeV distribution is piled up at small z . In addition, the high- z tail is further depleted by neutrino absorption. From the table entry, we see that N , z , and z^2 are slightly higher for antineutrinos, reflecting the smaller absorption probability. The reason why the difference between ν_μ and $\bar{\nu}_\mu$ is not bigger is that, after the c -hadrons slow to ≈ 250 GeV, the resulting neutrino energies are ≈ 80 GeV, which is significantly less than either E_{Abs}^ν or $E_{\text{Abs}}^{\bar{\nu}}$.

We performed runs injecting 10^4 $b\bar{b}$ and $t\bar{t}$ pairs, and the results are listed in table 1. The neutrino spectra are qualitatively similar to figs. 6 and 7. In all cases the distributions are peaked at small z , with a few high- z stragglers that escape absorption. At 60 GeV we found that prompt neutrinos (e.g. $t \rightarrow \mu^+ \nu_\mu b$) dominated the signal rates, Nz and Nz^2 , over secondary neutrinos (e.g. $t \rightarrow c\bar{s}b$, $c \rightarrow \mu^+ \nu_\mu s$). At 1000 GeV this effect is smaller as the prompt neutrinos are preferentially absorbed.

Finally, we do not present any runs where we injected W^+W^- pairs or Z^0 's. If these annihilation products are important, then one may use standard $SU(2) \times U(1)$ branching ratios for the decay of $W, Z \rightarrow$ fermions in combination with the results given above.

4. EVENT-RATE CALCULATIONS

In this section we use our results to predict event rates for a generic dark-matter candidate, which we shall refer to as χ . First, we must determine the neutrino flux at the Earth. The differential flux

of neutrino species i is

$$\left(\frac{d\psi}{dE}\right)_i = \frac{C}{8\pi R_{ES}^2} \sum_f \Gamma_f \left(\frac{dN}{dE}\right)_{fi} \quad (23)$$

where C is the capture rate of χ by the Sun and $R_{ES} = 1 \text{ AU} = 1.50 \times 10^{13} \text{ cm}$. The sum is over fermion species f ; Γ_f is the branching ratio for annihilation into $f\bar{f}$ pairs; and $(dN/dE)_{fi}$ is the differential yield of neutrino species i from injected fermion f . In writing eq. (23) we assumed that the abundance of χ in the Sun has reached equilibrium and that two χ 's are destroyed in an annihilation event, so that the annihilation rate is equal to half the capture rate.

We use a capture rate adapted from Gould [25]:

$$C = 1.1 \times 10^{25} \left(\frac{\rho_{0.4}^\chi \sigma_{40}}{m_\chi \bar{v}_{300}} \right) f(m_\chi, m_c) \quad (24)$$

which improves earlier results [26] by making a more accurate evaluation of integrals and by accounting for the Sun's motion through the halo. In eq. (24) $\rho_{0.4}^\chi$ is the local mass density of χ 's in the halo (at the Sun's position) scaled to a density of 0.4 GeV/cm^3 , σ_{40} is the χp elastic cross-section in units of 10^{-40} cm^2 , m_χ is the χ mass in GeV, and $\bar{v}_{300}$ is the velocity dispersion of the χ 's in the halo in units of 300 km/s. The function $f(m_\chi, m_c)$ expresses the kinematic efficiency of capture by elastic scattering. If $m_\chi < m_\chi^c$ then almost any χp elastic scattering event results in capturing the χ ; whilst if $m_\chi > m_\chi^c$, only a fraction, $\sim m_\chi^c/m_\chi$, of the incoming flux of χ 's may lose enough energy in an elastic collision to be captured. For simplicity we have taken $f = [m_c^a/(m_c^a + m_\chi^a)]^{1/a}$ with $a = 1.5$, which agrees with Gould's more accurate result to better than 5%. In eq. (24) we have explicitly assumed that capture is due to elastic

scattering off protons. This is appropriate for a Majorana particle; however, in other cases a more general form should be used [5-7]. For scattering off protons [25], $m_c = 20.4 m_p$.

In general, the event rate is given by

$$R = \int \sigma_D(E) \frac{d\varphi}{dE} dE \quad (25)$$

where σ_D is an effective area for the detector. We will consider two types of events. First, we consider contained events where a charge-exchange interaction, converting a neutrino to an observed lepton, takes place in the detector. In that case, σ_D is proportional to the number of nucleons in the detector and the charge-exchange cross-section on a given nucleon. Neglecting detector thresholds, the latter is proportional to the neutrino energy, so that the event rates are proportional to $\int (dN/dE)E dE = m_\chi \langle N \rangle \langle z \rangle$ [6]. Using eqs. (23) to (25) we find

$$R_{\text{cont}} = 3.6 \times 10^{-2} \frac{M_{\text{kt}} \rho_{0.4}^\chi \sigma_{40} f(m_\chi, m_c)}{\bar{N}_{300}} \sum_i a_i \sum_f \left[\prod_f \langle N \rangle \langle z \rangle_{f_i} \right] \gamma_i^{-1} \quad (26)$$

where M_{kt} is the mass of the detector in kilotonnes. The a_i are isospin-averaged neutrino scattering coefficients (see subsection 2.2). For an isospin neutral detector, $a_\nu = 6.8$, $a_\nu^- = 3.1$ for ν_μ 's and ν_e 's. For energies above 100 GeV, ν_τ 's should be included at almost full strength. For low energies, their interactions are greatly suppressed. To take them into account properly involves doing an integral over the neutrino distribution for each value of m_χ [6]. For these reasons we shall omit any contribution from ν_τ 's to the event rate, although in some cases they may be important. The differential fluxes given in the Appendix may be used for doing these integrals.

The second type of event is a through-going muon [27], i.e. a muon which is produced in the rock outside the detector and which subsequently passes through it. Now the effective area of the detector is proportional to E^2 . One factor of E comes from the νN cross-section, the other comes from the range of the muons. Integrating over the muon energy, but ignoring detector thresholds, the effective range of the muons is

$$L_\mu = \frac{\langle E_\mu \rangle}{\epsilon \rho} = \frac{b_\mu E_{\nu_\mu}}{2.5 \times 10^3 \rho} \quad (27)$$

where $b_\mu = \langle E_\mu \rangle / E_{\nu_\mu} \simeq 0.51$ and similarly $b_\mu^- \simeq 0.67$ for rock which is half neutrons and half protons. We have used a muon stopping power of $dE/dx = \epsilon \rho$, where ρ is the rock density, and $\epsilon \approx 2.5 \times 10^{-3} \text{ (g)}^{-1} \text{ cm}^2$ is the average stopping power for muons with energies $10 \text{ GeV} < E_\mu < 150 \text{ GeV}$ [28]. Combined with our other results, the rate for through-going muons is

$$R_{\text{through}} = 1.4 \times 10^{-2} \frac{A_6 \rho_{0.4}^\chi \sigma_{40} m_\chi}{\bar{\nu}_{300}} f(m_\chi, m_c) \sum_i a_i b_i \sum_f \Gamma_f \langle N \rangle_{f_i} \langle z^2 \rangle_{f_i} \gamma_r^{-1} \quad (28)$$

where A_6 is the cross-sectional area of the detector in units of 10^6 cm^2 . Here the sum over neutrino flavours includes only ν_μ and $\bar{\nu}_\mu$. In principle, ν_τ 's will produce some muons as well, but after considering branching ratios and energy degradation, their efficiency at producing through-going muons is at most 0.06 that for ν_μ 's.

Now we need to determine $\langle N_{if} \rangle$, $\langle z_{if} \rangle$, and $\langle z_{if}^2 \rangle$ at an injection energy equal to m_χ . We can, of course, run our Monte Carlo at any desired energy, but there is an easier way which works quite well for

$m_\chi < 100$ GeV. We give an analysis for injected quarks that is trivially extended to include injected τ 's.

In section 3 we noted that, neglecting interactions and heavy fermion masses, the dN/dz_i (and therefore $\langle z \rangle$, $\langle z^2 \rangle$, etc.) were independent of m_χ . Let us expand on this idea, first including mass effects and then interactions. We will pretend that hadron masses will depend only on the heavy quark, i.e. we will ignore mass splittings such as those between D^+ , D^0 , Λ_c , and D_s . Our results will not be accurate close to any threshold, $m_\chi = m_Q$.

Consider the neutrino distribution, dN/dy , in the rest frame of the decaying hadron, where y is the scaled energy $y = 2E_\nu/m_H$ and $0 < y < 1$. To determine the distribution of neutrino energies in the 'lab.' frame where the hadron energy is E_H we use

$$\frac{dN}{d\tilde{z}} = \int_{y_-}^{y_+} \frac{dN}{dy} \frac{1}{\beta y} dy \quad (29)$$

where $y_\pm = \min(1, 2\tilde{z}/(1 \pm \beta))$, $\tilde{z} = E_\nu/E_H$, and β is the hadron's velocity in the lab. frame. It is straightforward to show that several useful relations hold. First, for very relativistic hadrons ($\beta \rightarrow 1$),

$$\frac{dN}{d\tilde{z}} = \int_{\tilde{z}}^1 \frac{dN}{dy} \frac{1}{y} dy \quad (30)$$

Next, the moments in the rest frame and lab. frame are related by

$$\langle N \rangle = \langle N \rangle \quad (31a)$$

$$\langle \tilde{z} \rangle = \frac{1}{2} \langle y \rangle \quad (31b)$$

$$\langle \tilde{z}^2 \rangle = \frac{1}{3} \frac{(3+\beta^2)}{4} \langle y^2 \rangle \quad (31c)$$

$$\langle \tilde{z}^3 \rangle = \frac{1}{4} \frac{(1+\beta^2)}{2} \langle y^3 \rangle \quad (31d)$$

and in the relativistic regime $\langle \tilde{z}^n \rangle = \langle y^n \rangle / (n + 1)$. We give the third moment because it is useful for computing the corrections due to interactions. The rest frame moments $\langle y^n \rangle$ are listed in table 2.

We want the neutrino energy moments as a function of m_χ . We use $z = \tilde{z}z_H$, $z_H = E_H/m_\chi$, and the relations $(3 + \beta^2)/4 = 1 - (m_H^2/4E_H^2)$, $(1 + \beta^2)/2 = 1 - (m_H^2/2E_H^2)$ to find

$$\langle z \rangle = \frac{1}{2} \langle y \rangle \langle z_H \rangle \quad (32a)$$

$$\langle z^2 \rangle = \frac{1}{3} \langle y^2 \rangle \langle z_H^2 - \frac{1}{4} z_m^2 \rangle \quad (32b)$$

$$\langle z^3 \rangle = \frac{1}{4} \langle y^3 \rangle \langle z_H^3 - \frac{1}{2} z_m^2 z_H \rangle \quad (32c)$$

where we define $z_m = m_H/m_\chi$. It is apparent that we also need the moments of the hadron energy distribution. In the absence of interactions $z_H = z_F$, the hadron energy after fragmentation. We estimate $\langle z_F \rangle$ and $\langle z_F \rangle^2$ in two ways: directly, by collecting data on z_F while running our Monte Carlo; and indirectly, by using eqs. (32a,b) and comparing our rest-frame moments from table 2 with our 60 GeV non-interacting data from table 1. The two methods agree to 2%. To determine $\langle z_F^3 \rangle$ we neglect any skewness in the fragmentation function and use $\langle z_F^3 \rangle \approx \langle z_F \rangle (3\langle z_F^2 \rangle - 2\langle z_F \rangle^2)$. Similarly, we use $\langle z_F^4 \rangle \approx \langle z_F \rangle^2 (6\langle z_F^2 \rangle - 5\langle z_F \rangle^2)$. The fragmentation moments are given in table 3. For the t-quark we actually show the moments of a 160 GeV fragmentation function. The reason for this is to avoid phase-space effects; at threshold the fragmentation function is forced to 1 by energy conser-

vation. To take the threshold effect into account, we have modified the fragmentation functions in table 3 by

$$\langle z_F^n \rangle \longrightarrow \langle z_F^n \rangle + (1 - \langle z_F^n \rangle) z_m^n. \quad (33)$$

We choose this modification out of an infinite class of such modifications because it is simple, has the correct behaviour as $z_m \rightarrow 0$ or 1, and improves the comparison between our Monte Carlo results and the analytic extension being described here. We emphasize that neither our Monte Carlo nor our analytic technique are valid immediately at production thresholds for heavy quarks. In principle, the fragmentation moments also depend on m_χ through the running of the QCD coupling constant. We will neglect this effect in our analytic results.

Until now we have neglected interactions in our analytic procedure. In fact, for most plausible models this will be an excellent approximation, and we can use the results embodied in eqs. (29)-(33) to predict event rates. To confirm this we study the effects of interactions in the 'low'-energy regime, where $m_\chi < E_C, E_{Abs}$.

We alter the non-interacting results by applying a stopping coefficient to the hadrons and then an interaction coefficient to the neutrinos. We are ultimately interested in the moments after the neutrinos escape from the Sun, $\langle Nz \rangle_A$ and $\langle Nz^2 \rangle_A$. (We have subscripted these moments with an 'A' because the major effect of neutrino interactions is absorption, not stopping.) These absorbed moments may be expressed in terms of the neutrino energy moments after hadronic stopping and decay but before absorption: $\langle Nz \rangle_S$, $\langle Nz^2 \rangle_S$ and $\langle Nz^3 \rangle_S$. We use the integral relation $\langle Nz^n \rangle_A = \int (dN/dz)_S z^n f(z) dz$, where $f(z)$ embodies both absorption due to charged currents and stopping due to neutral currents. For small neutrino energies (and small m_χ) we may expand $f(z) = 1 - (z/z_A) - (z/z_S)$, where $z_A = E_{Abs}/m_\chi$ and $z_S = 1/\tau m_\chi$. E_{Abs} and τ are defined in subsection 2.2. Thus, we find

$$\langle N z^n \rangle_A = \langle N z^n \rangle_S z_{Abs}^n \quad (34)$$

with the n^{th} absorption coefficient defined by

$$z_{Abs}^n = 1 - \frac{\langle N z^{n+1} \rangle_S}{\langle N z^n \rangle_S z_A} \quad (35)$$

Similarly, to find the moments of the stopped but unabsorbed flux, we need to know the moments of the hadron distributions when they decay. In the low-energy regime, the energy at decay, z_H , is given in terms of the injection energy after fragmentation, z_F , by $z_H = z_F[1 - (z_F/z_c)]$, where $z_c = E_c/m_\chi$. Then the hadron moments after stopping are given by

$$\langle z_H^n \rangle = \frac{1}{N} \int \frac{dN}{dz_F} \left[z_F^n - \frac{n z_F^{n+1}}{z_c} \right] dz_F \quad (36)$$

or $\langle z_H^n \rangle = \langle z_F^n \rangle (1 - z_{stop}^n)$, where the n^{th} stopping coefficient is given by

$$z_{stop}^n = \frac{n \langle z_F^{n+1} \rangle}{\langle z_F^n \rangle z_c} \quad (37)$$

Actually, this only gives the effects of stopping the injected quark, whereas secondary quarks (such as the c -quark in $b \rightarrow c \rightarrow \nu_\mu$ decay) may also feel the effects of interactions. To account for this we supple-

ment the n^{th} stopping coefficient for type-a quarks to produce type-i neutrinos by additional terms for 'secondary stopping'

$$\frac{\langle z_H^{n+1} \rangle}{\langle z_H^n \rangle} = \sum_q \left(z_{\text{stop}}^n \right)^{g_i} \frac{f_{aqi}}{3} \quad (38)$$

where f_{aqi} is the fraction of i -type neutrinos produced in the decay chain $a \rightarrow q \rightarrow i$, and q is some secondary quark. The factor of $1/3$ is roughly the average energy of a q -quark from a weak decay. The only non-zero f_{aqi} are $f_{bc\mu} = 0.45$, $f_{tb\tau} = 0.31$, $f_{tb\mu} = 0.58$, and $f_{tc\mu} = 0.08$, which we estimate from the branching ratios within the Lund program. If we apply eqs. (36) to (38) in a hierarchical fashion we automatically include a small tertiary stopping term for the decay chain $t \rightarrow b \rightarrow c \rightarrow \mu$.

We summarize our method of correcting for interactions. We use eqs. (34) and (35) to find the fully corrected moments in terms of the stopped but unabsorbed moments, which in turn are determined using eq. (32) with hadron moments given by eq. (36) to adjust for stopping. The various parameters are collected in tables 3 and 4 for convenience. We note that the values for E_c , E_{Abs}^V , and τ are not fitted but taken from the estimates and calculations in section 2.

We test the technique in two ways. At 60 GeV we performed runs for injected $\tau\bar{\tau}$ and $c\bar{c}$ pairs both with and without interactions. Using our procedure we may use the results without interactions to predict the results with interactions. The predicted results for injected $\tau\bar{\tau}$ with interactions are $\langle Nz \rangle_\mu = 0.044$, $\langle Nz^2 \rangle_\mu = 0.018$, $\langle Nz \rangle_\tau = 0.302$, and $\langle Nz^2 \rangle_\tau = 0.152$. For injected $c\bar{c}$'s we predict $\langle Nz \rangle_\mu = 0.98 \times 10^{-2}$ and $\langle Nz^2 \rangle_\mu = 0.17 \times 10^{-2}$. All our predicted results are lower than the actual Monte Carlo results. This is to be expected since the second-order corrections are of the opposite sign. However, the prediction

for $\langle Nz^2 \rangle_\mu$ from $c\bar{c}$ pairs is substantially lower. Upon investigation, the problem is that the second-order corrections to the stopping coefficients are substantial. They are

$$Z_{\text{stop}}^{n(n+3)} = \frac{n(n+3)}{2} \frac{\langle Z_F^{n+2} \rangle}{\langle Z_F^n \rangle Z_C^2} \quad (39)$$

and are 30-50% of the first-order corrections. If we use stopping coefficients that are accurate to second order, we find $\langle Nz \rangle_\mu = 1.00 \times 10^{-2}$ and $\langle Nz^2 \rangle_\mu = 2.1 \times 10^{-2}$, which is much better.

Care must be taken when using higher-order stopping coefficients because the large argument approximation to the exponential integral, $E_1(x) \simeq (1/x) \exp[-x](1 - 1/x + 2/x^2 - \dots)$, is an asymptotic expansion; recall that $x = z_C/z_F$ (see subsection 2.5). The second-order correction to the absorption, $(Nz^{n+2})_S/2z_A^2(Nz^n)_S$, is numerically smaller: first, by $n(n+3)$, and also because the energy of the neutrinos is 1/3 to 1/6 of the energy of the parent hadron so that the expansion parameter is smaller. We will therefore not include second-order absorption corrections.

The second method for testing our determination of $\langle Nz \rangle$ and $\langle Nz^2 \rangle$ involves using an actual model. After describing the model and calculating branching ratios and elastic pX scattering, we will evaluate $\langle Nz \rangle$ and $\langle Nz^2 \rangle$ using both our Monte Carlo and the scheme just described, and compare the results.

5. A SPECIFIC EXAMPLE

The dark-matter candidate we consider is the lightest supersymmetric particle (LSP) in a phenomenological model inspired by the superstring [11, 12]. This model has been discussed extensively in refs. [11-13]. We remind the reader of its salient features. The LSP is primarily a linear combination of six spin-1/2 fields: three

neutral gauginos, two Higgsinos, and the spin-1/2 partner of a right-handed neutrino. Which linear combination makes up the LSP is determined by diagonalizing the 6×6 neutralino mass matrix [13]. Making plausible choices for most of the unknown parameters [11-13] we retain as the one free parameter the primordial gaugino mass $m_{1/2}$. We find that the mass of the LSP, m_χ , is approximately $m_{1/2}/6$. Phenomenological bounds from sparticle masses and neutral-current measurements suggest that $m_{1/2} \gtrsim 100$ GeV [11, 12], whilst cosmology in the form of primordial nucleosynthesis favours $m_{1/2} \gtrsim 400$ GeV [29]. Figure 2 of ref. 13 shows that the relic density of χ particles is likely to be close to the closure density for a large range of $m_{1/2} \gtrsim 100$. Therefore χ is a possible candidate for the content of the galactic halo.

To estimate the annihilation rate into neutrinos inside the Sun, we need the low-energy elastic scattering cross-section off protons and the branching ratios for annihilation into ordinary fermions. The branching ratios are given by $\Gamma_f = \tilde{a}_f / \Sigma_f \tilde{a}_f$. At the low temperatures present in the Sun,

$$\tilde{a}_f = \frac{\Theta(m_\chi - m_f)}{2\pi} \sqrt{1 - \frac{m_f^2}{m_\chi^2}} m_f^2 (A_f - B_f)^2. \quad (40)$$

The parameters A_f and B_f are combinations of couplings for χ annihilation into fermion-type f through sfermion and Z-boson exchange [13]. Implicit in the A_f and B_f coefficients are values for the left- and right-handed sfermions masses. The authors of ref. [11] have performed an analysis of how squark and slepton masses depend on $m_{1/2}$. We have used their figure 10 to find the rough empirical relationships

$$\begin{aligned} m_{\tilde{e}_R} &\simeq 0.48 m_{1/2} + 36 \text{ GeV} \\ m_{\tilde{e}_L} &\simeq 0.57 m_{1/2} + 43 \text{ GeV} \\ m_{\tilde{q}_L} = m_{\tilde{q}_R} &\simeq 1.77 m_{1/2} + 83 \text{ GeV} \end{aligned} \quad (41)$$

where $m_{\tilde{\chi}_R}$ is a right-handed slepton mass, and so on.

The elastic χp cross-section is given by [13, 30]

$$\sigma_{el} = \frac{3}{\pi} \frac{m_{\tilde{\chi}}^2 m_p^2}{(m_{\tilde{\chi}} + m_p)^2} \left[N_u^A (A_u - B_u) + N_d^A (A_d - B_d) \right]^2 \quad (42)$$

where $N_u^A(N_d^A)$ is the axial up (down) charge of a proton. Numerically, $g_A \equiv N_u^A - N_d^A$ is measured to be 1.25. The quark model then predicts [31] $N_u^A = 1$, $N_d^A = -1/4$. On the other hand, a phenomenological analysis of weak decays of n and Ξ based on $SU(3)$ flavour symmetry gives [30] $N_u^A = 0.85$, $N_d^A = -0.4$. We will use the quark model estimates; however our results are relatively insensitive to this choice.

Given the χp cross-sections, masses, and branching ratios implied above, we have evaluated the rates for contained and through-going events and show the results in fig. 8^{*}). The solid curves show the results using the moment analysis described in section 4, including second-order corrections for the stopping coefficients and first-order corrections for absorption. For comparison, the dotted curves show the results of a moment analysis without corrections for interactions with the solar medium. The interactions are just starting to be important at $m_{1/2} = 1000$ GeV, where $m_{\tilde{\chi}} = 168$ GeV. To check the moment analysis, we have run our Monte Carlo, with appropriate branching ratios etc., for various values of $m_{1/2}$. The data from these runs are plotted as solid symbols. Again, for comparison, we have run the Monte Carlo with interactions turned off, and plotted these results as open symbols. At

^{*}) Actually, in generating fig. 8 we implicitly assumed that capture and annihilation of χ 's reach equilibrium [6, 7]. In fact, because of the low elastic χp and annihilation cross-sections this is not always true for this model. At $m_{1/2} = 1000$ the true event rates are about half those given in the figure. For the other data points the correction is less than 10%.

the high-mass end it is clear that we are slightly overestimating the effects of interactions because we have neglected second-order effects due to absorption. We should point out that this example does not fully exercise our calculational methods for b- or c-quarks since below top threshold the annihilation is almost exclusively into $\tau\bar{\tau}$ pairs, whilst for $m_\chi > m_t$ the annihilation is almost always into $t\bar{t}$ pairs. This fact is reflected by the break in the curves at $m_{1/2} = 215$. In addition, the phase-space effects at values of m_χ just above threshold for top production, i.e. $z_f \rightarrow 1$, squeezing the top fragmentation function, become important. To show this, the dashed curves include interactions but do not include the modifications to the fragmentation moments in eq. (33).

Using the results in fig. 8, we point out the message that is implied. For this dark-matter candidate one would need a 1000 kilotonne detector to collect 1-10 contained events per year. Alternatively, one needs an area detector of 300 x 300 m for upward through-going muons. Besides collecting a non-zero signal, one must also worry about background. For these two classes of events the backgrounds have been estimated under some simplifying assumptions [7]. Assuming an angular area of 1 sr and no energy resolution except to apply a threshold E_T (in GeV) to the observed lepton energy, the background rate for contained events is roughly $R_{\text{cont}}^{\text{back}} \simeq (3.0 M_{\text{kt}}/E_T) \text{y}^{-1}$. For through-going muons the background [27] is roughly $R_{\text{through}}^{\text{back}} \simeq 6.5 A_6 \text{y}^{-1}$. Even with a 1000 kt detector, one needs an angular resolution of a few degrees and an energy cut of tens of GeV in order to see the signal. This is probably not feasible for $m_\chi \simeq 20 \text{ GeV}$ ($m_{1/2} = 120 \text{ GeV}$), but may be possible for $m_\chi = 170 \text{ GeV}$, where the mean energy of the prompt neutrinos is about 50 GeV. For through-going muons an energy cut is more difficult and only decreases the background by a logarithmic factor. An angular cut is possible, but we must consider the intrinsic angular

resolution, $\Delta\theta \approx 1/(2\sqrt{E_\nu})$ (E_ν in GeV), associated with the charged-current event which takes place unobserved in the rock outside the detector [27]. We conclude that this particular dark-matter candidate is currently unobservable via high-energy neutrinos from the Sun, and is likely to remain so.

6. CONCLUSIONS

We briefly summarize the content of this paper and comment on the uncertainties involved. The purpose has been to obtain an accurate estimate of the neutrino flux resulting from the annihilation of dark matter trapped by the Sun. The thrust of our research has been along two lines. First, we have considered the effects of the solar medium upon the propagation of neutrinos and their high-energy parents. Second, we have used the kinematics and branching ratios of the Lund Monte Carlo to obtain realistic differential neutrino fluxes.

The effects of environment are easily summarized. Almost all annihilations take place in a plasma of density 150 g/cm^3 . In this environment muons and light hadrons (u-, d-, and s- quarks only) will stop owing to either electromagnetic or strong interactions with the medium. Any neutrinos coming from such particles will have non-relativistic parents, and will consequently be quite low in energy. In addition, hadronic energy is efficiently absorbed by the solar medium so that the number of low-energy neutrinos is more than an order of magnitude lower than might be estimated by dividing the mass of the dark-matter particle by a pion mass. With relatively low energies and numbers, the neutrinos derived from muon or light hadron decays are unobservable.

Next we considered the decays of τ leptons and heavy hadrons (those containing a c-, b-, or t-quark). For dark-matter masses less than about 100 GeV the only relevant physics is the branching ratio and kinematics of neutrino-producing decays. For higher masses we must

consider neutrino absorption and dE/dx for hadronic particles. The optical depth of the Sun for neutrinos and antineutrinos depends only on their energy. The critical energy that gives a neutrino optical depth of 1 is $E_\nu \approx 198$ GeV. For antineutrinos $E_{\bar{\nu}} \approx 296$ GeV. We also considered the stopping of neutrinos due to neutral-current interactions and find this to be less important than absorption. Stopping power becomes a consideration for hadronic parents when the interaction rate with the medium exceeds the decay rate of the parent. This occurs at high energy when the Lorentz factor becomes large. For c-quarks we estimate that parent hadrons are unlikely to decay with energies greater than about 250 GeV. For b-hadrons the corresponding energy is of order 470 GeV. For t-hadrons the corresponding energy is so high that we treat them as decaying immediately with $dE/dx = 0$.

The effects of neutrino absorption and hadronic stopping may not seem very important because dark-matter particles in the TeV range are not currently popular. However, we point out that dark matter with weak interactions has three natural mass scales for which $\Omega = 1$. There are the two well-known cases of the ≈ 30 eV neutrino and the ≈ 10 GeV 'WIMP'. In addition, weakly interacting particles in the TeV range also produce $\Omega \approx 1$ because, at that energy scale, weak gauge bosons are effectively massless and the 'annihilation' cross-section has dropped to the point where $\Omega = 1$, naturally. We also note a trend among model builders (e.g. the values of $m_{1/2}$ in the superstring model considered here) towards higher-mass LSPs as accelerator limits become more stringent.

In order to obtain neutrino fluxes that are as realistic as possible, we have used a version of the Lund Monte Carlo to model the branching ratios and kinematics. Two effects became clear from this study. First, neutrinos from c- and b-quarks are considerably softer than those from τ leptons because of hadronic mass effects and energy lost in fragmentation. Muon neutrinos from τ leptons have $\langle z \rangle \simeq 0.30$,

i.e. a mean energy of 0.3 of the injected τ energy, whereas ν_μ 's from charm have $\langle z \rangle \simeq 0.12$, and for those parented directly by b's we find $\langle z \rangle \simeq 0.18$. Secondly, although the neutrinos that arise from secondary or tertiary decays (e.g. from the c-quark in the chain $b \rightarrow c \rightarrow \nu$ or $t \rightarrow b \rightarrow c \rightarrow \nu$) may dominate the total flux, they will not dominate the experimental signals because of their lower energy. For extremely high injection energies this conclusion must be modified because the more energetic primary neutrinos are more likely to be absorbed.

To make our Monte Carlo results accessible for use by others, we have compiled moments of the neutrino distributions in the rest frame of the parent particle. These may then be used to predict event rates for an arbitrary dark-matter candidate as described in section 4. As long as the mass of the dark-matter particle is less than 200 GeV, we can easily correct for interactions with the solar medium. In case detailed knowledge of the neutrino spectra is required, we give in the Appendix a polynomial fit to the neutrino distributions in the rest frame of the parent. These may be boosted to give neutrino spectra in the lab. frame.

As an example, we have applied our results to a particular dark-matter candidate arising from a superstring-inspired model. We look at the event rates that may be expected in detectors of charged-current events. For this model, one needs of order 10^3 kilotonne-years of observation to obtain a signal of order 10 events. Even with such a large amount of data it may not be possible to see the signal against the background from cosmic-ray air showers. We emphasize that this conclusion is not in contradiction with the results of refs. [5] to [7], where it is shown that a wide variety of dark-matter candidates are observable. The peculiar feature of the superstring-inspired model is relatively high squark masses. As a result, the elastic χp cross-section is small, and capture in the Sun is suppressed. None the less, we generally predict event rates lower than those given in previous

papers whenever a substantial fraction of the neutrino flux comes from heavy-quark decay.

We have made a number of approximations in our calculations. Throughout, we have ignored the τ mass and any polarization effects. We have suppressed details of the hadronic mass spectrum, so our results are not accurate very close (~ 1 GeV) to production threshold for a given quark. We should also comment on the garbage-in/garbage-out nature of our calculations. The Lund Monte Carlo is a mixture of theoretical expectations and experimental observations. To the extent that this input reproduces data in e^+e^- collisions, the input is consistent. In this respect we expect our results for c-quarks and τ leptons to be fairly accurate. For b-quarks we have less confidence, and for t-quarks we are clearly engaged in speculation. Similar remarks apply to our calculations of stopping power for high-energy c- and b-hadrons. There is no experimental data for fixed-target scattering of a beam of D mesons. Qualitatively, we believe our model is correct; quantitatively it is not difficult to imagine a factor of 2 error in the stopping power. In addition to the particle physics uncertainties we remind the reader of other uncertainties: astrophysics (halo density, velocity dispersion, capture rate in the Sun); model (m_χ , branching ratios, elastic χp cross-section); and detection (angular and energy resolutions, background, sensitivity to neutral currents).

Finally, we would like to comment on the possibility of observing annihilations of CDM trapped in the Earth [32]. It was argued in refs. [6] and [7] that the signal from the Sun would always be more significant than that from the Earth. However, Gould [25] has recently pointed out an error in that analysis, so that for dark-matter particles with coherent weak interactions, such as Dirac neutrinos or sneutrinos, the signal from the Earth dominates. With this in mind, we point out that the results of this paper may be applied directly to

the case of annihilations at the Earth's core. The density there is about $1/12$ of that in the Sun. This is still large enough to absorb light hadrons and muons. The 'optical' depth of the Earth is quite small ($\lesssim 0.01$ that of the Sun), so our non-interacting results should be an excellent approximation.

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Table 1

The total yield N and the moments $\langle z \rangle$ and $\langle z^2 \rangle$ of the neutrino energy distributions are tabulated for neutrino-type ν_{flavour} , with and without interactions with the solar medium, from the injection of τ , c , b , and t at 60 and 1000 GeV

E (GeV)	Parent	ν_{flavour}	N	$\langle z \rangle$	$\langle z^2 \rangle$	$N\langle z \rangle$ ($\times 10^{-2}$)	$N\langle z^2 \rangle$ ($\times 10^{-2}$)
60	$\tau\bar{\tau}$	μ_{NI}	0.173	0.295	0.131	5.1	2.3
		μ	0.167	0.258	0.106	4.3	1.8
		τ_{NI}	1.0	0.358	0.186	35.8	18.6
		τ	1.0	0.310	0.151	31.0	15.1
	$c\bar{c}$	μ_{NI}	0.087	0.138	0.032	1.2	0.28
		μ	0.084	0.111	0.022	0.93	0.18
	$b\bar{b}$	μ_{NI}	0.222	0.125	0.029	2.8	0.64
		τ_{NI}	0.094	0.110	0.020	1.0	0.19
	$t\bar{t}$	μ_{NI}	0.378	0.132	0.041	5.0	1.55
		μ_{tagged}	0.117	0.308	0.120	3.6	1.40
		τ_{NI}	0.303	0.144	0.043	4.4	1.30
1000	$\tau\bar{\tau}$	μ	0.165	0.083	0.015	1.4	0.25
		$\bar{\mu}$	0.198	0.093	0.021	1.8	0.42
		τ	1.0	0.072	0.013	7.2	1.3
		$\bar{\tau}$	1.0	0.122	0.032	12.2	3.2
	$c\bar{c}$	μ	0.065	0.036	0.0032	0.23	0.021
		$\bar{\mu}$	0.067	0.045	0.0049	0.30	0.033
	$b\bar{b}$	μ	0.170	0.036	0.0031	0.61	0.061
		$\bar{\mu}$	0.190	0.041	0.0044	0.78	0.084
		τ	0.100	0.035	0.0028	0.35	0.028
		$\bar{\tau}$	0.100	0.041	0.0036	0.41	0.036
	$t\bar{t}$	μ	0.286	0.039	0.0048	1.1	0.14
		$\bar{\mu}$	0.316	0.054	0.0102	1.7	0.32
		τ	0.300	0.046	0.0058	1.4	0.17
		$\bar{\tau}$	0.300	0.065	0.0119	2.0	0.36

Table 2

The total yield N and the rest-frame moments $\langle y \rangle$, $\langle y^2 \rangle$, and $\langle y^3 \rangle$ (where $y = 2E_\nu/m_H$) for neutrino-type ν_{flavour} from the injection of τ , c , b , and t .

Parent	ν_{flavour}	$\langle N \rangle$	$\langle y \rangle$	$\langle y^2 \rangle$	$\langle y^3 \rangle$
$\tau\bar{\tau}$	μ	0.173	0.596	0.394	0.279
	τ	1.0	0.717	0.559	0.458
$c\bar{c}$	μ	0.087	0.504	0.283	0.172
$b\bar{b}$	μ	0.222	0.353	0.163	0.088
	τ	0.094	0.306	0.107	0.041
$t\bar{t}$	μ	0.378	0.288	0.168	0.121
	τ	0.303	0.318	0.180	0.124

Table 3

Moments of the fragmentation function z_F
as calculated in section 4 of the text.

Fermion	$\langle z_F \rangle$	$\langle z_F^2 \rangle$	$\langle z_F^3 \rangle$	$\langle z_F^4 \rangle$	m_H	E_C
τ	1	1	1	1	1.78	∞
c	0.55	0.34	0.23	0.16	1.9	250
b	0.71	0.54	0.43	0.35	5.2	470
t	0.87	0.78	0.71	0.68	40	∞

Table 4

Values for non-zero f_{aqi} , the fraction of i-type neutrinos produced in the decay chain $a \rightarrow q \rightarrow i$, where a is a primary quark and q is a secondary quark [see eq. (38)]. The values of E_{Abs}^v used are also given

$E_{Abs}^v = 198$	$f_{bc\mu} = 0.45$	$f_{tb\mu} = 0.58$
$E_{Abs}^{\bar{v}} = 296$	$f_{tb\tau} = 0.31$	$f_{tc\mu} = 0.08$

Figure captions

- Fig. 1 : Relative probability densities for annihilation to occur as a function of solar density ρ for cold dark-matter masses of 3, 5, and 10 GeV.
- Fig. 2 : Differential neutrino yields from injecting 10^4 $\tau\bar{\tau}$ pairs at 60 GeV, using our Monte Carlo procedure. The results are displayed for bins of $\Delta z = 0.02$, where $z = E_\nu/E_{\text{inj}}$ (E_{inj} is the injection energy). The counts have been normalized so that the area within the histogram is the yield of a given neutrino type per injection event; a value of $dN/dz = 1$ for a bin corresponds to 200 counts. In this sample, interactions with the solar medium were turned off.
- Fig. 3 : Differential neutrino yield for 10^4 $c\bar{c}$ pairs at 60 GeV. The curves show that at 60 GeV the effects of interactions with the solar medium are small but noticeable.
- Fig. 4 : Differential neutrino yield for 10^4 $b\bar{b}$ pairs at 60 GeV, with interactions turned off. The smooth curves were derived from the procedure described in the Appendix, performing an appropriate Lorentz boost on our polynomial fit to the rest-frame differential yield.
- Fig. 5 : Differential neutrino yield from injecting 10^4 $t\bar{t}$ pairs at 60 GeV, with interactions turned off. A t -quark mass of 40 GeV was used. The crosses show a subset of the ν_μ data, where the ν_μ comes directly from $t \rightarrow b\mu^+\nu_\mu$. This subset is much harder than the whole data set. The maximum in dN/dz for this subset at $z \simeq 0.25$ reflects the non-relativistic nature of these t -quarks.

- Fig. 6 : Differential neutrino yields from injecting 10^4 $\tau\bar{\tau}$ pairs at $E_{\text{inj}} = 1000$ GeV. In fig. 6a, ν_μ and $\bar{\nu}_\mu$ yields with the effects of interactions turned on are compared with a similar data set but with interactions turned off. High- z neutrinos are absorbed. The surplus of low-energy neutrinos comes from the decay of τ leptons subsequent to a $\nu_\tau \rightarrow \tau$ charge-exchange event. Figure 6b shows similar results for ν_τ and $\bar{\nu}_\tau$ from the same Monte Carlo run.
- Fig. 7 : Differential neutrino yields for 10^4 $c\bar{c}$ pairs at 1000 GeV with interactions turned on. For comparison, we also show the 60 GeV data for $c\bar{c}$ pairs with interactions off. The pile-up at small z is due to the slowing down of c -quarks as they pass through the solar medium before decay.
- Fig. 8 : Event rates for the string-inspired model discussed in the text. The upper curves are the number of through-going events from eq. (28). The lower curves are for contained events, from eq. (26). Solid curves include corrections for interactions and adjusting the fragmentation moments near threshold in accordance with eq. (33). For the dotted curves, interactions have been turned off. For the dashed curves, interactions are included but the fragmentation moments are unadjusted. Solid symbols are our Monte Carlo results for 10,000 annihilation events for each value of $m_{1/2}$ (100, 150, 200, 218, 230, 260, 400, 700, 1000 GeV). The open symbols are the Monte Carlo results for 10,000 annihilation events, at each value of $m_{1/2}$, with interactions turned off.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

APPENDIX

In the main body of this paper we have concentrated on two points: i) qualitatively showing the effects of the solar medium on the differential neutrino flux; and ii) giving numerical values for the integrated event rates. However, it is quite possible that detailed information on the differential neutrino flux will be important in separating signal from background or in determining the CDM annihilation mechanism. With this in mind, we present some polynomial fits to our Monte Carlo data.

It is unreasonable and unwarranted to give differential neutrino fluxes from each hadronic species at different energies, e.g. from D^0 , D^+ , D_s , and Λ_c at many different energies. Rather, we will give analytic fits to the differential flux in the rest frame of the parent hadron. In this way we account for the branching ratios to produce, for example, D^0 , D^+ , D_s , Λ_c , but ignore the relatively small mass differences between hadrons containing a particular heavy quark. To obtain the differential flux in the lab. frame, we must boost the rest-frame distribution as in eq. (29). Since the hadron momenta are distributed according to the fragmentation function we should, in principle, integrate over a distribution of boosts. Using just the mean hadron energy after fragmentation will result in slightly underestimating the mean square neutrino energy, but will not seriously affect the shape of the neutrino spectrum. We fit the rest frame distributions to the form

$$\frac{dN}{dy} = \sum_{n=0}^4 a_n y^n \quad (A.1)$$

where the kinematic variable y is defined by $y = 2E_V/m_H$; so $0 \leq y \leq 1$. The results of these fits are listed in table A.1 along with $\langle N \rangle$, $\langle y \rangle$, $\langle y^2 \rangle$, and $\langle y^3 \rangle$. These values for the moments may be compared with those found directly from the data (table 2) and are found to be slightly on the low side.

For the case of ν_τ 's from τ decays, the fit was only done to those decays with more than two particles in the final state, representing 64.6% of the sample. In addition, the following two-body modes should be included: $\tau \rightarrow \nu_\tau \pi^-$ (10.3%); $\tau \rightarrow \nu_\tau e^-$ (22.1%); $\tau \rightarrow \nu_\tau K^-$ (1.3%); and $\tau \rightarrow \nu_\tau K^{*-}$ (1.7%). For the case of ν_μ 's from b-quarks, the parameter a_0 was set to zero. An idea of the quality of the fits may be obtained by considering ν_μ from τ decay. Here the coefficients may be computed from weak interaction theory: $a_0 = a_1 = a_4 = 0$, $-a_3 = a_2 = 12\langle N \rangle$.

As a final test of these fits, we take the rest frame distribution for b-quarks from table A.1, perform a boost to the lab. frame using eq. (29), and rescale the distribution by $z = \tilde{z}z_H$, $dN/dz = (1/z_H)(dN/d\tilde{z})$. The resulting dN/dz is plotted as a smooth curve and compared with our 60 GeV non-interacting data in fig. 4.

Table A.1

Fit parameters for the function $dN/dy = \sum_{n=0}^4 a_n y^n$ found for ν_μ 's and ν_τ 's from injected τ , c , b , and t fermions. The total yield N and the y moments obtained from this parametrization are also shown, and are to be compared with the values in Table 2.

Parent	ν_{flavour}	a_0	a_1	a_2	a_3	a_4
τ	μ	0.0005	-0.0104	2.1250	- 2.0750	- 0.0669
	τ	-0.0023	0.1311	6.3001	- 7.1656	1.3223
c	μ	0.0097	-0.3917	4.6487	- 8.6226	4.3834
b	μ	-0.0629	5.8391	-22.5507	32.5397	-16.5512
	τ	0	0.0009	12.4067	-41.0034	33.9340
t	μ	1.8118	-9.6799	18.3406	-12.3394	1.8472
	τ	1.0636	-4.1663	5.3022	- 0.0574	- 2.1818

Parent	ν_{flavour}	N	$\langle y \rangle$	$\langle y^2 \rangle$	$\langle y^3 \rangle$
τ	μ	0.172	0.594	0.392	0.277
	τ	0.636	0.636	0.450	0.341
c	μ	0.083	0.497	0.277	0.167
b	μ	0.218	0.355	0.164	0.088
	τ	0.093	0.306	0.108	0.042
t	μ	0.370	0.282	0.161	0.113
	τ	0.297	0.315	0.176	0.119