IMPLICATIONS OF RECENT MEASUREMENTS OF B MESON MIXING AND ϵ'/ϵ_K John Ellis^{a)}, John S. Hagelin^{b)}, S. Rudaz^{c)} and D.-D. Wu^{a,d)}A B S T R A C T

Within the Standard Model, the recent observation of $B_d^0-\bar{B}_d^0$ meson mixing provides a lower bound on the top quark mass which is $m_t \gtrsim 50$ to 100 GeV, depending on experimental and theoretical uncertainties. Conversely, flavour-conserving neutral current upper bounds on m_t in combination with $B_d^0-\bar{B}_d^0$ mixing provide non trivial lower bounds on hadronic current matrix elements. The lower bound on m_t is only marginally improved by incorporating as a constraint the CP violation ϵ_K observed in the $K^0-\bar{K}^0$ mass matrix, which allows in principle a determination of all the Kobayashi-Maskawa matrix elements. The preliminary measurement of ϵ'/ϵ_K falls comfortably within the range predicted using these other weak measurements, favouring $m_t \sim 100$ GeV within large errors. The Standard Model predicts large $B^0-\bar{B}^0$ mixing, but small CP-violating effects in both $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing ($\leq 0.3\%$), whereas non-standard models can give values as large as 6%. We comment on rare K decays such as $K^+ \rightarrow \pi^+ \bar{\nu}\nu$ in the light of $B_d^0-\bar{B}_d^0$ mixing. Finally, we review previous phenomenological proposals for the Kobayashi-Maskawa matrix elements and show that they can largely be excluded.

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1. INTRODUCTION

Recent observations^{1,2)} of mixing among neutral B mesons constitute an important step forward in the analysis of the electroweak interactions. Although the possibilities for $B_d^0-\bar{B}_d^0$ mixing in the context of the Standard Kobayashi-Maskawa Model had been discussed extensively³⁾ before its observation by the ARGUS collaboration²⁾, the observed magnitude of the mixing seemed at first sight surprisingly large. There have even been speculations that new physics beyond the Standard Model might be required, such as a fourth generation⁴⁾. However, it seems that the ARGUS data can in fact be reconciled with the Standard Model if the top quark mass m_t is large enough^{5,6)}. How large depends on the unknown magnitudes of hadronic operator matrix elements, and on uncertainties in other experimental constraints on Kobayashi-Maskawa matrix elements, such as measurements of the b quark lifetime⁷⁾ and upper limits on $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ ⁸⁾. Probably $m_t \gtrsim (50 \text{ to } 100) \text{ GeV}$ ⁵⁾, although the theoretical uncertainties prevent one from assigning a confidence level to any such bound.

Is a Standard Model interpretation of $B_d^0-\bar{B}_d^0$ mixing compatible with other electroweak measurements, e.g., of ϵ'/ϵ_K in the $K^0-\bar{K}^0$ system⁹⁾, and what predictions can be made for related weak processes? For example, how large should $B_s^0-\bar{B}_s^0$ mixing be? How large should CP-violating effects in $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing be? Are there interesting predictions for rare K decays, such as $K_L \rightarrow \mu^+\mu^-$ or $K^+ \rightarrow \pi^+\nu\bar{\nu}$? Which is the best way to pin down remaining uncertainties in the Standard Model, and perhaps even falsify it? Many of these questions have been addressed in a previous publication^{5,6)}. In this paper we expand on the previous analysis, introducing certain new considerations based on measurements of ϵ_K and ϵ'/ϵ_K in the $K^0-\bar{K}^0$ system and developing more predictions, in particular for CP violation in $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing. The structure of this paper is as follows.

In Section 2 we remind the reader of the logic behind the previous⁵⁾ lower bound on m_t . This is based on an m_t -dependent "determination" of $s_2 \equiv \sin\theta_2$ derived from the observed magnitude of $B_d^0-\bar{B}_d^0$ mixing²⁾, a related lower limit on $s_3 \equiv \sin\theta_3$ obtained from the b lifetime⁷⁾, and an upper limit on s_3 from experimental upper bounds⁸⁾ on $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$. We emphasize that s_3 is bounded away from zero for all except one exceptional value of m_t whose actual value depends on both theoretical and experimental uncertainties. The lower bound of 50 to 100 GeV which is obtained for m_t depends sensitively on theoretical uncertainties such as the B meson

decay constant f_{B_d} and the ratio B_{B_d} of $\langle B_d^0 | (\bar{b}\gamma_\mu(1-\gamma_5)d)^2 | \bar{B}_d^0 \rangle$ to its vacuum saturation value^{*}). The above argument can also be turned around, using the upper bound $m_t < 168$ GeV which comes from global analyses¹²⁾ of flavour-conserving neutral current data to derive a non-trivial lower bound $f_{B_d} B_{B_d}^{\frac{1}{2}} \gtrsim 65$ MeV.

In Section 3 we incorporate the CP violation ϵ_K seen¹³⁾ in the $K^0-\bar{K}^0$ mass matrix into the above analysis. This produces an improvement in our previous lower bound on m_t , which is however negligible compared to the previously mentioned theoretical uncertainties for plausible values $B_K = 0.33$ to 1.0 of the ratio between $\langle K^0 | (\bar{s}\gamma_\mu(1-\gamma_5)d)^2 | \bar{K}^0 \rangle$ and its vacuum saturation value. By fitting ϵ_K as well as $B_d^0-\bar{B}_d^0$ mixing and the classic weak interaction measurements, it is in principle possible to calculate all the Kobayashi-Maskawa matrix elements as functions of m_t , albeit with theoretical uncertainties which dwarf the experimental ones. It is therefore possible to calculate ϵ'/ϵ_K , albeit with additional theoretical uncertainties in hadronic matrix elements. The recent preliminary measurement of ϵ'/ϵ_K by the NA31 experiment⁹⁾ falls comfortably within the range so predicted, favouring $m_t \sim 100$ GeV within large errors.

Section 4 contains predictions for $B_s^0-\bar{B}_s^0$ mixing and for CP violation in $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing. As reported previously⁵⁾, we find that the Standard Model predicts large $B_s^0-\bar{B}_s^0$ mixing, though not necessarily maximal. We argue that this is consistent with upper bounds on inclusive like-sign dileptons from B meson mixing^{14), 1)}, particularly if one allows for the possibility of b quark fragmentation to baryons. We find the following bound on the CP-violating dilepton asymmetry a_d in B_d^0 decays:

$$a_d = \frac{\#(l^+l^+) - \#(l^-l^-)}{\#(l^+l^+) + \#(l^-l^-)} \leq 0.38 \quad (1.1)$$

in the Standard Model, and an even smaller value

$$a_s \leq 0.18 \quad (1.2)$$

for the corresponding CP-violating asymmetry in B_s^0 decays. In comparison, non-standard models such as superweak theory¹⁵⁾ or a fourth generation can give values as large as $a_{d,s} \lesssim 6\%$.

*) e^+e^- experiments¹⁰⁾ tell us that $m_t > 26$ GeV, which requires $f_{B_d} B_{B_d}^{\frac{1}{2}} \lesssim 300$ MeV, whilst the UAL Collaboration¹¹⁾ has a preliminary lower limit $m_t > 44$ GeV.

In Section 5 we comment on rare K decays, notably $K_L \rightarrow \mu^+\mu^-$ and $K^+ \rightarrow \pi^+\bar{\nu}\nu$, for which we update previous estimates^{16),5)} of the branching ratio.

In Section 6 we discuss previously-proposed phenomenological Ansätze^{17),18)} for the Kobayashi-Maskawa mixing matrix elements, developing their predictions and finding significant differences from previous analyses^{19),20)}. We then compare these predictions with the values inferred from $B_d^0-\bar{B}_d^0$ meson mixing and other experimental information, finding that the Ansätze can be excluded.

Finally, in Section 7 we summarize our conclusions. We emphasize that the Standard Model is now vulnerable as never before to future experiments including measurements of m_t , of $B_s^0-\bar{B}_s^0$ mixing, of a_d and a_s , and of $K^+ \rightarrow \pi^+\bar{\nu}\nu$ decay.

2. BOUNDS ON KOBAYASHI-MASKAWA MATRIX ELEMENTS AND m_t

We start with a recapitulation of the formalism for $B^0-\bar{B}^0$ mixing^{21),22)} which serves to introduce our notation and set the scene for our subsequent discussion of CP violation. We denote the phenomenological Hamiltonian in the flavour eigenbasis by

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12}^* - \frac{1}{2}i\Gamma_{12}^* \\ M_{12} - \frac{1}{2}i\Gamma_{12} & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} \quad (2.1)$$

which has mass eigenstates

$$B_{1,2} = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left[(1+\epsilon)B^0 \pm (1-\epsilon)\bar{B}^0 \right] \quad (2.2)$$

with eigenvalues

$$M_{1,2} = M \pm \frac{\Delta M}{2} : \Delta M = 2 \operatorname{Re} \sqrt{(M_{12} - \frac{1}{2}i\Gamma_{12})(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)} \quad (2.3a)$$

$$\Gamma_{1,2} = \Gamma \pm \frac{\Delta \Gamma}{2} : \Delta \Gamma = -2/m \sqrt{(M_{12} - \frac{1}{2}i\Gamma_{12})(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)} \quad (2.3b)$$

The CP impurity parameter ϵ in (2.2) is given by

$$\epsilon = \frac{-\text{Re} M_{12} + \frac{1}{2} i \text{Re} \Gamma_{12} + \sqrt{(M_{12} - \frac{1}{2} i \Gamma_{12})(M_{12}^* - \frac{1}{2} i \Gamma_{12}^*)}}{i \text{Im} M_{12} + \frac{1}{2} i \text{Im} \Gamma_{12}} \quad (2.4)$$

The b lifetime $\tau_b = \Gamma^{-1}$ is measured to be $(1.11 \pm 0.19) \times 10^{-12} \text{s}$ ⁷⁾. A non-zero value has recently been reported²⁾ for $r_d \equiv \Gamma(B_d^0 \rightarrow l^+ X) / \Gamma(B_d^0 \rightarrow l^- X)$. We assign to it the representative values $r_d = 0.20$ ("preferred") or 0.10 ("minimal"). In the standard theory of meson mixing,

$$r_d = \left| \frac{1-\epsilon}{1+\epsilon} \right|^2 \frac{(\Delta M)^2 + (\Delta \Gamma/2)^2}{2\Gamma^2 + (\Delta M)^2 - (\Delta \Gamma/2)^2} \quad (2.5)$$

In the approximation $(\Delta \Gamma/2)^2 \ll (\Delta M)^2$, which is justified in Ref. 22), $|(1-\epsilon)/(1+\epsilon)|$ is almost unity and Eq. (2.5) is well approximated by

$$r_d \simeq \frac{(\Delta M)^2}{2\Gamma^2 + (\Delta M)^2} \quad (2.6)$$

and the mass difference ΔM by²¹⁾

$$\Delta M \simeq 2 |M_{12}| \quad (2.7)$$

Note that the familiar approximation $\Delta M \simeq 2 \text{Re} M_{12}$, which is valid to lowest non-trivial order in $\text{Im} M_{12} / \text{Re} M_{12}$, does not apply to the $B_d^0 - \bar{B}_d^0$ system, since $M_{12} \propto e^{2i\delta}$ and $\sin \delta$ is not necessarily small, as we will see shortly.

In the standard six-quark model the charged weak current is given by

$$J^\mu = (\bar{u}, \bar{c}, \bar{s}) \gamma^\mu (1 - \gamma_5) V_{KM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.8)$$

where we parametrize the Kobayashi-Maskawa²³⁾ matrix in the standard form

$$V_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad (2.9)$$

Without loss of generality, the angles θ_1 , θ_2 and θ_3 may be chosen to lie in the first quadrant, in which case the value of δ is unconstrained a priori.

In this model, the off-diagonal mixing parameter M_{12} in the $B_d^0-\bar{B}_d^0$ mass matrix (2.1) is given by

$$M_{12} = \frac{G_F^2 f_{B_d}^2 B_{B_d} m_{B_d}}{12\pi^2} \left[(V_{td}^* V_{tb})^2 m_t^2 + O(m_c^2) \right] \quad (2.10)$$

where f_{B_d} is the B_d^0 meson decay constant and B_{B_d} is the ratio between $\langle B_d^0 | (\bar{b}\gamma^\mu(1-\gamma_5)d)^2 | \bar{B}_d^0 \rangle$ and its value in the vacuum saturation approximation. We will neglect in our subsequent calculations the $O(m_b^2)$ terms in (2.10), but include all the effects of QCD²²⁾ and effects of higher order in m_c^2/m_W^2 ^{22),24)}, which we express by the substitution $m_t \rightarrow \tilde{m}_t$ in (2.10). Using equations (2.6), (2.9) and (2.10), the experimental values of τ_b and of r_d , and a theoretical estimate of the product $f_{B_d} B_{B_d}^{1/2}$, we can estimate⁵⁾ s_2 as a function of $\tilde{m}_t$ (and hence of m_t):

$$s_2 = \frac{10.8 \text{ GeV}}{\tilde{m}_t} \left(\frac{r_d}{1-r_d} \right)^{1/4} \left(\frac{0.11 \text{ GeV}}{f_{B_d} B_{B_d}^{1/2}} \right) \left(\frac{1.11 \times 10^{-12} \text{ s}}{\tau_b} \right)^{1/2} \quad (2.11)$$

In writing this expression, we have neglected experimental uncertainties in $|V_{cb}|$ (assuming three-generation unitarity) and in s_1 (≈ 0.221), which are negligible compared to the uncertainties remaining in Eq. (2.11).

Given a value of s_2 , one can then constrain the value of s_3 using the measured b lifetime τ_b ⁷⁾, the measured semileptonic branching ratio $\text{BR}(b \rightarrow c l \bar{\nu}) \approx 11.6\%$ ²⁵⁾ and a simple parton model for semileptonic b decays. (This method avoids reliance upon somewhat uncertain strong interaction enhancement factors and phase space for the nonleptonic decay channels $b \rightarrow c \bar{u} d$, $b \rightarrow c \bar{c} s$, etc.). In this way one obtains

$$|V_{cb}| \approx |s_3 + s_2 e^{i\delta}| = \left[\frac{\text{BR}(b \rightarrow c l \bar{\nu})}{\tau_b} \left(\frac{192\pi^3}{G_F^2 m_b^5} \right) \frac{1}{F(m_c/m_b)} \right]^{1/2} \approx 0.050 \left(\frac{1.11 \times 10^{-12} \text{ s}}{\tau_b} \right)^{1/2} \quad (2.12a)$$

where $F(m_c/m_b)$ is the standard three-body phase space factor for $b \rightarrow c e \bar{\nu}$ decays:

$$F(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x \quad (2.12b)$$

and we have assumed $m_b = 4.7 \text{ GeV}$ and $m_c = 1.4 \text{ GeV}$ in evaluating (2.12a). For $s_2 > |V_{bc}|$, the minimum value of s_3 is obtained when $\cos\delta = -1$:

$$s_3 \geq s_3^{\min} = s_2 - 0.05 \left(\frac{1.11 \times 10^{-12} s}{\tau_b} \right)^{1/2} \quad (2.13a)$$

For $s_2 < |V_{bc}|$, the minimum value of s_3 is obtained when $\cos \delta = +1$:

$$s_3 \geq s_3^{\min} = 0.05 \left(\frac{1.11 \times 10^{-12} s}{\tau_b} \right)^{1/2} - s_2 \quad (2.13b)$$

Note that the extreme value $s_3 = 0$ is allowed only when $s_2 = |V_{bc}|$.

The final ingredient in bounding m_t and/or the combination $f_{B_d} B_{B_d}^{1/2}$ of hadronic matrix elements is the upper bound on s_3 deduced from the non-observation⁸⁾ of the $b \rightarrow u$ transition. Denoting $\bar{R} \equiv BR(b \rightarrow u l \bar{\nu}) / BR(b \rightarrow c l \bar{\nu})$, we have

$$\bar{R} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{F(m_u/m_b)}{F(m_c/m_b)} \quad (2.14)$$

with $F(x)$ given in (2.12b). Taking $m_u = 0$ and using (2.12a) for $|V_{cb}|$, we find

$$|V_{ub}| = s_1 s_3 = \bar{R}^{1/2} \left(\frac{192 \pi^3}{G_F^2 m_b^5} \right)^{1/2} \left(\frac{BR(b \rightarrow c l \bar{\nu})}{\tau_b} \right)^{1/2} \quad (2.15)$$

and hence

$$s_3 = 0.046 \sqrt{\frac{\bar{R}}{0.08}} \left(\frac{1.11 \times 10^{-12} s}{\tau_b} \right)^{1/2} \quad (2.16)$$

where we have again used $m_b = 4.7$ GeV in evaluating (2.16). At present we only have an upper bound on $\bar{R}^*$, which we denote by $\bar{R}^{\max}$, and hence an upper bound on s_3 denoted by $s_3^{\max}$.

The logic of the previous lower bound on m_t was to argue that the lower bound (2.13a) on s_3 was compatible with the upper bound $s_3^{\max}$ only if s_2 (2.11) was sufficiently small, and hence only if m_t was sufficiently large. Alternatively, it is conceivable that a comparison of the lower bound (2.13b) on s_3 with $s_3^{\max}$ could be used to provide an upper bound on m_t , although this is not the case if one uses the current value of $s_3^{\max}$.

The derivations of these bounds are subject to experimental uncertainties in τ_b , τ_b and $\bar{R}$, although the final upper and lower bounds on m_t and $f_{B_d} B_{B_d}^{1/2}$ mentioned

*) The charmless exclusive decay modes $B \rightarrow \bar{p} p \pi$ and $\bar{p} p \pi \pi$ have recently been reported²⁶⁾. They cannot be translated easily into lower limits on $|V_{ub}|$ and hence s_3 , but qualitatively favour values of $\bar{R}$ close to $\bar{R}^{\max}$.

previously are actually independent of τ_b . The bounds on m_t also depend on the theoretical value for the combination $f_{B_d} B_{B_d}^{\frac{1}{2}}$ of hadronic matrix elements. Although theoretical estimates of this are available, it is worth noting that one can obtain a non-trivial lower bound on it by demanding consistency between the lower bound on m_t which follows from Eqs. (2.11), (2.13a) and (2.16) and the upper bound $m_t < 168$ GeV obtained from global analyses¹²⁾ of flavour-conserving neutral current data.

Having outlined the principles of the available bounds on m_t and $f_{B_d} B_{B_d}^{\frac{1}{2}}$, we now discuss their numerical evaluation using plausible values for the necessary inputs. Recent estimates of the B_d^0 decay constant f_{B_d} using QCD sum rules^{27),28)}, the MIT bag model²⁹⁾ and various potential models³⁰⁾ give

$$90 \lesssim f_{B_d} / \text{MeV} \lesssim 130 \quad (2.17)$$

whilst recent estimates²⁹⁾ of the $\Delta B = 2$ matrix element $\langle B_d^0 | (\bar{b} \gamma_\mu (1 - \gamma_5) d)^2 | B_d^0 \rangle$ that do not rely on the vacuum saturation hypothesis yield similar values for $f_{B_d} B_{B_d}^{\frac{1}{2}}$. If we take the representative values $f_{B_d} B_{B_d}^{\frac{1}{2}} = 110 \text{ MeV}^*$ and $r_d = 0.20$ as a "preferred" scenario, we can compute $s_2(2.11)$ and $s_3^{\min}(2.13)$ as functions of m_t , as shown in Fig. 1a. If we demand consistency between $s_3^{\min}$ and $s_3^{\max}(2.16)$ obtained from $\bar{R}^{\max} = 0.08^8)$, we find

$$m_t \geq 107 \text{ GeV} \quad (\text{"preferred"}) \quad (2.18a)$$

If we adopt a more conservative stance towards the theoretical estimates of $f_{B_d} B_{B_d}^{\frac{1}{2}}$ and consider a "minimal" scenario in which $f_{B_d} B_{B_d}^{\frac{1}{2}} = 160 \text{ MeV}$, and take $r_d = 0.10$, then we obtain Fig. 1b, from which we infer

$$m_t \geq 53 \text{ GeV} \quad (\text{"minimal"}) \quad (2.18b)$$

Since both $s_3^{\min}(2.13a)$ and $s_3^{\max}(2.16)$ scale as $\tau_b^{-\frac{1}{2}}$, the bounds (2.18a,b) are independent of τ_b , and, apart from corrections due to higher order terms in m_t^2/m_W^2 and

*) This is also compatible with the first lattice calculations³¹⁾ which are now becoming available. In our normalization convention $f_\pi \approx 93 \text{ MeV}$.

QCD, scale inversely with $f_{B_d} B_{B_d}^{\frac{1}{2}}$. The dependence on $\bar{R}^{\max}$ is easily obtained by scaling $s_3^{\max}$ (2.16) in Figs. 1a, b.

One can also use the constraints (2.11), (2.13a) and (2.16) together with the upper bound $m_t < 168$ GeV obtained¹²⁾ from a global analysis of flavour-conserving neutral current data to obtain a non-trivial lower bound on $f_{B_d} B_{B_d}^{\frac{1}{2}}$. The results for different values of $\bar{R}^{\max}$ and r_d are summarized in Table 1. They approach some of the lower theoretical estimates²⁸⁾⁻³¹⁾ of $f_{B_d} B_{B_d}^{\frac{1}{2}}$.

3. CP VIOLATION IN THE K^0 SYSTEM

The bounds obtained in Section 2 can be improved slightly by incorporating constraints from the CP violation observed in the K^0 - $\bar{K}^0$ system. The CP impurity parameter ϵ_K corresponding to Eq. (2.4) is measured¹³⁾ to be $|\epsilon_K| = 2.27 \times 10^{-3}$. In the following, we adopt the standard phase convention³²⁾ which introduces a shift from the Kobayashi-Maskawa quark basis (2.9) to one in which A_0 , the amplitude for $K^0 \rightarrow \pi\pi$ ($I = 0$) is real. The dominant short-distance contribution to ϵ_K is then given in the Standard Model by^{22),24),33)}

$$\epsilon_K = e^{i\pi/4} \left\{ \left(\frac{G_F^2 f_K^2 B_K m_K}{12\sqrt{2}\pi^2 \Delta M_K} \right) \left(\ln \left[\eta_1 \lambda_c^2 \Delta_{cc} + \eta_2 \lambda_t^2 \Delta_{tt} + 2\eta_3 \lambda_c \lambda_t \Delta_{ct} \right] \right) + \sqrt{2} \Im \left(\text{Re } M_{12}^{sd} / \Delta M \right) \right\} \quad (3.1)$$

where B_K parametrizes the matrix element of the $(V-A)^2 \Delta S = 2$ operator relative to its vacuum insertion value $B_K = 1.0$, the $\lambda_q \equiv V_{qs}^* V_{qd}$ are products of Kobayashi-Maskawa matrix elements (2.9), the $\Delta_{qq'}$ are quark mass-dependent factors and $\eta_{1,2,3}$ are QCD correction factors. We use the exact expressions for the $\Delta_{qq'}$ and η_i given in Refs. 22),24),33). The recent preliminary experimental upper value⁹⁾ of $|\epsilon'/\epsilon_K|$ tells us that the second term in (3.1) is $\lesssim 10^{-4}$ so we will neglect it in our discussion of ϵ_K .

In principle, for any fixed value of m_t , the experimental values of r_d ²⁾, τ_b ⁷⁾ and ϵ_K ¹³⁾ now provide three equations for the three unknowns θ_2 , θ_3 and δ . The experimental upper bound on $\bar{R}$ ⁸⁾ and the value⁹⁾ of $|\epsilon'/\epsilon_K|$ also provide two constraints which can be useful for eliminating spurious solutions to these three equations. In particular, they may tell us that acceptable solutions only exist for a restricted range of m_t .

However, in addition to the theoretical uncertainties in f_{B_d} and B_{B_d} discussed previously, we must now cope with the added uncertainties associated with B_K . Non-perturbative methods are needed to calculate B_K . Most calculations³⁴⁾⁻³⁷⁾ give values in the range $0.33 < B_K < 1.0$, which is the range we use in our numerical studies. A value at the lower end of this range has been favoured by chiral symmetry and flavour SU(3)³⁵⁾. However, both these symmetries are only approximate. Recently a calculation³⁶⁾ based on the $1/N_c$ expansion and using effective Lagrangian techniques has yielded $B_K = 0.70 \pm 0.07$. Recent lattice calculations³⁷⁾ are also compatible with this value. Therefore we will sometimes use $B_K = 0.7$ for illustrative purposes.

Figures 1a,b include the results of adding to our previous analysis the CP violation constraint (3.1) for the favoured range $0.33 < B_K < 1.0$ (the cross-hatched region) and for the illustrative value $B_K = 0.7$. Values of s_3 at the boundary of the domain previously allowed are no longer permitted because $e^{i\delta} = \pm 1$. (Values of $\sin\delta$ as functions of m_t are plotted in Figs. 2a,b.) Our previous lower bound on m_t is consequently improved, although the improvement is marginal and dwarfed by the uncertainties in $f_{B_d} B_{B_d}^{\frac{1}{2}}$, r_d ²⁾ and $\bar{R}$ ⁸⁾. For $B_K = 0.7$ we find in the "preferred" scenario $f_{B_d} B_{B_d}^{\frac{1}{2}} = 110$ MeV, $r_d = 0.20$:

$$m_t \geq 108 \text{ GeV} \quad (107 \text{ GeV}) \quad (3.2a)$$

and in the "minimal" scenario $f_{B_d} B_{B_d}^{\frac{1}{2}} = 160$ MeV, $r_d = 0.10$:

$$m_t \geq 54 \text{ GeV} \quad (53 \text{ GeV}) \quad (3.2b)$$

where the figures in parentheses are the previous lower bounds derived in Section 2.

It is useful to view the above constraints in the (s_2, s_3) plane, where the constraints (2.13) from τ_b and (2.16) from $\bar{R}$ define a monokini-shaped wedge of allowed domain, as seen in Fig. 3. Within this domain, the $B_d^0 - \bar{B}_d^0$ mixing constraint (2.11) implicitly defines m_t as a function of s_2 , which in turn feeds into the ϵ_K constraint (3.1) and further restricts s_3 as a function of s_2 . Superimposed on this is the constraint $m_t < 168$ GeV from flavour-conserving neutral current data¹²⁾. We see that despite considerable uncertainties in the values of $f_{B_d} B_{B_d}^{\frac{1}{2}}$ ²⁸⁾⁻³¹⁾ and B_K ³⁴⁾⁻³⁷⁾, the combined application of r_d , ϵ_K and m_t constraints is very effective in restricting the allowed domain of (s_2, s_3) .

We now discuss the other CP-violating observable $\varepsilon'/\varepsilon_K$, for which we have the recent preliminary experimental result $|\varepsilon'/\varepsilon_K| < 0.0035 \pm 0.0015$ ⁹⁾. This falls within the expected range, and could in principle provide an additional constraint on m_t , but does not in practice, as we now see. In the Standard Model³⁸⁾

$$|\varepsilon'/\varepsilon_K| = 8.4 (C_2 S_2 S_3 \sin \delta) \left| \frac{\text{Im } \tilde{C}_6}{0.1} \right| \left| \frac{\langle \pi\pi(I=0) | Q_6 | K^0 \rangle}{1.4 \text{ GeV}^3} \right| R_\eta \quad (3.3)$$

where

$$Q_6 \equiv \left(\bar{S}_\alpha \gamma^\mu (1+\gamma_5) d_\beta \right) \left(\bar{u}_\beta \gamma_\mu (1+\gamma_5) u_\alpha + \bar{d}_\beta \gamma_\mu (1+\gamma_5) d_\alpha + \bar{S}_\beta \gamma_\mu (1+\gamma_5) S_\alpha \right) \quad (3.4)$$

and the Wilson coefficient $\text{Im } \tilde{C}_6$ is estimated³⁹⁾ to have the value $\text{Im } \tilde{C}_6 \approx -0.1$. A recent re-estimate⁴⁰⁾ of $\langle \pi\pi(I=0) | Q_6 | K^0 \rangle$ using the same $(1/N_c)$ calculational scheme as that leading to $B_K = 0.7$ suggests that

$$\langle \pi\pi(I=0) | Q_6 | K^0 \rangle \simeq (-0.38 \text{ to } -0.56) \text{ GeV}^3 \quad (3.5)$$

The factor R_η in Eq. (3.3) is an isospin-violating correction to the conventional SU(2) relation between η_{+-} , η_{00} and $\varepsilon'/\varepsilon_K$

$$\varepsilon'/\varepsilon_K = R_\eta \left(\varepsilon'/\varepsilon_K \right)_{\text{SU}(2)} \quad (3.6)$$

which arises from π^0 - η and π^0 - η' mixing. It has recently been estimated⁴¹⁾ to lie in the range

$$1.57 < R_\eta < 1.95 \quad (3.7)$$

This correction factor R_η largely compensates for the reduction in the estimate (3.5)⁴⁰⁾ relative to previous estimates³⁸⁾ of $\langle \pi\pi(I=0) | Q_6 | K^0 \rangle$. Lower bounds on $\varepsilon'/\varepsilon_K$ as a function of m_t have previously been obtained³⁸⁾ by fitting the CP impurity parameter ε_K in (3.1) while saturating the upper bounds on s_2 and s_3 from τ_b and $\bar{R}$. In principle, the incorporation of the new constraint (2.11) on s_2 from B_d^0 - $\bar{B}_d^0$ mixing permits an improved lower bound on $\varepsilon'/\varepsilon_K$ when $m_t > m_t^{\min}$ as defined in

(2.18), since in this case s_2 is forced below the maximum value allowed by τ_b and $\bar{R}$ (see Fig. 1). However, the value of $m_t^{\min}$ (2.18a,b) itself depends sensitively on $f_{B_d} B_{B_d}^{\frac{1}{2}}$ and r_d , and can even exceed the upper limit $m_t = 168$ GeV imposed by flavour-conserving neutral current data¹²⁾, for small (but not inconceivable) values of $f_{B_d} B_{B_d}^{\frac{1}{2}} \sim 80$ GeV as seen in Table 1. Hence the constraint from $B_d^0 - \bar{B}_d^0$ mixing does not permit an improved lower bound on ϵ'/ϵ_K . However, by placing a non-trivial lower bound on $m_t \gtrsim 50$ GeV, the observation of $B_d^0 - \bar{B}_d^0$ mixing effectively results in an upper bound on ϵ'/ϵ_K , since the latter decreases monotonically with m_t . The upper bounds on ϵ'/ϵ_K which follow from the inclusion of $B_d^0 - \bar{B}_d^0$ mixing are shown in Fig. 4 together with the previous lower bounds³⁸⁾ derived from ϵ_K , τ_b and $\bar{R}$, using the representative choices $\langle \pi\pi(I=0) | Q_6 | K^0 \rangle = -0.5 \text{ GeV}^3$ and $R_\eta = 1.75$, and the choices $B_K = 0.33$ and 1.0 (Fig. 4a) and $B_K = 0.70$ (Fig. 4b). The upper bounds follow from our "minimal" scenario $f_{B_d} B_{B_d}^{\frac{1}{2}} = 160$ MeV, $r_d = 0.10$, whilst a "preferred" estimate of ϵ'/ϵ_K which results from our "preferred" values $f_{B_d} B_{B_d}^{\frac{1}{2}} = 110$ MeV, $r_d = 0.20$ is also shown as a dashed line. The recently reported⁹⁾ preliminary measurement $\epsilon'/\epsilon_K = 0.0035 \pm 0.0015$, (shown by dotted lines) is highly compatible with this theoretical prediction of the Standard Model. However, it is clear from Fig. 4 that the measurement of ϵ'/ϵ_K does not yet give us any useful additional information on m_t .

4. $B_s^0 - \bar{B}_s^0$ MIXING AND CP VIOLATION IN B MESON MIXING

We now explore the consequences of $B_d^0 - \bar{B}_d^0$ mixing for $B_s^0 - \bar{B}_s^0$ mixing and for CP violation in the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ systems. First we discuss $B_s^0 - \bar{B}_s^0$ mixing, whose description is identical to Eqs. (2.1) to (2.10) with d subscripts replaced by s subscripts. The relevant combination of Kobayashi-Maskawa matrix elements

$$\begin{aligned} V_{ts}^* V_{tb} &= (c_1 s_2 c_3 + c_2 s_3 e^{i\delta}) (c_1 s_2 s_3 - c_2 c_3 e^{i\delta}) \\ &\simeq -(s_3 + s_2 e^{i\delta}) \simeq -V_{cb} \end{aligned} \quad (4.1)$$

is already fixed by the b lifetime (2.12) and is therefore insensitive to the new determination of s_2 from $B_d^0 - \bar{B}_d^0$ mixing (2.11). However, many of the same uncertainties appear in both $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing, and their relative magnitude is governed by a simple ratio of matrix elements:

$$\frac{M_{12}^{B_s^0}}{M_{12}^{B_d^0}} \approx \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} \left| \frac{V_{ts}}{V_{td}} \right|^2 \quad (4.2)$$

The magnitude of V_{ts} is fixed by τ_b (4.1), whereas $V_{td} = s_1 s_2$ is bounded above by τ_b and $\bar{R}$. From Eq. (2.12) we observe that s_2 is maximized when $s_3 = s_3^{\max}$ and $\cos\delta = -1$, from which we conclude

$$s_2 \leq s_2^{\max} = s_3^{\max} + 0.05 \left(\frac{1.11 \times 10^{-12} s}{\tau_b} \right)^{1/2} \quad (4.3)$$

For $\bar{R}^{\max} = 0.08$ we find, independently of τ_b , that

$$\left| \frac{V_{ts}}{V_{td}} \right| \geq 2.3 \quad (4.4)$$

Assuming $f_{B_s} B_{B_s}^{1/2} > f_{B_d} B_{B_d}^{1/2}$, the ratio (4.4) implies that

$$r_s \geq 0.77 \quad \text{for} \quad r_d = 0.10 \quad (4.5a)$$

and

$$r_s \geq 0.88 \quad \text{for} \quad r_d = 0.20 \quad (4.5b)$$

confirming previous expectations⁴²⁾ of large, but not necessarily maximal, $B_s^0 - \bar{B}_s^0$ mixing in the Standard Model. For $m_t > m_t^{\min}$ as defined in Eqs. (2.18), the $B_d^0 - \bar{B}_d^0$ mixing constraint (2.11) drives s_2 away from its maximum value (4.3), increasing the ratio $|V_{ts}/V_{td}|$ and hence r_s/r_d .

It is useful to view the results (4.4), (4.5) in the (r_s, r_d) plane shown in Fig. 5, on which various bounds on r_d and r_s from different experiments^{1),2),14)} have been superimposed. The shaded region represents the domain of (r_s, r_d) that is simultaneously compatible with the ARGUS measurement²⁾ of $B_d^0 - \bar{B}_d^0$ mixing and the Standard Model prediction (4.4), (4.5) of large $B_s^0 - \bar{B}_s^0$ mixing. There could appear to be a potential conflict between this region and the upper bound on neutral B meson mixing from the Mark II Collaboration¹⁴⁾. However, any such conclusion would depend sensitively upon experimentally untested hypotheses for the relative amounts of B_u^- , B_d^0 , B_s^0 and especially b-baryon production¹⁴⁾, as can be seen comparing curves (a), (b) and (c) in Fig. 5. It will nevertheless be interesting to monitor this

situation as the data improve. If the Standard Model prediction of large $B_s^0 - \bar{B}_s^0$ mixing were to be ruled out by experiment, this would be a clear indication of new physics in $B^0 - \bar{B}^0$ mixing. Such an observation could in principle be accommodated by a fourth generation⁴³⁾, which would invalidate the unitarity constraints leading to the relations (4.1) to (4.5).

The effect on $B_s^0 - \bar{B}_s^0$ mixing of adding the CP violation constraint (3.1) is rather indirect. The magnitude of the Kobayashi-Maskawa angles (4.1) responsible for $B_s^0 - \bar{B}_s^0$ mixing is fixed by the b lifetime and therefore insensitive to ε_K . But CP violation, by imposing $\sin\delta \neq 0$, prevents s_2 and thus V_{td} from reaching the maximal value (4.3). Hence the ratio $|V_{ts}/V_{td}|$ and hence r_s/r_d is increased relative to (4.4), resulting in an improved lower limit on r_s as a function of r_d . However, the improvement is again marginal: for the illustrative value $B_K = 0.7$ we find

$$r_s \geq 0.78(0.77) \text{ for } r_d = 0.10 \quad (4.6a)$$

and

$$r_s \geq 0.88(4) \text{ (0.88(3)) for } r_d = 0.20 \quad (4.6b)$$

where the figures in parentheses are the previous lower bounds (4.5) which were derived without the ε_K constraint. The "improved" bounds (4.6) are, in addition, rather sensitive to B_K and also depend on $f_{B_s} B_{B_s}^{\frac{1}{2}}$.

We now turn to CP violation in $B^0 - \bar{B}^0$ mixing. The like-sign dilepton charge asymmetry $a \equiv (\#l^+l^+ - \#l^-l^-)/(\#l^+l^+ + \#l^-l^-)$ depends on $\text{Re } \varepsilon_B$ through the exact, rephasing invariant expression⁴⁴⁾

$$a = \frac{-4\text{Re } \varepsilon (1 + |\varepsilon|^2)}{(1 + |\varepsilon|^2)^2 + 4(\text{Re } \varepsilon)^2} \quad (4.7)$$

which can also be expressed⁴⁵⁾ more simply in terms of M_{12} and Γ_{12} :

$$a = \frac{\text{Im}(M_{12}^* \Gamma_{12})}{|M_{12}|^2 + |\Gamma_{12}|^2/4} \simeq \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \quad (4.8)$$

where the final expression follows from the approximation $|\Gamma_{12}| \ll |M_{12}|$ introduced in Section 2. The width difference Γ_{12} is given in the Standard Model by

$$\Gamma_{12} = - \frac{G_F^2 f_B^2 B_B m_B}{8\pi} \left[\lambda_t^2 m_t^2 + \frac{8}{3} \lambda_c \lambda_t m_c^2 + O\left(\frac{m_c^4}{m_t^2}\right) \right] \quad (4.9)$$

where $\lambda_q \equiv (V_{qd}^* V_{qb})$ for $B_d^0 - \bar{B}_d^0$ mixing and $\lambda_q \equiv (V_{qs}^* V_{qb})$ for $B_s^0 - \bar{B}_s^0$ mixing. Inserting (4.9) into (4.8) and noting that the leading term in $\Gamma_{12} \propto m_b^2$ has the same dependence on Kobayashi-Maskawa angles as M_{12} (2.10), we obtain

$$a = -4\pi \frac{m_c^2}{m_t^2} \ln \left(\lambda_c / \lambda_t \right) \quad (4.10)$$

For the $B_d^0 - \bar{B}_d^0$ system, Eq. (4.10) implies

$$a_d = -4\pi \frac{m_c^2}{m_t^2} \frac{s_3 \sin \delta}{s_2} \quad (4.11a)$$

and

$$a_s = -4\pi \frac{m_c^2}{m_t^2} \ln \left[\frac{(c_1 c_2 c_3 - s_2 s_3 e^{-i\delta})(c_1 c_2 s_3 + s_2 c_3 e^{i\delta})}{(c_1 s_2 c_3 + c_2 s_3 e^{-i\delta})(c_1 s_2 s_3 - c_2 c_3 e^{i\delta})} \right] \quad (4.11b)$$

QCD radiative corrections to Γ_{12} (4.9) and M_{12} (2.10), which affect the numerator and denominator of (4.10) and (4.11), are described in Ref. 22) and have been included in our numerical calculations, along with corrections of higher order in m_c^2/m_W^2 . As discussed in Section 3, the constraints from r_d , τ_b and ε_K in principle allow a simultaneous determination of the three previously unknown Kobayashi-Maskawa parameters s_2 , s_3 and $\sin \delta$ as functions of m_t , and thereby permit a determination of (4.11). Figure 6 plots both a_d and a_s in the "preferred" and "minimal" scenarios introduced previously, for the allowed range $0.33 < B_K < 1.0$ and for the illustrative choice $B_K = 0.7$. We observe that a_d is negative, and that its magnitude is largest in the "minimal" scenario of Fig. 6b when $B_K = 0.33$. We find

$$0 < -a_d \lesssim 0.3 \text{ \%} \quad (4.12)$$

which is too small to be observed in the foreseeable future. We also observe that a_s is typically a factor of 5 to 10 smaller than a_d , and that its sign is positive.

Such small values of the charge asymmetries $a_{d,s}$ are a distinctive feature of the standard six-quark model. They result from the fact that Γ_{12} and M_{12} in (4.8) are both $\propto \lambda_t^2$ and thus have a common phase up to small SU(4) breaking corrections

$\propto ((m_c^2 - m_u^2)/m_b^2)$. This is in turn due to the unitarity of the Kobayashi-Maskawa matrix (2.9), which allows the replacement of $-(\lambda_u + \lambda_c)$ occurring in the calculation of Γ_{12} by the λ_t appearing in (4.9). If M_{12} were actually due to new physics, such as the exchange of a massive flavour-changing neutral Higgs or gauge boson, or even in the presence of a fourth generation so that the unitarity relation $\lambda_t = -(\lambda_u + \lambda_c)$ ceases to hold, then the relative phase of Γ_{12} and M_{12} would become arbitrary. The magnitude of $a_{d,s}$ would then be bounded only by the relative magnitude of Γ_{12} and M_{12} :

$$|a| \leq \left| \frac{\Gamma_{12}}{M_{12}} \right| \quad (4.8')$$

The width difference Γ_{12} for the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ systems would probably still be described by (4.9) with λ_t replaced by $-(\lambda_u + \lambda_c)$, since it is purely a low-energy phenomenon and therefore presumably due to known physics. The mass difference M_{12} is however predominantly a high-energy phenomenon which could be sensitive to the exchange of a fourth-generation t' quark or to some other new physics, and hence its phase is in general unknown. However, the magnitude of $M_{12}^{B_d^0}$ is approximately determined by the measurement of r_d :

$$|M_{12}^{B_d^0}| = 1.4 \times 10^{-13} \text{ GeV} \left(\frac{1.11 \times 10^{-12} \text{ s}}{\tau_b} \right) \left[\left(\frac{r_d}{1-r_d} \right) \left(\frac{0.1}{1-0.1} \right) \right]^{1/2} \quad (4.13)$$

Thus in this more general framework

$$|a_d| \leq \left| \frac{\Gamma_{12}^{B_d^0}}{M_{12}^{B_d^0}} \right| = \frac{1}{|M_{12}^{B_d^0}|} \frac{G_F^2 f_{B_d}^2 B_{B_d} m_{B_d}}{8\pi} \left| (\lambda_u + \lambda_c)^2 m_t^2 + O(m_c^2) \right| \quad (4.14)$$

where $|\lambda_u + \lambda_c| = s_1^2 s_2^2$ is bounded by τ_b and $\bar{R}$. The charge asymmetry (4.14) is maximized by taking $s_2 = s_2^{\max}$ (4.3) and $f_{B_d}^2 B_{B_d}$ large. Thus we obtain the model-independent bound^{*)}

$$|a_d| \leq 6.08 \left(\frac{f_{B_d} B_{B_d}}{160 \text{ MeV}} \right)^2 \left[\frac{1-r_d}{9r_d} \right]^{1/2} \quad (4.15)$$

Equation (4.15) includes a modest QCD enhancement factor $(2f_+ + f_-)/3 \approx 1.10$ associated with the leading term $\propto m_b^2$ in (4.14).

^{*)} A similar result for four generations has been obtained in Ref. 46).

Since $|M_{12}^{B_s^0}|$ is not experimentally determined, no model-independent upper bound on $|a_s|$ of the form (4.15) is possible. However, if for example the UA1¹⁾ or Mark II¹⁴⁾ data were eventually to permit a lower bound $r_s > r_d$, a bound comparable to (4.15) would also apply to $|a_s|$.

Although the maximum asymmetry (4.15) is not large, it is potentially observable, in contrast to the Standard Model prediction (4.12). If a CP-violating charge asymmetry $a_{d,s} > 0.3$ (0.1)% were measured, this would be clear evidence for new physics in $B_d^0-\bar{B}_d^0$ ($B_s^0-\bar{B}_s^0$) mixing.

5. RARE K DECAYS

In this section we comment on flavour-changing K decays in the light of $B_d^0-\bar{B}_d^0$ mixing.

First, we note that the short-distance top quark contribution ΔM_K^t to the K_L-K_S mass difference is now known to be negligible compared to the charm quark contribution ΔM_K^c . They are given by

$$\Delta M_K^t \propto (V_{ts}^* V_{td})^2 \tilde{m}_t^2 \sim s_1^2 s_2^2 \tilde{m}_t^2 \left[0.05 \left(\frac{1.1 \times 10^{-12} s}{\tau_b} \right)^{1/2} \right]^2 \quad (5.1a)$$

$$\Delta M_K^c \propto (V_{cs}^* V_{cd})^2 \tilde{m}_c^2 \sim s_1^2 m_c^2 \quad (5.1b)$$

where the final expression in (5.1a) follows from (4.1) and (2.12). The product $s_2^2 \tilde{m}_t^2$ in (5.1a) is fixed by the $B_d^0-\bar{B}_d^0$ mixing constraint (2.11):

$$s_2^2 \tilde{m}_t^2 = (10.8 \text{ GeV})^2 \left(\frac{r_d}{1-r_d} \right)^{1/2} \left(\frac{0.11 \text{ GeV}}{f_{B_d} B_{B_d}^{1/2}} \right)^2 \left(\frac{1.1 \times 10^{-12} s}{\tau_b} \right) \quad (5.2)$$

If one maximizes the top quark contribution ΔM_K^t by choosing r_d ($f_{B_d} B_{B_d}^{1/2}$) as large (small) as possible consistent with Table 1, one still finds $\Delta M_K^t \lesssim 0.1 \times \Delta M_K^c$. We therefore conclude that the K_L-K_S mass difference is predominantly due to the charm quark and/or long-distance contributions.

Turning to the amplitude for $K_L \rightarrow \mu^+ \mu^-$, the top quark contribution⁴⁷⁾ is proportional to $(V_{ts}^* V_{td}) m_t^2$ and therefore becomes effectively linear in m_t once the reciprocal dependence of $V_{td} = s_1 s_2$ on m_t is taken into account. Including higher order terms in m_t^2/m_W^2 ^{22), 24)} and QCD effects¹⁶⁾, we find that the top quark

contribution to the $K_L \rightarrow \mu^+ \mu^-$ amplitude is small compared to that allowed by subtracting the calculable absorptive $K_L \rightarrow \gamma\gamma \rightarrow \mu^+ \mu^-$ contribution from the measured $\gamma\gamma$ branching ratio⁴⁸⁾, and also small compared to the dispersive contribution expected from off-shell intermediate photons, for all $m_t < 168$ GeV as allowed by the flavour-conserving neutral current data. We therefore conclude that there are no additional useful bounds on mixing angles or m_t to be gained from $K_L \rightarrow \mu^+ \mu^-$.

A detailed analysis of the rare decay $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ was given in Ref. 5), based on the ARGUS measurement of r_d and the constraints (2.12) and (2.16) due to τ_b and $\bar{R}$. Here we simply comment on the additional constraints derived from CP violation (ϵ_K) and from flavour-conserving neutral current data ($m_t < 168$ GeV). The only effect of CP violation on the rate for $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ is to fix the relative phase of the short-distance charm and top quark contributions. We recall that

$$\begin{aligned} BR(K^+ \rightarrow \pi^+ \bar{\nu} \nu) &= \frac{0.61 \times 10^{-6}}{|V_{us}|^2} \left| \sum_{j=c,t} V_{js}^* V_{jd} D(x_j) \right|^2 \\ &\simeq 0.61 \times 10^{-6} \left| D(x_c) + s_2 (s_2 + s_3 e^{i\delta}) D(x_t) \right|^2 \end{aligned} \quad (5.3a)$$

where $x_j \equiv m_j^2/m_W^2$ and

$$D(x) = \frac{1}{8} \left[1 + \frac{3}{(1-x)^2} - \frac{(4-x)^2}{(1-x)^2} \right] x \ln x + \frac{x}{4} - \frac{3}{4} \frac{x}{1-x} \quad (5.3b)$$

The effect on our previous bounds of taking $e^{i\delta} = 1$ in (5.3a) is minor since the top quark dominates the charm quark contribution throughout most of the allowed domain of masses and mixing angles, and also since $\text{Re}(s_2 + s_3 e^{i\delta}) > 0$ for $\bar{R} > 0.08$ ³⁸⁾ so that the interference between the charm and top quark contributions is essentially constructive, as was already assumed in Ref. 5). The effect of the constraint $m_t < 168$ GeV is to strengthen slightly the previous upper bounds on $BR(K^+ \rightarrow \pi^+ \bar{\nu} \nu)$, as is easily seen from the diagrams of Ref. 5). Including higher order terms in m_t^2/m_W^2 ²⁴⁾ and QCD corrections¹⁶⁾, we now find

$$BR(K^+ \rightarrow \pi^+ \bar{\nu} \nu) = (1.67) \times 10^{-10} \left((1.68) \times 10^{-10} \right) \quad (5.4)$$

for three generations of neutrinos, where the number in parentheses represents the previous bound⁵⁾. As has been pointed out by Nir⁶⁾, the minimum value of $B(K^+ \rightarrow \pi^+ \bar{\nu} \nu)$ actually occurs in the interior of the range of s_3 allowed by Section 2. With our choice of phenomenological parameters, the minimum corresponds to $B_K \sim 0.33$ and is $\sim 10\%$ lower than on the boundary of the s_3 domain. Increasing the allowed range of f_{B_d} decreases the lower bound in (5.4).

6. PHENOMENOLOGICAL MODELS FOR THE KOBAYASHI-MASKAWA MATRIX

In this section we review the predictions of two phenomenological models for the quark mass mixing matrices and Kobayashi-Maskawa matrix, and examine them for consistency with the determinations of s_2 , s_3 and $\sin\delta$ made in Section 3.

We start with the Stech Ansatz¹⁷⁾, which may be phrased in the form

$$V_{KM} M_D V_{KM}^T = p M_U + i A \quad (6.1)$$

where V_{KM} is the Kobayashi-Maskawa matrix (2.9), M_D and M_U are the diagonalized charge $-1/3$ and charge $+2/3$ quark mass matrices:

$$M_D = \begin{pmatrix} m_d & 0 & 0 \\ 0 & -m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \quad M_U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & -m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad (6.2)$$

p is an arbitrary real number, and

$$A = -A^T = A^* \quad (6.3)$$

is a real, antisymmetric matrix. Since the Ansatz (6.1) contains four parameters, and there are a priori four unknown Kobayashi-Maskawa parameters s_1 , s_2 , s_3 and $\sin\delta$ as well as the unknown m_t , we cannot hope that the Ansatz (6.1) will determine everything uniquely without additional information. Since the relation (6.1) is highly non-linear, it is convenient to use the Wolfenstein⁴⁹⁾ parametrization of V_{KM} , in which phenomenological observations about the relative magnitudes of different terms are already incorporated. To leading non-trivial order in a small real parameter λ it is

$$V_{KM} \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (6.4)$$

where A , ρ and η are real and $O(1)$. Comparing (6.4) with (2.9), we see that a different phase convention is being used for the quarks. In the Kobayashi-Maskawa phase convention, the Wolfenstein parametrization (6.4) takes the following form:

$$V_{KM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & -\lambda & -A\lambda^3 \sqrt{\rho^2 + \gamma^2} \\ \lambda & 1 - \frac{\lambda^2}{2} & \frac{A\lambda^2(\rho + i\gamma)}{\sqrt{\rho^2 + \gamma^2}} \\ A\lambda^3 \sqrt{(1-\rho)^2 + \gamma^2} & \frac{A\lambda^2(1-\rho + i\gamma)}{\sqrt{(1-\rho)^2 + \gamma^2}} & -e^{i\delta} \end{pmatrix} \quad (6.4')$$

from which we see that

$$s_1 = \lambda, \quad s_3 = A\lambda^2 \sqrt{\rho^2 + \gamma^2}, \quad s_2 \approx A\lambda^2 \sqrt{(1-\rho)^2 + \gamma^2}$$

$$\sin \delta \approx \frac{\gamma}{\sqrt{(1-\rho)^2 + \gamma^2} \sqrt{\rho^2 + \gamma^2}} \quad (6.5)$$

Since the Wolfenstein parametrization is not exact, one must choose by convention which of the matrix elements in (6.4) and (6.4') are to be taken as exact. We take the three upper right-hand off-diagonal entries as exact, and make series expansions of the other entries as necessary. Since we will be examining the Ansatz using an expansion in powers of λ , it is useful to have in mind the following phenomenological orders of magnitude:

$$\frac{m_d}{m_s}, \frac{m_s}{m_b} = O(\lambda^2); \quad \frac{m_u}{m_d} = O(1); \quad \frac{m_u}{m_c} = O(\lambda^4); \quad \frac{m_c}{m_t} = O(\lambda^2) \quad (6.6)$$

After these preliminaries, we are now ready to study the phenomenological consequences of the Ansatz (6.1).

Looking at the off-diagonal terms in the matrix equation (6.1), note that $A_{12}A_{23}A_{31}$ is real and rephasing-invariant, implying that

$$\text{Re} \left[(m_d V_{ud} V_{cd}^* - m_s V_{us} V_{cs}^* + m_b V_{cb}^* V_{ub}) \cdot (m_d V_{td}^* V_{cd} - m_s V_{ts}^* V_{cs} + m_b V_{cb} V_{tb}^*) \cdot (m_d V_{ud}^* V_{td} - m_s V_{us}^* V_{ts} + m_b V_{ub}^* V_{tb}) \right] = 0 \quad (6.7)$$

Inserting either (6.4) or (6.4') into (6.7) we find

$$\rho \approx -\frac{m_s}{m_b} \left(1 + \gamma^2 \left(\frac{m_b}{m_s} \right)^2 |V_{cb}|^2 \right) \quad (6.8)$$

The terms in (6.8) are of the same order of magnitude if $\eta = O(1)$, but as we will see there is no information on η from the Stech Ansatz. We turn now to the diagonal entries in the two sides of (6.1), which tell us

$$|V_{ud}|^2 m_d - |V_{us}|^2 m_s + |V_{ub}|^2 m_b = p m_u \quad (6.9a)$$

$$|V_{cd}|^2 m_d - |V_{cs}|^2 m_s + |V_{cb}|^2 m_b = -p m_c \quad (6.9b)$$

$$|V_{td}|^2 m_d - |V_{ts}|^2 m_s + |V_{tb}|^2 m_b = p m_t \quad (6.9c)$$

We can determine p in terms of m_t by adding together the three equations (6.9):

$$p = \frac{m_b - m_s + m_d}{m_t - m_c + m_u} \quad (6.10)$$

This leaves us with two independent relations remaining among the equations (6.9). We can take these to be (6.9a) which tells us that

$$\lambda^2 = m_d/m_s + O(\lambda^4) \quad (6.11)$$

and (6.9c) which tells us that

$$(A\lambda^2)^2 = \left(\frac{m_s - m_d}{m_b} - \frac{m_c}{m_t} \right) + O(\lambda^6) \quad (6.12)$$

Equations (6.11) and (6.12) both have significant differences from previous analyses of the Stech Ansatz. The relation (6.11) is sometimes stated¹⁹⁾ to be exact without the $O(\lambda^4)$ correction, but we see no reason for such an identity. The relation (6.12) has previously been given²⁰⁾ without the $(-m_d/m_b)$ term on the right-hand side, which induces a multiplicative error of $O(1)$ in the formula for $A\lambda^2 = |V_{cb}|$, since Eq. (6.6) tells us that $(-m_d/m_b)$ is $O(\lambda^4)$ which is the same order as the left-hand side. Equation (6.12) can be rewritten as

$$m_t \simeq m_c \left(\frac{m_b}{m_s} \right) \frac{1}{1 - \frac{m_d}{m_s} - \frac{m_b}{m_s} |V_{cb}|^2} \quad (6.13)$$

which exhibits the leading order expression for m_t and two $O(\lambda^2)$ corrections which are of similar numerical size.

We now investigate whether the phenomenological consequences (6.8), (6.11), (6.12) or (6.13) of the Stech Ansatz are compatible with experiment. We assume the following numerical values for the "measured" ratios of quark masses⁵⁰⁾:

$$\begin{aligned} \frac{m_d}{m_s} &= 0.051 \pm 0.004, & \frac{m_u}{m_c} &= 0.0038 \pm 0.0012, \\ \frac{m_s}{m_b} &= 0.033 \pm 0.011 \end{aligned} \quad (6.14)$$

and take $m_c(1 \text{ GeV}) = 1.35 \pm 0.05 \text{ GeV}$. These, together with knowledge of $|V_{cb}|$ from τ_b , should in principle allow one to determine m_t using Eq. (6.13). Unfortunately, the errors in (6.14), particularly in (m_s/m_b) , allow a large range of m_t . Our strategy for analyzing the Stech Ansatz is to assume the central values for (m_d/m_s) and $(m_u/m_c)^{*)}$ given in (6.14), as well as $m_c = 1.35 \text{ GeV}$ and our phenomenological analysis in Section 3 for the Kobayashi-Maskawa matrix elements as functions of m_t . We then allow (m_s/m_b) to vary over the full range allowed in (6.14), and step (1) determine m_t using (6.13) and incorporating QCD corrections between $\mu = 1 \text{ GeV}$ and m_t , (2) determine s_2 using (2.11), and (3) determine η using (6.5), (6.8), i.e.,

$$\eta^2 \approx \frac{\frac{s_2^2}{|V_{cb}|^2} - \left(1 + \frac{m_s}{m_b}\right)^2}{1 + 2|V_{cb}|^2 \left(1 + \frac{m_b}{m_s}\right)} \quad (6.15)$$

and finally (4) determine s_3 and δ using (6.5), (6.8) and (6.15). In this way we arrive at a locus in the (m_t, s_3) and (m_t, s_δ) planes, which can be compared with the results of our phenomenological analysis in Section 3 for s_2 and s_3 , as shown in Figs. 1,2,3.

We first note that the range of m_t predicted by the Stech Ansatz (6.13) for $0.022 < m_s/m_b < 0.044$ is

$$23 \text{ GeV} < m_t < 47 \text{ GeV} \quad (6.16)$$

which is already somewhat difficult to reconcile with our previous conclusion that $m_t > 53 \text{ GeV}$ in our "minimal" scenario (2.18b). Moreover, the bounds (2.18) were derived by setting s_3 equal to $s_3^{\max}$ - its maximal value consistent with s_2 and τ_b (2.13). However, the Stech Ansatz predicts a value (6.5) of s_3 which is considerably larger than $s_3^{\min}$, increasing further the inconsistency with $s_3^{\max}$ which results from $\bar{R}$. Figure 7 shows the Stech prediction for s_3 as a function of m_t in our "preferred" and "minimal" scenarios, and also for $f_{B_d} B_{B_d}^{\frac{1}{2}} = 200 \text{ MeV}$. We find that the Stech Ansatz is incompatible with experiment for plausible values of $f_{B_d} B_{B_d}^{\frac{1}{2}}$.

*) Although there is considerable uncertainty in (m_u/m_c) , our results are insensitive to this ratio.

The Fritzsch Ansatz¹⁸⁾ is that the mass matrices have the form:

$$M_{D,U} = \begin{pmatrix} 0 & A_{D,U} & 0 \\ A_{D,U} & 0 & B_{D,U} \\ 0 & B_{D,U} & C_{D,U} \end{pmatrix} \quad (6.16)$$

As has been discussed in Ref. 20), one can deduce from (6.16) that

$$|V_{us}| \approx \left| \sqrt{\frac{m_d}{m_s}} - e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}} \right| \quad (6.17a)$$

$$|V_{cb}| \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right| \quad (6.17b)$$

$$|V_{ub}| \approx \left| \frac{m_s}{m_b} \sqrt{\frac{m_d}{m_t}} - e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}} \left(\sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right) \right| \quad (6.17c)$$

$$\frac{\sin \delta}{\left| \frac{V_{us} V_{cb}}{V_{ub}} \right| - \cos \delta} \approx \frac{\sin \phi_1}{\cos \phi_1 - \sqrt{\frac{m_d m_c}{m_s m_u}}} \quad (6.17d)$$

where $\phi_{1,2}$ are two arbitrary phases. Equation (6.17b) then tells us that

$$m_t \leq \frac{m_c}{\left(\sqrt{\frac{m_s}{m_b}} - |V_{cb}| \right)^2} \quad (6.18)$$

which yields an upper limit on m_t . We therefore obtain an upper bound on m_t , using the lowest value of (m_s/m_b) allowed by (6.14), which is

$$m_t \leq 84 \text{ GeV} \quad (6.19)$$

This is not yet in obvious conflict with the phenomenological analysis of Sections 2 and 3, but an exhaustive numerical search of the parameter space for m_d/m_s , m_u/m_c , m_s/m_b , m_c/m_t , ϕ_1 and ϕ_2 using Eqs. (6.17) revealed no consistent solution. For each possible value of m_t , we use (6.17a) to fix ϕ_1 , (6.17b) to fix ϕ_2 and (6.17c) to fix s_3 . The resulting range of s_3 shown in Fig. 8a is incompatible with our earlier allowed range for $0.33 < B_K < 1.0$, and is probably too small to accommodate the observed $b \rightarrow u$ transitions²⁶⁾. Finally, we use (6.17d) to fix $\sin \delta$ as shown in Fig. 8b. Whereas Fig. 8a indicates that there is almost compatibility with the phenomenological constraints on s_3 for $m_t \sim 84 \text{ GeV}$ and $B_K \sim 1.0$, we see in Fig. 8b that this gives an unacceptable value of $\sin \delta$. To obtain compatibility with some value of B_K , one would need $f_{B_d} \gtrsim 190 \text{ MeV}^{(6)}$. We conclude that the Fritzsch Ansatz is inconsistent with the combined analysis of $B_d^0 - \bar{B}_d^0$ mixing and CP violation in Section 3.

7. SUMMARY AND PERSPECTIVE

In this paper we have reviewed how the recent observation of $B_d^0-\bar{B}_d^0$ mixing²⁾, coupled with upper limits on $BR(b \rightarrow ul\bar{\nu})$ ⁸⁾ and measurements of τ_b ⁷⁾, provide^{5),6)} a lower limit on m_t of order 50 to 100 GeV. The precise value of the limit depends on $BR(b \rightarrow ul\bar{\nu})$, the assumed amount of $B_d^0-\bar{B}_d^0$ mixing and uncertain hadronic matrix elements. The upper bound on m_t from flavour-conserving neutral currents¹²⁾ can be used to give a non-trivial lower limit to these matrix elements. We have also shown that the K_L-K_S mass difference can only be due to charm or to long-distance effects, while $K_L \rightarrow \mu^+\mu^-$ is mainly due to long-distance physics. The upper bound on m_t also reduces slightly the previous upper bound⁵⁾ on $BR(K^+ \rightarrow \pi^+\bar{\nu}\nu)$, which we calculate to be in the range $(1 \text{ to } 7) \times 10^{-10}$. A measurement of this branching ratio would be a useful constraint on the Kobayashi-Maskawa matrix elements and/or m_t . It is predicted that $B_s^0-\bar{B}_s^0$ mixing should be large, but not necessarily maximal.

The magnitude ε_K of the CP violation observed in the $K^0-\bar{K}^0$ mass matrix provides an additional constraint which improves only marginally the bounds of the previous paragraph. Any improvement it provides is in any case dwarfed by the other uncertainties mentioned above. However, ε_K is useful for constraining the Kobayashi-Maskawa matrix elements, including in particular the phase δ . One can then obtain a strong upper bound on $\varepsilon'/\varepsilon_K$, within which falls the recent preliminary measurement by the NA31 experiment⁹⁾. One can also calculate and put stringent bounds on the CP-violating dilepton asymmetries for $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing. These therefore become important probes of physics beyond the Standard Model.

We have also studied certain phenomenological models^{17),18)} for the quark mass matrices and for the Kobayashi-Maskawa matrix, and shown that both the Stech and Fritzsch Ansätze are incompatible with experiment.

We conclude by remarking that the Standard Model is vulnerable as never before to future experiments. These include in particular measurements of m_t , $B_s^0-\bar{B}_s^0$ mixing, CP-violating dilepton asymmetries in $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing, and $BR(K^+ \rightarrow \pi^+\bar{\nu}\nu)$. The electroweak sector of the Standard Model is about to be subjected to its most stringent tests.

$r_d \backslash \bar{R}^{\max}$	0.05	0.08	0.13
0.10	72	64	57
0.15	81	72	64
0.20	88	79	70

TABLE

Lower bounds on $f_{B_d} B_{B_d}^{\frac{1}{2}}$ (in MeV) which result from $B_d^0 - \bar{B}_d^0$ mixing in combination with the upper bound $m_t < 168$ GeV from flavour-conserving neutral current data¹²⁾.

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FIGURE CAPTIONS

Figure 1: a) Values of s_2 , minimum values of s_3 allowed by $\tau_b^{7)}$, and maximum values of s_3 allowed by upper limits on the $b \rightarrow u$ transition⁸⁾, as functions of m_t with our "preferred" choice of parameters: $f_{B_d} B_{B_d}^{\frac{1}{2}} = 110$ MeV and $r_d = 0.20$. Also shown is the range of s_3 required to fit $\epsilon_K^{13)}$ if $0.33 < B_K < 1.0$.
b) As for a), but with our "minimal" choice of parameters: $f_{B_d} B_{B_d}^{\frac{1}{2}} = 160$ MeV and $r_d = 0.10$.

Figure 2: a) The range of $\sin\delta$ in our "preferred" scenario for $0.33 < B_K < 1.0$, as a function of m_t .
b) The range of $\sin\delta$ in our "minimal" scenario for $0.33 < B_K < 1.0$, as a function of m_t .

Figure 3: The allowed domains of the (s_2, s_3) plane a) in our "preferred" scenario, b) in our "minimal" scenario.

Figure 4: Predictions for ϵ'/ϵ_K as a function of m_t , assuming a) $B_K = 0.33$ and 1.0 , b) $B_K = 0.7$.

Figure 5: Comparison of the regions of the (r_s, r_d) plane allowed by our Standard Model analysis (labelled KM), ARGUS²⁾, UAl¹⁾ and Mark II¹⁴⁾ data. The curves labelled a,b,c correspond to probabilities 0.35 (0.10), 0.375 (0.15), 0.40 (0.20) for the b quark to combine with a u or d (s) quark respectively.

Figure 6: Predicted values of the CP-violating parameters a_d, a_s as functions of m_t in a) our "preferred" scenario, and b) our "minimal" scenario.

Figure 7: Lower bounds on s_3 in the Stech model¹⁷⁾ for our "preferred" scenario (curve a) and our "minimal" scenario (curve b), and in our "minimal" scenario but with $f_{B_d} B_{B_d}^{\frac{1}{2}} = 200$ MeV (curve c).

Figure 8: The allowed range of a) s_3 and b) $\sin\delta$ in the Fritzsch model¹⁸⁾ (shaded) with those favoured by the phenomenological analysis of Sections 2 and 3.

Fig. 1a

Fig. 1b

m_t (GeV)
Fig. 2a

Fig. 2b

Fig. 3a

Fig. 3b

m_τ (GeV)
Fig. 4a

Fig. 4b

Fig. 5

Fig. 6a

Fig. 6b

m_t (GeV)
Fig. 7

m_f (GeV)
Fig. 8a

m_t (GeV)
Fig. 8b