CERN-TH.4875/87

Top Quark Production at HERA

U. Baur

J.J. van der Bij

CERN - Geneva

Top quark production in ep -collisions at HERA energies is studied. In particular the reaction $ep \rightarrow \nu \bar{t} b X$ is analyzed. The exact analytic expression for the single top production cross-section including all quark mass effects is derived and evaluated for different values of the t -quark mass m_t . The results are compared with the rates obtained from $e\bar{b} \rightarrow \nu \bar{t} X$ where the b -quark is regarded as a sea quark in the proton and also with the rates for top-quark pair production. At HERA, single production of t -quarks is found to dominate over $t\bar{t}$ production for $m_t \gtrsim 60 \text{ GeV}$. The exact pair production rate for top quarks agrees with the one in the Weizsäcker-Williams approximation up to $\sim 15 \%$. A TeV ep -collider will give access to singly produced top quarks with masses up to several hundred GeV .

CERN-TH.4875/87

October 1987

One of the main physics goals of the accelerators which are presently operating or are under construction is to search for the top quark. If the t -quark mass m_t is less than 30 GeV , clear evidence should come from TRISTAN. SLC and LEP will be able to extend the mass range which can be explored up to $m_t \approx 50 \text{ GeV}$. Much heavier top quarks are in principle accessible at the CERN $p\bar{p}$ -collider and at the TEVATRON. However, in contrast to e^+e^- machines, a large hadronic background considerably complicates their detection at hadron colliders.

A possibility to search for t -quarks heavier than 50 GeV will also be offered by the ep -collider HERA ($\sqrt{s} \approx 314 \text{ GeV}$, $\mathcal{L} \approx 2 \cdot 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$) which is presently under construction at DESY. Top quarks can be produced in ep -collisions either pairwise in the neutral current reaction $ep \rightarrow e t \bar{t} X$, or singly in the charged current process $ep \rightarrow \nu \bar{t} b X$.¹ The corresponding Feynman diagrams are shown in figures 1a and 1b. There are several enhancement and suppression factors present in the charged as well as the neutral current process:

- i) Due to the t -channel photon exchange in $ep \rightarrow e t \bar{t} X$ pair production is enhanced with respect to single top quark production.
- ii) For heavy top quarks $t\bar{t}$ production is suppressed by phase space compared to $\bar{t}b$.
- iii) At large m_t the process

$$ep \rightarrow \nu \bar{t} b X \quad (1)$$

has a logarithmic enhancement factor of the form $\ln(m_t^2/m_b^2)$ associated with a collinear singularity which would be present for massless b -quarks.

For relatively small values of m_t one, therefore, expects that pair production will be the dominant source of top quarks in ep -collisions, whereas $\bar{t}b$ production may contribute significantly for large masses and, eventually, will dominate. Singly produced t -quarks will result in a simpler final state configuration and thus might be more easily reconstructed than $t\bar{t}$ events. This will be of particular interest at large m_t where the rates are small.

¹The reactions $ep \rightarrow \nu \bar{t} s X$ and $ep \rightarrow \nu \bar{t} d X$ are strongly suppressed due to the smallness of the ts and td weak mixing matrix elements and, therefore, will not be discussed in this paper.

Since recent experimental results^[ref.1] favor $m_t \gtrsim 50 \text{ GeV}$, the process (1) might be an important source for t -quarks at HERA. This has motivated us to study single top quark production in ep -collisions somewhat more in detail.

The differential cross-section for (1) can be represented in terms of the charged current structure functions F_2 , F_3 and F_L :

$$\begin{aligned} \frac{d^2\sigma}{dx dy}(ep \rightarrow \nu \bar{t} b X) = & |V_{tb}|^2 \frac{G_F^2}{2\pi} s \frac{m_W^4}{(Q^2 + m_W^2)^2} [(1-y + \frac{1}{2}y^2)F_2^{W^*g \rightarrow \bar{t}b}(x, Q^2) \\ & - \frac{1}{2}y^2 F_L^{W^*g \rightarrow \bar{t}b}(x, Q^2) + (y - \frac{1}{2}y^2)x F_3^{W^*g \rightarrow \bar{t}b}(x, Q^2)] . \end{aligned} \quad (2)$$

Here x is the usual Bjorken scaling variable,

$$y = \frac{Q^2}{xs} , \quad (3)$$

s and Q^2 denote the center of mass energy squared and the negative square of the momentum transfered to the $\bar{t}b$ system, respectively, m_W is the W -boson mass, and V_{tb} is the tb weak mixing matrix element which we assume to be equal to one. The variables x and y are bounded by (m_b denotes the b -quark mass)

$$0 \leq x \leq 1 - \frac{(m_t + m_b)^2}{ys} \quad (4)$$

and

$$\frac{(m_t + m_b)^2}{s} \leq y \leq 1 . \quad (5)$$

Since the expression for the longitudinal structure function $F_L = F_2 - 2xF_1$ turns out to be less complicated than the one for the usual charged current structure function F_1 , we have eliminated F_1 in favor of F_L in Eq. (2).

Calculating the $W^*g \rightarrow \bar{t}b$ structure functions is in principle straightforward. The F 's can be written as ($i = 2, 3, L$)

$$F_i^{W^*g \rightarrow \bar{t}b}(x, Q^2) = \int_{xa}^1 dz G(z, \frac{1}{2}\hat{s}) f_i^{W^*g \rightarrow \bar{t}b}(\frac{x}{z}, Q^2) \quad (6)$$

where $G(z, q^2)$ is the gluon distribution function and z is the momentum fraction of the proton carried by the gluon .

$$a = 1 + \frac{(m_t + m_b)^2}{Q^2} , \quad (7)$$

and

$$\hat{s} = Q^2 \left(\frac{z}{x} - 1 \right) \quad (8)$$

is the square of the invariant mass of the $\bar{t}b$ system. For f_L , f_2 and f_3 we find after some algebra

$$\begin{aligned} f_L^{W^*g \rightarrow \bar{t}b}(w, Q^2) = & \frac{\alpha_s(\frac{1}{2}\hat{s})}{\pi} \left\{ rw^2(1-w)[- \gamma(1-2\beta) + \gamma^2 - \beta(1-\beta) + 2] + \right. \\ & 2w^2L_1(1-2w)(\gamma-\beta) + wL_2 \left[\frac{1}{2}\gamma\left(\frac{1}{2} - w\gamma\right)(1-2w) + \gamma w^2\left(\frac{1}{2} - 4\beta - \right. \right. \\ & \left. \left. \frac{1}{2}(\beta^2 + \gamma\beta - \gamma^2)\right) + \beta w^2\left(\frac{1}{2}\beta^2 + \beta - \frac{7}{2}\right) + \beta w\left(\frac{3}{2} + \gamma - \frac{1}{2}\beta\right) + \frac{1}{4}\beta\right] \right\} , \quad (9) \end{aligned}$$

$$\begin{aligned} f_2^{W^*g \rightarrow \bar{t}b}(w, Q^2) = & \frac{\alpha_s(\frac{1}{2}\hat{s})}{\pi} \left\{ rw[w(1-w)(-\beta + \beta^2 - \gamma + \gamma^2 - 2\beta\gamma + 4) - \frac{1}{2}] + \right. \\ & 3w^2L_1(1-2w)(\gamma-\beta) + wL_2 \left[\frac{1}{4}(1+\gamma+\beta) + w(-\gamma(1-\beta) + 2\beta - \frac{1}{2}(1+\beta^2+\gamma^2)) + \right. \\ & \left. \left. \frac{1}{2}w^2(-10\gamma\beta + 3\gamma - \gamma\beta^2 + \gamma^2(3-\beta) + 1 + \gamma^3 + \beta^3 + 3\beta^2 - 9\beta)\right] \right\} , \quad (10) \end{aligned}$$

and

$$\begin{aligned} x f_3^{W^*g \rightarrow \bar{t}b}(w, Q^2) = & \frac{\alpha_s(\frac{1}{2}\hat{s})}{\pi} \left\{ rw^2(1-w)(\beta - \gamma) + wL_1\left(\frac{1}{2} - w + w^2\right) + \right. \\ & \left. wL_2\left[-\frac{1}{4} + \frac{1}{2}w(1-\beta+\gamma) + \frac{1}{2}w^2(\beta^2 + 2\beta - \gamma^2 - 2\gamma - 1)\right] \right\} \quad (11) \end{aligned}$$

where

$$\gamma = \frac{m_t^2}{Q^2} , \quad (12)$$

$$\beta = \frac{m_b^2}{Q^2} , \quad (13)$$

$$r = \frac{1}{\hat{s}} [m_b^4 - 2m_b^2(\hat{s} + m_t^2) + (\hat{s} - m_t^2)^2]^{1/2}, \quad (14)$$

$$L_1 = \ln \frac{(\hat{s}(1+r) + m_t^2 - m_b^2)^2}{4m_t^2\hat{s}} \quad (15)$$

and

$$L_2 = L_1 + \ln \frac{(\hat{s}(1+r) + m_b^2 - m_t^2)^2}{4m_b^2\hat{s}}. \quad (16)$$

Note the logarithmic singularity in L_2 which would be present for $m_b = 0$, resulting from the exchange of a massless fermion in the t-channel.

Our results for the total integrated $\bar{t}b$ cross-section versus m_t at $\sqrt{s} = 314 \text{ GeV}$ are presented in fig. 2 (solid line). For the parton distributions set 2 of ref.(2) was used. Here and in the following we took $\alpha = \frac{1}{137}$ and $\sin^2\theta_w = 0.23$. As one can see, the single top quark production rate is expected to be rather small at HERA, but is only slowly varying with m_t . Equations 2-16 can be compared with the corresponding formulae in the literature on charm production in charged current processes^[ref.3,4,6]. Our results differ from the ones in ref.(4) because we have explicitly kept the mass of the b-quark unequal to zero. We disagree with ref.(3), but agree with ref.(6) for $m_b \rightarrow 0$.

Inspection of the diagram for reaction (1) (see fig. 1a) shows that this process can also be interpreted in a different way. At the lower vertex a gluon splits into a $b\bar{b}$ pair with the b -quark then being converted into a top quark at the upper vertex. This process can, therefore, be also interpreted as top production from a W -boson fusing with a b -quark from the proton sea, where the bottom sea is generated by gluons splitting into $b\bar{b}$ pairs. The differential cross-section for $ep \rightarrow \nu\bar{t}X$ in the limit of a massless b -quark is given by

$$\frac{d^2\sigma}{dx dy}(ep \rightarrow \nu\bar{t}X) = q_{\bar{b}}(x, q^2) |V_{tb}|^2 \frac{\pi}{4} \frac{\alpha^2}{\sin^4\theta_W} (1-y) \frac{x(1-y)s - m_t^2}{(xys + m_W^2)^2} \quad (17)$$

where $q_{\bar{b}}(x, q^2)$ is the distribution function for $\bar{b}$ -quarks inside the proton. x and y are constrained by

$$\frac{m_t^2}{s} \leq x \leq 1 \quad (18)$$

and

$$0 \leq y \leq 1 - \frac{m_t^2}{xs} . \quad (19)$$

Using the b -quark distribution function of set 2 of ref.(2), we have calculated the total cross-section for $ep \rightarrow \nu \bar{t} X$ for two different values of q^2 . Our results are represented by the dashed lines in fig. 2. The upper dashed curve gives the total $\bar{t}$ production rate if q^2 is chosen to be equal to the center of mass energy squared xs of the $e\bar{b}$ system, whereas the lower one corresponds to $q^2 = xys$, the square of the momentum transferred by the virtual W . As one can see, the results for $e\bar{b} \rightarrow \nu \bar{t}$ using the $\bar{b}$ quark distribution function are in qualitative agreement with the ones obtained for process (1). Choosing $q^2 = xys$ underestimates the single top production rate, in particular for small values of m_t . On the other hand, $q^2 = xs$ overestimates the cross-section somewhat for $m_t \geq 30 \text{ GeV}$ but fits the exact result at small top quark masses better than $q^2 = xys$. These results agree well with the analogous calculation for hadron machines^[ref.5].

In order to better understand the relation between the two descriptions of $\bar{t}b$ production we have plotted in fig. 3 the differential cross section $\frac{d\sigma}{dz}$ for the process $W^*g \rightarrow \bar{t}b$ with z the momentum fraction of the proton carried by the gluon. In the same figure we also show $\frac{d\sigma}{dx}$ for the process $W^*b \rightarrow \bar{t}$ with x the momentum fraction of the proton carried by the bottom-quark. We used two different choices of the effective value of q^2 in the structure function of the bottom quark. The peak of the cross section located at $z \approx 0.14$ for the gluon, translates into a peak at about $x \approx 0.08$ for the bottom quark, since the gluon has to distribute its momentum among the b and $\bar{b}$ quarks inside the proton. The difference in the cross-sections between the two values of q^2 reflects the fact that for higher values of q^2 there are more heavy quarks in the proton than at low values of q^2 . In the figure we used $\sqrt{s} = 314 \text{ GeV}$ and $m_t = 60 \text{ GeV}$. For different values of m_t curves of similar shape are obtained with the peaks shifted towards lower (higher) values of z and x for smaller (larger) top quark masses.

Finally, in order to assess the importance of $\bar{t}b$ production as a top quark source at HERA, we compare our results for single top production with the corresponding ones for $\bar{t}t$ production. In analogy to the charged current

reaction (1) the differential cross-section for pair production of t -quarks in ep -collisions can be written as^[ref.6] :

$$\frac{d^2\sigma}{dx dy}(ep \rightarrow e\bar{t}tX) = \frac{4\pi\alpha^2 s}{Q^4} \left[(1-y+\frac{y^2}{2}) F_2^{\gamma^*g \rightarrow \bar{t}t}(x, Q^2) - \frac{1}{2} y^2 F_L^{\gamma^*g \rightarrow \bar{t}t}(x, Q^2) \right] \quad (20)$$

where ($i = 2, L$)

$$F_i^{\gamma^*g \rightarrow \bar{t}t}(x, Q^2) = \int_{x\tilde{a}}^1 dz G(z, \frac{1}{2}\hat{s}) f_i^{\gamma^*g \rightarrow \bar{t}t}(\frac{x}{z}, Q^2), \quad (21)$$

$$\tilde{a} = 1 + 4 \frac{m_t^2}{Q^2}, \quad (22)$$

and

$$\hat{s} = Q^2 \left(\frac{z}{x} - 1 \right) \quad (23)$$

is the square of the invariant mass of the $\bar{t}t$ system. For f_2 and f_L we find

$$f_2^{\gamma^*g \rightarrow \bar{t}t}(w, Q^2) = \frac{4}{9} \frac{\alpha_s(\frac{1}{2}\hat{s})}{\pi} \left\{ v[4w^2(1-w) - \frac{1}{2}w - 2\frac{m_t^2}{Q^2}w^2(1-w)] + \right. \\ \left. [\frac{1}{2}w - w^2(1-w) + 2\frac{m_t^2}{Q^2}w^2(1-3w) - 4\frac{m_t^4}{Q^4}w^3] \ln\left(\frac{1+v}{1-v}\right) \right\} \quad (24)$$

and

$$f_L^{\gamma^*g \rightarrow \bar{t}t}(w, Q^2) = \frac{4}{9} \frac{\alpha_s(\frac{1}{2}\hat{s})}{\pi} \left\{ 2w^2(1-w)v - 4\frac{m_t^2}{Q^2}w^3 \ln\left(\frac{1+v}{1-v}\right) \right\} \quad (25)$$

with the center of mass frame velocity of the top quark v which is given by

$$v^2 = 1 - 4 \frac{m_t^2}{Q^2} \frac{w}{1-w}. \quad (26)$$

The parameters x and y are constrained by the inequalities

$$y(1-x) \geq 4 \frac{m_t^2}{s} \quad (27)$$

and

$$xys = Q^2 \geq Q_{min}^2 \quad (28)$$

where

$$Q_{min}^2 = -W^2 - \frac{1}{2s} \left[(m_e^2 - s)(s + W^2 - m_e^2) + (s - m_e^2) \{ (s - W^2)^2 - 2m_e^2(s + W^2) + m_e^4 \}^{1/2} \right]. \quad (29)$$

Here, m_e denotes the electron mass and

$$W^2 = sy(1 - x) \quad (30)$$

is the invariant mass squared of the hadronic system in the final state. Due to the $1/Q^4$ behaviour of the differential cross-section (20) it is crucial to keep the electron mass in Eq. (29) in order to arrive at a finite result for the total rate.

A useful approximate form of the exact total cross-section for large values of $\sqrt{s}$ is provided by the Weizsäcker-Williams (WW) approximation, where the production rate is described by a convolution of the probability for emitting a photon from the electron with the corresponding real photon cross-section for the reaction $\gamma g \rightarrow \bar{t}t$. For top quark pair production the WW approximation then gives^[ref.7] :

$$\sigma_{WW}(ep \rightarrow e\bar{t}tX) = \int_{4m_t^2/s}^1 d\tau \int_{\tau}^1 \frac{dx}{x} f_{\gamma/e}(x, \frac{1}{2}\sqrt{s}) G(\frac{\tau}{x}, \frac{\tau s}{2}) \hat{\sigma}^{\gamma g \rightarrow \bar{t}t}(\tau s) \quad (31)$$

where

$$\hat{\sigma}^{\gamma g \rightarrow \bar{t}t}(\tilde{s}) = \frac{8\pi}{9\tilde{s}^2} \alpha \alpha_s(\frac{\tau s}{2}) \left\{ \ln\left(\frac{1+\hat{\beta}}{1-\hat{\beta}}\right) [\tilde{s} + 4m_t^2 - 8\frac{m_t^4}{\tilde{s}^2}] - \hat{\beta}[\tilde{s} + 4m_t^2] \right\} \quad (32)$$

is the constituent cross-section with

$$\hat{\beta} = \sqrt{1 - 4\frac{m_t^2}{\tilde{s}}} \quad (33)$$

and

$$f_{\gamma/e}(x, E) = \frac{\alpha}{\pi} \left\{ \frac{1 + (1-x)^2}{x} \left[\ln\left(\frac{E}{m_e}\right) - \frac{1}{2} \right] + \frac{x}{2} \left[\ln\left(\frac{2}{x} - 2\right) + 1 \right] + \frac{(2-x)^2}{2x} \ln\left(\frac{2-2x}{2-x}\right) \right\} \quad (34)$$

is related to the probability of finding a photon in the electron^[ref.8] .

The exact and the WW approximation result for the total $\bar{t}t$ production rate are shown by the dotted and by the dash-dotted line in fig. 2. From there we observe that :

- i) the WW approximation overestimates the top quark pair production cross-section by about 15 %,
- ii) single top production dominates over $\bar{t}t$ production for $m_t \gtrsim 60 \text{ GeV}$.

The above formulae can also be used to calculate the production of $\bar{b}b$ pairs . For the total $\bar{b}b$ production cross-section we find $3.9 \pm 0.9 \text{ nb}$. The major source of the uncertainty in the calculation of this cross-section is due to the fact that the bottom-mass is not well known. For our estimate we have taken the bottom-mass in the range $4.7 \text{ GeV} \leq m_b \leq 5.3 \text{ GeV}$. Because of this large cross-section and the possibility of top and bottom misidentification the production of $\bar{b}b$ pairs is actually a major source of background in a search for the top-quark at HERA. From fig. 2 it follows that the number of top quarks produced at HERA will not be very large, of the order of 20 per year for a top mass of 60 GeV. Therefore it is important that one does not only consider the semi-leptonic decay of the top quark, but also tries to identify its purely hadronic decays. A rather naive way to search for top quark events in the purely hadronic mode is to look for a peak in the invariant mass spectrum of the decay products. In order to test this possibility we calculated in fig. 4. the invariant mass spectrum for $\bar{b}b$, $\bar{t}b$ and $\bar{t}t$ final states for $m_t = 60 \text{ GeV}$. It is seen from this figure that the $\bar{t}t$ cross section is approximately of the same size as the background from $\bar{b}b$ pairs . The $\bar{t}b$ production is however completely drowned in the background . Similar results are found for other values of m_t . In order to identify $\bar{t}b$ events one needs therefore a good discrimination between top and bottom quarks. Whether this will be possible at HERA can probably only be settled by a full detector Monte Carlo .

From the above discussion we see that there is no hope to detect a top quark at HERA when it is heavier than 70 GeV. A much larger mass range should however be accessible at a collider with a $\sqrt{s}$ in the TeV region , like the proposed ep-option for LEP-LHC ($\sqrt{s} \approx 1.4 \text{ TeV}$, $\mathcal{L} \approx$

$10^{32} \text{cm}^{-2} \text{s}^{-1})^{[ref.8]}$. The cross-sections for single and pair production of top quarks at $\sqrt{s} = 1.4 \text{ TeV}$ are shown in fig. 5 . In this case one has cross-sections of a picobarn for top quarks up to a mass of 400 GeV , which puts one well into the range of a 4th generation. However in this range hadron colliders like the SSC or LHC, where one should be able to look for top up to a mass of 1.0 and 0.8 TeV respectively, seem to be a better option . We remark that the cross-over point between the dominance of the $\bar{t}b$ versus the $\bar{t}t$ production is approximately independent of the machine energy .

In summary, we have studied the production of top-quarks in electron proton collisions, emphasizing the possibility of single top production in charged current processes. The production of single top quarks was studied in the basic process of $\bar{t}b$ production by annihilation of a gluon and a virtual W-boson, keeping all mass effects. The result was compared with an effective description on the basis of bottom quark structure functions. Qualitative agreement between the two methods was found . Single top quark production is dominated by $\bar{t}t$ production up to $m_t \approx 60 \text{ GeV}$ at which point the production rate becomes marginal at HERA . Since the $\bar{b}b$ cross-section is three to four orders of magnitude larger, a good experimental discrimination between top and bottom quarks will be essential for detecting top at ep-colliders . If this can be realized, an ep-collider with a $\sqrt{s}$ in the TeV region, like the proposed ep option for LEP-LHC, will be able to search for singly produced t-quarks with a mass up to several hundred GeV, which is however still roughly a factor two below the potential of LHC and SSC .

ACKNOWLEDGEMENT

We would like to thank E. Eichten, K. Ellis, Z. Kunszt and C. Quigg for discussions. We thank Fermilab ,where a large part of this work was done, for financial support and hospitality. U.B. would like to acknowledge partial financial support of the Max-Kade-Foundation, New York.

References

- [1] Argus Collaboration , Phys. Lett. B192 , 245(1987).
P. Erhard , in the "Proceedings of the Uppsala conference" (1987) .
- [2] E. Eichten, I. Hinchliffe, K. Lane and C. Quigg : Rev. Mod. Phys. 56, 579(1984);
Err. Rev. Mod. Phys. 58, 1065(1986).
- [3] E. Witten : Nucl. Phys. B104, 445(1976).
- [4] Thomas Gottschalk : Phys. Rev. D23 , 56(1981).
- [5] Scott S. Willenbrock and Duane A. Dicus : Phys. Rev. D34 , 155(1986).
- [6] M. Glück, E. Hoffmann and E. Reya : Z. Phys. C13, 119(1982).
- [7] M. Drees and K. Grassie : Z. Phys. C28, 451(1985).
- [8] S. J. Brodsky, T. Kinoshita and H. Terazawa : Phys. Rev. D4 , 1532(1971).
- [9] G. Altarelli , in "Proceedings of the workshop on physics at future accelerators", La Thuile (Italy) and Geneva (Switzerland) ,7-13 January 1987,CERN yellow report 87-07.

Figure Captions

- Fig. 1: a) diagrams contributing to single top production in ep colliders.
b) diagrams contributing to top pair production in ep colliders.
- Fig. 2: The total top quark production cross-section σ versus the t-quark mass m_t at HERA energies. The W-mass M_W used for the charged current process $ep \rightarrow \nu \bar{t} b X$ and $ep \rightarrow \nu \bar{t} X$ is 82 GeV. In $ep \rightarrow \nu \bar{t} b X$ a bottom mass of 5 GeV was taken .
- Fig. 3: $\frac{d\sigma}{dz}$ for the process $W^* g \rightarrow \bar{t} b$ and $\frac{d\sigma}{dx}$ for $e \bar{b} \rightarrow \bar{t} \nu$ at HERA energies, M_W and m_b are as in fig. 2 .

Fig. 4: The invariant mass distribution for heavy quark production at HERA , M_W and m_b as in fig. 2 .

Fig. 5: The total top quark production cross-section σ versus the top quark mass in a 1.4 TeV electron-proton collider.

Fig. 1a

Fig. 1b

Fig. 2

Fig. 3

Fig. 4

Fig. 5