CERN-TH.4863/87

HIGGS PHENOMENON IN STRING THEORIES

I. Antoniadis^{*)}
CERN - Geneva

C. Bachas
Centre de Physique Théorique,
Ecole Polytechnique
F-91128 Palaiseau Cedex, France

and C. Kounnas^{**)}
Department of Physics
U.C. Berkeley and
Lawrence Berkeley Laboratory,
Berkeley, CA 94720, U.S.A.

A B S T R A C T

We show how to construct consistent four-dimensional string theories in the spontaneously broken, Higgs phase. The symmetry breaking scale is a free, classically undetermined parameter, due to the existence of flat directions in the scalar potential.

^{*)} On leave of absence from Centre de Physique Théorique, Ecole Polytechnique, 91128 Palaiseau Cedex, France.

^{**)} On leave of absence from LPT, Ecole Normale Supérieure, Paris, France.

It is at present widely believed that there are just a few fundamental string theories [1,2], but with a lot of classically degenerate ground states [3-8]. Nevertheless, our understanding of even this classical string landscape and of its symmetries, which might offer the key to the solution of long-standing problems of grand-unification, is still rather rudimentary.

In order to enhance this understanding, we will reconsider here the freedom of continuously deforming the string vacuum by Lorentz boosting the charge lattice of string states. Narain, Sarmadi and Witten [9] showed that for certain models obtained by compactification of the ten-dimensional supersymmetric strings, this freedom amounts to changing radii and background gauge and antisymmetric fields. Here we will show that these continuous deformations of the above as well as many other models correspond, from the four-dimensional point of view, to a standard Higgs phenomenon; gauge symmetries are broken by vacuum expectation values of Higgs fields sliding along flat directions of the scalar potential. The symmetry-breaking scales are thus classically undetermined parameters, which may in particular be hierarchically smaller than the Planck mass. We will first study this Higgs phenomenon in the effective low-energy theory, by calculating the scalar potential up to and including quartic terms. We will then show how to explicitly construct the corresponding model in the Higgs phase, by an appropriate Lorentz boost of the charge or momentum lattice of a more symmetric ground state. A corollary of the exercise will be to show that string calculations are often considerably simpler than field theory calculations.

An attempt to construct explicitly [10] string models with spontaneous gauge and supersymmetry breaking was also made recently by using a Scherk-Schwarz type compactification [11] from five dimensions. In what concerns the breaking of supersymmetry, it is not clear whether these models are consistent, because world-sheet supersymmetry has not a priori been respected [12]. In what concerns the breaking of gauge symmetries on the other hand, the standard Higgs phenomenon we will describe here is both simpler and, as can be seen by counting parameters, considerably more general. Furthermore our approach is in accordance with the spirit that the extra quantum numbers, necessary for the cancellation of string anomalies, need not be interpreted as extra space-time dimensions.

In order to avoid unnecessary complications, we will describe the phenomenon in the simplest four-dimensional heterotic string model. Let us recall that in the fermionic construction one has, in addition to the four space-time co-ordinates X^μ and their left-moving fermionic superpartners ψ^μ , an extra 18 left-moving and 44 right-moving free fermions χ^a and η_A respectively. World sheet supersymmetry is non-linearly realized among the χ^a , which must transform in the adjoint representation of a semi-simple group G [4,13]; the supercharge is

$$T_F = \psi^\mu \partial X_\mu + f_{abc} \chi^a \chi^b \chi^c \quad (1)$$

where the normalization of the structure constants is such that $f_{acd} f_b{}^{cd} = \frac{1}{2} \delta_{ab}$. There are just three possible choices for G, namely $SU(2)^6$, $SU(3) \times SO(5)$ and $SU(4) \times SU(2)$.

The theory is further specified by a set Ξ of mutually commuting boundary conditions diagonalized in some, generally complex basis for the fermions $\{f\}$ [8]. A boundary condition is described by a vector $\alpha(f)$, such that $f \rightarrow -e^{i\pi\alpha(f)} f$ under parallel transport around the string; Ξ is closed under vector addition modulo 2, and can be generated by linear combinations of some canonical basis $\{b_1, \dots, b_k\}$. To every element α of Ξ there corresponds a sector $\mathcal{H}_\alpha$ of the string Hilbert space $\mathcal{H}$, and to every generator b_i a generalized GSO projection:

$$\mathcal{H} = \bigoplus_{\alpha \in \Xi} \prod_{i=1}^k \left\{ e^{i\pi b_i \cdot F} = \delta_\alpha c^* \left[\begin{smallmatrix} \alpha \\ b_i \end{smallmatrix} \right] \right\} \mathcal{H}_\alpha \quad (2)$$

Here F is the vector of two-dimensional fermion numbers, δ_α is the space-time fermion parity of the states in $\mathcal{H}_\alpha$ and the dot product is Lorentzian: left minus right. The choice of the phase factors $c \left[\begin{smallmatrix} \alpha \\ b_i \end{smallmatrix} \right]$, as well as of the additive group Ξ is severely restricted by the requirement of multi-loop modular invariance [8].

The states in the sector $\mathcal{H}_\alpha$ are constructed by acting on the vacuum $|0\rangle_\alpha$ with the free bosonic and/or fermionic oscillators, with frequencies $\{1 + [\alpha](f)\}/2 + \text{integer}$ and $\{1 - [\alpha](f)\}/2 + \text{integer}$ for f and f^* respectively; here $[\alpha]$ stands for the representative of the vector α , whose components have been reduced modulo 2 to lie in the interval $(-1, +1]$. Physical states satisfy the mass-shell conditions:

$$\begin{aligned} M^2 &= -\frac{1}{2} + \frac{1}{8} [\alpha_L] \cdot [\alpha_L] + \sum_{\text{left-movers}} (\text{frequencies}) \\ &= -1 + \frac{1}{8} [\alpha_R] \cdot [\alpha_R] + \sum_{\text{right-movers}} (\text{frequencies}) \end{aligned} \quad (3)$$

with α_L and α_R the left- and right pieces of the vector α .

The simplest possible model has only two sectors: $\Xi = \{0, 1\}$, i.e., the fermions are either all anti-periodic (Neveu-Schwarz), or else all periodic (Ramond). The spectrum contains a tachyon in the vector representation of $SO(44)$:

$$T_A : \quad \eta_{A, \frac{1}{2}} |0\rangle_0 \quad M^2 = -\frac{1}{2} \quad (4a)$$

massless gauge bosons of $G \times SO(44)$:

$$W_{AB}^{\mu} : \quad \psi_{\frac{1}{2}}^{\mu} \eta_{A, \frac{1}{2}} \eta_{B, \frac{1}{2}} |0\rangle_0$$

$$W^{\mu, a} : \quad \chi_{\frac{1}{2}}^a (\bar{\psi} \chi^{\mu})_1 |0\rangle_0 \quad (4b)$$

and massless scalars in the adjoint representation of both G and $SO(44)$:

$$\Phi_{AB}^a : \quad \chi_{\frac{1}{2}}^a \eta_{A, \frac{1}{2}} \eta_{B, \frac{1}{2}} |0\rangle_0 \quad (4c)$$

There are also, of course, the graviton, dilaton and antisymmetric tensor. All other states are massive. We will use this simple toy model as well as a non-tachyonic $N=4$ supersymmetric theory to illustrate the Higgs phenomenon. The mechanism we will describe applies, however, to a large variety of other models.

Higgs Mechanism in the Low-Energy Limit

We may calculate the potential of the massless scalars, by coupling constant background fields to the corresponding vertex operators at zero momentum. This amounts to adding to the free two-dimensional Lagrangian the properly supersymmetrized, generalized Thirring interactions:

$$\begin{aligned} \mathcal{L}_I^{(2d)} &= \Phi_{ab}^{AB} \int d^2z d\bar{z} \int d\theta (\chi^a + \theta f_{bc}^a \chi^b \chi^c) \eta_A \eta_B \\ &= \tilde{\Phi}_{bc}^{AB} \int d^2z d\bar{z} (\chi^b \chi^c) (\eta_A \eta_B) \end{aligned} \quad (5)$$

where we have here defined $\tilde{\phi}_{bc}^{AB} = f_{bc}^a \phi_a^{AB}$. The classical equations of motion for the string are given by the condition that the two-dimensional β -functions vanish [14]:

$$\beta_{\tilde{\phi}_a^{AB}} = \frac{\partial V}{\partial \tilde{\phi}_a^{AB}} = 0. \quad (6)$$

Now it is known that Thirring interactions, coupling any number of mutually commuting left to mutually commuting right currents, do not break conformal invariance [15]. This means that we may give arbitrary non-vanishing vacuum expectation values to as many as $22 \times (\text{rank } G)$ of the massless scalars, without altering the value of the potential. Put differently, the scalar potential has lots of flat directions which can be used to break $SO(44) \times G$ at classically undetermined scales. Note that the presence of these flat directions is a priori totally unexpected in a non-supersymmetric theory.

In order to be more explicit, we may calculate the potential of the scalars ϕ_a^{AB} , up to and including quartic terms. There is a single one-loop diagram and five two-loop diagrams as shown in Fig. 1. Diagram (b) is actually finite, while (c) and (d) do not contribute to the β -function. The contributions of (e) and (f), on the other hand, are equal but opposite if one forgets the flow of group indices; this, of course, is expected, since these diagrams should exactly cancel for Abelian current-current couplings. The final result for the potential is:

$$\begin{aligned} V(\phi) &= \frac{g}{3\sqrt{\alpha'}} \tilde{\phi}_{AB}^{ab} \tilde{\phi}_{BC}^{bc} \tilde{\phi}_{CA}^{ca} + \\ &+ \left\{ 2 \tilde{\phi}_{AB}^{ab} \tilde{\phi}_{BC}^{cd} \left(\tilde{\phi}_{CD}^{ab} \tilde{\phi}_{DA}^{cd} - \tilde{\phi}_{CD}^{cd} \tilde{\phi}_{DA}^{ab} \right) + (a,b,\dots \leftrightarrow A,B,\dots) \right\}_{(7)} + O(\sqrt{\alpha'}) \\ &= \frac{g}{3\sqrt{\alpha'}} \text{Tr} \left(\tilde{\phi}^{ab} \tilde{\phi}^{bc} \tilde{\phi}^{ca} \right)_{SO(44)} + \\ &+ \text{Tr} \left[\tilde{\phi}^{ab}, \tilde{\phi}^{cd} \right]_{SO(44)}^2 + \text{Tr} \left[\tilde{\phi}_{AB}, \tilde{\phi}_{CD} \right]_G + O(\sqrt{\alpha'}) \end{aligned}$$

where repeated indices are summed, and in the second equation the trace and commutators are with respect to either the $SO(44)$ or the G indices as indicated. In what follows, we will restrict ourselves to $G = SU(2)^6$, so that:

$$\begin{aligned}
 V(\Phi) = & \frac{1}{3\sqrt{\alpha'}} \varepsilon_{ijk} \sum_I \text{Tr}(\Phi^{I,i} \Phi^{I,j} \Phi^{I,k}) + \frac{1}{4} \sum_{\substack{I,J \\ i,j}} \text{Tr}[\Phi^{I,i}, \Phi^{J,j}]^2 \\
 & - \frac{1}{8} \sum_{\substack{I \\ i,j}} \left(\text{Tr} \Phi^{I,i} \Phi^{I,j} \right)^2 + \frac{1}{8} \sum_I \left(\sum_i \text{Tr} \Phi^{I,i} \Phi^{I,i} \right)^2
 \end{aligned} \quad (8)$$

where $I = 1, \dots, 6$ labels the different $SU(2)$'s and $i, j = 1, 2, 3$.

The above potential, as advertized, has lots of flat directions. Sliding along them, scalar fields can acquire vacuum expectation values thus breaking the gauge group at classically undetermined scales. Before describing this in detail, we should however, first point out that the $SO(44)$ tachyonic theory is doubly sick: indeed the cubic term in the potential (8) signals an additional vacuum instability, other than the presence of the tachyon. Both of these problems do not, however, alter significantly the phenomenon we want to describe. Besides, they can be easily cured by modifying our theory with extra GSO-type projections. Adding for instance to the generators of $\mathbb{E}$ the vector [4]

$$S = \left(\underbrace{1 \dots 1}_{\psi^\mu, \chi^{I,3}} \quad \underbrace{0 \dots 0}_{\text{rest}} \right) \quad (9)$$

leads to a $SO(44)$ $N = 4$ supersymmetric theory. The tachyon T^A and the scalars $\phi_{AB}^{I,1}$ and $\phi_{AB}^{I,2}$ are truncated away, so that the scalar potential reduces simply to the second term of Eq. (8); this is precisely the D-term of the $SO(44)$ vector multiplet^{*)}, and is explicitly positive semi-finite.

With these remarks in mind, we now return to the original tachyonic theory. We will illustrate the Higgs phenomenon by choosing one particular flat direction, along which $SU(2)_L^6 \times SO(44)_R$ ^{**) is broken down to $U(1)_L^6 \times U(1)_R \times SU(22)_R$. Accordingly we introduce the notation $W_{AB}^\mu \equiv W_a^\mu \lambda_{AB}^a$ and $\Phi_{AB}^{I,i} = \Phi_a^{I,i} \lambda_{AB}^a$, where $\{\lambda^a\}$ is a choice of}

*) Comparison with the unique $N=4$ Lagrangian fixes the overall normalization of the potential, as in Eq. (8), in units where the gauge coupling constant is one. Note also that from (8) we can, by truncation, derive the low-energy potential of any theory whose massless scalars come exclusively from the Neveu-Schwarz sector.

**) The subscripts L and R simply indicate that the corresponding Kac-Moody currents are left- or right-moving on the world-sheet.

basis for the generators of $SO(44)_R$ corresponding to a maximal embedding of $U(22)_R \approx U(1)_R \times SU(22)_R$; furthermore the index $i = 1, 2, 3$ for each $SU(2)_L$ is traded, as usual, for $i = +, -, 3$. Now if λ^0 is the $U(1)_R$ generator of $U(1)_R \times SU(22)_R$, we set $\langle \Phi_0^{I,3} \rangle = v$, and all other scalar vacuum expectation values equal to zero. It is easy to see that the potential (8) remains zero for all values of v , which is therefore classically undetermined, and that for $v \neq 0$ the gauge group is broken down as advertized. A bit more crunching is required to work out the entire spectrum of low-lying states. Using the fact that $\text{tr}(\lambda^a \lambda^b) = \delta^{ab}$ for canonically normalized kinetic terms, and

$$[\lambda^0, \lambda^a] = \begin{cases} 0 & \text{if } a \in U(22) \\ \pm \frac{1}{\sqrt{11}} \lambda^a & \text{if } a \in SO(44)/U(22) \end{cases}$$

and assuming the Higgs scale is much lower than the Planck mass ($v \ll 1$ in units where $2\alpha' = 1$), we find the following spectrum up to terms of order v^2 :

$$M^2 = 0 \quad W_{U(22)}^P; \quad W^{P, I, 3}; \quad \Phi_{U(22)}^{I, 3} \\ \sum_{I=1}^6 \Phi_{SO(44)/U(22)}^{I, 3}; \quad \Phi_{U(1)}^{I, \pm} \quad (10a)$$

$$M^2 = \frac{1}{2}v^2 \quad W^{P, I, \pm}; \quad \Phi_{SU(22)}^{I, \pm} \quad (10b)$$

$$M^2 = 6/11v^2 \quad W_{SO(44)/U(22)}^P \\ \left\{ \sum_{I=1}^n \Phi_{SO(44)/U(22)}^{I, 3} - n \Phi_{SO(44)/U(22)}^{n+1, 3} \right\} \quad n = 1, \dots, 5 \quad (10c)$$

$$M^2 = \pm \sqrt{2/11}v + (23/22)v^2 \quad \Phi_{SO(44)/U(22)}^{I, \pm} \quad (10d)$$

Note that the mass matrix mixes the $\Phi_{SO(44)/U(22)}^{I, 3}$; a single linear combination of them, together with the $\Phi_{U(1)}^{I, \pm}$ are the massless would-be Goldstone bosons which in a

unitary gauge are eaten by the longitudinal components of the massive gauge bosons $W_{SO(44)/U(22)}^\mu$ and $W^{\mu,I,\pm}$ respectively. Note also that the trilinear coupling renders some previously massless particles, the $\Phi_{SO(44)/U(22)}^{I,-}$, tachyonic, and that the tachyons T_A of the unbroken theory, not shown in the above Table, shift to a higher mass due to the presence of a $T_A(\Phi^{I,i})_{AB}^2 T_B$ coupling. Tachyons would of course be altogether absent, had we started with a healthier theory.

Apart from the spurious complications of the large gauge group and many scalars, there is one crucial difference between the spectrum (10) and that of any common field theory in the Higgs phase. Indeed note first that, as discussed above, we can obtain from (10) the spectrum of an $N=4$ supersymmetric $SO(44)$ theory, broken down to $U(22)$, by truncating away the states (10b, d). In such a theory it is not surprising that scalars have mass degeneracies with corresponding vectors, since they belong to the same vector multiplets, and that in particular some scalar masses vanish, reflecting the existence of flat directions. What is surprising, is that these features also characterize the non-supersymmetric string theory; they hold, as we will see, to all orders in the v -expansion and are perhaps the signal of some larger string-symmetry.

Technical Interlude

So far we have studied the Higgs phenomenon in the effective low-energy limit. We will now show how to construct directly consistent string models in the Higgs phase, using only free fermions or bosons on the world-sheet. Although the fermionic and bosonic languages are essentially equivalent [6], they have complementary merits. Since in what follows we will find it convenient to employ the latter language, in which the existence of continuous spectrum deformations is manifest, we will first briefly review in this section the dictionary allowing one to pass from fermions to bosons and vice versa.

Let $\{b_1, \dots, b_K\}$ generate the set of sectors $\mathbb{E}$ of a consistent fermionic model. We assume that all spin-structure assignments are simultaneously diagonalized in a complex fermion basis^{*)}, so each b_i is a vector with ten transverse left and 22 right components. To any state of the Hilbert space (2) one may assign a charge or momentum vector: $P = \frac{1}{2}[\alpha] + F$, where $\alpha = n_1 b_1 + \dots + n_K b_K$ and F specify the sector and fermion numbers of the state. The lowest-lying state with a given momentum P can be obtained by acting on the vacuum $|0\rangle_\alpha$ with the $F(f)$ lowest-lying oscillators of type f . Hence its "left-moving mass" is

*) Since this need not be always the case, the fermionic language is actually a bit more general than the language of momentum lattices; this is however, irrelevant for the purpose of this letter.

$$L_0 = \sum_{\text{left } f} \left\{ \frac{1}{8} [\alpha](f)^2 + \frac{1 + [\alpha](f)}{2} + \dots + \frac{2F(f) - 1 + [\alpha](f)}{2} \right\} - \frac{1}{2}$$

$$= \frac{1}{2} P_L^2 - \frac{1}{2} \quad (11)$$

and likewise its "right-moving mass" $\bar{L}_0 = \frac{1}{2} P_R^2 - 1$. Furthermore, acting on this state with the integer-frequency currents $f f^*$, we can build the entire tower of states with the same momentum P . Thus we can rewrite the partition function of a fermionic model as:

$$Z = (\text{Im} \tau)^{-\frac{d-2}{2}} \eta^{-12} \bar{\eta}^{-24} \sum_{P \in \Gamma} e^{2i\pi \Delta \cdot P} e^{i\pi \tau P_L^2 - i\pi \tau^* P_R^2}$$

(12)

where here η is the Dedekind eta-function, the $(\text{Im} \tau)^{-(d-2)/2}$ comes from the integration of the X^μ -zero modes, and the spin-statistics vector $\Delta = (\frac{1}{\phi^\mu} 0 \dots 0)$ has been defined so that:

$$e^{2i\pi \Delta \cdot P} = \delta_\alpha = \pm 1$$

according as the states in the sector α are space-time bosons or fermions.

We must still characterize the set Γ of momenta over which the sum in (12) should run. To this end, note that in any sector $\alpha = n_1 b_1 + \dots + n_K b_K$, the GSO projections enforce for all j :

$$e^{i\pi b_j \cdot F} = \delta_\alpha c^* \begin{bmatrix} \alpha \\ b_j \end{bmatrix}$$

$$= \delta_\alpha \delta_{b_j}^{\sum n_i - 1} e^{\frac{i\pi}{2} b_j \cdot \{\alpha - [\alpha]\}} \prod_{i=1}^K c^* \begin{bmatrix} b_i \\ b_j \end{bmatrix}^{n_i} \quad (13)$$

where to derive the second line one must use the rules of multi-loop modular invariance [8]. It is now easy to rewrite (13) as a set of conditions that characterize momenta $p \in \Gamma$:

$$b_j \cdot (P + \Delta) = \sum_{i=1}^k n_i \left\{ \frac{1}{2} b_i + \phi_{ij} + \delta_{b_i} - \delta_{b_j} \right\} \mod 2 \quad (14)$$

where here $C_{[b_j]}^{b_i} = e^{-i\pi\phi_{ij}}$. The crucial observation is that Γ is a lattice shifted by the constant vector Δ , since if P_1 and P_2 satisfy conditions (14), then so does $P_1 + P_2 + \Delta$. The lattice $\tilde{\Gamma} \equiv \Gamma - \Delta$ is actually a direct sum:

$$\tilde{\Gamma} = \Xi' \oplus \tilde{\Gamma}_{NS} \quad (15)$$

where:

a) $\tilde{\Gamma}_{NS}$ is the sublattice of shifted momenta in the Neveu-Schwarz sector, i.e., contains all integer component vectors $\tilde{p}$ such that $\tilde{p} \cdot b_j = 0 \mod 2 \forall j$, and

b) $\Xi' = \{ \frac{1}{2}(m_1 b_1' + \dots + m_K b_K') \mid m_i = 1, \dots, N_i \}$, where $b_i' = b_i + 2F_i + \Delta$, F_i is some fermion-number vector allowed in the sector b_i , and N_i is the smallest integer such that $N_i b_i = 0 \mod 2$. Note that the assumption that the basis $\{b_i\}$ be canonical [6,8] means that no vector in Ξ' has all of its components integer. Ξ' is in fact isomorphic to the set of all sectors Ξ .

This then completes the transformation of the partition function of a consistent fermionic string model into a sum over momenta on a shifted lattice $\Gamma = \tilde{\Gamma} + \Delta$, as in Eq. (12). Invariance under the modular transformations $\tau \rightarrow \tau + 1$ and $\tau \rightarrow -1/\tau$ then guarantees that the lattice $\tilde{\Gamma}$ should be integer ($\tilde{p}, \tilde{p}' \in \mathbb{Z}, \forall \tilde{p}, \tilde{p}' \in \tilde{\Gamma}$), self-dual (if $x, \tilde{p} \in \mathbb{Z}, \forall \tilde{p} \in \tilde{\Gamma}$, then $x \in \tilde{\Gamma}$) and that Γ should be odd ($p^2 = 1 \mod 2 \forall p \in \Gamma$). This follows from a slight modification of Narain's arguments [5]. Conversely, given any shifted lattice Γ with the above properties we may always rewrite the Hilbert space of the ensuing string theory in the form of Eq. (2), in two steps: first the set of vectors Ξ is the set of equivalence classes of lattice momenta, two momenta being equivalent if they differ by an integer vector. Secondly, choosing some canonical basis $\{b_i\}$ for Ξ we may easily reconstruct the coefficients $c_{[b_j]}^{b_i}$ by means of Eq. (14). Now when lattice momenta have all their components rational, Ξ is a finite set of cardinality $c(\Xi)$ equal to the total number of sectors. The whole partition function can be then rewritten as a sum over spin-structure assignments in $\Xi \times \Xi$, of products of free fermionic determinants or Θ -functions normalized by $1/c(\Xi)$. This expression is, however, ill-defined when some lattice momenta become irrational, since both $c(\Xi)$ and the number of spin-structures diverge.

This technical difficulty shows clearly that it is unnatural to try and put in evidence the continuous parameters of the string vacuum in the fermionic language. In the lattice formulation, on the other hand, the existence of continuous spectrum deformations is manifest, as we now proceed to demonstrate.

Higgs Mechanism in the String Theory

Let us return now to the $SU(2)_L^6 \times SO(44)_R$ theory of the first section. We choose a complex fermion basis:

$$\left\{ f_1, \dots, f_{10} \mid \bar{f}_1, \dots, \bar{f}_{22} \right\} =$$

$$= \left\{ \psi + i\psi^*, \underbrace{\chi^{I,3} + i\chi^{I+3,3}}_{I=1,2,3}, \underbrace{\chi^{I,+}}_{I=1,\dots,6} \mid \underbrace{\bar{\eta}_A = \eta_A + i\eta_{A+22}}_{A=1,\dots,22} \right\}. \quad (16)$$

The vectors P of the charge or momentum lattice Γ have ten left and 22 right components, which are just the charges of the corresponding states with respect to the $U(1)$ currents $f_\ell f_\ell^*$. Thus P_ℓ is the fermion number $F(f_\ell)$ in the Neveu-Schwarz (0) sector, and $\frac{1}{2} + F(f_\ell)$ in the Ramond (1) sector.

Consider now a "Lorentz" transformation $\Lambda \in SO(6,22)$ of the lattice, that leaves the first four co-ordinates $P_1, \dots, P_4$ unchanged. Since inner products are preserved, the transformed lattice $\Gamma(\Lambda)$ remains odd, and $\Gamma(\Lambda) - \Delta$ integer and self-dual. Furthermore, since the boundary conditions of the ϕ^μ and $\chi^{I,3}$ are left unchanged, the supercharge

$$T_F = \psi^\mu \partial X_\mu + \sum_{I=1}^6 \chi^{I,3} (\chi^{I,+} \chi^{I,-})$$

is still either periodic or antiperiodic, so that world-sheet supersymmetry is respected. Thus $\Gamma(\Lambda)$ is the charge lattice of a new consistent string model, in which the masses of all states are shifted by:

$$\delta M^2 = \frac{1}{2} \left[(\Lambda P)_L^2 - P_L^2 \right] = \frac{1}{2} \left[(\Lambda P)_R^2 - P_R^2 \right] \quad (17)$$

Clearly these mass-shifts do not vanish only for Lorentz boosts $\Lambda \in SO(6,22)/SO(6) \times SO(22)$; $SO(6) \times SO(22)$ rotations amount simply to redefinitions of the basis of left and right currents, and do not yield distinct string models. The number of

available continuous parameters^{*)} is thus 6×22 , which is precisely the number of flat directions in the scalar potential as shown in the beginning of this paper.

A particular class of transformations are Lorentz boosts on the plane spanned by one left and one right unit vector, $\hat{u}_L$ and $\hat{u}_R$ respectively:

$$\begin{aligned} \Lambda p \cdot \hat{u}_L &= p \cdot \hat{u}_L \cosh \theta + p \cdot \hat{u}_R \sinh \theta \\ \Lambda p \cdot \hat{u}_R &= p \cdot \hat{u}_L \sinh \theta + p \cdot \hat{u}_R \cosh \theta \end{aligned} \quad (18)$$

The co-ordinates $Q_L = p \cdot \hat{u}_L$ and $Q_R = p \cdot \hat{u}_R$ are the charges with respect to the left and right $U(1)$ currents: $J_L = \sum \hat{u}_{L,i} f_i f_i^*$ and $J_R = \sum \hat{u}_{R,i} \bar{f}_i \bar{f}_i^*$. From (17) and (18) we see that the effect of such a boost is to shift the mass of all states by an amount that depends only on the charges of the state with respect to these two $U(1)$ currents:

$$\delta M^2 = \frac{1}{2} (Q_L^2 + Q_R^2) \sinh^2 \theta + Q_L Q_R \sinh \theta \cosh \theta. \quad (19)$$

Let us choose in particular:

$$\begin{aligned} \hat{u}_L &= \left(\underbrace{0 \dots 0}_4 \quad \underbrace{\frac{1}{\sqrt{6}} \dots \frac{1}{\sqrt{6}}}_6 \mid 0 \dots 0 \right) \\ \hat{u}_R &= \left(0 \dots 0 \mid \frac{1}{\sqrt{22}} \dots \frac{1}{\sqrt{22}} \right) \end{aligned} \quad (20)$$

so that $J_L = 1/\sqrt{6} \sum_I \chi^{I,+} \chi^{I,-}$, the only choice that respects the permutation symmetry of the six $SU(2)$'s, while $J_R = 1/\sqrt{22} \sum_A \eta_A^{22-} \eta_A^{*-}$ is precisely the $U(1)$ generator in a maximal embedding of $U(1) \times SU(22)$ in $SO(44)$. The low-lying spectrum of the "boosted" $SU(2)_L^6 \times SO(44)_R$ theory can now be derived easily with the help of Eq. (19). The result is:

^{*)} Assuming we leave the first four co-ordinates unchanged.

$$M^2 = 0 \quad W_{U(22)}^\mu : \psi^\mu \bar{\eta}_A \bar{\eta}_B^* |0\rangle_0 \quad (21a)$$

$$W^{r,I,3} : (\bar{\psi} X^r) \chi^{I,3} |0\rangle_0$$

$$\Phi_{U(22)}^{I,3} : \chi^{I,3} \bar{\eta}_A \bar{\eta}_B^* |0\rangle_0 \quad (21b)$$

$$M^2 = \frac{1}{12} s h^2 \theta \quad W^{r,I,+} : \chi^{I,+} (\bar{\psi} X^r) |0\rangle_0$$

$$\Phi_{SU(22)}^{I,+}, \Phi_{U(1)}^{I,+} : \chi^{I,+} \bar{\eta}_A \bar{\eta}_B^* |0\rangle_0 \quad (21c)$$

$$M^2 = \frac{1}{14} s h^2 \theta \quad W_{SO(44)/U(22)}^\mu : \psi^\mu \bar{\eta}_A \bar{\eta}_B |0\rangle_0$$

$$\Phi_{SO(44)/U(22)}^{I,3} : \chi^{I,3} \bar{\eta}_A \bar{\eta}_B |0\rangle_0 \quad (21d)$$

$$M^2 = \pm \frac{1}{\sqrt{33}} s h \theta \cosh \theta + \frac{23}{132} s h^2 \theta \quad \Phi_{SO(44)/U(22)}^{I,\pm} : \chi^{I,\pm} \bar{\eta}_A \bar{\eta}_B |0\rangle_0 \quad (21e)$$

$$M^2 = -\frac{1}{2} + \frac{1}{44} s h^2 \theta \quad T_A : \bar{\eta}_A |0\rangle_0$$

We have here indicated the form of the states in the unboosted theory, from which one reads off the charges, omitting of course complex conjugate states.

Clearly the effect of the Lorentz boost was to break down the $SU(2)_L^6 \times SO(44)_R$ gauge symmetry to $U(1)_L^6 \times U(22)_R$. The spectrum (21) agrees to lowest order with the one derived in the low-energy theory, Eq. (10), provided we identify the vacuum expectation value of the Higgs field with the angle of the Lorentz boost:

$$s h \theta = \sqrt{6} v + O(v^2) \quad (22)$$

It is one more illustration of the beauty of strings that, as opposed to (10), we can derive the spectrum (21) with practically no sweat in a few lines; yet (21) is the exact string spectrum in the Higgs phase, taking into account all higher-order terms of the potential.

A series of remarks are here in order. Firstly, the would-be Goldstone bosons which remained massless in the spectrum of the effective field theory, Eq. (10), appear in (21) directly as the massive longitudinal components of the corresponding gauge-bosons; this is expected since we are in a unitary light-cone gauge. Secondly, if we perform the same Lorentz boost on the charge lattice of the $SO(44)$ $N=4$ supersymmetric theory discussed earlier, we find a low-lying spectrum that consists of states (21a,c) and their fermionic partners. The observed degeneracy between vector and scalar masses and, in particular, the presence of massless scalars signalling the existence of flat directions, could then be explained as a conse-

quence of space-time supersymmetry. Since these features persist, however, in the non-supersymmetric models, we believe some deeper stringy symmetry should be responsible for them^{*}). Finally, let us note that the shift in the charges of string states accompanying the Higgs phenomenon, is due to higher-order, non-renormalizable terms in the action.

As discussed in the previous section, the boosted charge lattice completely defines the string theory in the Higgs phase. If so desired, it is straightforward to reconstruct the set of sectors and GSO projections in a fermionic language description of the model. The vacuum expectation values of the Higgs fields are then hidden in the choice of boundary conditions for the free complex world-sheet fermions. For instance, the above discussed tachyonic $SU(2)^6 \times SO(44)$ theory in the spontaneously broken phase has a set of sectors $\mathbb{E}$ generated by $\{b_0 = 1, b_1, b_2\}$ where

$$b_1 = \left(\underbrace{0 \dots 0}_4 \quad \underbrace{\frac{ch\theta-1}{3} \dots \frac{ch\theta-1}{3}}_6 \mid \frac{sh\theta}{\sqrt{33}} \dots \frac{sh\theta}{\sqrt{33}} \right)$$

$$b_2 = \left(0 \dots 0 \quad \frac{sh\theta}{\sqrt{33}} \dots \frac{sh\theta}{\sqrt{33}} \mid \frac{ch\theta-1}{11} \dots \frac{ch\theta-1}{11} \right)$$

and the coefficients $c \begin{bmatrix} b_i \\ b_j \end{bmatrix}$ can be computed from Eq. (14).

The above analysis can be extended easily both to a variety of other models, and to more general Lorentz boosts, provided they leave invariant the momentum components which correspond to charges that do not commute with the world-sheet supercurrent T_F , Eq. (1). This last restriction is severe for two different reasons: firstly, because in a space-time supersymmetric model these are the only charges that do not vanish for the massless spin-3/2 states, and one would like to boost precisely in their direction, in order to break space-time supersymmetry spontaneously. Secondly, because in models with chirality it can be shown [4] that no left-moving charge $\int dz f_L(z) f_L^*(z)$ in the basis diagonalizing the spin-structure assignments commutes with T_F . Consequently, we cannot obtain these models in the Higgs phase by a simple Lorentz boost of their charge lattice. A closely related problem is that a Lorentz boost cannot reduce the rank $r(G)$ of the gauge group, since the $r(G)$ massless $U(1)$ gauge bosons are always neutral and have therefore vanishing mass-shifts. These issues will be further discussed elsewhere [12].

^{*}) Another curious "accident", from the field theory point of view, is the fact that the mass-ratio of states (b) and (c) is identical in (10) and (21), i.e., does not receive corrections from higher-order terms in the potential.

We here conclude with a remark concerning the mass formula, Eq. (19). For large Lorentz boosts this reads:

$$SM^2 = \frac{1}{2} (Q_L + Q_R)^2 \operatorname{sh}^2 \theta \left\{ 1 + \mathcal{O}(e^{-\theta}) \right\} \quad (23)$$

so that the mass of any given state of the $SU(2)^6 \times SO(44)$ model, including the original tachyon, can be made positive at large enough angle θ . In general, however, some previously massive states shift to lower mass, and it is not clear whether the entire spectrum can be even made non-negative. If it can, the ensuing model would be non-tachyonic and yet (marginally) unstable since there would exist excitations lowering the vacuum energy without barrier penetration. Demanding the absence of such instabilities could thus provide an additional selection criterion for four-dimensional models.

ACKNOWLEDGEMENTS

Two of us, C. Bachas and C. Kounnas, wish to thank the CERN Theoretical Group for its hospitality while part of this work was being completed.

REFERENCES

- [1] J.H. Schwarz, Phys. Rep. 89 (1982) 223;
M.B. Green, J.H. Schwarz and E. Witten, "Superstring Theory", Cambridge University Press (1987).
- [2] D.J. Gross, J.A. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54 (1985) 502; Nucl. Phys. B256 (1985) 253 and B267 (1986) 75.
- [3] L. Dixon, J.A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261 (1985) 678 and B274 (1986) 285.
- [4] I. Antoniadis, C. Bachas, C. Kounnas and P. Windey, Phys. Lett. 171B (1986) 51;
I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. B289 (1987) 87.
- [5] K.S. Narain, Phys. Lett. 169B (1986) 41.
- [6] H. Kawai, D.C. Lewellen and S.H.H. Tye, Phys. Rev. Lett. 57 (1986) 1832 and Nucl. Phys. B288 (1987) 1.
- [7] W. Lerche, D. Lust and A.N. Schellekens, Nucl. Phys. B287 (1987) 477.
- [8] I. Antoniadis and C. Bachas, CERN preprint TH.4767/87 (1987).
- [9] K.S. Narain, M.H. Sarmadi and E. Witten, Nucl. Phys. B279 (1987) 369;
See also P. Ginsparg and C. Vafa, Nucl. Phys. B289 (1987) 414.
- [10] S. Ferrara, C. Kounnas and M. Porrati, LBL preprints 23594 and 24087 (1987),
and CERN preprint TH.4801/87 (1987);
See also E.T. Tomboulis, UCLA preprint TEP/29 (1987).
- [11] J. Scherk and J.H. Schwarz, Phys. Lett. 82B (1979) 60; Nucl. Phys. B153 (1979) 61;
R. Rohm, Nucl. Phys. B237 (1984) 553;
N.C. Tsamis, CERN preprint TH.4777/87 (1987).
- [12] I. Antoniadis, C. Bachas, C. Kounnas and T. Tomaras, in preparation.
- [13] P. Goddard, W. Nahm and D. Olive, Phys. Lett. 160B (1985) 111;
P. Di Vecchia, V.G. Knizhnik, J.L. Petersen and P. Rossi, Nucl. Phys. B253 (1985) 701;
P. Goddard and D. Olive, Nucl. Phys. B257 (1985) 226;
P. Goddard, A. Kent and D. Olive, Commun. Math. Phys. 103 (1986) 105.
- [14] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. 158B (1985) 316;
C.G. Callan, D. Friedan, E.J. Martinec and M.J. Perry, Nucl. Phys. B262 (1985) 593;
A. Sen, Phys. Rev. D32 (1985) 2102 and Phys. Rev. Lett. 55 (1985) 1846.
- [15] G.F. Dell'Antonio, Y. Frishman and D. Zwanziger, Phys. Rev. D6 (1972) 988;
see also J. Bagger, D. Nemeschansky, N. Seiberg and S. Yankielowicz, Nucl. Phys. B289 (1987) 53.

FIGURE CAPTION

The one-(a) and two-loops (b-f) diagrams that could in principle contribute to the Thirring-model beta-functions. Thick and thin lines stand for left- and right-movers respectively. Only diagrams (a), (e) and (f) do actually contribute. The flow of group indices is shown on one of the diagrams, whose contribution to $\beta_{\frac{ad}{CD}}$ is proportional to $\frac{ab}{AB} \frac{bc}{BA} \frac{cd}{CD}$.

Fig. 1