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THE ASYMPTOTIC BEHAVIOUR OF THE $(\bar{P}P - PP)$ FORWARD AMPLITUDE*

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Abstract

We have investigated from a new point of view the crossing-odd amplitude $F^{(-)}$ using dispersion relations (DR). It is shown that the existing data favour clearly asymptotic Regge behaviour which implies the validity of an unsubtracted DR although more complicated asymptotic behaviours involving odderon terms cannot be ruled out presently. Precise Coulomb interference data at ISR energies would constitute a crucial test. The DR applied at threshold leads to a sum rule between the pp and $\bar{p}p$ scattering lengths on one side and the cross-section data on the other side. It turns out that this sum rule is well satisfied.

Recently, the ratio $\rho_{\bar{p}p}$ of real to imaginary part of the $\bar{p}p$ forward scattering amplitude has been measured at $\sqrt{s} = 546$ Gev in an experiment at the CERN $S\bar{p}pS$ collider [1]. The value $\rho_{\bar{p}p} = 0.24 \pm 0.04$ found in this experiment is larger than expected from analyticity according to which $\rho_{\bar{p}p}$ should lie between 0.11 and 0.15 [2]. This result has led to a lot of speculations.

- (i) One may be tempted to take this large value of $\rho_{\bar{p}p}$ as evidence for the odderon [3,4]. The price one has to pay for accepting this explanation is that $\sigma_{\bar{p}p}$ and σ_{pp} cross at some energy leading to $\sigma_{pp} > \sigma_{\bar{p}p}$ at high energies. This consequence is unavoidable as has been derived by Fischer [5] for a large class of odderon-type amplitudes.

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- (ii) Leader [6] and Martin [2] have pointed out that an unexpected large rise of the crossing-even part of the total cross section above the energy of the CERN collider may cure the above disease. This rise, perhaps, may be related to a new threshold. Hadjitheodoridis and Kang [7] have studied such threshold models in more detail and derived a general bound of the total cross section for both $\bar{p}p$ and pp at a given high energy constrained only by the UA4 datum [1].
- (iii) The usual method of analyzing the Coulomb interference data bases on a very simple parametrization of the nuclear amplitude which if far from reality may lead to wrong results for ρ [8].

In this article we want to test whether or not an unsubtracted DR for the crossing-odd (spin averaged) forward amplitude for pp and $\bar{p}p$ scattering:

$$F^{(-)}(\omega) = \frac{1}{2} (F_{\bar{p}p}(\omega) - F_{pp}(\omega)) , \quad (1)$$

is compatible with the data. As a by-product of this analysis we investigate a sum rule - similar to the famous Goldberger- Miyazawa- Oehme one [9] for πN scattering - which relates the scattering lengths to an integral over the difference $\sigma^{(-)}$ of total cross sections for $\bar{p}p$ and pp scattering (ω lab. energy)

$$\sigma^{(-)} = \frac{4\pi}{\sqrt{\omega^2 - m^2}} \text{Im } F^{(-)} . \quad (2)$$

The numerical evaluation of this sum rule became feasible only recently by virtue of the advent of the LEAR $\bar{p}p$ data on total cross sections [10] and $\rho_{\bar{p}p}$ [11] at low energies and of the $\bar{p}p$ scattering lengths obtained from measurements of the shift and width of the 1s level of antiprotonic hydrogen [12-14].

Most convenient for our purpose is to write down a DR for $F^{(-)}$ by integrating along the real axis from $\omega = -N$ to N , with N being an appropriately chosen energy, and closing the contour by a semi circle in the upper half ω plane. Hence, for $\omega < N$

$$\begin{aligned} \text{Re}F^{(-)}(\omega) = & -\frac{\mu^2}{8m^2} (g_\pi^2/4\pi) \frac{\omega}{\omega^2 - \omega_\mu^2} + \frac{\omega}{\pi} \int_{\omega(2\mu)}^m d\omega' \frac{\text{Im}F_{pp}^{(-)}(\omega')}{\omega'^2 - \omega^2} \\ & + 2 \frac{\omega}{\pi} P \int_m^N d\omega' \frac{\text{Im}F^{(-)}(\omega')}{\omega'^2 - \omega^2} + \text{Re}I(\omega) \quad . \end{aligned} \quad (3)$$

Here $m(\mu)$ is the nucleon (pion) mass, $\omega(2\mu) = -m + 2\mu^2/m$ the tip of the unphysical part of the $\bar{p}p$ cut and $\omega_\mu = -m + \mu^2/m$ the position of the π pole and g_π its coupling constant. $I(\omega)$ is the integral over the semi circle

$$I(\omega) = \frac{1}{\pi i} \int_{C_1} d\omega' \frac{F^{(-)}(\omega')}{\omega' - \omega} \quad . \quad (4)$$

This method originally proposed by Lam and Truong [15] has the advantage as compared to standard DR that the contribution from the real axis can be calculated independently on the asymptotic behaviour of $F^{(-)}$. It is straightforward to show that if

$$\lim_{\omega \rightarrow \infty} \int_{-\infty}^{\omega} \text{Im}F^{(-)}(\omega') d\omega' / \omega'^2 \quad (5)$$

exists, $I(\omega)$ equals the integral from $-\infty$ to N and from N to ∞ along the real axis, i.e., (3) is equivalent to the usual unsubtracted DR. If $\text{Im}F^{(-)}$ grows stronger than allowed by (5) our method is mathematically equivalent to the use of Regge subtracted amplitudes, an example of which is the P' sum rule proposed in Ref[16] long time ago.

Now we are going to evaluate the various terms appearing in Eq.(3). For the special case $\omega = m$ the results are compiled in Table 1.

(i) Scattering length. Whereas the pp scattering length is known to a very high precision for a very long time, namely a $(^1S_0 = -85.5 \pm 1.0 \text{ GeV}^{-1}$ [17], the spin-averaged $\bar{p}p$ scattering length, both its real as well as its imaginary part, has been determined only recently by three different LEAR experiments [12-14]. Combining with pp , $\text{Re}F^{(-)}(m)$ is obtained .

(ii) π pole term. The tabulated value is computed with $g_\pi^2/4\pi = 14.28 \pm 0.20$ [17].

(iii) Unphysical part of the cut ($\omega(2\mu)$ to m) UPC. In a series of papers, Grein et al. [18] have studied the six singly subtracted nucleon-nucleon forward DR, utilizing the data on total cross sections in pure spin states besides the usual spin averaged one. With the aid of the discrepancy method, they were able to determine reasonably well the discontinuity of the UPC for $u \leq 120\mu^2$. In terms of dynamics, the discontinuity is built up by the 2π contribution (including the ρ meson) - which actually has been calculated from the $\pi\pi \rightarrow \bar{p}p$ partial waves given by Gustafson et al. [19] - the 3π continuum and the ω and A_1 pole terms. The recent measurements of $\sigma_{\bar{p}p}$ and $\rho_{\bar{p}p}$ [10,11] carried out at LEAR rendered possible to also determine the discontinuity of the UPC for $120\mu^2 \leq u \leq 4m^2$, the physical threshold [20]. Since only spin-averaged information is available at present, no detailed dynamical interpretation of this part of the discontinuity is possible as yet, it is rather to be understood as the net effect of many channels. Making use of these results we can compute the integral over the UPC. The error essentially originates from the region near the physical $\bar{p}p$ threshold. In order to reduce it, precise data at very small energies (kinetic energies less than 20 MeV) are needed.

(iv) Threshold region. The principal-value integral around threshold ($m-1MeV$ to $m+1MeV$) is computed with the aid of threshold expansions. For pp scattering this is the usual effective range approximation with the scattering length $a(^1S_0)$ as given before and the effective range $r(^1S_0) = 14.20 \pm 0.15 GeV$ [17]. For $\bar{p}p$ scattering we use the expansion

$$\text{Im}F_{\bar{p}p}^-(\omega) = \text{Im}F_{\bar{p}p}^-(m) + B(\omega-m) \quad (6)$$

with B obtained from an interpolation between threshold and the low-energy data of Ref. [10] ($B = 330 GeV^{-2}$). The value of the integral over the threshold region is obtained from these two expansions. It is very precise since it is dominated by pp scattering.

(v) σ^- . The integral over the physical part of the cut is readily calculated from all the precise total cross-section data for $\bar{p}p$ and pp scattering measured up to ISR energies. For technical reasons we take N slightly larger than the highest energy at which data exists ($\omega = 2064 GeV$). In fact, we take $N = 2500 GeV$ a choice causing an error much smaller than any other one

quoted in Table 1.

(vi) Re I(ω). What remains is to compute the contribution of the semi circle from a presumed asymptotic behaviour of $F^{(-)}$. The most simple ansatz is Regge behaviour

$$R: \quad F_R(\omega) = \gamma \omega^\alpha (i + \tan \pi\alpha/2) \quad (7)$$

the imaginary part of which fits nicely the $\sigma^{(-)}$ data between 10 and 2064 GeV (see Fig 1) with $\gamma = 5.313 \text{ GeV}$ and $\alpha = 0.42$. Evaluating Re I(ω) for (7) one finds ($\omega < N$)

$$\text{Re } I_R(\omega) = - \frac{2\gamma}{\pi} N^\alpha \sum_{m=1,3,\dots} \left(\frac{\omega}{N}\right)^m \frac{1}{\alpha-m} . \quad (8)$$

Inserting the values of the parameters γ and α , one obtains for $\omega = m$ the result quoted in Table 1.

Inspection of Table 1 reveals that the sum rule is well satisfied, demonstrating the internal consistency of all the elements used in the evaluation of the right-hand side of the sum rule. One may turn the arguments around and use the sum rule to determine one or the other piece entering it. However, since the error of the right-hand side is rather large - to reduce it LEAR data at very small energies are needed - one does not obtain any spectacular result at present. For instance, one may use the sum rule to determine the real part of the $\bar{p}p$ scattering length. One finds $\text{Re } F_{\bar{p}p}(m) = -5.5 \pm 7.1 \text{ GeV}^{-1}$ which agrees with the LEAR results [12-14] within the errors. This is the usual way how the GMO sum rule [9] is utilized; it provides the most accurate determination of the πN scattering length $a_1 - a_3$.

With asymptotic Regge behaviour the right-hand side of the sum rule is practically independent of the contribution from the unmeasured asymptotic region. However, one may think of amplitudes rising stronger than Regge for $\omega \rightarrow \infty$, such that an unsubtracted DR does not hold. Will the sum rule be spoilt in such cases? This indeed is not so. Let us consider two examples of such behaviours. One is the simple $J=1$ pole

$$P: \quad F_P = c\omega \quad (9)$$

i.e., a purely real contribution. Obviously,

$$\text{Re } I_P(\omega) = c\omega \quad (10)$$

As we will see below, the high energy ρ data put severe limits on c ($c \leq 0.08 \text{ GeV}^{-2}$). Hence, the contribution to such a pole term to the sum rule is negligibly small.

The other possibility is the (maximal) odderon advocated for in Ref[3] -

$$O: \quad F_O(\omega) = a\omega \ln^2 \left(-i \frac{\omega}{\omega_0} \right) \quad (11)$$

leading to

$$\begin{aligned} \text{Re } I_O(\omega) = & a\omega [\ln^2 N / \omega_0 - \pi^2 / 12] \\ & + 2aN \sum_{m=3,5,\dots} \left(\frac{\omega}{N} \right)^m \frac{1}{1-m} \left[\ln N / \omega_0 - \frac{1}{1-m} \right] \end{aligned} \quad (12)$$

It turns out that any reasonable odderon, i.e., one which is compatible with the ISR $\sigma^{(-)}$ data, contributes little to the sum rule. An example of such an amplitude is that proposed by Bernard et al. [3] which in fact is a superposition of all three types of terms, Regge (R), $J=1$ pole (P) and odderon (O). For this amplitude we find $\text{Re} I(m) = 0.191 \text{ GeV}^{-1}$ a value being negligible as compared with the errors of the other contributions to the sum rule. We note that for this example $\text{Re} I(m)$ is strongly dominated by the P term and not by the odderon as one may expect.

To investigate the asymptotic behaviour of $F^{(-)}$ it is therefore necessary to exploit Eq. (3) at high energies such as attained at FNAL or ISR. For $\text{Re} F^{(-)}$ data are available at some energies from Coulomb interference mea-

measurements for $\bar{p}p$ and pp scattering. The existing high energy data are shown in Fig. 2 where for graphical purposes only, we have divided $ReF^{(-)}$ by $ImF_R(cf.Eq.(7))$ with the parameters taken from the fit to $\sigma^{(-)}$. The right-hand side of Eq.(3) - except for $ReI(\omega)$ - is computed as described above. The results are shown in Fig. 2 as a dashed line (DR). We note that they are insensitive to the contribution from the low energy region. The UPC, for instance, contributes less than 1% at $\omega > 70GeV$. Consequently, the errors of the results are of the size of those of $\sigma^{(-)}$, i.e., small as compared to the errors of $ReF^{(-)}$.

In estimating the contribution of the semi circle we adopt the following strategy. For each example R,O or combinations, we fix the parameters from fits to $\sigma^{(-)}$ in an energy region as large as possible. The strength of the $J=1$ pole we obtain by adjusting c such that the predictions for $ReF^{(-)}$ are still tolerable with regard to their experimental errors. Thus one may consider the value of c we found this way rather as an upper bound than a best fit. Since the odderon contributes negatively the $J=1$ pole can be stronger with an additional odderon than without. The parameters for the various terms used to build up the asymptotic behaviour of $F^{(-)}$ are listed in Table 2 where also the $\chi^2/data$ point for the different parametrizations is given in order to facilitate the comparison.

Pure Regge behaviour (R in Fig.2) is by far the best description of the crossing-odd amplitude. It requires only two parameters which are precisely determined from the σ data for energies between 10 and 2000 GeV. $ReF^{(-)}/ImF_R$ has exactly the Regge value, namely $\tan(\pi\alpha/2) = 0.776$. With Regge behaviour, however, the large value of $\rho_{\bar{p}p}$ measured by UA4 [1] is not explained. One has to consider other explanations as for instance are offered in Refs. [2,6,7].

Keeping just Regge behaviour for $F^{(-)}$ the strength of an additional $J=1$ pole has to be quite small, $c \leq 0.08GeV^{-2}$ (RP in Fig. 2 with $c = 0.08GeV^{-2}$). In fact too small to account for the UA4 datum. A pure odderon seems rather unlikely. Neither a reasonable fit to $\sigma^{(-)}$ in the ISR energy region can be achieved nor $Re F^{(-)}$ reproduced well (O in Fig. 2). Acceptable fits to the ISR $\sigma^{(-)}$ data can only be obtained by a combination of odderon and Regge. The line in Fig. 2 denoted by RO is such an example. Adding a $J=1$ pole (RPO in Fig. 2) a description very similar to that proposed by Bernard et al.

[3] (B), is obtained. With these to latter examples it is possible to explain the UA4 datum by the behaviour of $F^{(-)}$.

In conclusion, we stress that the present data on $F^{(-)}$ favour clearly asymptotic Regge behaviour, implying the validity of an unsubtracted DR. More complicated asymptotic behaviours of $F^{(-)}$ are not excluded by the present data. However, in these cases the description of $\text{Re } F^{(-)}$ in the FNAL-ISR region is worse than for Regge behaviour despite the fact that a larger number of parameters is involved. More $\text{Re } F^{(-)}$ data of the quality of the UA6 datum [24] would be extremely helpful in pinning down the asymptotic behaviour of $F^{(-)}$.

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Remarks	Contribution GeV^{-1}
<div> <div>ReF⁽⁻⁾(m)</div> <div> PS 171 [12] PS 174 [13] PS 175 [14] average </div> </div>	<div>-46.80 $\pm$ 0.99</div> <div>-46.95 $\pm$ 0.58</div> <div>-46.55 $\pm$ 0.90</div> <div>-46.80 $\pm$ 0.48</div>
<div>π pole term</div> <div>Unphysical part of $\bar{p}p$ cut</div> <div>threshold region</div> <div>integration of $\Delta\sigma^{(-)}$</div> <div>ReI(m) for Ragge behaviour</div>	<div>-1.91 $\pm$ 0.03</div> <div>-24.14 $\pm$ 3.0</div> <div>-26.91 $\pm$ 0.13</div> <div>7.43 $\pm$ 0.60</div> <div>0.071 $\pm$ 0.001</div>
Sum	-45.46 $\pm$ 3.5

Table 1 - Contributions to the sum rule (3) ($\omega=m$)

asymptotic behaviour	parameters	$\chi^2/\text{data point}$
$I(\omega) = 0$		1.32
R	$\gamma = 5.313 \text{ GeV}^{-1}$ $\alpha = 0.42$	0.76
RP	γ, α as before $c = 0.08 \text{ GeV}^{-2}$	1.53
O	$a = -0.0508 \text{ GeV}^{-2}$ $\omega_0 = 3000 \text{ GeV}$	2.2
RO	$\gamma = 3.07 \text{ GeV}^{-2}$ $\alpha = 0.42$ $a = 0.025 \text{ GeV}^{-2}$ $\omega_0 = 1800 \text{ GeV}$	1.34
RPO	$\gamma, \alpha, a, \omega_0$ as before $c = 0.15 \text{ GeV}^{-2}$	1.81

Table 2 - Parameters and $\chi^2/\text{data points}$ for the various examples of asymptotic behaviours

FIGURE CAPTIONS

Fig. 1 Difference of $\bar{p}p$ and pp total cross sections vs. ω . Solid line is a Regge fit according to Eq.(7). For comparison the odderon fit of Ref. [3] is also shown (dash-dot-dotted line) Data are taken from Refs. [21-23].

Fig. 2 The real part of the crossing-odd amplitude. For graphical purposes $ReF^{(-)}$ is divided by $5.313GeV^{-1}\omega^{0.42}$. Data are taken from Refs. [21,24]. For other symbols it is referred to the text.

Fig. 1

Fig.2