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DOUBLE LAYERS

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Abstract

Electrostatic double layers have been proposed as an acceleration mechanism in solar flares and other astrophysical objects. They have been extensively studied in the laboratory and by means of computer simulations. The theory of steady state double layers implies several existence criteria, in particular the Bohm criteria, restricting the conditions under which double layers may form. In the present paper several already published theoretical models of different types of double layers are discussed. It is shown that the existence conditions often imply current driven instabilities in the ambient plasma, at least for strong double layers, and it is argued that such conditions must be used with care when applied to real plasmas. Laboratory double layers, and by implication those arising in astrophysical plasmas, often produce instabilities in the surrounding plasma and are generally time dependent structures. Naturally occurring double layers should therefore be far more common than the restrictions deduced from idealised time independent models would imply. In particular it is necessary to understand more fully the time dependent behaviour of double layers. In the present paper the dynamics of weak double layers is discussed. Also a model for a moving strong double layer, where an associated potential minimum plays a significant role, is presented.

1. Introduction

A double layer (DL) is a local region in a plasma which can sustain a potential difference. Essentially it consists of two adjacent layers with equal and opposite net charge. The layer as a whole is globally neutral but has an internal electric field. The structure is determined in a self consistent manner by the particle dynamics in the electric field set up by the net charge distribution produced by the particles. This is indicated schematically in Figure 1 (see also Figure 3). In general a DL requires both free and reflected ion and electron components. The free particles carry current through the layer and lead to emerging beams of accelerated particles. The hatched areas in Figure 1 denote regimes of ion and electron acceleration. The potential across the DL must in general be maintained by external sources which are the source of energy for the particle acceleration.

Double layers are of interest in astrophysics as a direct means of accelerating particles (Alfvén, 1981 and 1986). They can sustain a local region of parallel electric field leading to the magnetohydrodynamic relaxation of a large scale magnetic field. The globally stored magnetic energy is then released both as accelerated particles and in the mass motions set up by the untwisting motions of the magnetised plasma (Raadu, 1984). The power supplied to accelerated particles is simply the product of the current crossing the double layer and the potential across it. The formation of a DL in a current carrying loop in the solar corona has been proposed as a mechanism for energy release in a solar flare (Jacobsen and Carlqvist, 1964; Alfvén and Carlqvist, 1967; Hasan and ter Haar, 1978). Carlqvist (1979) has also argued that the associated motions in the coronal loop can explain the occurrence of solar surges. For suitable conditions the released magnetic energy can go predominantly to the surge mass ejection rather than particle acceleration at the DL. Hénoux (1986) argues that multiple DLs can explain the high rate of energy release observed in solar flares.

In the magnetosphere the first indications of parallel electric fields leading to electrostatic acceleration of particles were provided by electron measurements in a rocket experiment (McIlwain, 1960). Further evidence was provided by the discovery of the inverted-V structures and associated electric field patterns (Frank and Ackerson, 1971; Gurnett and Frank, 1973). There is evidently a close relationship between parallel electric fields and the aurora (e.g. Mozer et al., 1980 and 1985; Chiu et al., 1983; Fälthammar, 1983). DLs may well be responsible for sustaining the necessary parallel electric field. There is some observational evidence for multiple weak ion-acoustic DLs (e.g. Temerin et al., 1982; Kellogg et al., 1984). The theory of the acceleration of auroral particles by DLs has been treated in a number of papers (e.g. Block, 1978; Kan et al., 1979; Goertz, 1979). Wagner et al. (1980) have made a two dimensional computer simulation of an ionospheric DL (see also Singh et al., 1987). Applications have also been made to Jupiter's magnetosphere (Shawhan, 1976; Smith and Goertz, 1978) and to accretion onto neutron stars (Williams et al., 1986).

In the laboratory the existence of DLs has been for many years a well established fact. Recent theoretical and experimental work has been reported at three symposia on DLs (Michelsen and Rasmussen, 1982; Schrittwieser and Eder, 1984; Williams and Moorehead, 1986). See also recent reviews by Sato (1984), Hershkowitz (1985) and Schamel (1986). Clear evidence for the formation of DLs has been shown in numerical simulations, and stationary solutions have been shown to exist theoretically for a variety of choices of the velocity distributions of the involved particle species.

Near the boundaries of a DL self consistency between the electric field and electrical charges requires that the net charge on the low and high potential sides are negative and positive respectively (for a monotonic DL). These requirements take the form of a generalisation of the Bohm criterion for wall

sheaths. For a strong DL the incoming free particles must then have velocities exceeding their thermal velocities (Block, 1972a). In particular on the low potential side of the double layer this implies that the electron drift velocity should be sufficiently high for electrostatic two-stream instabilities to occur. Similarly ion-acoustic instabilities are expected on the high potential side. Alfvén and Carlqvist (1967) argued that a current driven instability (i.e. the two-stream instability in the long wave length regime (Carlqvist, 1973)) can produce localised extreme reductions in plasma density leading to the formation of a DL in an initially homogeneous plasma. Such a localised evacuation should occur for all temperature ratios and, in one particular case, was followed well into the non-linear regime (Raadu and Carlqvist, 1981).

Smith and Goertz (1978) have argued that the ponderomotive force of unstable waves produced by current driven instabilities can lead to DL formation. DeGroot et al. (1977) ran a simulation with hot electrons drifting at about their thermal velocity through cool ions. Many weak DLs were formed. The electrons accelerated by the potential jumps were thermalised by electron-electron two-stream instabilities in the intervening field free regions. Sato and Okuda's (1981) simulations also showed the formation of weak (ion-acoustic) DLs as a result of current driven instabilities for drift velocities below the electron thermal velocity. Smith and Priest (1972) argued that anomalous resistivity originating from plasma turbulence rather than DL formation should result from current driven instabilities. At the time it was not appreciated that the evacuation mechanism of Alfvén and Carlqvist (1967) was essentially identical to the two-stream instability in the long wavelength, non-dispersive, regime.

It should be stressed that very few attempts have been made to investigate the stability of the DL itself taking into account the spatial structure. A stationary DL may be regarded as a particular example of a non-linear wave as for example des-

cribed by the method of Bernstein et al. (1957) for deriving stationary solutions of the Vlasov-Poisson equations (the BGK method). This method was for example used by Knorr and Goertz (1974). In principle therefore the stability analysis of non-linear plasma waves described by Infeld (1972) should be possible to extend to treat DLs. A general study of the stability of stationary solutions of the Vlasov-Poisson equations has been presented by Lewis and Symon (1979). Wahlberg (1979) demonstrated that linearly unstable trapped Langmuir modes may exist within a DL. This leads to a stability requirement that the DL must have a thickness smaller than a critical value. A related study has recently been made by Teichmann (1987). Schamel's (1983) analysis for a very weak DL shows linear stability with respect to longitudinal one-dimensional perturbations, but instability for transverse perturbations.

Furthermore, except for Torvén's (1981) treatment of the dynamics of a weak DL by modelling the evolution with a modified Korteweg-deVries equation (see also Raadu and Chanteur 1986, and Chanteur and Raadu, 1987), very little work has been done to describe the dynamics of DLs. The movement of strong DLs has been treated by applying the stationary description in the frame of reference moving with the DL. The motion is interpreted as a consequence of current imbalance. Moving DLs have, for example, been discussed by Bergeron and Wright, (1978); Singh and Schunk (1982); Iizuka et al. (1982 and 1985) and by Raadu and Rasmussen (1984). Carlqvist (1984) has described a number of mechanisms that can influence the geometry of double layers and in particular lead to expansion. The properties of the surrounding plasma and the effects of the whole circuit containing the DL are here of great significance.

It has become clear that the requirement that DLs be strictly time independent structures is an over restriction on the theory (cf. Smith, 1985, and Borovsky, 1986). Rather, there is a strong need for theoretical investigations of DL stability and dynamics. A completely stable DL has not yet been observed ex-

perimentally. DLs are always subject to fluctuations in their profile, and the current through the system usually also shows some oscillations which however become relatively small for extremely strong DLs (Torvén, 1982; Sato et al., 1983). Experiments with an inductive external circuit have been shown to lead to the transient formation of strong DLs maintained by inductive voltage pulses accompanying rapid current decreases (Torvén et al., 1985). Even in computer simulations, where the boundary conditions can be fully controlled to meet the requirements for a theoretically stationary DL, the DLs produced are not at all stationary. Recurring formation and disruptions of a DL accompanied by a series of other phenomena are usually observed (cf. Smith 1982a, 1982b and 1985, and Borovsky, 1984). This indicates on the one hand that stationary DL solutions are most likely unstable and on the other hand that "real plasmas" can adjust "self-consistently" to limit these instabilities making the existence of almost stationary structures possible.

In the present paper we consider the existence conditions for a DL and in particular the generalised Bohm criteria. For strong DLs these criteria have implications for the stability of the adjacent uniform plasma regions. We take this as a further indication that DLs should be regarded as dynamical time dependent structures, so that, for example, the Bohm criteria probably only apply in some average sense. These considerations lead us on to the presentation of a simplified model of a moving dynamical DL, which we hope may lead to some new interpretations of earlier work (e.g. numerical simulations) and indicate the need for further work on DL-dynamics and stability. This paper is not intended to be a thorough review of DLs, and although the reference list is extensive it is far from complete.

2. Stationary Double Layers

In principle the structure of steady state plane parallel DLs may be found by a straightforward integration of the Poisson equation and the time-independent Vlasov equations for the particle species. In general a first integral may then be obtained in the form

$$\frac{1}{2} \epsilon_0 E^2 + V(\phi) = 0 \quad (2.1)$$

where E and ϕ are the electric field and potential. This is possible since stationary solutions of the Vlasov equation are given by arbitrary functions of the particle total energy. The function $V(\phi)$, the classical or Sagdeev potential, is to within an additive constant equal to minus the total particle pressure, defined using the second moments of the distribution functions (cf. Andrews and Allen, 1971, and Torvén, 1980). Thus the first integral of the equations is completely equivalent to the condition of stress balance within the DL:

$$-\frac{1}{2} \epsilon_0 E^2 + \sum_{i,e} \int m_{i,e} v^2 f_{i,e}(v) dv = \text{const.} \quad (2.2)$$

where by assumption any magnetic field is constant within the DL so that the magnetic stress term arising from the Maxwell stress tensor may be dropped. Here $m_{i,e}$ and $f_{i,e}$ denote respectively the mass and distribution function of ions and electrons. The form of the classical potential $V(\phi)$ is therefore determined by the choice of the particle distributions. A second integration gives the spacial structure of the DL. This is the explicit procedure followed by Andrews and Allen (1971), and by Schamel and Bujarbarua (1983). Schamel (1982) argues that it is more natural to specify all the particle distributions and derive the DL structure than to predetermine the potential profile and then to artificially adjust one of the particle distributions to obtain a solution i.e. BGK method (cf. Knorr and Goertz, 1974). However to correctly represent the physical situation care should be taken to define the various distributions at the potentials of their sources.

The properties of the DL solutions depend significantly on the potential ϕ_0 across the DL. In order to classify DLs we introduce a typical effective temperature T which represents the mean energy of the reflected particles on either side of the DL. Then for a strong DL $e\phi_0 \gg kT$ and the dynamical pressures of the accelerated particle beams dominate. Such a DL is to a good approximation described by Langmuir's (1929) theory (Levine and Crawford, 1980). For weak DLs $e\phi_0 \approx kT$ and a variety of solutions are possible. Schamel (1983) distinguishes two types of weak DLs (slow electron or ion acoustic DLs). Strong DLs are included among what he calls beamtype DLs. We note the existence of other types of weak DLs such as those described by Torvén (1981) for cold ions and non-Maxwellian electrons which move at close to an ion acoustic velocity. Also there are DLs which we may refer to as intermediate DLs. These are characterised by the presence of particle components produced by secondary physical processes. An example is given by the work of Andersson and Sørensen (1983) who include the effects of ionisation. Also Kan and Lee (1980) include an electron population built up by scattering processes in their magnetospheric DL model. We note that what have been referred to as asymmetric electron and ion holes also have some of the characteristics of DL's and for example can sustain a net potential difference (cf. Schamel, 1986).

3. Double Layer Existence Conditions

We now consider existence conditions for DL solutions in general terms. First since $E^2 > 0$, a necessary condition for the existence of DL solutions is that the classical potential $V(\phi) < 0$ within the DL. This condition is particularly sensitive at the edges of the DL ($\phi = 0$ and $\phi = \phi_0$, where ϕ_0 is the DL potential). There the normal matching conditions between the external plasma and the DL require $V(0) = V(\phi_0) = 0$ (zero electric field) and $V'(0) = V'(\phi_0) = 0$ (no net charge). A power series expansion then gives $V''(0) < 0$ and $V''(\phi_0) < 0$

so that V has local maxima at $\phi = 0, \phi_0$. These are the generalised forms of the Bohm criteria which ensure appropriate net charges near the edges of the DL (cf. Introduction) and may also be written $\{Z e n'_i(\phi) - e n'_e(\phi)\} < 0$ for $\phi = 0$ and $\phi = \phi_0$, where Z is the ion charge number and $n_{i,e}$ the number density of ions and electrons respectively. Since $E = 0$ at the edges of the double layer it follows that $V(0) = V(\phi_0) = 0$. This is simply the condition that the total particle pressures on either side of the DL are equal. The DL is in pressure equilibrium. For a strong DL the dynamical pressures of the emerging accelerated particle beams dominate and pressure equilibrium requires that $|j_e/j_i| \approx \sqrt{m_i/m_e}$, the Langmuir condition on j_e and j_i , the electron and ion current densities through the DL. Thus the pressure balance condition ($V(0) = V(\phi_0)$) may be regarded as the appropriate generalisation of the Langmuir condition.

Figure 2 illustrates the general required form of the classical potential $V(\phi)$, incorporating the existence conditions for DL solutions. The minimum value corresponds to the maximum electric field E inside the DL.

Many of the considerations that apply to DLs have counterparts in the theory of wall sheaths, where it is also of interest to find how the Bohm criterion can be satisfied and the stability properties of the sheath for different relative potentials between the plasma and the wall. A brief discussion of wall sheaths may be found in Appendix A.

Block (1972a) discusses the application of the Bohm sheath criterion to a double layer. Considering the predominant particle species he derives simple conditions for the ion and electron injection velocities v_{i0}, v_{e0} namely,

$$m_i v_{i0}^2 \geq kT_e \quad ; \quad m_e v_{e0}^2 \geq kT_i \quad (3.1)$$

where T_e , T_i are the trapped thermal electron and ion temperatures on the appropriate sides of the double layer. He argues that there must be a certain preacceleration of beam particles to meet these conditions before the final acceleration in the DL.

Levine and Crawford (1980) have presented a series of fluid models for double layers using four particle species and successively introducing thermal effects. They note the condition that the classical potential (in their notation $P(\phi)$ or $\Pi(\phi)$) must be negative and in particular that the second derivative must be negative at the edges of the DL, i.e. generalised Bohm criteria.

The generalised Bohm criteria may be expressed directly in terms of the distribution functions $f_{i,e}(v)$ of the ions and electrons. For time independent conditions the Vlasov Equations are satisfied if the distribution functions are

$$f_i = F_i [\text{sgn}(v) (v^2 + 2 q_i \phi / m_i)^{1/2}] \quad (3.2)$$

$$f_e = F_e [\text{sgn}(v) (v^2 + 2 q_e (\phi - \phi_0) / m_e)^{1/2}] \quad (3.3)$$

where the functions $F_{i,e}$ are defined by the ion distribution at $\phi = 0$ and by the electron distribution at $\phi = \phi_0$. The ion and electron particle charges are $q_i = Ze$ and $q_e = -e$, where for hydrogen and singly charged ions $Z = 1$. From the ion and electron densities given by

$$n_{i,e} = \int_{-\infty}^{+\infty} f_{i,e} dv \quad (3.4)$$

it follows that if $q(\phi)$ is the net charge then

$$q'(\phi) = \sum_{i,e} \frac{q_i^2}{m_i} \int_{-\infty}^{\infty} \frac{1}{v} \frac{\partial f_{i,e}}{\partial v} dv \quad (3.5)$$

using the functional dependence of $f_{i,e}$ on ϕ and v defined above through $F_{i,e}$. Now using distribution functions normalised to unit density the Bohm criteria (defined by $\rho'(\phi) < 0$ as $\phi \rightarrow 0$ and ϕ_0) are given by:

$$\beta \equiv \sum_{i,e} \omega_{pi,e}^2 \int_{-\infty}^{\infty} \frac{1}{v} \frac{\partial f_{i,e}}{\partial v} dv < 0 \text{ as } \phi \rightarrow 0+, \phi_0^- \quad (3.6)$$

This is closely related to the sheath criterion found by Allen (1976) (see Appendix A). It is of interest to compare this expression with the dispersion relation for electrostatic waves:

$$K^2 = \sum_{i,e} \omega_{pi,e}^2 \left[P \int_{-\infty}^{\infty} \frac{1}{v-a} \frac{\partial f_{i,e}}{\partial v} dv + i\pi f'_{i,e}(a) \right] \quad (3.7)$$

where $a = \omega/K$ is the phase velocity of waves with frequency ω and wave number K . From the expression for the distribution functions inside the DL given above we notice that $f'_{i,e}(0) = 0$. It then follows that for zero phase velocity ($a = 0$) $K^2 = \beta$. The Bohm criteria are then seen to be equivalent to requiring that the stationary electrostatic wave is spatially evanescent (K is imaginary). For a Maxwellian plasma $\beta = K^2$ ($a=0$) $= -\lambda_D^{-2}$ where λ_D is the Debye shielding length. In general we may then say that the Bohm criteria are requirements that the plasma just inside the edges of the DL acts to shield charge.

In general we may allow for discontinuities in the distribution functions at the separatrix between trapped and free particles. The separatrix is given by

$$v = \pm v_i \equiv \pm (2q_i(\phi_0 - \phi)/m_i)^{1/2} \text{ for the ions and}$$

$$v = \pm v_e \equiv \pm (-2q_e\phi/m_e)^{1/2} \text{ for the electrons. Ions and}$$

electrons with velocities v with $|v| < v_i$ and $|v| < v_e$,

respectively, are reflected somewhere inside the DL. The dis-

tribution functions are therefore symmetric inside these ranges

and in particular $f_i(v_i - \epsilon) = f_i(-v_i + \epsilon)$ and similar-

ly for electrons (where $\epsilon \rightarrow 0+$). In contrast, for free par-

ticles we expect no relationship between the distributions for $v > v_{i,e}$ and $v < -v_{i,e}$, since the particles come in general from widely separated, unrelated sources to the left and right of the DL respectively. To derive the Bohm criteria including the possible discontinuities in the distributions, the integration range in the definition of $n_{i,e}$ should be divided up before taking derivatives. The final result is then found to be unchanged if we treat $f'_{i,e}(v)$ as a δ -function at the discontinuities so that for example $f'_{i,e}(v) = \{f_i(v_{i+}) - f_i(v_{i-})\} \delta(v - v_i)$ for $v \approx v_i$. The Bohm criteria are then given by the expression already given above.

If we now consider the Bohm criterion as $\phi \rightarrow 0$ the critical velocity for electron trapping tends to zero i.e. $v_e \rightarrow 0$. Depending on the discontinuities in $f_e(v)$ at $v = -v_e$ (Δf_1) and at $v = +v_e$ (Δf_2) the electron contribution to β can be divergent. The δ -function contribution then dominates so that

$$\beta \approx \omega_{pe}^2 \left[\frac{\Delta f_1}{-v_e} + \frac{\Delta f_2}{v_e} \right] = \frac{\omega_{pe}^2 (\Delta f_2 - \Delta f_1)}{\sqrt{2e\phi/m_e}} \quad (3.8)$$

and if $\Delta f_2 \neq \Delta f_1$, $\beta \rightarrow \pm\infty$ as $\phi \rightarrow 0$. Then the Bohm condition $\beta < 0$ reduces to

$$0 > \Delta f_2 - \Delta f_1 = f_e(0+) + f_e(0-) - 2f_{et}(0) \quad (3.9)$$

where $f_{et}(0)$ is the value of the trapped electron distribution just inside the DL. This means that either $f_{et}(0)$ must exceed a critical value to satisfy the Bohm criterion (this can lead to a critical trapped particle density requirement as for Kan and Lee's (1980) model) or that $f_e(v) \rightarrow 0$ as $v \rightarrow \pm 0$. If $\Delta f_2 = \Delta f_1$ or if there are no discontinuities the singularity is removed and we must use the full expression for β to find the Bohm criterion. Similar considerations apply at $\phi = \phi_0$ for the ions.

4. Bohm Criteria and Instabilities

The existence of a double layer in current carrying plasma imposes requirements on the incoming particle distributions (e.g. generalised Bohm criteria) and also implies the presence of emerging particle beams accelerated by the DL. It is therefore necessary to ask if the uniform plasma outside the DL is stable. A distinction should be drawn between two possible sources of instability. Those instabilities, which may be driven by emerging beams, propagate away from the DL and affect it only indirectly by modifying the incoming particle distributions, even if the beams themselves are destroyed. In this respect such instabilities are relatively harmless to the continued existence of the DL. On the other hand instabilities generated by incoming particle beams might be dangerous as they may substantially modify or even disperse the beams so that they no longer fulfill the matching conditions, e.g. generalised Bohm criteria, at the edges of the DL.

Having derived the Bohm criteria in terms of the distribution functions it is now clear from a comparison with the dispersion relation that there is no simple relationship with the criteria for instability (Penrose, 1960) (see Equations (3.6) and (3.7)). We now proceed instead to consider several one dimensional DL models which have been presented in the literature and comment on the stability and Bohm criteria.

4.1 Strong Double Layers

In view of the special role that discontinuities can play we note that a fluid component in a DL model (e.g. Block, 1972b) is in one dimension completely equivalent to a water bag distribution (Bertrand and Feix, 1968).

Block (1972b) presented a DL model with fluid and Maxwellian distributed particle components. This model explicitly shows the matching of a strong DL onto the surrounding plasma. Colli-

sions are included, but for the present purposes we discuss the collisionless limit only. In this limit the fluid components are completely equivalent to water bag distributions (Bertrand and Feix, 1968), in one dimension and for the ratio of specific heats $\gamma = 3$. On the low potential side from the general analysis presented here we then find the Bohm criterion

$$0 > \beta = - \frac{n_{io} e^2}{\epsilon_o T_{io}} + \frac{\omega_{pe}^2}{v_e^2 - c_e^2} - \frac{n_{eo} e^2 e^{-\phi_o/T_{io}}}{\epsilon_o T_{eo}} + \frac{\omega_{pib}^2}{v_{ib}^2 - c_{ib}^2} \quad (4.1)$$

The notation is as follows: n_{io} , n_{eo} are the densities of the Maxwellian distribution of ions and electrons on the low ($\phi=0$) and high ($\phi=\phi_o$) potential sides of the DL respectively with corresponding temperatures T_{io} , T_{eo} ; v_e , c_e and ω_{pe} are the velocity, sound velocity and plasma frequency of the incoming fluid electrons on the low ($\phi=0$) potential side; v_{ib} , c_{ib} and ω_{pib} are the corresponding quantities for the fluid ions which have been accelerated by the DL potential ϕ_o and emerge as a beam on the low potential side. For a strong DL the last two terms in the Bohm criterion are negligible as well as the corresponding densities. The Bohm condition may be reduced and rewritten as

$$v_e > \sqrt{\frac{T_{io}}{m_e} + c_e^2} \quad (4.2)$$

in agreement with Block's (1972b) equation in the collisionless limit. An analogous result holds on the high potential side.

The emerging low density ion beam on the low potential side of the DL should drive instabilities with phase velocities close to the beam velocity, propagating away from the DL. More dangerous instabilities may be driven by the incoming fluid electrons moving through the Maxwellian distribution of ions i.e. just those components which dominate the Bohm criterion.

Schamel and Bujarbarua (1983) have derived several DL models by self consistently solving the stationary Vlasov equations together with Poisson's equation. They prescribe trapped electron and ion distribution functions in terms of Maxwellians and free electron and ion distribution functions in terms of drifting Maxwellians. They further assume that the distributions are continuous everywhere, i.e. without a jump at the separatrix between reflected and free particles (cf. the discussion in Section 3). This assumption provides an extra restriction on the solutions.

In the cases where the DL potential is much greater than the trapped particle temperatures (measured in electron volts) their distributions in the ambient plasma are essentially full Maxwellians. The drift velocities of the injected distributions are much larger than their thermal velocities because they have to match onto the small tails of the distributions with high energetic particles that have managed to overcome the DL potential.

Hence looking for instabilities driven by the injected distributions the situation is that of two relative drifting Maxwellian distributions (neglecting the low density emerging beams). The results of Stringer (1964) for electrostatic instabilities in a uniform plasma may then be applied to the ambient plasma regions. In Fig. 4 the instability diagram of Stringer (1964) is plotted. Clearly the models of Schamel and Bujarbarua (1983) imply instability in the ambient plasmas at least for strong DLs. Such instabilities are also to be expected for models involving distributions with discontinuities at the separatrix between reflected and free particles (cf. Section 3). The instabilities certainly appear when the incoming electrons have truncated distributions, with finite truncation velocity. Note that Jentsch (1976) has generalized the instability analysis of Stringer (1964) by allowing the relatively drifting distribution functions to have a continuous range of forms including Maxwellian, waterbag and resonance functions as limiting cases.

4.2 Intermediate Double Layers

The auroral DL proposed by Kan and Lee (1980) is an example of an intermediate DL. Their DL is "strong" if we consider only the primary particles i.e. ions and electrons from the plasma sheet and ionosphere. The DL potential energy $e\Phi_0$ is not limited and may be much greater than the typical energies of the primary particles. Kan and Lee note that with no extra particle components the Bohm conditions derived by Block (1972b) would imply that on the low potential side the incoming electrons would have to be preaccelerated by some mechanism up to energies corresponding to the temperatures of the hot plasma sheet ions i.e. a few keV. (This is already the energy of auroral electrons so that the DL acceleration is secondary.) They argue for an extra electron population trapped between the DL and the magnetic mirror. These electrons consist of degraded beam electrons and secondary electrons produced by collisions with neutral particles. This "extra" population has energies up to $e\phi_0$. They may therefore penetrate the DL and if there are enough they ensure that there is a net negative charge at the low potential side of the DL. This ensures that the Bohm criterion is satisfied. The DL is "weak" with respect to these secondary electrons. Kan and Lee's DL model successfully explains how low energy incoming electrons can be accelerated up to auroral energies. Since they assume a water bag distribution extending down to zero velocity for incoming electrons there are no dangerous instabilities propagating toward the DL. The emerging ion beam may be expected to drive instabilities, but these propagate upwards away from the DL. However depending on the effective external circuit global instabilities are possible and, for example, a negative resistance instability of the Kan and Lee model was shown by Silevitch (1981) and Raadu and Silevitch (1983).

In some experiments the energetic electrons emerging from a DL significantly ionise the neutral gas on the high potential side. This allows the production of DLs with only one plasma

source located on the low potential side of the DL: energetic electrons produce the plasma on the other side (Torvén and Andersson, 1979; Torvén and Lindberg, 1980; Andersson, 1981). Andrews and Allen (1971) discussed the structure of a DL pre-sheath with ionisation. We note that if their integrals are rewritten (using integration by parts) to avoid singularities it follows from the analysis that the ionisation rate has to go smoothly down to zero at the edge of the DL. In principle we might expect the Bohm condition to apply just at the inner edge of such a sheath at the DL. There is then a better chance that the presheath and external plasma region may be stable. This may also apply if collisions (classical or anomalous) in the external plasma are taken into account (cf. Borovsky, 1986). Comparisons may here be made with work on plasma wall sheaths as referred to in Appendix A.

Andersson and Sørensen (1983) have obtained time independent solutions of DLs where the ions on the high potential side are created by ionisation due to the accelerated electrons. They use their results to explain experiments and to suggest the possibility of DLs with ionisation effects in the ionosphere. The density and velocity distribution of the free ions are found assuming that they are created by ionisation at zero velocity and then freely accelerated in the electric field. The authors remark that the predominance of low energy ions in the distribution would seem to violate the Bohm criterion; but since ionisation is included here the densities are not functions of the potential only and the collisionless Bohm criteria are not applicable. However, on the low potential side the incoming electrons must satisfy the normal Bohm criterion, here, as for strong DLs, we can expect instabilities propagating towards the DL. In the relevant experiments (Torvén and Lindberg, 1980) however there is a potential minimum on the low potential side, beyond which the potential is gently increasing. Then even if the Bohm criterion is applicable at the DL it need not imply instability in the adjacent plasma where the particle distributions are functions of the changing potential.

The strong potential DLs discussed by Knorr and Goertz (1974) are only "strong" in the sense applied to Kan and Lee's (1980) models. Following the method of Bernstein et al. (1957), Knorr and Goertz derive a necessary distribution of trapped electrons for a given potential profile and free particle distribution functions. There are no trapped ions in their models. The trapped electrons have distributions extending up to the DL potential energy. The solutions have the property that the uniform plasmas on either side of the DL are Penrose stable i.e. there are no electrostatic current driven instabilities. The electrons have everywhere a mean velocity equal to zero. The generalised Bohm criterion on the low potential side is met by the trapped electrons which penetrate through the DL as in the model of Kan and Lee. In fact the strong DL of Knorr and Goertz have some of the properties characterising Schamel's (1983) Slow Electron and Ion Acoustic DLs (SEADL and SIADL). On the high potential side there is a minimum in the electron distribution at zero velocity as for a SEADL. However the density is largest on the high potential side (since there are no trapped ions) as is the case for a SIADL. Knorr and Goertz (1974) also obtain weak DL solutions which may be made asymptotically Penrose stable. These are constructed so that the electron distribution has a single maximum. Choosing an ion frame of reference on the low potential side, the velocity of these weak DLs must be less than the ion acoustic velocity (cf. their Equation (43), for the case of zero temperature free ions).

It is apparent that the extra particle distributions assumed in intermediate DLs help to avoid strict Bohm criteria. This has the consequence that the external plasmas may be less unstable or even completely stable. A departure from strictly one dimensional models would lead to an even stronger modification of DL properties. In experiments radial losses are probably important and may even be affected by instabilities resulting in a complex interdependence between the time averaged structure and time dependent fluctuations. Borovsky and Joyce (1983) have simulated DL structures in two dimension including the effects

of magnetic fields. They also present a simple one-dimensional oblique DL model including a uniform magnetic field. In particular they derive explicit Bohm criteria which clearly show a dependence on the angle between the DL electric field and the magnetic field.

5. Weak Double Layers

Weak double layers are characterized by having a potential jump $e\phi_0/kT \ll 1$ implying that the densities and thereby the function $V(\phi)$ may be expressed as a power expansion in ϕ . This of course simplifies the analysis considerably, and it has for instance been possible to derive evolution equations for such weak DLs (cf. Torvén, 1981). While the strong DL-solutions require the existence of four distinct particle types (see Section 4.1) for the weak DL-solutions three types of particles suffice. We shall also see that these solutions in general are stable with respect to beam instabilities, i.e. the generalized Bohm criterion do not require inflow of particles with velocities exceeding the critical velocities for instabilities.

Recently the existence of weak DLs has been shown for many models corresponding to different combinations of the involved particle species. These models may roughly be divided into two main types.

1) Models based on a fluid description, where typically the plasma consists of two groups of electrons with distinctly different temperatures and cold ions (cf. Torvén, 1981; Goswami and Bujarbarua, 1985; Raadu and Chanteur, 1986; Bharuthram and Shukla, 1986a; Chanteur and Raadu, 1987)

ii) Models based on a kinetic description of at least one of the involved particle species (cf. Perkins and Sun, 1981; Kim, 1983; Schamel, 1983; Torvén, 1986).

In addition we should also mention the so called ion acoustic double layers (Sato and Okuda, 1981; Hasegawa and Sato, 1982; Schamel, 1982) which essentially should be classified as weak DLs in the sense that the net potential jump across them: $e\phi_0/kT_e \leq 1$. However, by employing the terminology of Schamel (1983, 1986) these structures should rather be referred to as asymmetric ion holes. Similarly asymmetric electron holes also exist (Schamel, 1982, 1986) (See also Pécseli (1984) for a review of electron and ion holes). We shall not give a detailed description of these structures here, but just mention that e.g. the asymmetric ion holes are observed in simulations of current carrying plasmas (cf. Sato and Okuda, 1981; Borovsky, 1984). Furthermore they are observed experimentally (Chan et al., 1984; Sekar and Saxena, 1985) and above the aurora (cf. Temerin et al., 1982; Mozer et al., 1985), where a series of them are believed to be responsible for the acceleration of the auroral electrons.

To illustrate how the weak DL-solutions are constructed and how the different restrictions come about we consider a model of type i). The DL-solution, stationary in a frame moving with the velocity M , is obtained from the first integral of the Poisson equation (see Section 2) in the following normalized form,

$$\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + V(\phi) = 0 \quad (5.1)$$

where $-dV/d\phi \equiv (n_e(\phi) - n_i(\phi))$. Here ϕ is normalized with kT/e , $n_{e,i}$ with the undisturbed densities n_0 , x with $(\epsilon_0 T / e^2 n_0)^{1/2}$. For small amplitude DLs both n_e and n_i may be expanded in powers of ϕ resulting in

$$-V(\phi) = A_1 \phi^2 + A_2 \phi^3 + A_3 \phi^4 + \dots \quad (5.2)$$

where we have taken $V(0) = 0$. The coefficients A_i are functions of the velocity M and the parameters of the involved particle species.

Referring to Section 3 it is easily seen that we have to retain up to the fourth power in the expansion of $V(\phi)$, i.e. up to the third power in the expansions of n_e , n_i . Furthermore a DL-solution to (5.2) requires that

$$A_3 > 0 \quad (5.3)$$

(the generalized Bohm criterion) and that the polynomial

$$\phi^2 + \frac{A_2}{A_3} \phi + \frac{A_1}{A_3} = 0 \text{ has a double root i.e.}$$

$$A_2^2 = 4 A_1 A_3 \quad (5.4)$$

The DL potential is then given by

$$\phi_0 = - \frac{A_2}{2A_3} \quad (5.5)$$

and for consistency we must require that

$$\left| \frac{A_2}{2A_3} \right| \ll 1 \quad (5.6)$$

For the case where the plasma consists of cold ions and a hot and cold electron component it is possible to fulfil the conditions (5.3) (5.4) and (5.6) (cf. Torvén, 1981; Goswami and Bujarbarua, 1985), and the DL-solution is expressed as:

$$\phi = \frac{1}{2} \phi_0 \left\{ 1 - \tanh \left[\left(\frac{A_3}{2} \right)^{1/2} \phi_0 (x - Mt) \right] \right\} \quad (5.7)$$

where ϕ_0 as given by (5.5) is positive (negative) when A_2 is negative (positive). Thus both compressional ($\phi_0 > 0$) and rarefactive ($\phi_0 < 0$) DLs may exist. It is evident that these DL-solutions do not require streaming particle populations which could lead to instabilities. However, certain restrictions must be fulfilled for their existence, and in particular

(5.6), essentially ensuring a sufficiently small DL potential, must be satisfied for the expansion (5.2) to be consistent. The relation (5.6) is satisfied for certain ratios between the temperatures and the densities of the hot and cold electrons, respectively. Note that Bharuthram and Shukla (1986b) recently constructed DL solutions with finite potential (i.e. without employing the expansion (5.2) for the above discussed model).

When the condition (5.6) applies, it has been shown that the dynamical evolution of small amplitude disturbances, is governed by a modified KdV equation (cf. Torvén, 1981; Raadu and Chanteur, 1986; Chanteur and Raadu, 1987, and references therein):

$$\partial_t \varphi + \alpha \varphi \partial_x \varphi + \beta \varphi^2 \partial_x \varphi + \frac{1}{2} \partial_{xxx} \varphi = 0 \quad (5.8)$$

Here α corresponds to $-3A_2$ and β to $-6A_3$ in (5.2), which may easily be seen by looking for the stationary solution of (5.8). Note that (5.8) may be transformed into the canonical form of the mKdV by using the linear transformation

$$\varphi = \left(\frac{3}{|\beta|} \right)^{1/2} \psi - \frac{1}{2} \frac{\alpha}{\beta} \quad (5.9)$$

together with a change of the frame of reference and rescaling the time by dividing by two.

$$\partial_t \psi + 6 \text{sign}(\beta) \psi^2 \partial_x \psi + \partial_{xxx} \psi = 0 \quad (5.10)$$

The transformation (5.9) simply corresponds to a shift of the reference level of the potential. Only when $\beta < 0$ will the mKdV equation have DL-type solutions (cf. 5.3), and for consistency $\alpha = o(\beta)$ for the same parameters, corresponding to (5.6). In that connection we should mention that for a plasma consisting of positive and negative ions and isothermal electrons an mKdV equation has also been derived (Watanabe, 1984). However, in the validity region of this mKdV equation (i.e. $\alpha \approx o(\beta)$) $\beta > 0$ and DL-type solutions do not exist.

Chanteur and Raadu (1987) have presented a detailed study of the mKdV equation (5.10) with $\beta < 0$ solving it analytically, by means of the inverse scattering transformation, as well as numerically. The general results are briefly summarized as follows: Suppose that initially $\psi(-\infty) = \psi_0^-$ and $\psi(+\infty) = \psi_0^+$. Then a DL-like structure forms if and only if $\psi_0^- \psi_0^+ < 0$ and has a potential jump $2|\psi_0^+|$. In that case, for $|\psi_0^-| < |\psi_0^+|$ a train of solitons moving away from the DL appears and for $|\psi_0^-| > |\psi_0^+|$ a non-dispersive ramp forms behind the DL. Individual solitons may also appear depending on the form of the initial profile (ibid). The DL-like solutions to the mKdV equation are stable since they are obtained as the asymptotical evolution of a given initial condition by means of the inverse scattering method. The stability of these weak DL-solutions with respect to perturbations transverse to their propagation is however still an open question. Such a stability study could be based on a two dimensional generalization of the mKdV equation as e.g. derived by Bharuthram and Shukla (1986a) for a magnetized plasma.

For the models of type ii) the plasma usually consists of free ions and electrons and reflected ions and/or reflected electrons. Schamel (1983) has characterized these weak DLs as either slow electron acoustic DL (SEADL), when reflected electrons are involved, i.e. a tuning-fork-like phase space configuration of the electrons, or slow ion acoustic DL (SIADL) when reflected ions are involved. These DLs may even exist without any net current (Perkins and Sun, 1981). The SEADL at rest in the ion-frame exist for $T_i/T_e > 3.5$ while the SIADL correspondingly requires $T_e/T_i > 3.5$. We point out that the SEADL (SIADL) essentially are "half" electron- (ion-) holes. To obtain the solutions for the small amplitude DLs both the electron and ion densities may be expanded in powers of ϕ resulting in:

$$-V(\phi) = A_1 \phi^{3/2} + A_2 \phi^2 + A_3 \phi^{5/2} + A_4 \phi^3 + \dots \quad (5.11)$$

Due to the presence of reflected particles governed by a kinetic description the expansion includes half-powers in φ . The coefficients are complicated functions of the parameters of the involved particle species. A_1 in particular is related to the jump in the distribution function at the separatrix between free and reflected particles (cf. Torvén, 1986). Schamel (1983) assumes that the distributions are continuous everywhere and argues that this is a necessary requisite for a physically acceptable theory. In this case $A_1 = 0$ and $A_4\varphi^3$ must be retained to have DL-solutions. Torvén (1986) argues that if the free and reflected particles have different origins (and the plasma is collisionless) then a discontinuity at the separatrix is most probable (cf. Figure 3). We should stress that even if the distribution has a discontinuity the "physical" quantities, the moment of the distribution, may certainly be continuous as they of course should be. Referring to the discussion of model i) it is easily seen that a DL-solution requires

$$A_3 > 0 \quad (A_1 \neq 0); \quad A_4 > 0 \quad (A_1 = 0) \quad (5.12)$$

Furthermore:

$$A_2^2 = 4 A_1 A_3 \quad (A_1 \neq 0); \quad A_3^2 = 4 A_2 A_4 \quad (A_1 = 0) \quad (5.13)$$

and

$$\varphi_0 = \left(\frac{A_2}{2A_3}\right)^2 \quad (A_1 \neq 0); \quad \varphi_0 = \left(\frac{A_3}{2A_4}\right)^2 \quad (A_1 = 0) \quad (5.14)$$

The DL-solution reads

$$\begin{aligned} \varphi &= \varphi_0 \tanh^4(\gamma(x-Mt)) \quad (A_1 \neq 0) \\ \varphi &= \frac{1}{4} \varphi_0 (1+\tanh(\kappa(x-Mt)))^2 \quad (A_1 = 0) \end{aligned} \quad (5.15)$$

where the coefficient γ may be found in Torvén (1986) and $\kappa = (\varphi_0/8)^{1/2}$. Note that the first solution in (5.15) is defined in the half plane $-\infty < x - Mt \leq 0$. Torvén (1986) derived a modified KdV equation with half power non-linearities describing the DL-evolution (for $A_1 \neq 0$).

Concerning the stability of these types of DLs Schamel (1983) argues that both the SEADL and SIADL are stable with respect to beaming instabilities but unstable with respect to transverse perturbations.

In closing this section we mention that a weak DL of the SIADL type has been observed experimentally (Chan et al., 1986), while there is evidence for the existence of the SEADL in numerical simulations (Schamel, 1984).

6. Dynamics of Strong Double Layers

Double layers are inherently dynamical structures as has been strongly emphasised in the previous sections. It has also become evident that due to their strongly nonlinear nature it is hard to conceive of an analytical model for the dynamical evolution of strong and intermediate DLs. For weak DLs, on the other hand, tractable evolution equations have been derived taking the form of modified KdV-equations as discussed in Section 5.

Certainly a lot of insight into the dynamics of strong DLs has been gained from numerous computer simulations (e.g. Smith, 1982b; Singh and Schunk, 1982; Borovsky, 1984; Iizuka and Tanaka, 1985; Yamamoto, 1985; and references in these papers) and from experimental investigations (e.g. Sato, 1984; Hershkowitz, 1985). Furthermore some attempts to describe the dynamics of strong DLs have been made by investigating the consequences of varying some of the parameters of the stationary DL-solutions. For example the movement of a DL was treated by applying the stationary description in the moving frame of reference (e.g. Singh, 1979; Singh and Shunk, 1982; Raadu and Rasmussen, 1984), and Carlqvist (1984) considered different effects, that would give rise to an expansion or contraction of a DL. In this section we discuss some of these attempts and relate the results to the dynamical features observed in simulations and experiments. We emphasize that the dynamics of the

DL is strongly dependent on the properties of the surrounding plasma and the whole circuit, in the general sense, that contains the DL. We intend to demonstrate this through specific examples in what follows.

In particular we consider the movement of a strong DL. Moving double layers have been observed in most of the numerical simulations performed (see the above references) and also in many experiments (e.g. Lutsenko et al., 1976; Leung et al., 1980; Sato, 1984; Iizuka et al., 1985). In most cases the DL moves towards the high potential side, "rarefactive movement"; but also movement towards the low potential side, "compressional movement", has been observed (Coakley and Hershkovitz, 1981). In the following we describe a simple model for the rarefactive movement of a strong DL. We shall restrict ourselves to a one dimensional model and assume that the DL moves as a "rigid body" in accordance with the observations in experiments and simulations. Our argument is based on the pressure balance across the DL, which for strong DLs (non-relativistic) reduces to the Langmuir condition (see Section 3).

$$|j_e/j_i| = \sqrt{m_i/m_e} \equiv \mu \quad (6.1)$$

The DL is allowed to move in the laboratory frame with the velocity v_{DL} , positive in the direction of the electron beam velocity, i.e. from the low to the high potential side of the DL (see Fig.5). Since the DL is assumed to conserve its shape during the propagation (6.1) must be satisfied in the moving frame, resulting in:

$$v_{DL} = \frac{n_{e1}v_{e1} - \mu n_{i0}v_{i0}}{n_{e1} + \mu n_{i0}} \quad (6.2)$$

where 0 labels the parameters on the high potential side and 1 those on the low potential side, $v_{e1,i0}$ are the averaged drift velocities of electrons and ions entering the DL, respectively, while $n_{e1,i0}$ are the respective densities. When (6.1) is satisfied for the currents injected at the boundaries of the

DL region the DL is at rest, but when a current imbalance exists the DL may compensate by moving. Similar arguments were presented by several authors (see e.g. Singh, 1979; Singh and Schunk, 1982).

Imagine now a stationary DL formed in the interior of a plasma produced by two sources at different potentials at each end of a linear device. Then the plasmas emitted by the two sources satisfy: $n_{e1}v_{e1} = \mu n_{i0}v_{i0}$. At, say $t = 0$, the density on the high potential side is decreased, but the rest is left unchanged, i.e.:

$$n'_{i0} = n_{i0} - \delta n_0 \text{ and } n'_{et} = \delta n_{et} - \delta n_0$$

and we still have the plasma condition on the high potential side $n'_{i0} = n'_{et} + n'_{eb}$, where t labels the trapped (reflected) population and b the beam particles accelerated through the DL. A very small adjustment of the densities on the low potential side is also necessary to maintain plasma conditions since the density of the beam ions n_{ib} here is decreased by $\delta n_0 \cdot v_{i0} / (2e\phi_{DL}/M)^{1/2}$, which for the very strong DL is negligible. Neglecting the influence of any transient phenomena we see from (6.2) that the DL starts moving with the velocity:

$$v_{DL} = \frac{\mu \delta n_0 v_{i0}}{n_{e1} + \mu n'_{i0}} \quad (6.3)$$

i.e. the DL moves towards the high potential side. How does this movement effect the other particle species? The trapped electron population n'_{et} will be compressed, but the high potential source may easily regulate that, and the effects on the trapped electrons will be negligible. The trapped ions, on the other hand, may lag behind the moving DL if v_{DL} is larger than the expansion velocity of the plasma produced by the low potential source. This velocity will be around the ion acoustic velocity c_s . Thus the moving DL will be followed by an expanding plasma. For $c_s < v_{DL}$ an electron surplus will appear between the expansion front and the DL resulting in the formation of a negative potential dip (see also the more detailed

discussion in Appendix B) at the low potential tail of the DL as sketched in Fig.5 and clearly observed in simulations and experiments (e.g. Singh and Schunk, 1982; Iizuka and Tanaca, 1985; Iizuka et al., 1985). This dip in turn reflects some of the electrons leading to a decrease in the electron current. Denoting the decrease in the electron beam density and mean velocity at the bottom of the dip by δn_{e1} and δv_{e1} , respectively, we obtain:

$$v_{DL} = \frac{-\delta n_{e1} v_{e1} - \delta v_{e1} n_{e1} + \delta n_{e1} \delta v_{e1} + \mu \delta n_o v_{io}}{n'_{e1} + \mu n'_{io}} \quad (6.4)$$

If the distribution of the beam electrons are known then δv_e and δn_e can be calculated as functions of the depth of the dip. For our present argument it is sufficient to note that v_{DL} decreases. The potential dip cannot exist self-consistently in a stationary frame because the adjacent DL is very strong ($e\phi_{DL} \gg T$), and the ions will cancel the negative space charge connected with the dip (see Appendix B). Thus, when v_{DL} decreases, the dip potential decreases too and the electron current increases. From these considerations we can argue that the DL velocity cannot be much faster than the expansion velocity of the low potential plasma. The potential dip regulates the electron current through the system and is an effective current limiting mechanism. However, the dip is formed because of the movement of the DL which is initiated by the current imbalance. By using (6.2) we see that the (electron-) current through the DL is:

$$j_e \equiv en_{e1} v_{e1} = e\mu n_{io} (v_{io} + v_{DL}) + ev_{DL} n_{e1} \quad (6.5)$$

thus for $n_{io} > \mu^{-1} n_{e1}$ the current is mainly determined by the density of the high potential plasma and the DL velocity. We remark that the Bohm condition, in the generalized sense, ensuring that there is a net charge of the correct (self-consistent) sign on entering the DL (cf. Section 3), is easily verified on either side of the moving DL. This further supports the assumptions of "rigid" DL movement.

This simple model describes the essential features of the rarefactive moving DLs observed in experiments (e.g. Leung et al., 1980; Sato, 1984; Iizuka et al., 1985) and simulations (e.g. Singh and Schunk, 1982; Iizuka and Tanaca, 1985; Yamamoto, 1985). In particular it emphasizes the role of the negative potential dip which always seems to form on the low potential tail of a moving DL, and which effectively limits the current through the system. Related investigations of the role of an associated potential minimum for current disruptions caused by a DL have been reported by Torvén et al. (1985) and Carpenter and Torvén (1986).

In closing the discussion of the rarefactive moving DL we point out that the negative potential dip forming at the low potential side of the DL is of quite different origin than the solitary potential structure of the so called ion acoustic DL (Sato and Okuda, 1981; Hasegawa and Sato, 1982; Chanteur et al., 1983) (cf. Section 5) although they act in a similar way with respect to reflecting electrons and limiting the current. The dip following the moving strong DL can only exist in the moving frame and appears essentially as an ambipolar potential in front of an expanding DL, it does not contain trapped ions and is usually relatively broad (many Debye lengths λ_{De}). The potential dip connected with the ion acoustic double layer is narrow (a few λ_{De}) and may exist in a stationary frame where ion trapping results in the formation of an ion hole (cf. Pécseli, 1984). The scenario for the formation of an ion acoustic DL preceded by electron reflection from a developed ion hole is clearly revealed in numerical simulations (cf. Sato and Okuda, 1981; Borovsky, 1984). Actually even strong transient DLs have been observed to develop due to electron reflection presumably from an ion hole developed in a Buneman unstable plasma (Belova et al., 1980; Galeev et al., 1981). Note that while the simulations resulting in ion acoustic DLs employed periodic boundary condition, Belova et al. (1981) used "open" boundary conditions.

It is obvious that the model described for the moving DL assumes a particular form of the boundary condition imposed on the plasma and essentially also of the external circuit. That is, we assumed that the inflow of plasma could be regulated by some external agency. Bergeron and Wright (1978), on the other hand, argue that a pressure difference across the DL will not lead to any acceleration of the DL in a collisionless plasma, but rather be compensated by regulating the electron and/or ion beam through the DL. Thus they implicitly assume that their boundary conditions and external circuit allows for such a "self-consistent" regulation of the plasma inflow. They further argue that an acceleration of the DL will be the result of other processes such as for example collisions or beam plasma interactions. With the above discussion in mind we add that external constraints fixing for example the current through the system would also imply a movement.

The compressional movement of a DL as a "rigid" body is much more rarely observed than the above described rarefactive movement. Coakely and Hershkovitz (1981) explain their observations in terms of the theory for electrostatic shocks taking into account the effects of reflected electrons on the high potential side of the shock (Montgomery and Joyce, 1969). In that connection we also refer to the paper by Hershkovitz (1981) where shocks are distinguished from DLs by the direction of the flow of free ions: For shocks they flow toward the high potential side in the shock frame, while for DLs they flow toward the low potential side. Attempting to apply a model similar to the one used for the rarefactive movement, to the compressional movement of a strong DL, it may become difficult for the ions to fulfil the Bohm condition on the high potential side of the DL. If this condition is not fulfilled we expect the rigidity of the DL to break down and the DL profile to relax. For the electrostatic shock Hershkovitz (1981) argues that the fulfilment of the Bohm criterion on the high-potential side restricts the mach number M (shock velocity normalized with the ion acoustic velocity)

$$M^2 \geq 1 + 2e\phi_{\text{shock}}/T_e \quad (6.6)$$

This implies that the ions on the high potential side must be injected with a velocity

$$v_{i0} \geq c_s + (1 + 2e\phi_{\text{shock}}/T_e)^{1/2} c_s.$$

Extrapolating this to strong DLs obviously leads to unrealistically high ion velocities.

Finally we briefly discuss the work of Carlqvist (1984), who suggests different effects leading to relaxation of the DL-profile. Again we emphasize that these results are the outcome of particular assumptions on the whole circuit including the DL-structure. For instance Carlqvist considers a situation in which a strong DL is suddenly formed in a current carrying plasma. It is assumed that the current is kept constant by the external circuit. By demanding the fulfilment of the Langmuir condition (6.1) it is argued that the high potential edge of the DL move into the high potential plasma while the low potential edge is at rest. As a result the DL expands with velocity

$$v_{\text{exp}} \approx |v_{e1}|/\mu \quad (6.7)$$

where v_{e1} is the velocity of the electrons entering on the low potential side. Carlqvist also proposes an expansion mechanism due to collisions between the emitted beams and the surrounding plasma. Furthermore electromagnetic forces are shown to provide a contraction of the DL under particular conditions.

7. Discussion

DLs and turbulent plasma conditions are alternative outcomes for the same initial unstable conditions. The generalised Bohm criteria and the condition for electrostatic instability (Penrose criterion) are often identical; but there is a class of asymptotically stable DLs (e.g. the models of Knorr and Goertz, 1974). However, strong DLs appear to always require unstable external plasmas and are therefore always time dependent in some way.

The arguments presented in this paper show that many strictly time independent DL solutions seem impossible to realise because the incoming particle beams lead to two-stream instabilities when they are required to satisfy the Bohm criteria. Smith and Priest (1972) presented similar arguments and concluded that anomalous resistivity should occur rather than DL formation once the critical current drift velocity was exceeded. We agree with their diagnosis but with hindsight, knowing that DLs may in fact be produced both in the laboratory and in computer simulations, we instead argue for the DL as a dynamical, intrinsically time dependent phenomenon. The stability and dynamics of DLs should take into account the effects of realistic boundary conditions and appropriate effective external circuits.

Future developments of the theory of DL dynamics will probably include more realistic treatments of extra effects of the type we have discussed here e.g. ionisation. More three dimensional models are needed including a magnetic field. Global effects of the external medium should be considered. Electromagnetic aspects have received little attention apart from discussions concerning the mhd-relaxation of the external plasma. Goertz (1983) has argued for electron acceleration by kinetic Alfvén waves in the magnetosphere (see also Goertz and Boswell, 1979). There may perhaps be some direct interrelationship between kinetic Alfvén waves and DLs through the electric field component parallel to the magnetic field.

Finally we note that so far there has been no complete stability analysis for a strong DL taking into account its spatial structure. One reasonable approach would be to use a computer simulation starting from carefully prepared initial conditions determined by a stationary DL solution.

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Appendix A

The Bohm criterion for the wall-sheath

The criterion for the formation of a stationary sheath connecting a biased (or floating) conducting wall was originally considered in the classical papers of Langmuir (1929). It was later precisely stated in the work by Bohm (1949), from where the often quoted name "Bohm-criterion" originates. The criterion demands that the accelerated mono-energetic particles at the sheath edge stream towards the wall, having an energy greater than the thermal energy of the reflected particles.

We shall here briefly illustrate how a Bohm-criterion is derived. We also discuss the stability of the wall-sheath and discuss the dynamical evolution when the Bohm-criterion is not satisfied. Consider a sheath connecting a biased, perfectly absorbing wall with a one-dimensional, semi-infinite plasma. The plasma is unmagnetized, collisionless and without internal ionization. Using the Poisson equation the Bohm-criterion for both an electron and ion sheath reads:

$$[d(n_e(\varphi) - n_i(\varphi))/d\varphi]_{\varphi=\varphi(x_0)} > 0 \quad (\text{A.1})$$

where x_0 is the location of the sheath edge. Outside the sheath region in the uniform plasma with density n_0 we take $\varphi = 0$, thus $\varphi(x_0) = 0$. The condition (A.1) ensures the right sign of the charge when the sheath is entered. Let us consider the ion sheath (i.e. for a negatively biased wall). The electrons are assumed to be in Boltzman equilibrium with a temperature T_e , thus

$$n_e(\varphi) = n_0 \exp(e\varphi/T_e) \quad (\text{A.2})$$

The ion density is determined from the distribution function, which is a stationary solution of the Vlasov equation:

$$f_i(u, \varphi) = n_0 F(u^2 + \frac{2e\varphi}{m_i}) H(u - \sqrt{u_{tr}^2 - \frac{2e\varphi}{m_i}}) \quad (\text{A.3})$$

where H is the unit step function. Assuming that the truncation velocity $u_{tr} > 0$ we obtain

$$n_i'(\varphi=0) = \frac{n_o e}{m_i} \left[\frac{F(u_{tr}^2)}{u_{tr}} + \int_{u_{tr}}^{\infty} 2 F'(u^2) du \right] \quad (A.4)$$

By means of partial integrations we get

$$n_i'(\varphi=0) = \frac{n_o e}{m_i} \int_{u_{tr}}^{\infty} \frac{F(u^2)}{u^2} du \equiv \frac{n_o e}{m_i} \left\langle \frac{1}{u^2} \right\rangle \quad (A.5)$$

On using (A.5) and (A.2) the Bohm criterion reads:

$$\left\langle u^{-2} \right\rangle^{-1} > T_e / M \quad (A.6)$$

The Bohm-criterion in this form was first derived by Harrison and Thompson (1959). Allen (1976) argues that the criterion (A.6) should have an equality sign replacing the inequality sign. He further provides an alternative physical meaning of the criterion, showing that the criterion implies that an ion acoustic wave is unable to travel from the wall into the plasma, because its velocity in the laboratory frame is reduced to zero. With the inequality sign in (A.6) an ion acoustic wave is certainly also hindered in propagating into the plasma.

We emphasize that the criterion (A.6) not only demands a certain averaged drift velocity of the ions but also requires that the distribution function goes rapidly to zero for $u \rightarrow 0$. This is certainly the case for truncated distributions as considered by e.g. Hu and Ziering (1966). For collisional plasmas with internal ionization the analysis gets much more involved than the simple arguments presented above (cf. Riemann, 1981). Almost all analyses of sheath formation have been concerned with ion sheaths although the Bohm-criterion for the formation of a stationary electron sheath is in principle straightforward to obtain by interchanging the roles of electrons and ions in the analysis of ion sheaths.

Now we turn to the question of the stability of the wall sheaths. It is generally found that an electron-rich sheath (i.e. for a positive wall-bias) is unstable at least in a collisionless plasma. This is clearly seen in, for example, the simulations of Skøelv and Rasmussen (1984) and also in the simulations of the plasma diode (Gray et al., 1984; Iizuka and Tanaka, 1985). The ion-rich sheath (i.e. for negative wall-bias) on the other hand, is found to be generally stable, in particular when the Bohm-criterion is fulfilled (Skøelv and Rasmussen, 1984). Even when this criterion is not satisfied an almost stationary sheath, a few Debye-lengths wide, is formed. But a quasi-neutral region connecting the sheath to the undisturbed plasma develops. This region expands continuously and the velocity of its leading edge is the ion acoustic velocity (cf. Cipolla and Silevitch, 1981; Skøelv and Rasmussen, 1984). This quasi-neutral region is usually referred to as the presheath. It accelerates the ions to a velocity at the sheath edge which is just sufficient to fulfil the Bohm-criterion. In an ideal semi-infinite collisionless plasma the presheath will never become stationary but continue to expand. However, in a bounded plasma the presheath will reach an extension determined by the geometry (cf. Emmert et al., 1980; Meassick et al., 1985). In a collisional plasma the extension of a stationary presheath will be determined by the mean free-path.

Appendix B

First we show that a one dimensional time independent strong DL preceded by a potential dip on the low potential side (together forming a "triple layer") cannot be described by using the Vlasov-Poisson system. Consider the first integral of this system i.e. the stress balance condition. On the low potential side within the plasma outside the DL $\phi = 0$ and there is no electric field. Between the potential minimum and the DL there is a point where again $\phi = 0$. ($V(\phi)$ is now a multivalued function.) Here the electron total pressure must be lower since some electrons are reflected by the DL and do not reach the point $\phi = 0$ (those that do have the same distribution as they had in the plasma) and for a strong DL there are practically no trapped electrons that can penetrate the DL from the high potential side. If there is an electric field it contributes a negative pressure increasing the stress imbalance further. Hence the assumption of a potential minimum is untenable since for a strong DL stress balance cannot be maintained. Intuitively the stress imbalance might be expected in a dynamical development to steepen and close up the potential minimum region.

To model a strong DL with a potential minimum it is therefore necessary to relax the idealised assumptions. For example radial ion losses from the plasma in a DL experiment are often significant; the situation is not strictly one-dimensional. Fujita et al. (1984) argue that ions are scattered inside the DL in the region beyond the potential minimum. This they model by reducing the corresponding ion density by a factor depending exponentially on the increasing potential on the high potential side of the minimum. This introduces an asymmetry in the ion density about the minimum and self consistent solutions are then found for a DL with a potential minimum. Also other processes than those implied by the Vlasov-Poisson system must apply for broad potential minima with quasi-neutrality (Carpenter and Torvén, 1986).

Here and in a previous paper (Raadu and Rasmussen, 1984) we have discussed a simple non-steady model to explain the essential features of moving DLs. During one phase of the propagation we argue that ions lag behind the DL. Since the ion dynamical time

scale greatly exceeds that for the electrons, on intermediate time scales the ion density may be treated as fixed and the electrons may be assumed to have steady approximately time independent motion. As a simple model we will now consider the situation indicated in Fig.6. The ion density n_i is constant except in a region around the potential minimum where it is approximated to zero. The injected electrons entering from the low potential side have a water-bag (WB) distribution and density n_{eWB} . On the high potential side there are reflected thermal electrons with low temperatures ($T \ll e\Delta\phi$) and density n_{eT} .

Suppose that the injected electron water-bag distribution function $f_{eWB}(v)$ has a value f_0 for $v_{10} < v < v_{20}$ and is zero outside this range. This is equivalent to a fluid (in one dimension) with velocity $(v_{10} + v_{20})/2$ and sound speed $(v_{20} - v_{10})/2$ (cf. Bertrand and Feix, 1968). The density is then given by

$$n_{eWB} = f_0 [(v_{20}^2 + 2e\phi/m)^{1/2} \mp (v_{10}^2 + 2e\phi/m)^{1/2}] \quad (B.1)$$

where the water-bag extends over $v_1 < v < v_2$ and the velocities v_2, v_1 are given respectively by the square-root terms in Eq.(B1). For $v_1 < 0$ the fluid motion is subsonic and here we assume $v_{10} < 0$ so that the injected electrons are subsonic. This corresponds to a plus sign in Eq.(B1). As the potential ϕ decreases the density also decreases until $v_1 = 0$ at the sonic point $x = x_s$ in Fig.6. If this also corresponds to the minimum potential ϕ_s the potential increases beyond this point and choosing $v_1 > 0$ i.e. supersonic motion, minus sign in Eq.(B1), the density will continue to fall. The potential minimum acts as a Laval nozzle in aerodynamics allowing a transition from sub- to supersonic flow. The electron density is therefore asymmetric about the potential minimum as is also the total pressure.

$$P_{eWB} = \frac{1}{3} m f_0 [(v_{20}^2 + 2e\phi/m)^{3/2} \mp (v_{10}^2 + 2e\phi/m)^{3/2}] \quad (B.2)$$

However, the total pressure has a minimum value at the sonic point where the electric field $E = 0$. Now, neglecting the other

particle populations, the steady state dynamics of the water-bag electrons in the self consistent electric field is given by Eq.(2.2) as for a steady state DL. On the high potential side of the minimum when the motion is supersonic (minus sign in Eq.(B.2) the pressure is less than on the subsonic side at the same potential. From Eq.(2.2) the electric field E is correspondingly larger.

The model so far is a particular case of a "virtual cathode". To produce a DL potential profile as shown in Fig.6 it is only necessary to add the contributions of the other particle species. Thus the fixed ions on the low-potential side provide a region of net positive charge ($n_i > n_{eWB}$) cancelling the electric field and providing a match to plasma conditions ($E = 0, n_i = n_e$). On the high potential side the electron density is essentially given by the thermal electrons $n_e \approx n_{eT}$ since the water-bag electrons emerge as a low density beam. These thermal electrons then provide a Debye shielding of the DL electric field and produce the required match to plasma conditions over a few Debye lengths λ_{De} .

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Figure Captions

Fig. 1 The potential distribution $\phi(x)$, ion and electron phase space distributions across a double layer with potential ϕ_0 and internal electric field E . As indicated by the arrows and the separatrixes ("dashed" and "barred") there are phase space regions of transmitted and reflected particles for both species. It is physically important to distinguish between particles originating from different sources (at $x \rightarrow \pm\infty$) as indicated by the hatched and clear areas in phase space: at the boundaries ("barred" contours) there can be a sharp jump in phase space density, in principle a discontinuity, but in reality smoothed to some extent by fluctuations and diffusion processes. Outside the narrow transition region indicated by the vertical bars the phase space densities on either side have no physical relation, being related to particles from widely separated sources.

Fig. 2 Example of the classical potentials $V(\phi)$. The function plotted is:

$$V(\phi) = -5 \phi^2(1-\phi^2)(0.05 + \phi^4)$$

ϕ is normalized to the DL potential ϕ_{DL}

Fig. 3 The ion and electron distribution functions $f_i(v)$ and $f_e(v)$ within a double layer. These should be compared with the phase space plots of Figure 1. The hatched regions here correspond with those of Figure 1. Note the near discontinuities marked by bars which correspond to the barred contours of the previous figure. They mark the boundaries between particles from distinct sources at $\pm \infty$

Fig. 4 Stability diagram for electrostatic current driven instabilities taken from Stringer (1964),

$$V_{Te} \equiv \sqrt{\frac{T_e}{m_e}} . \text{ The contours show growth rate of the}$$

fastest growing wavenumber. The number labelling the curves is $\gamma_{\max}/\Omega_i$. Ω_i is the ion plasma frequency.

Note that a correction has here been made for a small error in the original scale on the vertical axis.

Fig. 5 Snapshot of a moving DL with negative potential dip followed by an expanding plasma.

(a) potential

(b) electron phase space. V_{DL} is much smaller than the thermal velocities of both streaming and reflected electrons.

(c) ion phase space, showing the expanding ions coming in from the left.

It should be noted that everything is in the laboratory frame of reference.

Fig. 6 For a time shorter than that for ion dynamics a local region of low ion density ($n_i \approx 0$) is crossed by electrons with a water bag distribution (n_{eWB}) moving in a self consistent potential $\phi(x)$. The potential minimum at x_s acts as a Laval nozzle for the fluid electrons. The electrons reach their sound speed at x_s beyond which their density drops as they are accelerated by the increasing potential. Thermal electrons (n_{eT}) reflected by the potential maintain charge neutrality to the right.

Fig.1

Fig. 2

Fig. 3

Fig.4

Fig.5

Fig.6

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DYNAMICAL ASPECTS OF ELECTROSTATIC DOUBLE LAYERS

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Electrostatic double layers have been proposed as an acceleration mechanism in solar flares and other astrophysical objects. They have been extensively studied in the laboratory and by means of computer simulations. The theory of steady state double layers implies several existence criteria, in particular the Bohm criteria, restricting the conditions under which double layers may form. In the present paper several already published theoretical models of different types of double layers are discussed. It is shown that the existence conditions often imply current driven instabilities in the ambient plasma, at least for strong double layers, and it is argued that such conditions must be used with care when applied to real plasmas. Laboratory double layers, and by implication those arising in astrophysical plasmas, often produce instabilities in the surrounding plasma and are generally time dependent structures. Naturally occurring double layers should therefore be far more common than the restrictions deduced from idealised time independent models would imply. In particular it is necessary to understand more fully the time dependent behaviour of double layers. In the present paper the dynamics of weak double layers is discussed. Also a model for a moving strong double layer, where an associated potential minimum plays a significant role, is presented.

Key words: Double layers, Particle acceleration, Existence criteria, Bohm criteria, Electrostatic instabilities, Dynamics, Propagation, Potential minima, Electron Dynamics, Wall-sheaths