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LOCAL SUPERSYMMETRY AND
DIRAC PARTICLE PROPAGATOR
AS A PATH INTEGRAL

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Annotation

Proceeding from reparametrization - invariant and locally supersymmetric action for the Dirac particle, the path integral representation for the Green function, satisfying the first order equation in the external Maxwell field is obtained.

I. Introduction

The supersymmetric extension of the Poincare group [1] has greatly influenced modern views on possible structure of space-time and the nature of motion and interaction of elementary particles. In particular we have gained greater knowledge about classical and first quantized particles. The local generalization of supersymmetry is especially useful in gravitation and in attempts to obtain the propagator in string field theory, where second-quantized action is in many cases unknown. Therefore our attention is attracted to the papers [2] where the local reparametrization and supersymmetric invariant action of the Dirac particle was obtained and first-quantized. This action is actually the action of one-dimensional supergravity.

The aim of this paper (developing [9]) is to obtain the path integral representation for the Dirac particle propagator satisfying the first order equation in the external Maxwell field. The representation for free particle was obtained in [4]. Our representation is based on local supersymmetry, that is the difference of our approach as compared with the earlier papers on the subject. The first representation for the Dirac particle propagator as a sum over paths was obtained in [5]. The papers [6] were devoted to constructing the representation of the Dirac particle propagator in the external Maxwell field, starting with the second order equation, ^{and} Grassman variables were first used to describe a spinning particle.

(See also [7].) The path representation of the Dirac propagator based on integration over Grassman variables was proposed in [8]. In the first and the last papers of ref. [8] the global supersymmetry of the action for the spinning particle was pointed out.

Now let us turn to the content of the present paper. Section 2 is devoted to the locally supersymmetric action for free Dirac particle and its generalization to the case of the particle in the external Maxwell field. The covariant path integral and boundary conditions are discussed in Section 3. The representation for the Dirac particle propagator in the external Maxwell field is obtained and substantiated in Section 4. Section 5 contains a brief discussion. Some useful formulae are given in the Appendix.

We use in the present paper the following notations:

$\eta_{\mu\nu} = \text{diag}(-++\dots+)$; $\mu, \nu = 0, 1, \dots, D-1$
 the dot stands for the differentiation with respect to the parameter of a path: $\dot{f} \equiv \frac{df}{dt}$

Then: $\psi_5 \equiv \psi_{D+1}$, $\gamma_5 \equiv \gamma_{D+1}$, $\gamma_5^2 = 1$
 $\delta/\delta\alpha$ means right variation derivative with respect to Grassman variables

Dirac matrices Γ_μ : $[\Gamma_\mu, \Gamma_\nu]_+ = 2\eta_{\mu\nu}$.

Then: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

In Euclidian formulation:

$e(t) \rightarrow -ie_E(t)$, $\chi(t) \rightarrow -i\chi_E(t)$
 $\eta_{\mu\nu} \rightarrow \delta_{\mu\nu}$, $iS_{\text{Mink}} \rightarrow -S_E$;

$$A \overset{\leftrightarrow}{\frac{\partial}{\partial x_\mu}} B = A \frac{\partial B}{\partial x_\mu} - \frac{\partial A}{\partial x_\mu} B; \quad \partial_\mu \equiv \frac{\partial}{\partial x_\mu}$$

$$\square = \delta_{\mu\nu} \partial_\mu \partial_\nu, \quad \not{\partial} = \gamma_\mu \partial_\mu$$

2. Action for the spin - $\frac{1}{2}$ particle and local supersymmetry

Let us consider the first-quantized action for the massless spinning particle. The action invariant under local symmetries is the action of one dimensional supergravity [2] (ψ^μ and χ are Grassman variables):

$$S_0 = \int_0^1 dt \frac{1}{2} \eta_{\mu\nu} [\dot{e}^{-1} \dot{x}^\mu \dot{x}^\nu - i \dot{\psi}^\mu \dot{\psi}^\nu - i \dot{e}^{-1} \dot{\chi} \dot{\chi} \psi^\nu] \quad (I)$$

The action (I) is invariant under local reparametrization (parameter ε) and under local supersymmetry (Grassman parameter α):

$$\begin{aligned} \delta_\varepsilon x^\mu &= \varepsilon(t) \dot{x}^\mu & \delta_\varepsilon e &= \frac{d}{dt} [\varepsilon(t) e] \\ \delta_\varepsilon \psi^\mu &= \varepsilon(t) \dot{\psi}^\mu & \delta_\varepsilon \chi &= \frac{d}{dt} [\varepsilon(t) \chi] \end{aligned} \quad (2a)$$

$$\begin{aligned} \delta_\alpha x^\mu &= i \alpha(t) \psi^\mu & \delta_\alpha e &= i \alpha(t) \chi \\ \delta_\alpha \psi^\mu &= \alpha(t) \left[\dot{x}^\mu - \frac{i}{2} \chi \dot{\psi}^\mu \right] e^{-1}, & \delta_\alpha \chi &= 2 \dot{\alpha}(t) \end{aligned} \quad (2b)$$

Under (2b) the Lagrangian is changed on a total derivative:

$$\delta_\alpha S_0 = \int_0^1 dt \frac{d}{dt} \left(\frac{i}{2e} \alpha(t) \dot{x}^\mu \psi_\mu \right) \quad (3)$$

In the case of a massive spinning particle, one has to add some terms, depending upon new Grassman field $\psi_5(t)$ and

containing mass m as a parameter:

$$S_m = S_0 - \int_0^1 dt \frac{1}{2} [em^2 + i\psi_5 \dot{\psi}_5 + im\chi\psi_5]$$

$$\delta_\varepsilon \psi_5 = \varepsilon(t) \dot{\psi}_5, \quad \delta_\alpha \psi_5 = m\alpha(t) + \frac{i}{me} \alpha(t) \psi_5 \quad (4)$$

$$\times (\dot{\psi}_5 - \frac{1}{2} m\chi), \quad \delta_\alpha S_m = \delta_\alpha S_0 - \int_0^1 dt \frac{d}{dt} (im\alpha(t) \psi_5)$$

The system (I) or (4) can be quantized in the gauge $e=1, \chi=0$ imposing commutation relation on canonical variables [2]. The action (I) and the equation of motion in the chosen gauge have the following form:

$$S_0 \Big|_{\substack{\text{gauge} \\ \text{fixed}}} = \int_0^1 dt \frac{1}{2} [\dot{\chi}^\mu \dot{\chi}_\mu - i\dot{\psi}_\mu \dot{\psi}^\mu] \quad (5a)$$

$$\ddot{\chi}^\mu = 0, \quad \dot{\psi}^\mu = 0, \quad \dot{\chi}^\mu \dot{\chi}_\mu = 0, \quad \psi^\mu \dot{\chi}_\mu = 0 \quad (5b)$$

The solution of the equation of motion (5b) for ψ^μ is: $\psi^\mu = \gamma^\mu = \text{const}$, where γ^μ is some initial condition.

Because of the second-class constraints:

$$\pi_\mu + \frac{i}{2} \psi_\mu = 0, \quad \pi_\mu = S_0 \Big|_{\substack{\text{gauge} \\ \text{fixed}}} \overleftarrow{\frac{\delta}{\delta \psi^\mu}} \quad (6)$$

one should use Dirac brackets to change them by operator anticommutation relations: Dirac brackets for ψ^μ variables are:

$$\{\psi^\mu, \psi^\nu\}_D = i\eta^{\mu\nu} \quad (7)$$

Then, in accordance with the quantization rules $i\{A, B\}_D \rightarrow [A, B]_+$, objects γ^μ should satisfy the following relations:

$$[\gamma_\mu, \gamma_\nu]_+ = -\eta_{\mu\nu} \quad (8a)$$

The same can be written with the help of usual Dirac matrices:

$$\gamma_\mu = \frac{i}{\sqrt{2}} \Gamma_\mu, \text{ where } [\Gamma_\mu, \Gamma_\nu]_+ = 2\eta_{\mu\nu} \quad (8b)$$

The generator of Lorentz transformations can be obtained from (5a):

$$M_{\mu\nu} = x_\mu \dot{x}_\nu - x_\nu \dot{x}_\mu + \frac{i}{2} [\gamma_\mu, \gamma_\nu]_- \quad (9)$$

The expression (9) can be naturally represented as the sum of orbital and spin terms. Taking into account (8) the spin term in (9) corresponds to the spin 1/2. In the massive case the solutions of the equation of motion for ψ are:

$$\psi_\mu = i\gamma_5 \gamma_\mu, \quad \psi_5 = \gamma_5 \frac{i}{\sqrt{2}} \quad (10)$$

where γ_μ satisfy (8) as in the massless case. Although free particle is considered in this paper the main point of interest for us is the particle in the external Maxwell field. The Lagrangian of interaction of the spinning particle with the external Maxwell field in the gauge $e=1$, $\chi=0$ was proposed in [2],

$$L_A = g [\dot{x}^\mu A_\mu(x(t)) + \frac{i}{2} F_{\mu\nu}(x(t)) \psi^\mu \psi^\nu] \quad (II)$$

where g is the electric charge. The Lagrangian (II) can be trivially generalized to the case of local supersymmetry:

$$S_{int} = \int_0^1 g [\dot{x}^\mu A_\mu + \frac{i}{2} e F_{\mu\nu} \psi^\mu \psi^\nu] dt$$

$$\delta_\alpha S_{int} = \int_0^1 dt \frac{d}{dt} [ig\alpha(t) \psi^\mu A_\mu(x(t))] \quad (I2)$$

The computations below will be carried out in the Euclidian formulation. After changing: $e \rightarrow -ie_E$, $\chi \rightarrow -i\chi_E$, $\eta_{\mu\nu} \rightarrow \delta_{\mu\nu}$ in D space-time dimension one has:

$$S_o^E = -iS_o = \int_0^1 dt \frac{1}{2} \left[e_E^{-1} \dot{x}_\mu^2 - \psi_\mu \dot{\psi}_\mu - e_E^{-1} \chi_E \dot{x}_\mu \psi_\mu \right] \quad (I3a)$$

$$S_{int}^E = -iS_{int} = - \int_0^1 dt ig \left[\dot{x}_\mu A_\mu + \frac{e_E}{2} F_{\mu\nu} \psi_\mu \psi_\nu \right] \quad (I3b)$$

The expressions (I3) will be used when quantizing the system by means of Euclidian path integral, then the index "E" will be omitted.

3. The path integral and boundary conditions

The expression for propagator by means of covariant path integral is [3]:

$$G(x_1, x_0; m^2) = \int_{X(0)=X_0}^{X(1)=X_1} \mathcal{D}x \exp \left[- \int_0^1 dt \frac{1}{2} \left(\dot{x}_\mu^2 + em^2 \right) \right] \quad (I4)$$

The boundary conditions for $X^\mu(t)$ in (I4) are fixed values of the coordinates on the ends of the path. The propagator of the spinning particle is defined in a similar way:

$$\hat{G}(x_1, x_0) = \int \mathcal{D}e \mathcal{D}\chi \mathcal{D}\psi \mathcal{D}x \exp [-S_o] \quad (I5)$$

$$\begin{matrix} X(0)=X_0 \\ X(1)=X_1 \end{matrix} ; \quad \psi^\mu(t) = \gamma^\mu + \phi^\mu(t)$$

where S_o is defined by (I3a), and $\phi^\mu(t)$ is the quantum part of the Grassman field $\psi^\mu(t)$, satisfying boundary conditions appropriate for one-dimensional fermions.

Let us define the covariant integration measure in (I5)

starting from covariant norm (following [3]):

$$\int De D\chi \exp \left[-\frac{1}{2} (\|\delta e\|^2 + \|\delta\chi\|^2) \right] = 1 \quad (I6)$$

where norms are reparametrisation invariants:

$$\langle \delta_1 e, \delta_2 e \rangle = \int dt e^{-1} (\delta_1 e) (\delta_2 e), \quad \|\delta e\|^2 = \langle \delta e, \delta e \rangle \quad (I7a)$$

$$\langle \delta_1 \chi, \delta_2 \chi \rangle = \int dt e^{-1} (\delta_1 \chi) (\delta_2 \chi), \quad \|\delta\chi\|^2 = \langle \delta\chi, \delta\bar{\chi} \rangle \quad (I7b)$$

The square-norms for variations (2) are computed by putting (2) into (I7), after that the obtained expressions are continued into Euclidian space: $i \|\delta e\|_{Mink}^2 \rightarrow -\|\delta e\|_E^2$

The gravitino χ has in one dimension the only component, accordingly to define the norm one should introduce the conjugated variable $\bar{\chi}$ (see [4]). The integration on e and χ will be reduced to the integral over groups (2) and also over parameters which are analogous to moduli and supermoduli in string theory. To obtain the result let us consider the total variations (taking into account that the Jacobian factors:

$$J\left(\frac{e, \chi}{\varepsilon, \alpha}\right) \sim J\left(\frac{e}{\varepsilon}\right) J\left(\frac{\chi}{\alpha}\right):$$

$$\delta e = \delta_\varepsilon e + \tilde{\delta} e \quad (I8a)$$

$$\delta\chi = \delta_\alpha \chi + \tilde{\delta}\chi \quad (I8b)$$

where $\tilde{\delta}$ means here some arbitrary variations. We require that they should be orthogonal to the transformations (2) in the sense of (I7). Then, taking into account boundary conditions $\varepsilon(0) = \varepsilon(1) = 0$, from (I8a), (I7a) and (2a) one obtains:

$$\langle \delta_\varepsilon e, \tilde{\delta} e \rangle = 0 = - \int dt \varepsilon(t) \frac{d}{dt} [e^{-1} \tilde{\delta} e] \quad (I9)$$

Then in the gauge:

$$e(t) = T = \text{const} = \int_0^1 e(t) dt \quad (20)$$

we can write:

$$\tilde{\delta} e = \delta T \quad (20')$$

Considering the orthogonality of $\delta_\alpha \chi$ and $\tilde{\delta} \chi$ by analogy we obtain:

$$\begin{aligned} \langle \delta_\alpha \chi, \tilde{\delta} \chi \rangle &= 0 = \int_0^1 dt e^{-1} 2\dot{\alpha} \tilde{\delta} \chi = \\ &= 2\alpha e^{-1} \tilde{\delta} \chi \Big|_0^1 - 2 \int_0^1 dt \alpha(t) \frac{d}{dt} (e^{-1} \tilde{\delta} \chi) \end{aligned} \quad (21)$$

Let us discuss the boundary conditions, which $\alpha(t)$ must satisfy. The invariance of action under local supersymmetry (3) requires:

$$\alpha(0) = 0, \quad \alpha(1) = 0 \quad (22)$$

It is boundary condition (22) that we shall use (in disagreement with [4]). Then the boundary terms in (21) disappear and one obtains:

$$\frac{d}{dt} (e^{-1} \tilde{\delta} \chi) = 0 \quad (23)$$

and in the gauge (20):

$$\tilde{\delta} \chi = \delta \chi, \quad \chi = \int_0^1 dt \chi(t) \quad (24)$$

Finally we choose the following gauge:

$$e(t) = T = \text{const}, \quad \chi(t) = \chi = \text{const} \quad (25)$$

The gauge (25) will be used below.

Now let us compute the integration measure $De D\chi$ by conventional way. Starting from (2), (16), (17), (18) one has:

$$\begin{aligned} 1 &= \int De e^{-\frac{1}{2} \|\delta e\|^2} \int D\chi e^{-\frac{1}{2} \|\delta\chi\|^2} = \\ &= J\left(\frac{e}{\varepsilon, T}\right) \int D(\delta\varepsilon) d(\delta T) e^{-\frac{1}{2} \|\delta e\|^2} \cdot J\left(\frac{\chi}{\alpha, \bar{\chi}}\right) \int D(\delta\alpha) d(\delta\bar{\chi}) \times \\ &\quad \times \exp\left(-\frac{1}{2} \|\delta\chi\|^2\right) \end{aligned} \quad (26)$$

Taking into consideration, that:

$$\begin{aligned} \|\delta e\|^2 &= \int dt e^{-1} \left[\frac{d}{dt}(e\varepsilon) \right]^2 + \int dt e^{-1} (\delta T)^2 \\ \|\delta\chi\|^2 &= \int dt e^{-1} (2\dot{\alpha})(2\dot{\bar{\alpha}}) + \int dt e^{-1} \delta\chi \delta\bar{\chi} \end{aligned} \quad (27)$$

from (26), (27) we obtain:

$$\int De D\chi \sim \int D\varepsilon D\alpha dT d\bar{\chi} J\left(\frac{e}{\varepsilon, T}\right) J\left(\frac{\chi}{\alpha, \bar{\chi}}\right) \quad (28)$$

where using the normalization:

$$\begin{aligned} \|\delta e\|^2 &= \int dt (\delta\varepsilon)^2 e^3, \quad \int D(\delta\varepsilon) \exp\left(-\frac{1}{2} \|\delta e\|^2\right) = 1 \\ \|\delta\alpha\|^2 &= \int dt e \delta\alpha \delta\bar{\alpha}, \quad \int D(\delta\alpha) \exp\left(-\frac{1}{2} \|\delta\alpha\|^2\right) = 1 \end{aligned} \quad (29)$$

from (26), (27) if $e(t) = T$ it follows:

$$J\left(\frac{e}{\varepsilon, T}\right) \sim \left(T^{-1} \det -\frac{1}{T^2} \frac{d^2}{dt^2}\right)^{1/2} \quad (30a)$$

$$J\left(\frac{\chi}{\alpha, \bar{\chi}}\right) \sim T \left(\det -\frac{1}{T^2} \frac{d^2}{dt^2}\right)^{-1} \quad (30b)$$

The determinants (30) are calculated by means of regularisation with zero boundary conditions on $\varepsilon(t)$ and $\alpha(t)$.

In this way:

$$\left(\det -\frac{1}{T^2} \frac{d^2}{dt^2}\right) \sim T$$

So, omitting trivial integrations $\int D\varepsilon \int D\alpha$, one obtains:

$$\int D\varepsilon D\chi D\psi D\chi \exp(-S) \sim \int dT d\chi D\psi D\chi \exp(-S|_{\varepsilon=T, \chi=\chi}) \quad (31)$$

Now we can compute the path integrals (I4) and (I5). Substituting:

$$x^\mu(t) = x_0^\mu + (x_1^\mu - x_0^\mu)t + y^\mu(t) \quad (32a)$$

$$\psi^\mu(t) = \delta^\mu + \phi^\mu(t) \quad (32b)$$

into (I5), where $y^\mu(t)$ satisfy zero boundary conditions and $\phi^\mu(t)$ satisfy antiperiodic boundary conditions, we have:

$$G(x_1, x_0; m^2) \sim \int_0^\infty dT e^{-\frac{1}{2} T m^2} e^{-\frac{1}{2} \frac{(x_1 - x_0)^2}{T}} \int_{y(0)=y(1)=0} D y e^{-\int_0^T \frac{\dot{y}^2}{2T} dt} \sim \int_0^\infty \frac{dT}{T^{D/2}} e^{-\frac{1}{2} (T m^2 + \frac{(x_1 - x_0)^2}{T})} = 2 \int \frac{d^D p}{(2\pi)^{D/2}} \frac{e^{ip(x_1 - x_0)}}{p^2 + m^2} \quad (33)$$

For (I5) one obtains:

$$\begin{aligned} \hat{G}(x_1, x_0) &\sim \int dT d\chi D\psi D\chi e^{-\frac{1}{2} \int dt \left\{ \frac{\dot{\chi}^2}{T} - \psi_\mu \dot{\chi}_\mu - \frac{\chi}{T} \dot{\chi}_\mu \psi_\mu \right\}} = \\ &= \int \frac{dT}{2T} \delta_\lambda(x_1 - x_0)_\lambda e^{-\frac{1}{2} \frac{(x_1 - x_0)^2}{T}} \int D y D \phi e^{-\frac{1}{2} \int dt \left(\frac{\dot{y}^2}{T} - \phi_\mu \dot{\phi}_\mu \right)} \sim \\ &\sim \frac{1}{2} \int \frac{dT}{T^{D/2}} \frac{\delta_\mu(x_1 - x_0)_\mu}{T} e^{-\frac{(x_1 - x_0)^2}{2T}} = \int \frac{d^D p}{(2\pi)^{D/2}} e^{ip(x_1 - x_0)} \frac{-i \delta_\mu p_\mu}{p^2} \quad (34) \end{aligned}$$

When computing (I4), (I5) we kept in mind, that:

$$\int_{y(1)=0}^{y(0)=0} D y e^{-\int_0^T \frac{\dot{y}^2}{2T} dt} = \left(\det -\frac{1}{T^2} \frac{d^2}{dt^2} \right)^{-D/2} \sim T^{-D/2} \quad (35a)$$

Later on the path integral (15) is meant as a Weyl symbol of operator in the sense of [7].

$$\int \mathcal{D}\phi \, e^{\int \frac{1}{2} \bar{\phi}_\mu \dot{\phi}_\mu dt} = \left(\det -\frac{1}{T^2} \frac{d^2}{dt^2} \right)^{D/2} = \text{const} \quad (35b)$$

The expressions (35) can be obtained proceeding from the covariant definition of integration measure:

$$\begin{aligned} \int \mathcal{D}x \exp \left[-\frac{1}{2} \int dt \, e (\delta x_\mu)^2 \right] &= 1 \\ \int \mathcal{D}\psi \exp \left[-\frac{1}{2} \int dt \, e \delta \bar{\psi}_\mu \delta \psi_\mu \right] &= 1 \end{aligned} \quad (36)$$

and also by means of discrete representation of path integral (see Appendix 2).

The representation for the propagator of the massive Dirac particle can be obtained by analogy. Then we must begin from (4) and add in (15) the integration over anticommuting variable $\psi_5(t)$, satisfying antiperiodic boundary conditions:

$$\psi_5(t) = \frac{i\gamma_5}{\sqrt{2}} + \phi_5(t) \quad (37)$$

We must also change (32b) for:

$$\psi^\mu(t) = i\gamma_5 \gamma^\mu + \phi^\mu(t) \quad (38)$$

Then, as in (34) one obtains:

$$\int \mathcal{D}e \mathcal{D}q \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S_m) \sim \int \frac{d^D p}{(2\pi)^{D/2}} e^{ip(x_1 - x_0)} \times \frac{\gamma_5}{\sqrt{2}} \frac{m + i\gamma_\mu p_\mu}{p^2 + m^2} \quad (39)$$

It is easy to see that changing formula (38) for: $\psi_\mu(t) = \gamma_\mu + \phi_\mu(t)$ one can get for the massive Dirac particle propagator instead of (39) the following expression:

$$\hat{G}(x_1, x_0; m) = \int \frac{d^D p}{(2\pi)^{D/2}} e^{ip(x_1 - x_0)} \frac{1}{\sqrt{2}} \frac{m\gamma_5 + i\gamma_\mu p_\mu}{p^2 + m^2} \quad (40)$$

The formula (39) is more convenient for computations, because multiplying (39) by γ_5 factor we obtain the conventional Dirac particle propagator. However the expression (40) in the limit $M \rightarrow 0$ coincides with (34).

4. The propagator in the external Maxwell field

The representations (I4) and (I5) can be generalized to the case of propagators of spinless and spinning particles in the external Maxwell field:

$$G(1,0/A) = \int D\epsilon D\chi \exp \left[- \int_0^1 dt \left(\frac{\dot{x}_\mu^2}{2e} + \frac{1}{2} em^2 - ig \dot{x}_\mu A_\mu \right) \right]_{(41a)}$$

$$\hat{G}(1,0/A) = \int D\epsilon D\chi D\psi D\bar{\psi} \exp \left[-S_0 + ig \int_0^1 dt (\dot{x}_\mu A_\mu + \frac{e}{2} F_{\mu\nu} \psi_\mu \psi_\nu) \right]_{(41b)}$$

where S_0 is defined in (I3a). The boundary conditions in (4I) are those of (I4) and (I5) respectively. To prove the correctness of these representations one has to compute the variation derivatives of (4I) with respect to Maxwell field

$A_\mu(x)$:

$$j_\mu(1,0/x) = -\frac{i}{g} \frac{\delta G(1,0/A)}{\delta A_\mu(x)} \Big|_{A=0} = \left\langle \int_0^1 dt \dot{x}_\mu(\tau) \delta(x-x(\tau)) \right\rangle_{e,x} \quad (42a)$$

$$\hat{j}_\mu(1,0/x) = -\frac{i}{g} \frac{\delta \hat{G}(1,0/A)}{\delta A_\mu(x)} \Big|_{A=0} = \left\langle \int_0^1 dt \left\{ \dot{x}_\mu(\tau) + e(\tau) \psi_\mu(\tau) \psi_\nu(\tau) \frac{\partial}{\partial x^\nu} \right\} \delta(x-x(\tau)) \right\rangle_{e,x,\psi,\bar{\psi}} \quad (42b)$$

where averaging is made in the sense of (4I). Let us note

that we can immediately compute:

$$\frac{\partial}{\partial x^\mu} j_\mu(1,0/x) = \left\langle - \int_0^1 dt \frac{d}{dt} \delta(x-x(\tau)) \right\rangle_{e,x} =$$

$$= G(1, x) \delta(x, 0) - \delta(1, x) G(x, 0) \quad (43a)$$

$$\begin{aligned} \frac{\partial}{\partial x_\mu} \hat{J}_\mu(1, 0|x) &= \left\langle - \int_0^1 d\tau \frac{d}{d\tau} \delta(x - x(\tau)) \right\rangle_{e, \chi, x, \psi} = \\ &= \hat{G}(1, x) \delta(x, 0) - \delta(1, x) \hat{G}(x, 0) \end{aligned} \quad (43b)$$

Therefore the result of (42) must satisfy (43). In the case of (42a) we obtain

$$\hat{J}_\mu(1, 0|x) = - \frac{1}{2(2\pi)^{3/2}} G(1, x) \overleftrightarrow{\frac{\partial}{\partial x_\mu}} G(x, 0) \quad (44)$$

The computation of (42b) will be propounded in detail. Choosing the gauge (25) we integrate over Grassmann variable χ , then:

$$\begin{aligned} \hat{J}_\mu(1, 0|x) &\sim \left\langle \int_0^\infty \frac{dT}{2T} \int_0^1 dt \psi_\lambda(t) \dot{x}_\lambda(t) \int_0^1 d\tau \left\{ \dot{x}_\mu(\tau) + \right. \right. \\ &\left. \left. + T \psi_\mu(\tau) \psi_\nu(\tau) \frac{\partial}{\partial x_\nu} \right\} \delta(x - x(\tau)) \right\rangle_{\chi, \psi} \end{aligned} \quad (45)$$

where averaging with respect to $\frac{1}{2} X$ and ψ means:

$$\langle F \rangle_X = \int \mathcal{D}X e^{-\int dt \frac{1}{2T} F} F \quad (46a)$$

$$\langle \Psi \rangle_\psi = \int \mathcal{D}\psi e^{\int \frac{1}{2} \psi_\mu \dot{x}_\mu dt} \Psi \quad (46b)$$

with the corresponding boundary conditions on X and ψ (15).

Taking into account that (see Appendix I):

$$\langle \psi_\lambda(t) \rangle_\psi = \delta_\lambda \quad (47)$$

and in correspondence with antiperiodic boundary conditions we have:

$$\langle \phi_\lambda(t) \phi_\mu(\tau) \rangle_\psi = - \frac{1}{2} \varepsilon(t - \tau) \delta_{\lambda\mu}$$

$$\text{and } \langle \psi_\lambda(t) \psi_\mu(\tau) \psi_\nu(\tau) \rangle_\psi = \frac{1}{2} \varepsilon(\tau-t) (\delta_\nu \delta_{\lambda\mu} - \delta_\mu \delta_{\lambda\nu}) + \frac{1}{2} (\delta_\lambda \delta_\mu \delta_\nu - \delta_\nu \delta_\mu \delta_\lambda) \quad (48)$$

Substituting (47) and (48) into (45) and integrating on t , one finds

$$\hat{J}_\mu(1,0/x) = \frac{1}{2} \left\langle \int \frac{dT}{T} \int d\tau \left\{ \delta_\lambda (x_1 - x_0)_\lambda \dot{x}_\mu(\tau) + \frac{T}{2} [\delta_\lambda (2x^\lambda - x_1^\lambda - x_0^\lambda) (\delta_\nu \delta_{\lambda\mu} - \delta_\mu \delta_{\lambda\nu}) + (x_1 - x_0)_\lambda (\delta_\lambda \delta_\mu \delta_\nu - \delta_\nu \delta_\mu \delta_\lambda)] \frac{\partial}{\partial x_\nu} \right\} \delta(x - x(\tau)) \right\rangle_x \quad (49)$$

As it is shown in Appendix 2:

$$\langle \dot{x}_\mu(\tau) \delta(x - x(\tau)) \rangle_x = \frac{T}{2} \left(\frac{\partial}{\partial x_0^\mu} - \frac{\partial}{\partial x_1^\mu} \right) \langle \delta(x - x(\tau)) \rangle_x =$$

$$= \frac{T}{2} \left(\frac{\partial}{\partial x_0^\mu} - \frac{\partial}{\partial x_1^\mu} \right) \exp \left[-\frac{(x - x_0)^2}{2T\tau} - \frac{(x_1 - x)^2}{2T(1-\tau)} \right] (T\tau)^{-D/2} (T(1-\tau))^{-D/2} \frac{1}{(2\pi)^{D/2}} \quad (50)$$

Therefore, introducing new variables:

$$T\tau \equiv u, \quad T(1-\tau) \equiv v$$

we obtain (see Appendix 3):

$$\hat{J}_\mu(1,0/x) = \frac{1}{4(2\pi)^{D/2}} \int \frac{dT}{T} \int du \int dv \delta(u+v-T) \times \quad (51)$$

$$\times [2 \delta_\lambda \delta_\mu \delta_0(u+v) + \delta_\mu (u \square_0 + v \square_1 + 2)] J(u,v)$$

where

$$J(u,v) = (uv)^{-D/2} \exp \left[-\frac{(x-x_0)^2}{2u} - \frac{(x_1-x)^2}{2v} \right]$$

The term proportional $(u+v)$ in (51) leads to the correct

result:

$$\hat{J}_\mu(1,0/x) = \frac{1}{2(2\pi)^{D/2}} \delta_\lambda \delta_\mu \delta_0 G(1,x) G(x,0) =$$

$$= \frac{2}{(2\pi)^{D/2}} \hat{G}(1,x) \delta_\mu \hat{G}(x,0) \quad (52)$$

where $G(x_1, x_0)$ is defined by formula (33) when mass is equal to zero.

The sum of last three terms in (51) is equal to zero

(see Appendix 3). When computing this result the following representation was used:

$$\int_0^\infty du \int_0^\infty dv \delta(u+v-T) J(u,v) = \frac{1}{2\pi} \int_{-\infty}^\infty d\lambda e^{i\lambda T} \mathcal{H}(\beta_0, \lambda) \mathcal{H}(\beta_1, \lambda)$$

where function:

$$\mathcal{H}(\beta, \lambda) = \int_0^\infty \frac{du}{u^{3/2}} e^{-\frac{\beta^2}{2u} - i\lambda u}, \quad \begin{aligned} \beta_0 &= x - x_0 \\ \beta_1 &= x - x_1 \end{aligned}$$

is the solution of the equation:

$$(\square - 2i\lambda) \mathcal{H}(x, \lambda) = -2 (2\pi)^{1/2} \delta^D(x)$$

It is easy to see, that $\int_\mu$ in (52) satisfy (43b).

5. Discussion

1. The results of previous Section can be naturally generalized to the case of the massive Dirac particle. We should start with (4) and (I2) and then, using the covariant path integral representation and some formulae analogous to those of the Appendix, we obtain:

$$-\frac{\gamma_5}{\sqrt{2}} \frac{1}{4(2\pi)^{D/2}} (m + \Gamma_\lambda \partial_\lambda^1) \Gamma_\mu (m - \Gamma_\nu \partial_\nu^0) G(x, 0; m^2) G(x, 0; m^2)$$

This expression is the generalization of the result (52).

2. To get (34) we have used the antiperiodic boundary conditions for quantum component Φ_μ of ψ_μ field. However, to compute the determinant in (30b) we proceeded from zero boundary condition on $\alpha(t)$. It is easy to prove that an attempt to use symmetric boundary conditions with respect to quantum fermion field ϕ_μ : $\phi_\mu(0) = \phi_\mu(1)$ leads to an in-

consistency, because an arbitrary source η_μ should then satisfy the following condition:

$$\int_0^1 \eta_\mu(t) dt = 0$$

3. In the case of the Dirac particle propagator in external non Abelian gauge field the following generalization of the path integral (4I) appears naturally:

$$\int D\epsilon D\chi \exp\left[-\int_0^1 dt \frac{1}{2}\left(\frac{\dot{x}_\mu^2}{e} + e m^2\right)\right] P \exp\left[ig \int_0^1 dt \dot{x}_\mu(t) A_\mu(x(t))\right]$$

for scalar particle, and

$$\int D\epsilon D\chi D\psi D\bar{\psi} \exp[-S] P \exp\left[ig \int_0^1 dt (\dot{x}_\mu A_\mu + \frac{e}{2} F_{\mu\nu} \psi_\mu \psi_\nu)\right]$$

for Dirac particle, where $A_\mu = A_\mu^a \tau^a$, $F_{\mu\nu} = F_{\mu\nu}^a \tau^a$, τ^a is the generator of the non Abelian group.

4. We can use the representation (4Ib) to obtain the exact Green function of the Dirac equation (taking into account all radiative corrections). Then it is necessary to average (4Ib) with respect to all $A_\mu(x)$, using the following measure:

$$\int DA \exp\left[-\frac{1}{4} \int (F_{\mu\nu})^2 dx - \frac{1}{2\alpha_0} \int (\partial_\mu A_\mu) dx\right]$$

Then the effective action of particle appears, which has the form:

$$S_{\text{eff}}(e, \chi, x_\mu, \psi_\mu) = \frac{1}{2} \int j_\mu(x) D_{\mu\nu}(x, y) j_\nu(y) dx dy + S_0$$

where

$$j_\mu(x) = \frac{1}{2} g \int_0^1 dt (\dot{x}_\mu(t) + e(t) \psi_\mu(t) \psi_\nu(t) \frac{\partial}{\partial x_\nu}) \delta(x - x(t))$$

and $D_{\mu\nu}(x, y)$ is the propagator of free Maxwell field in the

α_0 gauge.

Appendix I

The integration on ψ_μ is propounded according to the following formulas:

$$\begin{aligned}\langle \psi_\lambda(t) \rangle_4 &= \int \mathcal{D}\psi e^{\frac{1}{2} \int dt \psi_\mu \dot{\psi}_\mu} \psi_\lambda(t) = \\ &= \int \mathcal{D}\phi e^{\frac{1}{2} \int dt \phi_\mu \dot{\phi}_\mu} (\delta_\lambda + \phi_\lambda(t)) = \\ &= \delta_\lambda \int \mathcal{D}\phi e^{\frac{1}{2} \int dt \phi_\mu \dot{\phi}_\mu} = \delta_\lambda \cdot \text{const}\end{aligned}$$

because the determinant, calculated with antiperiodic boundary conditions, is constant. This constant included in all expressions in the same way will be omitted everywhere. Then:

$$\begin{aligned}\langle \psi_\lambda(t) \psi_\mu(\tau) \psi_\nu(\tau) \rangle_4 &= \delta_{[\lambda} \delta_\mu \delta_{\nu]} + \langle \phi_\lambda(t) \phi_\mu(\tau) \rangle_4 \delta_\nu - \\ &\quad - \langle \phi_\lambda(t) \phi_\nu(\tau) \rangle_4 \delta_\mu\end{aligned}$$

where the square brackets mean the antisymmetrization with respect to all indices. It is necessary when proceeding to δ -matrices, because $\psi_\mu(t)$ are anticommuting objects and δ_μ are not always anticommuting

$$\begin{aligned}\langle \phi_\lambda(t) \phi_\rho(\tau) \rangle_4 &= \int \mathcal{D}\phi e^{\frac{1}{2} \int \phi_\mu \dot{\phi}_\mu} \phi_\lambda(t) \phi_\rho(\tau) = \\ &= -\frac{1}{2} \delta_{\lambda\rho} \varepsilon(t-\tau), \text{ where } \varepsilon(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}\end{aligned}$$

Moreover, using (8a), one can obtain:

$$\delta_{[\lambda} \delta_\mu \delta_{\nu]} = \frac{1}{2} (\delta_\lambda \delta_\mu \delta_\nu - \delta_\nu \delta_\mu \delta_\lambda)$$

The Green function of anticommuting fermion fields is chosen

proportional to antisymmetric function $\mathcal{E}(t)$ in accordance with boundary conditions (15).

Appendix 2

The integration on x_μ is propounded with the help of substitution (47):

$$\begin{aligned} \langle \dot{x}_\mu(\tau) \delta(x - x(\tau)) \rangle_x &= \int \mathcal{D}x e^{-\int \frac{\dot{x}^2}{2T} dt} \dot{x}_\mu(\tau) \delta(x - x(\tau)) = \\ &= \int \mathcal{D}x e^{-\int dt \frac{\dot{x}^2}{2T}} \frac{1}{2} \left(\frac{x - x_0}{\tau} + \frac{x_1 - x}{1 - \tau} \right)_\mu \delta(x - x(\tau)) = \\ &= \frac{1}{(2\pi)^{D/2}} (T\tau)^{-D/2} (T(1-\tau))^{-D/2} T \left(\frac{\partial}{\partial x_0} - \frac{\partial}{\partial x_1} \right)_\mu e^{-\frac{(x-x_0)^2}{2T\tau} - \frac{(x_1-x)^2}{2T(1-\tau)}} \end{aligned}$$

Let us derive this formula, considering the path integral as a limit of repeated (discrete) integral. The Green function of scalar particle (14) ($m^2 = 0$) has the following form:

$$G(x_1, x_0) = \lim_{N \rightarrow \infty} \int_0^T dT \int \prod_{i=1}^N d^D y_i \exp \left[- \sum_{i=1}^{N+1} \frac{(y_i - y_{i-1})^2}{2\epsilon} \right] \mathcal{E}^{(D/2)(N+1)}$$

$$\epsilon = \frac{T}{N+1}, \quad y_0 = x_0, \quad y_{N+1} = x_1, \quad d^D y_i = (2\pi)^{-D/2} dy_i^{(1)} \dots dy_i^{(D)}$$

Changing variables, we can write this expression in the following way:

$$\begin{aligned} G(x_1, x_0) &= \lim_{N \rightarrow \infty} \int_0^T dT \int \frac{\prod_{i=1}^N d^D y_i}{\mathcal{E}^{(D/2)(N+1)}} e^{-\frac{(x_1 - x_0)^2}{2\epsilon(N+1)} - \sum_{i=1}^N \frac{y_i^2}{2\epsilon i}} = \\ &= \lim_{N \rightarrow \infty} \int_0^T dT T^{-D/2} e^{\frac{(x_1 - x_0)^2}{2T}} = \int_0^T \frac{dT}{T^{D/2}} e^{-\frac{(x_1 - x_0)^2}{2T}} \end{aligned}$$

This formula is correct for any finite number N . For the derivative of Green function of scalar particle one has:

$$\begin{aligned}
j_\mu(1,0|x) &= \int dT \int_0^T d\tau \int_{x_0}^{\bar{x}_1} dy e^{-\int_0^T dt \frac{1}{2} \dot{y}^2(t)} \dot{x}_\mu(\tau) \delta(x - x(\tau)) = \\
&= \lim_{N \rightarrow \infty} \frac{1}{(2\pi)^{D/2}} \int_0^\infty dT \sum_{j=1}^N \varepsilon \left(\prod_{i \neq j}^N dy_i \frac{(y_{j+1} - x + x - y_{j-1})_\mu}{2\varepsilon} \right) e^{-\sum_{i=1}^{N+1} \frac{(y_i - y_{i-1})^2}{2\varepsilon}} = \\
&= \lim_{N \rightarrow \infty} \frac{1}{(2\pi)^{D/2}} \int_0^\infty dT \sum_{j=1}^N \varepsilon^{1 - \frac{D}{2}(N+1)} \left(\prod_{i \neq j}^N dy_i \frac{1}{2} \left\{ \left(\partial_{y_j}^\mu e^{-\sum_{i=j+1}^{N+1} \frac{(y_i - y_{i-1})^2}{2\varepsilon}} \right) e^{-\sum_{i=1}^j \frac{(y_i - y_{i-1})^2}{2\varepsilon}} - \right. \right. \\
&\quad \left. \left. - e^{-\sum_{i=j+1}^{N+1} \frac{(y_i - y_{i-1})^2}{2\varepsilon}} \left(\partial_{y_j}^\mu e^{-\sum_{i=1}^j \frac{(y_i - y_{i-1})^2}{2\varepsilon}} \right) \right\} \right) \Big|_{y_j=x} = \lim_{N \rightarrow \infty} \frac{1}{(2\pi)^{D/2}} \sum_{j=1}^N \int_0^\infty dT \varepsilon^{1-D/2} \times \\
&\quad \times (\partial_0 - \partial_1)_\mu (\sqrt{j} - \sqrt{N-j+1})^{-D} \exp \left[-\frac{(x-x_0)^2}{2\varepsilon j} - \frac{(x_1-x)^2}{2\varepsilon(N-j+1)} \right]
\end{aligned}$$

Taking the limit and defining $d\tau \equiv \frac{\varepsilon}{T}$ ($0 \leq \tau \leq 1$)

one has:

$$\begin{aligned}
j_\mu(1,0|x) &= \frac{1}{2(2\pi)^{D/2}} \int_0^\infty dT \int_0^1 d\tau (T\tau)^{-D/2} [T(1-\tau)]^{-D/2} \times \\
&\quad \times (\partial_0 - \partial_1)_\mu \exp \left[-\frac{(x-x_0)^2}{2T\tau} - \frac{(x_1-x)^2}{2T(1-\tau)} \right]
\end{aligned}$$

We can easily obtain (44) from the last expression. It is necessary to pay attention to the symmetrical definition of the derivative $\dot{x}_\mu(\tau)$ in discrete and continuous cases.

Appendix 3

Using (49), (50), we have:

$$\begin{aligned}
\hat{j}_\mu(1,0|x) &= \frac{1}{2} \int_0^\infty du \int_0^\infty dv \frac{1}{T} \left\{ \delta_\lambda \frac{1}{2} (\partial_0 - \partial_1)_\mu (u\partial_0 - v\partial_1)_\lambda + \right. \\
&\quad + \delta_\mu + \frac{1}{2} \frac{\partial}{\partial x_\nu} (\delta_\lambda \delta_\mu \delta_\nu - \delta_\nu \delta_\mu \delta_\lambda) (u\partial_0 - v\partial_1)_\lambda + \frac{1}{2} \frac{\partial}{\partial x_\nu} (\delta_\nu \delta_\lambda \mu - \\
&\quad \left. - \delta_\mu \delta_\lambda \nu) (u\partial_0 + v\partial_1)_\lambda \right\} \frac{\exp \left[-\frac{(x-x_0)^2}{2u} - \frac{(x_1-x)^2}{2v} \right]}{(uv)^{D/2}}
\end{aligned}$$

Hence it follows:

$$\begin{aligned}
 \hat{J}_\mu(1,0|x) &= \frac{1}{8} \int \frac{du dv}{T} \left\{ \delta_\lambda (\partial_0 - \partial_1)_\mu (\partial_0 - \partial_1)_\lambda (u+v) + \right. \\
 &+ \delta_\lambda (\partial_0 - \partial_1)_\mu (\partial_0 + \partial_1)_\lambda (u-v) + 4\delta_\mu - (\partial_0 + \partial_1)_\nu (\delta_\lambda \delta_\mu \delta_\nu - \\
 &- \delta_\nu \delta_\mu \delta_\lambda) (\partial_0 - \partial_1)_\lambda (u+v) - (\partial_0 + \partial_1)_\nu (\delta_\nu \delta_\lambda \delta_\mu - \delta_\mu \delta_\lambda \delta_\nu) \times \\
 &\times (\partial_0 + \partial_1)_\lambda (u+v) - (\partial_0 + \partial_1)_\nu (\delta_\nu \delta_\lambda \delta_\mu - \delta_\mu \delta_\lambda \delta_\nu) (\partial_0 - \partial_1)_\lambda (u-v) \left. \right\} \times \\
 &\times \frac{\exp \left[-\frac{(x-x_0)^2}{2u} - \frac{(x_1-x)^2}{2v} \right]}{(uv)^{3/2}}
 \end{aligned}$$

It follows from the formula (51) directly.

Appendix 4

Let us consider three last terms in (51):

$$\begin{aligned}
 &\int_0^\infty \frac{dT}{T} \int_0^\infty du \int_0^\infty dv \delta(u+v-T) (u \square_0 + v \square_1 + 2) J(u, v) = \\
 &= \frac{1}{2\pi} \int_0^\infty \frac{dT}{T} \int_{-\infty}^\infty d\lambda e^{i\lambda T} \left[-\left(\frac{\partial}{\partial i\lambda} \square_0 Y(\beta_0, \lambda) \right) Y(\beta_1, \lambda) - Y(\beta_0, \lambda) \times \right. \\
 &\times \left(\frac{\partial}{\partial i\lambda} \square_1 Y(\beta_1, \lambda) \right) + 2 Y(\beta_0, \lambda) Y(\beta_1, \lambda) = -\frac{1}{2\pi} \int_0^\infty \frac{dT}{T} d\lambda e^{i\lambda T} \times \\
 &\times \frac{\partial}{\partial i\lambda} (2i\lambda Y(\beta_0, \lambda) Y(\beta_1, \lambda)) = 2 \int_0^\infty dT \frac{\partial}{\partial T} \int du dv \times \\
 &\times \delta(u+v-T) (uv)^{-3/2} \exp \left[-\frac{(x-x_0)^2}{2u} - \frac{(x_1-x)^2}{2v} \right] = 0
 \end{aligned}$$

which is necessary for obtaining (52) from (51).

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References

- 1 Yu.A.Gelfand, E.P.Likhtman JETP Lett. 13 (1971) 323.
D.V.Volkov, V.P.Akulov JETP Lett. 16 (1972) 438.
J.Wess, B.Zumino Phys. Lett. 49B (1974) 52.
- 2 L.Brink, P.Di Vecchia, P.Howe, S.Deser, B.Zumino Phys.
Lett. 64B (1976) 435.
L.Brink, P.Di Vecchia, P.Howe Nucl. Phys. B118 (1977) 76.
- 3 A.Cohen, G.Moore, P.Nelson, J.Polchinski Nucl. Phys.
B267 (1986) 143.
- 4 V.I.S.Dotsenko Nucl. Phys. B265 [FSI9] (1987) 45
- 5 W.Tobocman Nuovo Cim. 3 (1956) 1213.
G.V.Riazancev Soviet Physics JETP 6 (1958) 1107.
- 6 E.S.Fradkin Nucl. Phys. 76 (1966) 588, I.A.Batalin, E.S.Fradkin
Teor. i Mat. Fiz. 5 (1970) 190
- 7 F.A.Berezin, M.S.Marinov JETP Lett. 21 (1975) 320.
Ann. of Phys. 104 (1977) 336.
- 8 P.Di Vecchia, P.Ravndal Phys. Lett. 73A (1979) 391
F.Bordi, R.Casalbuoni Phys. Lett. 93B (1980) 308.
M.Kennesaux, G.Teitelboim Ann. of Phys. 143 (1982) 127.
- 9 A.V.Marshakov, V.Ya.Fainberg JETP Lett., in press.
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