CERN-TH.4830/87

WILSON FLUX BREAKING AND COSET SPACE DIMENSIONAL REDUCTION

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Abstract

Higher dimensional gauge theories lead, after dimensional reduction on coset spaces, to four dimensional gauge theories usually with the natural emergence of a Higgs sector which is completely determined. However the Higgs fields never appear in the adjoint representation which in many G.U.T.'s could lead to a successful spontaneous symmetry breaking towards the low energy gauge group. As an alternative we suggest that the breaking of the four dimensional G.U.T.'s obtained from C.S.D.R. could be provided by the Wilson flux breaking and we discuss some semirealistic examples. We also speculate on the possibility that the breaking of the electroweak sector has dynamical origin.

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In recent years the idea that the desirable unification of all interactions might be realized in more than four space-time dimensions has become very popular. In order to circumvent the non observability of the extra dimensions it is assumed that they belong to a spacelike compact manifold B with very small radius. However the most ambitious attempts based on the old Kaluza-Klein program⁽¹⁾ are not able to reproduce the observed low energy chiral world, while the assumed space-time configurations cannot be solutions of the higher dimensional theory. In principle a way out is to introduce matter fields such as Yang-Mills in the higher dimensional theory⁽²⁾, a solution which is even better motivated in superstring theories⁽³⁾. Nevertheless it is very important to examine to which extent the full theory really can approach the standard model in low energies. Fortunately there exists an attractive scheme, which permits the study of various features of the effective four-dimensional theory the Coset-Space-Dimensional-Reduction (C.S.D.R.)⁽⁴⁻⁸⁾. In this scheme some gravitational solution, is assumed which is carried over into a pure Yang-Mills theory with gauge group G which is coupled to fermions. Then one tries to find a suitable combination of Yang-Mills and fermion sectors in higher dimensions which would reproduce in four dimensions the standard model. Eventually one has to generate stable, compactifying solutions of the full Einstein Yang-Mills equations compatible with the assumed manifold⁽⁷⁾.

As one can easily imagine that the model building in the C.S.D.R. scheme is a very restricted game, and it is highly non trivial to find the right gauge group and fermion multiplets in higher dimensions that could lead to a reasonable four dimensional effective theory. A lot of work has been done in this direction, no-go theorems have been established and despite the important progress which has been achieved the attempts so far presented are still far from providing a model in high dimensions that reproduces the standard model in four dimensions. In the present paper we would like to show that the use of topological and dynamical symmetry breaking mechanisms in addition to the geometrical one can lead to an almost realistic low energy theory.

Let us first recall some features of the C.S.D.R. scheme. One introduces a pure gauge field theory based on a gauge group G which is defined on a space time manifold M which is assumed to be

$$M = M_4 \times B \quad (1)$$

where M_4 is a four-dimensional Minkowski space-time and $B=S/R$ is a compact coset space and the metric block diagonal. The gauge fields $A_{\alpha=1,\dots,D}$ of the gauge group G are considered symmetric in the sense that any symmetry transformation in S acting on S/R can be compensated by a gauge transformation in G . Once the coset space $B=S/R$ is chosen, one can find the dependence of $A_{\mu=1,\dots,4}$ on the coset space coordinates and by integrating the Yang-Mills Lagrangian over them obtains a Yang-Mills-Higgs theory in four dimensions. Some features of the four-dimensional theory are: (i) The gauge group, which we denote by H , is the centralizer in G of a subgroup R_G (isomorphic to R) of G . (ii) The gauge fields, which become scalars in four dimensions, transform under R as a vector as specified by the embedding:

$$\text{ad} S = \text{ad} R + \mathbf{u} \quad (2)$$

and $\mathbf{u} = \sum s_k$, where each s_k is an irreducible representation of R . Then the multiplets under H of the genuine Higgs fields can be found by decomposing the adjoint representation of G under $R_G \times H$:

$$\text{ad} G = \sum (r_k, h_k) \quad (3)$$

where r_k is some irreducible representation of R_G and h_k is some irreducible representation of H . For each pair of (r_k, s_j) , where r_k and s_j are identical irreps, there is a Higgs multiplet h_k in the four dimensional theory. The Higgs potential can also be determined but in the models we are going to discuss here there are no Higgs fields surviving in four dimensions as one can check by applying eqs (2),(3) in these models. In general Higgs fields survive in four dimensions and can break further the gauge group H . However usually the symmetry breaking does not drive to phenomenologically interesting subgroups of H . In particular given the rules in eqs. (2),(3) for the surviving Higgs it is not

clear how one can obtain in four dimensions Higgs fields belonging in the adjoint representations of H . As is well known the adjoint representations usually lead to successful spontaneous symmetry breakings in many G.U.T's. On the other hand the Higgs that usually survive in four dimensions from C.S.D.R. have the right quantum numbers to break the electroweak group which in turn means that if this breaking takes place the radius of the compact space is related to the Fermi rather than the Planck scale. Since we do not find very attractive this possibility we prefer to examine here models in which Higgs of geometrical origin do not survive at all in four dimensions. Therefore the only breaking related to geometry here will be the one that breaks G to $H=C_R(G)$ by embedding R in G .

We also recall that in superstring theories a topological mechanism has been used that mimics the spontaneous breakdown of a theory due to Higgs in the adjoint representation. This mechanism known as "flux breaking"⁽⁹⁾ is applicable in non-simply connected manifolds and it can provide the missing ingredient for an interesting spontaneous symmetry breaking within the C.S.D.R. scheme. Thus we are led to consider instead of the simply connected coset spaces $B=S/R$ the non-simply connected B/T , where T is a suitable discrete symmetry group. One picks for each $t \in T$ an element $U_t \in H$ and expects that non trivial gauge configurations U_t can be trapped in the non-simply connected manifold B/T even though, the gauge field strength vanishes in the vacuum. The group elements U_t form a discrete subgroup U of H and break H to the centralizer of U in H , $K=C_H(U)$.

In order to incorporate fermions in the C.S.D.R. scheme one has to add a Dirac Lagrangian to the pure Yang-Mills one. Next one demands that the action of S on the spinors is compensated by a gauge transformation and a local Lorentz rotation in the tangent space. After dimensional reduction, the theory becomes four dimensional with spinors being various multiplets of H . In order to identify the four-dimensional fermion multiplets one has to embed R in the Lorentz group $SO(N)$ of S/R in such a way that the N of $SO(N)$ has the branching rule $N=\mathbf{v}$ where $\mathbf{v}$ is given by eq(2). Then one takes the spinor σ of $SO(N)$ and decomposes it into irreps of R ,

$$\sigma = \sum \sigma_K \quad (4)$$

and has to decompose the representation F of the gauge group G to which the fermions are assigned under $R_G \times H$,

$$F = \sum (r_K, h_K) \quad (5)$$

It turns out that for each pair of (r_K, σ_K) where r_K and σ_K are identical irreps, there is an h_K multiplet of spinor fields in the four-dimensional theory.

An attractive feature of the fermion sector of the theory is that it is possible to obtain parity violation in the dimensionally reduced theory starting with Weyl fermions in a complex representation of G provided, that, $\text{rank } S = \text{rank } R$. Even more interesting is the possibility to obtain chiral fermions in four dimensions starting with real (if $D=2+8n$) or pseudoreal (if $D=6+8n$) representation of G by requiring the fermions to obey both Weyl and Majorana conditions.

Let us now discuss some specific examples of dimensional reduction of ten-dimensional model employing the "flux breaking" mechanism for the first step of the spontaneous symmetry breaking of the four-dimensional gauge group.

a) $G = SU_8$; $B = S/R = SU_4/SU_3 \times U_1$; $H = SU_5 \times U_1$

The $SU_3 \times SU_5 \times U_1$ branching rule of the adjoint of SU_8 is:

$$63 = (1,1)_0 + (8,1)_0 + (3,\bar{5})_{-4} + (3,5)_4 + (1,24)_0 \quad (6)$$

From this decomposition we find that the four-dimensional gauge group is $H = C_{SU_8}(SU_3 \times U_1) = SU_5 \times U_1$. The $SU_3 \times U_1$ content of $SU_4/SU_3 \times U_1$ vector is $\bar{3}_2 + 3_{-2}$ and spinor is $1_3 + 3_{-1}$. Then according to the rules mentioned before there are no Higgs fields of geometrical origin surviving in four dimensions. If we choose to assign fermions in $28 + \bar{28}$ representations of SU_8 , since the $SU_3 \times SU_5 \times U_1$ branching rule of 28 is:

$$28 = (\bar{3}, 1)_{-5} + (1, 10)_3 + (3, 5)_{-1}, \quad (7)$$

we obtain in four dimensions left-handed fermions transforming as $\bar{5}_1 + 10_3$ if we apply Weyl and Majorana conditions on the spinors in ten dimensions. Next we would like to employ the Wilson lines in order to break further the SU_5 gauge group. The way that this can be done is known^(9,10). The Wilson line U that leaves the standard model unbroken must be an element of the $U(1)$ group generated by the hypercharge. The hypercharge in SU_5 is

$$Y=2/3 \quad \begin{pmatrix} & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & -2 & \\ & & & & 3 \\ & & & & & 3 \end{pmatrix} \quad (8)$$

and therefore U has to be of the form

$$U = \begin{pmatrix} & & & & \\ & \alpha & & & \\ & & \alpha & & \\ & & & \alpha & \\ & & & & \beta \\ & & & & & \beta \end{pmatrix} \quad (9)$$

where $\beta = \alpha^{-3/2}$ and $\alpha^n = \beta^n = 1$. The U can have the form (9) if we consider instead of the coset space $B = SU_4/SU_3 \times U_1$ the non-simply connected B/Z_n and we connect the α and β in eq.(9) to elements of Z_n (here in particular Z_3 ⁽¹⁰⁾). Then the Wilson lines will break SU_5 to the $C_{SU_5}(U) = SU_3 \times SU_2 \times U_1$ and the gauge group after breaking will have an extra U_1 since the four dimensional gauge group was $H = SU_5 \times U_1$.

So far we have obtained the standard model with the right fermions just enlarged by one U_1 . In order to break further the gauge group to the observed

$SU_3 \times U_{1e.m.}$ we suggest to use a dynamical mechanism for the final breaking. The dynamical mechanism is based on high SU_{3c} representations which, according to the high colour scenario⁽¹¹⁾ and to the findings in lattice calculations⁽¹²⁾, can have chiral symmetry breaking at scales much higher than ordinary quarks. In order to use this mechanism we should introduce new fermions in the original theory in 10 dimensions. Instead of $28 + \overline{28}$ we introduce the multiplets $420 + \overline{420}$; $36 + \overline{36}$. Their decomposition under $SU_3 \times SU_5 \times U_1$ is

$$\begin{aligned}
 SU_8 &\supset SU_3 \times SU_5 \times U_1 \\
 36 &= (6, 1)_{-5} + (1, 15)_3 + (3, 5)_{-1} \\
 420 &= (\overline{3}, 1)_{-5} + (1, \overline{5})_{-9} + (1, 10)_3 + (3, 5)_{-1} \\
 &\quad + (\overline{6}, 5)_{-1} + (\overline{3}, \overline{10})_7 + (1, 40)_3 + (\overline{3}, 24)_{-5} \\
 &\quad + (8, 10)_3 + (3, 45)_{-1}
 \end{aligned} \tag{10}$$

From the above fermion multiplets survive in four dimensions only the $2 \cdot \overline{5}_1, 10_3, 15_3, 40_3, \overline{45}_1$. This fermionic content contains a desired combination of SU_5 fermionic representations advertised in ref (13) for $SU_{2L} \times U_1$ breaking plus an extra family $\overline{5}_1 + 10$. We refer to ref.(13) for a discussion of the fermionic content of the model analysed under the standard model. What we would like to note here is that the chiral symmetry breaking of the SU_{3c} sextets would be adequate for the electroweak breaking if the extra U_1 was not there.

Therefore this model has many attractive features but the various kinds of symmetry breaking mechanisms are not enough to make it totally realistic.

b) $G=E_6$; $B=S/R=Sp_4/SU_2 \times U_1$; $H=SU_5 \times U_1$

The $R=SU_2 \times U_1$ is chosen as follows. The SU_2 is taken from the decomposition of E_6 under the maximal subgroup

$$E_6 \supset SU_2 \times SU_6$$

and the $U(1)$ is taken from

$$SU_6 \supset SU_5 \times U_1$$

Then the $SU_2 \times SU_5 \times U_1$ branching rule of the adjoint of E_6 is:

$$78 = (3, 1)_0 + (1, 1)_0 + (1, 5)_6 + (1, \bar{5})_{-6} + (1, 24)_0 + (2, 10)_{-3} + (2, \bar{10})_3 \quad (11)$$

The $SU_2 \times U_1$ content of the $SP_4/SU_2 \times U_1$ vector is $1_2 + 1_{-2} + 2_1 + 2_{-1}$ and the spinor is $1_2 + 1_0 + 2_{-1}$. From the decomposition (11) we find that the four dimensional gauge group $H = C_{E_6}(SU_2 \times U_1) = SU_5 \times U_1$ and there are no surviving Higgs in four dimensions. On the other hand if we assign fermions in the 27 representation of E_6 the $SU_2 \times SU_5 \times U_1$ branching rule is

$$27 = (2, 1)_5 + (2, \bar{5})_{-1} + (1, 5)_{-4} + (1, 10)_2 \quad (12)$$

and therefore the surviving fermions are $5_{-1} + 10_2$. Next in order to break the SU_5 down to the standard model we proceed like in the previous example by considering instead of $B = Sp_4/SU_2 \times U_1$ the B/Z_n where the elements of Z_n (in particular Z_2)⁽¹⁴⁾ correspond to discrete elements of the hypercharge in SU_5 . So as in the previous example the gauge group after the breaking with Wilson lines is $H = SU_3 \times SU_2 \times U_1 \times U_1$. Then in an attempt to break further the group we replace the fermionic multiplet 27 with the $78 + 351 + \bar{351}$ in the ten-dimensional theory and we apply Weyl and Majorana conditions. Then, we find that the surviving fermions in four dimensions are under $SU_5 \times U_1$: $2 \cdot \bar{5}_{-1} + 10_2 + 15_2 + 40_2 + \bar{45}_{-1} + 24_0 + 1_0$. Again we have the set of ref(13) plus another ordinary family and $24_0 + 1_0$. Here in the spontaneous symmetry breaking of H' contribute in addition to the chiral symmetry breakings of the SU_{3C} sextets the corresponding of the octets. The details of this last breaking depend crucially on the values of the condensates which are not known quantitatively with confidence till now. But there exists the possibility that the sextet and octet condensates can break H' to $SU_3 \times U_{1em}$.

$$c) G=SO_{13} ; B=S/R=Sp_4/SU_2 \times U_1 ; H=SU_5 \times U_1$$

The $R=SU_2 \times U_1$ is chosen as follows. The SU_2 is taken from the decomposition of SO_{13} under the maximal subgroups

$$SO_{13} \supset SU_2 \times SO_{10}$$

and the U_1 is taken from the

$$SO_{10} \supset SU_5 \times U_1$$

The $SU_2 \times SU_5 \times U_1$ branching rule of the adjoint of SO_{13} is:

$$78 = (3,1)_0 + (1,1)_0 + (1,10)_4 + (1,\bar{10})_{-4} + (1,24)_0 + (3,5)_2 + (3,\bar{5})_{-2} \quad (13)$$

From eq.(13) we conclude that the four-dimensional gauge group is $H=SU_5 \times U_1$, which has no surviving Higgs. Then we assign fermions in ten dimensions in the $13+64$ representations which have the following decompositions under $SU_2 \times SU_5 \times U_1$:

$$\begin{aligned} 13 &= (3,1)_0 + (1,5)_2 + (1,\bar{5})_{-2} \\ 64 &= (2,1)_{-5} + (2,\bar{5})_3 + (2,10)_{-1} + (2,1)_5 + (2,5)_{-3} + (2,\bar{10})_1 \end{aligned} \quad (14)$$

If we apply the Weyl and Majorana conditions in ten dimensions we obtain $\bar{5}_{-2} + 10_{-1}$ fermions in four dimensions.

The "flux mechanism" breaks the SU_5 like in the previous examples.

We can attempt to use also in this model the dynamical mechanism of the high colour scenario in order to break the electroweak gauge group. By introducing the fermionic multiplets $78+286+768$ and applying the Weyl and Majorana conditions in the ten-dimensional theory we obtain in four dimensions the same

fermionic content as in model (b).

In conclusion we have suggested that the use of the "flux breaking" mechanism in the C.S.D.R. in addition to other spontaneous symmetry breaking mechanisms can help in the construction of unified gauge theories in higher dimensions with the right low energy gauge group. Our suggestion was supported with the construction of semirealistic models of this kind.

I would like to thank S. Ferrara and P. Forgacs for interesting discussions and for reading the manuscript.

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