Radiative Corrections to the Process

$$e^+ e^- \rightarrow \nu \bar{\nu} \gamma$$

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Abstract

We present a detailed calculation of the process $e^+ e^- \rightarrow \nu \bar{\nu} \gamma$. For the lowest order, the calculation is done keeping a finite mass for the W boson and the results apply for unpolarized as well as longitudinally polarized beams. One loop QED corrections are calculated and, together with the hard bremsstrahlung process $e^+ e^- \rightarrow \nu \bar{\nu} \gamma \gamma$ are implemented in an unweighted Monte Carlo generator of events. Results for a realistic experimental set-up are presented, amounting to corrections of the order of 17 %.

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1 Introduction

A basic question both in particle physics and cosmology is the determination of the number of generations. As proposed a long time ago [1,2], this can be achieved in a direct and clean way by the study of the final state $\nu\bar{\nu}\gamma$ in e^+e^- collisions under the assumption that the neutrinos are massless. Although for centre-of-mass energies above the Z^0 threshold the cross-section for this process is small, it is measurable. This cross-section is more sensitive to the number of light neutrinos than the decay width of the Z^0 boson. The existence of an additional light neutrino will raise the total Z^0 width by $\sim 6\%$, whereas the cross-section of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ will increase by $\sim 33\%$.

At least, this is the expectation on the basis of a lowest order calculation of the cross-section. For an actual experiment one has to face at least two questions. One, whether the experimental set-up can handle the backgrounds for the single photon detection. Another, whether radiative corrections to the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ affect the size of the signal.

The question of backgrounds has been discussed already some time ago [3]. The main source is radiative Bhabha scattering. Various recent assessments exist, which agree in their results [4,5,6]. Also, the radiative corrections to this background have been considered and turn out to be small [6,7].

The question how the $\nu\bar{\nu}\gamma$ signal itself is affected by radiative corrections is the subject of this paper. Some of our results have been presented before in the context of conclusions of a LEP 200 Study Group [8]. In the present paper, a detailed account of the methods and results of the evaluation will be given.

It should be stressed that we consider here the single γ signal from the $\nu\bar{\nu}\gamma$ final state. The question whether other undetectable neutral objects (e.g. SUSY partners of known particles) are produced in conjunction with a photon will not be discussed.

The outline of the paper is as follows. In the next section we discuss the lowest-order process and give some analytic formulae valid for the finite W mass. In section 3 virtual corrections and soft bremsstrahlung are considered. Section 4 then deals with the hard bremsstrahlung corrections, i.e. with the $\nu\bar{\nu}\gamma\gamma$ final state. The treatment of the phase space for the evaluation of the correction and an efficient generation of unweighted events is done in section 5, whereas the last section deals with numerical results for a realistic experimental situation. Some useful formulae are listed in an appendix.

2 The Process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ in lowest order

In this section, analytic formulae for the lowest order cross-section of the reaction

$$e^+(p_+) + e^-(p_-) \rightarrow \bar{\nu}(q_+) + \nu(q_-) + \gamma(k) \quad (1)$$

are given. In contrast to the expressions in [2,4] we keep a finite rather than infinite value for the mass M_W of the W boson like in [9]. The results apply to unpolarized and longitudinally polarized beams. Since we are interested in a situation in which a photon is emitted in a direction different from the beam, the electron mass can be neglected in the process (1). The fact that all external masses vanish simplifies the calculation. Using the Weyl - van der Waerden spinor calculus [10] one easily obtains expressions for the helicity amplitudes $M(\lambda_+, \lambda_-, \lambda_\gamma)$, where λ denotes successively the helicity of the e^+ , e^- and γ . These quantities are listed in the appendix.

For the production of a $\nu_e\bar{\nu}_e$ pair they lead to the following non-vanishing squared matrix elements, summed over the photon polarizations

$$\sum_{\lambda_\gamma} |M(-, +, \lambda_\gamma)|^2 = \frac{e^2 g^4 (g_v - g_a)^2}{2 c_w^4 |Z|^2} \cdot \frac{s'}{\kappa_+ \kappa_-} \cdot (t^2 + t'^2) \quad (2)$$

$$\begin{aligned} \sum_{\lambda_\gamma} |M(+, -, \lambda_\gamma)|^2 = & 2 e^2 g^4 \frac{s'}{\kappa_+ \kappa_-} \left[u^2 \left| \frac{g_v + g_a}{2 c_w^2 Z} + \frac{1}{W} \right|^2 + \right. \\ & + u'^2 \left| \frac{g_v + g_a}{2 c_w^2 Z} + \frac{1}{W'} \right|^2 - \frac{u^2 t' \kappa'_+ \kappa_- + u'^2 t \kappa'_- \kappa_+}{|W W'|^2} - \\ & - u^2 \operatorname{Re} \left(\left(\frac{g_v + g_a}{2 c_w^2 Z} + \frac{1}{W} \right)^* \frac{1}{W W'} (s' \kappa_- - t' \kappa'_+ + u' \kappa'_- - 4i\epsilon(q_+ q_- p_- k)) \right) - \\ & \left. - u'^2 \operatorname{Re} \left(\left(\frac{g_v + g_a}{2 c_w^2 Z} + \frac{1}{W'} \right)^* \frac{1}{W W'} (s' \kappa_+ - t \kappa'_- + u \kappa'_+ - 4i\epsilon(q_+ q_- p_+ k)) \right) \right] \end{aligned} \quad (3)$$

In these expressions, the following notation is used

$$\begin{aligned} s &= (p_+ + p_-)^2 & s' &= (q_+ + q_-)^2 \\ t &= (p_+ - q_+)^2 & t' &= (p_- - q_-)^2 \\ u &= (p_+ - q_-)^2 & u' &= (p_- - q_+)^2 \\ \kappa_\pm &= 2 p_\pm \cdot k & \kappa'_\pm &= 2 q_\pm \cdot k \end{aligned}$$

$$\begin{aligned} Z &= s' - M_Z^2 + i M_Z \Gamma_Z, \\ W &= t - M_W^2 + i M_W \Gamma_W, \\ W' &= t' - M_W^2 + i M_W \Gamma_W, \\ \varepsilon(a b c d) &= \varepsilon_{\mu\nu\rho\delta} a^\mu b^\nu c^\rho d^\delta \end{aligned} \quad (4)$$

with $\varepsilon_{0123} = 1$. Moreover, as usual in the standard model, one has relations involving the weak mixing angle ϑ_w , boson masses and the Fermi coupling constant G

$$\begin{aligned} c_w &= \cos \vartheta_w = \frac{M_W}{M_Z} \\ \frac{G}{\sqrt{2}} &= \frac{g^2}{8 M_W^2} \\ g_v &= -1/2 + 2 \sin^2 \vartheta_w, \quad g_a = -1/2 \end{aligned} \quad (5)$$

It should be noted that the expression (3) consists of two sets of three terms. These sets transform into each other by the interchange of $p_+ \leftrightarrow p_-$, $q_+ \leftrightarrow q_-$ and the change of $i\varepsilon$ into $-i\varepsilon$. The diagrams leading to these matrix elements are depicted in fig. 1. If only Z^0 exchange contributes, we get still eq. (2) but a simplified expression (3), i.e. only parts of the first and second terms.

Thus, one has for $\nu_\mu \bar{\nu}_\mu$ pair production

$$\sum_{hel} |M|^2 = \frac{e^2 g^4}{2 c_w^4 |Z|^2} \frac{s'}{\kappa_+ \kappa_-} \left((g_v - g_a)^2 (t^2 + t'^2) + (g_v + g_a)^2 (u^2 + u'^2) \right) \quad (6)$$

When one wants to calculate the cross-section differential in the photon energy k and $\cos \vartheta_\gamma$ in the range of LEP energies, one can make the following approximations. The last three terms in eq. (3) having at least three propagators, two of which are due to the W boson, can be neglected with respect to the first two terms. In the remaining terms Γ_W can be neglected.

The resulting expression is

$$\frac{d\sigma}{d \cos \vartheta_\gamma d k} = \frac{\alpha}{12 \pi^2} G^2 M_W^4 \frac{s' k}{s \kappa_+ \kappa_-} \left[\eta_+^2 F(\eta_+) + \eta_-^2 F(\eta_-) \right] \quad (7)$$

with

$$\eta_\pm = \frac{s - \kappa_\pm}{M_W^2} \quad (8)$$

$$\begin{aligned} F(\eta_\pm) &= N_\nu \frac{1}{2} \left((g_v + g_a)^2 + (g_v - g_a)^2 \right) \frac{M_Z^4}{|Z|^2} \\ &+ 3(g_v + g_a) \frac{M_Z^2 \operatorname{Re} Z}{|Z|^2} \frac{1}{\eta_\pm} \left(3 + \frac{2}{\eta_\pm} - 2(1 + 1/\eta_\pm)^2 \ln(1 + \eta_\pm) \right) \\ &+ \frac{6}{\eta_\pm^2} \left((1 + \eta_\pm) \left(1 - \frac{2}{\eta_\pm} \ln(1 + \eta_\pm) \right) + 1 \right) \end{aligned} \quad (9)$$

where N_ν denotes the number of neutrinos. The last expression is written in such a way that the contributions from the $+-$ and $-+$ helicity combinations, the latter being

proportional to $(g_v - g_a)^2$, are clearly visible. Thus, polarization asymmetries can be read off from this formula.

One could further approximate (7) by setting

$$\eta_+ = \eta_- = \frac{s}{M_W^2} \quad (10)$$

The result is the same expression one gets making a soft-photon approximation. Both ways of evaluating the cross section give, on the percent level, agreement with the exact calculation. Neglecting W exchange altogether can be done in the Z^0 region, e.g. for $\sqrt{s} = 100 \text{ GeV}$ a typical total cross-section gets only a $\sim 3\%$ contribution from W exchange. Nearer to the W-pair production threshold ($\sqrt{s} = 150 \text{ GeV}$) the contribution is $\sim 20\%$. These numbers justify the approach of the following sections, i.e. the radiative corrections to the Z^0 exchange diagrams will be considered and not those to W exchange. The necessary apparatus to integrate the last three terms in (3), neglecting Γ_W , is given in [9].

Finally, it should be noted that in the $M_W \rightarrow \infty$ limit, one recovers the expressions of refs. [2] and [4].

3 Virtual and soft corrections

In this section, the virtual and soft-bremsstrahlung corrections to reaction (1) will be considered. In doing so, several approximations are made. Firstly, only corrections to the Z^0 -exchange diagrams of fig. 1 will be taken into account. In terms of equation (7), it is the part proportional to N_ν to which corrections are applied. For the LEP energy range this is a reasonable approach. Secondly, when correcting the Z^0 -exchange diagrams, in principle all electroweak loop diagrams contribute. From the corrections to mu-pair production [11] we know that the dominant part of the weak corrections can be obtained by replacing the Z^0 propagator by the electroweakly corrected propagator

$$Z^{-1}(s') = (s' - M_Z^2 + iM_Z\Gamma_Z) \rightarrow (s' - M_Z^2 + \Sigma^Z(s'))^{-1} \quad (11)$$

Therefore, we replace everywhere in our calculation $Z^{-1}(s')$ by the corrected propagator (also in the hard-bremsstrahlung part). Apart from this replacement, only the photonic (or QED) corrections are considered. Those are depicted in fig. 2. This procedure is justified since for the subprocess

$$e^+ e^- \rightarrow \gamma Z^0 \quad (12)$$

both the photonic [12,13] and the complete [14] calculations exist. Again, in the energy range considered, the QED part is the most important part [14].

As far as the photonic corrections are concerned, we take the results of ref. [12] and write

$$d\sigma = d\sigma_Z^0 (1 + \delta_{v+s}) + d\sigma_{T-Z}^0 \quad (13)$$

where $d\sigma_Z^0$ is the Z^0 part of the lowest-order cross-section, $d\sigma_{T-Z}^0$ are the remaining terms of the lowest-order cross-section and δ_{v+s} is the sum of the virtual and soft corrections given by:

$$\begin{aligned} \delta_{v+s} = \frac{\alpha}{\pi} & \left[2 \left(\ln \frac{s}{m^2} - 1 \right) \ln \frac{k_m}{E} + \frac{\pi^2}{3} - 2 + \frac{3}{2} \ln \frac{s'}{m^2} + 2 \left(\frac{s^2 + s'^2}{\kappa_+ \kappa_-} - 2 \right)^{-1} \right. \\ & \left(\frac{s^2 + (s - \kappa_+)^2}{2\kappa_+ \kappa_-} \left(-Li_2 \left(1 - \frac{s}{s'} \right) - \ln \frac{s}{s'} \ln \frac{\kappa_-}{s'} + Li_2 \left(\frac{\kappa_-}{s'} \right) + \ln \left(\frac{\kappa_-}{s'} \right) \ln \left(1 + \frac{\kappa_-}{s'} \right) \right) \right. \\ & + \ln \frac{\kappa_-}{s'} \left(1 - \frac{\kappa_- - 2\kappa_+}{2(s' + \kappa_-)} - \frac{\kappa_- \kappa_+}{4(s' + \kappa_-)^2} \right) + \ln \frac{s'}{s} \left(\frac{s}{s - s'} - \frac{s'}{2(s - s')^2} \right) \\ & \left. \left. - \frac{s}{2(s - s')} + \frac{s}{4(s' + \kappa_-)} + \frac{s}{4\kappa_-} - \frac{\kappa_+}{4\kappa_-} + (\kappa_+ \leftrightarrow \kappa_-) \right) \right] \quad (14) \end{aligned}$$

Here, $k_m \ll E_b$ denotes the maximum energy for the soft-bremsstrahlung emission. One can take for instance 1% of the beam energy. As expected, δ_{v+s} will give a large and negative contribution. The hard bremsstrahlung, to be discussed in next section, will reduce the correction.

4 Hard-bremsstrahlung corrections

In this section we will discuss the process

$$e^+(p_1) + e^-(p_2) \rightarrow \nu(p_3) + \bar{\nu}(p_4) + \gamma(p_5) + \gamma(p_6) \quad (15)$$

as the hard-photon correction to the process (1)[15]. In order g^4 , the reaction (15) is mediated by a Z^0 boson in the s-channel and by a W in the t-channel. As we have discussed in section 2, for energies close to the Z^0 mass the contribution of the diagrams with W is very small and can be neglected. We are therefore left with the 6 Feynman diagrams shown in figure 3. Clearly, they form a gauge-invariant subset and we have grouped them in two topologically different classes (a and b in fig. 3) such that, by a permutation of particles, a diagram of one class can be transformed into any other one

belonging to the same class but can not be obtained from any one of the other class. It is therefore sufficient to calculate the transition amplitude for one characteristic diagram of each group.

We have evaluated the transition amplitude for this process using the helicity-amplitude technique which provides a fast and simple way of evaluating the Feynman diagrams in a suitable way for numerical calculations. The method is described in great detail in references [5,15 and 16].

With the basic definitions given in reference [5], one can express the amplitude for the diagram 3.a1 as

$$iM_{a1} = \frac{i e^4}{\sin 2\vartheta_w} \cdot T_{a1} \cdot G(p_1, p_5, p_6) \cdot F(p_1, p_5) \cdot D(q^2) \quad (16)$$

where

$$T_{a1}(p_i, \lambda_i) = [\bar{v}(p_2, \lambda_2) \gamma^\mu (C_L P_L + C_R P_R) (b_1 \not{p}_1 + m_1) + b_5 \not{p}_5 + b_6 \not{p}_6] \cdot \quad (17)$$

$$\cdot [\varepsilon^*(p_6, \lambda_6) (b_1 \not{p}_1 + m_1) + b_5 \not{p}_5] \varepsilon^*(p_5, \lambda_5) u(p_1, \lambda_1)] \cdot [\bar{u}(p_3, \lambda_3) \gamma_\mu P_L v(p_4, \lambda_4)]$$

$$G(p_i, p_j, p_k) = \frac{1}{2(b_i b_j (p_i \cdot p_j) + b_i b_k (p_i \cdot p_k) + b_j b_k (p_j \cdot p_k))}$$

$$F(p_i, p_j) = \frac{1}{2b_i b_j (p_i \cdot p_j)}$$

$$D(q^2) = \frac{1}{q^2 - M_Z^2 + \Sigma^Z(q^2)} \quad \text{with} \quad q^2 = 2(p_3 \cdot p_4) \quad (18)$$

b_i being a dichotomic variable equal to $+1$ (-1) if the momentum p_i corresponds to an ingoing (outgoing) particle, and the mass m_i is $+m$ or $-m$ depending on whether it corresponds to a particle or an antiparticle. In a similar way, one can express the amplitude for the diagram 3.b1 as

$$iM_{b1} = \frac{-i e^4}{\sin 2\vartheta_w} \cdot T_{b1} \cdot F(p_1, p_5) \cdot F(p_2, p_6) \cdot D(q^2) \quad (19)$$

where

$$T_{b1}(p_i, \lambda_i) = [\bar{v}(p_2, \lambda_2) \varepsilon^*(p_6, \lambda_6) (b_2 \not{p}_2 + m_2) + b_6 \not{p}_6] \gamma^\mu (C_L P_L + C_R P_R) \cdot$$

$$\cdot [b_1 \not{p}_1 + m_1 + b_5 \not{p}_5] \varepsilon^*(p_5, \lambda_5) u(p_1, \lambda_1)] \cdot [u(p_3, \lambda_3) \gamma_\mu P_L v(p_4, \lambda_4)] \quad (20)$$

If we introduce the $Z(p_i, \lambda_i)$ functions defined as the product of two bilineals with the following notation

$$Z(p_i, \lambda_i; p_j, \lambda_j; p_k, \lambda_k; p_l, \lambda_l; C_L; C_R; C'_L; C'_R) \equiv$$

$$\equiv [\bar{u}(p_i, \lambda_i) \Gamma^\mu u(p_j, \lambda_j)] [\bar{u}(p_k, \lambda_k) \Gamma'_\mu u(p_l, \lambda_l)] \quad (21)$$

where C_L (C_R) and C'_L (C'_R) are the left (right) components of the Γ^μ and Γ'_μ couplings, and the polarization vector of the photon as

$$\varepsilon_\mu^*(k, \lambda) = N \bar{u}(k, \lambda) \gamma_\mu u(p, \lambda) \quad (22)$$

with p any four-momentum occurring in the process and N the normalization factor, we can write T_{a1} and T_{b1} as

$$\begin{aligned} T_{a1} = & Z (p_2, \lambda_2; p_1, \lambda'; p_3, \lambda_3; p_4, \lambda_4; C_L, C_R, 1, 0) \cdot b_1 \cdot \\ & \cdot (Z(p_1, \lambda'; p_5, \lambda; p, \lambda_6; p_6, \lambda_6; 1, 1, 1, 1) \cdot b_5 \cdot Z_1 + \\ & + Z(p_1, \lambda'; p_1, \lambda; p, \lambda_6; p_6, \lambda_6; 1, 1, 1, 1) \cdot b_1 \cdot Z_2) + \\ + & Z (p_2, \lambda_2; p_5, \lambda'; p_3, \lambda_3; p_4, \lambda_4; C_L, C_R, 1, 0) \cdot b_5 \cdot \\ & \cdot (Z(p_5, \lambda'; p_5, \lambda; p, \lambda_6; p_6, \lambda_6; 1, 1, 1, 1) \cdot b_5 \cdot Z_1 + \\ & + Z(p_5, \lambda'; p_1, \lambda; p, \lambda_6; p_6, \lambda_6; 1, 1, 1, 1) \cdot b_1 \cdot Z_2) + \\ + & Z (p_2, \lambda_2; p_6, \lambda'; p_3, \lambda_3; p_4, \lambda_4; C_L, C_R, 1, 0) \cdot b_6 \cdot \\ & \cdot (Z(p_6, \lambda'; p_5, \lambda; p, \lambda_6; p_6, \lambda_6; 1, 1, 1, 1) \cdot b_5 \cdot Z_1 + \\ & + Z(p_6, \lambda'; p_1, \lambda; p, \lambda_6; p_6, \lambda_6; 1, 1, 1, 1) \cdot b_1 \cdot Z_2) \end{aligned} \quad (23)$$

$$\begin{aligned} T_{b1} = & Z (p_2, \lambda_2; p_6, \lambda'; p, \lambda_6; p_6, \lambda_6; 1, 1, 1, 1) \cdot b_6 \cdot \\ & \cdot (Z(p_6, \lambda'; p_1, \lambda; p_3, \lambda_3; p_4, \lambda_4; C_L, C_R, 1, 0) \cdot b_1 \cdot Z_2 + \\ & + Z(p_6, \lambda'; p_5, \lambda; p_3, \lambda_3; p_4, \lambda_4; C_L, C_R, 1, 0) \cdot b_5 \cdot Z_1) + \\ + & Z (p_2, \lambda_2; p_2, \lambda'; p, \lambda_6; p_6, \lambda_6; 1, 1, 1, 1) \cdot b_2 \cdot \\ & \cdot (Z(p_2, \lambda'; p_1, \lambda; p_3, \lambda_3; p_4, \lambda_4; C_L, C_R, 1, 0) \cdot b_1 \cdot Z_2 + \\ & + Z(p_2, \lambda'; p_5, \lambda; p_3, \lambda_3; p_4, \lambda_4; C_L, C_R, 1, 0) \cdot b_5 \cdot Z_1) \end{aligned} \quad (24)$$

with

$$Z_1 = Z(p_5, \lambda; p_1, \lambda_1; p, \lambda_5; p_5, \lambda_5; 1, 1, 1, 1) \quad (25)$$

$$Z_2 = Z(p_1, \lambda; p_1, \lambda_1; p, \lambda_5; p_5, \lambda_5; 1, 1, 1, 1) \quad (26)$$

and where the summation over λ and λ' is understood.

Given a set of four-momenta and a particular helicity configuration, one can easily evaluate the Z functions obtaining an explicit value for the transition amplitude of diagrams a1 and b1 and consequently, with the appropriate permutation of indices and conjugations, the corresponding ones for the remaining diagrams. Then, the total transition probability, averaged over initial spins, is given by

$$\frac{1}{4} \sum_{\text{spins}} |iM_T|^2 = \frac{N_\nu}{8} \sum_{\lambda} \left| \sum_j iM_j \right|^2 \quad (27)$$

where the indices j and λ run over all contributing diagrams and helicity configurations respectively, N_ν stands for the number of light neutrinos and the extra factor 2 in the denominator comes from the symmetrization of the final state photons.

5 The phase space

In the process under study, the only particles in the final state which are experimentally detected are the photons. To ensure their observability, we should require a minimum energy and polar angle with respect to the beam direction. It is therefore natural to choose a phase space parametrization in terms of photonic kinematic variables, i.e. the solid angle Ω_γ and the energy E_γ . For the single bremsstrahlung process

$$e^+(p_1) + e^-(p_2) \rightarrow \nu(p_3) + \bar{\nu}(p_4) + \gamma(k) \quad (28)$$

this is trivially done and leads to the three-body phase space

$$dR_3 = \frac{E_b^2}{8} \frac{x_k x_3}{2 + x_k (\cos \vartheta_{3k} - 1)} \cdot dx_k \cdot d\Omega_k \cdot d\Omega_3 \quad (29)$$

with $x_i = E_i/E_b$. For the double bremsstrahlung process

$$e^+(p_1) + e^-(p_2) \rightarrow \nu(p_3) + \bar{\nu}(p_4) + \gamma(k_1) + \gamma(k_2) \quad (30)$$

one can again express in a simple way the phase space in terms of the kinematical variables of the photons [15]. However, in order to have an efficient generator of events, one would like to include the most relevant peaks of the differential cross-section in the actual definition of the integration variables in such a way that the integrand becomes a smooth function. In practice, this can be implemented by applying the Importance Sampling technique, that is, by generating phase-space points distributed according to an approximative

function easy to handle and having identical peaking structures as the function we want to study and correcting afterwards the obtained distribution to get the exact result.

In the case of single bremsstrahlung, the integration region can be trivially defined in terms of the integration variables introduced above, that is

1. $E_\gamma > E_D$
2. $|\cos\vartheta_\gamma| < \cos\vartheta_D$

where E_D and ϑ_D are the minimum photon detection energy and polar angle with respect to the beam direction. The relevant structures of the cross-section for the single bremsstrahlung case can be expressed in the form of the following approximative distribution function

$$d\sigma_{np} \propto \frac{(1-x_k) dx_k}{x_k \left[(x_k - 1 + M_Z^2/s)^2 + M_Z^2 \Gamma_Z^2/s^2 \right]} \cdot \frac{d\cos\vartheta_k}{1 - \cos^2\vartheta_k} \cdot d\phi_k \cdot d\Omega_3 \quad (31)$$

which can be easily generated leading to an efficient integration of the differential cross-section.

The process with double bremsstrahlung is dominated, for energies around the Z^0 peak, by the s-channel Z^0 diagrams. It would be therefore more desirable to factorize the phase space onto the production of an off-shell Z^0 boson (with mass M_{Z^*}) and its subsequent decay. Thus, we can write

$$dR_4 = dM_{Z^*}^2 \cdot dR_3(s; M_{Z^*}^2, 0, 0) \cdot dR_2(M_{Z^*}^2; 0, 0) \quad (32)$$

with

$$dR_2(M_{Z^*}^2; 0, 0) = \frac{d\Omega_3^*}{8} \quad (33)$$

and

$$\begin{aligned} dR_3(s; M_{Z^*}^2, 0, 0) &= \frac{1}{8} \frac{k_1 k_2}{\sqrt{s} + k_1(\cos\vartheta_{\gamma_1\gamma_2} - 1)} \cdot dk_1 \cdot d\Omega_{\gamma_1} \cdot d\Omega_{\gamma_2} \\ &= \frac{s}{128} \frac{x_{k_1} x_{k_2}^2}{4(1-x_{k_1}) - x_{M_{Z^*}}^2} \cdot dx_{k_1} \cdot d\Omega_{\gamma_1} \cdot d\Omega_{\gamma_2} \end{aligned} \quad (34)$$

where the superscript $*$ refers to the Z^0 rest frame. Then, the phase space factor can be written as

$$dR_4 = \frac{s^2}{2^9} \frac{x_{k_1} x_{k_2}^2}{4(1-x_{k_1}) - x_{M_{Z^*}}^2} \cdot dx_{M_{Z^*}}^2 \cdot dx_{k_1} \cdot d\Omega_{\gamma_1} \cdot d\Omega_{\gamma_2} \cdot d\Omega_3^* \quad (35)$$

The integration limits for the variables chosen are somewhat more complicated than in the case of single bremsstrahlung. There are basically two situations which lead to a single-photon signature:

1. One photon (detected) fulfils the detection conditions and the second one (undetected) is lost because, although being $E_{\gamma_2} > E_{min}$ (maximum soft photon energy) it is such that

i) $E_{\gamma_2} < E_V$ or

ii) $|\cos\vartheta_{\gamma_2}| > \cos\vartheta_V$

E_V and ϑ_V being the veto energy and polar angle.

2. Both photons are collinear; that is, $\cos\vartheta_{\gamma_1\gamma_2} > \cos\vartheta_{RES}$ being ϑ_{RES} the minimum angle necessary to separate photons and

i) $E_{\gamma_1} + E_{\gamma_2} > E_D$ (with $E_{\gamma_i} > E_{min}$; $i = 1, 2$)

ii) $|\cos\vartheta_{\gamma_1}| < \cos\vartheta_D$

It should be pointed out that, since neutrinos are not detected, the integration region for $d\Omega_3^*$ is the full solid angle.

In order to integrate the photonic variables over these phase space regions, it is useful to consider separately the following exclusive and exhaustive regions :

a) Both photons are in the observable angular region; that is, $|\cos\vartheta_{\gamma_i}| < \cos\vartheta_D$

b) Only one photon is in the aforementioned region

In case a), due to the symmetry of the integration domain it is sufficient to integrate half of the allowed region by requiring $E_{\gamma_1} > E_{\gamma_2}$ and therefore multiply by 2 the final result.

Here we can distinguish between two cases:

1. If the two photons are not collinear (that is, $\cos\vartheta_{\gamma_1\gamma_2} < \cos\vartheta_{RES}$), since $\vartheta_{\gamma_1} > \vartheta_{\gamma_2}$ we have the integration limits (fig. 4a)

i) $E_{\gamma_1} > E_D$

ii) $E_{min} < E_{\gamma_2} < E_V$

2. If both photons are collinear (so $\cos\vartheta_{\gamma_1\gamma_2} > \cos\vartheta_{RES}$), since $E_{\gamma_1} > E_{\gamma_2}$ and $E_{\gamma_1} + E_{\gamma_2} > E_D$ the integration limits are (fig. 4b)

i) $E_{\gamma_1} > E_D/2$

ii) $E_{min} < E_{\gamma_2} < E_{\gamma_1}$

It is clear from the above conditions that the values of E_{min} and E_D have to be such that $E_{min} \leq E_D/2$. On the other hand, E_D has to be sufficiently large so that higher-order correction terms are not necessary.

Let us finally analyze the situation b). If we require the photon 1 to be inside the detectable angular region and photon 2 outside (and therefore, since we could exchange $1 \leftrightarrow 2$ we shall multiply by 2) the integration limits are (fig. 4c):

1. $E_{\gamma_1} > E_D$

2. $E_{\gamma_2} > E_{min}$

In addition, since the expected behaviour of the matrix element squared goes as

$$|M|^2 \propto \frac{1}{(x_{M_{Z^*}}^2 - M_Z^2/E_b^2)^2 + M_Z^2\Gamma_Z^2/E_b^4} \cdot \frac{1}{x_{\gamma_1}^2(1 - \beta^2\cos\vartheta_{\gamma_1})} \cdot \frac{1}{x_{\gamma_2}^2(1 - \beta^2\cos\vartheta_{\gamma_2})} \quad (36)$$

taking into account the phase space density we can use as an approximant

$$d\sigma_{app} \propto \frac{dx_{M_{Z^*}}^2}{(x_{M_{Z^*}}^2 - M_Z^2/E_b^2)^2 + M_Z^2\Gamma_Z^2/E_b^4} \cdot \frac{dx_{\gamma_1}}{x_{\gamma_1}(1 - x_{\gamma_1} - x_{M_{Z^*}}^2/4)} \cdot \frac{d\cos\vartheta_{\gamma_1}}{1 - \beta^2\cos\vartheta_{\gamma_1}} \cdot \frac{d\cos\vartheta_{\gamma_2}}{1 - \beta^2\cos\vartheta_{\gamma_2}} \cdot d\Phi_{\gamma_1} \cdot d\Phi_{\gamma_2} \cdot d\Omega_3^* \quad (37)$$

which, like in the case of single bremsstrahlung, can be easily generated by using standard Monte Carlo procedures.

6 Numerical Results

In this last section, we shall present some numerical results ¹ of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ under a hypothetical experimental set-up defined by

1. veto angle of $\vartheta_V = 5^\circ$

¹The relevant input parameters used for this evaluation are $\alpha = 1/137.03604$, $M_Z = 93 \text{ GeV}$, $\Gamma_Z = 2.7 \text{ GeV}$, and $M_W = 82 \text{ GeV}$

2. tagging angle of $\vartheta_D = 20^\circ$
3. minimum photon energy to be detected of $E_D = 1\text{GeV}$
4. two photons separation angle of $\vartheta_S = 1^\circ$.

The dependence of the observed cross-section for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ with the centre-of-mass energy under the above experimental conditions is shown in figure 5.1 for 3 generations. The solid line corresponds to the corrected cross-section; the dashed line to the lowest-order cross-section and the dotted one to the lowest-order cross-section with the Z^0 self-energy correction included. The effect of the radiative corrections amounts to $\sim 17\%$, that is, about half the effect of an additional neutrino generation.

For a centre-of-mass energy of $\sqrt{s} = 100\text{GeV}$ and again 3 generations, the differential cross-section with respect to the energy of the observed photon is depicted in figure 5.2 (solid line) as compared to the lowest-order expectation. One can clearly see that the effect of the radiative corrections is to shift the photon-energy distribution to lower values than the ones obtained with the first-order calculation and to change in the same way the P_t distribution shown in figure 5.3, making therefore less efficient the separation of the signal from the QED background [15]. Finally, the differential cross-section with respect to the photon polar angle ϑ_γ is shown in figure 5.4.

We are grateful to W. Hollik for interesting discussions and suggestions.

Appendix A

In this appendix, we list the four helicity amplitudes for the lowest-order process in terms of the spinorial inner-products

$$\langle p \cdot q \rangle = p_A \epsilon^{AB} q_B \quad (\text{A.1})$$

where

$$\epsilon^{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (\text{A.2})$$

and the spinor p_A belonging to a light-like vector p_μ is given by

$$p_A = \begin{pmatrix} \sqrt{p_0 - p_3} \exp(-iw/2) \\ \sqrt{p_0 + p_3} \exp(iw/2) \end{pmatrix} \quad (\text{A.3})$$

with

$$\cos w = \frac{p_1}{\sqrt{p_1^2 + p_2^2}}, \quad \sin w = \frac{p_2}{\sqrt{p_1^2 + p_2^2}} \quad (\text{A.4})$$

When squaring the helicity amplitudes, the following property (usual four-dimensional Minkowski inner product) is useful

$$|\langle p \cdot q \rangle|^2 = 2 p \cdot q \quad (\text{A.5})$$

Using the techniques of ref. [10], we find

$$M(-++) = \frac{ieg^2(g_v - g_a)}{\sqrt{2}C_w^2 Z} \frac{\langle p_- q_- \rangle^2 \langle q_+ q_- \rangle^*}{\langle p_+ k \rangle \langle p_- k \rangle} \quad (\text{A.6})$$

$$M(-+-) = \frac{ieg^2(g_v - g_a)}{\sqrt{2}C_w^2 Z} \frac{\langle p_+ q_+ \rangle^{*2} \langle q_+ q_- \rangle}{\langle p_+ k \rangle^* \langle p_- k \rangle^*} \quad (\text{A.7})$$

$$M(++-) = -ieg^2\sqrt{2} \frac{\langle p_+ q_- \rangle^2}{\langle p_+ k \rangle \langle p_- k \rangle} \cdot \left[\langle q_+ q_- \rangle^* \left(\frac{g_v + g_a}{2C_w^2 Z} + \frac{1}{W} \right) + \frac{\langle q_+ k \rangle^* \langle k p_- \rangle \langle p_- q_- \rangle^*}{W W'} \right] \quad (\text{A.8})$$

$$\begin{aligned}
&= -ieg^2\sqrt{2}\frac{\langle p_+q_- \rangle^2}{\langle p_+k \rangle \langle p_-k \rangle \langle q_+q_- \rangle} \cdot \\
&\cdot \left[s' \left(\frac{g_v + g_a}{2C_w^2 Z} + \frac{1}{W} \right) - \frac{1}{W W'} \text{Tr}(q_- p_- k q_+ \frac{1+\gamma^5}{2}) \right] \quad (\text{A.9})
\end{aligned}$$

$$\begin{aligned}
M(+--)&= ieg^2\sqrt{2}\frac{\langle p_-q_+ \rangle^{*2}}{\langle p_+k \rangle^* \langle p_-k \rangle^*} \cdot \\
&\cdot \left[\langle q_-q_+ \rangle \left(\frac{g_v + g_a}{2C_w^2 Z} + \frac{1}{W'} \right) + \frac{\langle q_-k \rangle \langle kp_+ \rangle^* \langle p_+q_+ \rangle}{W W'} \right] \quad (\text{A.10})
\end{aligned}$$

$$\begin{aligned}
&= ieg^2\sqrt{2}\frac{\langle p_-q_+ \rangle^{*2}}{\langle p_+k \rangle^* \langle p_-k \rangle^* \langle q_-q_+ \rangle^*} \cdot \\
&\cdot \left[s' \left(\frac{g_v + g_a}{2C_w^2 Z} + \frac{1}{W'} \right) - \frac{1}{W W'} \text{Tr}(q_+ p_+ k q_- \frac{1+\gamma^5}{2}) \right] \quad (\text{A.11})
\end{aligned}$$

In all of these expressions, the standard bremsstrahlung factor

$$[\langle p_+ \cdot k \rangle \langle p_- \cdot k \rangle]^{-1} \quad (\text{A.12})$$

is present. In eqs. (A.6) and (A.7) it multiplies the lowest-order amplitude $M(-+)$ written in a suitable form. In eqs. (A.8) and (A.10) it multiplies the sum of two terms. In the first term, one can recognize $M(+ -)$ after choosing one of the ways $M(+ -)$ can be written. The second term is due to the diagram of fig. 1.e. It vanishes for soft photons, such that the usual soft-bremsstrahlung factorization arises.

Using eqs. (5) one can introduce G . Where Z^{-1} , W^{-1} and W'^{-1} occur, we now get M_Z^2/Z , M_W^2/W and M_W^2/W' except for the double propagator where we are left with $M_W^2 W^{-1} W'^{-1}$. Thus, in the $M_W^2 \rightarrow \infty$ limit, the last terms in A.8 and A.10 vanish, whereas both M_W^2/W and M_W^2/W' become -1 .

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FIGURE CAPTIONS

- 1.— Feynman diagrams of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ in first order.
- 2.— Photonic corrections of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$.
- 3.— Feynman diagrams of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ with Z^0 in s - channel.
- 4.— Phase space integration domain.
- 5.1— Total cross-section for single photon within the experimental cuts referred to in section 6.
- 5.2— Differential cross-section with respect to the photon energy.
- 5.3— Differential cross-section with respect to the photon transverse momentum.
- 5.4— Differential cross-section with respect to the photon polar angle.

Fig. 1

+ interchange of $\gamma \leftrightarrow Z^0$

Fig. 2

Fig. 5.1

Fig. 5.2

Fig. 5.3

Fig. 5.4