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VECTOR BOSON SCATTERING AT e^+e^- COLLIDERS
AS A TEST OF $W^\pm$, Z_0 , γ SELF-INTERACTIONS^{*})

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ABSTRACT

We investigate the potential of a high-energy e^+e^- collider (e.g., CLIC, $E_{e^+} = E_{e^-} = 1$ TeV) for determining trilinear and quadrilinear vector boson self-interactions in various vector boson scattering processes which can be measured in reactions of the type $e^+e^- \rightarrow (e^+e^-, \nu e^-, \bar{\nu} e^+, \bar{\nu} \bar{\nu}) + VV'$. Our analysis is based upon a recently suggested single parameter Lagrangian model for vector boson self-interactions incorporating global SU(2) weak isospin symmetry broken by electromagnetism and a minimal increase of vector boson scattering tree amplitudes as a function of the energy. The results are compared with the $SU(2)_L \times U(1)_Y$ predictions for the cases of a light and a heavy Higgs boson. We find that the cross-section for the production of a vector boson pair, VV' , is very sensitively dependent on the magnitude of the single free parameter, κ , the anomalous magnetic dipole moment of the $W^\pm$. The cross-section changes by approximately one order of magnitude, even near production threshold, if κ is varied by one unit around the $SU(2)_L \times U(1)_Y$ value of $\kappa = 1$.

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1. Introduction

Much attention has recently been devoted to theoretical studies of vector boson ($W^\pm, Z, \gamma$) scattering^{1,2)}, mainly in connection with plans for a new generation of colliders, a 20 TeV proton-proton or proton-antiproton collider in the LEP tunnel (the LHC), a 40 TeV proton-proton collider (the SSC) in the USA and a 2 TeV electron-positron linear collider (the CLIC) in Europe. A detailed experimental investigation of reactions such as $(\gamma\gamma, \gamma Z, ZZ) \rightarrow WW, ZW \rightarrow ZW, WW \rightarrow WW$ and $ZZ \rightarrow ZZ$ will become feasible, if such powerful facilities indeed become reality.

Theoretical studies of vector boson scattering have so far been focused almost exclusively on analysing these reactions with respect to the effects of the Higgs scalar particle as the essential missing link within the $SU(2)_L \times U(1)_Y$ spontaneously broken electroweak theory³⁾. In this connection, the existence and magnitude of the trilinear and quadrilinear self-couplings of the vector bosons have usually been taken for granted, as they appear as a strict consequence of the local non-Abelian gauge principle underlying the $SU(2)_L \times U(1)_Y$ electroweak theory. It is well known, however, that a theoretical derivation of the empirically verified parts of the $SU(2)_L \times U(1)_Y$ electroweak theory, in particular of the neutral current structure and the values of the vector boson masses, does not necessarily require a local $SU(2)$ gauge principle and spontaneous symmetry breaking to start with. All empirically known facts on electroweak interactions may equally well be derived from global $SU(2)_{WI}$ weak isospin symmetry broken by electromagnetism, in particular γW^3 mixing⁴⁾, in such a manner that W-dominance^{5,6)} is fulfilled for the interactions of photons with leptons and quarks. The generalization of W-dominance from leptons and quarks to the charged vector bosons seems natural, and in fact it leads^{7,8)} to the standard $SU(2)_L \times U(1)_Y$ couplings also for the vector boson self-interactions. It should be stressed, nevertheless, that this latter generalization amounts to an additional assumption on the properties of the $W^\pm$ and Z bosons, which ought to be tested empirically, thus making the experimental study of vector boson self-interactions in itself a subject worth considerable attention.

Empirically, no direct experimental information^{*)} is available so far on trilinear and quadrilinear vector boson self-interactions. A detailed experimental study of the trilinear couplings will start as soon as LEP reaches its design energy of about 200 GeV in the nineties. The fact that at LEP 200 the threshold energy for the reaction $e^+e^- \rightarrow W^+W^-$ will be surpassed by only about 40 GeV, will nevertheless lead to a limited accuracy^{12,13)} as regards the results to be

*) The agreement⁹⁾ with experiment of the radiatively corrected predictions for the $W^\pm$ and Z_0 masses implies that vector boson loop corrections cannot be arbitrarily large, but present empirical evidence can hardly be considered¹⁰⁾ as an empirical "proof" of gauge theory couplings (compare also Ref. 11).

obtained when extracting various trilinear couplings, such as the $Z_0 W^+ W^-$ coupling constant, g_{ZWW} , and the magnetic and weak anomalous dipole moments, κ and κ_Z , of the charged vector bosons.

Two alternatives can be envisaged, once experimental results at LEP 200 become available:

(i) Measurements of the cross-section and angular distribution of $e^+ e^- \rightarrow W^+ W^-$ fully agree within errors with the $SU(2)_L \times U(1)_Y$ predictions. If this turns out to be the case a further refinement of the accuracy achieved in the determination of the various coupling constants will almost certainly be desirable.

(ii) Some discrepancy between the $SU(2)_L \times U(1)_Y$ predictions and experiments is found. It goes without saying that a further experimental study of vector boson scattering will become unavoidable in order to identify the origin of the discrepancy.

We thus conclude that a study to assess the potential of a 2 TeV electron-positron collider for determining vector boson self-interactions is in any case well motivated and will be pursued within the present paper.

The large number of different vector boson scattering amplitudes contributing to measurable processes of the form $e^+ e^- \rightarrow (e^+ e^-, \nu e^-, \bar{\nu} e^+, \nu \bar{\nu}) + VV'$, where V and V' stand for the vector bosons (compare Fig. 1), as well as the dependence of these amplitudes on both trilinear and quadrilinear couplings leads to a richness and complexity of these processes which considerably surpasses the much simpler case of $e^+ e^- \rightarrow W^+ W^-$. To start from a purely kinematical analysis of the various amplitudes in terms of (a very large number of) arbitrary coupling constants to be determined by scattering experiments is excluded a priori by the mentioned complexity of the physical situation. It seems mandatory that the analysis is based on an underlying Lagrangian model for trilinear and quadrilinear self-couplings. The model should be sufficiently general on the one hand to allow for significant deviations from and a test of the $SU(2)_L \times U(1)_Y$ predictions, while on the other hand it should be restricted by those principles for which good empirical support is available from other studies of electroweak interactions. Otherwise no significant conclusions can be drawn from the measurements, once these will become available.

The present study of vector boson scattering at very high energy $e^+ e^-$ colliders will be based on the KMSS Lagrangian model^{14,13)} for vector boson self interactions. It rests upon global $SU(2)_{WI}$ weak isospin symmetry broken by electromagnetism as a basic assumption. Electromagnetic interactions of the charged vector bosons are assumed to be P and C conserving and, for simplicity, only dimensionless couplings are allowed to be present, thus suppressing dimension-six quadrupole terms^{*)}.

*) For a different point of view, in particular a treatment of additional quadrupole interactions, compare Ref. 7).

The resulting Lagrangian depends upon two arbitrary trilinear and two arbitrary quadrilinear coupling constants, and it is the most general one under the assumptions mentioned. For a special choice of the free parameters one obtains the $SU(2)_L \times U(1)_Y$ self-interactions.

The number of four free parameters in the KMSS Lagrangian is reduced to a single one, if one imposes¹⁵⁾ the additional requirement that those terms in the vector boson scattering amplitudes which grow most strongly with energy (such as s^2) are absent. This requirement assures¹⁵⁾ that the critical energy, $\sqrt{s_{\text{crit}}}$, at which the growth of the tree level amplitudes threatens to violate the partial wave unitarity bounds, is of the order of $\sqrt{s_{\text{crit}}} \cong 2 \text{ TeV}$ for any reasonable value of the remaining free parameter, for which the anomalous magnetic moment, κ , of the charged vector bosons, $W^\pm$, has been chosen. Within the present paper we will restrict ourselves to an analysis of the mentioned processes as a function of this single free parameter, κ . We will compare our results with those obtained within the $SU(2)_L \times U(1)_Y$ theory for two different choices of the mass of the Higgs scalar.

In section 2, we will summarize the relevant formulae for treating the processes $e^+e^- \rightarrow (e^+e^-, \nu e^-, \bar{\nu} e^+, \nu\bar{\nu}) + VV'$ to be studied subsequently. The helicity amplitudes for vector boson scattering following from the mentioned Lagrangian as well as the $SU(2)_L \times U(1)_Y$ amplitudes including the Higgs particle will be deferred to Appendix A. In section 3, we will present and discuss our numerical results. Section 4 will contain a few summarizing conclusions.

2. Scattering Amplitudes and Cross-Sections

We are interested in the e^+e^- scattering process $e^+e^- \rightarrow l l' + V_3 V_4$ depicted in Fig. 1, where l and l' stand for electrons or neutrinos and V_1, V_2, V_3 and V_4 for vector bosons. Each incoming $e^\pm$ thus emits a $W^\pm, Z$ or γ and emerges as a final $(\bar{\nu}_e^-)$ or $e^\pm$ predominantly in the forward direction. The sub-process $V_1 V_2 \rightarrow V_3 V_4$ is a two-body vector-boson interaction. For sufficiently high $e^\pm$ energies (i.e., $E_e \gg M_W$), the reactions of Fig. 1 are expected to be the dominating^{1,2,16,17)} e^+e^- reactions.

Generalizing the analyses of $\gamma\gamma$ collision processes¹⁷⁾, one approximates^{16,18)} the cross-section for $e^+e^- \rightarrow l l' + V_3 V_4$ as a sum of products of boson-boson luminosity factors, $L_{V_1 V_2, i}(\tau)$, and cross-sections, $\hat{\sigma}_i^{V_1 V_2 \rightarrow V_3 V_4}(s)$, i.e.,

$$\frac{d\sigma}{d\tau} (e^+e^- \rightarrow l l' + V_3 V_4) = \sum_{V_1 V_2, i} L_{V_1 V_2, i}(\tau) \hat{\sigma}_i^{V_1 V_2 \rightarrow V_3 V_4}(s), \quad (2.1)$$

where $\tau \equiv s/4 E_e^2$ is the ratio of the sub-energy squared, s , to the square of the

e^+e^- centre-of-mass energy, $4 E_e^2$. The summation in (2.1) is to be extended over the contributing vector bosons $V_1 V_2$ and over the polarization index, i , which refers to the various helicity combinations of the intermediate vector bosons V_1 and V_2 . In general there are nineteen helicity combinations, but dealing with parity and time reversal invariant processes and integrating over angles leaves five terms only, two transverse-transverse ones (to be denoted by TT and TT'), a longitudinal-longitudinal one (LL) and two longitudinal-transverse ones (LT and TL). For a full discussion of (2.1) we refer to Refs. 17) and 18). The effective W approximation (EWA) consists of taking for $\hat{\sigma}^{V_1 V_2 \rightarrow V_3 V_4}(s)$ the physical cross-section for on-mass-shell vector bosons. The corresponding luminosity factors are model dependent, as they depend on the assumptions made for the off-mass-shell behaviour of the subprocesses. Minimal assumptions led to simple analytical forms^{16,18)} for $L_{V_1 V_2, i}(\tau)$.

The mentioned approximations are sufficiently accurate for the present purpose of providing an overview on the effects caused by a departure of the self-interaction from the standard ones. A detailed analysis of future experiments with respect to deducing vector boson self-couplings will require a refined calculation including contributions due to other diagrams and a study of the effects caused by the virtuality of the initial bosons V_1 and V_2 . Alternatively it may be possible to assure the dominance of the diagrams of Fig. 1 by appropriate cuts in the phase space of the final vector bosons V_3 and V_4 .

In the limit of very high e^+e^- energies ($E_e \gg M_W$), the leading terms of the luminosity functions read

$$\begin{aligned}
 L_{TT}(\tau) &= V_{12} \left(\frac{\alpha}{2\pi} \right)^2 \frac{1}{\tau} \left[(2+\tau)^2 \ln \frac{1}{\tau} - 2(1-\tau)(3+\tau) \right] \ln \left(\frac{4E_e^2}{M_1^2} \right) \ln \left(\frac{4E_e^2}{M_2^2} \right), \\
 L_{LL}(\tau) &= V_{12} \left(\frac{\alpha}{\pi} \right)^2 \frac{1}{\tau} \left[(1+\tau) \ln \frac{1}{\tau} - 2(1-\tau) \right], \\
 L_{TL}(\tau) &= V_{12} \left(\frac{\alpha}{2\pi} \right)^2 \frac{2}{\tau} \left[2(1+\tau) \ln \frac{1}{\tau} - \frac{1}{2}(1-\tau)(7+\tau) \right] \ln \left(\frac{4E_e^2}{M_1^2} \right), \\
 L_{LT}(\tau) &= V_{12} \left(\frac{\alpha}{2\pi} \right)^2 \frac{2}{\tau} \left[2(1+\tau) \ln \frac{1}{\tau} - \frac{1}{2}(1-\tau)(7+\tau) \right] \ln \left(\frac{4E_e^2}{M_2^2} \right), \\
 L_{TT'}(\tau) &= 4v_1 a_1 v_2 a_2 \left(\frac{\alpha}{2\pi} \right)^2 \left[(4+\tau) \ln \frac{1}{\tau} - 4(1-\tau) \right] \ln \left(\frac{4E_e^2}{M_1^2} \right) \ln \left(\frac{4E_e^2}{M_2^2} \right),
 \end{aligned} \tag{2.2}$$

where for simplicity we have dropped the subscripts V_1 and V_2 , and V_{12} is related to the vector and axial vector couplings, v_1, a_1 and v_2, a_2 , (in units of e) of the vector bosons V_1 and V_2 to the electron by

$$V_{12} = (v_1^2 + a_1^2)(v_2^2 + a_2^2). \tag{2.3}$$

For completeness we note that $v_\gamma = 1$, $a_\gamma = 0$, $v_W = a_W = \frac{1}{2\sqrt{2}\sin\theta_W}$, and finally

$$v_Z = \frac{-1+4\sin^2\theta_W}{2\sin 2\theta_W} \quad \text{and} \quad a_Z = \frac{-1}{2\sin 2\theta_W}.$$

The above leading log terms apply when there is no tagging restriction. In this case, for a massless photon, the masses M_1 or M_2 in (2.2) have to be replaced by the electron mass. As a consequence, photon luminosity factors will be much larger than massive vector boson ones^{16,18)}, which contain the vector boson masses for M_1 or M_2 or both. When tagging restrictions are imposed, the log terms in (2.2) have to be replaced by $\ln(\theta_{\max}/\theta_{\min})$, and consequently all vector boson luminosity factors will be of the same order of magnitude. Finally, we note that longitudinal luminosities in (2.2) are smaller than transverse ones, as the longitudinal ones do not have logarithmic enhancements. Also, as a consequence of the different vector and axial vector couplings, the $W^\pm$ luminosity factors are larger than the Z_0 luminosity factors.

The reactions $e^+e^- \rightarrow l\bar{l}' + V_3V_4$ involve twelve different vector boson subprocesses which are listed explicitly in the Table.

$e^+e^- \rightarrow$	subprocesses contributing
$e^+e^- + W^+W^-$ $\nu\bar{\nu} + W^+W^-$	$(\gamma\gamma, \gamma Z, ZZ) \rightarrow W^+W^-$ $W^+W^- \rightarrow W^+W^-$
$e\nu + \gamma W$	$(\gamma W, ZW) \rightarrow \gamma W$
$\nu\bar{\nu} + \gamma\gamma$	$W^+W^- \rightarrow \gamma\gamma$
$e^+e^- + ZZ$ $\nu\bar{\nu} + ZZ$	$ZZ \rightarrow ZZ$ $W^+W^- \rightarrow ZZ$
$e\nu + ZW$	$(\gamma W, ZW) \rightarrow ZW$
$\nu\bar{\nu} + \gamma Z$	$W^+W^- \rightarrow \gamma Z$

Table: Processes $e^+e^- \rightarrow l\bar{l}' + V_3V_4$ and subprocesses contributing

In second order in the electroweak couplings these vector boson - vector boson scattering reactions proceed via single vector boson (and possibly Higgs boson) exchange and via quadrilinear couplings as depicted in Fig. 2. Apart from the

Higgs contribution, vector boson scattering subprocesses are therefore directly sensitive to and solely dependent on the trilinear and quadrilinear self-couplings of the vector bosons. In this respect vector boson scattering is different from $e^+e^- \rightarrow W^+W^-$, which depends on trilinear vector boson couplings only and involves a lepton exchange contribution which often dominates. Vector boson scattering thus provides a unique possibility to study the trilinear and quadrilinear self-couplings.

As mentioned, our investigation of vector boson scattering will be based on the KMSS model¹³⁻¹⁵⁾ for vector boson self-interactions. The Lagrangian for the self-interactions, L_{SI} , is given by

$$\begin{aligned}
 L_{SI} = & i e A_\mu (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) + i e \kappa F_{\mu\nu} W^{+\mu} W^{-\nu} \\
 & + i e \frac{\kappa - \sin^2\theta_W}{\sin\theta_W \cos\theta_W} (Z_\mu (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) + \frac{\kappa \cos^2\theta_W}{\kappa - \sin^2\theta_W} Z_{\mu\nu} W^{+\mu} W^{-\nu}) \\
 & - e^2 (A_\mu A^\mu W_\nu^+ W^{-\nu} - A_\mu A_\nu W^{+\mu} W^{-\nu}) \\
 & - 2e^2 \frac{\kappa - \sin^2\theta_W}{\sin\theta_W \cos\theta_W} (A_\mu Z^\mu W_\nu^+ W^{-\nu} - \frac{1}{2} A_\mu Z_\nu (W^{+\mu} W^{-\nu} - W^{-\mu} W^{+\nu})) \\
 & - e^2 \frac{(\kappa - \sin^2\theta_W)^2}{\sin^2\theta_W \cos^2\theta_W} (Z_\mu Z^\mu W_\nu^+ W^{-\nu} - Z_\mu Z_\nu W^{+\mu} W^{-\nu}) \\
 & + \frac{1}{2} \frac{e^2}{\sin^2\theta_W} \kappa^2 (W_\mu^- W_\nu^+ W^{-\mu} W^{+\nu} - W_\mu^- W_\nu^+ W^{+\mu} W^{-\nu}) \quad .
 \end{aligned} \tag{2.4}$$

We are restricting our analysis to this form of the Lagrangian¹⁵⁾, which contains the anomalous magnetic dipole moment, κ , of the $W^\pm$ as the only free parameter. It is obtained from the more general four-parameter case¹⁴⁾ by requiring¹⁵⁾ vanishing s^2 terms in vector boson scattering amplitudes. The content of (2.4) may be summarized by noting that L_{SI} interpolates between the minimal electromagnetic substitution case of $\kappa = 0$ and the W -dominance (or, equivalently $SU(2)_L \times U(1)_Y$) case of $\kappa = 1$ in such a manner that the mentioned s^2 terms in vector boson scattering are absent.

The full expressions for the invariant amplitudes for the twelve vector boson scattering processes listed in the Table are given in Appendix A, together with the helicity basis necessary to compute the cross-sections, $\hat{\sigma}_i^{V_1 V_2 \rightarrow V_3 V_4}(s)$, ($i = TT, TT', LL, LT, TL$), for each of the subprocesses contributing to (2.1).

The amplitudes in Appendix A degenerate into the standard ones for $\kappa = 1$. In addition, using the Lagrangian for Higgs boson interactions given by

$$L_H = e \frac{M_W}{\sin \theta_W} (H W_\mu^+ W^{\mu-} + \frac{1}{2 \cos^2 \theta_W} H Z_\mu Z^\mu) , \quad (2.5)$$

the Higgs boson contributions to the various amplitudes have been computed and are also given in Appendix A.

3. Sensitivity under Variation of the Vector Boson Self-couplings

We will first study the twelve vector boson scattering subprocesses listed in the Table, and subsequently turn to their manifestations in e^+e^- collisions.

3.1 Numerical Results for Vector Boson Scattering

The twelve vector boson scattering processes, $V_1 V_2 \rightarrow V_3 V_4$, of the Table can be classified in four groups, with similar structure of the amplitudes (compare Appendix A) in each group. The reactions $(\gamma\gamma, \gamma Z, ZZ) \rightarrow W^+ W^-$ are characterized by $W^\pm$ exchange in the t and u channels, a quadrilinear term and possibly a Higgs exchange contribution in the s -channel for $ZZ \rightarrow W^+ W^-$. The processes $(\gamma W, ZW) \rightarrow (\gamma W, ZW)$ have $W^\pm$ exchange contributions in the s and u channels, a quadrilinear term and possibly Higgs exchange in the t channel for $ZW \rightarrow ZW$. The process $W^+ W^- \rightarrow W^+ W^-$ occurs through γ, Z (and possibly Higgs) exchange in the s and t channels and a quadrilinear term. Finally, $ZZ \rightarrow ZZ$ only receives Higgs exchange in the s, t and u channels. It completely vanishes if there is no Higgs contribution except in more general non-standard cases, where a $ZZZZ$ quadrilinear term is allowed^{13,14)}.

We will mainly discuss the energy dependence of the integrated cross-sections $\hat{\sigma}_i^{V_1 V_2 \rightarrow V_3 V_4}(s)$ for various choices of κ . All processes involving t or u channel exchange diagrams lead to a strong forward or backward peak in the angular distribution. Due to photon exchange, this is especially the case for $W^+ W^- \rightarrow W^+ W^-$. In the numerical computation we systematically put an angular cut allowing for $10^\circ \leq \theta^* \leq 170^\circ$ in the boson-boson centre-of-mass frame.

Results are presented in Figs. 3 to 6 and Appendix B for typical values of the anomalous magnetic dipole moment, κ , of the $W^\pm$ in the energy range extending from the threshold of about 200 GeV up to several TeV. We note that the curves become unrealistic for energies $\sqrt{s} \gtrsim 2$ TeV, where partial wave unitarity starts to be violated¹⁵⁾ by the tree diagrams. Standard model results ($\kappa=1$) with two choices of the Higgs mass (100 GeV and 1 TeV) are shown for comparison. They agree with earlier^{2,19)} results. We will concentrate our discussion on the transverse - transverse and the longitudinal - longitudinal cross-sections $\hat{\sigma}_{TT}(s)$ and $\hat{\sigma}_{LL}(s)$ shown in Figs. 3 to 6. Additional results for different reactions but with features similar to the ones shown in Figs. 3 to 6 are deferred to Appendix B.

The sensitivity of the cross-sections under variation of κ is striking at all energies, even near threshold. Cross-sections change by a full order of magnitude when one departs from $\kappa = 1$ by only one unit. This is due to the direct proportiona-

lity of the amplitudes to the squares of the trilinear couplings and to the quadri-linear ones. For example, the dependence on κ of $\hat{\sigma}^{\gamma\gamma \rightarrow W^+W^-}$ near threshold (c.m. momentum $p' \rightarrow 0$) in Fig. 3 is given by

$$\hat{\sigma}_{\text{TT}}^{\gamma\gamma \rightarrow W^+W^-} = \frac{\alpha\pi p'}{16M_W^3} (5(1+\kappa)^4 + 12\kappa^4 + (12\kappa^2-4)(1+\kappa)^2 + 8\kappa^2 + 20). \quad (3.1)$$

This explains the order of the curves corresponding to different values of κ in the limit of low values of s .

When the energy, $\sqrt{s}$, increases, a different dependence on κ becomes dominant which corresponds to the asymptotic behaviour studied in¹⁵⁾. The transverse - transverse and longitudinal - transverse cross-sections $\hat{\sigma}_{\text{TT}}(s)$ and $\hat{\sigma}_{\text{LT}}(s)$ decrease with increasing s for $\kappa = 1$, while for $\kappa \neq 1$ terms growing as s are present. As a consequence, the mentioned violations of partial wave unitarity occur for energies of approximately $\sqrt{s} \approx 2 \text{ TeV}$, the exact values depending¹⁵⁾ on the process under consideration and the precise value of κ . In the channels in which a longitudinal - longitudinal cross-section, $\hat{\sigma}_{\text{LL}}(s)$, exists, i.e., in the channels $W^+W^- \rightarrow W^+W^-$ (Fig. 4b), $W^+W^- \leftrightarrow ZZ$ (Fig. 5b) and $ZW \rightarrow ZW$ (Fig. 6b), the well-known growth of $\hat{\sigma}_{\text{LL}}(s)$ with s even for $\kappa = 1$ is present. In the $SU(2)_L \times U(1)_Y$ theory the associated violation of tree unitarity is cured by the exchange of a sufficiently light Higgs boson. This is seen explicitly in the curves for $m_H = 100 \text{ GeV}$ in Figs. 4b, 5b and 6b. When the Higgs mass enters the TeV range (see curves for $m_H = 1 \text{ TeV}$), standard model cross-sections and alternative ones have comparable magnitude but (obviously) different shapes as a function of $\sqrt{s}$. Altogether this drastically displays the need for a unitarizing mechanism, and it shows how the Higgs boson fulfils this purpose in a simple way provided we have $\kappa = 1$.

Finally, we observe that channels involving Z-bosons are more sensitively dependent on κ than channels involving photons. This is seen by comparing $WW \rightarrow ZZ$ (Figs. 5a,b) and $WZ \rightarrow WZ$ (Figs. 6a,b) with $\gamma\gamma \rightarrow WW$ (Fig. 3) and $\gamma W \rightarrow \gamma W$ (Fig. B1 in Appendix B), respectively. The stronger dependence on κ , if photons are replaced by Z-bosons, can be traced back to the κ dependence of the associated coupling constants in our Lagrangian (2.4).

3.2 Electron - Positron Collisions

Using the various vector boson scattering cross-sections calculated from the invariant amplitudes of Appendix A and displayed in Figs. 3 to 6 and B1 to B4 in Appendix B, the invariant mass distributions for the production of vector boson pairs in $e^+e^- \rightarrow l l' + V_3 V_4$ may now be computed with the help of the basic formula (2.1). In the no-tagging case the sum over $V_1 V_2$ in (2.1) is to be extended over all subprocesses leading to a given pair of vector bosons, $V_3 V_4$ (i.e. W^+W^- , ZZ , $ZW^\pm$, $\gamma W^\pm$, γZ , $\gamma\gamma$), in the final state. The subprocesses were listed in the Table.

Because of the particularly large photon luminosities, WW , ZW and γW production are dominated by $\gamma\gamma \rightarrow WW$, $\gamma W \rightarrow ZW$ and $\gamma W \rightarrow \gamma W$, respectively. In order to see the influence of the other subprocesses contributing to the production of these final states, and even more so, if one wishes to disentangle the various reactions leading to a given final boson pair, experiments under variable tagging (or anti-tagging) conditions are necessary. In this way one could perhaps have separate access to all the twelve subprocesses listed in the Table and studied in Sect. 3.1. In the following we will restrict ourselves to the six sums of processes of the Table and discuss the sensitivity of the e^+e^- cross-sections under variation of κ .

Invariant mass distributions, $d\sigma/d\tau$, at an e^+e^- centre-of-mass energy of 2 TeV are shown in Figs. 7 to 10 for the typical values of κ chosen in Sect. 3.1. The general trend of these distributions directly reflects the properties of the cross-sections for the subprocesses modified by the luminosity factors $L_{V_1 V_2, i}(\tau)$ given in (2.2). These factors are characterized by a strong decrease with increasing τ and the following relative order^{*)} (see Sect. 2 and Refs. 16) - 18)),

$$L_{TT} > L_{TL, LT} > L_{LL},$$

and

$$L_{\gamma\gamma} > L_{\gamma W} > L_{\gamma Z} > L_{WW} > L_{WZ} > L_{ZZ}.$$

The largest cross-section is obtained for W^+W^- pair production. It is shown in Figs. 7a to 7c. The change by approximately one order of magnitude under variation of κ by one unit in Fig. 7a reflects the dependence on κ of the cross-section of the subprocess $\gamma\gamma \rightarrow W^+W^-$ (Fig. 3). The dominating role of this subprocess can explicitly be seen in Figs. 7b and 7c, where the contributions of the various initial states, $\gamma\gamma$, γZ , WW and ZZ to WW production are plotted separately for fixed values of $\kappa = 1$ and $\kappa = 0$, respectively. The dominance of $\gamma\gamma \rightarrow W^+W^-$ is so strong that, for example, the presence of a 1 TeV Higgs boson in $(ZZ, WW) \rightarrow WW$ just manifests itself in a minor disturbance at the corresponding value of τ (Fig. 7b).

The cross-sections for $e^+e^- \rightarrow \gamma\gamma + \dots$ and $e^+e^- \rightarrow \gamma W + \dots$ in Figs. 8a,b are similar to the cross-section for $e^+e^- \rightarrow W^+W^- + \dots$ as regards the κ dependence, but they are smaller in magnitude. This is due to the close relationship of the processes $WW \rightarrow \gamma\gamma$, $\gamma\gamma \rightarrow WW$ and $\gamma W \rightarrow \gamma W$, and to the smaller magnitude of the

*) We will not concentrate on the precise values of these luminosities, as they are somewhat model dependent, as mentioned in Sect. 2, and as we are mainly concerned with the sensitivity of the various processes under variation of κ .

luminosities $L_{\gamma W}$ and L_{WW} , compared with $L_{\gamma\gamma}$.

Cross-sections for reactions in which Z-bosons are produced, i.e., $e^+e^- \rightarrow ZZ+\dots$, $e^+e^- \rightarrow ZW+\dots$ and $e^+e^- \rightarrow \gamma Z+\dots$, show a particularly strong dependence on κ , but are smaller in magnitude than the cross-section for $e^+e^- \rightarrow W^+W^-+\dots$. This is true in the low τ as well as in the high τ ($\tau \geq 0.1$) regions. The strong variation with κ is due to the particularly strong dependence on κ of the corresponding subprocesses already discussed in Sect. 3.1 (Figs. 5 and 6). The different κ dependence of ZZ and ZW compared with WW pair production is explicitly shown in Fig. 11, where the ratios

$$\frac{d\sigma}{d\tau} (e^+e^- \rightarrow e\nu + ZW) / \frac{d\sigma}{d\tau} (e^+e^- \rightarrow (e^+e^-, \nu\bar{\nu}) + WW)$$

and

$$\frac{d\sigma}{d\tau} (e^+e^- \rightarrow (e^+e^-, \nu\bar{\nu}) + ZZ) / \frac{d\sigma}{d\tau} (e^+e^- \rightarrow (e^+e^-, \nu\bar{\nu}) + WW)$$

are plotted. They vary by approximately an order of magnitude for low as well as for high values of τ . These ratios of cross-sections are expected to be less sensitive to systematic experimental errors and theoretical ambiguities in the interpretation of the data.

4. Conclusions

We end with the following summarizing remarks:

- (i) Due to the structure of the Feynman diagrams describing the scattering of vector bosons on each other, it is a priori clear that a measurement of the cross-sections for the various processes should be particularly well suited for a study and a precise determination of the vector boson self-couplings. In contrast to the amplitude for $e^+e^-(q\bar{q}) \rightarrow W^+W^-$, which depends on trilinear vector boson couplings only and also receives contributions due to lepton exchange which often dominate, vector boson scattering solely depends on trilinear and quadrilinear boson couplings.
- (ii) Evaluating the cross-sections for vector boson ($W^\pm, Z_0, \gamma$) scattering, $V_1V_2 \rightarrow V_3V_4$ within a recently suggested model for self-interactions which is constrained by reasonable physical assumptions to contain a single free parameter, one finds extremely strong sensitivity under variation of this free parameter, κ (the anomalous magnetic dipole moment of the $W^\pm$). Cross-sections change by approximately one order of magnitude, if κ is varied by one unit around its standard value of $\kappa = 1$.

(iii) The strong sensitivity of vector boson scattering under variation of κ is reflected in the behaviour of the (measurable!) reaction $e^+e^- \rightarrow \ell\ell' + VV'$. Considering the no-tagging (as regards $\ell\ell'$) case at an e^+e^- energy of 2 TeV (CLIC), one finds that also the cross-section for $e^+e^- \rightarrow \ell\ell' + VV'$ changes by an order of magnitude, if κ is varied by one unit around $\kappa = 1$. This is true in the low τ region, where cross-sections are large, as well as in the high τ region, where possibly interesting effects due to a unitarizing mechanism (e.g. a heavy Higgs boson) appear. This change by an order of magnitude is to be compared with a change by a factor of 2 in the cross-section for $e^+e^- \rightarrow W^+W^-$ at LEP 200.

(iv) As the cross-section for $e^+e^- \rightarrow W^+W^-$ can also be measured in the TeV energy range at CLIC, a brief comparative remark on the processes $e^+e^- \rightarrow W^+W^-$ and $e^+e^- \rightarrow \ell\ell' + VV'$ seems mandatory. If one assumes $SU(2)_L \times U(1)_Y$ couplings, the logarithmically increasing cross-section for $e^+e^- \rightarrow \ell\ell' + VV'$ equals¹⁹⁾ the decreasing (such as s^{-1}) cross-section for $e^+e^- \rightarrow W^+W^-$ at an energy of about 1 TeV, and the magnitude of the cross-sections at this energy is approximately 1 pb. The strong increase²⁰⁾ of the integrated cross-section for $e^+e^- \rightarrow W^+W^-$ by about two orders of magnitude when changing κ from $\kappa = 1$ by one unit, may allow for a precise determination of κ even at the 1% level, provided one finds agreement with the $SU(2)_L \times U(1)_Y$ predictions and is capable of coping with the small rate and the extremely strong forward peaking of the cross-section. An empirical study of the process $e^+e^- \rightarrow \ell\ell' + VV'$, on the other hand, will be particularly rewarding and practically unavoidable, if deviations from the $SU(2)_L \times U(1)_Y$ predictions are found in the reaction $e^+e^- \rightarrow W^+W^-$. Such deviations cannot uniquely be attributed to the trilinear boson couplings, but may originate from both the vector boson self-couplings and the vector boson lepton couplings contributing to $e^+e^- \rightarrow W^+W^-$ via neutrino exchange. Even more so, as deviations in the vector-boson self-couplings are likely to be accompanied also by deviations in the vector boson lepton couplings (e.g., if the vector bosons are of composite nature²¹⁾), an empirical study of both processes is needed to disentangle the effects in the bosonic and fermionic sectors.

(v) Effects similar to the ones discussed in the present paper for e^+e^- interactions are expected to be present in $p\bar{p}$ interactions at SSC energies. Technically, the vector boson luminosities in e^+e^- beams have to be replaced by the vector boson luminosities in $p\bar{p}$ beams. Background problems are expected to be more severe, however.

(vi) The approach of the present paper is conservative insofar as the underlying Lagrangian is constrained to contain a single free parameter only, and no other interactions apart from the boson self-interactions are assumed to be present. Many entirely new phenomena (partners of the $W^\pm$, Z , excited vector bosons etc.) might actually be hidden in the TeV energy range and make their appearance in vector boson scattering. Nevertheless, the present study indicates the strong potential of an e^+e^- collider in the TeV energy range to reveal the true nature of the intermediate vector bosons.

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Appendix A

In this Appendix we will explicitly give the matrix elements for the twelve vector boson scattering processes listed in the Table. For generality, the amplitudes are computed within the general four-parameter KMSS model¹⁴⁾ according to Feynman rules given in¹⁴⁾. The Lagrangian for the vector boson self-interactions contains two arbitrary trilinear couplings, $\hat{g}$ and κ , and two arbitrary quadri-linear ones, $\hat{g}$ and $\tilde{g}$ and reduces to the single parameter (κ) Lagrangian¹⁵⁾ (2.4) upon inserting the relations

$$\hat{g} = \frac{e}{\sin\theta_W} \kappa ,$$

$$\hat{g} = \hat{g}^2 , \quad (A1)$$

$$\tilde{g} = 0.$$

For the numerical evaluation of the cross-sections we will use (A1) in the expressions for the scattering amplitudes to be given below, thus reducing the number of free parameters to a single one, κ . The amplitudes will be written in terms of the coupling constants $g_{ZWW} \equiv g_Z$ and κ_Z related to $\hat{g}$ and κ via

$$g_Z = \frac{\hat{g}}{\cos\theta_W} - e \tan\theta_W, \quad (A2)$$

$$\kappa_Z g_Z = \frac{\hat{g}}{\cos\theta_W} - e \kappa \tan\theta_W.$$

Standard notation will be used for the kinematics. Four-momenta and polarization vectors of the vector bosons appearing in

$$V_1 + V_2 \rightarrow V_3 + V_4 \quad (A3)$$

will be denoted by p_i ($i = 1, \dots, 4$) and ε_i ($i = 1, \dots, 4$), respectively. The usual invariants s , t and u are related to the momenta by

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2,$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2, \quad (A4)$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2.$$

In the c.m. frame of reaction (A3), where the momenta have the components

$$\begin{aligned}
 p_1^\mu &= (p_1^0, 0, 0, p), \\
 p_2^\mu &= (p_2^0, 0, 0, -p), \\
 p_3^\mu &= (p_3^0, p' \sin \theta, 0, p' \cos \theta), \\
 p_4^\mu &= (p_4^0, -p' \sin \theta, 0, -p' \cos \theta),
 \end{aligned}
 \tag{A5}$$

the polarization vectors for the transverse ($\lambda = \pm 1$) and longitudinal ($\lambda = 0$) helicity states are given by

$$\begin{aligned}
 \epsilon_1^\mu(\lambda_1 = \pm 1) &= \frac{1}{\sqrt{2}} (0, -\lambda_1, -i, 0), \\
 \epsilon_1^\mu(\lambda_1 = 0) &= \frac{1}{M_1} (p, 0, 0, p_1^0), \\
 \epsilon_2^\mu(\lambda_2 = \pm 1) &= \frac{1}{\sqrt{2}} (0, \lambda_2, -i, 0), \\
 \epsilon_2^\mu(\lambda_2 = 0) &= \frac{1}{M_2} (-p, 0, 0, p_2^0), \\
 \epsilon_3^{*\mu}(\lambda_3 = \pm 1) &= \frac{1}{\sqrt{2}} (0, -\lambda_3 \cos \theta, i, \lambda_3 \sin \theta), \\
 \epsilon_3^{*\mu}(\lambda_3 = 0) &= \frac{1}{M_3} (p', p_3^0 \sin \theta, 0, p_3^0 \cos \theta), \\
 \epsilon_4^{*\mu}(\lambda_4 = \pm 1) &= \frac{1}{\sqrt{2}} (0, \lambda_4 \cos \theta, i, -\lambda_4 \sin \theta), \\
 \epsilon_4^{*\mu}(\lambda_4 = 0) &= \frac{1}{M_4} (-p', p_4^0 \sin \theta, 0, p_4^0 \cos \theta).
 \end{aligned}
 \tag{A6}$$

The amplitudes of the vector boson scattering processes are conveniently expressed in terms of the functions $f^S(\kappa)$, $f^t(\kappa)$ and $h^S(\kappa, \kappa_Z)$ and $h^t(\kappa, \kappa_Z)$ defined by

$$\begin{aligned}
 f^S(\kappa) &= [(\epsilon_1 \epsilon_2)(p_2 - p_1)_\lambda - (1 + \kappa)(\epsilon_1 p_2)_\lambda \epsilon_2 + (1 + \kappa)(\epsilon_2 p_1)_\lambda \epsilon_1] \\
 &\quad \cdot [(\epsilon_3^* \epsilon_4^*)(p_3 - p_4)^\lambda - (1 + \kappa)(\epsilon_4^* p_3)^\lambda \epsilon_3^* + (1 + \kappa)(\epsilon_3^* p_4)^\lambda \epsilon_4^*] \\
 f^t(\kappa) &= \{p_2 \leftrightarrow -p_3, \epsilon_2 \leftrightarrow \epsilon_3^* \text{ in } f^S(\kappa)\}
 \end{aligned}
 \tag{A7}$$

$$\begin{aligned}
 h^t(\kappa, \kappa_Z) = & [2(\epsilon_1 p_3) \epsilon_{3\lambda}^* - (\epsilon_1 \epsilon_3^*)(p_3 + \kappa p_1)_\lambda + (1 + \kappa)(p_1 \epsilon_3^*) \epsilon_{1\lambda}] \\
 & \cdot [2(\epsilon_2 p_4) \epsilon_{4\lambda}^* - (\epsilon_2 \epsilon_4^*)(p_4 + \kappa_Z p_2)_\lambda + (1 + \kappa_Z)(p_2 \epsilon_4^*) \epsilon_{2\lambda}^*] \\
 & + [(\epsilon_1 \epsilon_3^*) \{\kappa p_1^2 + p_3^2 - (1 - \kappa)(p_1 p_3)\} + (\epsilon_1 p_3)(\epsilon_3^* p_1)] \\
 & \cdot [(\epsilon_2 \epsilon_4^*) \{\kappa_Z p_2^2 + p_4^2 - (1 - \kappa_Z)(p_2 p_4)\} + (\epsilon_2 p_4)(\epsilon_4^* p_2)] / M_W^2
 \end{aligned} \tag{A7}$$

$$h^u(\kappa, \kappa_Z) = \{p_3 \leftrightarrow p_4, \epsilon_3^* \leftrightarrow \epsilon_4^* \text{ in } h^t(\kappa, \kappa_Z)\}$$

The amplitudes for the various processes now become:

$$\begin{aligned}
 A(W^+ W^- \rightarrow W^+ W^-) \\
 = & -ie^2 \{f^S(\kappa)/s + f^t(\kappa)/t\} \\
 & -ig_Z^2 \{f^S(\kappa_Z)/(s - M_Z^2) + f^t(\kappa_Z)/(t - M_Z^2)\} \\
 & + i\hat{g} \{2(\epsilon_1 \epsilon_4^*)(\epsilon_2 \epsilon_4^*) - (\epsilon_1 \epsilon_2)(\epsilon_3^* \epsilon_4^*) - (\epsilon_1 \epsilon_3^*)(\epsilon_2 \epsilon_4^*)\} \\
 & - i\tilde{g} \{2(\epsilon_1 \epsilon_2)(\epsilon_3^* \epsilon_4^*) + (\epsilon_1 \epsilon_3^*)(\epsilon_2 \epsilon_4^*)\}, \\
 A(\gamma\gamma \rightarrow W^+ W^-) \\
 = & -ie^2 \{h^t(\kappa, \kappa)/(t - M_W^2) + h^u(\kappa, \kappa)/(u - M_W^2)\} \\
 & -ie^2 \{2(\epsilon_1 \epsilon_2)(\epsilon_3^* \epsilon_4^*) - (\epsilon_1 \epsilon_3^*)(\epsilon_2 \epsilon_4^*) - (\epsilon_1 \epsilon_4^*)(\epsilon_2 \epsilon_3^*)\},
 \end{aligned} \tag{A8}$$

$$\begin{aligned}
 A(\gamma Z \rightarrow W^+ W^-) \\
 = & -ieg_Z \{h^t(\kappa, \kappa_Z)/(t - M_W^2) + h^u(\kappa, \kappa_Z)/(u - M_W^2)\} \\
 & -ieg_Z \{2(\epsilon_1 \epsilon_2)(\epsilon_3^* \epsilon_4^*) - (\epsilon_1 \epsilon_3^*)(\epsilon_2 \epsilon_4^*) - (\epsilon_1 \epsilon_4^*)(\epsilon_2 \epsilon_3^*)\},
 \end{aligned}$$

$$\begin{aligned}
 A(ZZ \rightarrow W^+ W^-) \\
 = & -ig_Z^2 \{h^t(\kappa_Z, \kappa_Z)/(t - M_W^2) + h^u(\kappa_Z, \kappa_Z)/(u - M_W^2)\} \\
 & -i(g_Z^2 + \frac{\hat{g} - \hat{g}^2}{\cos^2 \theta_W}) \{2(\epsilon_1 \epsilon_2)(\epsilon_3^* \epsilon_4^*) - (\epsilon_1 \epsilon_3^*)(\epsilon_2 \epsilon_4^*) - (\epsilon_1 \epsilon_4^*)(\epsilon_2 \epsilon_3^*)\},
 \end{aligned}$$

$$A(W^+W^- \rightarrow \gamma\gamma)$$

$$= \{ p_1 \Leftrightarrow -p_3, p_2 \Leftrightarrow -p_4, \varepsilon_1 \Leftrightarrow \varepsilon_3^*, \varepsilon_2 \Leftrightarrow \varepsilon_4^* \text{ in } A(\gamma\gamma \rightarrow W^+W^-) \},$$

$$A(W^+W^- \rightarrow \gamma Z)$$

$$= \{ p_1 \Leftrightarrow -p_3, p_2 \Leftrightarrow -p_4, \varepsilon_1 \Leftrightarrow \varepsilon_3^*, \varepsilon_2 \Leftrightarrow \varepsilon_4^* \text{ in } A(\gamma Z \rightarrow W^+W^-) \},$$

$$A(W^+W^- \rightarrow ZZ)$$

$$= \{ p_1 \Leftrightarrow -p_3, p_2 \Leftrightarrow -p_4, \varepsilon_1 \Leftrightarrow \varepsilon_3^*, \varepsilon_2 \Leftrightarrow \varepsilon_4^* \text{ in } A(ZZ \rightarrow W^+W^-) \},$$

$$A(\gamma W^\pm \rightarrow \gamma W^\pm)$$

$$= \{ p_2 \Leftrightarrow -p_3, \varepsilon_2 \Leftrightarrow \varepsilon_3^* \text{ in } A(\gamma\gamma \rightarrow W^+W^-) \},$$

$$A(\gamma W^\pm \rightarrow ZW^\pm)$$

$$= \{ p_2 \Leftrightarrow -p_3, \varepsilon_2 \Leftrightarrow \varepsilon_3^* \text{ in } A(\gamma Z \rightarrow W^+W^-) \},$$

$$A(ZW^\pm \rightarrow \gamma W^\pm)$$

$$= \{ p_1 \Leftrightarrow -p_3, p_2 \Leftrightarrow -p_4, \varepsilon_1 \Leftrightarrow \varepsilon_3^*, \varepsilon_2 \Leftrightarrow \varepsilon_4^* \text{ in } A(\gamma W \rightarrow ZW) \},$$

$$A(ZW^\pm \rightarrow ZW^\pm)$$

$$= \{ p_2 \Leftrightarrow -p_3, \varepsilon_2 \Leftrightarrow \varepsilon_3^* \text{ in } A(ZZ \rightarrow W^+W^-) \},$$

$$A(ZZ \rightarrow ZZ)$$

$$= - \frac{i2\tilde{g}}{\cos^4\theta_W} \{ (\varepsilon_1 \varepsilon_2) (\varepsilon_3^* \varepsilon_4^*) + (\varepsilon_1 \varepsilon_3^*) (\varepsilon_2 \varepsilon_4^*) + (\varepsilon_1 \varepsilon_4^*) (\varepsilon_2 \varepsilon_3^*) \}$$

The amplitudes for the $SU(2)_L \times U(1)_Y$ self-interactions are obtained by inserting (A1) with $\kappa = 1$ in the expressions above and adding the Higgs scalar exchange contributions. These are given by

(A9)

$$\begin{aligned}
 & -ie^2 \frac{M_W^2}{\sin^2 \theta_W} \left(\frac{\varepsilon_1 \cdot \varepsilon_2 \varepsilon_3 \cdot \varepsilon_4}{s - m_H^2} + \frac{\varepsilon_1 \cdot \varepsilon_3 \varepsilon_2 \cdot \varepsilon_4}{t - m_H^2} \right), & \text{for } W^+ W^- \rightarrow W^+ W^-, \\
 & -ie^2 \frac{M_W^2}{\sin^2 \theta_W \cos^2 \theta_W} \frac{\varepsilon_1 \cdot \varepsilon_2 \varepsilon_3 \cdot \varepsilon_4}{s - m_H^2}, & \text{for } ZZ \leftrightarrow W^+ W^-, \\
 & -ie^2 \frac{M_W^2}{\sin^2 \theta_W \cos^2 \theta_W} \frac{\varepsilon_1 \cdot \varepsilon_2 \varepsilon_3 \cdot \varepsilon_4}{t - m_H^2}, & \text{for } ZW^\pm \rightarrow ZW^\pm, \\
 & -ie^2 \frac{M_W^2}{\sin^2 \theta_W \cos^4 \theta_W} \left(\frac{\varepsilon_1 \cdot \varepsilon_2 \varepsilon_3 \cdot \varepsilon_4}{s - m_H^2} + \frac{\varepsilon_1 \cdot \varepsilon_3 \varepsilon_2 \cdot \varepsilon_4}{t - m_H^2} + \frac{\varepsilon_1 \cdot \varepsilon_4 \varepsilon_2 \cdot \varepsilon_3}{u - m_H^2} \right), \\
 & & \text{for } ZZ \rightarrow ZZ.
 \end{aligned}$$

Cross-sections for given helicity states are then computed according to

$$\hat{\sigma}(\lambda_1 \lambda_2 \lambda_3 \lambda_4) = \frac{p'}{64 \pi^2 s p} \int d\Omega |A(\lambda_1 \lambda_2 \lambda_3 \lambda_4)|^2. \quad (A10)$$

The combinations occurring in the definition of $\frac{d\sigma}{d\tau}$ in eq. (2.1) are the following ones¹⁸⁾

$$\begin{aligned}
 \hat{\sigma}_{TT} &= \frac{1}{4} [\sigma(++++) + \sigma(----) + \sigma(+--+) + \sigma(-+-+)] \\
 \hat{\sigma}_{LL} &= \sigma(0000) \\
 \hat{\sigma}_{TL} &= \frac{1}{2} [\sigma(+0+0) + \sigma(-0-0)] \\
 \hat{\sigma}_{LT} &= \frac{1}{2} [\sigma(0+0+) + \sigma(0-0-)] \\
 \hat{\sigma}'_{TT} &= \frac{1}{4} [\sigma(++++) + \sigma(----) - \sigma(+--+) - \sigma(-+-+)].
 \end{aligned} \quad (A11)$$

Appendix B

In this Appendix we collect in graphical form the numerical results for a few additional vector boson scattering processes.

Fig. B1: The cross-section for γW scattering for transversely (a) and longitudinally (b) polarized W-bosons in the initial state.

Fig. B2: The cross-sections for $WW \rightarrow \gamma\gamma$ for transversely (a) and longitudinally (b) polarized W-bosons.

Fig. B3: The cross-sections $\gamma W \rightarrow ZW$ for transversely (a) and longitudinally (b) polarized W-bosons.

Fig. B4: The cross-sections for $\gamma Z \rightarrow W^+W^-$ for transverse (a) and longitudinal (b) Z polarization.

Fig. B1a

Fig. B1b

Fig. B2a

Fig. B2b

Fig. B3a

Fig. B3b

Fig. B4a

Fig. B4b

Figure Captions

- Fig. 1.: The reaction $e^+e^- \rightarrow l l' + V_3 V_4$ proceeding via the subprocess $V_1 V_2 \rightarrow V_3 V_4$.
- Fig. 2.: Vector boson scattering via trilinear and quadrilinear self-interactions and exchange of the Higgs scalar.
- Fig. 3.: Cross-section for production of W^+W^- pairs by transversely polarized photons as a function of the c. m. energy, $\sqrt{s}$, for various values of the single free parameter, κ , in the Lagrangian (2.4). The parameter κ is identical with the anomalous magnetic dipole moment of the $W^\pm$, but all trilinear and quadrilinear vector boson couplings in (2.4) are functions of κ . In the $SU(2)_L \times U(1)_Y$ electroweak theory, $\kappa = 1$. Due to violations of unitarity bounds, the curves in Figs. 3 to 6 become unrealistic for $\sqrt{s} \gtrsim 2$ TeV.
- Fig. 4.: Cross-sections for W^+W^- scattering for transversely and longitudinally polarized W^+W^- pairs in the initial state for various values of κ . For comparison, the $SU(2)_L \times U(1)_Y$ predictions, assuming Higgs boson masses of $m_H = 100$ GeV and $m_H = 1$ TeV, are also shown.
- Fig. 5.: Cross-sections for production of Z boson pairs by transversely and longitudinally polarized $W^\pm$ bosons for various values of κ . For comparison, the $SU(2)_L \times U(1)_Y$ predictions, assuming Higgs boson masses of $m_H = 100$ GeV and $m_H = 1$ TeV, are also shown.
- Fig. 6.: Cross-sections for ZW scattering for transversely and longitudinally polarized initial particles for various values of κ . For comparison, the $SU(2)_L \times U(1)_Y$ predictions, assuming Higgs boson masses of $m_H = 100$ GeV and $m_H = 1$ TeV, are also shown.
- Fig. 7a.: Cross-section for $e^+e^- \rightarrow (e^+e^-, \nu\bar{\nu}) + W^+W^-$ as a function of $\tau \equiv s/4E_e^2$ for various values of κ , including the standard model with $\kappa = 1$ and a mass of the Higgs boson of $m_H = 100$ GeV.
- Fig. 7b,c: Cross-sections for the various subprocesses contributing to $e^+e^- \rightarrow (e^+e^-, \nu\bar{\nu}) + W^+W^-$ for the cases of $\kappa = 1.0$ (Fig. 7b) and $\kappa = 0.0$ (Fig. 7c).

- Fig. 8.: Cross-sections for $e^+e^- \rightarrow \nu\bar{\nu} + \gamma\gamma$ (Fig. 8a) and for $e^+e^- \rightarrow e\nu + \gamma W$ (Fig. 8b) for various values of κ .
- Fig. 9.: Cross-sections for $e^+e^- \rightarrow (e^+e^-, \nu\bar{\nu}) + ZZ$ and for $e^+e^- \rightarrow e\nu + ZW$ for various values of κ .
- Fig. 10.: Cross-sections for $e^+e^- \rightarrow \nu_e\bar{\nu}_e + \gamma Z$ for various values of κ .
- Fig. 11.: The ratios of ZZ to WW (Fig. 11a) and of ZW to WW production (Fig. 11b) for various values of κ .

Fig. 1

Fig. 2

Fig. 3

Fig. 4a

Fig. 4b

Fig. 5a

Fig. 5b

Fig. 6a

Fig. 6b

Figure 7a

Fig. 7b

Fig. 7c

Fig. 8a

Fig. 8b

Figure 9

Fig. 10

Fig. 11a

Fig. 11b