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**PHENOMENOLOGY OF SUPERSTRINGS
AND
SEARCHES FOR SUPERSYMMETRIC DARK MATTER**

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1. LOW-ENERGY EFFECTIVE THEORY FROM THE HETEROTIC STRING

1.1 Strategies for String Phenomenology

I believe in string theory¹ because it is the only framework we have which seems able to reconcile gravity with quantum mechanics. String theory also seems rich enough to incorporate all the known elementary particles and their interactions. However, the manner in which these are incorporated is as yet highly ambiguous. The road from some two-dimensional conformal field theory formulated on the world sheet at the Planck scale to four-dimensional physics in real space-time at energies ≤ 1 TeV is very long and tortuous, with many forks where one does not know which turning to take. It may be that this ambiguity will be resolved purely theoretically by an understanding of the mechanism whereby string dynamics selects a ground state. Even in this case, one must learn how to do phenomenology with this understanding. Alternatively, it is possible that purely phenomenological arguments and model-building will help us towards the correct string vacuum, much as studies of pseudoscalar meson dynamics helped us understand the QCD vacuum.

There are many possible strategies for string phenomenology, some of which are charted in Fig. 1. Two of these lectures are concerned with the extraction of a low-energy effective field theory from an underlying string theory. The first treats strategies based on the ten-dimensional heterotic superstring,² either reduced to a ten-dimensional supergravity theory by expanding the theory in powers of the slope parameter α' and then compactifying the six surplus dimensions on a manifold,³ or else by compactifying the heterotic superstring directly on an orbifold.⁴ The second lecture describes some formulations of string theories directly in four dimensions,⁵⁻⁹ and the extraction from them of an effective $N = 1$, $D = 4$ supergravity field theory. The breaking of local supersymmetry and its communication to the observable sector will be discussed only briefly, as they form the main subject of lectures given here by Mary K. Gaillard.¹⁰ My third lecture discusses the possible phenomenology which could emerge from some Calabi-Yau manifold compactification of the heterotic string,³ including searches for additional neutral currents, gauge bosons, and matter particles. Superstring-inspired models, like most other supersymmetric models, predict the existence of a stable supersymmetric relic particle left over from the Big Bang.¹¹ My fourth lecture discusses the particle physics and astrophysical motivations for supersymmetric dark matter, and how to search for it. The recent measurement by the European Muon Collaboration (EMC) of the spin-dependent electroproduction structure function¹² could have a dramatic impact on these searches, as will be discussed at the end of the fourth lecture.

A common element in all the strategies for string phenomenology charted in Fig. 1 is the appearance of $N = 1$, $D = 4$ no-scale supergravity,¹³⁻¹⁶ whose structure I now review as a prelude to my lectures and to those of Mary K. Gaillard.¹⁰ A generic $N = 1$, $D = 4$ supergravity theory with at most two space-time derivatives of fields is characterized by two functions.¹⁷ One is the Kähler potential $\mathcal{G}$, a real function of the complex chiral superfields ϕ which might include

$$G \ni |\phi|^2, \ln(\lambda\phi^3 + \mu\phi^2)^2, \dots \quad (1.1)$$

The other function is the gauge kinetic function f_{ab} , which is a holomorphic function of the complex chiral superfields ϕ (a, b are indices representing gauge group generators). Possible forms for f_{ab} include

$$f_{ab} \ni \delta_{ab}, \delta_{ab}\phi, \dots \quad (1.2)$$

The Kähler potential G plays many roles,¹⁷ including that of determining the kinetic energy terms for the chiral fields via

$$\mathcal{L}_k = G_i^j \partial_\mu \phi^i \partial^\mu \phi_j^* : G_i^j \equiv \frac{\partial^2 G}{\partial \phi^i \partial \phi_j^*} \quad (1.3)$$

For example, the first term in (1.1) would give a canonical kinetic energy term. The Kähler potential also determines the tree-level effective potential:

$$V = e^G [G_i (G^{-1})^i_j G^j - 3] + (\text{D-terms}) : G_i \equiv \frac{\partial G}{\partial \phi^i} \quad (1.4)$$

from which we see that the second term in (1.1) would give the F-term associated with a ϕ^3 superpotential. The gauge kinetic function f also plays key roles,¹⁷ for example that of determining the gauge coupling strength

$$g^2 = \frac{1}{\text{Re} f} \quad (1.5)$$

and the vacuum angle parameter

$$\theta = \text{Im} f / \text{Re} f \quad (1.6)$$

which violates CP.

Fig. 1. Possible strategies for superstring phenomenology.

The signature for supersymmetry breaking is a non-zero gravitino mass

$$m_{3/2} = e^{G/2} \quad (1.7)$$

which can arise from the super-Higgs mechanism, whereby a helicity $\pm 3/2$ massless gravitino eats a massless helicity $\pm 1/2$ Goldstone fermion

$$\chi = G_i f^i \quad (1.8)$$

Once supersymmetry is broken, supersymmetry-breaking masses $m_{1/2}$ for the gauginos,

$$m_{1/2} = \frac{e^{G/2}}{4 \text{Re} f} G_i (G'')^{-1}_{ij} \frac{\partial f^j}{\partial \phi_i^*} \quad (1.9)$$

and $(m_0^2)_i = \partial^2 V / \partial \phi^i \partial \phi_j$ for the scalar particles, are possible, as well as other soft supersymmetry-breaking terms in the low-energy effective potential. The generic possibility is¹⁸

$$V \simeq \sum_i \left| \frac{\partial P}{\partial \phi_i} \right|^2 + (\text{D-terms}) \\ + \sum_i m_0^2 |\phi_i|^2 + \left(\sum_\lambda A_\lambda \lambda \phi^3 + \sum_\mu B_\mu \mu \phi^2 + \text{h.c.} \right) \quad (1.10)$$

where the A_λ and B_μ are model-dependent coefficients with the dimensions of masses.

In the special class of no-scale supergravity models recurring in low-energy effective field theories derived from strings, the effective potential (1.4) or (1.10) is flat in at least some field directions. The simplest case^{13,15} is that of a single chiral field z for which

$$V = e^{G/2} \left(G_z (G_z^*)^{-1} G_z^* - 3 \right) = 9 e^{\frac{4G/3}{G_z^*}} \left(\partial_z \partial_z^* e^{-G/3} \right) \quad (1.11)$$

The expression (1.11) vanishes if

$$G = -3 \ln (f(z) + f^*(z^*)) \quad (1.12)$$

which we can always reparametrize as

$$G = -3 \ln (z + z^*) \quad (1.13)$$

In this case the field z parametrizes a non-compact $SU(1,1)/U(1)$ coset manifold, where the $SU(1,1)$ transformations are

$$z \rightarrow \frac{\alpha z + i\beta}{i\delta z + i\epsilon} : \alpha\delta + \beta\epsilon = 1 \quad (1.14)$$

A second example of a no-scale model with more fields is¹⁶

$$G = -3 \ln (z + z^* + g(\phi^i, \phi_i^*)) \quad (1.15)$$

which also has a potential $V = 0$. The simplest variant of (1.15), which is unique to first non-trivial order in the ϕ^i appearing in g , is

$$g = -\phi^i \phi_i^* : i = 1, 2, \dots, N \quad (1.16)$$

which provides a theory on an $SU(N+1, 1)/SU(N+1) \times U(1)$ manifold. The global $SU(N+1)$ symmetry of this manifold can be made manifest by the holomorphic field redefinitions

$$z = \frac{1}{2} \frac{1 - y^0/\sqrt{3}}{1 + y^0/\sqrt{3}}, \quad \phi^i = \frac{y^i/\sqrt{3}}{1 + y^0/\sqrt{3}} \quad (1.17)$$

Fig. 2. Radiative corrections may drive $m_H^2 < 0$ triggering gauge symmetry breaking, while m_q^2, m_l^2 remain > 0 .

in terms of which

$$G = -3 \ln \left(1 - \frac{1}{3} y^I y_I^* \right) + \ln |P|^2, \quad I=0,1,\dots,N \quad (1.18)$$

Other, more general, no-scale models are also known.¹⁹

No-scale supergravity models first attracted attention a few years ago because they had fields whose vacuum expectation values (v.e.v.'s) were undetermined at the classical level, and one could hope to fix them by quantum corrections at an energy scale very different from the Planck mass present in the theory at the tree level.¹⁴ This strategy²⁰ for model-building exploits renormalization group-improved²¹ radiative corrections to the soft supersymmetry-breaking parameters ($m_0, m_{1/2}, A_\lambda, B_\mu$) in the effective low-energy theory (1.10) obtained from some Theory of Everything (TOE) as illustrated in Fig. 2. As we decrease the renormalization scale Q from close to the Planck mass m_p where initial values for these parameters are fixed, m_q^2, m_l^2 , and m_H^2 vary logarithmically. We can hope that m_H^2 is driven negative at some small scale Q , so that weak gauge symmetry breaking is possible via the Higgs mechanism and

$$m_w \sim Q \sim m_p \exp(-O(1)/\alpha) \quad (1.19)$$

Gauge fields tend to increase m_0^2 as Q decreases, whilst Yukawa couplings λ tend to drive $m_0^2 < 0$ at low Q . A typical scenario^{14-16,20} is that the α in (1.19) is

$$\alpha_\lambda = \frac{\lambda^2}{4\pi} : \lambda H q \bar{q} \Rightarrow m_q = \lambda \langle 0|H|0 \rangle \quad (1.20)$$

where q is the heaviest quark, most likely the top quark with

$$m_t/m_w = 2^{0 \pm 1} \quad (1.21)$$

as favoured by experiment.²²⁻²⁵ This scenario can be implemented in superstring-inspired models, and will lie behind some of the Calabi-Yau phenomenology in Lecture 3.

1.2 Manifold Compactification of the Heterotic String

Historically, this was the first phenomenological strategy³ of those in Fig. 1 to be pursued, and it is still one of the most developed. Although it can be discussed directly at the string level, here we will approach it by first taking the zero-slope limit, i.e. by making a series expansion in α' where the string mass squared spectrum is

$$M^2 = M_0^2 + n/\alpha' \quad (1.22)$$

with $M_0 = 0$, at least as a first approximation. The massless particles of the 10-dimensional heterotic string are just those of $N = 1, D = 10$, supergravity.²⁶ They include the following bosons: a graviton $g_{\mu\nu}$, a dilaton ϕ , an antisymmetric tensor field $B_{\mu\nu}$ and Yang-Mills fields A_μ . It is well known that the

Lagrangian of $N = 1$, $D = 10$ supergravity is unique to second order in the space-time derivatives of the fields. Its purely bosonic part is

$$\mathcal{L}_{N=1, D=10} = -\frac{1}{2} R^{(10)} - \frac{9}{16} \left(\frac{\partial \phi}{\phi} \right)^2 - \frac{3}{4} \phi^{-3/4} F_{\mu\nu} F^{\mu\nu} - \frac{3}{4} \phi^{-3/2} H_{\mu\nu\rho} H^{\mu\nu\rho} \quad (1.23)$$

where $R^{(10)}$ is the familiar Einstein term in ten dimensions, $F_{\mu\nu}$ is the usual Yang-Mills field strength, and $H_{\mu\nu\rho}$ is a field strength for the antisymmetric tensor field

$$H_{\mu\nu\rho} = \frac{1}{3} \left[\partial_\mu B_{\nu\rho} - \frac{1}{4} (\omega_{3Y})_{\mu\nu\rho} + \frac{1}{4} (\omega_{3L})_{\mu\nu\rho} + \text{cyclic perms.} \right] \quad (1.24)$$

where ω_{3Y} and ω_{3L} are the familiar Yang-Mills and Lorentz 3-forms.^{26,27}

In view of the uniqueness of (1.23), we do not need to derive it once we know that the massless modes in string theory (1.22) have the right spectrum and symmetries. However, it is instructive and useful for the subsequent discussion of non-perturbative effects to sketch a technique^{28,29} for deriving the $N = 1$, $D = 10$ Lagrangian (1.23) from the string using the Polyakov³⁰ non-linear σ -model approach. In this method, we make a functional integral over string configurations with the action

$$S = \frac{1}{4\pi} \int d^2 z \left[\frac{1}{2} (\det \gamma)^{\frac{1}{2}} \gamma^{mn} G_{\mu\nu}(X) \partial_m X^\mu(z) \partial_n X^\nu(z) + \frac{1}{2} \epsilon^{mn} B_{\mu\nu}(X) \partial_m X^\mu(z) \partial_n X^\nu(z) + \alpha' \sqrt{\det \gamma} R^{(2)} \Phi(X) + \dots \right] \quad (1.25)$$

where z is a complex variable parametrizing the world-sheet located in 10-space at $X^\mu(z)$, γ^{mn} is the world-sheet metric, $G_{\mu\nu}(X)$, $B_{\mu\nu}(X)$, and $\Phi(X)$ are external fields, and the dots represent extra terms involving Yang-Mills fields, fermions, etc. We can use two-dimensional conformal field theory to compute the β -functions for the couplings of $G_{\mu\nu}(X)$, $B_{\mu\nu}(X)$, etc., treated as external fields. For this purpose it is convenient to work in the conformal gauge: $\gamma^{mn} = e^{2\sigma} \delta^{mn}$. In this way we find²⁹

$$\beta_{\mu\nu}^G = R_{\mu\nu} - \frac{1}{4} H_{\mu\lambda\sigma} H_\nu^{\lambda\sigma} + 2 \nabla_\mu \nabla_\nu \Phi + \dots \quad (1.26a)$$

$$\beta_{\mu\nu}^B = \nabla_\lambda H_{\mu\nu}^\lambda - 2 (\nabla_\lambda \Phi) H_{\mu\nu}^\lambda + \dots \quad (1.26b)$$

$$\beta^\Phi = \frac{1}{16\pi^2} \left\{ 4 (\nabla \Phi)^2 - 4 \nabla^2 \Phi - R - \frac{1}{12} H^2 \right\} + \dots \quad (1.26c)$$

Since string theory is finite, we must have

$$\beta_{\mu\nu}^G = 0, \quad \beta_{\mu\nu}^B = 0, \quad \beta^\Phi = 0, \quad \text{etc.} \quad (1.27)$$

These are precisely the equations of motion for the $N = 1$, $D = 10$ supergravity theory (1.23) with $\Phi = 0$. This technique can be extended to calculate the higher-order terms indicated by dots in (1.26), in a systematic expansion in α' . For example, to next order in α' we find²⁹

$$\delta \beta_{\mu\nu}^G = \frac{\alpha'}{2} (R_{\mu\lambda\sigma\tau} R_\nu^{\lambda\sigma\tau} - \text{tr } F_{\mu\lambda} F_\nu^{\lambda\tau}) \quad (1.28)$$

telling us that the Lagrangian (1.23) must be supplemented by a term

$$\delta \mathcal{L}_{N=1, D=10} = \frac{1}{4} \phi^{-3/4} R_{\mu\nu} R^{\mu\nu} \quad (1.29)$$

which is crucial for the consistency of the compactification scheme we now discuss.

In their pioneering work on compactification of the heterotic string, Candelas, Horowitz, Strominger and Witten³ proposed the following selection criteria:

- A) $M_{10} \rightarrow M_4 \times K$, where M_4 is some maximally symmetric four-dimensional space (i.e. Minkowski, de Sitter, or anti de Sitter), and K is some compact six-dimensional manifold.
- B) $N = 1$ supersymmetry should be retained in four dimensions, so as to aid in solving the gauge hierarchy problem.
- C) The residual four-dimensional gauge group should be 'realistic' with the possibility of chiral fermion representations.

The following appealing but non-unique solution to these conditions was given in Ref. 3:

- i) they showed that K is a Calabi-Yau manifold, i.e. it is a complex Kähler manifold with a metric

$$ds^2 = \frac{\partial^2 K}{\partial z_i \partial \bar{z}_j} dz_i d\bar{z}_j \quad : i=1,2,3 \quad (1.30)$$

which is Ricci flat: $R_{ij} = 0$, and has $SU(3)$ holonomy;

- ii) they imposed $H_{\mu\nu\rho} = 0$;
- iii) they showed that M_4 is Minkowski, i.e. the cosmological constant $\Lambda = 0$;
- iv) they showed that the $SO(32)$ gauge group could only give real fermions, whilst the $E_8 \times E_8$ gauge group could give chiral fermions. This is because in the compactification, we lose the $SU(3)$ factor in the maximal subgroup decomposition $E_8 \rightarrow SU(3) \times E_6$, since the $SU(3)$ gauge factor must be identified with the $SU(3)$ of holonomy: $F^2 \propto R^2$ so that (1.28) vanishes. The chiral fermions are obtained from the adjoint **248** of $D = 10$ gauge bosons of E_8 , which breaks up into

$$\underline{248} = (\underline{1}, \underline{78}) + (\underline{3}, \underline{27}) + (\bar{\underline{3}}, \bar{\underline{27}}) + (\underline{8}, \underline{1}) \quad (1.31)$$

under $SU(3) \times E_6$, the fermions being in **27** (and perhaps $\bar{\underline{27}}$ and **1**) representations of E_6 with a number of generations N_g :

$$N_g = \#_{\underline{27}} - \#_{\bar{\underline{27}}} = \frac{1}{2} |\chi| \quad (1.32a)$$

where χ is the Euler characteristic of K . The individual numbers of **27**, $\bar{\underline{27}}$, and **1** representations are determined by other topological numbers of the manifold, such as the Betti numbers:

$$\#_{\underline{27}} = b_{2,1} \quad , \quad \#_{\bar{\underline{27}}} = b_{1,1} \quad (1.32b)$$

These fields can all be associated with geometrical deformations of the manifold.

Here are two examples of Calabi-Yau manifolds whose phenomenology will be discussed in more detail in Lecture 3. The Yau model³¹ is a subspace of $CP^3 \times CP^3$ defined by the simultaneous vanishing of three polynomials of bidegrees (3,0), (0,3), and (1,1):

$$\sum_{i=0}^3 x_i^3 + \dots = 0 \quad (1.33a)$$

$$\sum_{i=0}^3 y_i^3 + \dots = 0 \quad (1.33b)$$

$$x_0 y_0 + \dots = 0 \quad (1.33c)$$

with an identification under the following discrete symmetry group $G \cong Z_3$:

$$(x_0, x_1, x_2, x_3) \times (y_0, y_1, y_2, y_3) \rightarrow (\alpha x_0, \alpha^2 x_1, \alpha x_2, \alpha x_3) \times (\alpha y_0, \alpha y_1, \alpha^2 y_2, \alpha^2 y_3) \quad (1.34)$$

where $\alpha \equiv \exp(2\pi i/3)$. This manifold has Euler characteristic $\chi = -18/\dim G = -6$, corresponding to $N_g = \frac{1}{2}|\chi| = 3$. The other model, due to Strominger and Witten,³² is a subspace of CP^7 defined by the simultaneous vanishing of four quadratics, with an identification under the 16-element non-Abelian discrete symmetry group G generated by $z_i \rightarrow z_{3i+1}$, $z_i \rightarrow \beta^i z_i$; $i = 0, \dots, 7$ and $\beta^4 = 1$. It has Euler number $\chi = -144/\dim G = -8$, implying $N_g = 4$. Its interest lies in the possibility offered by a non-Abelian discrete symmetry group of giving a rank-5 gauge group in four dimensions, which will be investigated in Lecture 3.

1.3 Alternative Compactifications

We can divide these into two categories: alternative manifolds, and alternatives to manifolds, i.e. orbifolds.

There exist manifold compactifications yielding Polyakov non-linear σ -models with different numbers of world-sheet supersymmetries.³³ We can see these by reformulating the Polyakov action (1.25) in the light-cone gauge and including fermions:

$$S = \frac{1}{4\pi} \int d\sigma d\tau \left[(g_{ij} + B_{ij}) \partial_+ x^i \partial_- x^j + i \psi^A (\partial_- \delta^{AB} + A_i^{AB} \partial_- x^i) \psi^B + i \lambda^a (\nabla_+^{(\pm)} \lambda)^a + \frac{1}{2} F_{ab}^{AB} \lambda^a \lambda^b \psi^A \psi^B \right] \quad (1.35)$$

where $i, j = 1, \dots, 8$, $\partial_\pm = \partial/\partial\tau \pm \partial/\partial\sigma$, the ψ^A are right-moving fermions, the $\lambda^a = e_i^a \lambda^i$ (where e_i^a is the zweibein) are left-moving fermions, and A_i^{AB} is a gauge field. The covariant derivatives are defined by

$$(\nabla_\pm^{(\pm)})^{ab} = \partial_\pm \delta^{ab} + \partial_\pm \omega_i^{(\pm)ab} \quad (1.36a)$$

with torsions

$$\omega_\pm^{(\pm)ab} = \omega_\mu^{ab}(e) \mp H_{\mu\nu\rho} e^{\nu a} e^{\rho b} \quad (1.36b)$$

Since the left- and right-moving fermions in two dimensions (represented by fields depending on $\sigma \pm \tau$, respectively) are independent, we can have independent left- and right-moving supersymmetry algebras

$$\{Q_\pm^I, Q_\pm^J\} = \delta^{IJ} P_\pm \quad (1.37)$$

and hence in general (p, q) supersymmetries, where p and q are independent integers.³³ The σ -model (1.35) automatically has $(0, 1)$ supersymmetry. This is extended to $(0, 2)$ supersymmetry if the compactification manifold has a suitable complex structure. In general, such a complex manifold can have a larger holonomy group [e.g. $SU(4)$ or $SU(5)$], and hence will not leave as much residual four-dimensional gauge symmetry as Calabi-Yau compactification: $E_8 \rightarrow SU(4) \times SO(10)$ or $SU(5) \times SU(5)$. We can obtain $(1, 1)$ supersymmetry for the σ -model (1.35) by imposing

$$A_\mu = \omega_\mu^{(\pm)} \quad (1.38)$$

If the compactification manifold has both (1.38) and a suitable complex structure, the resulting σ -model will have $(1, 2)$ supersymmetry, $SU(4)$ holonomy and an $SO(10)$ gauge group in general. Finally, we can get $(2, 2)$ supersymmetry by imposing

$$A_\mu = \omega_\mu^{(+)} = \omega_\mu^{(-)} \quad (1.39)$$

which is clearly seen from the definition (1.36b) of the torsions $\omega_\mu^{(\pm)}$ to require $H_{\mu\nu\rho} = 0$, as is the case for a Calabi-Yau manifold.³ World-sheet instanton effects on these different classes of manifolds will be discussed in subsection 1.5, and some comments on GUT model-building using $(0, 2)$ and $(1, 2)$ manifolds will be made in subsection 2.5.

Orbifolds⁴ are spaces which are smooth (like manifolds) everywhere except at a discrete set of points, the simplest example being a cone. Most orbifolds studied so far have been obtained from flat n -dimensional spaces R^n divided by some discrete group S of spatial transformations. These would generically include discrete translations—corresponding to the identification of points separated by a space lattice L —and a point group P of rotations and/or reflections. Dividing R^n by a discrete translation group gives a torus $T^n \cong R^n/L$, and a generic orbifold $\mathcal{O} \cong T^n/P \cong R^n/S$. A simple example³⁴ is obtained from R^2 using the spatial lattice defined by the points $(1, 0)$ and $(0, 1)$, and a point group $P \cong Z_2$ generated by reflections $R(x, y) = (-x, -y)$. As seen in Fig. 3, this orbifold has four fixed points given by x and $y = 0, 1/2$. To retain supersymmetry, the orbifold analogue of $SU(3)$ holonomy must be some discrete subgroup of $SU(3)$. Chiral fermions and a realistic number of

Fig. 3. Simple orbifold³⁴ based on R^2 , indicating string states which become legal on T^2 and on T^2/P . The crosses denote fixed points under reflections R .

generations, as well as a gauge group of the form $SU(3)_c \times SU(2)_L \times U(1)^{n>1}$, can be obtained with non-trivial gauge fields on orbifolds, by analogy with the Hosotani mechanism to be described in subsection 3.1.³⁵ Orbifolds are distinctively 'stringy' compactifications, since field theory is not consistent on an orbifold. Generic classes of consistent string states on orbifolds are illustrated in Fig. 3. The closed loop is an example of a string state that is legal already on R^2 , while the line crossing from $(0,y)$ to $(1,y)$ becomes legal only on the torus $T^2 \equiv R^2/\{(0,1), (1,0)\}$. These are called the untwisted sector. Finally, the configuration around the point $(0, 1/2)$ is legal only on T^2/P . It is one of four so-called twisted sectors — one for each fixed point.³⁴

The orbifold concept has been extended to include the class of asymmetric orbifolds,³⁶ which use different space groups S for the holomorphic (left-moving) and antiholomorphic (right-moving) fields:

$$\lambda(z) = \lambda(S_L z), \quad \psi(\bar{z}) = \psi(S_R \bar{z}) \quad (1.40)$$

where $S_L \neq S_R$. These 'compactifications' defy any simple space-time interpretation of the internal degrees of freedom on the world-sheet — an issue to which we return in Lecture 2.

1.4 Effective $N = 1$, $D = 4$ Supergravity Theory

We will now see what forms for the basic functions $\mathcal{G}$ (Kähler potential) and f_{ab} (gauge kinetic function) appear in different compactification schemes. The first approach taken was that of simple truncation of $N = 1$, $D = 10$ supergravity to $N = 1$, $D = 4$, which provided a model for Calabi-Yau compactification with one generation.³⁷ The various pieces in the $N = 1$, $D = 10$ supergravity Lagrangian (1.23) provide the following terms in $N = 1$, $D = 4$:

$$-\frac{1}{2}R^{(10)} \rightarrow -\frac{1}{2}R^{(4)} - 3(\partial_\mu \sigma)(\partial^\mu \sigma) \quad (1.41)$$

where the 'compacton' σ scales the metric of the compactified dimensions:

$$g_{IJ} = \delta_{IJ} e^\sigma \quad (1.42)$$

The dilaton kinetic energy term $-(9/16)(\partial_\mu \phi/\phi)^2$ in (1.23) remains unchanged. The gauge terms provide several contributions:

$$\begin{aligned} -\frac{1}{4}\phi^{-3/4}F_{\mu\nu}F^{\mu\nu} &\rightarrow -\frac{1}{4}\phi^{-3/4}e^{3\sigma}F_{\mu\nu}F^{\mu\nu} - 3e^{-\sigma}\phi^{-3/4}|D_\mu C|^2 \\ -\frac{8}{3}\phi^{-3/4}e^{-5\sigma}|\partial P/\partial C|^2 &+ (\text{D-terms}): P = 852d_{ijk}C^i C^j C^k \end{aligned} \quad (1.43)$$

where the components of A_μ which have coordinates in the six compactified dimensions yield chiral scalars C , and $F_{\mu\nu}F^{\mu\nu}$ gave them kinetic terms, F-terms, and D-terms. Finally, the kinetic term of the antisymmetric tensor fields in (1.23) becomes

$$-\frac{3}{4}\phi^{-3/4}H_{\mu\nu\rho}H^{\mu\nu\rho} \rightarrow -\frac{3}{2}e^{-2\sigma}\phi^{-3/2}(\partial_\mu a + \dots)^2 - \frac{9}{2}\phi^{3/2}e^{-6\sigma}(\partial_\mu D)^2 \quad (1.44)$$

where the spin-zero fields are defined by

$$B_{\pm 5} = \epsilon_{\pm 5} a \quad (1.45)$$

in the compactified dimensions, and

$$H_{\mu\nu\rho} = e^{-6\sigma}\phi^{3/2}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma D \quad (1.46)$$

in the four uncompactified dimensions. The results (1.41) to (1.46) can be expressed in the canonical form of subsection 1.2 after constructing two complex chiral fields out of the four real fields σ , ϕ , a , and D :

$$S = e^{3\sigma}\phi^{-3/4} + 3i\sqrt{2}D, \quad T = e^{\sigma}\phi^{3/4} - i\sqrt{2}a + |C|^2 \quad (1.47)$$

in terms of which

$$G = -\ln(S+S^*) - 3\ln(T+T^* - 2|C|^2) - \ln|P|^2 \quad (1.48)$$

and

$$f_{ab} = S_{ab} S \quad (1.49)$$

Notice the double no-scale structure: the chiral fields S and (T, C) parametrize an

$$\frac{SU(1,1)}{U(1)} \otimes \frac{SU(N_c+1,1)}{SU(N_c+1) \times U(1)} \quad (1.50)$$

Kähler manifold.

Alternative compactification schemes give different non-compact no-scale Kähler manifolds. For example, a maximal truncation of $N = 1$, $D = 10$ to $N = 1$, $D = 4$, which is a model for multi-generation Calabi-Yau manifolds and exact for compactification on a T^6/Z_3 orbifold, gives³⁸ an

$$\frac{SU(1,1)}{U(1)} \otimes \frac{SU(3, 3+3n)}{SU(3) \times SU(3+3n) \times U(1)} \quad (1.51)$$

manifold. A three-generation subset of this maximal truncation corresponding to a T_6/Z_{12} orbifold gives an

$$\frac{SU(1,1)}{U(1)} \otimes \left(\frac{SU(1, 1+n)}{SU(1+n) \times U(1)} \right)^3 \quad (1.52)$$

manifold.³⁸ We note that the $SU(1,1)/U(1)$ nature of the S dilaton sector of the Kähler potential and the kinetic function (1.49) are universal features of all these models.

The repeated reappearance of generalized no-scale models may seem surprising at first sight. It is in fact a reflection of the result of Ref. 3 that the cosmological constant is zero in a supersymmetric truncation. This is true for all possible deformations of the manifold, and hence over the full ranges of the field variables (T, C) which correspond to such deformations, and thus requires a no-scale flat potential for these fields. Only models without such fields—which are not interesting for phenomenology—would not have such a no-scale structure.

What of supersymmetry breaking in these models? It has been suggested that^{39,40} (at least a subset of⁴¹) the hidden sector E_8 gauginos may condense, providing

$$\delta P = h e^{-\lambda S} \quad (1.53)$$

where λ depends on the unbroken subgroup of E_8 in which condensation occurs. This is not the full story, since in order to obtain a stable non-trivial minimum with $\langle S \rangle \neq 0$ we need another contribution to $\mathcal{G}(S)$.^{40,42}

There are alternatives to this scheme. For example, some models with an anomalous $U(1)$ factor in the four-dimensional gauge group can yield Fayet-Iliopoulos D-type supersymmetry breaking, albeit with $\Lambda = O(m_P^4)$.⁴³ Alternatively, it has recently been shown⁴⁴ how to realize the Scherk-Schwarz mechanism⁴⁵ in a five-dimensional string theory while retaining $\Lambda = 0$. This mechanism can be expressed in terms of topologically non-trivial configurations for auxiliary $U(1)$ gauge fields on the world-sheet.⁴⁶

1.5 Corrections to the Low-Energy Effective Theory

So far, we have only treated the low-energy theory derived from $D = 10$ strings in a lowest-order approximation. In this subsection we discuss qualitatively three classes of corrections to the theory (1.48) and (1.49): those due to world-sheet instantons, string loop effects, and quantum loops in the low-energy theory.

World-sheet instantons⁴⁷ are configurations of the string world-sheet which are embedded in a topologically non-trivial way in the background space-time manifold K . The space-time position $X^\mu(z)$ of a point z on the string can be regarded as a map from the world-sheet Σ into the manifold K , as seen in Fig. 4. Here Σ is a sphere S^2 in the tree-level of string perturbation theory, a torus T^2 at the one-loop level, etc. The map $X^\mu: \Sigma \rightarrow K$ is analogous to the gauge map $A^\mu: E^4 \rightarrow$ gauge group whose non-trivial sectors yield conventional gauge instantons. To infer the effective action for the low-mass fields T , C^i , etc., we must make a functional integral over string configurations:

$$\begin{aligned} Z(g^{ij}, B^{ij}, \dots) &= \int \mathcal{D}g \int \mathcal{D}X \\ &\exp - \int_{\Sigma} d^2z \left(\sqrt{\det g} \delta^{mn} g^{ij}(X(z)) + \epsilon^{mn} B^{ij}(X(z)) \right) \partial_m X^i(z) \partial_n X^j(z) \end{aligned} \quad (1.54)$$

Symmetries of the effective theory are characterized by equivalences between different backgrounds g^{ij} , B^{ij} and $g^{ij'}$, $B^{ij'}$:⁴⁸

$$\int_{\Sigma} d^2z (g^{ij'} - g^{ij}) \partial_m X^i \partial_n X^j = 0, \quad \int_{\Sigma} d^2z (B^{ij'} - B^{ij}) \partial_m X^i \partial_n X^j = 0 \quad (1.55)$$

in conformal gauge. Let us consider, for example^{*}, the field T (1.47) and its generalization to more fields in the case of realistic Calabi-Yau manifolds, which are associated with deformations of the manifold K that do not change its Kähler structure. [Clearly the simple T of (1.47) belongs to this

Fig. 4. The possibility of world-sheet instanton configurations on a manifold K . A is a topologically trivial configuration, whilst B is topologically non-trivial.

^{*}) Other categories of world-sheet instanton effects are discussed in Ref. 48.

category since its real part corresponds to an overall rescaling (1.42) of K .] In general, the real parts of the different T fields are associated with elements of $H_{1,1}(K)$, whilst their imaginary parts are related to components of $B_{\mu\nu}$ which are associated with elements of $H_2(K)$.⁴⁸ The embedding of Σ in K induces pull-backs $H_{1,1}(\Sigma)$ and $H_2(\Sigma)$. These may be non-trivial if $H_{1,1}(K)$, $H_2(K) \neq 0$, in which case the equalities (1.57) are not obeyed in general because, there are deformations of K , i.e. transformations of the T fields, which are not symmetries of the low-energy effective field theory. In the tree-level case $\Sigma = S^2$, this can occur if there are non-contractible maps $X: \Sigma \rightarrow K$, i.e. if $\Pi_2(K_6) \neq 0$. This mathematics is visualized in Fig. 4, where K is portrayed as a Swiss cheese with bubbles inside it. There exist string S^2 world-sheet configurations which surround bubbles and are non-contractible, and others which do not surround any bubble and can be contracted. The former give a non-trivial effective potential for the (generalized) T field describing the size of the bubble. This is because the string action is always e^{-A} , where in this case A is the area of the non-contractible surface, which depends in turn on the size of the bubble and hence of the T field.

The nature of these world-sheet instanton effects can often be deduced using techniques borrowed from gauge-instanton physics.^{47,49,50} For example, instantons have fermionic zero modes which lead to interactions obtained by matching them with the fermion operators appearing in the vertex operators for low-mass fields.⁵¹ The number of left- (right-)moving zero modes is twice the number of left- (right-)supersymmetries for a holomorphic instanton. The vertex operators for T fields contain two right-moving fermions but no left-movers, whilst the operators for the C^i contain one of each. The operators for the S field and for four-dimensional gauge fields A_μ contain only $D = 4$ Minkowski indices, and therefore instantons in the six compactified dimensions generate no interactions among them.^{49,50} From these elementary properties we see that manifolds with $(0,2)$ world-sheet supersymmetry generate only T^n interactions. These imply that such a manifold is not a true solution to the equations of motion for all values of T , and will tend to collapse in some direction: the world sheet in Fig. 4 tries to squeeze the bubble. In general it is thought⁵⁰ that this instability tends to turn a $(0,2)$ manifold into an orbifold, and this argument is supported by explicit orbifold calculations.⁵² Manifolds with $(2,2)$ world-sheet supersymmetry cannot generate T^n interactions, and so do not have this instability, but can acquire C^n couplings, which may give masses or interactions for states obtained from the gauge fields in ten dimensions. Finally, manifolds with $(1,2)$ supersymmetry do not have either T^n or C^n interactions.⁵³ In summary, we have $\delta f = 0$, $\delta g(S) = 0$ in all cases, $\delta g(T) \neq 0$, $\delta g(C) = 0$ for $(0,2)$ models, $\delta g(T) = 0$, $\delta g(C) \neq 0$ for $(2,2)$ models, and $\delta g(T) = 0$, $\delta g(C) = 0$ for $(1,2)$ models.

Next we turn to perturbative calculations of string-loop effects. The previous sketch in subsection 1.2 of the calculation of $\mathcal{L}^{(10)}$ was carried out in the tree-level $\Sigma = S^2$ sector of the theory. We might expect the low-energy theory obtained from n -loop terms ($\Sigma = T^2$, etc.) to differ in general. We expect for these the general scaling laws⁵⁴

$$\mathcal{L}_{n\text{-loop}} \rightarrow t^{4(1-n)} r^{-\frac{1}{2}} \mathcal{L}_{n\text{-loop}} \quad (1.56)$$

where

$$\begin{aligned} T &\rightarrow r^{\frac{1}{2}} T, & T &\rightarrow t^0 T \\ S &\rightarrow r^{-\frac{1}{2}} S, & S &\rightarrow t^4 S \end{aligned} \quad (1.57)$$

This means that the loop expansion parameter is effectively $\text{Re } T / \text{Re } S = (T + T^*) / (S + S^*)$, and we might expect⁵⁴

$$\delta g = g\left(\frac{T+T^*}{S+S^*}\right), \quad \delta f = \alpha T \quad (1.58)$$

However, explicit one-loop string calculations⁵⁵ show that there are no corrections to the two- and three-point functions, for the simple reason that string-loop calculations involve traces over all modes, each external field introduces two fermionic zero modes S_0 , and

$$\text{Tr } S_0^4 = \text{Tr } S_0^6 = 0 \quad (1.59)$$

This observation tells us that

$$\delta G_i^i = 0, \quad \delta f = 0 \quad (1.60)$$

which is confirmed by one-loop studies of four-point functions.⁵⁵ Indeed, the only bosonic corrections to the $N = 1$, $D = 10$, supergravity Lagrangian are fourth order in the bosonic field curvatures, which do not contribute to the effective $N = 1$, $D = 4$ supergravity theory discussed in subsection 1.1.⁵⁶ This argument can probably also be extended to multiloop diagrams.

Finally, we comment on loops in the low-energy effective field theory. Spontaneously broken supersymmetry [as obtained, for example from (1.53) etc.] would give $m_{3/2} \neq 0$ at the tree level. However, m_0 would still vanish at the tree level (the potential is still flat) and also $m_{1/2} = 0$ at the tree level, since it is proportional [(1.9), (1.48), (1.49)] to G_S :

$$m_{1/2} = \frac{e^{G/2}}{4 \text{Re } S} \frac{G_S}{G_S^*} \quad (1.61)$$

whilst

$$V_0 = e^G \frac{G_S G_S^*}{G_S^*} \quad (1.62)$$

at the tree level. One could hope that m_0 and $m_{1/2} \neq 0$ in higher orders of perturbation theory. One-loop diagrams in the low-energy $N = 1$, $D = 4$ theory make a correction to the effective potential (1.62) of the form

$$\delta V_{1\text{-loop}} = \text{Str} \left(f(m_{3/2}^2) \right) = \text{Str} \left(f(e^G) \right) \quad (1.63)$$

so that the (T, C) -dependence is always of the form

$$\delta V_{1\text{-loop}} = F(T + T^* - 2|C|^2) \quad (1.64)$$

in the toy model (1.48). Hence

$$\frac{\partial^2 V}{\partial C \partial C^*} = \frac{\partial^2 (\delta V_{1\text{-loop}})}{\partial C \partial C^*} = -2F' = -2 \frac{\partial (\delta V_{1\text{-loop}})}{\partial T} = -2 \frac{\partial V}{\partial T} \quad (1.65)$$

which vanishes at an extremum with respect to variation in T .⁵⁷ Therefore

$$\delta m_{0_c}^2 (1\text{-loop}) = 0 \quad (1.66)$$

However, adding $\delta V_{1\text{-loop}}$ (1.63) to (1.62) gives a shifted extremum with respect to S : $\partial V / \partial S = 0$, where $V = V_0 + \delta V_{1\text{-loop}}$ implies $\delta S \neq 0$ and hence makes a contribution⁵⁸

$$\delta m_{1/2} (1\text{-loop}) = O(\hbar m_{3/2}^{5/3} / m_P^{2/3}) \quad (1.67)$$

in the simple model (1.53). There are also other contributions⁵⁹ whose relative magnitude is uncertain.^{10,60,61} The first contribution to m_0^2 seems to appear at the two-loop level, where Fig. 5 and ten other diagrams contribute:⁶²

$$m_{0_c}^2 = O(\hbar^2 m_{3/2}^4 / m_P^2) \quad (1.68)$$

Comparing (1.68) with (1.69), we see that $m_{1/2}$ may be the dominant source of supersymmetry breaking in the observable sector. Accordingly, $m_{1/2} \gg m_0$ should be used as the initial condition in Fig. 2 for the renormalization group analysis of gauge symmetry breaking in models along the lines described at the end of subsection 1.1. This assumption will be used in phenomenological estimates in subsection 3.5.

Fig. 5. Typical two-loop diagram contributing to the scalar-mass squared.

2. LOW-ENERGY EFFECTIVE THEORY FROM FOUR-DIMENSIONAL STRINGS

2.1 Critique of the Ten-Dimensional Approach

As we have seen in Lecture 1, although there is only one phenomenologically acceptable heterotic string in ten dimensions,² there is considerable ambiguity in its compactification: a Calabi-Yau manifold? if so, which of many? if not, some other manifold? or an orbifold? perhaps an asymmetric orbifold? It is also unclear how string dynamics chooses between these alternatives. It is not even known how the overall scale of a Calabi-Yau manifold (as measured by $\text{Re } T$) can be fixed. Consistency arguments tell us that the compactification radius must be $O(1/m_P)$: $\text{Re } S = 1/g^2 = e^{3\sigma} \phi^{-3/4} = O(1)$, whilst $\text{Re } T = e^{\sigma} \phi^{3/4}$ must be $O(1)$ in order to avoid a large range of energies where the effective theory is $D = 10$ supergravity, and hence $e^{\sigma} = O(1)$.⁶³ With internal dimensions that are so small, it is not clear that we gain by calling them space-time dimensions. Our concept of space-time is an infrared one, derived from the fact that the graviton with zero mass is much lighter than all the other force-carrying particles, so that the size of space is much larger than their Compton wavelengths. This is no longer the case when the compactification scale is $O(1/m_P)$. The 'compactified' variables can be represented in other ways: for example, as internal gauge degrees of freedom, or even by fermions. Some 'compactifications' such as asymmetric orbifolds³⁶ have in fact no simple space-time interpretation at all, because the left- and right-moving coordinates are identified using different space groups (1.40). The most economical approach, which includes all 'compactifications' of ten-dimensional strings and may provide a suitable framework for formulating the dynamics which discriminates between them, is to formulate superstrings directly in four dimensions with the appropriate number of internal degrees of freedom.

2.2 Formulations of Superstrings Directly in Four Dimensions

The basic requirements for a consistent superstring theory are that it be conformally invariant—needed for finiteness and to remove negative norm states, and modular invariant—so that loop diagrams are counted correctly. As was already mentioned, there are equivalent bosonic^{5,6} and fermionic⁷⁻⁹ formulations of four-dimensional strings. I am more familiar with the latter, and so will describe it here. Fermions on the world-sheet may be given combinations of periodic ($\psi \rightarrow \psi$: Ramond), antiperiodic ($\psi \rightarrow -\psi$: Neveu-Schwarz), or more general ($\psi \rightarrow e^{i\alpha}\psi$) boundary conditions. They may, or may not, have quartic Thirring interactions $\psi\psi\psi\psi$. Thus there is considerable ambiguity in the formalism, but also many equivalences. In addition to the boson-fermion equivalence already mentioned, we may trade-off $e^{i\alpha}$ boundary conditions against Thirring interactions, etc.⁶⁴ In what follows we will make three steps towards the most general formulation: first with just periodic and antiperiodic boundary conditions,^{7,8} then with general rational⁹ phases $e^{2\pi i/N}$, and finally mentioning yet another possibility going beyond simple boson or fermion building blocks.

We start with the following field content in the light-cone gauge. The left-movers consist of two transverse bosonic coordinates $X^i(z)$, their two supersymmetric fermion partners $\psi^i(z)$, and eighteen Majorana-Weyl fermions to cancel the conformal anomaly. These latter are divided into three sets of six each: x^m , y^m , and ω^m : $m = 1, 2, \dots, 6$, which enable supersymmetry to be realized non-linearly with the supercurrent⁶⁵

$$G = \gamma^i \partial_z X_i + \sum_{m=1}^6 x^m y^m \omega^m \quad (2.1)$$

The right-movers consist of two transverse bosonic coordinates $X^i(\bar{z})$ and forty-four real fermions $\bar{\psi}^A(\bar{z})$ to cancel the conformal anomaly.

Modular invariance is imposed by giving ± 1 boundary conditions to the world-sheet fermions according to rules⁸ which we now describe. The fermions are allocated to $(P+1)$ sets $B \equiv \{B_0, B_1, \dots, B_P\}$, where $B_0 \equiv F \equiv \{\text{all 64 left- and right-moving fermions}\}$ and the B_i ($i = 1, \dots, P$) are subsets of F chosen subject to the following conditions:

i) No subset B_j is the product $B_j \circ B_k$ in the sense

$$B_j \circ B_k \equiv B_j \cup B_k - B_j \cap B_k \quad (2.2)$$

of any two or more of the others. This condition just establishes B as a basis.

ii) The fermion parity operators $(-1)^{B_i}$ either commute or anticommute with G (2.1):

$$[G, (-1)^{B_i}] = 0 \quad \text{or} \quad \{G, (-1)^{B_i}\} = 0 \quad (2.3)$$

This condition guarantees world-sheet supersymmetry.

iii) For any subsets B_i in B , $n(B_i) = \#(\text{left-movers}) - \#(\text{right-movers})$:

$$n(B_i) = 2 \cdot n(B_i \cap B_j) = 4 \cdot n(B_i \cap B_j \cap B_k \cap B_l) = 0, \text{ mod } 8 \quad (2.4)$$

which guarantees modular invariance at the one- and multi-loop levels.

We now construct the set Ξ of products [in the sense (2.2)] of elements B_i of B , and introduce for any member $X \in \Xi$ a sector $R_X(NS)_{F \circ X}$: all the fermions in X are periodic and all the others are antiperiodic. The physical states of the spectrum are then selected according to whether

$$(-1)^{B_i} = \delta_X C(X|B_i) \quad (2.5)$$

where $\delta_X = -1$ ($+1$) if a fermion is inside (outside) the set X , and the coefficients $C(X|B_i) = \pm 1$ can be chosen in any way that is consistent with the following rules:

$$\begin{aligned} C(X|Y) &= e_{X \cap Y}^2 C(Y|X) = -e_X C(X|F \circ X \circ Y) : e_X \equiv \exp\left[\frac{i\pi}{8} n(X)\right] \\ C(X|Y_1 \circ Y_2 \circ \dots \circ Y_k) &= (\delta_X)^{k-1} C(X|Y_1) C(X|Y_2) \dots C(X|Y_k) \end{aligned} \quad (2.6)$$

Thus we arrive at the following fermionic contribution to the partition function:⁸

$$Z_F = \sum_{X \in \Xi} \delta_X \pi \left[\frac{1 + \delta_X C(X|B) (-1)^B}{2} \right] R_X(NS)_{F \circ X} \quad (2.7)$$

and the bosonic contribution to the partition function is conventional.

The resulting theory has space-time supersymmetry if B contains the set $S \equiv \{\psi^i, x^m : m = 1, 2, \dots, 6\}$ and $C(S|Y) = -1$ for any $Y : Y \cap S = \emptyset$. The generic form of an $N = 4$ supersymmetric theory is

$$B_{N=4} = \{F, S, \mathcal{Z}_i\} : \mathcal{Z}_i \cap S = \emptyset \quad (2.8)$$

The generic $N = 2$ supersymmetric theory has

$$B_{N=2} = \{F, S, \mathcal{U}_1, \mathcal{Z}_i\} : \mathcal{U}_1 \cap S = \{\psi^i, x^1, x^2\} \quad (2.9)$$

and the generic $N = 1$ supersymmetric theory has

$$B_{N=1} = \{F, S, \mathcal{U}_1, \mathcal{U}_2, \mathcal{Z}_i\} : \mathcal{U}_2 \cap S = \{\psi^i, x^3, x^4\} \quad (2.10)$$

A generic $N = 1$, $D = 4$ field theory can be arrived at in two steps: the $N = 1$ graviton, gauge, and associated matter supermultiplets can be obtained by truncations of the full $N = 4$ spectrum obtained by first augmenting the set $B_{N=4}$ (2.8) to $B_{N=2}$ (2.9) and finally to $B_{N=1}$ (2.10).^{66,67} The remaining $N = 1$ matter supermultiplets can be obtained by truncation of another generic $N = 2$ theory (2.9).^{66,68}

We will return to the construction of a generic $N = 1, D = 4$ supergravity theory using this framework in the next section, after a discussion of more general formulations of $D = 4$ superstring models.

We next discuss a fermionic formulation with rational⁹ phase boundary conditions: $\psi \rightarrow e^{2\pi i/N} \psi$. This can be characterized by an additive group of vectors describing the different fermion boundary conditions: $\mathcal{E} = Z_{N_1} \otimes Z_{N_2} \otimes \dots \otimes Z_{N_k}$ which is generated by the basis $\{G_1, G_2, \dots, G_k\}$, where

- i) $\sum_i m_i b_i = 0$ if and only if $m_i = 0 \pmod{N_i}$ for all $i = 1, \dots, k$;
- ii) $\frac{1}{2} N_i b_i = 1$;
- iii) $N_{ij} b_i \cdot b_j = 0 \pmod{4}$, where N_{ij} is the LCM of N_i and N_j ;
- iv) $N_i b_i^2 = 0 \pmod{8}$, if N_i is even;
- v) the number of fermions which are periodic simultaneously under any set of four boundary conditions $b_{i,j,\ell,m}$ is always even;
- vi) the boundary conditions generate an automorphism of the Lie algebra used in defining the supercharge, so that one has space-time supersymmetry.

The full set of physical states is obtained, as before, by adding together sectors with all these boundary conditions, using projection coefficients $C(b_i|b_j)$ analogous to those introduced (2.6) for the ± 1 boundary conditions. The coefficients $C(b_i|b_j)$ must be chosen to obey the following consistency conditions:

$$C(b_i|b_j) = \delta_{b_i} \exp \frac{2\pi i}{N_j} n = \delta_{b_j} e^{i\frac{\pi}{2} b_i \cdot b_j} \exp \frac{2\pi i}{N_i} m \quad (2.11a)$$

where n and m are arbitrary integers, and

$$C(b_i|b_i) = \pm e^{i\pi/4 b_i^2} \quad (2.11b)$$

$$C(\alpha|\alpha) = e^{i\pi/4 (\alpha \cdot \alpha + 1 \cdot 1)} C(\alpha|b_i)^{N_i/2} \quad (2.11c)$$

$$C(\alpha|\beta) = e^{i\pi/2 \alpha \cdot \beta} C(\beta|\alpha)^* \quad (2.11d)$$

$$C(\alpha|\beta + \gamma) = \delta_\alpha C(\alpha|\beta) C(\alpha|\gamma) \quad (2.11e)$$

The fermionic contribution to the partition function is finally obtained⁹ by a sum analogous to (2.7).

This is by no means the most general string theory. Certainly the group of possible boundary conditions could be made infinite by choosing to have one or more sectors with irrational phases α : $\psi \rightarrow e^{i\alpha} \psi$.⁶⁹ Also, the possibility of a non-commuting algebra of boundary conditions has yet to be considered. It is also possible to imagine constructing string theories out of building blocks which are not just free bosons or fermions.⁷⁰ What we seek is the most general two-dimensional conformal invariant theory on the world sheet, subject to the constraints of modular invariance. The conformal theory must be a unitary representation of the Virasoro algebra

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{c}{12} m(m^2-1) \delta_{n,-m} \quad (2.12)$$

with central charge $c = 26$ (for a non-supersymmetric sector, to cancel the ghost contribution) or $c = 15$ (for a supersymmetric sector, to cancel the ghosts and the superghosts). Free bosons with a Lagrangian

$$\mathcal{L}_B = \sum_{i=1}^d \partial_m X^i \partial^n X^i \quad (2.13)$$

contribute $\Delta c = d$, whilst free fermions

$$\mathcal{L}_F = \sum_{j=1}^{N_F} i \bar{\psi}_j \not{\partial} \psi_j \quad (2.14)$$

contribute $\Delta c = N_{F/2}$. However, in addition to these there is a discrete series of other representations with $c < 1$:⁷¹

$$c = 1 - \frac{6}{m(m+1)} \quad ; \quad m = 3, 4, 5, \dots \quad (2.15)$$

each with many possible choices of the highest weight h :

$$L_0 |h\rangle = h |h\rangle \quad (2.16)$$

given by

$$h = \frac{((m+1)p - mq)^2 - 1}{4m(m+1)} \quad ; \quad \begin{matrix} 1 \leq p \leq m-1 \\ 1 \leq q \leq p \end{matrix} \quad (2.17)$$

The first examples are for $m = 3$: $c = 1/2$; $h = 0, 1/16, \text{ or } 1/2$, and for $m = 4$: $c = 7/10$; $h = 0, 3/80, 4/10, 7/16, 3/5, 3/2$. We can now imagine tensoring these models to get a total central charge,

$$c = \sum_{m=3}^{\infty} n_m c_m = 22 \quad (\text{or } 9) \quad (2.18)$$

where the total is made up to 26 (or 15) by four flat space-time coordinates X^μ and their fermionic superpartners.

Are these in fact different from the previous models made out of free bosons or fermions? The answer is apparently that some are, and some are not.⁷⁰ For an example of one that is not, consider the set of $c = 1$ models which includes the tensor product of two Ising models, each of which has $c = 1/2$. The $(\text{Ising})^2$ model can be regarded as a particular case of a $Z_2 \times Z_2$ orbifold looking like a semicircle with radius $R_0 = 1$. Figure 6 shows a chart of all the known $c = 1$ models including these orbifolds, for

Fig. 6. Classification^{70,72,73} of $c = 1$ models, with major symmetries indicated. There are in addition 3 models obtained by dividing $SU(2)$ by the discrete tetrahedral, octahedral and icosahedral groups.

which different radii: $R_0, R_{02} = 1/2$, are equivalent; and circles, which have in general a $U(1) \times U(1)$ symmetry and are equivalent on radii: $R_c, R_{c2} = 1/2$, with the $R_c = 1/\sqrt{2}$ having an augmented $SU(2) \times SU(2)$ symmetry.^{70,72} Strings on circles of different radii can equivalently be regarded as free-fermion models with Thirring-type $\psi\psi\psi\psi$ interactions, or generalized $e^{i\alpha}$ boundary conditions. In this approach we see that (Ising)² is just one point in a continuum, although there are other isolated $c = 1$ models, obtained by dividing the $SU(2)$ group manifold by the discrete tetrahedral, octahedral, and icosahedral groups.⁷³ To get isolated models from tensor products of $1/2 < c < 1$ models (2.15), it suffices to consider the product of $m(m+1)/6$ copies of the $c = 1 - [6/m(m+1)]$ model for $m \geq 7$, which contains a singlet field, but one that cannot be identified as a radius-changing 'compacton' field analogous to σ (1.42), because it has a finite mass.⁷⁰

There is still a long way to go before the most general string theory is formulated. Yet another ambiguity is in the realization of any internal symmetry. Its Lie algebra generated by charges J^a can be extended to a Kac-Moody algebra

$$[J_m^a, J_n^b] = i f^{abc} J_{m+n}^c + \frac{kn}{2} \delta_{m,-n} \delta^{ab} \quad (2.19)$$

where k is an arbitrary positive integer called the level.⁷⁴ A realization of the Lie algebra using simple fermions has $k = 1$, but one can in principle construct models with arbitrary integer $k \leq c$. In the fullness of time, presumably string theories realizing any or all of the above possibilities will be found.

2.3 Extraction of Low-Energy Effective Theory

To determine the interactions between the $m \approx 0$ states in four-dimensional string models,^{66,67} we must first find the states. They are obtained by applying the boson and fermion creation operators [subject to the projection constraints (2.5) etc. of the previous subsection] to the string ground state. Here we work explicitly with the fermionic formulation using ± 1 boundary conditions. The left-(right-) moving ground state in such a heterotic string model has $(\text{mass})^2$ level -1 (-2), and each boson (antiperiodic fermion) operator raises the $(\text{mass})^2$ by 2 (1). We look first for particles associated with the graviton supermultiplet, which can be found in the $R_\phi(\text{NS})_F$ sector of (2.7). This is best approached by starting from the generic $N = 4$ model (2.8), and then decimating its states using the projection operators introduced in the $N = 2$ model (2.9) and finally the $N = 1$ model (2.10). The projections $(-1)^S = -1$ and $(-1)^{\zeta_i} = +1$ of the $N = 4$ model (2.8) leave us the following massless states:

- a₄): $|\psi\rangle_L |X^j\rangle_R$, which has two space-time indices and includes the graviton, antisymmetric 2-index tensor field, and dilaton, represented here by the fields ϕ_α : $\alpha = 1, 2$;
- b₄): $|x^m\rangle_L |X^j\rangle_R$, which has one space-time index and includes the gauge bosons of the $N = 4$ graviton supermultiplet;
- c₄): $|\psi\rangle_L |\tilde{\psi}^A \tilde{\psi}^B\rangle_R$, which also has one space-time index but internal (right-moving) indices, and hence represents the gauge bosons for some gauge group $G \subset O(44)$ with field strengths $F_{\mu\nu}^I$;
- d₄): $|x^m\rangle_L |\tilde{\psi}^A \tilde{\psi}^B\rangle_R$, which are the six scalar partners of the gauge bosons of G in an $N = 4$ Yang-Mills supermultiplet, henceforward denoted by Φ_{ab}^I , where the indices $a, b = 1, 2, 3, 4$ are antisymmetrized.

The $N = 4$, $D = 4$ supergravity Lagrangian is unique, and takes the form⁷⁵

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} R + D_\mu \phi_\alpha D^\mu \phi^\alpha - \frac{1}{2} \text{Tr} \mathcal{G}_\mu \mathcal{G}^\mu \mathcal{F}_{ab} \mathcal{F}^{ab} \\ & - \frac{1}{4} \text{Re} \left(\frac{\phi^1 - \phi^2}{\phi^1 + \phi^2} \right) \text{Tr} (F_{\mu\nu} F^{\mu\nu}) - \frac{1}{4} \text{Im} \left(\frac{\phi^1 - \phi^2}{\phi^1 + \phi^2} \right) \text{Tr} (F_{\mu\nu} \tilde{F}^{\mu\nu}) \\ & + |\phi^1 + \phi^2|^2 \left[\frac{1}{2} \text{Tr} [\mathcal{F}_{ab}, \mathcal{F}^{bc}]^2 - \frac{4}{9} \left| \text{Tr} \phi^{bd} [\mathcal{F}_{ab}, \mathcal{F}_{dc}] \right|^2 \right] + \dots \end{aligned} \quad (2.20)$$

where the dilaton fields obey the conditions

$$\phi_\alpha = \gamma_{\alpha\beta} (\phi^\beta)^*, \quad \phi^\alpha \phi_\alpha = \phi^1 \phi_1^* - \phi^2 \phi_2^* = 1 \quad (2.21)$$

and parametrize an $SU(1,1)/U(1)$ coset manifold, and the other scalar fields Φ_{ab}^I obey the conditions

$$\begin{aligned} \Phi_{ab}^I &= -\Phi_{ba}^I, \quad (\Phi_{ab}^I)^* = -\Phi^{abI} = -\frac{1}{2}\epsilon^{abcd}\Phi_{cd}^I, \\ \text{Tr}(\Phi_{ab}\Phi^{cd}) &= -\frac{1}{2}\delta_{[a}^c\delta_{b]}^d \end{aligned} \quad (2.22)$$

and parametrize an $O(6,n)/O(6) \times O(n)$ coset manifold, where $n = \dim G$.

We now make the first decimation by projecting down to an $N = 2$ theory (2.9) using the projection $(-1)^{b_1} = -1$. Henceforward we denote by $\bar{\psi}^{A1}$ the right-moving fermions contained in the set b_1 , and by $\bar{\psi}^{A'}$ those not contained in b_1 . The $N = 2$ decimation reduces the previous states (a₄) to (d₄) to:

- a₂): $|\psi^j\rangle_L |X^j\rangle_R$, which still includes the graviton and dilaton fields as before;
- b₂): $|x^{1,2}\rangle_L |X^j\rangle_R$, which includes vector bosons in the dilaton and graviton $N = 2$ supermultiplets;
- c₂): $|\psi^j\rangle_L |\bar{\psi}^{A1}\bar{\psi}^{B1}, \bar{\psi}^{A'}\bar{\psi}^{B'}\rangle_R$ which are the gauge bosons of some gauged subgroup G' of G , and the $|x^{1,2}\rangle_L |\bar{\psi}^{A1}\bar{\psi}^{B1}, \bar{\psi}^{A'}\bar{\psi}^{B'}\rangle_R$ which are their paired scalar partners in $N = 2$ vector supermultiplets;
- d₂): $|x^{3,4,5,6}\rangle_L |\bar{\psi}^{A1}\bar{\psi}^{B'}\rangle_R$ which are scalar $N = 2$ supermultiplets.

The manifolds parametrized by these $N = 2$ scalar fields do not have the most general possible form⁷⁶ but are restricted by their origin in an $N = 4$ theory. The dilaton fields parametrize an $SU(1,1)/U(1)$ manifold as before. The scalars in the $N = 4$ vector multiplet, denoted by Φ_0 , are subject to the constraints

$$\text{Tr}(\Phi_0\Phi_0^*) = -\frac{1}{2}, \quad \text{Tr}\Phi_0^2 = 0 \quad (2.23)$$

and parametrize an $O(2,m)/O(2) \times O(m)$ manifold, where $m = \dim G'$. The $N = 2$ scalar multiplet fields Φ_1, Φ_2 are subject to the constraints

$$\text{Tr}\left(\frac{\Phi_1}{\Phi_2}\right)(\Phi_1^*\Phi_2^*) = -\frac{1}{2}\mathbb{1}, \quad \text{Tr}\left(\frac{\Phi_1}{\Phi_2}\right)(\Phi_1\Phi_2) = 0 \quad (2.24)$$

and parametrize a quaternionic space $O(4,p)/O(4) \times O(p)$.

Finally we decimate the $N = 2$ theory to obtain an $N = 1$ theory by making the second projection $(-1)^{b_2} = -1$ in the generic $N = 1$ model (2.10). We denote by $\bar{\psi}^{\hat{A}}$ the right-moving fermions in the set b_2 , and by $\bar{\psi}^{\hat{A}'}$ those not in b_2 . The remaining states are:

- a₁): $|\psi^j\rangle_L |X^j\rangle_R$, which includes the graviton and dilaton as before;
- b₁): $|\psi^j\rangle_L |\bar{\psi}^{\hat{A}1}\bar{\psi}^{\hat{B}1}, \bar{\psi}^{\hat{A}'}\bar{\psi}^{\hat{B}'}, \bar{\psi}^{\hat{A}'}\bar{\psi}^{\hat{B}'}\rangle_R$, which are the gauge bosons of some gauge subgroup $G'' \subset G' \subset G$;
- c₁): $|x^{1,2}\rangle_L |\bar{\psi}^{\hat{A}1}\bar{\psi}^{\hat{B}1}, \bar{\psi}^{\hat{A}'}\bar{\psi}^{\hat{B}'}\rangle_R$, which describe two scalars in some representation R_0 of G'' ;
- d₁): $|x^{3,4}\rangle_L |\bar{\psi}^{\hat{A}1}\bar{\psi}^{\hat{B}'}, \bar{\psi}^{\hat{A}'}\bar{\psi}^{\hat{B}'}\rangle_R$ and $|x^{5,6}\rangle_L |\bar{\psi}^{\hat{A}1}\bar{\psi}^{\hat{B}'}\rangle_R$, which each describe two scalars in representations R_1 and R_2 respectively of G'' .

We know from subsection 1.1 that the scalars of a generic $N = 1, D = 4$ supergravity theory must parametrize a Kähler manifold, which is in general arbitrary, but is now specified by the $N = 4$ to $N = 2$ to $N = 1$ reduction above. The dilaton fields still parametrize a Kähler $SU(1,1)/U(1)$ manifold as before. The scalars Φ_0 still obey the same constraints (2.23) as before, and parametrize an $O(2,m_0)/O(2) \times O(m_0)$ Kähler manifold, where $m_0 = \dim R_0$. The scalars $\Phi_{1,2}$ are now dissociated, with the constraints (2.24) becoming

$$\text{Tr}\Phi_i\Phi_i^* = -\frac{1}{2}, \quad \text{Tr}\Phi_i\Phi_i = 0 \quad (i=1,2) \quad (2.25)$$

They now parametrize the direct product $[O(2,m_1)/O(2) \times O(m_1)] \otimes [O(2,m_2)/O(2) \times O(m_2)]$ of two Kähler manifolds with $m_{1,2} = \dim R_{1,2}$.

Because of the factorized form of the Kähler manifold, the Kähler potential $\mathcal{G}$ can be written as a sum of terms:^{66,67}

$$\mathcal{G} = \mathcal{G}_D + \mathcal{G}_0 + \mathcal{G}_1 + \mathcal{G}_D + \ln|W|^2 + \mathcal{G}_C \quad (2.26)$$

where the first four terms correspond to the dilaton and $\Phi_{0,1,2}$ sectors introduced above, W is the superpotential, and $\mathcal{G}_C$ characterizes the couplings of the chiral sector which will be introduced shortly. We can write $\mathcal{G}_D$ in the standard form [cf. equation (1.13)],

$$\mathcal{G}_D = -\ln(z + z^*) \quad (2.27)$$

by introducing the variable $z \equiv (\phi_1 - \phi_2)/(\phi_1 + \phi_2)$. To describe each of the different $O(2,m)/O(2) \times O(m)$ sectors, we write

$$\Phi_i = (\gamma^0, \gamma^1, \gamma^k)_i : i = 0, 1, 2 \quad (2.28)$$

subject to the conditions (2.25) which can be written as

$$-|\gamma^0|^2 - |\gamma^1|^2 + |\gamma^k|^2 = \frac{1}{2}, \quad -(\gamma^0)^2 - (\gamma^1)^2 - (\gamma^k)^2 = 0 \quad (2.29)$$

These enable us to write

$$\gamma^0 = \frac{1}{\sqrt{2}} (1 - |\hat{\gamma}^1|^2 + |\hat{\gamma}^k|^2)^{-\frac{1}{2}} : \hat{\gamma}^{1,2} = -1 + \hat{\gamma}^{k^2} \quad (2.30)$$

where $\hat{\gamma}^{1,i} \equiv \gamma^{1,i}/\gamma^0$. In terms of these variables, we have

$$\mathcal{G}_{0,1,2} = 2\ln\gamma_{0,1,2}^0 = -\ln \left(1 + \sqrt{(1 - \hat{\gamma}_{0,1,2}^i)^2 (1 - \hat{\gamma}_{0,1,2}^{i^*})^2 - \hat{\gamma}_{0,1,2}^i \hat{\gamma}_{0,1,2}^{i^*}} \right) \quad (2.31)$$

To fix the superpotential W in (2.26), we look at the gravitino mass term in the effective Lagrangian, which is

$$-\frac{2}{3}(\phi_1 + \phi_2) \text{Tr} \Phi^{ab} [\Phi_{a1}, \Phi_{b1}] \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu \quad (2.32)$$

for one of the gravitinos in the $N = 4$ theory (2.8). When reduced to the $N = 1$ theory (2.10), equation (2.32) becomes

$$-4(\phi_1 + \phi_2) \text{Tr} \Phi_0 [\Phi_1, \Phi_2] \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu \quad (2.33)$$

and on identifying the coefficient with $-e^{g/2}$ (1.7), we find

$$W = 2 \text{Tr} \hat{\gamma}_0 [\hat{\gamma}_1, \hat{\gamma}_2] \quad (2.34)$$

Of course, in addition to the Kähler potential (2.26), the effective low-energy theory requires for its specification the gauge kinetic function f_{ab} (1.2), which takes the simple form

$$f_{ab} = \delta_{ab} z \quad (2.35)$$

for all four-dimensional string models.

To obtain an effective action for chiral fields,^{66,68} we can start from a generic $N = 2$ model of the form (2.9) with, for example,

$$b_1 = \{ \psi^i, \chi^1, \chi^2, \psi^3, \dots, \psi^6, \bar{\psi}^1, \dots, \bar{\psi}^6 \} \quad (2.36)$$

The projections $(-1)^S$ and $(-1)^{b_1}$ then yield scalars which parametrize an $SU(2,M)/SU(2) \times SU(M) \times U(1)$ manifold if we neglect interactions with the non-chiral sector described above. We can then reduce down to an $N = 1$ model (2.10) by making another projection, using, for example,

$$b_2 = \{ \psi^i, \chi^3, \chi^4, \omega^1, \omega^2, \omega^5, \omega^6, \bar{\psi}^1, \dots, \bar{\psi}^{10}, \bar{\psi}^{17}, \dots, \bar{\psi}^k \} \quad (2.37)$$

$k = 22, 30 \text{ or } 38$

where $k = 22, 30$, or 38 . In this case we get a set of chiral $O(10)$ generations $C = \sum_{\alpha} C^{\alpha}$ parametrizing a product of $SU(1,M)/SU(M) \times U(1)$ manifolds in the absence of couplings to the non-chiral sector. We have a chiral field contribution to the Kähler potential (2.26) of the form⁶⁸

$$G_c = \sum_{\alpha} G^{\alpha}, \quad \delta W = W_c \quad (2.38)$$

where

$$G^{\alpha} = -\ln(1 - C^{\alpha} \cdot C_{\alpha}^{\dagger} f(\gamma^0, z)) \quad (2.39)$$

and

$$W_c = 2\text{Tr} \Phi_0 [C, C] \quad (2.40)$$

It should be emphasized that the precise forms of G_c and W_c are quite model-dependent, although the general features of (2.39) and (2.40) may be expected to persist in more general models.

2.4 Comments on $D = 4$ Phenomenology

The phenomenology of string models formulated directly in four dimensions is not so specific as, and is less well-developed than, the phenomenology of Calabi-Yau compactification, which was introduced in Lecture 1 and will be worked out in more detail in Lecture 3. Accordingly, these comments are rather sketchy and intended to provoke further effort.

Just how general are the results (2.26), (2.27), (2.31), (2.34), (2.35), and (2.38)–(2.40)? and how related are they to the Calabi-Yau³⁷ and orbifold³⁸ results (1.48) to (1.52)? At least the dilaton sector is universal. There is always an $SU(1,1)/U(1)$ piece in the Kähler potential, and the gauge kinetic function $f_{ab} = \delta_{ab} z$ is always the same. All the manifolds for gauge non-singlet fields in (1.50)–(1.52) and (2.31) are obtained from truncation of an $N = 4$, $D = 4$ supergravity model, and hence are given by Kähler submanifolds of its uniquely prescribed $O(6,n)/O(6) \times O(n)$ coset manifold (2.22). It seems⁷⁷ that all Kähler submanifolds of this space can be obtained from this by some four-dimensional string, either formulated directly or after some compactification from $D = 10$ (which should be included in the most general four-dimensional formulation).

It is certainly possible to get observable sector gauge groups of rank < 6 , including $O(10)$ and $SU(5) \times U(1)$ as will be discussed shortly. However, no example is yet known of a model with an $SU(3)_c \times SU(2)_L \times U(1)^{1 \text{ or } 2}$ gauge group. Quite often a $U(1)$ subgroup is anomalous, which makes possible the D-term mechanism for supersymmetry breaking⁴³ mentioned in subsection 1.4.

Looking at the couplings (2.40), the possibility suggests itself⁶⁶ that whereas the matter fields must be among the chiral fields C , the Higgses might be among the Φ_0 . Note that since the Φ_0 parametrize a real $O(2, m_0)/O(2) \times O(m_0)$ manifold, it must contain a self-conjugate representation of the gauge group, for example a $\mathbf{5} + \bar{\mathbf{5}}$ of $SU(5)$.

Note also that it is difficult, if not impossible, to obtain any matter or Higgs fields in an adjoint representation of the low-energy gauge group. The genesis of this problem can be seen in the particle

sets (b_i) , (c_i) , and (d_i) of subsection 2.3: none of the scalar fields (c_i, d_i) has all the same right-moving fermion content as the gauge fields (b_i) . It was also true that the manifold compactifications of Lecture 1 never gave adjoint Higgs fields: only **27**'s of E_6 , only **16**'s and **10**'s of $O(10)$, only **5**'s, **10**'s, $\bar{\mathbf{5}}$'s, and $\bar{\mathbf{10}}$'s of $SU(5)$, etc. This is embarrassing for would-be constructors of Grand Unified Theories (GUTs), since they always needed adjoint Higgs representations of the GUT group: **78** of E_6 , **45** of $SO(10)$, **24** of $SU(5)$, etc.

It was while contemplating this difficulty for GUT model-building that we stumbled serendipitously across the flipped $SU(5)$ GUT⁷⁸ described in the next subsection.

2.5 A Flipped $SU(5)$ GUT Inspired by Strings

The sort of $SU(5)$ model suggested by strings contains only $\mathbf{5}$, $\bar{\mathbf{5}}$, $\mathbf{10}$, and $\bar{\mathbf{10}}$ representations of Higgses. The $\mathbf{5}$ and $\bar{\mathbf{5}}$ are welcome for the purpose of breaking $SU(2)_L$, but we cannot give a v.e.v. to any component of the $\mathbf{10}$ or $\bar{\mathbf{10}}$ of conventional $SU(5)$ without breaking $U(1)_{em}$. The way around this difficulty is to start from an $SU(5) \times U(1)$ gauge group⁷⁹ in which $U(1)_{em}$ is a non-trivial superposition of the extra $U(1)$ and of the $U(1)$ of $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)$. Such an extra $U(1)$ can appear naturally in a string-inspired model,^{53,80} as we shall see later.

The matter fields are $F_i = (\mathbf{10}, 1)$, $\bar{F}_i = (\bar{\mathbf{5}}, -3)$, and $\ell_i^c = (\mathbf{1}, 5)$ of $SU(5) \times U(1)$, where $i = 1, 2, \dots, N_g$. The assignments of (u, d) , (u^c, d^c) , (ν, e) , and (ν^c, e^c) in this model are flipped by comparison with those in conventional $SU(5)$: their assignments become⁷⁹

$$F_i = \begin{pmatrix} 0 & d_B^c & -d_Y^c & d_R & u_R \\ -d_B^c & 0 & d_R^c & d_Y & u_Y \\ d_Y^c & -d_R^c & 0 & d_B & u_B \\ -d_R & -d_Y & -d_B & 0 & \nu^c \\ -u_R & -u_Y & -u_B & -\nu^c & 0 \end{pmatrix}_L, \quad \bar{F}_i = \begin{pmatrix} u_R^c \\ u_Y^c \\ u_B^c \\ \nu \\ e \end{pmatrix}_L, \text{ etc.} \quad (2.41)$$

The Higgs fields are $H \equiv (\mathbf{10}, 1)$, $\bar{H} \equiv (\bar{\mathbf{10}}, -1)$, $h \equiv (\mathbf{5}, -2)$, and $\bar{h} \equiv (\bar{\mathbf{5}}, 2)$ of $SU(5) \times U(1)$, and the model needs $(N_g + 1)$ singlet fields $\phi_{0,1} = (\mathbf{1}, 0)$ of $SU(5) \times U(1)$. The superpotential of the model is⁷⁸

$$W = \underbrace{\lambda_1 F F h}_{m_d} + \underbrace{\lambda_2 F \bar{F} \bar{h}}_{m_u} + \underbrace{\lambda_3 \bar{F} \ell^c h}_{m_l} + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} + \lambda_6 F \bar{H} \phi + \lambda_7 h \bar{h} \phi + \lambda_8 \phi^3 \quad (2.42)$$

where we have indicated the roles of some of the Yukawa couplings, all of which should be regarded as tensors in flavour space. The superpotential (2.42) is the most general one respecting the global Z_2 symmetry: $H \leftrightarrow -H$. It is easy to see that the superpotential (2.42) has a flat direction in field space along which one can develop a large v.e.v. without generating a large vacuum energy. It corresponds to the ν^c component of the H and the $\bar{\nu}^c$ component of the $\bar{H}$ acquiring large v.e.v.'s: $\langle 0 | \nu_H^c, \bar{\nu}_H^c | 0 \rangle \approx m_{GUT}$: $SU(5) \times U(1) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ as desired.

This $SU(5) \times U(1)$ model has two other interesting phenomenological properties.⁷⁸ One is that it solves naturally and very simply the doublet-triplet mass-splitting problem. As in any other $SU(5)$ GUT, the $\mathbf{5}$ and $\bar{\mathbf{5}}$ of Higgses contain colour **3**, charge $|^{1/3}|$ particles: $h = (D_R, D_Y, D_B, h^0, h^-)$, $\bar{h} =$

$(D_R^c, D_Y^c, D_B^c, h^0, h^+)^T$. These have baryon-number-violating combinations of couplings to $\bar{q}q$ (qq) and to $q\ell$ ($\bar{q}\bar{\ell}$), and would mediate rapid proton decay if they were light. The doublet-triplet mass-splitting problem is to make the D and D^c very heavy while keeping (h^0, h^-) and (h^0, h^+) light. This happens naturally in the $SU(5) \times U(1)$ model (2.41), (2.42), because the λ_4 and λ_5 terms give large Dirac masses $O(m_{GUT})$ by coupling D to a d^c -like particle in the $\mathbf{10}_H$, and the D^c to a $\bar{d}^c$ -like particle in the $\mathbf{\bar{10}}_{\bar{H}}$:

$$(\nu, \nu^c, \phi) \begin{pmatrix} 0 & m_u & 0 \\ m_u & 0 & \lambda_5 M_{GUT} \\ 0 & \lambda_5 M_{GUT} & 0 \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ \phi \end{pmatrix} \quad (2.43)$$

People who remember traditional GUTs will appreciate the simplicity of this mechanism: previous attempts at natural doublet-triplet mass splitting used $\mathbf{75}$, $\mathbf{50}$, and $\mathbf{\bar{50}}$ representations of Higgs fields.⁸¹ The second elegant feature of the model is its see-saw neutrino mass matrix:

$$(\nu, \nu^c, \phi) \begin{pmatrix} 0 & m_u & 0 \\ m_u & 0 & \lambda_5 M_{GUT} \\ 0 & \lambda_5 M_{GUT} & 0 \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ \phi \end{pmatrix} \quad (2.44)$$

which gives a massless neutrino and a heavy neutrino with $m \approx m_{GUT}$ in each generation.

Although this flipped $SU(5)$ model was inspired by the superstring, it has not yet been derived from it. However, there are some indications as to how this might be done.⁵³ The first comment is that one cannot hope to obtain it from a conventional $SO(10)$ GUT, since the Clebsch-Gordan relations of $SO(10)$ applied to the desirable couplings λ_4, λ_5 generate unwanted couplings which give large masses to the doublet components of h and $\bar{h}$. However, these Clebsch-Gordan relations can in principle⁵³ be violated in a string $SO(10)$ model by the Hosotani mechanism to be discussed in subsection 3.1. A Calabi-Yau compactification will not work, though, since the λ_2 coupling would require the $\bar{h}$ to be in a $\mathbf{27}$ of E_6 , while the λ_5 coupling would require it to be in a $\mathbf{\bar{27}}$ of E_6 . The only hope using manifold compactification appears to be some alternative manifold of the type discussed in subsection 1.3, which can give a Hosotani-broken $SO(10)$ GUT. The $(0,2)$ compactifications are not desirable because of the instability induced by world-sheet instantons^{49,50} which was discussed in subsection 1.5. The $(1,2)$ compactifications have no such problems: world-sheet instantons would neither generate an instability for the T field, nor generate large non-perturbative masses for the singlet fields $\phi_{0,i}$.⁵³ An identikit picture of the desired manifold has been given,⁵³ including the topological properties needed to get the desired light $\mathbf{5}$, $\mathbf{\bar{5}}$, $\mathbf{10}$, $\mathbf{\bar{10}}$, and $\mathbf{1}$ fields, and the couplings (2.42) appear to be consistent with the Hosotani symmetry-breaking mechanism, but no manifold with the desired properties has yet been found.

Since most of this Lecture has been concerned with direct four-dimensional formulations of string models, it is natural to ask how to obtain flipped $SU(5)$ from such a model. It is indeed possible⁸⁰ to obtain the $SU(5) \times U(1)$ gauge group using the generalized $\psi_i \rightarrow e^{i\pi\alpha_i}\psi_i$ boundary conditions discussed in subsection 2.2. This is done by having one basis vector of boundary conditions with $\alpha_i = 1$ for 10 of the right-moving fermions [which gives an $SO(10)$ gauge group], and another basis vector with $\alpha_i = +1/2$ for five of these fermions and $\alpha_i = -1/2$ for the other five [which breaks $SO(10) \rightarrow SU(5) \times U(1)$]. The trick is to extend these conditions to a set which gives the correct numbers of $\mathbf{5}$, $\mathbf{\bar{5}}$, $\mathbf{10}$, $\mathbf{\bar{10}}$, and $\mathbf{1}$ fields needed in the flipped $SU(5)$ model. Maybe when we get back from the beach...?

3. CALABI-YAU PHENOMENOLOGY

3.1 Breaking Gauge Symmetry

We recall from Lecture 1 that the maximal gauge symmetry in four dimensions after Calabi-Yau compactification is the subgroup of $E_8 \times E_8$ that commutes with the $SU(3)$ factor identified with the holonomy group, i.e. $E_6 \times E_8$. The observable gauge interactions are to be found in the E_6 factor, whilst the E_8 factor constitutes a hidden sector where strong dynamics may cause gauginos to condense and thereby trigger supersymmetry breaking. Both the E_6 and E_8 factors may be broken down to subgroups by the Hosotani mechanism,⁸² which is a non-Abelian analogue of the Bohm-Aharonov

Fig. 7. (a) The traditional Aharonov-Bohm effect,⁸³ and (b) its analogue⁸² in compactified field theory.

effect⁸³ illustrated in Fig. 7a. In its original form, we imagine transporting a charged particle around a circuit C which is penetrated by magnetic flux lines—for example a circuit just outside a solenoidal magnet (such as one of the LEP experiments). After this transport around C, the wave function of the particle acquires a phase $e^{ie\phi}$: $\phi = \oint_C \mathbf{A} \cdot d\mathbf{x}$. Since the phase factor must be $+1$ for a boson and ± 1 for a fermion, we obtain from the known electric charge a quantization condition on the magnetic flux. In principle, we could also imagine doing the reverse: if we had first measured a quantum of magnetic flux ϕ , we could then have deduced from it what values of the electric charge were allowed by the requirement that $e^{ie\phi} = \pm 1$. The Hosotani mechanism⁸² works in this reversed way: first, imagine a manifold of compactification with non-contractible loops C in it (you can imagine a manifold with tunnels through it, like those containing accelerators at CERN) as in Fig. 7b. Mathematically, we consider a manifold $G \equiv \Pi_1(K) \neq 0$. Suppose then that particle wave functions acquire phases $e^{i\phi}$ when transported around these non-contractible loops (as if there were magnetic fields running along the CERN tunnels). The set of phase factors would constitute a finite representation of the discrete group G : $\{e^{i\phi}\} = U_G$, operating on the matter fields representing the original gauge group (in our case E_6). The residual observable gauge group in four dimensions would be the subgroup V of the original group which commutes with U_G :

$$[V, U_G] = 0 \quad (3.1)$$

Gauge fields and/or matter fields which were not left invariant by U_G would not be legal, in just the same way that particles with non-integral electric charge would not be legal according to the reversal of the history of the Bohm-Aharonov effect that we imagined above.

This mechanism could be used to break any gauge symmetry obtained from higher dimensions, such as the $SO(10) \rightarrow SU(5) \times U(1)$ breaking of subsection 2.4, but here we concentrate on its application to E_6 .⁸⁴ Since we want an observable gauge group including $SU(3)_c \times SU(2)_L \times U(1)_Y$, it is convenient to analyse the breaking of E_6 using the decomposition via maximal subgroup $SU(3)_c \times SU(3)_L \times SU(3)_R$. Clearly $SU(3)_c$ can be identified with the first $SU(3)$ factor and $SU(2)_L$ can be embedded in the second $SU(3)$ factor. On the other hand, the hypercharge Y can only be represented by a superposition of generators $Y = \frac{1}{2}(Y_L + Y_R)$:

$$Y_L = (0) \times \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix} \times (0) \quad (3.2a)$$

$$Y_R = (0) \times (0) \times \begin{pmatrix} \frac{4}{3} & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix} \quad (3.2b)$$

The most general possible group element U_G that commutes with $SU(3)_c \times SU(2)_L \times U(1)_Y$ is therefore

$$U_G = (1) \times \begin{pmatrix} \alpha_G & 0 & 0 \\ 0 & \alpha_G & 0 \\ 0 & 0 & \alpha_G^{-2} \end{pmatrix} \times \begin{pmatrix} \beta_G & 0 & 0 \\ 0 & V_G & 0 \\ 0 & 0 & 1 \end{pmatrix} : \det V_G = \beta_G^{-1} \quad (3.3)$$

It is clear that U_G conserves separately both Y_L and Y_R , and therefore that the observable four-dimensional gauge group is at least as large as $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_E$, where the extra hypercharge assignments are completely specified for the minimal rank-5 gauge group. To realize this requires a non-Abelian 2×2 matrix V_G , hence a non-Abelian discrete group U_G and a non-Abelian fundamental group $G = \Pi_1(K)$. Other possible four-dimensional gauge groups which can be, and have been, considered are all of rank 6, and include $SU(3) \times SU(2) \times U(1)^3$, $SU(3) \times SU(2)^2 \times U(1)^2$, $SU(3)^3$, $SU(4) \times SU(2) \times U(1)^2$, etc.⁸⁴ These can be obtained from Abelian groups G .

Since the rank of the Standard Model gauge group is 4, it is clear that to get down to it we need more symmetry breaking by the conventional Higgs mechanism. In Calabi-Yau compactification, the available candidates for Higgs fields are in the following representations:

$$\begin{aligned} 27|_{E_6} &= (\underline{16} + \underline{10} + \underline{1})|_{SO(10)} \\ &= ((\underline{10} + \bar{\underline{5}} + \underline{1}) + (\underline{5} + \bar{\underline{5}}) + \underline{1})|_{SU(5)} \end{aligned} \quad (3.4)$$

The first $\underline{10} + \bar{\underline{5}}$ in the bottom line of (3.4) can be the quarks and leptons in a conventional $SU(5)$ GUT, whilst the first $\underline{1}$ is equivalent to a conjugate neutrino ν^c in a conventional $SO(10)$ GUT. The next $\underline{5} + \bar{\underline{5}}$ contain two doublet Higgses H and H^c , and their colour-triplet partners D and D^c . Finally, there is another $\underline{1}$ of $SU(5)$ which is also a $\underline{1}$ of $SO(10)$, called here N . The $SU(3)_c \times SU(2)_L \times U(1)_Y$ -invariant fields which can break the symmetry down to the Standard Model are the ν^c and the N . If the Hosotani mechanism has provided us with a rank-5 gauge group, then we need *either* $\langle 0|\nu^c|0 \rangle \neq 0$ *or* $\langle 0|N|0 \rangle \neq 0$, whereas if we have a rank-6 gauge group, then we need *both* $\langle 0|\nu^c|0 \rangle \neq 0$ *and* $\langle 0|N|0 \rangle \neq 0$. In the rest of this subsection we discuss various phenomenological considerations bearing on the choice of these v.e.v.'s.

The Yukawa couplings of E_6 models obtained from Calabi-Yau compactification take the form⁸⁴

$$\begin{aligned} W = & \underbrace{\lambda_2 H^c l l^c}_{m_l} + \underbrace{\lambda_3 H^c q d^c}_{m_d} + \underbrace{\lambda_1 H q u^c}_{m_u} + \underbrace{\lambda_4 H l \nu^c}_{m_\nu!} \\ & + \underbrace{\lambda_4 H H^c N}_{HH^c \text{ mixing}} + \underbrace{\lambda_5 D D^c N}_{m_D} \\ & + \underbrace{(\lambda_6 D q q + \lambda_7 D^c u^c d^c)}_{D \rightarrow \bar{q} \bar{q}, D^c \rightarrow q q} + \underbrace{(\lambda_8 D l^c u^c + \lambda_9 D^c l q + \lambda_{10} D \nu^c d^c)}_{D \rightarrow \bar{q} l, D^c \rightarrow \bar{q} \bar{l}} \end{aligned} \quad (3.5)$$

Indicated under each term is the role it can play in the effective low-energy theory. Note that if terms in both of the parentheses on the last line are present, then the exchange of light D/D^c particles can mediate rapid proton decay.

As was already mentioned, if the Hosotani mechanism leaves us with a rank-6 gauge group, we need both $\langle 0|\nu^c|0\rangle \equiv y$ and $\langle 0|N|0\rangle \equiv x \neq 0$, and the vacuum has $\Delta L \neq 0$. In this case the $d/D/d^c/D^c$ mass matrix takes the form^{85,86}

$$(d, D) \begin{pmatrix} \lambda_3 v^c & 0 \\ \lambda_{10} y & \lambda_5 x \end{pmatrix} \begin{pmatrix} d^c \\ D^c \end{pmatrix} \quad (3.6)$$

and the mass eigenstates are in general mixtures of d with D , and of d^c with D^c . Moreover, the λ_v term in (3.5) gives us H/ℓ mixing. As a result, the generic model of this type has $\Delta B \neq 0$ and $\Delta L \neq 0$ interactions, and we will study a specific example of this difficulty in subsection 3.2. On the other hand, if the Hosotani mechanism leaves us with a rank-5 gauge group, we only need one of $\langle 0|\nu^c|0\rangle = y$, $\langle 0|N|0\rangle = x \neq 0$, which we may take without loss of generality to be $x \neq 0$. In this case the vacuum has $\Delta L = 0$, and the low-energy theory has no ΔB or $\Delta L = 0$ interactions if *either* the interactions in the first parenthesis on the last line of (3.5) *or* those in the second parenthesis are present, but *not both*. We will study in subsection 3.3 whether this scenario can be achieved in realistic models.

In order to minimize the problems with $\Delta B \neq 0$ interactions in rank-6 models by making the dangerous exchanged particles very heavy, it is often desired that either $y = \langle 0|\nu^c|0\rangle$ or $x = \langle 0|N|0\rangle$ (or both) be very large, $\sim m_1 \gtrsim 10^{16}$ GeV. If just one of these is very large, then the light states left over by the $d/D/d^c/D^c$ mixing matrix (3.6) are either essentially pure d/d^c or d/D^c combinations. On the other hand, if both x and y are large, the light, charge $+1/3$ antiquarks are mixtures of d^c and D^c . If both x and y were small, there would be significant d/D mixing as well as d^c/D^c mixing, and flavour-changing interactions would become a problem.^{85,86} The only chance for rank-6 models is to have at least one of x and y very large.

We can generate one or more large v.e.v.'s, $x, y \gg v, v^c$, by exploiting some flat direction in the low-energy effective potential.⁸⁴ The F-terms associated with the superpotential (3.5) are flat if y and/or x is large. To get flat D-terms we must postulate the presence of conjugate fields $\bar{\nu}$ and/or $\bar{N}$ from some $\overline{27}$ representation of E_6 , as appear naturally in most Calabi-Yau compactifications. The residual scalar potential along such a flat direction (call it ϕ) can only contain a soft supersymmetry-breaking (mass)² term and non-renormalizable terms:

$$V = \tilde{m}^2 \phi^2 + \frac{\phi^{4+n}}{m_p^n} \quad : n > 1 \quad (3.7)$$

The non-renormalizable term in (3.7) could perhaps come from the exchanges of massive Kaluza-Klein modes, or from non-perturbative effects (although some specific calculations⁸⁷ have not found such terms). If the soft supersymmetry-breaking coefficient $\tilde{m}^2$ in the potential (3.7) is negative, the minimum is obtained for

$$\langle 0|\phi|0\rangle = \phi_0 \sim \left((-\tilde{m}^2) m_p^n \right)^{1/2+n} \quad (3.8)$$

One expects that $|\tilde{m}^2| \approx m_W^2$, so $\langle 0|\phi|0\rangle$ in (3.8) is above 10^{16} GeV only if the power of non-renormalizability n in (3.7) and (3.8) is very large.

Large v.e.v.'s, $\langle 0|\phi|0\rangle$, as required in rank-6 models, lead to the danger of excess entropy generation when the phase transition to $\langle 0|\phi|0\rangle \neq 0$ (3.8) is completed,^{88,89} as we now see. The effective potential (3.7) is shown in Fig. 8, together with the following corrections which appear at finite temperature,

$$\delta V(\phi, T) \simeq \# T^4 + \lambda^2 \phi^2 T^2 \quad (3.9)$$

where $\#$ is the number of light degrees of freedom, and the second term is effectively damped when $\phi \approx T$. What is actually shown as the coefficient of the T^4 term in Fig. 7 is the difference

$$\Delta \# = \#(\langle 0|\phi|0\rangle \neq 0) - \#(\langle 0|\phi|0\rangle = 0) \quad (3.10)$$

Fig. 8. Sample movie of the evolution with temperature of the effective potential $V_{\text{eff}}(\Phi, T)$ in a strongly-coupled $SU(5) \times U(1)$ model⁹¹ when $T/\Lambda_c = O(1)$.

which is generally negative when we calculate in perturbation theory.⁸⁸ Therefore, as seen in Fig. 8, there is a local minimum in the potential at the origin, where the ϕ field is trapped during the early history of the Universe. Eventually the ϕ field will tunnel out to a large value of ϕ , where $V(\phi) + \delta V(\phi, T) \leq V(0)$, but this tunnelling is in general very delayed. When it finally occurs, a large energy density $V(0) - V(\phi_0)$ is released into the Universe, which tends to be much larger than the energy in light relativistic particles. This surplus vacuum energy is converted into a large amount of entropy.^{88,89} If cosmological baryosynthesis occurred at high temperature, as in the traditional British scenario (out-of-equilibrium decay of a heavy particle), then a reasonable upper bound on this entropy increase implies^{88,89}

$$m_I \lesssim 10^7 \text{ GeV} \quad (3.11a)$$

whereas if baryosynthesis occurs at low temperatures,⁹⁰

$$m_I \lesssim 10^{10} \text{ GeV} \quad (3.11b)$$

could be allowed. Both of the bounds (3.11) are incompatible with the desired value $m_I \gtrsim 10^{16} \text{ GeV}$.

One way around this difficulty is to postulate⁹¹ strong-coupling condensation effects in the $\langle 0|\phi|0\rangle = 0$ theory, which might *either* break supersymmetry and generate $V(\phi) > 0$ for ϕ below some condensation scale Λ_c , *or* reduce the number of effective degrees of freedom so that $\Delta\#$ in (3.10) becomes positive. In either case, one obtains an effective bump instead of hole at small ϕ in Fig. 8, the ϕ field never gets trapped near the origin, and there is no danger of excess entropy generation. This scenario⁹¹ applies to the $SU(5) \times U(1)$ model discussed in subsection 2.5, but is not applicable if the maximal semi-simple gauge subgroup factor is $SU(3)$, as in the next subsection.

3.2 Phenomenology of the Yau Manifold

This is the first of two subsections in which we discuss the possible phenomenology which might be obtained from the two specific Calabi-Yau manifolds introduced in subsection 1.2. The Yau manifold³¹ has three generations and an Abelian fundamental group G . Therefore, by the analysis at the beginning of subsection 3.1, the Hosotani mechanism⁸² can only break E_6 down to a rank-6 group. The case studied in Ref. 92 was $E_6 \rightarrow SU(3) \times SU(3) \times SU(3)$, although other possibilities could in principle be considered. The spectrum of light gauge non-singlet particles then contains, in addition to three full $\mathbf{27}$ representations of E_6 , also six complete sets of colour-singlet leptons and of antileptons from split $\mathbf{27}$ and $\overline{\mathbf{27}}$ representations of E_6 denoted generically by L and $\bar{L}$, and four complete sets of colour non-singlets from split $\mathbf{27}$ and $\overline{\mathbf{27}}$ representations denoted generically by Q and $\bar{Q}$.⁹² The Yau manifold

could in principle be subject to many types of deformation, to which correspond the different massless particles. If the configuration ground-state of the Yau manifold is particularly symmetric, the fields describing these massless particles will have discrete symmetries which force many Yukawa couplings to vanish. These leave many flat directions in the potential, along which some large non-zero $\langle 0 | \bar{\nu}_i^c | 0 \rangle$ and/or $\langle 0 | \bar{N}_j | 0 \rangle$ may develop. The flatness can in principle be broken by higher-order couplings such as $LL\bar{L}\bar{L}$, which are allowed by the discrete symmetries.⁹² It is an open question which of any of these are actually generated by Kaluza–Klein exchanges and/or non-perturbative effects. Even if we include all the $LL\bar{L}\bar{L}$, $LQ\bar{L}\bar{Q}$, and $QQ\bar{Q}\bar{Q}$ and higher-order couplings allowed by the discrete symmetries, there is considerable ambiguity in the choice of non-zero $\langle 0 | \bar{\nu}_i^c | 0 \rangle$ and $\langle 0 | \bar{N}_j | 0 \rangle$. The direction which was studied initially⁹² was the one where

$$\langle 0 | \bar{\nu}_1^c - \bar{\nu}_2^c | 0 \rangle, \langle 0 | \bar{\nu}_3^c | 0 \rangle, \langle 0 | \bar{\nu}_6^c | 0 \rangle \sim 10^{14} \text{ GeV} \quad (3.12a)$$

and

$$10^{14} \text{ GeV} \gg \langle 0 | N_8 + N_9 | 0 \rangle, \langle 0 | \bar{N}_1 | 0 \rangle, \langle 0 | \bar{N}_2 | 0 \rangle \gg 10^{11} \text{ GeV} \quad (3.12b)$$

Getting these orders of magnitude requires the appropriate powers of n in the non-renormalizable terms in the effective potential (3.7). Given the v.e.v.'s (3.12), the effective low-energy theory has no $\Delta B \neq 0$ or $\Delta L \neq 0$ interactions. In general, we require $m_D \geq 10^{16} \text{ GeV}$ in order to avoid their mediating proton decay too rapidly, so the v.e.v.'s (3.12) might not be large enough—but this is a relatively minor quibble.

In the absence of a clear dynamical criterion favouring the flat direction (3.12) over any other, we might ask whether the absence of $\Delta B \neq 0$ and $\Delta L \neq 0$ interactions is a generic feature of all (or most) flat directions. We⁹³ have looked for flat directions with

$$\langle 0 | \bar{\nu}_1^c | 0 \rangle, \langle 0 | \bar{\nu}_6^c | 0 \rangle, \langle 0 | \bar{\nu}_8^c | 0 \rangle \neq 0 \quad (3.13a)$$

and a single $\langle 0 | \bar{N}_i | 0 \rangle \neq 0$. We found the possibilities

$$\langle 0 | N_{1,2,4,5,6,7,9} | 0 \rangle, \langle 0 | \bar{N}_1 | 0 \rangle, \langle 0 | \bar{N}_2 | 0 \rangle \neq 0 \quad (3.13b)$$

With the combination of v.e.v.'s (3.13), we expect, as argued in subsection 3.1, that there will be light states which are mixtures of $d^c/D^c/\bar{D}$, of $D/\bar{d}^c/\bar{D}^c$, and of $\ell/H^c/\bar{H}$. This is indeed what happens.⁹³ We have then studied whether these models (3.13) have enough $m_u \neq 0$, enough $m_d \neq 0$, enough $m_\ell \neq 0$, small m_ν , and an absence of $\Delta B \neq 0$, $\Delta L \neq 0$, and flavour-changing interactions in the effective low-energy theory. As seen in Table 1, the answers are ‘No’ for most of the flat directions (3.13).

We are therefore left with the conclusions that there is great ambiguity in the phenomenological interpretation of the Yau model, that it has one good candidate vacuum (3.12)⁹² and many bad ones

Table 1 Possible problems in $n(\bar{1}, \bar{2})$ flat directions

n	$3m_u$ + mirrors	$3m_d$ + mirrors	$3m_\ell$ + mirrors	$m_\nu = 0$	HH ^c mixing	No HL mixing	No $\Delta B \neq 0$	No $\Delta L \neq 0$	Absence of FCNC due to				
									Hqu ^c	H ^c qd ^c	H ^c le ^c	Dqq	D ^c u ^c d ^c
1	X(1)	X(2)	X(1)	X(1)	✓ ⁺	✓ ^x	X	X	✓	X	X	✓	X
2	X(2)	X(2)	X(2)	X(2)	✓ ⁺	✓ ^x	✓	X	✓	✓	✓	✓	✓
4	X(3)*	✓	X(2)	X(3)	✓ ⁺	✓ ^x	X	X	X	X	X	X	X
5	X(3)*	X(4)*	X(1)	X(1)	✓ ⁺	✓	X	X	X	X	X	X	X
6	X(3)*	X(1)*	X(3)*	X(1)	X ^x	✓	X	X	✓	X	X	X	X
7	X(3)*	X(2)*	✓	X(2)	X ^x	✓	X	X	✓	X	✓	X	X
9	✓	✓	X(2)	✓	✓ ⁺	✓ ^x	✓	X	X	X	X	✓	X

The numbers in parentheses indicate the number of massive states:

*: not all mirror particles acquire masses;

+ : mixing in order $\langle \rangle^5$;

x: mixing in order $\langle \rangle^3$;

Problems are denoted by X, their absence by ✓.

(3.13).⁹³ The problem is to find some dynamical criterion for choosing between them, and whether there is reason to think that a bad choice can be avoided.

3.3 Phenomenology of a Manifold with a Non-Abelian Fundamental Group

We have recently studied⁹⁴ the possible phenomenology of the Strominger–Witten manifold³² which has four generations and a non-Abelian fundamental group G , which can be used to break the gauge symmetry down to the minimal rank-5 possibility $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_E$. As in the case of the Yau manifold discussed in the previous subsection, there is considerable ambiguity in the discrete symmetries of the model. One possibility is a Z_2 symmetry S ,

$$Q \rightarrow -Q, (\nu^c, N) \rightarrow -(\nu^c, N); (L, H, H^c, e^c) \rightarrow (L, H, H^c, e^c) \quad (3.14a)$$

for the first two generations, denoted hereafter by F , and

$$Q \rightarrow Q, (\nu^c, N) \rightarrow (\nu^c, N); (L, H, H^c, e^c) \rightarrow -(L, H, H^c, e^c) \quad (3.14b)$$

for the last two generations, denoted hereafter by L . The resulting mass matrices have the forms

$$\begin{aligned} & (d_F, d_L) \begin{pmatrix} \nu_F^c & \nu_L^c \\ \nu_L^c & \nu_F^c \end{pmatrix} \begin{pmatrix} d_F^c \\ d_L^c \end{pmatrix}; (u_F, u_L) \begin{pmatrix} \nu_F^c & \nu_L^c \\ \nu_L^c & \nu_F^c \end{pmatrix} \begin{pmatrix} u_F^c \\ u_L^c \end{pmatrix}; (D_F, D_L) \begin{pmatrix} \chi_L & \chi_F \\ \chi_F & \chi_L \end{pmatrix} \begin{pmatrix} D_F \\ D_L \end{pmatrix}; \\ & (h_F, h_L) \begin{pmatrix} \nu_F^c & \nu_L^c \\ \nu_L^c & \nu_F^c \end{pmatrix} \begin{pmatrix} e_F^c \\ e_L^c \end{pmatrix}; (\chi_F, \chi_L) \begin{pmatrix} \nu_L & \nu_F \\ \nu_F & \nu_L \end{pmatrix} \begin{pmatrix} \nu_F^c \\ \nu_L^c \end{pmatrix}; (H_F, H_L) \begin{pmatrix} \chi_L & \chi_F \\ \chi_F & \chi_L \end{pmatrix} \begin{pmatrix} H_F^c \\ H_L^c \end{pmatrix} \end{aligned} \quad (3.15)$$

If all the v.e.v.'s $\nu_{F,L}$, $\nu_{F,L}^c$, and $\chi_{F,L}$ were $\neq 0$, the low-energy theory would have maximal $\Delta B \neq 0$ interactions and many other disasters. However, there are consistent subsets of possible non-zero v.e.v.'s:

$$\left. \begin{aligned} & (\chi_L, \nu_F, \nu_F^c) \neq 0; (\chi_L, \nu_L, \nu_L^c) \neq 0 \\ & (\chi_F, \nu_F, \nu_F^c) \neq 0; (\chi_F, \nu_L, \nu_L^c) \neq 0 \end{aligned} \right\} \quad (3.16)$$

As well as the ambiguity (3.16), there is also a two-fold ambiguity in the assignment of the u-quark (among the u_F or the u_L ?) and of the d-quark (d_F or d_L ?), making a total of 16 inequivalent cases to study. We have found two interesting cases:⁹⁴

$$(\chi_F, \nu_F, \nu_F^c) \neq 0, u \in u_F, d \in d_F; (\chi_L, \nu_F, \nu_F^c) \neq 0, u \in u_L, d \in d_L \quad (3.17)$$

In each of these there are only two non-zero neutrino masses, which we may take to be those of the ν_τ and of the fourth-generation ν_4 , and the only $\Delta B \neq 0$ interactions involving the u-quark and the d-quarks are to these neutrinos and to two charged leptons, which we may take to be the fourth-generation ℓ_4 and the τ . Kinematics forbids $p, n \rightarrow \ell_4, \tau$ and maybe $\nu_4 + X$ decay, leaving $p, n \rightarrow \nu_\tau + X$ as the only $\Delta B \neq 0$ embarrassment in leading order. Even this could have been avoided if $m_{\nu_\tau} > m_{p,n}$, although this is excluded by experiment. A second problem is that there is no Cabibbo–Kobayashi–Maskawa mixing between the first two and the last two generations.

It should be clear from the discussion of this subsection that the phenomenology of the Strominger–Witten manifold³² is also ambiguous and unsatisfactory. The ambiguity is less extreme than in the Yau manifold,³¹ where one had to choose among a galaxy of flat directions with large v.e.v.'s fixed by postulating uncomputed non-renormalizable terms in the effective potential. Nevertheless, the hunt for 'the' Calabi–Yau manifold is not at an end.

3.4 Neutral Currents and an Additional Z'

In subsection 3.1 we saw that at least one additional gauge boson is expected in the four-dimensional gauge theory obtained from Calabi-Yau compactification.⁸⁴ As was seen in subsections 3.2 and 3.3, no convincing Calabi-Yau model exists, and we do not know what the mass of such an additional Z' might be. Nevertheless, we can ask, How do we look for it experimentally? and What are the present bounds on its mass? The answer to the first question is quite model-independent, whereas the answer to the latter is very model-dependent. In this Lecture we will illustrate the general principles of Z'-hunting with the minimal rank-5 example.^{95,96} The cases of some other models can be found in Refs. 24, 25, and 97.

The extra hypercharges Y_E of the light particles are completely fixed in the rank-5 $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_E$ example and take the values shown in Table 2. The normalization factor of $\sqrt{3/5}$ has been introduced so that $\alpha_E = \alpha_2 = \alpha_3$ at the GUT scale where the Hosotani mechanism operates, which is related to the compactification scale. One can then make a prediction for the renormalized value of α_E at present energies,

$$\alpha_E = 0.016 \quad (3.18)$$

if there are no light survivors from split $27 + \bar{27}$ representations of E_6 . All is now known except the mass of the extra Z', which is fixed by diagonalizing the squared-mass matrix^{95,96}

$$M^2 = m_{Z^0}^2 \begin{pmatrix} 1 & \frac{1}{3} \sin \theta_W \frac{4v^2 - v'^2}{v^2 + v'^2} \\ \frac{1}{3} \sin \theta_W \frac{4v^2 - v'^2}{v^2 + v'^2} & \frac{1}{9} \sin^2 \theta_W \frac{25x^2 + 16v^2 + v'^2}{v^2 + v'^2} \end{pmatrix} \quad (3.19)$$

Approximate eigenvalues of the matrix (3.19) in the limit $x^2 \gg v^2, v'^2$ are

$$m_Z^2 = m_{Z^0}^2 (1 - O(v^2, v'^2/x^2)) \quad (3.20)$$

and

Table 2 Possible neutral currents in superstring models [97]

	T_{3L}	$\sqrt{(5/3)}Y$	$\sqrt{(5/3)}Y'$	Y''
u_L^d	$\pm 1/2$	$1/6$	$1/3$	0
u_L^f	0	$-2/3$	$1/3$	0
d_L^f	0	$1/3$	$-1/6$	$1/2$
$(\nu, \ell)_L$	$\pm 1/2$	$-1/2$	$-1/6$	$1/2$
e_L^f	0	1	$1/3$	0
D_L	0	$-1/3$	$-2/3$	0
D_L^f	0	$1/3$	$-1/6$	$-1/2$
ν_L^f	0	0	$5/6$	$-1/2$
N_L	0	0	$5/6$	$1/2$
$(H^+, H^0)_L$	$\pm 1/2$	$1/2$	$-2/3$	0
$(\bar{H}^+, \bar{H}^0)_L$	$\pm 1/2$	$-1/2$	$-1/6$	$-1/2$

Model A: Z' couples to Y'

Model B: Z' couples to $(\sqrt{(3/5)} Y' - \sqrt{(5/8)} Y'')$

Model C: Z' couples to $(\sqrt{(3/5)} Y' + \sqrt{(5/8)} Y'')$

$$m_{Z'}^2 \simeq m_{Z^0}^2 \cdot \frac{\sin^2 \theta_w}{9} \cdot \frac{25\pi^2}{v^2 + v'^2} \quad (3.21)$$

where $m_{W^*}^2 = m_{Z^0}^2 \cos^2 \theta_w = \frac{1}{2} g_2^2 (v^2 + v'^2)$. The Z^0 and the Z_E mix to give the mass eigenstates Z, Z' with eigenvalues (3.20) and (3.21) via a mixing angle θ_3 :⁹⁷

$$\begin{pmatrix} m_Z^2 & 0 \\ 0 & m_{Z'}^2 \end{pmatrix} = \begin{pmatrix} c_3 & -s_3 \\ s_3 & c_3 \end{pmatrix} \begin{pmatrix} 1 & b \\ b & a \end{pmatrix} \begin{pmatrix} c_3 & s_3 \\ -s_3 & c_3 \end{pmatrix} m_{Z^0}^2 \quad (3.22)$$

The name of the game is now to bound θ_3 and $m_{Z'}$ or equivalently x .

One approach to this problem is a global analysis of all available neutral-current data including: νN and νe scattering; parity violation in atoms; asymmetries in eN and μN scattering; $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$, and $\bar{q}q$; and measurements of m_{W^*} and $m_{Z'}$.^{24,25} Two analyses of this type have recently been made, and here I will discuss the one²⁵ in which I participated. Our first step was to analyse separately the νq , lq , ll , and vector boson sectors, making model-independent fits to the neutral-current interaction parameters. Next we made combined fits using the parameters ($\sin^2 \theta_w$, $\rho \equiv m_{W^*}^2/m_{Z^0}^2 \cos^2 \theta_w$) of the Standard Model. Finally, we established bounds on the parameters ($m_{Z'}$, θ_3) of each of three superstring-inspired Z' models.

We found that the Standard Model fitted the data very (too?) well. In a one-parameter fit with ρ fixed at 1 as predicted by the Standard Model with only 1 = $\frac{1}{2}$ Higgses, we found²⁵

$$\sin^2 \theta_w = 0.228 \pm 0.004 \quad (3.23)$$

with $m_c = 1.5$ GeV (as preferred by our fit) and $m_t = 40$ GeV. The assumed value of m_t affects the radiative corrections to which the data are now sensitive, and we obtained from them an upper limit²⁵ of

$$m_t < 168 \text{ GeV} \quad (90\% \text{ c.l.}) \quad (3.24)$$

The value (3.24) of $\sin^2 \theta_w$ seems to rule out minimal SU(5) GUT models⁹⁸ of the type already troubled by the absence of $p \rightarrow e^+ \pi^0$ decay. However, supersymmetric SU(5) GUT models⁹⁹ cannot be ruled out. A two-parameter fit with ρ left free as well as $\sin^2 \theta_w$ led to²⁵

$$\sin^2 \theta_w = 0.229 \pm 0.006, \quad \rho = 1.002 \pm 0.007 \quad (3.25)$$

consistent with the usual choice of Higgs representations in the Standard Model.

Since there is no indication of any deviation from the Standard Model, we are not going to get any upper bounds on $m_{Z'}$! In the case of the minimal rank-5 model,^{95,96} we find²⁵

$$m_{Z'} > 129 \text{ GeV} \quad (90\% \text{ c.l.}) \quad (3.26)$$

if no extra assumptions are made about the Higgs v.e.v.'s, v and v' . In fact, all known models have $v'/v \leq 1$, and if we impose this constraint on the model we find a much more stringent lower limit on $m_{Z'}$, whose value depends sensitively on the confidence level sought. Examples are:

$$m_{Z'} > 375 \text{ GeV} \quad \text{if } 0 < v'/v < 1 \quad (90\% \text{ c.l.}) \quad (3.27a)$$

$$m_{Z'} > 506 \text{ GeV} \quad \text{if } 0.2 < v'/v < 0.6 \quad (90\% \text{ c.l.}) \quad (3.27b)$$

In the cases of two other superstring-inspired Z' models derived from rank-6 models that we studied, the lower bounds on $m_{Z'}$ were considerably stronger than (3.26), and less sensitive to assumptions on v'/v , or to the required confidence level.²⁵

An independent way of bounding $m_{Z'}$ using existing data is from its non-observation at the CERN $p\bar{p}$ Collider. Compiling the published results of UA1¹⁰⁰ and UA2,¹⁰¹ we find¹⁰²

$$\sigma(B(Z' \rightarrow e^+e^-)) < 1.8 \text{ pb} \quad (90\% \text{ c.l.}) \quad (3.28)$$

with only a small dependence on $m_{Z'}$ in the range m_Z to 150 GeV. To interpret (3.28) we must¹⁰² formulate a hypothesis on the available decay modes of the Z' . The simplest hypothesis is that it can only decay into conventional quarks and leptons, which gives the largest estimate of $B(Z' \rightarrow e^+e^-)$ and hence the least conservative lower bound on $m_{Z'}$. The most conservative hypothesis that the various unobserved particles (spartners of the known particles, E_6 exotica, etc.) can have any masses consistent with present direct experimental limits. These are $m_{\pm} > 20$ GeV for all charged particles, $m_{\tilde{q}} \geq 50$ GeV for the $\tilde{u}$, $\tilde{d}$, $\tilde{s}$, $\tilde{c}$, and $\tilde{b}$, and $m_t > 44$ GeV, but there are no direct limits on the masses of unseen neutral particles.¹⁰² The present experimental upper limits on $\Gamma(Z \rightarrow \text{all})$ and $\Gamma(W^* \rightarrow \text{all})$ do not give any further constraints on the masses of these unseen particles. Allowing masses to take any values above the lower mass limits and using the upper limit (3.28) we find the lower limit¹⁰²

$$m_{Z'} > 118 \text{ GeV} \quad (90\% \text{ c.l.}) \quad (3.29)$$

It is striking that the bounds (3.26) from neutral-current data and (3.29) from the UA1/UA2 Z' searches are closely comparable. We can combine them to obtain¹⁰²

$$(3.30)$$

which is the most stringent conservative bound that can be established using present data.

One of the most interesting future searches for indirect effects of the Z' will be with precision measurements on the Z peak at the SLC or LEP. Because of the mixing (3.19) between the Z and the Z' , and the resulting shift downwards (3.20) in m_Z , there can be a mismatch between the values of $\sin^2 \theta_w$ deduced from m_Z assuming the Standard Model and from measurements of the neutral-current parameters on the Z peak.^{103,104} The most sensitive of these is likely to be the polarization asymmetry¹⁰⁵

$$A_{LR} = \frac{\sigma(\leftarrow) - \sigma(\rightarrow)}{\sigma(\leftarrow) + \sigma(\rightarrow)} \quad (3.31)$$

which may be measured with a precision $\delta A_{LR} = 0.003$. Figure 9 shows contours of $m_{Z'}$ and of δA_{LR} in the x/v , v^c/v plane,¹⁰⁵ from which we see that measurements of the polarization asymmetry could well be sensitive to $m_{Z'} \approx 500$ GeV to 1 TeV.

Fig. 9. Sensitivity of A_{LR} measurements¹⁰⁵ to parameters in the minimal rank-5 superstring-inspired model, showing also contours of $m_{Z'}$.

Fig. 10. Cross-sections times branching ratios for the three superstring-inspired models described in ref. 97. The higher (lower) curves are for Z' decays into conventional particles (all particles and sparticles in three 27 's of E_6).

The obvious tool for further exploration is a high-energy hadron-hadron collider such as the LHC ($\sqrt{s} = 17$ TeV) or the SSC ($\sqrt{s} = 40$ TeV). Figure 10 shows calculations of $\sigma \cdot \text{BR}(Z' \rightarrow \ell^+ \ell^-)$ at the LHC,^{96,106} assuming either decays into quarks and leptons alone, or into all particles and sparticles in three generations, each containing a 27 of E_6 . We see that with an integrated luminosity of 10^{40} cm^{-2} it should be possible to reach $m_{Z'} \approx 4$ TeV in the minimal rank-5 model.

A high-energy e^+e^- machine could be a very useful Z' factory.^{106,107} It is still even conceivable that LEP II could turn out to be a Z' factory, although this is a long shot. In general, at the peak of the Z' resonance we have

$$\frac{\sigma(e^+e^- \rightarrow Z' \rightarrow X)}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)} = \frac{9}{\alpha^2} B(Z' \rightarrow e^+e^-) B(Z' \rightarrow X) \quad (3.32)$$

Typical e^+e^- branching ratios are in the range

$$B(Z' \rightarrow e^+e^-) = (0.6 \text{ to } 6) \% \quad (3.33)$$

If we assume $\text{BR}(Z' \rightarrow e^+e^-) = 1\%$, formula (3.32) gives

$$\sigma(e^+e^- \rightarrow Z' \rightarrow \text{all}) = \frac{0.13 \text{ nb}}{(m_{Z'}(\text{TeV}))^2} \quad (3.34)$$

Taking parameters characteristic of the high-energy e^+e^- CERN Linear Collider (CLIC) project, namely $m_{Z'} = (1 \text{ to } 4) \text{ TeV}$ and a luminosity $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, equation (3.34) gives event rates of 1 every 8 s to every 2 min. These event rates should be reduced by a factor of a few,^{106,108} because the beam energy spread is likely to be somewhat wider than the natural Breit-Wigner width of the Z' resonance, but event samples of 10^4 to 10^5 Z' decays appear to be attainable. These would permit detailed studies of the Z' couplings, and searches for exotic Z' decays (analogous to those planned for the Z at the SLC and LEP), perhaps after discovery of the Z' at the LHC or SSC (analogous to the discovery of the Z at the CERN $p\bar{p}$ Collider).

3.5 Additional Matter Particles

Phenomenological models inspired by Calabi-Yau compactification contain particles in 27 representations of E_6 . As discussed above, these include the conventional quarks and leptons, the ν^c , the Higgses H and H^c , the additional charge- $1/3$ colour 3 and $\bar{3}$ particles D and D^c (on which we concentrate here), and a second $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)$ -singlet field N . Bounds on the masses of the additional particles are rather model-dependent, with the masses of the sparticles sensitive, in particular, to the favoured mechanism of supersymmetry breaking in the observable sector.⁹⁶ If the dominant source is a gaugino mass $m_{1/2}$ as suggested in subsection 1.5, then all the soft supersymmetry-breaking sparticle masses are proportional to one another, with ratios which depend on the low-energy gauge group. If this is the minimal rank-5 possibility, then a renormalization group analysis of the type discussed in subsection 1.1 gives the ratios⁹⁶

$$m_{\tilde{q}} : m_{\tilde{g}} : m_{\tilde{t}_L} : m_{\tilde{t}_R} : m_{\tilde{g}} \approx 1.95 : 1 : 0.7 : 0.4 : 1/7 \quad (3.35)$$

and the physical masses also have D-term contributions. Using (3.35), we can combine the limits on different particle and sparticle masses to obtain 'absolute' lower bounds on the conventional sparticle masses and also on additional particle masses.⁹⁶ In this way, we find¹⁰⁹ that of the three spin- $1/2$ D particles, two could have masses as low as 20 GeV, whilst the third has a mass which is probably ≥ 100 GeV. The masses of their scalar partners D_0 could all be as low as 20 GeV, although model calculations push one of them above 100 GeV and two of them above 170 GeV. We have found that either of the sequences $m_{D_0} < m_{D_{1/2}}$ or $m_{D_{1/2}} < m_{D_0}$ could be realized.¹⁰⁹

The cross-sections for producing the D_0 and $D_{1/2}$ in hadron-hadron collisions are simply given by QCD:

$$\sigma(\bar{p}p \rightarrow D_{1/2} \bar{D}_{1/2} + X) = \sigma(\bar{p}p \rightarrow \bar{t}t + X) \quad (3.36)$$

for equal masses $m_{D_{1/2}} = m_t$, and

$$\sigma(\bar{p}p \rightarrow D_0 \bar{D}_0 + X) = \frac{3}{5} \sigma(\bar{p}p \rightarrow \tilde{q}\tilde{q} + X) \quad (3.37)$$

for equal masses $m_{D_0} = m_{\tilde{q}}$, where we have assumed five degenerate flavours of $\tilde{q}_L$ and $\tilde{q}_R$, and the three degenerate D_0 , D_0^* —one from each generation. The cross-sections (3.36) and (3.37) are large enough for m_{D_0} , $m_{D_{1/2}} \lesssim 60$ GeV to be accessible in present UA1/UA2 collider data. The signatures would be $D_0 \rightarrow \bar{q}q^*$, or $q\nu$ and $q\ell$. The first of these has a very large QCD jet background, the second would give final states with missing energy similar to those from $\tilde{q}$ production followed by $\tilde{q} \rightarrow q\tilde{\gamma}$

Fig. 11. Total cross-sections¹⁰⁹ for $\bar{p}p \rightarrow D_0 \bar{D}_0 + X$ at $\sqrt{s} = 630$ GeV, followed by (a) $D_0 \rightarrow q\nu$, and (b) $D_0 \rightarrow q\mu$ decay, incorporating detector effects.

*) If this coupling is present, there could also be single D_0 production followed by $D_0 \rightarrow \bar{q}q$ decay, but this would be drowned in QCD two-jet events.

Fig. 12. Detectable ranges of m_{D_0} in ep collisions,¹¹⁰ for different decay modes and values of the coupling strengths $F_{3,4} \equiv (\lambda_{3,4}^2/4\pi/\alpha_{em})$, for HERA ($\sqrt{s} = 314$ GeV) and for two LHC ep options ($\sqrt{s} = 1.4$ and 1.8 TeV).

decay, and the third may be the easiest to detect. Typical cross-sections¹⁰⁹ at the CERN $p\bar{p}$ Collider are shown in Fig. 11. The analogous $D_{1/2}$ decays would be $D_{1/2} \rightarrow \bar{q}q\tilde{\gamma}$ (as detectable as $\tilde{g} \rightarrow q\bar{q}\tilde{\gamma}$ decay?), or $D_{1/2} \rightarrow q\nu\tilde{\chi}$ (also with a missing energy signature) and $q\ell\tilde{\gamma}$ (as detectable as semileptonic heavy quark decay?) The first search for leptoquark pair-production in hadron-hadron collisions has recently been reported by the UA1 Collaboration,¹⁰⁰ which has looked for $D_0 \rightarrow q\nu$ and/or $D_0 \rightarrow q\mu$ decays. They find a lower bound

$$m_{D_0} > 33 \text{ GeV} \quad (\text{for } q\nu, q\mu \text{ decays}) \quad (3.38)$$

which may be substantially improved by forthcoming CERN and/or FNAL $p\bar{p}$ collider data.

Leptoquarks can also be produced singly in ep collisions via lepton-quark fusion: $e + q \rightarrow D_0$.¹¹⁰ The cross-section for this reaction depends sensitively on the corresponding Yukawa coupling λ . However, the process appears to be detectable even if $\lambda^2/4\pi \approx 10^{-4}\alpha_{em}$. If $\lambda^2/4\pi \approx \alpha_{em}$, this mechanism can be used to search for leptoquarks weighing almost as much as the centre-of-mass energy, e.g. $m_{D_0} \lesssim 250$ GeV at HERA with $\sqrt{s} = 314$ GeV,¹¹⁰ as seen in Fig. 12.

4. SEARCHES FOR SUPERSYMMETRIC DARK MATTER

4.1 A Supersymmetric Relic from the Big Bang

Most supersymmetric theories contain one supersymmetric particle which should be present today as a cosmological relic from the Big Bang.¹¹ This is because they have a multiplicatively conserved quantum number called R-parity,¹¹¹ with the assignments

$$R = \begin{cases} +1 & \text{for all particles} \\ -1 & \text{for all sparticles} \end{cases} \quad (4.1)$$

It is easy to check that R-parity is conserved by all the interaction vertices in a conventional supersymmetric theory, such as $\tilde{f}\tilde{f}G$, $\tilde{f}\tilde{f}H$, $|\tilde{f}^2|^2$, $|\tilde{f}H|^2$, etc. This is because the conservation of R-parity is linked to those of baryon number B and lepton number L, via the representation

$$R = (-1)^{3B+L+2S} \quad (4.2)$$

We can see from this representation that R conservation could be violated by processes with ΔB and/or $\Delta L \neq 0$. Such a violation could arise spontaneously, e.g. from $\Delta L \neq 0$ in the vacuum due to $\langle 0|\bar{\nu}^c|0\rangle \neq 0$

0, or explicitly, e.g. from an $H\ell$ coupling in the superpotential. However, these sources of $\Delta L \neq 0$ are severely constrained by experiment,¹¹² and are absent in most phenomenologically acceptable models.*)

There are three important implications of R conservation, namely

- i) sparticles are always produced in pairs, e.g. $e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$, $\bar{p}p \rightarrow \bar{q}q + X$, etc.;
- ii) heavier sparticles always decay into lighter sparticles, e.g. $\tilde{e} \rightarrow e\tilde{\gamma}$, $\tilde{q} \rightarrow q\tilde{g}$, $\tilde{g} \rightarrow q\bar{q}\tilde{\gamma}$, etc.;
- iii) the lightest supersymmetric particle (LSP) is stable, because it has no available decay mode.

Any candidate for a cosmological relic particle must probably have no strong or electromagnetic interactions. If it did, then presumably it would have dissipated energy and condensed along with ordinary matter into galaxies, stars, and planets. We could then have detected it in searches for anomalous heavy isotopes. However, none has been seen, with limits¹¹³

$$\frac{n(\text{relic})}{n(p)} \lesssim 10^{-15} \text{ to } 10^{-30} \quad (4.3)$$

for $1 \text{ GeV} < m_{\text{relic}} < 10 \text{ TeV}$ as shown in Fig. 13. These upper limits conflict with the calculated abundances expected¹¹⁴ for heavy relics:

$$\frac{n(\text{relic})}{n(p)} \sim \frac{10^{-10}}{\alpha^2} \sim \begin{cases} 10^{-10} & \text{if strongly interacting} \\ 10^{-6} & \text{if electromagnetically interacting} \end{cases} \quad (4.4)$$

(More details of the calculations of these abundances will be given in subsection 4.3.) The conflict between (4.3) and (4.4) can only be resolved if the dark matter particles have not condensed along with ordinary matter, which would be the case if they had neither strong nor electromagnetic interactions, but only weak interactions.

Fig. 13. Compilation of upper limits¹¹³ on exotic heavy isotope abundances.

*) Models with such effects, such as many models⁹³ based on the Yau manifold as described in subsection 3.2, generally violate these experimental constraints.

Any LSP candidate for this relic particle must therefore be electromagnetically neutral and have no strong interactions, which gives us the following options, in order of increasing spin: the sneutrinos $\tilde{\nu}_{e,\mu,\tau}$ of spin 0; some mixture of the photino $\tilde{\gamma}$, the higgsinos $\tilde{H}$, and the zino $\tilde{Z}$ of spin $1/2$; or the gravitino of spin $3/2$. Before examining these candidates in more detail, we first review the astrophysical evidence for dark matter, and estimate its density, both locally in our galactic halo and globally averaged over all the Universe. We will then see how different LSP candidates can meet these requirements.

4.2 Astrophysical Evidence for Dark Matter

One of the best pieces of astrophysical evidence for dark matter in the Universe is the fact that many galaxies have flat rotation curves, i.e. the velocities of neutral hydrogen orbiting around the galaxies at radii larger than the visible spirals is roughly constant.¹¹⁵ If all the matter in a galaxy were concentrated in the volume spanned by the visible spirals, the rotation velocities would fall in accordance with Kepler's law. The observed constant velocities suggest that there is invisible dark matter outside the visible spirals, and that

$$m_{DM}/m_{vis} \gtrsim 5 \quad (4.5)$$

Another piece of evidence comes from elliptic galaxies that contain clouds of hot gas.¹¹⁶ These would have evaporated if the potential wells of the elliptic galaxies were only those calculated from the visible luminous matter. There must be additional, invisible matter to deepen the potential wells and keep in the hot gas. Another piece of evidence comes from galactic clusters, where the velocities of galaxies which are apparently bound are often much larger than those suggested by the virial theorem:

$$v^2 \sim m_{vis}/r \quad (4.6)$$

One example of this is the Coma cluster;¹¹⁷ another is our own infall towards the nearby Virgo cluster.¹¹⁸ There is also weaker evidence for dark matter on smaller scales, with the suggestion that

$$m_{DM}/m_{vis} \approx 10 ? \quad (4.7)$$

in dwarf galaxies.¹¹⁹ Finally, there is also tentative evidence for dark matter on very large scales, since recent analyses of IRAS data¹²⁰ and of distant sources¹²¹ both suggest that

$$\Omega \equiv \rho/\rho_c \approx 1 ? \quad (4.8)$$

at very large scales, where ρ_c is the critical density required to close the Universe,

$$\rho_c \approx 1.8 \times 10^{-29} h_0^2 \text{ g cm}^{-3} \quad (4.9)$$

with h_0 being the present-day Hubble expansion rate in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Cosmological nucleosynthesis¹²² suggests that the baryon density

$$\Omega_B \equiv \rho_B/\rho_c \leq 0.2 \quad (4.10)$$

which is not enough to give (4.8), and that even the dark matter (4.5) in galactic haloes would be insufficient to reach (4.8).

What might this dark matter be? The uncertainties in ρ_B as inferred from cosmological nucleosynthesis would permit galactic haloes to be made out of baryons, but then why did such baryonic dark matter not condense into the galactic disk? Arguments have been given against the possibility that the galactic haloes are made of baryons in the form of gas, dust, or 'snowballs'.¹²³ The possibilities that the galactic haloes are made of Jupiters or black holes have also been disfavoured,¹²³ but not excluded. The favoured possibility seems to be that the dark matter is composed of some species of non-dissipative particle.

Fig. 14. Spectra of mass perturbations $\delta M/M$ after enhancement (a) by cold dark matter, (b) by hot dark matter.

Such dark-matter particles can be divided into two generic categories: hot (cold), according to whether the particles were relativistic (non-relativistic) when their density was equal to that of conventional matter in the Universe. Models of the evolution of structures in the Universe give arguments for and against hot dark matter (HDM) or cold dark matter (CDM). These models can again be divided into two classes: i) those based on a randomly fluctuating Gaussian field of density perturbations—either adiabatic as favoured by inflation,¹²⁴ or isothermal; ii) those based on discrete seeds—such as cosmic strings.¹²⁵ In the first case, one expects the field to have similar fluctuations $\delta\rho/\rho$ on all distance scales.^{124,126} These may then be magnified on certain scales by gravitational instabilities, yielding the spectra of $\delta M/M$ shown in Figs. 14a and b for CDM and HDM, respectively.¹²⁷ Neither can give big $\delta M/M$ on scales larger than the horizon scale at recombination, which means that we only have large $\delta M/M$ for $M < M_{cl}$, a typical cluster mass. In the case of HDM particles, the fact that they are initially relativistic means that they can escape from small perturbations, reducing the power in $\delta M/M$ on scales $< M_g$, a typical galaxy mass. In this case, galaxies can only be formed by the fragmentation of larger structures such as clusters, whereas they can be formed earlier in a CDM scenario. This is an advantage of CDM and a problem for HDM, since quasar data suggest that galaxies form at red shifts $z \gtrsim 4$. However, if we want to understand the observed clustering of galaxies, it seems that only ‘large’ perturbations¹²⁸

$$\left(\frac{\delta\rho}{\rho}\right) \gtrsim \nu \left(\frac{\delta\rho}{\rho}\right)_{\text{RMS}} \quad (4.11)$$

can succeed in evolving to become galaxies. This is known as biased galaxy formation, and the biasing factor ν is usually thought to be $\sim (2 \text{ to } 3)$. If perturbations grow from a Gaussian field of fluctuations $\delta\rho/\rho$, CDM with biasing seems able to accommodate the observational data on large voids,¹²⁹ and also those on large-scale streaming motions discussed below.¹³⁰ However, neither this nor any other scenario could accommodate hyperclustering on scales $\gg M_{cl}$,¹³¹ should this be confirmed.

The above preference for CDM is reversed if structures in the Universe grow from discrete seeds such as loops of cosmic string. In this case, the tendency of CDM to make small perturbations grow, causes the radii of galactic haloes to become too small.¹³² Therefore HDM seems to be favoured in a seed scenario.

Since the observed large-scale streaming motions raised some doubts about CDM a while ago, I take this opportunity to describe how these observations (sketched in Fig. 15) can be reconciled with CDM if galaxy formation is biased. The only confirmed anisotropy of the microwave background radiation is a dipole¹³³ that is usually interpreted as our motion relative to the last scattering surface with a velocity $v_{\text{dip}} \approx 610 \text{ km s}^{-1}$. Some of this motion can be explained by our infall velocity of $\sim 250 \text{ km s}^{-1}$ towards the Virgo cluster,¹¹⁸ leaving $\sim 450 \text{ km s}^{-1}$ to be explained as the motion of the whole Virgo cluster towards the Hydra–Centaurus supercluster. Our distance from Virgo is about $15 h_0^{-1} \text{ Mpc} \approx 25 \text{ Mpc}$, and its distance from Hydra–Centaurus is about $(30 \text{ to } 50) h_0^{-1} \text{ Mpc} \approx 50 \text{ Mpc}$, where we have used the reasonable guess $h_0 \approx 1/2$ to 1. Therefore we have to explain the three pieces of large-scale velocity data in the first row of Table 3. They can indeed be easily fitted with HDM, assuming a random Gaussian field with $\delta\rho/\rho \approx 10^{-4}$ on all scales,¹³⁴ as seen in the second row of Table 3. This and the third row, which is for CDM with a biasing factor ν , contain 90% upper and lower limits on the velocities due to Gaussian random fields of perturbations. Note that CDM¹³⁴ can also fit the data if $\nu \approx 2$ to 3.

Fig. 15. Impressionistic sketch of large-scale streaming motions in our corner of the Universe.

Table 3 Large-scale streaming motions

	Velocities (km s^{-1})		
	V_{dip}	$V_{25 \text{ Mpc}}$	$V_{50 \text{ Mpc}}$
Observed	610	250	450
Hot dark matter	400 to 1885	215 to 1025	120 to 571
Cold dark matter	$(163 \text{ to } 770)_\nu$	$(90 \text{ to } 430)_\nu$	$(60 \text{ to } 260)_\nu$

Since there is no confirmed contradiction between CDM and observation, we may go on to estimate the local density of the dark matter in our galactic halo.¹³⁵ To do this, we first assume that the protogalaxy was an isothermal sphere containing CDM and conventional matter. We assume that the galaxy's angular momentum is due to tidal torques induced by neighbouring structures. We next assume that as the conventional matter collapsed into the galactic disk, the CDM evolved adiabatically. We then demand that the resulting rotation curve be flat or rising at the Sun's distance from the galactic centre. With these assumptions it has been found that almost certainly¹³⁵

$$0.2 < \rho^{\text{CDM}} / \text{GeV cm}^{-3} < 0.43 \quad (4.12)$$

and probably

$$0.3 < \rho^{\text{CDM}} / \text{GeV cm}^{-3} < 0.43 \quad (4.13)$$

with the best guess being

$$\rho^{\text{CDM}} \simeq 0.3 \text{ GeV cm}^{-3} \simeq 5.4 \times 10^{-25} \text{ g cm}^{-3} \quad (4.14)$$

The estimates (4.12) and (4.14) will be our guide in judging the searches for supersymmetric dark matter discussed in subsection 4.4.

4.3 Estimation of Relic Density in the Universe

In subsection 4.1 we saw that the LSP in most supersymmetric theories is a natural candidate to be dark matter, and in subsection 4.2 that dark matter is wanted by astrophysicists: CDM if structures

arise from a random Gaussian field of perturbations, or HDM if structures arise from discrete seeds such as cosmic strings. We will now see if the LSP can have the desired density and other properties. The time evolution of the LSP number density $n(t)$ is controlled by the equation

$$\frac{dn}{dt} = -3 \frac{\dot{R}}{R} n - \langle \sigma_{\text{ann}} v \rangle (n^2 - n_0^2) \quad (4.15)$$

where the first term expresses the natural dilution due to the Hubble expansion $H = \dot{R}/R$ which we include subsequently in $\dot{n} \equiv (dn/dt) + 3(\dot{R}/R)n$, and the second term describes the effects of annihilations and inverse annihilations, with σ_{ann} the annihilation cross-section, v the relative velocity, and n_0 the equilibrium density. In most cases of interest, the annihilations are mainly non-relativistic, and one can approximate¹¹

$$\langle \sigma_{\text{ann}} v \rangle \simeq a + bx : x \equiv \frac{\langle v^2 \rangle}{6} \quad (4.16)$$

A good approximate solution¹³⁶ to (4.15) is obtained by assuming $n = n_0$ until

$$n_0 \simeq \langle \sigma_{\text{ann}} v \rangle n_0^2 \quad (4.17)$$

which occurs at some freeze-out temperature T_f . Thereafter one approximates (4.16) by

$$\dot{n}_0 \simeq \langle \sigma_{\text{ann}} v \rangle n^2 \quad (4.18)$$

which leads to

$$\rho \simeq m_\chi T_0^3 / a x_f + \frac{1}{2} b x_f^2 \quad (4.19)$$

where T_0 is the effective temperature of the relics today, which is given¹³⁷ by

$$\left(\frac{T_0}{T_f} \right)^3 = \left[\frac{n_{\text{eff}}(1 \text{ MeV})}{n_{\text{eff}}(T_f)} \right] \quad (4.20)$$

where $N_{\text{eff}}(T_f)$ is the number of effective degrees of freedom at the freeze-out temperature T_f . In useful units, and introducing a factor of 0.8 to correct for the naïve freeze-out approximation (4.17) and (4.18), one arrives¹¹ at a relic density

$$\rho \simeq 4.9 \times 10^{-40} \left(\frac{T_0}{2.8 g} \right)^3 (n_{\text{eff}}(T_f))^{1/2} \left(\frac{\text{GeV}^{-2}}{a x_f + \frac{1}{2} b x_f^2} \right) \text{g cm}^{-3} \quad (4.21)$$

to be computed for different particles with different σ_{ann} parametrized by a, b (4.16).

Let us first consider spin- $1/2$ LSP candidates such as gauginos and higgsinos. The dominant contributions to their annihilations are given by $\tilde{f}$ exchange for gauginos (e.g. $\tilde{\gamma}$) and by Z^0 exchange for higgsinos. These give an effective four-fermion interaction of the form

$$\mathcal{L}_{\text{eff}} = (\tilde{\chi} \gamma_\mu \gamma_5 \tilde{\chi})(\tilde{f} \gamma_\mu (A_f L + B_f R) f) \quad (4.22)$$

where the form of the first factor is decreed by the Majorana nature of the spin- $1/2$ particles $\tilde{\chi}$:

$$A_f^{\tilde{f}} = \frac{(\alpha T_f^3 g_2 + \beta \gamma_{f_L} g_1)^2}{2 m_{\tilde{f}_L}^2} + O\left(\frac{\delta^2}{m_w^2}, \frac{\delta^2}{m_{\tilde{f}}^2}\right) \quad (4.23a)$$

$$B_f^{\tilde{f}} = - \frac{(Y_{fR} \beta g_i)^2}{2m_{\tilde{f}_R}^2} + O((\delta^2, \delta^2) \frac{m_f^2}{m_{\tilde{f}_R}^2} \frac{1}{m_{\tilde{f}_L}^2}) \quad (4.23b)$$

where we write the composition of the $\tilde{X}$ as

$$\tilde{X} = \alpha \tilde{W}^3 + \beta \tilde{B} + \gamma \tilde{H}^0 + \delta \tilde{H}^\pm \quad (4.24)$$

and

$$a = \sum_f \theta(m_{\tilde{X}} - m_f) \frac{p_f}{2\pi m_{\tilde{X}}} m_f^2 (A_f - B_f)^2 \quad (4.25a)$$

$$b = \sum_f \theta(m_{\tilde{X}} - m_f) \frac{p_f}{2\pi m_{\tilde{X}}} \left[(A_f + B_f)^2 \left(4m_{\tilde{X}}^2 - \frac{13}{4} m_f^2 \right) + \frac{21}{2} A_f B_f m_f^2 + \frac{3}{4} m_f^2 (A_f - B_f)^2 / (1 - m_f^2/m_{\tilde{X}}^2) \right] \quad (4.25b)$$

The annihilation cross-section $\sigma_{\text{ann}} = \sum_f \sigma(\tilde{X}\tilde{X} \rightarrow f\bar{f})$ is given in terms of A_f and B_f by equation (4.16) and¹¹

$$a = \sum_f \theta(m_{\tilde{X}} - m_f) \frac{p_f}{2\pi m_{\tilde{X}}} m_f^2 (A_f - B_f)^2 \quad (4.26a)$$

$$b = \sum_f \theta(m_{\tilde{X}} - m_f) \frac{p_f}{2\pi m_{\tilde{X}}} \left[(A_f + B_f)^2 \left(4m_{\tilde{X}}^2 - \frac{13}{4} m_f^2 \right) + \frac{21}{2} A_f B_f m_f^2 + \frac{3}{4} m_f^2 (A_f - B_f)^2 / (1 - m_f^2/m_{\tilde{X}}^2) \right] \quad (4.26b)$$

It is clear from these equations that $q_{\tilde{X}} = n_{\tilde{X}} m_{\tilde{X}}$, where $n_{\tilde{X}} \propto 1/\sigma_{\tilde{X}}$ (4.21), with $\sigma_{\tilde{X}} \propto m_{\tilde{X}}^2/m_f^4$ in the case of a gaugino.¹³⁸

The general approach up to equation (4.21) can also be applied to sneutrino candidates for the LSP, but the form of the annihilation cross-section parameters a , b is different from equations (4.22) to (4.26).¹³⁹ We do not go through the details here, because the $\tilde{\nu}$ are not favoured LSP candidates. Theoretically, arranging one of them to be the LSP requires a bizarre choice of model parameters. Experimentally, they would have coherent interactions with heavy nuclei,¹⁴⁰ which would have been detected by the experiments¹⁴¹ described in subsection 4.4 if their local density was in the range (4.12) suggested by astrophysics and

$$20 \text{ GeV} \lesssim m_{\tilde{\nu}} \lesssim 1 \text{ TeV} \quad (4.27)$$

Moreover, $\tilde{\nu}_\mu \tilde{\bar{\nu}}_\mu$ annihilations inside the Sun would have given an unacceptably large flux of ν_μ (see subsection 4.4) if¹⁴²

$$m_{\tilde{\nu}_\mu} \gtrsim 4 \text{ GeV} \quad (4.28)$$

and a similar argument applied to $\tilde{\nu}_e$ may also exclude $m_{\tilde{\nu}_e} \gtrsim 4 \text{ GeV}$.¹⁴² Therefore we will not discuss further the possibility that the LSP is a $\tilde{\nu}$.

To make the gravitino $\tilde{G}$ the LSP also requires a rather special choice of model parameters, and most models have $m_{\tilde{G}} \gtrsim m_{\tilde{t}}, m_{\tilde{q}}$, maybe even $m_{\tilde{G}} \gg m_{\tilde{t}}, m_{\tilde{q}}$. The gravitino has such weak interactions at low energies that the formalism (4.16) to (4.26) is not useful. Instead, the abundance of relic gravitinos today is simply given, in the absence of inflation, by the primordial density diluted by the subsequent expansion of the Universe. If we assume an initial thermal distribution when $T \approx m_P$, then the closure density ρ_c is obtained for¹⁴³

$$m_{\tilde{G}} \sim 1 \text{ keV} \quad (4.29)$$

This would constitute so-called 'warm' dark matter with properties that are intermediate between those of HDM and CDM discussed in subsection 4.2. If the Universe underwent inflationary expansion at some epoch, the primordial gravitino density would be exponentially suppressed¹⁴⁴ and the number

density would be partially restored by the subsequent reheating to some temperature $T < m_P$. The critical density ρ_c would then be obtained for¹⁴⁵

$$m_{\tilde{\gamma}} \simeq 100 \text{ GeV} \times \left(\frac{10^{13} \text{ GeV}}{T} \right) \quad (4.30)$$

In this case the gravitino would be CDM. However, as mentioned at the beginning of this paragraph, the gravitino is not the LSP in most models.

We are left with the spin- $1/2$ candidates $\tilde{\gamma}$, $\tilde{H}$, etc., as the most plausible LSPs, and so discuss them in more detail. It is apparent from a comparison of equations (4.24) and (4.25) that the $\tilde{\gamma}\tilde{\gamma}$ annihilation cross-section is much larger than the $\tilde{H}\tilde{H}$ annihilation cross-section so that an $\tilde{H}$ LSP would be over-dense unless the $\tilde{H}$ is heavier than some massive quark such as the b or t.¹¹ In this case it is possible to achieve closure density for some special value of the Higgs v.e.v.'s. However, in most models $m_{\tilde{H}} > m_{\tilde{\gamma}}$. It is evident from (4.25) that the relic $\tilde{\gamma}$ density is sensitive to $m_{\tilde{t}}$ and $m_{\tilde{q}}$. Encouragingly, these masses are such that one naturally gets a relic density $\rho_{\tilde{\gamma}} \approx \rho_c$ in the minimal supersymmetric Standard Model, with $m_{\tilde{t}}$ and $m_{\tilde{q}}$ renormalized along the lines discussed in subsection 1.1, as seen in Fig. 16a. Using the UA1 constraints on $m_{\tilde{q}}$ and $m_{\tilde{g}}$, the ASP constraints on $m_{\tilde{\gamma}}$ and $m_{\tilde{g}}$, and particularly the absence of $e^+e^- \rightarrow \tilde{e}^+\tilde{e}^-$, one finds¹⁴⁶ in this model

$$\Omega_{\tilde{\gamma}} h_0^2 \gtrsim 0.025 \quad (4.31)$$

Fig. 16. (a) A minimal supergravity model¹⁴⁵ gives $\Omega_{\tilde{\gamma}} = 1$ for the indicated values of the present-day Hubble constant in units of $\text{km s}^{-1} \text{ Mpc}^{-1}$. (b) A minimal rank-5 superstring-inspired model¹⁴⁶ also gives $\Omega_{\tilde{\gamma}} \sim 1$.

with $\Omega_{\tilde{\gamma}} = 1$ for $h_0 = 1/2$ being obtained for

$$m_{\tilde{q}} \simeq 44 \text{ GeV} + 5.5 m_{\tilde{g}} \quad (4.32)$$

when $m_{\tilde{\gamma}}$ is in the range of 5 to 20 GeV. A minimal rank-5 superstring model,¹⁴⁷ with $m_{1/2} \neq 0$ the dominant source of supersymmetry breaking in the observable sector, gets even closer:

$$\Omega_{\tilde{g}} \left(h_0 / 1/2 \right)^2 = \left(\frac{1}{2} \text{ to } 2 \right) \quad (4.33)$$

for $30 \text{ GeV} < m_{1/2} < 100 \text{ GeV}$ as favoured in this model, as seen in Fig. 16b. Therefore photinos are favoured over many other CDM candidates, for which $\Omega \approx 1$ is possible but by no means natural.

4.4 The Detection of Stable Supersymmetric Relics

We start with the detection of relic particles at a distance, via their annihilation in the galactic halo; then closer to home, via their annihilation in the Sun; and finally discuss detection in the laboratory, via the scattering off nuclei in a cryogenic detector.

4.4.1 Annihilation in the galactic halo? It has been suggested¹⁴⁷ that annihilations of LSPs in the halo may produce fluxes of cosmic rays that are observable at the Earth's surface, e.g. $\tilde{\gamma}\tilde{\gamma} \rightarrow \bar{p}, e^+, \gamma + X$. Fluxes of $\bar{p}$ and e^+ have already been observed. In the case of the $\bar{p}$, their overall ratio to the p flux is not surprisingly large, but there is an apparent excess of $\bar{p}$ at low energies $< 1 \text{ GeV}$: $\phi_{\bar{p}} \approx 3 \times 10^{-6} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$.¹⁴⁸ The flux of e^+ , $\phi_{e^+} \approx 10^{-3} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$, is also not anomalously high, but a substantial fraction of it could have an exotic origin. As an example of what LSP annihilation could do, one calculation of $\tilde{\gamma}\tilde{\gamma}$ annihilation with $m_{\tilde{\gamma}} = 4.2 \text{ GeV}$, $m_{\tilde{t}} = 50 \text{ GeV}$, and $m_{\tilde{q}} = 100 \text{ GeV}$ gave¹⁴⁹

$$\begin{aligned} \phi_{\bar{p}} &\sim 1.1 \times 10^{-6} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}, \quad \phi_{e^+} \sim 3 \times 10^{-4} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}, \\ \phi_{\tilde{g}} &\sim 0.9 \times 10^{-7} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1} \end{aligned} \quad (4.34)$$

It could also be possible to see $\tilde{H}\tilde{H} \rightarrow \bar{p} + X$ if the ratio of Higgs v.e.v.'s were $v^c/v \geq 2$.¹⁵⁰ However, it is not expected to be possible to see $\tilde{\nu}\tilde{\nu} \rightarrow p + X$, since most of their annihilation products are $\nu\bar{\nu}$ pairs.

4.4.2 Annihilation in the Sun? Any species of CDM particle χ has some probability of being trapped in the Sun after losing energy in elastic scattering: $\chi p \rightarrow \chi p$. The trapping rate Γ_T is given by¹⁵¹

$$\Gamma_T \simeq (\pi R_\odot^2 n_\chi \bar{v}) \cdot f_g \cdot f_I \cdot f_T \quad (4.35)$$

where the factor in parentheses is simply geometrical, f_g is a gravitational enhancement factor for bending trajectories into the Sun, f_I is the probability of a χ particle interacting while passing through the Sun, and f_T is the probability of the particle losing enough energy in one elastic collision for it to be trapped in the Sun's gravitational field. Once trapped in this field, the χ particle will in general have repeated collisions in the Sun until it eventually joins an isothermal distribution in the core of the Sun. The population of this distribution is controlled by two effects. One is evaporation, which can be treated in much the same way as the loss of light elements from planetary atmospheres. The evaporation rate varies exponentially rapidly, and is negligible for masses $m_\chi > m_E$ (dominant for masses $< m_E$), where¹⁵²

$$\begin{aligned} \Gamma_E(m_E) &= \Gamma_T : \\ \Gamma_E(m_{\tilde{g}}) &= 0.66 \times 10^{-2} \left(\frac{m_{3.75}}{T_{1.4}} \right)^2 \exp \left(-32.9 \frac{m_{3.75}}{T_{1.4}} \right) \\ m_{3.75} &= \frac{m_{\tilde{g}}}{3.75 \text{ GeV}}, \quad T_{1.4} = T / 1.4 \times 10^7 \text{ K} \end{aligned} \quad (4.36)$$

Fig. 17. Illustration of the cosmion idea¹⁵³ for cooling the centre of the Sun.

where T is the temperature of the isothermal distribution, Γ_A is the annihilation rate, $v_\odot \approx 617 \text{ km s}^{-1}$ is the escape velocity from the surface of the Sun, $M_\odot$ is the solar mass.

In the case of the $\tilde{\gamma}$ to be discussed later, $m_E \approx 2.5 \text{ GeV}$, and other LSP candidates have m_E in a similar range. When evaporation is unimportant, the solar χ population is controlled by annihilation, which occurs at an equilibrium rate

$$\Gamma_A = \frac{1}{2} \Gamma_T \quad (4.37)$$

since two χ particles disappear in each annihilation event.

One of the possible observable consequences of the trapping of relic particles in the Sun is a cooling of the central core¹⁵³ due to the isothermal relic distribution transferring heat to outer regions of the Sun, as indicated in Fig. 17. A reduction in the central core temperature by as little as 7% would reduce the flux of solar ^8B neutrinos sufficiently to explain the low flux seen. This resolution of the solar neutrino problem requires

$$54 \text{ eV} \lesssim m_\chi \lesssim 304 \text{ eV} \quad (4.38)$$

so that the isothermal χ distribution is neither too central nor too broad, and

$$\sigma_{el}(\chi p \rightarrow \chi p) \sim 10^{-36} \text{ cm}^2, \quad \sigma_{ann} \lesssim 10^{-4} \sigma_{el} \quad (4.39)$$

The disparity (4.39) between σ_{el} and σ_{ann} is difficult to arrange, and impossible for an LSP or a conventional heavy neutrino. One proposal¹⁵⁴ for a particle with the properties (4.38) and (4.39) is a heavy Dirac neutral lepton L^0 whose charged partner $L^\pm$ is only slightly heavier:

$$m_{L^\pm} - m_{L^0} \lesssim 24 \text{ eV} \quad (4.40)$$

This propinquity has two motivations: it gives the L^0 a large magnetic dipole moment which enhances σ_{el} in the desired manner (4.39), and it prevents the $L^\pm$ from being detected in e^+e^- annihilation, since $L^\pm \rightarrow L^0 + (e\nu, \mu\nu, \text{ or } \bar{q}q)$ decay would give low-energy events which have not yet been excluded by searches for heavy leptons in e^+e^- annihilation.

Even though relic LSPs could not have this cooling effect, their annihilations could produce observable energetic neutrinos.¹⁵⁵ When two relics annihilate in the solar medium, most annihilation products are absorbed, but prompt neutrinos from heavy fermions ($\chi\chi \rightarrow \tau^+\tau^-$, $c\bar{c}$, $b\bar{b}$, or $t\bar{t}$, followed by $\tau, c, b, \text{ or } t \rightarrow \nu + X$) escape and could be detected on Earth. They may produce either contained events inside a proton detector, or interact in the rock and produce an upward-going muon. The rate for contained neutrino events is

$$\begin{aligned} \#/\text{Ktony} &= 1.8 \times 10^{-2} \left(\frac{\rho_\odot}{0.34 \text{ eV cm}^{-3}} \right) \left(\frac{300 \text{ km s}^{-1}}{v} \right) \\ &\left(\frac{\sigma_{el}(\chi p \rightarrow \chi p)}{10^{-40} \text{ cm}^2} \right) f(m_\chi/m_c) \sum_f \sum_i BR_f \langle N_e \rangle_{fi} a_i \end{aligned} \quad (4.41)$$

where $\rho_{\tilde{\gamma}}$ is the local photino density [cf. (4.12)], $\bar{v}$ is the local r.m.s. halo particle velocity (for which the best guess is 300 km s^{-1}), $f(m_{\tilde{\chi}}/m_c)$ is a kinematic factor which can be approximated by¹⁵²

$$f(m_{\tilde{\chi}}/m_c) = \left(\frac{m_c^2}{m_c^2 + m_{\tilde{\chi}}^2} \right)^{1/2} : \alpha = 1.5, m_c = 20.4 \text{ GeV} \quad (4.42)$$

for χp scattering, BR_i is the branching ratio for $\chi\chi \rightarrow f\bar{f}$, N is the number of prompt neutrinos of species i with scaled energy fraction z produced in each $f\bar{f}$ event, and the a_i parametrize $\nu_i N$ scattering cross-sections. The most reliable way to calculate $\langle Nz \rangle_{fi}$ is to simulate $f\bar{f}$ final states using an e^+e^- annihilation Monte Carlo generator.¹⁵⁶ Quark fragmentation into heavy hadrons reduces $\langle z \rangle_{b,c}$ by factors of ~ 2 to 3 .

To calculate¹⁵⁷ with (4.41), we need an estimate of $\sigma_{el}(\chi p \rightarrow \chi p)$. In the cases $\chi = \tilde{\gamma}$ or $\tilde{H}$, we can do this by assuming closure density: $\Omega_{\tilde{\gamma}}$ or $\Omega_{\tilde{H}} = 1$, and choosing the appropriate values of the model parameters to get the right value of Ω . This fixes $m_{\tilde{L},\tilde{Q}}$ as functions of $m_{\tilde{\gamma}}$, or v^c/v as a function of $m_{\tilde{H}}$. Then we calculate the elastic cross-section using¹⁵⁸

$$\sigma_{el}(\tilde{\gamma} p \rightarrow \tilde{\gamma} p) = \frac{3}{\pi} \left(\frac{4\pi\alpha}{m_{\tilde{\gamma}}^2} \right) \frac{m_{\tilde{\gamma}}^2 m_p^2}{(m_{\tilde{\gamma}} + m_p)^2} |M_{\tilde{\gamma}}|^2 \quad (4.43)$$

or

$$\sigma_{el}(\tilde{H} p \rightarrow \tilde{H} p) = \frac{6}{\pi} \frac{m_{\tilde{H}}^2 m_p^2}{(m_{\tilde{H}} + m_p)^2} \zeta_F^2 \cos^2(2\beta) \left[\sum_q I_{3q} \Delta q \right]^2 \quad (4.44)$$

where $\tan\beta = v/v^c$

The matrix elements $M_{\tilde{\gamma},\tilde{H}}$ are unfortunately subject to some uncertainty, as we will now discuss. As is shown in Fig. 18, $M_{\tilde{\gamma}}$ is given by $\Sigma_q Q_q^2 A(\tilde{\gamma} q \rightarrow \tilde{\gamma} q)$, which is in turn given by the effective Lagrangian (4.22) and (4.23) which becomes

$$\mathcal{L}_{eff} = \sum_q Q_q^2 \frac{(\tilde{\gamma} \delta_\mu \delta_s \tilde{\gamma})(\bar{q} \delta^\mu \delta_s q)}{m_{\tilde{q}}^2} \quad (4.45)$$

if $m_{\tilde{q}_L} = m_{\tilde{q}_R}$. We infer from (4.45) that

$$M_{\tilde{\gamma}} \propto \sum_q Q_q^2 \langle p | \bar{q} \delta_\mu \delta_s q | p \rangle \quad (4.46)$$

Now comes the connection¹⁵⁷ with the recent EMC measurement of the spin-dependent μp structure function.¹² The EMC Collaboration measures $\Sigma_x |\mu p(s) \rightarrow \mu' X|^2$, which is proportional to $\int \langle p, s | J_\mu^{em}(x) J_\nu^{em}(0) | p, s \rangle e^{iq \cdot x} d^4x$, and extracts the spin-dependent piece $\epsilon_{\mu\nu\alpha\beta} q^\alpha s^\beta g_1^p(X, Q^2)$, where q is the momentum transfer from the incoming muon, $Q^2 \equiv -q^2$ and $X = Q^2/2p \cdot q$. In the deep-inelastic

Fig. 18. The relation¹⁵⁷ between polarized deep inelastic μp scattering and elastic $\tilde{\gamma} p$ scattering, via the quark-parton model, the operator product expansion and the $\tilde{q}$ exchange diagram for $\tilde{\gamma} q \rightarrow \tilde{\gamma} q$.

limit $Q^2 \rightarrow \infty$ with X fixed, moments of $g_1^p(X, Q^2)$ measure the matrix elements of operators appearing in the operator product expansion for two electromagnetic currents:

$$\int_0^1 dx X^n g_1^p(x, Q^2) \approx \tilde{C}_n(Q^2) \langle p | (O_n)_Q | p \rangle \quad (4.47)$$

where

$$J_\mu^{em}(x) J_\nu^{em}(0) \underset{x \rightarrow 0}{\approx} \sum_n C_n(x) O_n(0) \quad (4.48)$$

and we have indicated in (4.47) that the operator O_n is renormalized at the scale Q . The particular case of (4.47) which interests us is

$$\int_0^1 dx g_1^p(x, Q^2) = C_0(Q) \langle p | \frac{1}{2} \sum_f Q_f^2 (\bar{q} \gamma_\mu \gamma_5 q)_Q | p \rangle / S_\mu \quad (4.49)$$

whose right-hand side is just $1/2 M_{\tilde{\gamma}}$ (4.46). The EMC Collaboration has recently measured¹²

$$\int_0^1 dx g_1^p(x, Q^2_{EMC} = 10 \text{ GeV}^2) = 0.113 \pm 0.012 \pm 0.025 \quad (4.50)$$

We are interested in $\langle p | (\bar{q} \gamma_\mu \gamma_5 q)_{Q \approx m_{\tilde{q}} = 100 \text{ GeV}} | p \rangle$, but this is independent of Q for SU(3) flavour non-singlet combinations of the $\bar{q} \gamma_\mu \gamma_5 q$ and proportional to $(1 + 2/3 \alpha_s(Q)/\pi)$ for the singlet combination,¹⁵⁹ whilst $C_0(Q) = 1 - \alpha_s(Q)/\pi$.¹⁶⁰ This Q -dependence can be neglected between $Q^2 \approx 3 \text{ GeV}^2$ and 10^4 GeV^2 in a first approximation, so we would infer from (4.50) that

$$M_{\tilde{\gamma}} = 0.226 \pm 0.024 \pm 0.050 \quad (4.51)$$

on the basis of the EMC data.

This estimate is to be contrasted with the other estimates in Table 4, which rely on naïve quark-model estimates of the $\langle p | \bar{q} \gamma_\mu \gamma_5 q | p \rangle$. The most sophisticated of those is the one in Ref. 161, which uses neutron decay data to fix $\langle p | \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d | p \rangle \propto g_A = 1.25$ and determines the SU(3) flavour-octet matrix element $\langle p | (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d - 2 \bar{s} \gamma_\mu \gamma_5 s) | p \rangle$ using the F/D ratio extracted from hyperon β -decay experiments.¹⁶² The SU(3) flavour-singlet matrix element $\langle p | (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s) | p \rangle$ was fixed by assuming

$$\langle p | \bar{s} \gamma_\mu \gamma_5 s | p \rangle = 0 \quad (4.52)$$

which has the physical interpretation that none of the proton's spin is carried by strange quarks: $\Delta s = 0$, where

$$\Delta q = \int_0^1 dx \Delta q(x) : \Delta q(x) = q_\uparrow(x) + \bar{q}_\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x) \quad (4.53)$$

and

Table 4 Estimates of $\bar{M}^2 \equiv (\sum_q Q_q^2 \Delta q)^2$

References	[140]	[158]	[157] using [161]	[157] using [12]
Assumptions	$\bar{u}$ only; naïve quark model	$\bar{u} + \bar{d}$; $\Delta s = 0$, old F/D	$\bar{u} + \bar{d}$; $\Delta s = 0$, F/D = 0.63	$\bar{u} + \bar{d} + \bar{s}$; EMC, F/D = 0.63
For protons	0.24	0.11	0.16	0.05
For neutrons	0.005	0.007	0.0004	0.04

$$g^p(x) = \frac{1}{2} \left(\frac{4}{9} \Delta u(x) + \frac{2}{9} \Delta d(x) + \frac{1}{9} \Delta s(x) \right) \quad (4.54a)$$

$$g_A = \Delta u - \Delta d \quad (4.54b)$$

$$3F-D = \Delta u + \Delta d - 2\Delta s \quad (4.54c)$$

with the proton spin

$$S = S_q + \dots : S_q = \frac{1}{2} (\Delta u + \Delta d + \Delta s) \quad (4.55)$$

The EMC data¹² seem to tell us that $\Delta s \neq 0$. If we combine (4.50), (4.54a) and (4.54b) naïvely without worrying about any Q -dependence, we obtain

$$\int_0^1 dx g_1^p(x, Q^2) = 0.20, \quad M_{\tilde{g}} = 0.40 \quad (4.56)$$

and the results in Table 4. Included among these is the surprising conclusion that a fraction (-0.09 ± 0.25) of the proton spin is carried by u -, d -, and s -quarks. The EMC result corresponds to an almost vanishing $SU(3)$ flavour-singlet matrix element, whereas a large non-zero value had previously been favoured by the assumption (4.52).

It may be misleading to ignore the Q -dependence in this way. The naïve quark model prediction (4.52) presumably applies to $Q^2 \ll 1 \text{ GeV}^2$, whereas the EMC measurement is at $Q^2 \gtrsim 3 \text{ GeV}^2$, and there could in principle be significant Q^2 -dependence in the $SU(3)$ flavour-singlet matrix element over this range.¹⁶³ However, the Q^2 -dependence visible in QCD perturbation theory is not very strong.

Until the result and interpretation of the EMC Collaboration are confirmed, it may be reasonable to treat the estimates (4.51) and (4.52) of $M_{\tilde{g}}$ as spanning a legitimate range of uncertainty. Accordingly, we¹⁵⁷ have used these in the general formulae (4.41) and (4.43) in comparisons with the Fréjus¹⁶⁴ and IMB¹⁶⁵ upper limits on contained ν events due to high-energy solar neutrinos. The Fréjus limit of

$$\#/\text{ktomy} < 1.6 \quad : E_{\text{vis}} > 2 \text{ GeV} \quad (4.57)$$

seems to be the more stringent, except for $m_{\tilde{g}} \lesssim 5 \text{ GeV}$.¹⁶⁶ In (4.43) we have used mainly two models for the squark masses which yield $\Omega_{\tilde{g}} = 1$, namely the renormalized minimal supersymmetric Standard Model choice for which $m_{\tilde{t}} \neq m_{\tilde{q}}$ is given by (4.32), and the equal-mass choice¹⁴²

$$m_{\tilde{t}} = m_{\tilde{q}} = 56 \text{ GeV} (1 + 0.04 m_{\tilde{g}}^2)^{1/4} \quad (4.58)$$

Rates obtained¹⁵⁷ under different assumptions are shown in Table 5. The Fréjus and IMB data can then be used to derive upper limits on the local halo density $\rho_{\tilde{g}}$ as a function of $m_{\tilde{g}}$, as seen in Fig. 19a. We see that in no case does the upper limit on $\rho_{\tilde{g}}$ fall below the astrophysically allowed domain (4.12). However, even the smaller EMC estimate (4.51) of $M_{\tilde{g}}$ only allows¹⁵⁷

$$\rho_{\tilde{g}} \lesssim 0(10) \times \rho_{\text{CDM}} \quad (4.59)$$

in the range $m_{\tilde{g}} \lesssim 10 \text{ GeV}$. Hence the upper limits on supersymmetric dark matter are much more stringent than those for axions.¹⁶⁷ A similar analysis has also been carried out for higgsinos, as seen in Fig. 19b, with similar results: no range of $m_{\tilde{H}}$ can yet be excluded.¹⁵⁷ Note, however, that $\tilde{\nu}_\mu$ and $\tilde{\nu}_e$ dark matter would give larger event rates, which would rule them out¹⁴² as the halo dark matter unless $m_{\tilde{\nu}_{e,\mu}} < m_E$ or $\gg m_E$ (4.36).

Table 5 Model-dependent estimates of R_ν from $\tilde{\gamma}\text{--}\tilde{\gamma}$ annihilation. These numbers and the Fréjus curves in Fig. 18 are calculated for $E_\nu > 2$ GeV.¹⁶⁶ The IMB curves in Fig. 18 are for $0.2 < E_\nu < 2$ GeV.¹⁶⁷

$m_{\tilde{X}}$ (GeV)	R_ν (events/kty) according to model			
	A	B	C	D
	$\Omega = 1$ MSM NQM	$\Omega = 1$ MSM EMC	$\Omega = 1$ EM NQM	$\Omega = 1/4$ MSM NQM
3	0.46	0.15	0.51	1.91
4	1.56	0.50	1.80	6.10
6	1.83	0.59	2.28	6.24
8	0.82	0.26	1.03	2.74
10	0.58	0.19	0.81	1.76
15	0.22	0.072	0.39	0.57
20	0.11	0.034	0.23	0.23
30	0.028	0.009	0.09	0.05

MSM denotes minimal supergravity model mass relations
EM denotes equal masses
NQM denotes naïve quark model matrix elements

Fig. 19. Upper limits on $q_{\tilde{X}}$, with $\tilde{X} = \tilde{\gamma}$, using Fréjus data¹⁶⁴ under the model assumptions A, B, and C of Table 5, and IBM data¹⁶⁵ under assumption A. The $\tilde{E}$ and $\tilde{F}$ curves are obtained from the full cross-section by removing the model-dependent factor of $\cos^2(2\beta)$. The horizontal lines indicate the astrophysically preferred value and probable limits¹³⁵ on $q_{\tilde{X}}$.

4.4.3 Elastic scattering in the laboratory? Finally, the most direct way to look for the LSP is via elastic scattering off heavy nuclei in the laboratory. The rate for such events has been estimated¹⁴⁰ to be

$$\begin{aligned} \#/\text{kg day} &\approx \left(\frac{100 \text{ GeV}}{m_{\tilde{q}}}\right)^4 \left(\frac{4m_{\tilde{g}}m_{Z,A}}{(m_{\tilde{g}} + m_{Z,A})^2}\right) \left(Qq/\frac{2}{3}\right)^4 \\ &\times \lambda^2 J(J+1) \times \left(\frac{\rho}{10^{-24} \text{ g cm}^{-3}}\right) \left(\frac{v}{200 \text{ km s}^{-1}}\right) \end{aligned} \quad (4.60)$$

where a simplified model with the exchange of just one squark with charge Q in Fig. 17 has been assumed, and the shell model is used to calculate the nuclear matrix element factor λ^2 . Previous estimates of this factor for different nuclei are shown in Table 6, together with modified predictions¹⁶⁸ based on the EMC result (4.51) and on the naïve quark-parton model prediction (4.52). We see that the rates for different nuclei may be drastically altered: rates involving scattering off protons in the shell model are reduced, whilst those off neutrons are greatly increased. This means that the preferred nuclei to be used in experiments may be completely changed.

Each elastic scattering event deposits a small amount of recoil energy E :

$$\langle E \rangle = \frac{m_{\tilde{g}}^2 m_{Z,A}}{(m_{\tilde{g}} + m_{Z,A})^2} v^2 \quad (4.61)$$

which is about 1 keV for $m_{\tilde{g}} \approx 4 \text{ GeV}$. Several cryogenic techniques for detecting this recoil energy have been proposed. One of them¹⁶⁹ uses superconducting grains embedded in a target material, placed in a magnetic field monitored by a SQUID, as seen in Fig. 20. The recoil energy may heat up a grain,

Table 6 Photino scattering on nuclei

Nucleus	Model			
	[140]	[158]	[157] using [161]	[157] using [12]
²⁷ Al	0.42	0.19	0.26	0.086
²⁹ Si	0.23	0.10	0.005	0.53
^{69,71} Ga	0.50	0.22	0.32	0.10
⁷³ Ge	0.09	0.041	0.002	0.22
¹¹¹ Cd	0.23	0.10	0.005	0.53
¹³⁹ La	0.24	0.11	0.152	0.049
²⁰⁷ Pb	0.025	0.011	0.0006	0.058

Fig. 20. Idea¹⁶⁹ for detecting dark matter through the nuclear recoil energy it deposits on a superconducting grain.

turning it normal, so that it ceases to exclude the magnetic field, causing a change in $\mathbf{B}$ which can be picked up by the SQUID. Another technique¹⁷⁰ would use superconducting strips whose resistivity would change when heated. Alternatively, one could use thermistors which measure the heating of a supercold block of silicon.¹⁷¹ Ballistic phonons might be detectable in a supercold material.¹⁷² Massive tunnel-junctions would also be heat-sensitive.¹⁷³ There are many ideas for detecting galactic-halo LSPs in the laboratory, but much work needs to be done before their feasibility is demonstrated.

4.5 Two Challenges

Many physicists and astrophysicists believe that not only are we not the centre of the Universe—as was realized by Copernicus—but also we may not be made of the same material as the bulk of the matter in the Universe. Most supersymmetric theories, including superstring-inspired models, have a natural candidate for this dark matter, namely the LSP. The challenge to experimentalists is to complete the second Copernican revolution by detecting LSPs from the galactic halo. The rates for LSP interactions depend on many things: astrophysics, fundamental particle physics, and mundane hadronic matrix elements. Theorists should not forget that even the most beautiful mathematical theory must be testable by experiment before it can be called physics. The challenge to theorists is to devise a clear-cut experimental test for the superstring. In so doing, they will have to keep in mind mundane phenomenological physics.

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