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The standard Z peak

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Abstract

The Standard Model prediction for the shape of the Z resonance in e^+e^- collisions is calculated. The initial-state radiation and the boson self-energies are taken into account. This is both necessary and sufficient to match the experimental precision of SLC and LEP. The dependence of the resonance shape on the top mass and the Higgs mass is studied.

Two basic measurements will be made in the near future: the determinations of the Z-resonance shape and of the asymmetries in e^+e^- collisions. Given accurate theoretical Standard Model predictions, these measurements allow for precision tests and could signal the presence of new physics beyond this model.

In this note we study the shape of the Z resonance for fermion-pair production in e^+e^- collisions. In lowest order the cross-section is given by

$$(1) \quad \sigma_0(s) = \frac{4\pi\alpha^2}{3s} \frac{N_c s^2}{4} \left\{ \left| \frac{g_e^- g_f^-}{s - M_Z^2 + iM_Z \Gamma_Z} + \frac{Q_e Q_f}{s} \right|^2 + \left| \frac{g_e^- g_f^+}{s - M_Z^2 + iM_Z \Gamma_Z} + \frac{Q_e Q_f}{s} \right|^2 \right. \\ \left. + \left| \frac{g_e^+ g_f^-}{s - M_Z^2 + iM_Z \Gamma_Z} + \frac{Q_e Q_f}{s} \right|^2 + \left| \frac{g_e^+ g_f^+}{s - M_Z^2 + iM_Z \Gamma_Z} + \frac{Q_e Q_f}{s} \right|^2 \right\}$$

where

$$(2) \quad g_f^- = (I_f^3 - Q_f \sin^2 \theta_W) / (\sin \theta_W \cos \theta_W) \quad \text{and} \quad g_f^+ = -Q_f \sin^2 \theta_W / (\sin \theta_W \cos \theta_W)$$

are the left- and right-handed Z-fermion-fermion couplings. For the electron, $Q_e = -1$, and $I_e^3 = -1/2$. The colour factor N_c is 3(1) for quarks (leptons). The lowest-order width Γ_Z in eq.(1) is the sum of all lowest-order partial decay widths into fermion pairs, which are given by

$$(3) \quad \Gamma_{Z \rightarrow f\bar{f}}^{(0)} = \frac{1}{6} N_c M_Z \sqrt{1 - \frac{4m_f^2}{M_Z^2}} \left((g_f^-)^2 + (g_f^+)^2 + \frac{m_f^2}{M_Z^2} (6g_f^- g_f^+ - (g_f^-)^2 - (g_f^+)^2) \right)$$

So far the initial-state radiation corrections to the cross-section and the self-energy contributions to the Z propagator for the determination of the Z-resonance peak have only been separately calculated. The purpose of this note is to give a realistic estimate of how the combination of all corrections together will affect the Z-resonance shape. In order to concentrate on the characteristics of those few corrections that really do matter, we discard many minor contributions which we found to have negligible influence in the renormalisation scheme we use.

Since the Z mass and width will be measured with an accuracy of 50 MeV, and maybe even 20 MeV [1], we will try to determine the position of the peak and the size of the width with a 10 MeV accuracy. The absolute scale is less important for the determination of the Z mass and width from the shape of the resonance [2], and we will be content here with an accuracy of a few per cent for the absolute height of the peak. Clearly, for the determination of branching ratios, which falls outside the scope of this note, one needs a better accuracy for the absolute height of the various fermion-pair production cross-sections.

We use the on-shell (OS) renormalisation scheme [3] with the boson masses M_W and M_Z , and the fine structure constant α (the on-shell eey-vertex coupling constant) as physical parameters. For

numerical discussions, the vector-boson masses have to be specified. We take $M_Z = 92\text{GeV}$, and determine M_W from α , M_Z and the Fermi coupling constant G_μ using the relation

$$(4) \quad M_W^2(1 - M_W^2/M_Z^2) = A/(1 - \Delta r) \quad \text{where} \quad A = \frac{\pi\alpha}{\sqrt{2}G_\mu} = (37.281\text{GeV})^2$$

By definition, $\cos\theta_W = M_W/M_Z$. Δr is a radiative correction which depends on the full parameter set of the theory, including α , M_W , M_Z , m_t and M_H . Δr has been calculated in a one-loop perturbative calculation and the masses of the light quarks have been adjusted such that the result agrees with that of Jegerlehner [4,5] ^{*}).

We estimate the error in Δr to be of the order of 0.0015. This corresponds to an error of 0.0005 in $\sin^2\theta_W = 1 - M_W^2/M_Z^2$. Note that the large QED part of Δr appears in the Z self-energy as well.

Electroweak corrections to eqs. (1)–(3) for fermion-pair production have been studied extensively [4–12]. The main non-photon effects are the replacement of the constant imaginary term $iM_Z\Gamma_Z$ by the energy-dependent self-energy $\Sigma_Z(s)$, and the electroweak and strong corrections to the width. Thus one obtains a modified cross-section $\sigma(s')$. In order to incorporate the dominant correction, which is photonic [13], $\sigma(s')$ should be folded with the initial-state radiation spectrum, which we will do later on.

Let us first determine $\sigma(s')$. Therefore we make the following replacements in the boson propagators

$$(5) \quad \frac{1}{s - M_Z^2 + iM_Z\Gamma_Z} \rightarrow \frac{1}{s - M_Z^2 + \text{Re}\Sigma_Z(s) + i\text{Im}\Sigma_Z(s)}$$

$$\frac{1}{s} \rightarrow \frac{1}{s(1 + \Pi_\gamma(s))}$$

Σ_Z and Π_γ have been calculated in one loop. In the OS scheme, we have $\text{Re}\Sigma_Z(M_Z^2) \equiv 0$. The Z width in this approach is defined by

$$(6) \quad M_Z\Gamma_Z \equiv \frac{\text{Im}\Sigma_Z(M_Z^2)}{1 + \Pi_Z(M_Z^2)}, \quad \Pi_Z(s) = (\partial/\partial s)\text{Re}\Sigma_Z(s)$$

A good approximation to the effect of Π_Z and Π_γ is the following: replace $\alpha = 1/137$ by $\alpha(M_Z^2) \approx 1/130$ in eqs. (1) and (3). There are various possibilities to define $\alpha(M_Z^2)$, of which the simplest amounts to eliminating α in favour of G_μ in the Born expressions:

^{*}) Note that Jegerlehner [5] averages over several renormalisation schemes, so that his published results cannot be directly compared with ours. We obtain values for $\sin^2\theta_W$ which are for all top and Higgs masses 0.0005 higher than those obtained by Lynn and Stuart [6].

$$(7) \quad \alpha \rightarrow \alpha(M_Z^2) = (\sqrt{2}G_\mu M_Z^2/\pi)\sin^2\theta_W\cos^2\theta_W = \alpha \sin^2\theta_W\cos^2\theta_W M_Z^2/A$$

Note that this $\alpha(M_Z^2)$ is not the conventional QED running α . Moreover, the above approximation does not take into account the effect of the s -dependence of $\text{Im}\Sigma_Z(s)$.

Qualitatively, the s -dependence of $\text{Re}\Sigma_Z$ and $\text{Im}\Sigma_Z$ is as follows: $\text{Re}\Sigma_Z \sim (s - M_Z^2)$ and $\text{Im}\Sigma_Z \sim s$. The s -dependence of $\text{Im}\Sigma_Z$ shifts the maximum of the peak some 35 MeV downwards with respect to the approximation with a constant imaginary part. This value of 35 MeV is not altered after including the folding with the initial-state radiation.

Off resonance the one-loop approximation $\text{Im}\Sigma_Z^{(1)}(s)$ is quite adequate, but at resonance one needs to know the Z width with a precision of about 10 MeV. This forces us to consider one-loop corrections to the Z width, which, according to eq. (6), correspond to two-loop corrections to $\text{Im}\Sigma_Z(M_Z^2)$. The one-loop s -dependent part $\text{Im}\Sigma_Z^{(1)}(s)$ and the correction to the Z width, which is proportional to the two-loop $\text{Im}\Sigma_Z^{(2)}(M_Z^2)$ are then combined as follows (cf. Ref. [9]):

$$(8) \quad \text{Im}\Sigma_Z(s) = \text{Im}\Sigma_Z^{(1)}(s) + \frac{s}{M_Z^2} \text{Im}\Sigma_Z^{(2)}(M_Z^2)$$

For $s \approx M_Z^2$ this gives an excellent approximation to the full self-energy; outside the resonance region where the approximation is not so good, such a high precision is not necessary. For details of the calculation of $\text{Im}\Sigma_Z^{(2)}$ see Ref. [14].

We have not calculated higher-order contributions to $(1 + \Pi_Z(M_Z^2))$. The hadronic error in our approximation for Π_Z is the same as in Δr . Estimates for the leading next-order corrections [15] to Δr show that they are negligible and we expect that those to Π_Z are smaller than 0.0005. Because they are correlated, the errors on $\sin^2\theta_W$ and $(1 + \Pi_Z)$ induce only an uncertainty of about 3 MeV on the total Z width.

The electroweak corrections to the width, apart from those arising from the common factor $(1 + \Pi_Z)$, are small. We have kept the mass dependence of the lowest-order partial $Z \rightarrow b\bar{b}$ and $Z \rightarrow t\bar{t}$ decay widths, see eq.(2), and also the top-mass dependence of the corrections to the $b\bar{b}$ width. The electroweak correction to the $Z \rightarrow t\bar{t}$ decay width, calculated in [16], has been neglected since it is very small and only plays a role if the toponium threshold is below the Z mass. Our results agree with those of Akhundov, Bardin and Riemann [17] and others [5,9,18,19] ^{*)}. The decrease of the width due to electroweak corrections (not including the $(1 + \Pi_Z)$ factor) varies in the OS scheme from less than 10 MeV for low and medium top quark masses till about 60 MeV for $m_t = 200\text{GeV}$.

^{*)} The differences are less than 2 MeV in the total width and less than 0.2% in the partial widths if we adjust the hadronic parts of Δr and Π_Z to that of Akhundov et al. [17]. The $\Gamma(Z \rightarrow t\bar{t})$ values in [5] have been updated [20] and now agree with our results.

The effect of the strong corrections is to increase the total width by some 70 MeV. Each partial quark decay width should be multiplied by an factor [21]

$$(9) \quad R = 1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 (1.98 - 0.115n_f)$$

where α_s should be evaluated at $Q^2 = M_Z^2$, and n_f is the number of quark flavours. The second-order term is of no numerical importance in view of the uncertainty of the exact value of R . We kept it here merely to show that we have used the same $\overline{MS}$ scheme as was used by PETRA experiments to arrive at the value of $\alpha_s((34\text{GeV})^2) = 0.138 \pm 0.023$ [22]. Letting the coupling constant run we arrive at $\alpha_s((92\text{GeV})^2) = 0.12 \pm 0.02$ (i.e. $R = 1.040 \pm 0.007$), both if the toponium threshold is below or above the Z mass. For the $Z \rightarrow t\bar{t}$ partial decay width the mass-dependent strong correction [23] has been used instead of the zero-mass approximation, eq. (9), which can be more than 50% too low in that case.

Table I gives partial decay widths for various values of the top and Higgs mass. Our ignorance of the precise value of the strong coupling constant induces an uncertainty of about 10 MeV in the total width. The uncertainty due to electroweak effects is of the order of 5 MeV.

In the case of quark-pair production, the cross-section should be multiplied by the factor R of eq. (9) as well, because of the strong corrections to the final state.

The largest correction to the cross-section comes from initial-state radiation. Its size is characterized by the parameter

$$(10) \quad \beta = 2 \left(\frac{\alpha}{\pi}\right) \left(\ln \frac{s}{m_e^2} - 1\right)$$

At $s = M_Z^2$, $\beta \approx 0.11$. Around the Z resonance, this correction is enhanced because there the Z propagator is highly sensitive to the invariant-mass squared of the final fermion-pair s' , and radiation by the initial electron or positron will cause big changes in the cross-section. Here multiple soft-photon effects must be taken into account [24].

The final result of this note for the total cross-section is given by the convolution of the modified cross-section $\sigma(s')$, as obtained from the propagator replacements eqs. (5) and (8), with the initial-state radiation spectrum:

$$(11) \quad \sigma(s) = \frac{1}{s} \int_{s_0}^s ds' \sigma(s') \left[\beta(s) (1 - s'/s)^{\beta(s)-1} (1 + \delta^{V+S}(s)) + \tilde{\delta}^H(s, s') \right]$$

Here the term between the square brackets is the initial-state radiation spectrum. The factor $\beta(1 - s'/s)^{\beta-1}$ represents a resummation of soft photons to all orders in perturbation theory, and is

responsible for the bulk of the effect: a reduction of the peak size by a factor $r \approx (\Gamma_Z/M_Z)^8 \approx 0.7$, and a shift $\Delta E \approx \pi\beta\Gamma_Z/8 \approx +100\text{MeV}$ in the position of the maximum [12]. For the results presented below for muon-pair production, we have used the analytic expressions for the order α and order α^2 contributions to δ^{V+S} and $\tilde{\delta}^H$ as calculated in Ref. [25], and we have taken $s_0 = 4m_\mu^2$, equal to its kinematical bound. At $s = M_Z^2$, the virtual/soft correction $\delta^{V+S}(s) \approx 0.08$ and the hard $\tilde{\delta}^H(s, s') \approx -0.05(1 + s'/s)$. Kuraev and Fadin[26], Nicrosini and Trentadue [27], and Greco[28] have calculated δ^{V+S} and $\tilde{\delta}^H$ in slightly different approximations. Numerically all these approaches agree [29]. Although the initial-state radiation constitutes by far the largest correction to the resonance shape, the uncertainty it induces is negligible: a few MeV in the position of the peak, and less than 1% in the height of the peak.

In *fig. 1* we compare the final result for muon-pair production based on eqs. (5) and (8) with the G_μ -Born approximation (i.e. with the replacement of eq. (7) for α in the α -Born approximation of eqs. (1) and (3)) and with an "improved" G_μ -Born approximation where moreover $iM_Z\Gamma_Z$ is replaced by $is\Gamma_Z/M_Z$ and the quark partial decay widths are multiplied with the factor R of eq.(9). All these cross-sections have been folded with the same initial-state spectrum from [25] as indicated in eq. (11). Remember that in the experimental situation it is much more easy to note differences in the shape than differences in the absolute scale. In *Table II*, we list some characteristic parameters of these curves: the maximum, the position of the maximum, and the values of $\sqrt{s}$ where $\sigma(s)$ attains half its maximum value.

In *figs. 2,3* and *Table III* we have compared the Z shape in muon-pair production for several values of M_H and m_t and a fixed value of $M_Z = 92\text{GeV}$. We observe that, also after folding with the initial-state radiation spectrum, the position of the maximum is hardly dependent on the values of m_t and M_H . Thus, if the corrections discussed in this note are taken into account, a precise determination of the Z mass from the measurement of the Z -peak shape is straightforward. *Fig. 2* shows that the shape of the peak is almost independent of the Higgs mass. *Fig. 3* signals that the measurement of the Z -resonance shape might give interesting limits on the top mass.

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Table 1

Partial and total Z widths for $M_Z = 92\text{GeV}$ and $\alpha_s = 0.12$.

m_t (GeV)	M_H (GeV)	$\sin^2\theta_W$	Γ_Z (MeV)	$\Gamma_{Z \rightarrow \nu\bar{\nu}}$ (MeV)	$\Gamma_{Z \rightarrow e^+e^-}$ (MeV)	$\Gamma_{Z \rightarrow d\bar{d}}$ (MeV)	$\Gamma_{Z \rightarrow u\bar{u}}$ (MeV)	$\Gamma_{Z \rightarrow b\bar{b}}$ (MeV)
40	100	0.2293	2638	170.4	85.8	395.0	306.7	390.8
60	100	0.2296	2560	170.5	85.8	395.2	306.9	391.3
90	10	0.2242	2563	170.4	85.8	396.0	307.8	391.3
90	100	0.2257	2566	170.8	86.0	396.3	308.0	391.6
90	1000	0.2288	2557	170.5	85.7	394.9	306.6	390.2
200	100	0.2109	2596	172.5	86.9	402.6	313.5	390.4

Table 2

The width, maxima and half-maxima of some approximations to the Z-resonance shape in muon-pair production. $M_Z = 92\text{GeV}$, $m_t = 90\text{GeV}$ and $M_H = 100\text{GeV}$, $\alpha_s = 0.12$. All the cross-sections have been folded with the initial-state radiation from Ref. [25].

	Γ_Z (MeV)	$\sigma_{\max}$ (nb)	$\sqrt{s}_{\max}$ (GeV)	$\sqrt{s}_{\max/2,-}$ (GeV)	$\sqrt{s}_{\max/2,+}$ (GeV)
α -Born, eqs.(1-3)	2351	1.52	92.118	90.889	93.613
G_μ -Born, eq.(7)	2502	1.53	92.127	90.817	93.720
"improved" G_μ -Born	2572	1.45	92.095	90.749	93.733
this note, eqs. (5,8)	2566	1.48	92.095	90.753	93.728

Table 3

Maxima and half-maxima of the Z-resonance shape in muon-pair production for various values of M_H and m_t . $M_Z = 92\text{GeV}$, $\alpha_s = 0.12$.

m_t (GeV)	M_H (GeV)	$\sigma_{\max}$ (nb)	$\sqrt{s}_{\max}$ (GeV)	$\sqrt{s}_{\max/2,-}$ (GeV)	$\sqrt{s}_{\max/2,+}$ (GeV)
40	100	1.39	92.094	90.713	93.774
60	100	1.47	92.094	90.755	93.723
90	10	1.48	92.095	90.754	93.726
90	100	1.48	92.095	90.753	93.728
90	1000	1.47	92.094	90.756	93.722
200	100	1.52	92.097	90.741	93.751

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FIGURE CAPTIONS

Figure 1: The Z peak in muon-pair production in various approximations. $M_Z=92\text{GeV}$, $m_t=90\text{GeV}$, $M_H=100\text{GeV}$, $\alpha_s=0.12$. Dashed line: G_μ -Born (see eq. (7)), dash-dotted line: "improved" G_μ -Born (see text), solid line: this note, eqs. (5) and (8). All these cross-sections have been folded with the initial-state radiation spectrum from Ref. [25] as indicated in eq. (11).

Figure 2: Higgs-mass dependence of the Z peak in muon-pair production. $M_Z=92\text{GeV}$, $m_t=90\text{GeV}$, $\alpha_s=0.12$. Solid line $M_H=10\text{GeV}$, dashed line $M_H=1000\text{GeV}$.

Figure 3: Top-mass dependence of the Z peak in muon-pair production. $M_Z=92\text{GeV}$, $M_H=100\text{GeV}$, $\alpha_s=0.12$. Upper solid line $m_t=200\text{GeV}$, lower solid line $m_t=60\text{GeV}$, dashed line $m_t=40\text{GeV}$.

- Figure 1 -

- Figure 2 -

- Figure 3 -