END EFFECTS IN QUANTUM BEAMSTRAHLUNG

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End effects in quantum beamstrahlung, in the case of a uniform deflecting field, in the extreme quantum limit, are calculated. The result is applied to beamstrahlung with uniform cylindrical bunches and small disruption.

1. INTRODUCTION

The quantum theory of beamstrahlung energy loss for highly relativistic particles in quasiclassical orbits was developed by Schwinger¹ and a series of Soviet theorists.²⁻⁶ In particular, the case of a uniform deflecting field was worked out in detail. In application to the beamstrahlung problem in linear colliders,^{7,8} the field was treated as locally uniform, and the results were averaged over the various strengths present in the bunch. The question arose as to on what scale the field must be approximately uniform for this treatment to be valid.

When the beamstrahlung process is largely classical, the important characteristic length is

$$L_C = \rho/\gamma, \quad (1)$$

where ρ is the radius of curvature of the track, and γ is the usual relativistic Fitzgerald contraction factor. A condition for the process to be largely classical is that the typical classical photon energy

$$\gamma^3/\rho$$

be small compared with the initial particle energy $\varepsilon = m\gamma$. When, on the contrary,

$$Y = (\gamma^3/\rho)/m\gamma = \gamma^2/(m\rho) \quad (2)$$

is large compared with 1, we have the "ultra quantum" limit,² with the photon typically taking a substantial fraction of the primary energy. For this extreme quantum case, it was pointed out (Ref. 9) that the characteristic length for the convergence of the integrals involved is not L_C , but the much greater

$$L_Q = Y^{1/3} L_C = Y^{1/3} \rho/\gamma = (\gamma/m)^{1/3} (\rho/\gamma)^{2/3}. \quad (3)$$

It was suggested therefore that nonuniformity and end effects would be relatively unimportant if the bunch were longitudinally uniform and long, on this scale.

The study of small nonuniformities has been initiated by Pisin Chen,¹⁰ who

calculates the corrections proportional to

$$(L_Q B^{-1} dB/dz)^2 \quad \text{and} \quad L_Q^2 B^{-1} d^2 B/dz^2, \quad (4)$$

where B is the deflecting force, and z is distance along the particle trajectory. However, we are concerned here with the large nonuniformities that arise at the ends of a region of uniform transverse acceleration, as occur in the beamstrahlung problem with sharply bounded uniform cylindrical space-charge bunches. The quantitative examination of this case has been started by Jacob and Wu.¹¹⁻¹³ They have discovered an end effect in the mean energy loss that increases logarithmically with Y . We confirm this finding, and calculate also the sublogarithmic terms for large Y . The general conclusion is that the end effects are indeed relatively unimportant when the bunch length is large compared with typical values of L_Q , except in very extreme conditions when even $\ln Y$ is large.

2. DEPARTURE

We start with the following formula⁶ for the mean energy dE radiated into a frequency range $d\omega$ and solid angle do by a relativistic electron:

$$dE[e^2\omega^2/(4\pi^2)] d\omega do \int dt \int dt' A e^{i\phi}, \quad (5)$$

with

$$A = [(\varepsilon^2 + \varepsilon'^2)/(2\varepsilon'^2)][v(t)v(t') - 1] + (1/2)(\omega/\varepsilon')^2(m/\varepsilon)^2, \quad (6)$$

$$\phi = (\varepsilon/\varepsilon')\omega\{(t - t') - n[r(t) - r(t')]\}, \quad (7)$$

where ε = initial electron energy, $\varepsilon' = \varepsilon - \omega$, $r(t)$ = position on classical orbit unperturbed by radiation, $v(t) = (d/dt)r(t)$, n = unit vector in photon direction, $e^2 = 1/137$, and $h/(2\pi) = c = 1$.

With

$$\delta = E/\varepsilon, \quad x = \varepsilon'/\varepsilon, \quad (1 - x) = \omega/\varepsilon, \quad (8)$$

the formulae can be rewritten

$$d\delta = dx do (e^2\omega^2/4\pi^2) \int dt \int dt' A e^{i\phi}, \quad (9)$$

$$A = [(1/2x^2) + (1/2)][v(t) \cdot v(t') - 1] + (1/2)[(1 - x)/x]^2/\gamma^2, \quad (10)$$

$$\phi = (\omega/x)(t - t' - n \cdot r), \quad (11)$$

$$r = r(t) - r(t'). \quad (12)$$

Performing the integration over solid angle, i.e., over all directions of n , gives

$$d\delta/dx = (e^2/\pi)\omega x \int dt \int dt' (A/r) \sin(\omega r/x) \cos[\omega(t - t')/x], \quad (13)$$

with

$$r = |r| (t - t')/|t - t'|. \quad (14)$$

In

$$\sin(\omega r/x) \cos[\omega(t - t')/x] = (1/2) \sin[\omega(r - t + t')/x] + \sin[\omega(r + t - t')/x] \quad (15)$$

the second term oscillates very quickly for a particle traveling with nearly the velocity of light. Neglecting the variation of v on this scale,

$$d\delta/dx = -(e^2/\pi)\omega x \int_{-\infty}^{\infty} dt' [(-\pi/2)A_0 + \int_{t'}^{\infty} dt (A/r) \sin \theta], \quad (16)$$

where, taking advantage of the symmetry of the integrand, the integral has been restricted to $t > t'$, with a compensating factor 2, and

$$\theta = (\omega/x)(t - t' - r), \quad A_0 = -1/(x\gamma^2). \quad (17)$$

We suppose that the unperturbed classical orbit deviates little from a straight line¹⁴ and drop terms of higher order than second in the deviation. Take Cartesian coordinates with the z axis in the direction of $v(t')$. Then, for large γ and small deviations

$$\begin{aligned} v(t') &= [0, 0, 1 - 1/(2\gamma^2)], \\ v(t) &= [v_x, v_y, 1 - 1/(2\gamma^2) - (v_x^2 + v_y^2)/2], \\ 1 - v(t) \cdot v(t') &= 1/\gamma^2 + (v_x^2 + v_y^2)/2, \\ r &= (z^2 + x^2 + y^2)^{1/2} = z + (1/2)(x^2 + y^2)/z, \\ (r - t) &= (z - t) + (1/2)(x^2 + y^2)/(t - t'). \end{aligned}$$

Then

$$A = -[(1 + x^2)/(2x^2)][1/\gamma^2 + (v_x^2 + v_y^2)/2] + (1 - x)^2/(2x^2\gamma^2), \quad (18)$$

$$\theta = \psi - (1/2)(\omega/x)(x^2 + y^2)/(t - t'), \quad (19)$$

$$\partial\psi/\partial t = (\omega/x)[1/(2\gamma^2) + (v_x^2 + v_y^2)/2], \quad \psi(t') = 0, \quad (20)$$

(where x is used in two different senses!).

It is convenient to introduce normalized variables, denoted by capital letters, such that

$$t = (\rho/\gamma)T, \quad (21)$$

$$v_x = V_x/\gamma, \quad v_y = V_y/\gamma, \quad (22)$$

$$(x, y) = (\rho/\gamma)(X, Y)/\gamma, \quad (23)$$

where ρ is some typical radius of curvature. Then

$$d\delta/dx = (e^2/\pi)\Omega \int dT' \left\{ \left[\int dT (F/\tau) \sin \theta \right] - (\pi/2)F_0 \right\} \quad (24)$$

$$T > T'$$

$$\tau = T - T', \quad (25)$$

$$F = [1 + (V_x^2 + V_y^2)/2](1 + x^2) - (1 - x)^2, \quad F_0 = 2x, \quad (26)$$

$$\theta = \psi - \Omega(X^2 + Y^2)/\tau, \quad (27)$$

$$\partial\psi/\partial T = \Omega(1 + V_x^2 + V_y^2), \quad (28)$$

$$\Omega = (1 - x)/(2xY), \quad Y = (\gamma^2/m\rho). \quad (29)$$

The scaling variable Y is essentially that of Noble.⁸ It measures the ratio of the typical photon frequency of the classical case, of order γ^3/ρ , to that typical in the quantum case, of order $m\gamma$. When Γ is large, quantum effects are essential. We use the terminology

$$Y \gg 1, \quad \text{“extreme quantum”} \quad (30)$$

$$Y \ll 1, \quad \text{“classical”}. \quad (31)$$

It remains to write equations of motion for X, Y, V_x, V_y :

$$dX/dT = V_x, \quad dV_x/dT = K_x \quad (32)$$

$$dY/dT = V_y, \quad dV_y/dT = K_y \quad (33)$$

$$(X, Y, V_x, V_y) = 0 \quad \text{at} \quad T = T'. \quad (34)$$

The quantity K , in general a function of T , is the deflecting force in units of that defining the typical radius of curvature ρ . In the present paper we will consider only a deflecting force uniform over a segment of the trajectory:

$$K_x = 0$$

$$K_y = 0 \quad \text{outside the field}$$

$$= 1 \quad \text{inside the field.}$$

3. CENTRAL EFFECT

For a sufficiently long field, and for T' sufficiently far from the ends,

$$(d^2\delta/dx dT') = (e^2/\pi)\Omega \left[\int_0^\infty (d\tau/\tau) F \sin \Omega(\tau + \tau^3/12) - (\pi/2)F_0 \right] \quad (35)$$

$$F = (1 + \tau^2/2)(1 + x^2) - (1 - x)^2, \quad F_0 = 2x. \quad (36)$$

This gives, in terms of Airy functions,

$$(d^2\delta/dx dT') = -e^2(1 - x)/(xY)[(1 + x^2)Ai'(f)/f + x \int_f^\infty dv Ai(v)], \quad (37)$$

with

$$f = [(1 - x)/(xY)]^{2/3}. \quad (38)$$

This is one form of the result of Sokolov and Ternov (Ref. 15, Eq. 8). As pointed out by them, it simplifies for large Y :

$$(d^2\delta/dx dT') = (e^2/\pi)[(1-x)/(xY)]^{1/3}(1+x^2)(-\pi Ai'(0)), \quad (39)$$

where

$$-\pi Ai'(0) = 0.813. \quad (40)$$

One comes directly to the asymptotic form [Eq. (39)] by noting in Eq. (35) that the integral converges when the phase oscillates rapidly. For large Y , and so large Ω (for x not very small), this happens only at large τ . The τ^3 term then dominates in the phase, and the τ^2 in F . Retaining only these gives Eq. (39).

More precisely, remembering that our unit of time, or distance, is $L_C = \rho/\gamma$, we see that in the extreme quantum case the characteristic time, or length, at which rapid oscillation sets in is of order

$$L_{Qx} = (2\Omega)^{-1/3} \rho/\gamma = (xY/(1-x))^{1/3} (\rho/\gamma),$$

which for x not very small or large is of order L_Q .

Integrating Eq. (39) over x , we have in the central region

$$d\delta/dt = 1.165(e^2/\pi)L_Q^{-1} = 1.165(e^2/\pi)(m\gamma/\rho^2)^{1/3}, \quad (41)$$

where t is time in ordinary units, in whatever reference frame is used.

For application to the beamstrahlung problem, with a uniform cylindrical bunch and small disruption, a further integration is required, over impact parameter b . Since ρ^{-1} is proportional to b , and therefore L_Q^{-1} to $(b^2)^{1/3}$, the appropriate average over b^2 gives a factor 3/4. The average fractional energy loss for a particle moving axially through a uniform cylindrical bunch is then

$$d\delta/dt = 0.874(e^2/\pi)L_{QB}^{-1}, \quad (42)$$

where L_{QB} is just L_Q evaluated at maximum impact parameter B . Note that if the laboratory frame of reference is used, the transit time is bunch length/2, and it is this by which Eq. (42) must be multiplied to get the integrated central effect. Following Himel and Siegrist⁷ formulae such as Eq. (42) were given by several authors. We checked against that of Blankenbecler and Drell.¹⁶

4. END EFFECTS

Near the ends we have to be more careful. Let us divide the ranges of T' and T in Eq. (24) into three parts, 1, 2, and 3, corresponding to before, during, and after the application of the field. Then the integral can be written with an obvious notation

$$I_{11} + I_{12} + I_{13} + I_{22} + I_{23} + I_{33}.$$

We can evaluate I_{11} from Eq. (24), if we are careful with the convergence of the integral at infinity, by introducing, for example, a convergence factor

$$e^{-\epsilon|T|}e^{-\epsilon|T'|},$$

where ε is very small. It is simpler to go back to Eq. (9):

$$\begin{aligned} I_{11} &= (e^2 \omega^2) / (8\pi^2 x^2) [(1+x^2)(v^2-1) + (1-x)^2/\gamma^2] \\ &\quad \times \int d\omega \left| \int_{-\infty}^0 dt \exp [i(\omega/x)(1-n \cdot v)t] \right|^2 \\ &= -(e^2 x) / (4\pi^2 \gamma^2) \int d\omega (1-n \cdot v)^{-2} \end{aligned} \quad (43)$$

or

$$I_{11} = -(e^2/\pi)x \quad (44)$$

For the next integral we go to Eq. (24) (leaving the $\pi/2$ term to I_{11} , I_{22} , and I_{33})

$$I_{12} = (e^2/\pi)\Omega \int_{-\infty}^0 dT' \int_0^\infty dT (F/\tau) \sin \theta, \quad (45)$$

where

$$\tau = T - T' \quad (46)$$

$$F = (1 + T^2/2)(1 + x^2) - (1 - x)^2 \quad (47)$$

$$\theta = \Omega [T - T' + T^3/3 - (1/4)T^4/\tau]. \quad (48)$$

This is some function of Ω , which we will calculate analytically only for Ω small. The T^2 term in F then dominates the other (by a factor of order $\Omega^{-2/3}$), and we have

$$I_{12} = (e^2/2\pi)(1 + x^2)(A + B), \quad (49)$$

where

$$A = \Omega \int_0^\infty d\tau/\tau \int_0^\tau dT T^2 \sin [\Omega(\tau + T^3/3)], \quad (50)$$

$$B = \Omega \int_0^\infty d\tau/\tau \int_0^\tau dT T^2 \{\sin [\Omega(\tau + T^3/3 - T^4/4\tau)] - \sin [\Omega(\tau + T^3/3)]\}. \quad (51)$$

In the following sequence we drop at each stage terms that vanish for small Ω ; the “=” sign should be qualified accordingly. The integration range near $\tau = 0$ has to be treated with care because of the factor $(1/\tau)$. Then

$$\begin{aligned} A &= \int_0^\infty (d\tau/\tau) \{\cos (\Omega\tau) - \cos [\Omega(\tau + \tau^3/3)]\} \\ &= \int_0^1 (d\tau/\tau) [\cos \tau - \cos (\tau + \Omega^{-2}\tau^3/3)] + \int_1^\infty (d\tau/\tau) \cos \tau \\ &= \int_0^1 (d\tau/\tau) \cos \tau [1 - \cos (\Omega^{-2}\tau^3/3)] + \int_1^\infty (d\tau/\tau) \cos \tau \\ &= \int_0^1 (d\tau/\tau) [1 - \cos (\Omega^{-2}\tau^3/3)] + \int_1^\infty (d\tau/\tau) \cos \tau \end{aligned}$$

$$\begin{aligned}
& + \int_0^1 (d\tau/\tau)(\cos \tau - 1)[1 - \cos(\Omega^{-2}\tau^3/3)] \\
& = (1/3) \int_0^G (d\tau/\tau)(1 - \cos \tau) + \int_1^\infty (d\tau/\tau) \cos \tau + \int_0^1 (d\tau/\tau)(\cos \tau - 1) \quad (52)
\end{aligned}$$

(where $G = \Omega^{-2}/3$)

$$\begin{aligned}
& = (1/3) \int_1^G (d\tau/\tau)(1 - \cos \tau) + \int_1^\infty (d\tau/\tau) \cos \tau + (2/3) \int_0^1 (d\tau/\tau)(\cos \tau - 1) \\
& = (2/3) \ln \Omega^{-1} - (1/3) \ln 3 + (2/3) \int_1^\infty (d\tau/\tau) \cos \tau \\
& \quad + (2/3) \int_0^1 (d\tau/\tau)(\cos \tau - 1) \\
& = (2/3) \ln \Omega^{-1} - (1/3) \ln 3 - (2/3)\gamma
\end{aligned}$$

(where γ is Euler's constant)

$$= (2/3)(\ln \Omega^{-1} - 1.127). \quad (53)$$

In the integral B , it is convenient to replace T by $\chi = T/\tau$:

$$\begin{aligned}
B & = \Omega \int_0^\infty d\tau \tau^2 \int_0^1 d\chi \chi^2 [\sin(\Omega\{\tau + (\tau^3/3)[\chi^3 - (3/4)\chi^4]\}) - \sin[\Omega(\tau + \tau^3\chi^3/3)]] \\
& = \int_0^\infty d\tau \tau^2 \int_0^1 d\chi \chi^2 \quad (54)
\end{aligned}$$

$$[\sin\{\Omega^{2/3}\tau + (1/3)\tau^3\chi^3[1 - (3/4)\chi]\} - \sin[\Omega^{2/3}\tau + (1/3)\tau^3\chi^3]]. \quad (55)$$

We now put $\Omega = 0$ and do the τ integration. Then

$$B = \int_0^1 (d\chi/\chi) \{ [1 - (3/4)\chi]^{-1} - 1 \} \quad (56)$$

$$\begin{aligned}
& = \int_0^1 d\chi (3/4) [1 - (3/4)\chi]^{-1} = \ln 4 \\
& = 1.386. \quad (57)
\end{aligned}$$

So

$$I_{12} = (e^2/\pi)(1 + x^2)[(1/3) \ln \Omega^{-1} + 0.3176], \quad (58)$$

where¹⁷ we have taken (with γ denoting Euler's constant)

$$(1/2) \ln 4 - (1/6) \ln 3 - \gamma/3 = 0.3176.$$

The integral I_{13} , if the field region is long compared with $L_\Omega = \Omega^{-1/3}(\rho/\gamma)$, will involve the sine of a large phase, which will oscillate rapidly with the parameters defining the situation. We can take then

$$I_{13} = 0$$

For I_{22} , when the field is sufficiently long, the main contribution is the first part of

$$I_{22} = I_{220} + \Delta I_{22}, \quad (59)$$

where

$$I_{220} = (e^2/\pi)\Omega \int_{-L}^0 dT' \left(\left\{ \int_{T'}^{\infty} dT(F/\tau) \sin [\Omega(\tau + \tau^3/12)] - F_0\pi/2 \right\} \right), \quad (60)$$

$$\Delta I_{22} = -(e^2/\pi)\Omega \int_{-L}^0 dT' \int_0^{\infty} dT(F/\tau) \sin [\Omega(\tau + \tau^3/12)]. \quad (61)$$

The first term is just the central effect [Eq. (42)] multiplied by field length L . The second term is the correction for having taken the T integral to infinity, rather than 0, in the first (we have shifted the origin of T' and T to the exit point). For L large enough, we can replace $-L$ as lower limit of T' in I_{220} by $-\infty$. Then

$$\Delta I_{22} = -(e^2/\pi)\Omega \int_{-\infty}^0 dT' \int_{-T'}^{\infty} d\tau(F/\tau) \sin [\Omega(\tau + \tau^3/12)] \quad (62)$$

$$= -(e^2/\pi)\Omega \int_0^{\infty} d\tau F \sin [\Omega(\tau + \tau^3/12)]. \quad (63)$$

For small Ω , taking

$$F = (1/2)\tau^2(1 + x^2), \quad (64)$$

we have, absorbing $\Omega^{1/3}$ into the integration variable,

$$\Delta I_{22} = -(e^2/2\pi)(1 + x^2) \int_0^{\infty} d\tau \tau^2 \sin [\tau^3/12] \quad (65)$$

$$\begin{aligned} &= -(e^2/2\pi)4(1 + x^2) \int_0^{\infty} d\tau \sin \tau \\ &= -2(e^2/\pi)(1 + x^2). \end{aligned} \quad (66)$$

The integral I_{23} looks different from I_{12} only because of our unsymmetric treatment of T and T' . We have

$$I_{23} = (e^2/\pi)\Omega^{-1} \int_{-\infty}^0 dT' \int_0^{\infty} dT(F/\tau) \sin \theta. \quad (67)$$

From Eqs. (33) and (34) in the range of the integral

$$V_Y = +T' \quad (68)$$

$$Y = -(1/2)T'^2 + T'T \quad (69)$$

$$\begin{aligned} \theta &= \Omega[-T' - (1/3)T'^3 + T(1 + T'^2) - Y^2/(T - T')] \\ &= \Omega[T - T' - (1/3)T'^3 - (1/4)T'^4/(T - T')]. \end{aligned} \quad (70)$$

Then with a change of variables replacing T' and T by $-T$ and $-T'$, respectively,

and comparing with Eqs. (47) and (48),

$$I_{23} = I_{12}.$$

Finally,

$$I_{33} = I_{11}.$$

Collecting terms, the total and effect is

$$\begin{aligned} (d\delta/dx)_{\text{end}} &= 2I_{11} + 2I_{12} + \Delta I_{22} \\ &= (e^2/\pi)[-2x + (1+x^2)((2/3)\ln \Omega^{-1} - 1.365)] \end{aligned} \quad (71)$$

$$= (e^2/\pi)[(2/3)(1+x^2)(\ln \Omega^{-1} - 2.047) - 2x]. \quad (72)$$

Integrating over x [assuming Y large enough that $xY/(1-x)$ is large over most of the range]

$$\delta_{\text{end}} = (e^2/\pi)(8/9)(\ln Y - 2.104). \quad (73)$$

In the case of a circular cylinder of uniform space charge, the deflecting field increases linearly with radius, and so then does Y , to a maximum value Y_B . Then the appropriate average is

$$\delta_{\text{end}} = (e^2/\pi)(8/9)(\ln Y_B - 2.604). \quad (74)$$

This last case is that considered by Jacob and Wu.¹¹⁻¹³ The logarithmic rise with very large Y is their discovery.

Comparing Eq. (72) with the central effect [Eq. (39)], the effective increase in field length is

$$\Delta L = 0.820[\ln \Omega^{-1} - 2.047 - 3x/(1+x^2)]L_{Qx} \quad (75)$$

with

$$L_{Qx} = [xY/(1-x)]^{1/3}(\rho/\gamma), \quad \Omega = (1-x)/(2xY).$$

Comparing Eq. (41) with Eq. (73) integrated over the photon spectrum, the end effect corresponds to an increase in effective length

$$\Delta L = 0.763L_Q(\ln Y - 2.104). \quad (76)$$

Finally, comparing Eq. (42) with Eq. (74), averaged over the impact parameter for a uniformly charged cylinder, the effective increase in length is

$$\Delta L = 1.017(\ln Y_B - 2.604)L_{QB}, \quad (77)$$

where subscript B indicates evaluation at the beam edge.

5. CONCLUSION

Our main result is Eq. (72). We used it here to discuss only mean energy loss. Clearly we could go on to discuss photon numbers and fluctuations, following Noble⁸ and Yokoya.¹⁸

It is natural to suppose that the “end effects” actually occur at the ends. Strictly speaking, in this approach only the total integral is significant, however convenient it may be to separate out (in a gauge-dependent way) different contributions in the calculation. It would be good to see an approach that explicitly keeps track of the approximate space-time development.

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