CERN-TH.4896/87

THREE LECTURES ON FLAVOUR MIXING

Guido Altarelli

CERN - Geneva

CONTENTS

1. Introduction
2. Basic Formalism
3. General Properties of M_{12} and Γ_{12}
4. The Cabibbo-Kobayashi-Maskawa Matrix
5. Calculation of the Box Diagrams
6. QCD Corrections to Box Diagrams
7. $B^0 - \bar{B}^0$ Mixing in the Standard Model
8. $B^0 - \bar{B}^0$ Mixing beyond the Standard Model
9. Conclusion

Lectures given at the
Cargèse 1987 School on Particle Physics

1. INTRODUCTION

The observation by ARGUS¹ at DESY of a relatively large amount of $B^0-\bar{B}^0$ mixing, following a previous positive signal of mixing by UA1², was the most important experimental result of the year in particle physics (together with the very recent result on ϵ'/ϵ by the NA31³ collaboration at CERN). The UA1 result was already known last year. The ARGUS result refers to the B_d^0 meson ($B_d^0 \equiv b\bar{d}$, $B_s^0 \equiv b\bar{s}$). In terms of $r \equiv P(B^0 \rightarrow \bar{B}^0)/P(B^0 \rightarrow B^0)$, i.e., the ratio of the probability for mixing and for no mixing, ARGUS finds:

$$r_d = 0.21 \pm 0.08 \quad (1.1)$$

The experimental method was described in the lectures by S.L. Wu⁴. On the theoretical side a large number of papers have been devoted on $B^0-\bar{B}^0$ mixing in the past⁵ and then recently⁶⁻¹⁰ after the UA1 and ARGUS results. These lectures are intended to an elementary introduction to flavour mixing in general and to $B^0-\bar{B}^0$ mixing in particular. Their purpose is to provide the reader with the essential background necessary to follow the current specialized literature.

2. BASIC FORMALISM

For a stable free particle at rest the quantum mechanics time evolution is given by $\psi \sim e^{-iMt}$. For an unstable particle at rest, this is modified into $\psi \sim e^{-i(M-i\Gamma/2)t}$ (in fact, $|\psi|^2 \sim e^{-\Gamma t} = e^{-t/\tau}$), with M and Γ real, positive numbers. For several coupled states M and Γ became Hermitian matrices with positive eigenvalues (i.e., the analogue of real, positive numbers). In particular for $B^0-\bar{B}^0$ (or any other similar system), we have:

$$H \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M - i\Gamma/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M - i\Gamma/2 \end{pmatrix} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} \quad (2.1)$$

Note that a) "H" is not Hermitian (since probability is not conserved within the $B^0-\bar{B}^0$ system, because of the decays); b) $H_{11} = H_{22}$ by CPT;

c) $H_{12} \neq 0$, $H_{21} \neq 0$ because of the weak interactions which violate the conservation of quark flavours. d) $\text{Im } M_{12} \neq 0$, $\text{Im } \Gamma_{12} \neq 0$ because of CP violation.

The eigenvalues of "H" can be written down in the form:

$$B_{1,2} = \frac{(1+\epsilon) B^0 \pm (1-\epsilon) \bar{B}^0}{\sqrt{2(1+|\epsilon|^2)}} \quad (2.2)$$

Note that B_1 and B_2 are not orthogonal because "H" is not Hermitian. If $\epsilon = 0$, CP is conserved in the wave functions. In general, ϵ depends on the phase convention chosen. Thus, for example, ϵ pure imaginary does not lead to any CP violation because it can be removed by a redefinition of the relative $B^0 - \bar{B}^0$ phase. A simple calculation immediately leads to the following results for ϵ and the eigenvalues of M and Γ :

$$\eta \equiv \frac{1-\epsilon}{1+\epsilon} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}} \quad (2.3)$$

$$M_{1,2} = M \pm \text{Re } Q \quad (2.4)$$

$$\Gamma_{1,2} = \Gamma \mp 2 \text{Im } Q \quad (2.5)$$

where M, Γ , M_{12} and Γ_{12} are defined in Eq. (2.1) and

$$Q = \sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)} \quad (2.6)$$

$B^0 - \bar{B}^0$ oscillations are caused by the different evolution in time of the eigenvectors B_1 and B_2 . Starting at $t = 0$ from a pure B^0 state:

$$|\psi(t=0)\rangle = |B^0\rangle = (|B_1\rangle + |B_2\rangle) \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2(1+\epsilon)}} \quad (2.7)$$

one obtains at time t:

$$|\psi(t)\rangle = \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2(1+\epsilon)}} \left[|B_1\rangle e^{-i(M_1 - \frac{1}{2}\Gamma_1)t} + |B_2\rangle e^{-i(M_2 - \frac{1}{2}\Gamma_2)t} \right] \quad (2.8)$$

By using Eq. (2.2) we can eliminate $|B_1\rangle$ and $|B_2\rangle$ and write $|\psi(t)\rangle$ as a superposition of B^0 and $\bar{B}^0$. The coefficients are the transition amplitudes: $A(B \rightarrow B)$ and $A(B \rightarrow \bar{B})$. One immediately obtains:

$$A(B \rightarrow B) = \frac{1}{2} \left[e^{-iM_1 t} e^{-\frac{1}{2}\Gamma_1 t} + e^{-iM_2 t} e^{-\frac{1}{2}\Gamma_2 t} \right]$$

$$A(B \rightarrow \bar{B}) = \frac{1-\epsilon}{1+\epsilon} \frac{1}{2} \left[e^{-iM_1 t} e^{-\frac{1}{2}\Gamma_1 t} - e^{-iM_2 t} e^{-\frac{1}{2}\Gamma_2 t} \right] \quad (2.9)$$

We define the ratio r of total (i.e., integrated over time) probabilities:

$$r = \frac{P(B \rightarrow \bar{B})}{P(B \rightarrow B)} = \frac{\int_0^T |A(B \rightarrow \bar{B})|^2 dt}{\int_0^T |A(B \rightarrow B)|^2 dt} \quad (2.10)$$

where T is a conveniently large time. One directly obtains from Eqs. (2.9) and (2.10):

$$r = \left| \frac{1-\epsilon}{1+\epsilon} \right|^2 \frac{\int_0^T dt [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2 e^{-\Gamma t} \cos(\Delta m t)]}{\int_0^T dt [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2 e^{-\Gamma t} \cos(\Delta m t)]} \quad (2.11)$$

where $\Gamma = \frac{1}{2}(\Gamma_1 + \Gamma_2)$ is the average width of B_1 and B_2 and $\Delta m = M_1 - M_2$ is their mass difference (we define 1 and 2 such that $\Delta m > 0$). By performing the integrals for $T \rightarrow \infty$, we finally obtain:

$$r = \left| \frac{1-\epsilon}{1+\epsilon} \right|^2 \frac{x^2 + y^2}{2 + x^2 - y^2} \quad (2.12)$$

where $x = \Delta m / \Gamma$ and $y = \Delta \Gamma / 2\Gamma$ with $\Delta \Gamma = \Gamma_1 - \Gamma_2$. Similarly, we could have obtained

$$\bar{r} = \left| \frac{1+\epsilon}{1-\epsilon} \right|^2 \frac{x^2 + y^2}{2 + x^2 - y^2} \quad (2.13)$$

where

$$\bar{r} \equiv \frac{P(\bar{B} \rightarrow B)}{P(\bar{B} \rightarrow \bar{B})} \quad (2.14)$$

Clearly, when CP violation effects are neglected

$$r = \bar{r} = \frac{x^2 + y^2}{2 + x^2 - y^2} \quad (2.15)$$

so that the asymmetry

$$a = \frac{r - \bar{r}}{r + \bar{r}} \quad (2.16)$$

is a well-known measure of CP violation. Note that

$$\begin{aligned} 0 &\leq x^2 \leq +\infty \\ 0 &\leq y^2 = \left(\frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2} \right)^2 \leq 1 \end{aligned} \quad (2.17)$$

Then, neglecting CP violation, we have

$$0 \leq r \leq 1 \quad (2.18)$$

An alternative parameter for B - $\bar{B}$ mixing, often used, is χ :

$$r = \frac{\chi}{1-\chi} \quad \text{or} \quad \chi = \frac{r}{1+r} \quad (2.19)$$

From Eq. (2.18), it follows that $0 < \chi < \frac{1}{2}$.

Typically r is measured through the ratio:

$$\rho = \frac{N(BB) + N(\bar{B}\bar{B})}{N(B\bar{B}) + N(\bar{B}B)} \quad (2.20)$$

where $N(BB)$ is the number of BB final states observed in a sample of events from a process where a $B\bar{B}$ pair is produced. $N(BB)$ is identified by some convenient final state (e.g., two negatively charged leptons: $\ell^-\ell^-$). $N(B\bar{B})$ and $N(\bar{B}B)$ are experimentally the same but are kept separate to remind us of the possibility of a double flip.

If the $B\bar{B}$ pair is produced incoherently (i.e., in each event the angular momentum, parity, etc., of the $B\bar{B}$ system are different) then, neglecting CP violation:

$$\rho = \frac{2r}{1+r^2} \quad (2.21)$$

because $N(B\bar{B}) = N(\bar{B}B) \sim \chi(1-\chi)$, $N(BB) \sim (1-\chi)^2$, $N(\bar{B}\bar{B}) \sim \chi^2$ where χ is the flip probability given in Eq. (2.19).

But if the $B\bar{B}$ pair is produced coherently (for example, at the T(4S) $B\bar{B}$ has $J = 1$, $C = -1$, etc.) then ρ is different. For J odd, one obtains

$$\rho = r \quad (J \text{ odd}) \quad (2.22)$$

This is because quantum mechanics is at work. Let us see how¹¹.

Coherence with $J = 1$, $C = -1$ means that the produced state is:

$$|B(k_1, t)\rangle |\bar{B}(k_2, t)\rangle - |\bar{B}(k_1, t)\rangle |B(k_2, t)\rangle \quad (2.23)$$

with (we always neglect CP violation):

$$\begin{aligned} |B(t)\rangle &= R(t) |B\rangle + C(t) |\bar{B}\rangle \\ |\bar{B}(t)\rangle &= C(t) |B\rangle + R(t) |\bar{B}\rangle \end{aligned} \quad (2.24)$$

$R(t)$, $C(t)$ are the amplitudes to "remain" or to "change", i.e., $R = A(B \rightarrow B) = A(\bar{B} \rightarrow \bar{B})$ and $C = A(B \rightarrow \bar{B}) = A(\bar{B} \rightarrow B)$. Here $|B(t)\rangle$ means the state that evolves with time starting from $|B\rangle$ at time $t = 0$. If the two decays take place at the times t_1 and t_2 , the corresponding superpositions of states are given by:

$$\begin{aligned} &(R_1 C_2 - C_1 R_2) |BB\rangle + (R_1 R_2 - C_1 C_2) |B\bar{B}\rangle + \\ &+ (C_1 C_2 - R_1 R_2) |\bar{B}B\rangle + (C_1 R_2 - C_2 R_1) |\bar{B}\bar{B}\rangle \end{aligned} \quad (2.25)$$

where $R_1 C_2$ mean $R(t_1)C(t_2)$, etc. Now:

$$\rho = \frac{\int |R_1 C_2 - C_1 R_2|^2 dt_1 dt_2}{\int |R_1 R_2 - C_1 C_2|^2 dt_1 dt_2} = \frac{\text{NUM}(\rho)}{\text{DEN}(\rho)} \quad (2.26)$$

where R and C are directly obtained from Eqs. (2.9). With few steps one obtains:

$$\begin{aligned} \frac{\text{DEN}(\rho)}{\text{NUM}(\rho)} &= \int \left[e^{-\Gamma_1 t_1} e^{-\Gamma_2 t_2} + e^{-\Gamma_2 t_1} e^{-\Gamma_1 t_2} \pm \right. \\ &\left. \pm 2 \text{Re} e^{-\Gamma t_1} e^{i \Delta m t_1} e^{-\Gamma t_2} e^{-i \Delta m t_2} \right] dt_1 dt_2 \quad (2.27) \end{aligned}$$

$$= \frac{2}{\Gamma_1 \Gamma_2} \pm 2 \text{Re} \frac{1}{\Gamma - i \Delta m} \frac{1}{\Gamma + i \Delta m} = \frac{2}{\Gamma^2 - \frac{\Delta \Gamma^2}{4}} \pm \frac{2}{\Gamma^2 + \Delta m^2}$$

Thus, one finally gets:

$$\rho = \frac{\Delta m^2 + \Delta \Gamma^2/4}{2\Gamma^2 + \Delta m^2 - \Delta \Gamma^2/4} = r \quad (2.28)$$

3. GENERAL PROPERTIES OF M_{12} and Γ_{12}

A double change of flavour, $\Delta b = 2$, as in $B^0 - \bar{B}^0$ mixing, clearly requires the action of two charged currents. The two emitted W 's must be reabsorbed, so that a total of four weak vertices are involved. Quite in general, including all effects of strong interactions, one can write:

$$M_{12} = \int dx dy dz D_{\mu\nu}(x-y) D_{\sigma\tau}(z) \langle B^0 | J^\mu(x) J^\nu(y) J^\sigma(z) J^\tau(0) | \bar{B}^0 \rangle \quad (3.1)$$

where $D_{\mu\nu}$ is the W propagator. In terms of Feynman diagrams, the above formula includes box diagrams with all possible gluon exchanges among quark legs. This is clearly a very complicated object. The situation would become simpler if one could exploit the fact that QCD is asymptotically free at short distances. The effective interaction induced by Eq. (3.1) involves four space-time points where the W 's are emitted or reabsorbed. If one can prove that the four points at some level of approximation can be confused with a single point, then the effective interaction becomes local. The QCD effects can then, at least in principle, be taken into account perturbatively. The ideal situation is for $m_{\text{ext}} \ll m_Q \ll M_W$, where m_{ext} is the external meson mass and m_Q is the exchanged virtual quark mass. The W propagator $D_{\mu\nu}(u)$ is significantly different from zero only at distances $u \lesssim 1/M_W$.

On the other hand, at short distances $x-y \lesssim 1/M_W$ an operator expansion is valid, and $J_\mu(x)J^\mu(y)$ approaches a local form fermion operator. Thus the four-point function in Eq. (3.1) can be reduced to a two-point function when terms of order m_{ext}^2/M_W^2 , m_Q^2/M_W^2 are neglected. For heavy enough virtual quarks the operator expansion technique can in an analogous way be applied to the resulting product of two local four fermion operators. The heavy virtual quark lines can also be shrunk down to a point. The effective Hamiltonian is then finally given in terms of a single local four fermion operator:

$$H_{\text{eff}} \approx G_F^2 \bar{d}_L \gamma_\mu b_L \bar{d}_L \gamma^\mu b_L c(m_b, m_Q, M_W, \dots) \quad (3.2)$$

where

$$q_L = \frac{1-\gamma_5}{2} q \quad (3.3)$$

and c is a coefficient function. Since at short distance the QCD coupling α_s is small, one can compute the coefficient function c in the approximation of neglecting the strong interactions. This corresponds to the evaluation of the box diagrams (Fig. 1). The QCD corrections are then, in principle, computable and are determined by the anomalous dimensions of the different operators which enter in the short distance expansion (see Section 6). When $m_Q \gtrsim M_W$ the dominance of short distances remains true. What is to be rediscussed is the appropriate limiting procedure (e.g., $M_W, m_Q \rightarrow \infty$ with m_Q/M_W fixed).

In order for the above strategy to apply, it is necessary to have an argument for the dominance of heavy virtual quarks in the case of interest. As we shall see, it is the structure of the quark mixing matrix that fixes the relative importance of the different virtual quark

exchanges. For example, for $B^0 - \bar{B}^0$ mixing, M_{12} is dominated by the top quark exchange, because $|V_{tb}V_{td}^*|$ is of the same order of $|V_{cb}V_{cd}^*|$. The same is also true for $B_s^0 - \bar{B}_s^0$ mixing. On the other hand, M_{12} would vanish if all exchanged quarks had the same mass, due to the GIM mechanism¹² (b and d quarks are coupled to orthogonal combinations of u, c and t quarks). Thus for $B^0 - \bar{B}^0$ mixing the ratio of t to c contributions is of order m_t^2/m_c^2 .

Even if, in a given case, the short distance expansion can be applied, one is still confronted with the problem of computing the matrix element of H_{eff} in Eq. (3.2) between the external meson-antimeson states. This is a different problem. It is precisely at this stage that we cannot avoid to cope with QCD at scales of order m_{ext} . Thus, if the external meson is light (e.g., the kaon) the problem is not easy. One tentative solution is to evaluate the matrix element by vacuum insertion (or saturation)¹³. This is a kind of valence approximation which will be discussed in Section 5. The vacuum saturation approximation is expected to work better and better as the mass of the external meson increases. This is because of the Zweig rule: gluon radiation from the quark legs is suppressed when the relevant transverse momentum of order of the meson mass is large.

In the standard model all complex phases enter through the Cabibbo-Kobayashi-Maskawa (CKM) matrix (see Fig. 1). The phases determine $\text{Im } M_{12}$ vs. $\text{Re } M_{12}$ and $\text{Im } \Gamma_{12}$ vs. $\text{Re } \Gamma_{12}$.

Γ_{12} is given by the absorptive part of the diagrams of Fig. 2. A cut indicates the on-shell particles of the final state after decay (with the integrations over the available phase space). The cut in Fig. 2a corresponds to spectator decays, while the cut in Fig. 2b has to do with W-exchange decay modes. Clearly only common final states for B and $\bar{B}$ decay contribute to Γ_{12} . Examples of common final states are given in Fig. 3. Obviously, only light quarks can appear in the final state. In the limit when all quarks are massless except for b and t quarks, Γ_{12} for B mesons turns out to be proportional to m_b^2 . Actually, in this limit

$$\Gamma_{12} \propto [V_{ub}V_{ud}^* + V_{cb}V_{cd}^*]^2 m_b^2 = (V_{tb}V_{td}^*)^2 m_b^2 \quad (3.4)$$

where the last equality is due to the unitarity of the 3x3 CKM matrix.

Two important consequences follow for B mesons: a) Γ_{12} has almost the same phase of M_{12} (both being determined by $V_{tb}V_{td}^*$); b) $|\Gamma_{12}| \ll |M_{12}|$ or $\Delta\Gamma \ll \Delta m$ [see Eq. (2.6)] in the same proportion as $m_b^2 \ll m_t^2$. Thus for B mesons a quite good approximation is

$$\gamma = \frac{x^2}{2 + x^2} \quad (3.5)$$

Also in the limit that M_{12} and Γ_{12} have the same phase, ϵ is purely imaginary as seen from Eq. (2.3). If $M_{12} = |M_{12}|e^{i\phi}$, $\Gamma_{12} = |\Gamma_{12}|e^{i\phi}$, then

$$\frac{1-\epsilon}{1+\epsilon} = e^{-i\varphi} \quad (3.6)$$

In this case $\text{Re } \epsilon = 0$ and the effect of ϵ can be rotated away by a phase redefinition. Thus, there are two ways of having small effects of CP violation: either $\text{Im } M_{12}$, $\text{Im } \Gamma_{12}$ are small, then $|\epsilon|$ is small; or M_{12}

Fig. 1 Box diagrams for $B_d^0 - \bar{B}_d^0$ mixing with top quark exchange.

Fig. 2 Contribution to Γ_{12} for the $B^0 - \bar{B}^0$ system.

Fig. 3 Examples of common final states in B^0 and $\bar{B}^0$ decays.

and Γ_{12} can have nearly the same phase. In the CKM phase convention the first option is realized for the kaon system, the second for the B meson system.

Going back to Eqs. (2.3)-(2.6), we see that in general

$$\begin{aligned}\Delta m &= 2 \operatorname{Re} Q \\ \Delta \Gamma &= -4 \operatorname{Im} Q\end{aligned}\quad (3.7)$$

For B mesons M_{12} and Γ_{12} have nearly the same phase. Then

$$Q \approx |M_{12}| - \frac{i}{2} |\Gamma_{12}| \quad (3.8)$$

so that

$$\begin{aligned}\Delta m_B &\approx 2 |M_{12}| \\ \Delta \Gamma_B &\approx 2 |\Gamma_{12}|\end{aligned}\quad (3.9)$$

For kaons the situation is different as we shall now summarize.

For kaons¹⁸:

$$\begin{aligned}\Delta m_K &= m_{K_L} - m_{K_S} = (3.521 \pm 0.014) 10^{-12} \text{ MeV} \\ \tau_S &= (0.8923 \pm 0.0022) 10^{-10} \text{ s} \\ \tau_L &= (0.5193 \pm 0.0040) 10^{-7} \text{ s}\end{aligned}\quad (3.10)$$

As a consequence of $\Gamma_S \gg \Gamma_L$ one has

$$\begin{aligned}\Gamma_K &= \frac{1}{2}(\Gamma_S + \Gamma_L) \approx \frac{1}{2} \Gamma_S \\ \Delta \Gamma_K &= \Gamma_L - \Gamma_S \approx -\Gamma_S\end{aligned}\quad (3.11)$$

and $y_K = \Delta \Gamma / 2\Gamma \approx -1$ (actually $y_K^2 \approx 1 - 4\Gamma_L / \Gamma_S \sim 1 - 7 \cdot 10^{-3}$). Also

$$x_K \approx \frac{\Delta m_K}{\Gamma_K} \sim 0.954 \quad (3.12)$$

Finally

$$r_K \approx \frac{x_K^2 + y_K^2}{2 + x_K^2 - y_K^2} \approx 1 - 7 \cdot 10^{-3} \quad (3.13)$$

Moreover, for the kaon system the CP violation parameter $|\epsilon|$ is given by:

$$|\epsilon| \approx (2.28 \pm 0.02) \cdot 10^{-3} \quad (3.14)$$

The small absolute value of ϵ implies that for kaons

$$\begin{aligned}\operatorname{Im} M_{12} &\ll \operatorname{Re} M_{12} \\ \operatorname{Im} \Gamma_{12} &\ll \operatorname{Re} \Gamma_{12}\end{aligned}\quad (3.15)$$

because $\epsilon = 0$ if M_{12} and Γ_{12} are both real [Eq. (2.3)]. From Eqs. (2.4)-(2.5) we see that for kaons

$$\begin{aligned}\Delta m_K &\approx 2 \operatorname{Re} M_{12} \\ \Delta \Gamma_K &\approx 2 \operatorname{Re} \Gamma_{12}\end{aligned}\quad (3.16)$$

Note the difference with Eq. (3.9) valid for the B system.

Starting from the general expression for ϵ given by Eq. (2.3) we can expand in the small imaginary parts $\operatorname{Im} M_{12}$, $\operatorname{Im} \Gamma_{12}$:

$$\frac{1-\epsilon}{1+\epsilon} \approx 1-2\epsilon = \frac{\text{Re } M_{12} - i \text{Im } M_{12} - \frac{i}{2} \text{Re } \Gamma_{12} - \frac{1}{2} \text{Im } \Gamma_{12}}{\text{Re } M_{12} + i \text{Im } M_{12} - \frac{i}{2} \text{Re } \Gamma_{12} + \frac{1}{2} \text{Im } \Gamma_{12}} \approx$$

$$\approx 1 - \frac{i \text{Im } M_{12} + \frac{1}{2} \text{Im } \Gamma_{12}}{\text{Re } M_{12} - \frac{i}{2} \text{Re } \Gamma_{12}} \quad (3.17)$$

Thus, for kaons:

$$\epsilon \approx \frac{i \text{Im } M_{12} + \frac{1}{2} \Gamma_{12}}{\Delta m - i \Delta \Gamma / 2} \quad (3.18)$$

We have already seen that $\Delta m_K \approx -\Delta \Gamma_K / 2$. Also for kaons $\text{Im } \Gamma_K \approx 0$. This follows because only u quark can contribute to kaon decay (as opposed to c and t quarks). But V_{ud} and V_{us} can be chosen to be real, so that $\text{Im } \Gamma_K \approx 0$ for kaons. Hence, one obtains

$$\epsilon \approx e^{i\pi/4} \frac{\text{Im } M_{12}}{\sqrt{2} \Delta m} \quad (3.19)$$

Actually the experimental definition of ϵ is different than that given by Eq. (2.2) because of a different phase convention. The measured value of ϵ [Eq. (3.14)] corresponds to the definition:

$$\eta_{+-} = \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K_S \rightarrow \pi^+ \pi^-)} = \epsilon + \epsilon' \quad (3.20)$$

$$\eta_{00} = \frac{A(K_L \rightarrow \pi^0 \pi^0)}{A(K_S \rightarrow \pi^0 \pi^0)} = \epsilon - 2\epsilon'$$

The two definitions of ϵ coincide for the phase choice $\text{Im } A_0 = 0$, where $A_I = |A_I| e^{i\delta_I} = A(K^0 \rightarrow (\pi\pi)_I)$, $I = 0, 2$ being the isospin. Instead, the phase definition chosen here was specified by the requirement that $|K^0\rangle = \text{CP}|\bar{K}^0\rangle$. By defining:

$$\xi = \frac{\text{Im } A_0}{\text{Re } A_0} \quad (3.21)$$

the correction to Eq. (3.17) is given by¹⁴:

$$\epsilon = \frac{e^{i\pi/4}}{\sqrt{2}} \left[\frac{\text{Im } M_{12}}{\Delta m} + \xi \right] \quad (3.22)$$

Similarly

$$\frac{\epsilon'}{\epsilon} = \frac{1}{\sqrt{2}} \frac{1}{\epsilon} e^{i(\delta_2 - \delta_0 + \pi/2)} \left[\frac{\text{Im } A_2}{\text{Re } A_0} - \frac{\text{Re } A_2}{\text{Re } A_0} \xi \right] \quad (3.23)$$

Note that experimentally $\delta_2 - \delta_0 + \pi/2 \approx (48 \pm 8)^\circ$ so that $e^{i(\delta_2 - \delta_0 + \pi/2)} \approx e^{i\pi/4}$. The experimental validity of the $\Delta I = \frac{1}{2}$ rule implies:

$$\omega = \frac{\text{Re } A_2}{\text{Re } A_0} \approx 0.045 \quad (3.24)$$

According to Eq. (3.23), ϵ' is determined by the imaginary parts of the $\Delta S = 1$ $K \rightarrow \pi\pi$ amplitudes. These are zero in the standard model at tree level. At one loop, imaginary parts are generated by the famous penguin diagrams, Fig. 4. These diagrams only contribute to the $\Delta I = \frac{1}{2}$ amplitudes (hence to A_0 and not to A_2) so that $\text{Im } A_2 = 0$. Thus

$$\left| \frac{\epsilon'}{\epsilon} \right| = \frac{1}{\sqrt{2}} \omega |\xi| \approx 14 |\xi| \quad (3.25)$$

(ϵ'/ϵ is nearly real and is positive for ξ negative, as is most probably the case in the standard model).

The very recent experimental result by the NA31 Collaboration³ at CERN provides us with the value:

$$\frac{\epsilon'}{\epsilon} = (3.5 \pm 0.7 \pm 0.4 \pm 1.2) 10^{-3} \quad (3.26)$$

or

$$\frac{\epsilon'}{\epsilon} = (3.5 \pm 1.5) 10^{-3} \quad (3.27)$$

if the errors are added in quadrature.

4. THE CABIBBO-KOBAYASHI-MASKAWA MATRIX

Given the experimental values of the quark mixing angles, the most convenient parametrization of the CKM matrix is the one proposed by Maiani¹⁵. In comparison, the original proposal by Kobayashi-Maskawa¹⁶ leads to cumbersome expressions for the physically relevant transition amplitudes.

We denote by d' , s' , b' the three down partners of u , c , t respectively in the weak charged current left doublets. We write, following Maiani,

$$|d'\rangle = c_\beta |d_c\rangle + s_\beta e^{i\varphi} |b\rangle \quad (4.1)$$

where $c_\beta \equiv \cos\beta$, $s_\beta \equiv \sin\beta$ (analogous shorthand notations will be used in the following). d_c is the Cabibbo rotated down quark

$$|d_c\rangle = c_\theta |d\rangle + s_\theta |s\rangle \quad (4.2)$$

Note that in a four quark model the Cabibbo angle fixes both the ratio of the u to d coupling with respect to the ν_μ to μ coupling and the ratio of the u to s and u to d couplings. In a six quark model one has to choose whether to keep the first or the second definition. Here the second is taken and in fact the u to d coupling is given by $\cos\beta \cos\theta_c$, i.e., it is no longer completely specified by θ_c . Also note that we can certainly fix the phases of u , d , s and b so that a real coefficient appears in front of d_c .

We now construct two orthonormal vectors, both orthogonal to d' . They can be chosen as the Cabibbo rotated strange quark:

$$|s_c\rangle = -s_\theta |d\rangle + c_\theta |s\rangle \quad (4.3)$$

and as

$$|u\rangle = -s_\beta e^{-i\varphi} |d_c\rangle + c_\beta |b\rangle \quad (4.4)$$

Fig. 4 Penguin diagram with gluon exchange.

The angle γ is now defined by the physical combinations of s_c and v coupled to c and t quarks:

$$\begin{aligned} |s'\rangle &= c_\gamma |s_c\rangle + s_\gamma (-s_\beta e^{-i\varphi} |d_c\rangle + c_\beta |b\rangle) \\ |b'\rangle &= -s_\gamma |s_c\rangle + c_\gamma (-s_\beta e^{-i\varphi} |d_c\rangle + c_\beta |b\rangle) \end{aligned} \quad (4.5)$$

From experiment

$$s_\theta \equiv \lambda = 0.221 \pm 0.002 \quad (4.6)$$

Also $s_\gamma \sim \lambda^2$ and $s_\beta \sim \lambda^3$. Thus, empirically s_γ and s_β are very small in the Maiani convention. Neglecting terms of order λ^4 , one obtains a considerable simplification:

$$\begin{pmatrix} |u\rangle \\ |d_c\rangle + s_\beta e^{i\varphi} |b\rangle \end{pmatrix} \begin{pmatrix} |c\rangle \\ |s_c\rangle + s_\gamma |b\rangle \end{pmatrix} \begin{pmatrix} |t\rangle \\ |b\rangle - s_\gamma |s_c\rangle - s_\beta e^{-i\varphi} |d\rangle \end{pmatrix} \quad (4.7)$$

or

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta & s_\beta e^{i\varphi} \\ -s_\theta & c_\theta & s_\gamma \\ s_\gamma s_\theta - s_\beta e^{-i\varphi} & -s_\gamma & 1 \end{pmatrix} \quad (4.8)$$

Finally, it is convenient to set, following Wolfenstein¹⁷

$$\begin{aligned} s_\gamma &= A\lambda^2 \\ s_\beta &= A\lambda^3 \rho \end{aligned} \quad (4.9)$$

Then V takes the simple form

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 \rho e^{i\varphi} \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3 (1 - \rho e^{-i\varphi}) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \quad (4.10)$$

which will be used in the following.

We now consider the available experimental information on A , ρ and ϕ [λ is given in Eq. (4.6)]. A is fixed by the b lifetime τ_B and the semi-leptonic branching ratio $B_{SL} = B(b \rightarrow e\nu X)$. The semi-leptonic width $\Gamma_{SL} = B_{SL}/\tau_B$ can be computed by the parton model improved by QCD and phase space corrections. One could also add non-perturbative corrections to the spectator picture, typical of the parton model. These terms are model-dependent, but are small for the totally inclusive semi-leptonic width. One obtains:

$$\tau_B |V_{bc}|^2 = \frac{B_{SL} 10^{-13} s}{Z_c \left(1 + \frac{\Gamma(b \rightarrow u)}{\Gamma(b \rightarrow c)}\right)} = (2.9 \pm 0.6) 10^{-15} s \quad (4.11)$$

where we used $B_{SL} = 0.117 \pm 0.006$ ¹⁸ and $Z_c = 4.0 \pm 0.6$, the latter value

being taken from Ref. 19. For $B^0 - \bar{B}^0$ mixing only the combination $\tau_B A^2$ is important, which can directly be obtained from Eq. (4.11) [$\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ is small, see Eq. (4.15)]. Otherwise one can obtain A by using²⁰
 $\tau_B = (1.11 \pm 0.16) 10^{-12} \text{s}$:

$$A = 1.05 \pm 0.17 \quad (4.12)$$

Note that, since $A \sim 1$, $|V_{cb}| \sim |V_{ts}| \sim A\lambda^2$ indeed turn out to be of order λ^2 .

ρ is fixed by $R = \Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ [where $\Gamma(b \rightarrow u)$ means $\Gamma(b \rightarrow u \text{ ev})$, i.e., the semi-leptonic width into charmless final states]:

$$(0.47 \pm 0.02) R = \left| \frac{V_{ub}}{V_{cb}} \right|^2 = (\lambda \rho)^2 \quad (4.13)$$

where the numerical factor is obtained from the parton model plus phase space and QCD corrections. R is determined by the electron spectrum near the end-point. Clearly:

$$\frac{d\Gamma}{dE_e} = \Gamma(b \rightarrow u) \left[\frac{1}{\Gamma(b \rightarrow u)} \frac{d\Gamma(b \rightarrow u)}{dE_e} \right] + \Gamma(b \rightarrow c) \left[\frac{1}{\Gamma(b \rightarrow c)} \frac{d\Gamma(b \rightarrow c)}{dE_e} \right] \quad (4.14)$$

A priori there is some model dependence in the calculation of the normalized spectra. Various models have been tried²¹. A posteriori the differences are found smaller than one could expect. Quoting from Ref. 21: "The mistaken impression has been given that spectator quark and valence quark models give substantially different results. As we shall see, this is not the case". At 90% c.l., one has²²:

$$\begin{aligned} R &< 0.13 && \text{(Crystal Ball)} \\ &< 0.12 && \text{(ARGUS)} \\ &< 0.04 && \text{(CLEO)} \end{aligned} \quad (4.15)$$

I obtained the last number from Table I of Ref. 23 (CLEO). The quoted limit comes from $R = (1.0 \pm 1.9)\%$ at 1.64σ . From the same table, one also sees that different models and procedures lead to comparable (and more restrictive) results. In conclusion, there could be model dependence in extracting R from the spectrum (but apparently there is not). But I claim that, once R is extracted from the spectrum, then there are no reasonable arguments for model-dependence in the relation Eq. (4.13) between R and $|V_{bu}|/|V_{bc}|$, that in fact everybody is always using outside this very particular discussion. It is true that Grinstein et al.²⁴ find ~ 1 instead of 0.47 ± 0.02 in Eq. (4.13) by trying to reconstruct the total $b \rightarrow u$ width from the sum of many exclusive channels estimated by modelling the relevant form factors. However, for the totally inclusive semi-leptonic widths $\Gamma(b \rightarrow u)$ and $\Gamma(b \rightarrow c)$, we have no reasons to cast doubts on the parton model, which works so well even for systems as light as the τ lepton (when only one hadronic current is involved and inclusive rates are concerned). Note that from Ref. 24 (see their Fig. 1 and 2), it appears that both the spectrum and the rate agree for the $b \rightarrow c$ semi-leptonic transition when computed in the parton model and by adding up the D^* and D exclusive contribution. So, if the parton model is right for the $b \rightarrow c$ transition which is dominated by the D^* and D final states, why should we not a fortiori trust the parton model for the $b \rightarrow u$ case, where a multitude of final states contribute and there is a lot of phase space?

And indeed the authors of Ref. 24 explicitly state that they view the discrepancy by a factor of two between their model and the parton model in the $b \rightarrow u$ case as a measure of the error in the procedure of addition of exclusive channels (while the model could still be appropriate for the shape of the electron spectrum near the end-point).

In conclusion, the CLEO result $R < 0.04$ implies $|V_{bu}|/|V_{bc}| < 0.14$ or $\rho < 0.6$ (we essentially agree with Ref. 21). However, to be conservative and let people do their choice, we shall, in the following, present results for $|V_{bu}|/|V_{bc}| < 0.20$ or $\rho < 0.9$.

Recently, ARGUS presented evidence for $V_{bu} \neq 0$:

$$\begin{aligned} B(B^\pm \rightarrow p \bar{p} \pi^\pm) &= (3.7 \pm 1.3 \pm 1.4) 10^{-4} \\ B(B^0 \rightarrow p \bar{p} \pi^+ \pi^-) &= (6.0 \pm 2.0 \pm 2.2) 10^{-4} \end{aligned} \quad (4.16)$$

(a signal of 32.7 ± 7.7 events over the background). The connection with $|V_{bu}|/|V_{bc}|$ is really model-dependent in this case. ARGUS indicates the limit

$$\left| \frac{V_{bu}}{V_{bc}} \right| > 0.07 \quad \text{or} \quad \rho > 0.3 \quad (4.17)$$

which we will tentatively assume in the following, where we shall use $0.9 < \rho < 0.3$.

On $\cos\phi$ we observe that:

$$\left| \frac{V_{td}}{V_{ts}} \right|^2 = \lambda^2 (1 + \rho^2 - 2\rho \cos\phi) \quad (4.18)$$

Thus $|V_{td}|$ is maximum for $\cos\phi = -1$:

$$\left| \frac{V_{td}}{V_{ts}} \right| < \lambda (1 + \rho) \simeq \frac{1}{2.4} \quad (4.19)$$

One might be interested in maximizing $|V_{td}|$ in order to make the predicted amount of $B_d^0 - \bar{B}_d^0$ mixing as large as possible. But for $\cos\phi = -1$ all CP violating effects vanish. Thus a compromise is necessary. In Figs. 5 and 6 we show the allowed intervals of $\cos\phi$, once the constraint provided by the measured value of $|\epsilon|$ is imposed, for different values of m_t , ρ , A and $B_k < 1$. The additional constraint from ϵ'/ϵ given by Eq. (3.27), not included in Figs. 5, 6, would only be barely visible at large values of m_t .

The values obtained for the mixing angles already allow some important semi-qualitative statements.

1) For kaons, the dominant charm contribution to ΔM_{box} is proportional to $(V_{cd}^* V_{cs})^2 m_c^2$, while the top quark contribution is proportional to $(V_{td}^* V_{ts})^2 m_t^2$. Since $|V_{cd}^* V_{cs}| \simeq \lambda$, while $|V_{td}^* V_{ts}| \simeq A^2 \lambda^5 |1 - \rho e^{-i\phi}|$ the top term is negligible for all practical values of m_t (i.e., for $m_t \lesssim m_c / \lambda^4 \sim 500-600$ GeV).

2) Going back to Eqs. (4.5) [or by Eq. (5.4) plus the fact that $V_{ud}^* V_{us}$ is real], we see that

$$\text{Im}(V_{cd} V_{cs}^*) = -\text{Im}(V_{td} V_{ts}^*) \simeq A^2 \lambda^5 \rho \sin\phi$$

Fig. 5 Limits on $\cos\phi$ obtained from the experimental value of the CP violating parameter $|\epsilon|$ for the kaon system, as functions of the top quark mass, m_t , for various values of ρ . The solid (dashed) lines include (do not include) the effect of the ξ term. Here we have taken $A = 1.05$ [the central value in Eq. (4.12)]. The parameters ϕ , ρ and A are defined in Eq. (4.10). The indicates values of $\rho = 0.9, 0.6$ and 0.3 correspond to $R = \Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) \approx 0.08, 0.04$ and 0.009 respectively.

Fig. 6 Same as Fig. 5, but with $A = 1.22$.

As

$$\text{Im } z^2 = 2 \text{Re } z \text{Im } z$$

then

$$\frac{\text{Im}(V_{td} V_{ts}^*)^2}{\text{Im}(V_{cd} V_{cs}^*)^2} = - \frac{\text{Re}(V_{td} V_{ts}^*)}{\text{Re}(V_{cd} V_{cs}^*)} \sim -A^2 \lambda^4 (1 - \rho \cos \phi) \quad (4.20)$$

As a consequence the top quark contribution to ϵ (i.e., to $\text{Im } M_{12}$) is important as soon as $m_t \gtrsim m_c / \lambda^2 \sim 25 \div 30$ GeV. Note that the short distance approximation is reasonably justified for ϵ and not so much for Δm_K .

3) For charmed mesons ($D^0 = c\bar{u}$), the $D^0 - \bar{D}^0$ mixing is predicted to be very small and cannot be reliably computed by the box diagram. In fact, the b term is proportional to $(V_{cb} V_{ub}^*)^2 m_b^2$, hence so small that the strange quark exchange of order of $\lambda^2 m_s^2$ can still win. Typical order of magnitude estimates lead to $(\Delta m / \Gamma)_D \sim 10^{-2} \div 10^{-3}$. Experimentally²⁵, at 90% c.l., $r_D \lesssim 1.4\%$ (ARGUS), $r_D \lesssim 0.6\%$ (TPS).

5. CALCULATION OF THE BOX DIAGRAMS

It is instructive to do the simple calculation of the box diagrams in Fig. 1. We shall explicitly consider the $B^0 - \bar{B}^0$ system where we can restrict the calculation to the contribution of the top quark. The extension to the general case where two different heavy quarks are exchanged on the two sides of the box is left as an exercise.

Of the two box diagrams in Fig. 1, it is sufficient to compute the first one. As we shall see, the second one can then be obtained for free. We shall do the computation in the unitary gauge where the W propagator is $(-g^{\mu\nu} + k^\mu k^\nu / M_W^2) / (k^2 - M_W^2)$ and there are no diagrams with unphysical scalars. In general, in a spontaneously broken gauge theory it is simpler and more rigorous to work in the so-called ξ -gauges. However, this particular problem is so simple and convergent that we can handle it directly. Thus, I do not have to invest time in explaining the ξ -gauge methods.

It is convenient to neglect external quark momenta. All their components in the B rest frame are of order M_B and can be ignored with respect to M_W and m_t . If two quarks i and j are exchanged, the box amplitude from diagram a in Fig. 1 is proportional to (for me $\hat{k} \equiv k_\mu \gamma^\mu$)

$$\begin{aligned} \text{Box} &\propto \int d^4 k \sum_{i,j} \bar{u}_d V_{id}^* \gamma_\mu \frac{1-\gamma_5}{2} \frac{\hat{k} + m_i}{k^2 - m_i^2} \gamma_\nu \frac{1-\gamma_5}{2} V_{ib} u_b \cdot \\ &\cdot \bar{u}_d V_{jd}^* \gamma_\sigma \frac{1-\gamma_5}{2} \frac{\hat{k} + m_j}{k^2 - m_j^2} \gamma_\rho \frac{1-\gamma_5}{2} V_{jb} u_b \cdot \left(-g^{\mu\rho} + \frac{k^\mu k^\rho}{M_W^2} \right) \cdot \\ &\cdot \left(-g^{\nu\sigma} + \frac{k^\nu k^\sigma}{M_W^2} \right) \frac{1}{(k^2 - M_W^2)^2} \end{aligned} \quad (5.1)$$

The quark masses in the numerator can be immediately dropped because they are killed by the chiral projector $(1-\gamma_5)/2$. Denoting

$$\lambda_i \equiv V_{ib} V_{id}^* \quad (5.2)$$

we directly obtain (an overall factor is omitted)

$$\begin{aligned} \text{Box} = \sum_{ij} \lambda_i \lambda_j \bar{v}_d \gamma_\mu \gamma_\tau \gamma_\nu \frac{1-\gamma_5}{2} u_b \cdot \bar{u}_d \gamma_\sigma \gamma_\lambda \gamma_\rho \frac{1-\gamma_5}{2} v_b \cdot \\ \cdot \int d^4 k \frac{k^\tau \left(-g^{\mu\rho} + \frac{k^\mu k^\rho}{M_W^2} \right) \left(-g^{\nu\sigma} + \frac{k^\nu k^\sigma}{M_W^2} \right) k^\lambda}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)^2} \end{aligned} \quad (5.3)$$

The integral is quadratically divergent. Fortunately the application of the GIM mechanism¹² directly leads to a convergent integral. In fact, note that

$$\sum_i \lambda_i = 0 \quad (5.4)$$

because the V matrix is unitary and therefore $\sum_i \lambda_i = 0$ [see Eq. (5.2)] being an off-diagonal element of the identity matrix. With an obvious notation, the box diagram is of the form $\text{Box} = \sum_{i,j} \lambda_i \lambda_j E(m_i, m_j)$. By using Eq. (5.3) we can then write:

$$\begin{aligned} \text{Box} = \sum_{ij} \lambda_i \lambda_j E(m_i, m_j) = \sum_{ij} \lambda_i \lambda_j \left[E(m_i, m_j) - \right. \\ \left. - E(0, m_j) - E(m_i, 0) + E(0, 0) \right] \end{aligned} \quad (5.5)$$

This is now much more convergent. To see this in a simple way, we now restrict ourselves to the case of interest. The dominant top contribution is given by

$$\text{Box} = \lambda_t^2 \left[E(m_t, m_t) - 2E(m_t, 0) + E(0, 0) \right] \quad (5.6)$$

where we used the fact, evident from Eq. (5.3), that $E(m_t, 0) = E(0, m_t)$. By observing that:

$$\begin{aligned} \frac{1}{(k^2 - m_t^2)^2} + \frac{1}{k^4} - \frac{2}{k^2(k^2 - m_t^2)} &= \left(\frac{1}{k^2 - m_t^2} - \frac{1}{k^2} \right)^2 = \\ &= \frac{m_t^4}{k^4(k^2 - m_t^2)^2} \end{aligned} \quad (5.7)$$

we see that the quadratically divergent integral in Eq. (5.3) after the GIM treatment is reduced to the more sensible form:

$$I = m_t^4 \int d^4 k \frac{k^\tau \left(-g^{\mu\rho} + \frac{k^\mu k^\rho}{M_W^2} \right) \left(-g^{\nu\sigma} + \frac{k^\nu k^\sigma}{M_W^2} \right) k^\lambda}{k^4 (k^2 - m_t^2)^2 (k^2 - M_W^2)^2} \quad (5.8)$$

Now there is a little bit of algebra to do with the Lorentz indices and the Dirac matrices. We take the numerator in the integral and we dot it into the fermion lines:

$$\gamma_\mu \gamma_\tau \gamma_\nu \frac{1-\gamma_5}{2} \otimes \gamma_\sigma \gamma_\lambda \gamma_\rho \frac{1-\gamma_5}{2} \left\{ g^{\mu\rho} g^{\nu\sigma} k^\tau k^\lambda - g^{\mu\rho} \frac{k^\tau k^\nu k^\sigma k^\lambda}{M_W^2} - \right. \\ \left. - g^{\nu\sigma} \frac{k^\tau k^\mu k^\rho k^\lambda}{M_W^2} + \frac{k^\tau k^\mu k^\rho k^\nu k^\sigma k^\lambda}{M_W^4} \right\} \quad (5.9)$$

where the symbol $\otimes$ separates the matrices from two different fermion lines. By using $\hat{k}\hat{k} = k^2$, $k^\tau k^\sigma = 1/4 g^{\mu\tau} k^2$ (because of symmetric integration in d^4k) and the identity:

$$\gamma_\mu \gamma_\tau \gamma_\nu \frac{1-\gamma_5}{2} \otimes \gamma^\nu \gamma^\tau \gamma^\mu \frac{1-\gamma_5}{2} = (g_{\mu\tau} \gamma_\nu - g_{\mu\nu} \gamma_\tau + \\ + g_{\tau\nu} \gamma_\mu + i \epsilon_{\mu\tau\nu\rho} \gamma^\rho) \frac{1-\gamma_5}{2} \otimes (g^{\mu\tau} \gamma^\nu - g^{\mu\nu} \gamma^\tau + \\ + g^{\tau\nu} \gamma^\mu - i \epsilon^{\mu\tau\nu\sigma} \gamma_\sigma) \frac{1-\gamma_5}{2} = 4 \gamma_\mu \frac{1-\gamma_5}{2} \otimes \gamma^\mu \frac{1-\gamma_5}{2} \quad (5.10)$$

we finally obtain

$$8\alpha x = \lambda_t^2 \bar{u}_d \gamma_\mu \frac{1-\gamma_5}{2} u_b \cdot \bar{u}_d \gamma^\mu \frac{1-\gamma_5}{2} u_b \left[I_2 - \frac{2I_4}{M_W^2} + \frac{I_6}{4M_W^4} \right] \quad (5.11)$$

where

$$I_{2,4,6} = m_t^4 \int d^4k \frac{k^2, k^4, k^6}{k^4 (k^2 - m_t^2)^2 (k^2 - M_W^2)^2} \quad (5.12)$$

We can set the following identities and definitions:

$$I_2 = m_t^4 \int d^4k \frac{1}{k^2 (k^2 - m_t^2)^2 (k^2 - M_W^2)^2} = \\ = m_t^4 \frac{\partial}{\partial m_t^2} \frac{\partial}{\partial M_W^2} \int d^4k \frac{1}{k^2 (k^2 - m_t^2) (k^2 - M_W^2)} = m_t^4 \frac{\partial}{\partial m_t^2} \frac{\partial}{\partial M_W^2} \mathcal{J} \quad (5.13)$$

$$I_4 = m_t^4 \int d^4k \frac{1}{(k^2 - m_t^2)^2 (k^2 - M_W^2)^2} = m_t^4 \frac{\partial}{\partial m_t^2} \int d^4k \frac{1}{(k^2 - m_t^2) (k^2 - M_W^2)^2} = \\ = m_t^4 \frac{\partial}{\partial m_t^2} \mathcal{J}_1 \quad (5.14)$$

$$I_6 = m_t^4 \int d^4 k \frac{k^2 - m_t^2 + m_t^2}{(k^2 - m_t^2)^2 (k^2 - M_W^2)^2} = m_t^4 J_1 + m_t^2 I_4 \quad (5.15)$$

In order to compute J and J₁ we use the famous identity:

$$\begin{aligned} \int d^4 k \frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)} &= \\ = 2 \int dx dy dz \delta(1-x-y-z) \int d^4 k \frac{1}{(k^2 - m_1^2 x - m_2^2 y - m_3^2 z)^3} \end{aligned} \quad (5.16)$$

Given the volume of the sphere of radius R in N dimensions

$$V_N = \frac{\pi^{N/2} R^N}{\Gamma(1+N/2)} \quad (5.17)$$

we obtain

$$V_4 = \pi^2 R^4 / 2 \rightarrow dV_4 = \pi^2 R^2 dR^2 \quad (5.18)$$

By a Wick rotation (which changes $k_0 \rightarrow ik_{0E}$) $\int d^4 k \rightarrow +i \int d^4 k_E = +i\pi^2 k_E^2 dk_E^2$ and one finally obtains:

$$\int \frac{d^4 k}{(k^2 - A^2)^3} = -i\pi^2 \int \frac{k_E^2 dk_E^2}{(k_E^2 + A^2)^3} = \frac{-i\pi^2}{2A^2} \quad (5.19)$$

From these relations the following table of integrals can easily be derived:

$$\begin{aligned} J &= \frac{+i\pi^2}{M_W^2} \frac{\ln \eta}{1-\eta} & J_1 &= \frac{-i\pi^2}{M_W^2} \frac{1}{1-\eta} \left(1 + \frac{\eta}{1-\eta} \ln \eta \right) \\ I_2 &= \frac{-i\pi^2}{M_W^2} \frac{\eta}{(1-\eta)^2} \left(1 + \eta + \frac{2\eta}{1-\eta} \ln \eta \right) \\ I_4 &= \frac{-i\pi^2}{(1-\eta)^2} \eta^2 \left(2 + \frac{1+\eta}{1-\eta} \ln \eta \right) \\ I_6 &= \frac{-i\pi^2 M_W^2 \eta^2}{(1-\eta)^2} \left(1 + \eta + \frac{2\eta}{1-\eta} \ln \eta \right) \end{aligned} \quad (5.20)$$

where

$$\eta = \frac{m_t^2}{M_W^2} \quad (5.21)$$

Finally,

$$\begin{aligned} I_2 - \frac{2I_4}{M_W^2} + \frac{I_6}{4M_W^4} &= \frac{i\pi^2}{M_W^2} \eta \left(\frac{1}{4} + \frac{9}{4} \frac{1}{1-\eta} - \frac{3}{2} \frac{1}{(1-\eta)^2} - \right. \\ &\quad \left. - \frac{3}{2} \frac{\eta^2 \ln \eta}{(1-\eta)^3} \right) \equiv \frac{i\pi^2}{M_W^2} A(\eta) \end{aligned} \quad (5.22)$$

By taking into account the overall factors left aside, the result can be cast into the form of an equivalent four fermion operator for the Hamiltonian:

$$H_{eff} \approx \left(\frac{g^2}{2}\right)^2 \frac{\lambda_t^2}{16\pi^4} \frac{\pi^2}{M_W^2} A(\eta) \frac{1}{2} \bar{d} \gamma_\mu \frac{1-\gamma_5}{2} b \bar{d} \gamma^\mu \frac{1-\gamma_5}{2} b \quad (5.23)$$

g is the SU(2) weak coupling ($g^2/8M_W^2 = G_F/\sqrt{2}$), $(2\pi)^{-4}$ is the factor associated with the loop integration, a factor $-i$ was dropped because $S \sim -iH$ where S is the S-matrix element, the spinors $\bar{v}_d, \dots$, have been replaced by the corresponding operator fields $\bar{d}, \dots$, and a factor of $\frac{1}{2}$ was added to compensate for exchange of the two identical $\bar{d} \gamma_\mu (1-\gamma_5)/2b$ factors. Finally, we can write the result in the form:

$$H_{eff} = \frac{G_F^2}{4\pi^2} m_t^2 \frac{A(\eta)}{\eta} \lambda_t^2 \bar{d} \gamma_\mu \frac{1-\gamma_5}{2} b \bar{d} \gamma^\mu \frac{1-\gamma_5}{2} b \quad (5.24)$$

Once the effective Hamiltonian has been written in operator form, there is no need of computing the second diagram. Note that, in fact, in the limit of vanishing external momenta the loop integrals are identical in the two diagrams. The contribution of the second diagram corresponds to a different contraction of the fields in the Hamiltonian with the external quarks. We will take both contributions into account when taking the matrix element $\langle B_0 | H_{eff} | B^0 \rangle$.

We now compute the above matrix element in the vacuum saturation (or valence) approximation. When the matrix element of a V-A current is taken between the vacuum and a pseudoscalar meson, only the axial current contributes (one cannot make a pseudovector out of only one momentum P_μ). For $\pi \rightarrow \mu\nu$ one defines:

$$\langle 0 | \bar{u} \gamma_\mu \gamma_5 d | \pi \rangle = \frac{i P_\mu f_\pi}{\sqrt{2} E_\pi} \quad (5.25)$$

With this normalization, experimentally $f_\pi \approx 130$ MeV (similarly $f_K \approx 160$ MeV). Thus, by restricting the sum over a complete set of intermediate states to the vacuum only, one obtains in the B^0 rest frame:

$$\langle B^0 | \bar{d} \gamma_\mu \gamma_5 d | B^0 \rangle = 2 \cdot \frac{1}{4} \left(1 + \frac{1}{3}\right) \frac{1}{2m_B} m_B^2 f_B^2 \quad (5.26)$$

We now explain one by one the factors which appear in the last expression. The factor of two is there because we can choose in two ways the current on the B^0 side (this factor compensates the factor of $\frac{1}{2}$ that we had inserted in the Hamiltonian). The factor of $1/4$ is there because each current contains the projector $(1-\gamma_5)/2$. The factor $4/3$ arises because the vacuum state can be inserted in two ways. This corresponds to the two original diagrams a and b in Fig. 1 which differ by the exchange of the two b fields. A (V-A) $\otimes$ (V-A) product of currents is invariant under Fierz rearrangement for colourless quarks. For coloured quarks the colour indices have also to be rearranged and one obtains: (see Eq. (3.3))

$$\begin{aligned} \bar{d}_{1L} \gamma_\mu b_{1L} \bar{d}_{2L} \gamma^\mu b_{2L} &= \frac{1}{3} \bar{d}_{1L} \gamma_\mu b_{2L} \bar{d}_{2L} \gamma^\mu b_{1L} + \\ &+ 2 \sum_A \bar{d}_{1L} \gamma_\mu \lambda^A b_{2L} \bar{d}_{2L} \gamma^\mu \lambda^A b_{1L} \end{aligned} \quad (5.27)$$

where λ_{ij}^A are the 3×3 colour generator matrices with normalization

$\text{Tr } \lambda^A \lambda^B = \frac{1}{2} \delta^{AB}$ [to derive Eq. (5.27) is a nice exercise for you to do]. The octet-octet term cannot contribute when each octet is sandwiched between the vacuum and the colour singlet B state. Thus the two contractions differing by $b_1 \leftrightarrow b_2$ would contribute $(1+1) = 2$ for colourless quarks and $(1+1/3) = 4/3$ for triplet coloured quarks. Thus for coloured quarks we set

$$\langle B^0 | (\bar{d} \gamma_\mu \frac{1-\gamma_5}{2} b)^2 | \bar{B}^0 \rangle = \frac{1}{3} B_B f_B^2 m_B^2 \quad (5.28)$$

where B_B is a factor which is inserted to take all possible deviations from the vacuum saturation approximation into account. Finally, we have

$$M_{12} = \frac{G_F^2 m_t^2}{12 \pi^2} B_B f_B^2 m_B \lambda_t^2 \frac{A(\eta)}{\eta} \quad (5.29)$$

$A(\eta)/\eta$ is a slowly decreasing monotonic function of η , plotted in Fig. 7, which is 1 at $\eta = 0$, $3/4$ at $\eta = 1$ and $1/4$ at $\eta \rightarrow \infty$.

For B mesons the top contribution is by far the only one that matters. For kaons the charm contribution, similar to Eq. (5.29), is sufficient to $\text{Re } M_{12}$, while for $\text{Im } M_{12}$ (which determines ϵ) one has also to compute the term with c and t quarks on the two sides of the box. The calculation is entirely analogous to the one performed here and the result can be found in the literature²⁶.

6. QCD CORRECTIONS TO BOX DIAGRAMS

The box diagram calculation of the last section is based on neglecting the strong interactions at short distances, which is valid if both M_W and the exchanged quark masses are large. The QCD corrections to the strong interaction-free result can be simply obtained in terms of the anomalous dimensions of the relevant operators in the short distance expansion. We shall now derive the result for $m_t < M_W$ and then comment on the case that $m_t > M_W$.

We start again from the general discussion in Section 2 of the two-step operator expansion. For $M_W \gg m_t$ the W propagators in Eq. (3.1) are only different from zero at short distances, so that the operator expansion for the product of two weak charged currents can be used²⁷ (including the effects of strong interactions):

$$J_\mu(z) J^\mu(0) \simeq C_+ O_+(0) + C_- O_-(0) + o\left(\frac{1}{M_W^2}\right) \quad (6.1)$$

where [recall Eq. (3.3)]

$$\begin{aligned} O_\pm &= \frac{1}{2} (\bar{d}_L \gamma_\mu t_L \bar{t}_L \gamma^\mu b_L \pm \bar{d}_L \gamma_\mu b_L \bar{t}_L \gamma^\mu t_L) = \\ &= \frac{1}{2} \left\{ (1 \pm \frac{1}{3}) \bar{d}_L \gamma_\mu t_L \bar{t}_L \gamma^\mu b_L \pm \right. \\ &\quad \left. \pm 2 \sum_A \bar{d}_L \gamma_\mu \lambda^A t_L \bar{t}_L \gamma^\mu \lambda^A b_L \right\} \end{aligned} \quad (6.2)$$

The second equality was obtained by Fierz rearrangement. $C_\pm$ are computable coefficients well known²⁷ from the physics of weak non-leptonic decays. At the leading logarithmic level:

Fig. 7 The function $A(\eta)/\eta$ ($\eta = m_E^2/M_W^2$) defined in Eq. (5.22).

$$c_{\pm} = \left[\frac{\alpha_s(\mu)}{\alpha_s(M_W)} \right]^{d_{\pm}} \quad \begin{cases} d_- = \frac{4}{b_0(f)} \\ d_+ = \frac{-2}{b_0(f)} \end{cases} \quad (6.3)$$

$$b_0(f) = 11 - \frac{2}{3}f$$

where α_s is the QCD running coupling and f is the number of excited flavours between μ and M_W (it can vary along the road, but we forget that for a moment). In the first step, when we shrink the W lines to a point we get:

$$JJJJ \rightarrow c_+^2 O_+ O_+ + 2c_+ c_- O_+ O_- + c_-^2 O_- O_- \quad (6.4)$$

Since $f = 6$ between m_t and M_W , we set $\mu = m_t$ and obtain ($b_0(6) = 7$):

$$c_+^2 \Rightarrow 2d_+ = -\frac{4}{7}, \quad c_+ c_- \Rightarrow d_+ + d_- = \frac{2}{7}, \quad c_-^2 \Rightarrow 2d_- = \frac{8}{7} \quad (6.5)$$

In the subsequent steps, we shrink the top quark lines to a point. We end up with a four-fermion operator of the form $\bar{d}_{LY\mu} b_L \bar{d}_{LY\mu} b_L$ or $\Sigma_A \bar{d}_{LY\mu} \lambda^{Ab} \bar{d}_{LY\mu} \lambda^{Ab}$ which can always be expressed in terms of $O_{\pm}$ [the symmetric and antisymmetric combinations completely analogous to Eq. (6.2)]. When finally the matrix element is taken, O_- does not contribute because the two modes of vacuum insertion cancel each other. Thus when going from m_t down to m_b , the relevant coefficient is c_+ (with $f = 5$, i.e., $b_0(5) = 23/3$). Up to now we have obtained the result

$$\eta_{QCD} \approx \left[\frac{\alpha_s(m_b)}{\alpha_s(m_t)} \right]^{-\frac{6}{23}} \left\{ a \left[\frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right]^{-\frac{4}{7}} + b \left[\frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right]^{\frac{2}{7}} + c \left[\frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right]^{\frac{8}{7}} \right\} \quad (6.6)$$

but the coefficients a , b and c remain to be determined. Of course, $a+b+c = 1$ because the curly bracket must approach 1 for $m_t \rightarrow M_W$ (no correction while running from M_W down to m_t if there is no distance!). a , b and c are obtained by colour traces. Before the top quark contractions we have (all indices are colour indices):

$$H \sim \bar{q}_a C_{ab} t_b \bar{t}_c C_{cd} q_d \bar{q}_s D_{st} t_t \bar{t}_u D_{uv} q_v \quad (6.7)$$

Here $C_{ab} C_{cd}$ ($D_{st} D_{uv}$) are the colour matrices in the two currents previously joined by a W line:

$$C_{ab} C_{cd} = C \otimes C = A \mathbb{1} \otimes \mathbb{1} + B \sum_A \lambda^A \otimes \lambda^A \quad (6.8)$$

C (or D) can correspond either to O_+ or to O_- . If $C \sim O_+$ then [see Eq. (6.2)] $A = 4/3$, $B = 2$ while for $C \sim O_-$ then $A = 2/3$, $B = -2$. We now contract t_b with $\bar{t}_u$ (giving δ_{bu}) and $\bar{t}_c$ with t_t (δ_{ct}) so that

$$H \sim \bar{q}_a (CD)_{av} q_v \bar{q}_s (DC)_{sd} q_d \quad (6.9)$$

Finally, we make the two possible projections over colour singlets by multiplying by either $\delta_{av} \delta_{sd}$ or $\delta_{ad} \delta_{vs}$ which leads to $\text{Tr} CD \text{Tr} DC + \text{Tr} CDDC$. If we denote by A' and B' the analogous coefficients for $D \otimes D$ of A and B for $C \otimes C$ [defined in Eq. (6.8)] we obtain:

$$M_{12} \simeq 12AA' + 4(BA' + AB') + \frac{22}{3} BB' \quad (6.10)$$

This leads to:

$$O_+ O_+ : O_+ O_- + O_- O_+ : O_- O_- = \frac{3}{2} : -1 : \frac{1}{2} \quad (6.11)$$

Thus with the right normalization:

$$\eta_{\text{QCD}} = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_t)} \right]^{-\frac{6}{23}} \left\{ \frac{3}{2} \left[\frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right]^{-\frac{4}{7}} - \left[\frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right]^{\frac{2}{7}} + \frac{1}{2} \left[\frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right]^{\frac{8}{7}} \right\} \quad (6.12)$$

This formula has been derived for $m_b \ll m_t \ll M_W$. Strictly speaking, perturbative QCD with a given number of massless flavours can only be applied far away from quark thresholds. In the regions across the thresholds we can only guess reasonable extrapolations. Note that in Eq. (6.12) for η_{QCD} , α_s runs with five flavours in the first factor (between m_b and m_t) and with six flavours in the second factor (between m_t and M_W). For example, we can impose that α_s is continuous in the two regions by setting

$$\alpha_s^{f=5}(Q^2) = 4\pi \left[b_0(5) \ln \frac{Q^2}{\Lambda^2} \right]^{-1}$$

$$\alpha_s^{f=6}(Q^2) = 4\pi \left[b_0(6) \ln \frac{Q^2}{\Lambda^2} + C \right]^{-1} \quad (6.13)$$

where C is fixed by $\alpha_s^{f=5}(m_t) = \alpha_s^{f=6}(m_t)$. This amounts to change Λ for each f . We put C in $\alpha_s^{f=6}$ so that our Λ is Λ (leading order) for $f = 5$ which is what we can measure below the top threshold.

When m_t increases up to M_W and beyond (not too much beyond because we know that $m_t \lesssim 2 \div 3 M_W$), the interplay between the logarithmic terms for the anomalous dimensions and the mass terms [the function $A(\eta)/\eta$ in Eq. (5.29)] becomes more complicated. The logs resummed by η_{QCD} are still there. The expression for η_{QCD} will change a bit but not much (for example, the running will all the way occur with $f = 5$). However, the function $A(\eta)/\eta$ could be deformed by additional QCD corrections because η_{QCD} is the right multiplying factor in the limit $\eta \rightarrow 0$. However, since $A(\eta)/\eta$ is a slowly varying function and η_{QCD} is almost a constant ($\eta_{\text{QCD}} \sim 0.80-0.85$) for $m_t \approx 40 \div 200$ GeV, it is presumably correct to use the product $\eta_{\text{QCD}}(A(\eta)/\eta)$ for the whole physically interesting range of m_t .

7. $B^0 - \bar{B}^0$ MIXING IN THE STANDARD MODEL

By collecting Eqs. (3.9) and (5.29), we finally obtain the top exchange contribution to $\Delta m/\Gamma$ for a B_q meson ($q = d, s$):

$$x_q \equiv \left(\frac{\Delta m}{\Gamma} \right)_{B_q} = \frac{G_F^2}{6\kappa^2} m_t^2 \tau_{B_q} B_{B_q} f_{B_q}^2 m_{B_q} |V_{tb} V_{tq}^*|^2 \frac{A(\eta)}{\eta} \eta_{QCD} \quad (7.1)$$

where $A(\eta)/\eta$ and η_{QCD} are given in Eqs. (5.22) and (6.12) respectively. Recall that [Eq. (3.5)] $r_q = x_q^2/(2+x_q^2)$. The first thing to observe is that the ratio x_d/x_s can be predicted with little ambiguity. From Eq. (7.1), one in fact obtains:

$$\frac{x_d}{x_s} = \left| \frac{V_{td}}{V_{ts}} \right|^2 (1 + 5u(3)_{\text{flavor breaking}}) \quad (7.2)$$

where the $d \leftrightarrow s$ symmetry breaking correction arises from all quantities which in Eq. (7.1) carry a q label, apart from the mixing angles which are explicitly factored out. Although the corrective term may well be sizeable (and is probably negative), it is safe to state that the main factor in Eq. (7.2) is the ratio $|V_{td}/V_{ts}|^2 \lesssim 1/5$. Namely x_d is Cabibbo suppressed with respect to x_s (in fact, the present bound roughly corresponds to $|V_{td}| \lesssim 2\theta_c |V_{ts}|$). This is why r_d was expected to be small. Then from the experimental value of r_d , one immediately deduces that r_s must be near one. Precisely $r_d > 0.13$ implies $r_s \gtrsim 0.75-0.80$ depending on the precise input maximum value of $|V_{td}/V_{ts}|$. The resulting allowed region in the r_d, r_s plane is plotted in Fig. 8 together with the available experimental information. One sees that the MARK II limits²⁸ are potentially dangerous for the Standard Model with three families. However, the interpretation of the MARK II results in terms of r_d, r_s requires an assumption on the probabilities P_d, P_s that the produced b quark picks up a $\bar{d}$ or $\bar{s}$ companion. For example the allowed region for the Standard Model with three families is not much restricted by the MARK II result if $P_d \approx 0.35, P_s \approx 0.10$ (which, however, leaves a rather generous 20% for $b\bar{c}$ and b -carrying baryons).

The value of x_d is more uncertain. One can write the approximate expressions valid for $m_t \lesssim M_W$:

$$x_d \approx 0.15 \left[\frac{\tau_B |V_{td}|^2}{3.3 \cdot 10^{-16} \text{ s}} \right] \left[\frac{B_{B_d} f_{B_d}^2}{(0.14 \text{ GeV})^2} \right] \left[\frac{m_t}{40 \text{ GeV}} \right]^2 \quad (7.3)$$

or

$$x_d = (0.26 \pm 0.05) \frac{1}{4} (1 + \rho^2 - 2\rho \cos \phi) \left[\frac{B_{B_d} f_{B_d}^2}{(0.14 \text{ GeV})^2} \right] \left[\frac{m_t}{40 \text{ GeV}} \right]^2 \quad (7.4)$$

Recall that experimentally ARGUS finds: $x_d = 0.73 \pm 0.18$. The main unknowns are $\tau_B |V_{td}|^2, B_{B_d}^2 f_{B_d}^2$ and m_t . The errors on τ_B and $|V_{td}|$ are related so that the two factors should be kept together. $|V_{td}|$ is maximum for an extremal value of the Kobayashi-Maskawa phase. However, at extremal values of the phase all CP violation effects vanish. The experimental values of ϵ and ϵ'/ϵ therefore impose some constraints on how large $|V_{td}|$ can be. Only mild restrictions are found (which become weaker with increasing m_t) if one takes $B_K \lesssim 1$ and $|V_{bu}/V_{bc}| \lesssim 0.2$ (see

Fig. 8 The experimentally allowed region¹⁰ in the x_d - x_s plane (χ is defined in Eq. (2.19) and the CKM model constraint. The shaded area is the region allowed by the present experiments and the CKM model.

Figs. 5 and 6). With these assumptions one finds for $\tau_B |V_{td}|^2$ the range⁷

$$\tau_B |V_{td}|^2 \approx (0.02 \div 6.2) 10^{-16} \text{ s} \quad (7.5)$$

The factor $B_B f_B^2$ contains all the uncertainties arising from the hadronic matrix element. The parameter B should approach one when the mass of the meson increases. In fact, the vacuum saturation approximation should be better when gluon emission from the quark legs is inhibited by large transferred momenta of order the meson mass (Zweig rule). This trend appears to be supported by some recent lattice evaluations of B_D and B_B ^{29,30}. The corresponding results are closer to unity than those obtained for B_K by several methods. The most reliable results on B are collected in Table 1^{29,33}. Similarly the pseudo-scalar decay constants f_K , f_D , f_B can be estimated mainly by QCD sum rules and lattice calculations. The available results are collected in Table 2^{34,40}. The tentative conclusion that was derived in Ref. 7 is given by:

$$B_B^{1/2} f_B = (140 \pm 40) \text{ MeV} \quad (7.6)$$

The implications on m_t are illustrated in Figs. 9 and 10⁷. One obtains that almost certainly $m_t > 45 \text{ GeV}$. The most likely range for m_t is given by $90 \text{ GeV} < m_t < 150 \text{ GeV}$.

In conclusion the ARGUS result can be accommodated in the Standard Model with three families provided that m_t is large. The first important experimental check is to measure the mixing parameter r_s for the B_s mesons. If r_s is not nearly maximal, the most obvious possibility is a fourth family.

It is interesting to remark that in the presence of a fourth generation, because of the more relaxed unitarity constraints on the CKM matrix elements, even the top quark contribution by itself (i.e., without invoking the additional terms from the new up-type quark t') can lead to drastically different values of both the ratio x_d/x_s and the quantity x_d . For example, the analysis of mixing angles in Ref. 41 leads to $V_{td} \lesssim 0.017$ [note that Eq. (7.5) corresponds to $V_{td} \lesssim 0.026$] for three generations. For four generations, their bound on V_{td} is relaxed by almost one order of magnitude -- $V_{td} \lesssim 0.15$ -- which implies that practically all values of r_d are possible.

Before closing this section, it is important to recall that for kaons the short distance contribution to Δm_K (which essentially arises from c quark exchange) is not sufficient to explain the observed value if $B_K \lesssim 1$. With very good approximation:

$$\frac{\Delta m_K(\text{box})}{\Delta m_K(\text{exp})} = 0.7 \left[\frac{m_c}{1.5 \text{ GeV}} \right]^2 B_K \quad (7.7)$$

Clearly, the range of relevant momenta between m_K and m_c is too low for a neglect of long distance effects. The situation is different for $\text{Im } M_{12}$, i.e., for ϵ . The CKM phases are only present in the heavy quark sector. The t quark exchange is also important. Thus in this case the range of relevant virtual momenta is between m_c and m_t and the dominance of short range effects is more plausible.

TABLE 1

B_K	
0.33 ± 0.2	SU(3)+chiral inv.+experiment [31]
0.33 ± 0.09	QCD sum rules [32]
$0.5 \pm \begin{smallmatrix} 0.37 \\ 0.17 \end{smallmatrix}$	Lattice QCD [29]
0.70 ± 0.07	1/N expansion [33]
B_D	
$1 \pm \begin{smallmatrix} 0.25 \\ 0.15 \end{smallmatrix}$	Lattice QCD [29]
0.98 ± 0.25	Lattice QCD [30]
B_{B_d}	
0.98 ± 0.25	Lattice QCD [30]

TABLE 2

$$f_\pi = 130 \text{ MeV}, f_K = 160 \text{ MeV}$$

f_D	
$\sim 220 \text{ (MeV)}$	QCD sum rules, Ref. [34]
~ 165	" Ref. [35]
~ 170	" Ref. [36]
170 ± 20	" Ref. [37]
220 ± 25	" Ref. [38]
180 ± 25	Lattice QCD, Ref. [29]
128 ± 25	" " Ref. [30]
< 290	Exp. MARK III, Ref. [39]
f_B	
~ 140	QCD sum rules, Ref. [34]
~ 95	" Ref. [35]
~ 130	" Ref. [36]
190 ± 30	" Ref. [40]
180 ± 20	" Ref. [37]
175 ± 30	" Ref. [38]

Fig. 9 Predicted values of $B_d^0 - \bar{B}_d^0$ mixing in the standard model with three fermion families, as a function of the top quark mass, for different choices of the relevant parameters⁷. The ARGUS [Eq. (1.1)] for r_d is also shown, which translates to $x_d = 0.73 \pm 0.18$ by the relation $r_d = x_d^2 / (2 + x_d^2)$. All curves are for $\rho < 0.9$ [or $\Gamma(b-u)/\Gamma(b-c) < 0.08$] where ρ is defined by $\lambda\rho = |V_{bu}|/|V_{bc}|$ with $\lambda \approx 0.221$. In the curves (a)-(e) we use the following values of the parameters $F = B_B^2 f_B$, $\cos\phi$ and $T = \tau_B |V_{bc}|^2 / 10^{-15} s$:

- a. $F = 180$ MeV, $T = 3.5$, and $\cos\phi = (\cos\phi)_{\min}$ (the largest negative value of $\cos\phi$ allowed by the $|\epsilon|$ constraint). This curve gives the lower bound $m_t > 45$ GeV quoted in the text.
- b. $F = 140$ MeV, $T = 3.5$, and $\cos\phi = (\cos\phi)_{\min}$.
- c. $F = 140$ MeV, $T = 2.9$ and $\cos\phi = 0$. This curve corresponds to about the central prediction of the standard model, and leads to the statement in the text that probably $m_t > 90$ GeV.
- d. $F = 100$ MeV, $T = 2.3$ and $\cos\phi = (\cos\phi)_{\max}$ (the largest positive value of $\cos\phi$ allowed by the $|\epsilon|$ constraint).
- e. $F = 100$ MeV, $T = 3.5$ and $\cos\phi = (\cos\phi)_{\max}$.

Fig. 10 Maximum value of x_d compatible with the standard model with three fermion families as a function of the top quark mass, for different values of ρ ⁷. ρ is related to $R = \Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ ($\rho = 0.9, 0.6$ and 0.3 correspond to $R \approx 0.08, 0.04$ and 0.009). The $\rho = 0.9$ curve corresponds to curve (a) of Fig. 9, i.e., $B_B^{\frac{1}{2}f_B} = 180$ MeV, $\tau_B |V_{bc}|^2 = 3.5 \times 10^{-15} \text{s}$, and $\cos\phi = (\cos\phi)_{\min}$ is the appropriate minimum value allowed by the $|\epsilon|$ constraints, which changes with ρ .

8. $B^0 \bar{B}^0$ MIXING BEYOND THE STANDARD MODEL

In this section we consider the effect on $B^0 - \bar{B}^0$ mixing (and in general on flavour mixing) of a number of simple generalizations of the standard model. We will discuss in detail the case of supersymmetry, charged Higgses, left-right models⁷. In all the generalizations of the standard model considered in the following the CKM suppression of r_d versus r_s is maintained. Thus the prediction of r_s near unity is not altered in these minimal models of new physics. This is a reflection of the fact that the experimental absence of flavour changing neutral currents imposes really stringent constraints. Thus all reasonable models are constructed in such a way as to preserve the validity of the GIM mechanism and of the CKM hierarchy of couplings.

We first consider minimal models of supersymmetry broken softly by gravity⁴². The dominant additional contribution to flavour mixing from virtual exchange of supersymmetric particles arises from box diagrams with gluino and down squark exchange, as shown in Fig. 11a. In fact, it is well known⁴² that flavour changing couplings $\tilde{g}-q-\tilde{q}$ (where $\tilde{g}$ and $\tilde{q}$ denote gluinos and squarks) are induced by charged Higgsino exchange in (left-handed) down squark self mass diagrams (Fig. 11b). The corresponding down squark mass squared matrix acquires a component proportional to M_u^2 :

$$m_{\tilde{d}_L}^2 = m^2 11 + M_d^2 - |c| V^\dagger M_u^2 V \quad (8.1)$$

where $M_{d,u}$ are the down, up quark mass matrices. We are systematically neglecting non-diagonal terms in the $\tilde{d}$ mass squared matrix connecting left and right squarks, since these presumably small effects can well be absorbed, at this exploratory state, in our ignorance of the other parameters. The parameter $|c|$ plays a crucial role in the following in that the interesting effect vanishes for $c = 0$ and increases rapidly with $|c|$. c is related to the Yukawa couplings of the Higgs particles and is expected to be of $O(1)$. The really relevant parameter is not c but $cm_t^2/m_{\tilde{d}}^2$. However, for $m_t \sim m_{\tilde{q}}$ in various simple models⁴², $|c|$ is found to be in the range 0.1 to 1. But there are no stringent arguments that prevent somewhat larger values of $|c|$. Thus, it appears reasonable to set $|c| = 1$ if we are interested in obtaining an estimate of how large the supersymmetric contribution can be at most for a given value of m_t . Results for $|c| = \frac{1}{2}$ will also be given.

The gluino box diagrams lead to the following expressions⁴² for ΔM_{ab} (QCD correction factors are not included at this stage):

$$\Delta M_{ab}^{\tilde{g}} = \frac{\alpha_s^2}{54 m_{\tilde{g}}^2} B_B f_B^2 m_B \sum_{ij} \lambda_{ab}^i \lambda_{ab}^j S_{ij}(z_i, z_j) \quad (8.2)$$

Here α_s is the QCD coupling at a scale of order $m_{\tilde{q}} \approx M_W$. We set in the following $\alpha_s \approx 0.12$. $\lambda_{ab}^i = V_{ia}^* V_{ib}$ and the index i can be u, c or t . $S_{ij}(z_i, z_j)$ is a kernel given by

$$S_{ij} = 11 K_{ij} + 4 I_{ij} \quad (8.3)$$

with

$$K_{ij} = \frac{1}{z_i - z_j} \left[\frac{z_i^2 \ln z_i}{(1 - z_i)^2} + \frac{1}{1 - z_i} - (i \leftrightarrow j) \right]$$

$$I_{ij} = \frac{1}{z_i - z_j} \left[\frac{z_i \ln z_i}{(1 - z_i)^2} + \frac{1}{1 - z_i} - (i \leftrightarrow j) \right] \quad (8.4)$$

(a)

(b)

Fig. 11 a) Box diagram with gluino and squark exchange.
b) Charged Higgsino and top quark exchange contributing to flavour non-diagonal b to d (left-handed) squark mass matrix elements.

where $z_i = m_{\tilde{d}_i}^2 / m_{\tilde{g}}^2$ ($\tilde{d}_i = \tilde{d}, \tilde{s}, \tilde{b}$ are the down type squarks). In the excellent approximation (0.1%) of neglecting all quark masses except m_t , one has $m_{\tilde{d}}^2 = m_{\tilde{s}}^2 = m_{\tilde{b}}^2 + |c|m_t^2$. Then, by using the unitarity of the CKM matrix, one can reduce Eq. (8.2) to the simpler, more explicit form:

$$x_{\tilde{d}}^{\tilde{g}} = \frac{\alpha_s^2}{54m_{\tilde{g}}^2} \tau_{B_d} B_{B_d} f_{B_d}^2 |V_{tb} V_{td}^*|^2 \Delta S_t(m_{\tilde{g}}, m_{\tilde{b}}, m_{\tilde{g}}) \quad (8.5)$$

where

$$\Delta S_t = S_{33}(z_3, z_3) + S_{11}(z_1, z_1) - 2S_{13}(z_1, z_3) \quad (8.6)$$

Other supersymmetric contributions arising from wino or Higgsino exchange are much suppressed, since α_s is relatively large and up squarks are nearly degenerate in mass.

In order to obtain numerical results, we need to input values for the gluino and squark masses. Taking into account the UAI limits as presented in Ref. 43, we consider $m_{\tilde{g}} \gtrsim 55-60$ GeV and $m_{\tilde{q}} \gtrsim 60-70$ GeV. In Figs. 12 we plot⁷, for different values of the relevant parameters, the ratio $x^{\text{TOT}}/x^{\text{ST}}$ of the total resulting $x^{\text{TOT}} = x^{\text{ST}} + x_{\tilde{g}}^{\tilde{g}}$ for the B_d^0, s mesons and the corresponding standard model contribution x^{ST} given in Eq. (7.4). This ratio is particularly interesting because it is independent of the values of CKM angles, of $B_B f_B^2$ and of τ_B . We see that the contribution of gluino box diagrams can indeed be important as r_d grows quadratically with x_d at small x_d . It is to be stressed, however, that the supersymmetric effect drops very rapidly with $m_{\tilde{g}}$ and $m_{\tilde{q}}$. The dependence on $|c|$ is also very sharp (the ratio $x_{\tilde{g}}^{\tilde{g}}/x^{\text{ST}}$ is approximately of the form $R = |c| f(|c|m_t^2)$). Note that the requirement of positivity of $m_{\tilde{b}}^2 = m_{\tilde{d}}^2 - |c|m_t^2$ implies an upper bound on m_t for given values of $m_{\tilde{d}}^2$ and $|c|$. The drop at large m_t of the supersymmetric contribution visible in Figs. 12 is due to this upper bound.

The effects of supersymmetry on $B^0 - \bar{B}^0$ mixing were also studied in Ref. 44. Unfortunately, their numerical results are marred by a sign error. Their Eq. (5) should be changed of sign on the right-hand side. As a consequence, in their Eq. (10) the coefficient $-16/27$ becomes $-8/27$ (and $+16/9$ becomes $-4/9$) so that the result given here and in Refs. 42 is reproduced. Once this error is corrected, we agree with them when the small values of c they use are taken into account.

In conclusion, supersymmetry could in principle explain the large value of r_d at moderate values of m_t , but only if gluino and squark masses are close to the present experimental bounds. On the other hand, we have checked that in all cases studied here the supersymmetric contribution to $M_{K_L} - M_{K_S}$ is completely negligible. Also, in the presence of supersymmetric effects, the constraints arising from the observed value of $|\epsilon|$ are further relaxed.

It is interesting to recall that the same flavour changing mechanism that contributes to $B^0 - \bar{B}^0$ mixing can lead to sizeable branching ratios for decay rates of the types $b \rightarrow s\gamma$ or $b \rightarrow sg$ which could be large enough to be observed, as discussed in Refs. 45.

Fig. 12 The ratio $x^{\text{TOT}}/x^{\text{ST}}$ for B_d^0 or B_s^0 mesons, where $x^{\text{TOT}} = x^{\text{ST}} + x^{\tilde{g}}$ is the sum of the standard term, Eq. (7.1), plus the supersymmetric contribution, given in Eq. (8.5). The solid lines correspond to $|c| = 1$, while the dashed line is computed for $|c| = \frac{1}{2}$, where $|c|$ is defined in Eq. (8.1). The gluino and down squark masses (in GeV) are indicated on the curves⁷.

We now consider the contribution of charged Higgses. In all the models based on supersymmetry, physical charged Higgs bosons are also predicted. Additional box diagrams contributions to $B^0-\bar{B}^0$ mixing obtained by replacing one or both of the W lines by virtual charged Higgses are therefore also present. We shall now discuss these terms briefly. Of course, charged Higgs contributions to $B^0-\bar{B}^0$ mixing can occur independently of supersymmetry, so that the following discussion is of more general validity. In Refs. 46,47 the box diagrams with charged Higgs exchange were computed, and their implications for $B^0-\bar{B}^0$ mixing were discussed. In a model with two Higgs doublets, the relevant couplings of charged Higgses to quarks are given by

$$H \approx \frac{g_2 H^\dagger}{2\sqrt{2} M_W} \bar{u}_R \frac{v_d}{v_u} M_u V d_L + \dots \quad (8.7)$$

where u and d indicate up- and down-type quarks respectively, g_2 is the SU(2) weak gauge coupling, M_u is the diagonal up quark mass matrix, and V is the CKM matrix. v_d and v_u are the Higgs vacuum expectation values of the Higgs doublets giving mass to the down-type and up-type quarks respectively; they are proportional to the down and up quark masses via the corresponding Yukawa coupling constants. The ellipsis indicates terms proportional to the down quark mass matrix which can be neglected in comparison with the top quark contribution. The exact result for x_d can be obtained from Ref. 46. An approximate form (adequate for the scope of the present discussion) of the correction to the standard result, due to charged Higgs exchange, is given by Ref. 47

$$x_d \approx x_d^{ST} \left[1 + \frac{1}{4} \left(\frac{v_d}{v_u} \right)^4 \left(\frac{m_t}{m_H} \right)^2 \right] \quad (8.8)$$

The limits of validity of this simple approximation obtained for $m_t \ll m_H$, and neglecting the box diagram with one Higgs and one W propagator, are discussed in Ref. 46. For $m_t \lesssim m_H$, the exact expression gives x_d larger, by about 10% or less. One sees, for example, that for $m_H \sim M_W$ and m_t as low as $m_t \sim 30$ to 40 GeV a ratio $v_d/v_u \sim 2$ to 4 is sufficient to yield a possible explanation for the observed value of r_d . Such values for the ratio of vacuum expectation values are perfectly admissible (see Ref. 46 for current limits). However, we note that the perhaps appealing requirement of using the duplication of doublets to decrease the differences among the Yukawa couplings of Higgs bosons to quarks would lead to $v_d/v_u < 1$ since up quarks are on the average heavier than down quarks. Also in minimal models of supersymmetry broken by gravity $v_u > v_d$ as a consequence of the running of the Yukawa up-couplings on the way from M_{Pl} down to M_W induced by the relatively heavy top quark.

We now review $B^0-\bar{B}^0$ mixing in the context of minimal left-right symmetric models⁴⁸ as discussed in Refs. 49 and 50 and add some observations. By minimal models we mean both a single multiplet of Higgs bosons coupled to quarks (transforming as $(\frac{1}{2}, \frac{1}{2}, 0)$ of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ and spontaneous CP violation with identical or complex conjugate CKM matrices for left-handed and right-handed quarks. It is well known that the contribution to the K_L-K_S mass difference arising from (right-handed) W_R exchange in the box diagrams are much larger than for the ordinary (left-handed) W_L of the same mass⁵⁰. Requiring that the short-distance contribution to Δm_K does not exceed the experimental value leads in fact to a strong limit on the W_R mass: $M_{W_R} > 1.5$ to 2.5 TeV. There are also additional contributions to Δm_K arising from tree-level diagrams with flavour

changing Higgs exchange, with masses M_H order M_{W_R} . Thus a lower limit on M_H of several TeV can be derived.

In the approximation of neglecting the small mixing ξ_{LR} between left and right components in the charged weak boson mass matrix, the charged current term in the Hamiltonian can be written

$$H = \frac{g_2}{\sqrt{2}} \left\{ W_L^{+\mu} \bar{u} V_L \gamma_\mu \frac{1-\gamma_5}{2} d + W_R^{+\mu} \bar{u} V_R \gamma_\mu \frac{1+\gamma_5}{2} d \right\} + h.c. \quad (8.9)$$

where V_L and V_R are the CKM matrices for left- and right-handed quarks respectively, which we assume to be the same in the following. The total resulting box diagram contributions to $x_{d,s}$, including the standard contribution x_{LL}^{LL} from W_L exchange plus the W_R and unphysical scalar exchange terms, is given by:

$$x_{d,s}^{\text{Tot}} = x_{d,s}^{LL} \left\{ 1 + \frac{3}{2} \frac{M_{W_L}^2}{M_{W_R}^2} \left(\frac{m_{B_{d,s}}^2}{(m_b + m_{d,s})^2} + \frac{1}{6} \right) \frac{1}{\eta_{\text{QCD}} A(\eta)} \right. \\ \left. \cdot \left[\eta_{t1}^{LR} \left(\frac{4-\eta}{1-\eta} + \frac{4-2\eta+\eta^2}{(1-\eta)^2} \ln \eta \right) + \eta_{t2}^{LR} \ln \frac{M_{W_L}^2}{M_{W_R}^2} \right] \right\} \quad (8.10)$$

Here η_{QCD} , η and $A(\eta)/\eta$ are defined in Eqs. (6.12), (5.21) and (5.22) and $\eta_{t1}^{LR} \approx \eta_{t2}^{LR} \approx 1.8$ are additional QCD correction factors. From the above formula, one can immediately derive the important results that

a) the amount of $B^0 - \bar{B}^0$ mixing is not very much altered with respect to the standard model result, once the constraints on M_{W_R} imposed by the observed value of Δm_K are taken into account.

b) The sign of the effect for $m_t \lesssim 250$ GeV goes in the direction of reducing the amount of mixing with respect to the standard model if one assumes that, after diagonalization of the mass matrix, all right-handed quark mass eigenvalues are positive in the CKM phase choice (as discussed in Ref. 49 this is not really necessary).

c) The absolute value of the correction slowly decreases with m_t .

In fact, compared with the $K^0 - \bar{K}^0$ system, the importance of the diagrams with left-right exchange is much decreased because $(m_B^2/(m_b + m_{d,s})^2) \sim 1$ is small in comparison with $(m_K^2/(m_s + m_d)^2)$. Moreover, the dominant contribution for the $K^0 - \bar{K}^0$ system arises from charm exchange. Then $\ln \eta \rightarrow \ln m_c^2/m_W^2$ in Eq. (8.10) and the corresponding term is quite large. As a consequence, for values of M_{W_R} large enough to be compatible with Δm_K , the corresponding correction for $B^0 - \bar{B}^0$ mixing is small.

Similarly, the Higgs exchange term is also negative, and smaller than for the $K^0 - \bar{K}^0$ system. Neglecting QCD corrections, one obtains:

$$X_{d,s}^{\text{Higgs}} = -X_{d,s}^{\text{LL}} \frac{\sqrt{2} 6\pi^2}{G_F m_H^2} \left(\frac{m_{B_{d,s}}^2}{(m_b + m_{d,s})^2} + \frac{1}{6} \right) \frac{\eta}{A|\eta|} \quad (8.11)$$

Since the additional logarithmic enhancement due to the relative smallness of the charm mass is not present in this case, the absolute value of the Higgs correction can be larger than that arising from the box diagrams.

9. CONCLUSION

The ARGUS result on $B_d^0 - \bar{B}_d^0$ mixing can be accommodated in the standard model with three families provided that m_t is large. The first important experimental check is to measure the mixing parameter r_s for the B_s^0 mesons. If r_s is not nearly maximal, the most obvious possibility is a fourth family. On the other hand, simplest extensions of the standard model tend to preserve the Kobayashi-Maskawa pattern. Thus if r_s is near one, then lower values of m_t can be allowed as an effect of charged Higgs exchange or of supersymmetry. In the former case, one needs relatively light charged-Higgs bosons and a ratio of vacuum expectation values $v_d/v_u > 1$ (which is perhaps counter-intuitive and not what minimal supersymmetry would predict), where $v_{d,u}$ are the vacuum expectation values of the Higgs doublet giving masses to the d or u type of quarks respectively. In the latter case, the supersymmetric contribution can only be important if the parameters are somewhat stretched to the most favourable situation. In particular squarks and gluinos should have relatively small masses, close to the experimental lower bounds. Finally, minimal left-right-symmetric models do not lead to important effects for $B^0 - \bar{B}^0$ mixing if the bounds from kaon mixing are taken into account.

REFERENCES

1. H. Albrecht et al. (ARGUS Collaboration), DESY Report DESY-87-029 (1987).
2. C. Albajar et al. (UA1 Collaboration), Phys. Lett. 186B (1987) 247.
3. I. Mannelli, Proceedings of the 1987 Int. Symposium on Lepton and Photon Interactions at High Energies, Hamburg (1987).
4. S.-L. Wu, these proceedings.
5. J.S. Hagelin, Nucl. Phys. B193 (1981) 123;
I.I. Bigi and A.I. Sanda, Phys. Rev. D29 (1984) 1393;
V. Barger and R.J.N. Phillips, Phys. Lett. 143B (1984) 259;
L.-L. Chau and W.Y. Keung, Phys. Rev. D29 (1984) 592;
E.A. Paschos and U. Türke, Nucl. Phys. B243 (1984) 29;
A. Ali, DESY Reports 85-107 (1985) and 86-108 (1986);
A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. B245 (1984) 369;
For a review, see also:
L.-L. Chau, Physics Reports 95 (1983) 1.
6. J. Ellis, J.S. Hagelin and S. Rudaz, CERN preprint TH.4679 (1987);
L.-L. Chau and W.Y. Keung, Univ. of California at Davis Report UCD-87-02 (1987);
A. Datta, E.A. Paschos and U. Türke, Univ. of Dortmund Report DO-TH-87/9 (1987);
D.F. Donoghue et al., SIN-PR-87-05 (1987);
H. Harari and Y. Nir, Stanford Report SLAC-PUB-4341 (1987);
W.S. Hou and A. Soni, Univ. of Pittsburgh Report PITT-87-06 (1987);
L. Angelini et al., Univ. of Bari Report BARI-GT/2-87 (1987);
V.A. Khoze and N.G. Ural'tsev, Leningrad Report 1290 (1987).
7. G. Altarelli and P. Franzini, CERN preprint TH.4745 (1987).
8. J. Maalampi and M. Roos, Univ. of Helsinki Report HU-TFT-87-12 (1987).
9. V. Barger, T. Han, D.V. Nanopoulos and R.J.N. Phillips, Univ. of Wisconsin Report MAD/TH/87-12 (1987);
J.R. Cudell et al., Univ. of Wisconsin Report MAD/PH/352 (1987).
10. A. Ali, Proceedings of the UCLA Workshop for a $B\bar{B}$ factory, DESY Report (1987).
11. See, for example:
R.L. Kingsley, Phys. Lett. 63B (1976) 329;
I. Bigi and A.I. Sanda, Phys. Lett. B171 (1986) 320.
12. S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.
13. M.K. Gaillard and B.W. Lee, Phys. Rev. D10 (1975) 897.
14. For a recent review of CP violation, see:
J.F. Donoghue, B.R. Holstein and G. Valencia, Int. Journal of Mod. Phys. 2 (1987) 319.
15. L. Maiani, Proceedings of the Int. Symposium on Lepton and Photon Interactions at High Energies, Hamburg (1977).

16. T. Kobayashi and M. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
17. L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.
18. Particle Data Group, Phys. Lett. 170B (1986) 1.
19. G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani and G. Martinelli, Nucl. Phys. B208 (1982) 365.
20. D.M. Ritson, Proceedings of the 23rd Int. Conference on High Energy Physics, Berkeley (1986).
21. E.W. Thorndike and R.A. Poling, University of Rochester preprint, UR-1022 (1987), submitted to Physics Reports.
22. W. Schmidt-Parzefall, Proceedings of the Int. Symposium on Lepton and Photon Interactions at High Energies, Hamburg (1987).
23. S. Behrends et al., (CLEO Collaboration), Phys. Rev. Lett. 59 (1987) 407.
24. B. Grinstein, M.B. Wise and N. Isgur, Phys. Rev. Lett. 56 (1986) 298.
25. See, for example;
K. Schubert, Proceedings of the EPS Conference on High Energy Physics, Uppsala, Sweden (1987).
26. T. Inami and C.S. Lim, Prog. Theor. Physics 65 (1981) 297.
27. M.K. Gaillard and B.W. Lee, Phys. Rev. Lett. 33 (1974) 108;
G. Altarelli and L. Maiani, Phys. Lett. B52 (1974) 351.
28. T. Schaad et al. (MARK II Collaboration), Phys. Lett. 160B (1985) 188.
29. M.B. Gavela et al., presented by O. Pène at the Proceedings of the EPS Int. Conference on High Energy Physics, Uppsala, Sweden (1987).
30. C. Bernard et al., private communication by A. Soni.
31. J.F. Donoghue, E. Golowich and B.R. Holstein, Phys. Lett. 119B (1982) 412.
32. A. Pich and E. de Rafael, Phys. Lett. 158B (1985) 477.
33. W.A. Bardeen, A.J. Buras and J.-M. Gérard, MPI Report MPI-PAE(PTH 22/87 (1987)).
34. E.V. Shuryak, Nucl. Phys. B198 (1982) 83.
35. V.L. Chernyak et al., Sov. J. Nucl. Phys. 38 (1983) 773.
36. T.A. Aliiev et al., Sov. J. Nucl. Phys. 38 (1983) 936.
37. S. Narison, CERN preprint TH.4746 (1987).
38. C.A. Dominguez and N. Paver, DESY Report 87-060 (1987).
39. J. Adler et al. (MARK III Collaboration), SLAC-PUB-4343 (1987).

40. L.J. Reinders, H. Rubinstein and S. Yazaki, Physics Reports 127 (1985) 1.
41. K. Kleinknecht and B. Renk, Z. Phys. C34 (1987) 209.
42. M.J. Duncan and J. Trampetic, Phys. Lett. 134B (1984) 439;
J.-M. Gérard et al., Phys. Lett. 140B (1984) 349;
J.-M. Gérard et al., Phys. Lett. 141B (1984) 79; Nucl. Phys. B253 (1985) 93;
P. Langacker and B. Sathiapalan, Phys. Lett. 144B (1984) 395; Phys. Lett. 144B (1984) 401;
M. Dugan, B. Grinstein and L. Hall, Nucl. Phys. B255 (1985) 413;
A. Bouquet, J. Kaplan and C.A. Savoy, Phys. Lett. 148B (1984) 69.
43. S. Geer, Proceedings of the EPS Int. Conference on High Energy Physics, Uppsala, Sweden (1987).
44. S. Bertolini, F. Borzumati and A. Masiero, Carnegie Mellon Report CMU-HEP 87-07 (1987).
45. S. Bertolini, F. Borzumati and A. Masiero, Carnegie-Mellon Univ. Reports CMU-HEP 86-19 (1986) and 87-03 (1987);
N.G. Deshpande et al., Univ. of Oregon Reports OITS-346 (1986).
46. L.F. Abbott, P. Sikivie and M.B. Wise, Phys. Rev. D21 (1980) 1393;
See also the recent paper by:
S.L. Glashow and E.J. Jenkins, HUTP-87/A047 (1987).
47. G.G. Athanasiu, P.J. Franzini and F.J. Gilman, Phys. Rev. D32 (1985) 3010.
48. J.C. Pati and A. Salam, Phys. Rev. D10 (1974) 275;
R.N. Mohapatra and J.C. Pati, Phys. Rev. D11 (1975) 566, 2558;
R.N. Mohapatra and G. Senjanovic, Phys. Rev. D12 (1975) 1502, Phys. Rev. Lett. 44 (1980) 912; Phys. Rev. D23 (1981) 165.
49. G. Ecker and W. Grimus, Nucl. Phys. B258 (1985) 3281 and Z. Phys. C30 (1986) 293.
See, also:
R. Decker and U. Turke, Z. Phys. C26 (1984) 117.
50. G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. 48 (1982) 848;
G. Ecker, W. Grimus and H. Neufeld, Phys. Lett. 127B (1983) 365;
R.N. Mohapatra, G. Senjanovic and M.D. Tran, Phys. Rev. D28 (1983) 546;
F.J. Gilman and M.H. Reno, Phys. Rev. D29 (1984) 937.