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Higgs boson pair production via gluon fusion

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We study pair production of Higgs bosons at hadron colliders via the gluon fusion process $gg \rightarrow HH$. This process is dependent on the three Higgs self coupling. We find that this coupling is an important effect in determining the absolute size of the cross section. However the shapes of the transverse momentum and the rapidity distributions are almost unaffected by its presence. At SSC energies we find cross-sections in the range $1pb - 1fb$, for Higgs boson masses of 50-250 GeV and top masses larger than 40 GeV. At LHC energies the cross sections are about a factor five smaller.

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I. Introduction

The standard model of weak interactions is by now well established. The main missing ingredient is the Higgs particle. If the Higgs is light enough ($\lesssim 50\text{GeV}$) it can be produced at LEP. If it is heavier one can try to look for it at pp colliders like the SSC(40 TeV) or the LHC(16 TeV). The two basic mechanisms for producing a Higgs particle at hadron colliders are W-fusion^[1] and gluon-fusion^[2] via a virtual quark loop. For Higgs masses in the range 50-250 GeV gluon fusion is the dominant process at SSC energies^[3].

However the discovery of the Higgs particle is not in itself sufficient to completely establish the standard model. An essential part is the existence of Higgs self-interactions which are necessary for the renormalizability of the theory. In order to establish the existence of the three Higgs self coupling one therefore has to consider the production of Higgs boson pairs. At electron machines Higgs pair production has been studied in ref. [4]. At hadron colliders there are again the two major production mechanisms of W-fusion and gluon-fusion. The W-fusion mechanism $WW \rightarrow HH$ has been considered in ref. [5,6]. Typical production cross sections are in the range $10^{-1} - 10^{-3} pb$ at the SSC for m_H in the range 50-250 GeV. For gluon fusion there are contributions from virtual box graphs and from virtual triangle graphs. In the triangle graphs an off-shell Higgs particle is produced, which subsequently decays into two real Higgs particles through the three Higgs coupling. In ref. [6] the triangle graph has been considered.

In this paper we compute the complete set of diagrams contributing to the process $gg \rightarrow HH$. We also establish an approximation that is valid for large masses of the quarks inside the loop, which is useful if a fourth generation exists. In section II we give the exact cross-section and in section III the large mass limit. Numerical results are presented in section IV. Conclusions are drawn in section V. The appendix contains some details of the calculation.

II. The cross section

The cross section for the process $gg \rightarrow HH$ is determined by two Lorentz and gauge invariant structure functions which we call `gauge1` and `gauge2`. The amplitude for given polarizations e_μ^1 and e_ν^2 of the gluons is:

$$\mathcal{M} = \frac{\alpha_s \alpha_w \delta^{ab}}{8M_W^2} \{ A^{\mu\nu} gauge1(\hat{s}, \hat{t}, \hat{u}) + B^{\mu\nu} gauge2(\hat{s}, \hat{t}, \hat{u}) \} e_\mu^1 e_\nu^2, \quad (1)$$

where,

$$A^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}, \quad (2)$$

and,

$$B^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2p_2 \cdot p_3 p_1^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\mu p_3^\nu}{p_T^2}. \quad (3)$$

In these formulae p_1 and p_2 are the gluon momenta, p_3 is the momentum of one of the Higgs particles and p_T its transverse momentum. a and b are the color indices of gluons 1 and 2 respectively. The Mandelstam variables of the parton process are $\hat{s}, \hat{t}, \hat{u}$. In terms of these variables one has $p_T^2 = (\hat{u}\hat{t} - m_H^4)/\hat{s}$. Using momentum conservation one can show that eq. (3) is actually symmetric under the interchange of the two Higgs momenta. In terms of helicity amplitudes, one has,

$$\mathcal{M}_{++} = \mathcal{M}_{--} = -gauge1, \quad (4)$$

$$\mathcal{M}_{+-} = \mathcal{M}_{-+} = -gauge2. \quad (5)$$

The spin and colour averaged subprocess cross section is:

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\alpha_w^2 \alpha_s^2}{2^{15} \pi M_W^4 \hat{s}^2} (|gauge1|^2 + |gauge2|^2) \quad (6)$$

To determine the formulae for gauge1 and gauge2 the diagrams in fig.[1] have to be calculated. The diagrams containing the triangle only contribute to gauge1:

$$gauge1(triangle) = \frac{12m_H^2 m_q^2}{\hat{s} - m_H^2} (2 + (4m_q^2 - \hat{s})C(p_1, p_2)) \quad (7)$$

The box graphs contribute both to gauge1 and gauge2. Using the methods of ref. [7] we find:

$$\begin{aligned}
gauge1(box) = & 4m_q^2 \{ \\
& m_q^2 (8m_q^2 - \hat{s} - 2m_H^2)(D(p_1, p_2, p_3) + D(p_2, p_1, p_3) + D(p_1, p_3, p_2)) \\
& + \frac{(\hat{u}\hat{t} - m_H^4)}{\hat{s}}(4m_q^2 - m_H^2)D(p_1, p_3, p_2) \\
& + 2 + 4m_q^2 C(p_1, p_2) \\
& + \frac{2}{\hat{s}}(m_H^2 - 4m_q^2)((\hat{t} - m_H^2)C(p_1, p_3) + (\hat{u} - m_H^2)C(p_2, p_3)) \}, \quad (8)
\end{aligned}$$

and,

$$\begin{aligned}
gauge2(box) = & 2m_q^2 \{ \\
& 2(8m_q^2 + \hat{s} - 2m_H^2)\{m_q^2[D(p_1, p_2, p_3) + D(p_2, p_1, p_3) + D(p_1, p_3, p_2)] - C(p_3, p_4)\} \\
& - 2\{\hat{s}C(p_1, p_2) + (\hat{t} - m_H^2)C(p_1, p_3) + (\hat{u} - m_H^2)C(p_2, p_3)\} \\
& + \frac{1}{\hat{u}\hat{t} - m_H^4} \left[\hat{s}\hat{u}(8\hat{u}m_q^2 - \hat{u}^2 - m_H^4)D(p_1, p_2, p_3) + \hat{s}\hat{t}(8\hat{t}m_q^2 - \hat{t}^2 - m_H^4)D(p_2, p_1, p_3) \right. \\
& + (8m_q^2 + \hat{s} - 2m_H^2)\{\hat{s}(\hat{s} - 2m_H^2)C(p_1, p_2) + \hat{s}(\hat{s} - 4m_H^2)C(p_3, p_4) \\
& \left. + 2\hat{t}(m_H^2 - \hat{t})C(p_1, p_3) + 2\hat{u}(m_H^2 - \hat{u})C(p_2, p_3)\} \right] \}. \quad (9)
\end{aligned}$$

In these formulae the C's are the basic triangle form factors given by:

$$C(p_1, p_2) = \int \frac{d^4 q}{i\pi^2} \frac{1}{(q^2 - m_q^2)((q + p_1)^2 - m_q^2)((q + p_1 + p_2)^2 - m_q^2)} \quad (10)$$

The D's are the basic four point functions given by:

$$D(p_1, p_2, p_3) = \int \frac{d^4 q}{i\pi^2} \frac{1}{(q^2 - m_q^2)((q + p_1)^2 - m_q^2)((q + p_1 + p_2)^2 - m_q^2)((q + p_1 + p_2 + p_3)^2 - m_q^2)} \quad (11)$$

All the momenta in the above formulae are taken to be incoming; p_1 and p_2 are the gluon momenta and p_3 and p_4 the Higgs momenta. Using the methods of ref. [8] these functions can be expressed in terms of Spence functions. This is done in the appendix. In principle the amplitude has to be summed over all existing quarks coupling to the Higgs. In practice only the contribution of the top quark is important.

III. The large mass limit

The expressions in section 2 are quite complicated. In order to gain some insight into the relative importance of the different terms it is useful to consider the limit in which the quark mass is much larger than all external momenta in the problem. Numerically this approximation will be bad for ordinary top quarks, but it would be useful if a fourth generation exists. In the limit $m_q \rightarrow \infty$ one can make an expansion in inverse powers of the quark mass. The leading terms are,

$$gauge1(triangle) = \frac{4m_H^2 \hat{s}}{\hat{s} - m_H^2}, \quad (12)$$

$$gauge1(box) = -\frac{4\hat{s}}{3}, \quad (13)$$

and,

$$gauge2(box) = -\frac{11\hat{s}p_T^2}{45m_q^2}. \quad (14)$$

From the above formulae we see that the cross section is dominated by the gauge1 piece, except for very large p_T , where the gauge2 piece is enhanced. Because of the relative unimportance of the gauge2 piece the

contribution of the box graph and the triangle graph have essentially the same structure. Therefore we expect that the presence or absence of the three Higgs coupling will not effect the shape of distributions much, even though it will be an important effect in determining the size of the cross sections. However we notice that the triangle and the box appear with an opposite sign and therefore interfere destructively. In the limit where the mass of the quark is strictly infinite we find that the square of the triangle graph is larger than the square of the total if $\hat{s} < 7m_H^2$. This is contrary to the assumption of ref. [6], where it is assumed that the triangle graph always gives a lower bound.

In order to see how the heavy quark mass limit is approached we have plotted in fig. [2] $|\mathcal{M}|^2$ for fixed values of $\hat{s}, \hat{t}, \hat{u}$ as a function of the quark mass. The large mass limit has been normalized to one. One notices that the large mass limit is not monotonically approached, but that there is a resonance for m_q of the order $1/2\sqrt{\hat{s}}$. This is very similar to the case for single Higgs production. In practice the resonance is smeared out by the integration over $\hat{t}$ and over the gluon distribution functions. At SSC energies for typical values of m_H and m_t of order 100 GeV, the cross section is dominated by the area where m_t is too small to be at the resonance. Therefore at these energies one expects a steep fall-off of the cross section as the top mass decreases.

IV. Numerical results

In fig. [3], we show the total Higgs pair production cross section for pp collisions at $\sqrt{s} = 40$ TeV as a function of the Higgs mass for various values of m_t . We used set 1 of the Q^2 dependent gluon distributions of Duke and Owens^[9] with $\Lambda = 0.2$ GeV. Both the gluon distributions and α_s were evaluated at the scale $Q^2 = p_T^2 + m_H^2$. The cross section lies in the range $1 - 10^{-3}$ pb for m_H in the range 50 – 250 GeV and $m_t > 40$ GeV. The cross section falls sharply with decreasing top quark mass, in accordance with the expectations of section III, however, for $m_t \geq 80$ GeV, the $gg \rightarrow HH$ cross section is larger than the $WW \rightarrow HH$ cross section quoted in refs. [5,6]. At fixed m_t , the cross section falls with increasing m_H due to both the diminishing luminosity and the decrease in the effective value of $m_H/\sqrt{\hat{s}}$.

The corresponding cross sections for pp collisions at $\sqrt{s} = 16$ TeV are shown in fig. [4]. The cross sections are typically a factor of five smaller. The average value of $\hat{s}$ is smaller and, as a consequence, for large values of m_t the cross section is sensitive to the resonance structure of the subprocess cross section. For example, for $m_t = 200$ GeV and $m_H \leq 150$ GeV, the exact cross section is larger than the large mass limit. In the following, we will consider only pp collisions at $\sqrt{s} = 40$ TeV.

In fig. [5], we try to determine the sensitivity of the cross section to the three Higgs coupling. We show the $gg \rightarrow HH$ cross section as a function of m_t both with and without the HHH coupling. For large m_t , the box graphs are much larger than the total due to the negative interference with the triangle graphs. However, for small top quark masses, the triangle graphs become more important

In fig. [6], we show a representative transverse momentum distribution of the Higgs boson. The contributions of the different pieces have been plotted separately. The cross section is largest at moderate p_T , where the gauge2 piece is unimportant. The box and triangle contributions to gauge1 have similar shapes, making the p_T distribution useless in determining the existence of the HHH coupling. At large p_T , the gauge2 piece dominates as naively predicted by the large mass limit in section III. The triangle contribution, which has no p_T dependence in the large mass limit, dies away very fast.

In fig. [7], we show the rapidity distribution of Higgs bosons for the same representative values of m_t and m_H . The contributions of the different pieces of the total have similar shapes rendering the rapidity distribution useless in determining the size of the triangle contribution.

V. Conclusions

We have calculated the cross section for the process $gg \rightarrow HH$ both exactly and in the heavy quark mass limit. The cross section lies in the range $1 - 10^{-3}$ pb for $50 < m_H < 250$ GeV and $m_t > 40$ GeV for pp collisions at $\sqrt{s} = 40$ TeV. These cross sections are at least comparable with those from WW fusion. We have therefore shown that the gluon fusion subprocess can not be ignored for Higgs pair production. The cross sections correspond to 10000-10 HH events per year at the SSC.

Even though the production cross section is quite sizeable, it is difficult to find an observable signature. If $m_H < 2m_t$, $2M_W$ one could try to look for $b\bar{b}\tau\bar{\tau}$ events which occur about 6% of the time. This leads to about 60 events per year at the SSC when, for example, $m_H = 100$ GeV, and $m_t = 100$ GeV. If $2m_t < m_H < 2M_W$, the situation is hopeless since the Higgs decays almost always into $t\bar{t}$ which one would expect to be overwhelmed by the QCD production of top quark quartets. For $m_H > 2M_W$, the signal is four gauge bosons. However, since the Higgs is already rather heavy, the rate is very small unless the top quark is very large. Therefore we conclude that the prospects for discovering Higgs pairs at the SSC are rather poor. If further generations of fermions exist, then the $gg \rightarrow HH$ cross section is increased. The large quark mass limit given in section III is a good approximation for quark masses of 500 GeV or more.

The $gg \rightarrow HH$ process is in principle sensitive to the presence of the three Higgs self coupling. We find that the box and triangle graphs contribute mainly to the same partial wave, the s-wave. Therefore, contrary to the statement in ref. [6], it is not possible to separate the two contributions by a partial wave analysis. Only the total cross section exhibits any strong dependance on the three Higgs coupling. In order to determine the three Higgs coupling, one would need a measurement of the total $pp \rightarrow HH$ cross section. However, uncertainties in the gluon structure functions, the scale of α , and the higher order corrections would render such a measurement inconclusive.

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Appendix

In order to describe the basic integrals to be evaluated we introduce two auxiliary functions:

$$I1(a, b, y_0) = \int_0^1 \frac{dy}{y - y_0} (\log(ay + b - i\epsilon) - \log(ay_0 + b - i\epsilon)), \quad (A.1)$$

$$I2(a, b, y_0) = \int_0^1 \frac{dy}{y - y_0} (\log(a - i\epsilon - by(1 - y)) - \log(a - i\epsilon - by_0(1 - y_0))). \quad (A.2)$$

In terms of Spence functions one has :

$$I1(a, b, y_0) = Sp\left(\frac{y_0}{y_0 - z}\right) - Sp\left(\frac{y_0 - 1}{y_0 - z}\right), \quad (A.3)$$

with $z = -(b - i\epsilon)/a$, and,

$$I2(a, b, y_0) = Sp\left(\frac{y_0}{y_0 - z_1}\right) - Sp\left(\frac{y_0 - 1}{y_0 - z_1}\right) + Sp\left(\frac{y_0}{y_0 - z_2}\right) - Sp\left(\frac{y_0 - 1}{y_0 - z_2}\right), \quad (A.4)$$

with $z_1 = \frac{1}{2}(1 + \sqrt{1 - 4(a - i\epsilon)/b})$ and $z_2 = 1 - z_1$.

We define :

$$C(p_1, p_2) = \int \frac{d^4 q}{i\pi^2} \frac{1}{(q^2 - m_q^2)((q + p_1)^2 - m_q^2)((q + p_1 + p_2)^2 - m_q^2)}, \quad (A.5)$$

and,

$$D(p_1, p_2, p_3) = \int \frac{d^4 q}{i\pi^2} \frac{1}{(q^2 - m_q^2)((q + p_1)^2 - m_q^2)((q + p_1 + p_2)^2 - m_q^2)((q + p_1 + p_2 + p_3)^2 - m_q^2)}. \quad (A.6)$$

All momenta are incoming. In the following we use the conventions p_1 and p_2 are the gluon momenta, $p_1^2 = p_2^2 = 0$, p_3 and p_4 are the Higgs momenta, $p_3^2 = p_4^2 = m_H^2$. The Mandelstam variables are $s = (p_1 + p_2)^2$, $t = (p_1 + p_3)^2$, $u = (p_2 + p_3)^2$. There are three basic triangle functions. With two external gluon lines:

$$C(p_1, p_2) = \frac{1}{s} I2(m_q^2, s, 0). \quad (A.7)$$

One external gluon and one external Higgs:

$$C(p_1, p_3) = \frac{1}{t - m_H^2} (I2(m_q^2, t, 0) - I2(m_q^2, m_H^2, 0)). \quad (A.8)$$

$C(p_2, p_3)$ is obtained by replacing t with u in eq. (A.8).

Two external Higgs lines:

$$C(p_3, p_4) = -\frac{1 - \alpha}{s(1 + \alpha)} (2 \cdot I2(m_q^2, m_H^2, \alpha/(1 + \alpha)) - I2(m_q^2, s, \alpha^2/(\alpha^2 - 1))), \quad (A.9)$$

with,

$$\alpha = 1 - \frac{s}{2m_H^2} (1 + \sqrt{1 - 4m_H^2/s}). \quad (A.10)$$

There are two basic four point functions.

The cross box:

$$\begin{aligned} D(p_1, p_3, p_2) = \frac{2}{X} \{ & 2 \cdot I2(m_q^2, m_H^2, x_+) - I2(m_q^2, u, x_+) - I2(m_q^2, t, x_+) \\ & + \log(-x_-/x_+) [2 \log(m_q^2 - i\epsilon - m_H^2 x_+ x_-) - \log(m_q^2 - i\epsilon - u x_+ x_-) \\ & - \log(m_q^2 - i\epsilon - t x_+ x_-)] \} \end{aligned} \quad (A.11)$$

with,

$$x_{\pm} = \frac{1}{2} (1 \pm X/(ut - m_H^4)), \quad (A.12)$$

and,

$$X = ((ut - m_H^4)^2 + 4sm_q^2(ut - m_H^4))^{1/2} \quad (A.13)$$

The other basic function is the straight box.

$$\begin{aligned}
D(p_1, p_2, p_3) = & \frac{1}{Y} \left\{ -I1(s, 0, \beta_1 + y_{1+}) + I1(s, 0, \beta_1 + y_{1-}) \right. \\
& + I1(m_H^2 - t, m_H^2 - u, y_{1+}/(1 - \beta_1)) - I1(m_H^2 - t, m_H^2 - u, y_{1-}/(1 - \beta_1)) \\
& - I1(u - m_H^2, m_H^2 - u, -y_{1+}/\beta_1) + I1(u - m_H^2, m_H^2 - u, -y_{1-}/\beta_1) \\
& + I1(s, 0, \beta_2 + y_{2+}) - I1(s, 0, \beta_2 + y_{2-}) \\
& - I1(m_H^2 - t, m_H^2 - u, y_{2+}/(1 - \beta_2)) + I1(m_H^2 - t, m_H^2 - u, y_{2-}/(1 - \beta_2)) \\
& + I1(u - m_H^2, m_H^2 - u, -y_{2+}/\beta_2) - I1(u - m_H^2, m_H^2 - u, -y_{2-}/\beta_2) \\
& - I2(m_q^2, m_H^2, y_{1+}/(1 - \beta_1)) + I2(m_q^2, m_H^2, y_{1-}/(1 - \beta_1)) \\
& + I2(m_q^2, u, -y_{1+}/\beta_1) - I2(m_q^2, u, -y_{1-}/\beta_1) \\
& - I2(m_q^2, s, y_{2+} + \beta_2) + I2(m_q^2, s, y_{2-} + \beta_2) \\
& + I2(m_q^2, m_H^2, y_{2+}/(1 - \beta_2)) - I2(m_q^2, m_H^2, y_{2-}/(1 - \beta_2)) \\
& \left. - I2(m_q^2, m_H^2, -y_{2+}/\beta_2) + I2(m_q^2, m_H^2, -y_{2-}/\beta_2) \right\}, \quad (A.14)
\end{aligned}$$

with,

$$\beta_1 = -u/(m_H^2 - u) \quad (A.15)$$

$$v = -\sqrt{1 - 4m_H^2/s} \quad (A.16)$$

$$\beta_2 = (1 + v)/2 \quad (A.17)$$

$$Y = (s^2 u^2 + 4m_q^2 s(ut - m_H^4))^{1/2} \quad (A.18)$$

$$y_{1\pm} = \frac{-u \pm Y/s}{2(u - m^2)} \quad (A.19)$$

$$y_{2\pm} = \frac{-m_H^2 + m_H^2 v - uv \pm Y/s}{2u - 2m_H^2 + s - sv} \quad (A.20)$$

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Figure Captions

- Fig. 1: The Feynman graphs for the process $gg \rightarrow HH$. The four graphs not shown may be obtained by crossing the gluon lines.
- Fig. 2: The exact $gg \rightarrow HH$ matrix elements squared as a function of $m_q/\sqrt{\hat{s}}$, at $\hat{s} = 6m_H^2$, $\hat{u} = \hat{t}$. The curve has been normalised to the large mass limit.
- Fig. 3: The total $gg \rightarrow HH$ cross section for pp collisions at $\sqrt{s} = 40$ TeV as a function of the Higgs mass. Curves for $m_t = 40, 80, 120, 160, 200$ GeV and the heavy quark limit are shown.

- Fig. 4: The total $gg \rightarrow HH$ cross section for pp collisions at $\sqrt{s} = 16$ TeV as a function of the Higgs mass. Curves for $m_t = 40, 80, 120, 160, 200$ GeV and the heavy quark limit are shown.
- Fig. 5: The total $gg \rightarrow HH$ cross section for pp collisions at $\sqrt{s} = 40$ TeV as a function of the m_t . Curves are shown both with (solid lines) and without (dashed lines) the HHH coupling for $m_H = 50, 150$ and 250 GeV.
- Fig. 6: The transverse momentum distribution of the Higgs boson for pp collisions at $\sqrt{s}=40$ TeV. The representative values $m_t = 80$ GeV and $m_H = 100$ GeV have been chosen. The contributions of the different pieces are plotted separately.
- Fig. 7: The rapidity distribution of the Higgs boson for the same representative case as in Fig. 6.

+ permutations of gluon lines

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.