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REALISTIC PREDICTIONS FOR THE DETECTION OF SUPERSYMMETRIC DARK MATTER

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ABSTRACT

We present new predictions for the detection of supersymmetric dark matter via its annihilation in the Sun and elastic scattering off heavy nuclei in the laboratory. Our predictions include many effects found in realistic models such as non-degenerate left- and right-squark masses, unequal supersymmetric Higgs v.e.v.s and photino/higgsino/zino mixing in the lightest supersymmetric particle (LSP). Hadronic matrix elements are estimated using either the naive quark model or the EMC measurement of the spin-dependent proton structure function and perturbative QCD. Nuclear matrix elements are calculated using the shell-model and the small effects of quark vector current operators are discussed. Previous predictions for the elastic LSP-proton scattering cross-section, and hence for high energy solar neutrinos from LSP annihilations, are reduced by the EMC estimate and by unequal squark masses, but may be increased by unequal Higgs v.e.v.s. Previous predictions for elastic photino scattering off nuclei with unpaired neutrons are greatly enhanced by the EMC estimate. As a result, preferences for the nuclei to be used in laboratory experiments to detect supersymmetric dark matter may be greatly altered.

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1. - INTRODUCTION

Most astrophysicists believe that the bulk of the matter in the Universe has not yet been seen [1]. The evidence for this belief is essentially indirect, being due to the observations that imply gravitational fields stronger than can be explained by visible baryonic matter on scales ranging through dwarf galaxies, spiral galaxies, elliptical galaxies, galactic clusters and perhaps even larger scales [1]. There are reasons to think that the invisible dark matter is not baryonic [2]. Some of these are direct, for example, searches reveal no trace of gas, dust clouds, "snowballs" or small stars in galactic halos that should have been seen if they were responsible for the high mass/light ratios implied by observations [3]. Other reasons are less direct, for example, galactic halos seem to be approximately spherical and extend far beyond galactic discs, indicating that the material in them dissipated less than conventional baryonic matter [4]. It is therefore thought that the halo dark matter may consist of some species of massive, neutral, weakly-interacting particle.

Many candidates for this particle have been proposed, but the two which seem best motivated from the point of view of particle physics are the axion [5] and the lightest supersymmetric particle (LSP) [6]. There are no strong theoretical reasons to expect the axion density to be in the small range allowed by astrophysics and cosmology. However, if it is in this range, the axion halo can probably be detected in the laboratory [7], and an experimental programme to search for it is now being initiated [8]. On the other hand, calculations of the relic LSP density suggest that it is quite likely to be in the range allowed by cosmology and favoured by astrophysics [9]. There are several ideas for detecting these relic LSPs, including searches for the products of LSP annihilation in the galactic halo [10] (e.g., $\tilde{\gamma}\tilde{\gamma} \rightarrow \bar{p}$ or $e^+ + X$), searches for high-energy neutrinos from LSP annihilation in the Sun [11] (e.g., $\tilde{\gamma}\tilde{\gamma} \rightarrow b\bar{b} \rightarrow \nu(\bar{\nu}) + X$), and searches for the elastic scattering of LSPs on nuclei in low-threshold laboratory detectors [12].

The relative merits of these detection schemes depend on the nature of the LSP. For example, spin-0 sneutrinos are relatively difficult to find via halo annihilation, much easier to find via solar annihilation and have large coherent elastic scattering cross-sections off heavy nuclei. Indeed, searches for energetic solar neutrinos and relic scattering in the laboratory already go a long way towards excluding a sneutrino as the LSP [13]. However, in many models the most natural candidate for the LSP is some Majorana spin- $\frac{1}{2}$ mixture of the photino $\tilde{\gamma}$, Higgsino $\tilde{H}$ and zino $\tilde{Z}$. This class of LSP is more difficult to detect via solar annihilation or elastic scattering in the laboratory, but searches for energetic solar neutrinos are already getting close to excluding some ranges of $\tilde{\gamma}$ and/or $\tilde{H}$ masses [14].

This paper is devoted to re-evaluation of the rates for spin- $\frac{1}{2}$ LSP detection via solar annihilation or elastic scattering in the laboratory. We include several effects that are found in realistic supersymmetric models, but have not always been included in previous rate calculations. We allow for the possibility that slepton and squark masses are different, including in particular the possibility of splitting between the supersymmetric partners of left- and right-handed quarks. Effects contributing to these splittings include unequal mass renormalization effects, and unequal vacuum expectation values (v.e.v.'s) for the two supersymmetric Higgs doublets. We find that unequal mass renormalizations tend to reduce event rates, whilst they may be increased by unequal v.e.v.'s. We also allow for realistic $\tilde{\gamma}/\tilde{H}/\tilde{Z}$ mixing in the LSP, which has only a small effect on event rates if the LSP mass $m_{\text{LSP}} \lesssim 20$ GeV.

We also present alternative estimates of the hadronic matrix elements for LSP-(p,n) scattering which govern the LSP capture rate in the Sun and elastic scattering off nuclei in the shell model. One estimate is based on the naive quark model (NQM), and the other uses a recent EMC measurement of the spin-dependent deep inelastic proton structure function [15], including effects related to differing renormalization scales. The NQM estimate for $\tilde{\gamma}p$ scattering is much larger than the EMC estimate, whilst $\tilde{\gamma}n$ scattering is much larger in the EMC estimate. We consider these two estimates as extremes bracketing the plausible range of unavoidable hadronic uncertainty.

We incorporate these different hadronic matrix elements into shell model calculations of elastic LSP scattering off nuclei, tabulating results for a wide range of nuclei which have been proposed for different detection techniques. We also discuss the reliability of the shell model results for different nuclei. We emphasize that the preferred nuclei for laboratory experiments are greatly altered if one uses the EMC estimates for $\tilde{\gamma}n$ and $\tilde{\gamma}p$ scattering instead of the NQM estimates. The EMC estimates give much larger rates for nuclei with unpaired neutrons.

The layout of this paper is as follows. In Section 2 we review expectations for sparticle masses in realistic models and show how the relic LSP density can be equal to the closure density for suitable choices of the model parameters (sparticle masses in the case of the $\tilde{\gamma}$, v.e.v.'s in the case of the $\tilde{H}$). In Section 3 we present our estimates of hadronic matrix elements, explaining in particular the EMC estimates and how the difference in renormalization scale between the EMC experiment and LSP scattering (which is a small effect) can be taken into account. In Section 4 we develop predictions for elastic LSP scattering off nuclei using the shell model. In particular, we demonstrate the negligibility of new coherent contributions to the amplitude which are present when the left- and

right-squarks have unequal masses. In Section 5 we discuss our results and present some conclusions on the prospects for LSP detection via energetic solar neutrinos and/or elastic scattering off nuclei in the laboratory.

2. - SPIN- $\frac{1}{2}$ SUPERSYMMETRIC DARK MATTER

2.1 Neutralino spectrum and the LSP

We consider first the composition of the LSP in the minimal standard model with softly-broken supersymmetry (susy). The scalar sector of the theory contains two light Higgs doublets H_1 and H_2 , of hypercharges +1 and -1 respectively, that give masses to the up (H_1) and down (H_2) quarks. There are four neutral, colourless, spin- $\frac{1}{2}$ susy fermions that mix in general. Their mass matrix is determined by soft susy-breaking mass terms [16]:

$$\mathcal{L}_{\text{mass}} = \varepsilon \varepsilon_{ij} \tilde{H}_1^i \tilde{H}_2^j - M_2 \tilde{W}_a \tilde{W}_a - M_1 \tilde{B} \tilde{B} \quad (2.1)$$

and the kinetic terms

$$\begin{aligned} \mathcal{L}_{\text{kin}}(H_k) = & \sqrt{2} H_{k,i}^\dagger i g_2 T_{ij}^a \tilde{W}^a \tilde{H}_{k,j} + \text{h.c.} \\ & + \sqrt{2} H_k^\dagger i g_1 \frac{1}{2} Y \tilde{B} \tilde{H}_k + \text{h.c.} \end{aligned} \quad (2.2)$$

where $k = 1, 2$, $\tilde{H}_k$ is the supersymmetric partner of H_k , $\tilde{W}_a$ and $\tilde{B}$ denote SU(2) and U(1) gauginos respectively and i, j (a) are doublet (triplet) SU(2) indices. Thus, the mass matrix for the four neutral fermions $\tilde{W}^3, \tilde{B}, \tilde{H}_1, \tilde{H}_2$ is given by:

$$[\tilde{W}^3, \tilde{B}, \tilde{H}_1, \tilde{H}_2] \begin{bmatrix} M_2 & 0 & -\frac{g_2 v_1}{\sqrt{2}} & \frac{g_2 v_2}{\sqrt{2}} \\ 0 & M_1 & \frac{g_1 v_1}{\sqrt{2}} & -\frac{g_1 v_2}{\sqrt{2}} \\ -\frac{g_2 v_1}{\sqrt{2}} & \frac{g_1 v_1}{\sqrt{2}} & 0 & \varepsilon \\ \frac{g_2 v_2}{\sqrt{2}} & -\frac{g_1 v_2}{\sqrt{2}} & \varepsilon & 0 \end{bmatrix} \begin{bmatrix} \tilde{W}^3 \\ \tilde{B} \\ \tilde{H}_1 \\ \tilde{H}_2 \end{bmatrix} \quad (2.3)$$

where $v_k^2 = \langle H_k^\dagger H_k \rangle$ and the W-boson mass is given by $M_W^2 = g_2^2(v_1^2 + v_2^2)/2 \approx (81.8 \text{ GeV})^2$; the ratio v_1/v_2 is undetermined, although the pattern of quark masses suggests

$v_1/v_2 > 1$ unless quark Yukawa coupling differ by more than an order of magnitude.

We shall assume that $M_1 = (5/3)(\alpha_1/\alpha_2)M_2$, which holds to leading order in the couplings $\alpha_j \equiv g_j^2/4\pi$ if the standard model is the low-energy effective theory of a higher energy, unified theory that embeds $SU(2) \times U(1)$ in a simple group. At the unification scale M_X , $g_2(M_X) = \sqrt{5/3} g_1(M_X) = g_{GUT}$ and the gaugino masses are equal: $M_1(M_X) = M_2(M_X) = M_{\frac{1}{2}}$. At scale M_W [17]:

$$M_1 \equiv M_1(M_W) = \frac{5}{3} \frac{\alpha_1(M_W)}{\alpha_{GUT}} M_{1/2} \quad (2.4a)$$

$$M_2 \equiv M_2(M_W) = \frac{\alpha_2(M_W)}{\alpha_{GUT}} M_{1/2} \quad (2.4b)$$

The mass eigenstates, χ_i , of the mass matrix (2.3) are in general complicated combinations of the fields

$$\chi_i \equiv \alpha_i \tilde{W}^3 + \beta_i \tilde{B} + \gamma_i \tilde{H}_1 + \delta_i \tilde{H}_2 \quad (2.5)$$

However, in the limit $\epsilon \ll M_2$, the LSP is the Higgsino $\tilde{H}$:

$$LSP = \tilde{H} \equiv (v_2 \tilde{H}_1 + v_1 \tilde{H}_2)/v; \quad m_{\tilde{H}} = \frac{2v_1 v_2}{v^2} \epsilon \quad (2.6)$$

where $v^2 = v_1^2 + v_2^2$. Similarly, in the limit $M_2 \ll \epsilon$ the LSP is the photino:

$$LSP = \tilde{\gamma} \equiv \frac{g_1 \tilde{W}^3 + g_2 \tilde{B}}{\sqrt{g_1^2 + g_2^2}}; \quad m_{\tilde{\gamma}} = \frac{g}{3} \frac{\partial_1^2}{\partial_1^2 + \partial_2^2} M_2 \quad (2.7)$$

The remaining two eigenstates for $\epsilon, M_2 \ll M_W$ are

$$\frac{1}{\sqrt{2}} (\tilde{Z} \pm \tilde{A}); \quad m \simeq M_2 = \sqrt{\frac{1}{2} (g_1^2 + g_2^2)} v \quad (2.8)$$

where

$$\tilde{Z} \equiv \frac{g_1 \tilde{B} - g_2 \tilde{W}^3}{\sqrt{g_1^2 + g_2^2}}$$

is the susy partner of the Z-boson and $\tilde{A} \equiv (v_1 \tilde{H}_1 - v_2 \tilde{H}_2)/v$.

For general values of ϵ and M_2 we diagonalize (2.3) exactly to determine the LSP and thus the coefficients in (2.5). These will be needed to evaluate rates of LSP interaction with matter as well as LSP annihilation in the following sections.

The processes are mediated by sfermion exchange and, thus, require the sfermion spectrum.

We assume that the softly-broken susy standard model is the low-energy effective theory of an $N = 1$ supergravity model. The general form of the scalar potential is then:

$$V = \sum_i \left| \frac{\partial W}{\partial \phi_i} + \phi_i^* M_0 \right|^2 + M_0 (A-3) (W+W^*) + \frac{1}{2} \sum_a D_a^2 \quad (2.9)$$

where the ϕ_i are scalar fields, W is the superpotential, $D_a = \Sigma_i \phi_i^* T^a \phi_i$ and A is a constant characterizing the local susy breaking. The scale masses M_0 and $M_{\frac{1}{2}}$ (2.4) determine, together with v_1/v_2 , the sfermion masses:

$$m_{\tilde{f}_{L,R}}^2 = M_0^2 (1 + C_{f_{L,R}} \xi^2) + m_f^2 \pm \frac{v_1^2 - v_2^2}{2} \left\{ \frac{1}{2} \gamma_{f_{L,R}}^2 \tilde{g}_1^2 - T_{f_{L,R}}^3 \tilde{g}_2^2 \right\} \quad (2.10)$$

where the $+$ ($-$) sign is for $\tilde{f}_L$ ($\tilde{f}_R$), the scalar susy partner of the left-(right-) handed component of the fermion f , and $\xi \equiv (M_{\frac{1}{2}}/M_0)$. At the unification scale M_X the sfermions are degenerate, $m_{\tilde{f}_{L,R}} = M_0$. Large radiative corrections split the degeneracy at scales $\sim M_W$ according to the coefficients $C_{f_{L,R}}$ given in Table 1 [17] (for exact global susy, $M_0 = M_{\frac{1}{2}} = 0$ and $v_1 = v_2$, thus $m_{\tilde{f}_{L,R}}^2 = m_f^2$ as expected).

In addition to the diagonal entries (2.10) in the squark mass matrix, there are in principle off-diagonal terms. However, these are of magnitude $= O(m_f \cdot M_0)$, and hence are small compared with $m_{\tilde{f}_L}^2 - m_{\tilde{f}_R}^2 = M_0^2 (C_{f_L} - C_{f_R}) \xi^2 + O(v_1^2 - v_2^2)$, at least for the $\tilde{u}$, $\tilde{d}$ and $\tilde{s}$ squarks of interest in this paper. Therefore $\tilde{f}_L$ - $\tilde{f}_R$ mixing is negligible for us, though it may well be important for $\tilde{t}$ and perhaps $\tilde{b}$ squarks.

Thus, for a given m_{LSP} , ϵ and v_1/v_2 we determine $M_{\frac{1}{2}}$ from (2.4) and M_0 will be determined by the requirement that the LSP cosmic density be, in units of the closure density, $\Omega_{LSP} = 1$ (see below; for the case $LSP \approx \tilde{H}$, $\Omega_{LSP} = 1$ determines v_1/v_2 instead). The determined squark masses will in turn fix the elastic scattering cross-section of LSP's off nuclei and, thus, the LSP detection rates in the laboratory and the high-energy neutrino flux from LSP annihilation in the Sun.

2.2 LSP Annihilation and Relic Abundance

Having determined the LSP composition (2.5), we now turn to the question of its relic abundance after the Big Bang [16]. At high temperatures, $T \gg m_{LSP}$, the

LSP is in thermal equilibrium with all "radiation" (relativistic degrees of freedom) until the temperature falls to the freeze-out value $T_f \sim m_{\text{LSP}}/20$; thereafter the LSP density rapidly decreases due to annihilation until the annihilation rate falls below the expansion rate of the Universe, at which point their density continues to be diluted only by the expansion of the Universe again. The present mass density is constrained by the present expansion rate of the Universe to be $\Omega_{\text{LSP}} \leq 4$ [18]. The mass density of the Universe is inferred to be $\Omega \sim 0.1-0.3$ on scales of up to $\sim 10h^{-1}$ Mpc if light is assumed to be a good tracer of mass [19] (here h is the present Hubble constant in units of $100 \text{ kms}^{-1} \text{ Mpc}^{-1}$). If the mass of the Universe is distributed like the sample of IRAS galaxies (dominated by late type galaxies), then the motion of the Local Group relative to the cosmic microwave background implies $\Omega \sim 1$ [20]. An entirely different method based on the number distribution of galaxies as a function of red-shift also yields $\Omega \sim 1$ [21]. Doubtless, these results have yet to be confirmed and understood. Naturalness and inflation favour $\Omega = 1$; moreover, if most of the halo of our galaxy is made of LSP's, then underground detector constraints on the flux of high-energy solar neutrinos imply large values of Ω_{LSP} for $m_{\text{LSP}} \sim 3-10 \text{ GeV}$ [14]. We shall assume hereafter that $\Omega_{\text{LSP}} = 1$.

The cosmic number density of the LSP, n_{LSP} , is governed by [16]

$$\frac{dn_{\text{LSP}}}{dt} = -3 \frac{\dot{R}}{R} n_{\text{LSP}} - \langle \sigma v_{\text{rel}} \rangle (n_{\text{LSP}}^2 - n_{\text{LSP}}^0{}^2) \quad (2.11)$$

where R is the scale factor of the Universe, $\langle \sigma v_{\text{rel}} \rangle$ is the thermally averaged product of the cross-section and relative velocity of the annihilating particles and n_{LSP}^0 is the LSP number density in thermal equilibrium. Equation (2.9) can be cast in the convenient form [16]

$$\frac{df}{dx} = \frac{m_{\text{LSP}}}{k^3} \left(\frac{8\pi^3 N_F G_N}{45} \right)^{-1/2} \langle \sigma v_{\text{rel}} \rangle (f^2 - f_0^2) \quad (2.12)$$

where $x \equiv kT/m_{\text{LSP}}$, $f(x) = n_{\text{LSP}}/T^3$, $f_0(x) = n_{\text{LSP}}^0/T^3$, k is the Boltzmann constant and N_F counts the effective number of relativistic degrees of freedom at a given temperature.

The LSP annihilation cross-section into fermion-antifermion pairs, $\sigma(\text{LSP}^2 \rightarrow f\bar{f})$, is determined by the low-energy effective Lagrangian [16]:

$$\mathcal{L} = \sum_f \bar{\text{LSP}} \gamma^\mu \gamma_5 \text{LSP} \bar{f} \gamma_\mu (A_f P_L + B_f P_R) f \quad (2.13)$$

where $A_f = A_f^Z + A_f^{\tilde{f}}$ and $B_f = B_f^Z + B_f^{\tilde{f}}$, with

$$A_f^z = (\gamma^2 - \delta^2) \frac{\gamma_1 \sin \theta_w + \gamma_2 \cos \theta_w}{4M_z^2} \left(\frac{1}{2} Y_{f_L} \gamma_1 \sin \theta_w - T_{f_L}^3 \gamma_2 \cos \theta_w \right) \quad (2.14a)$$

$$B_f^z = (\gamma^2 - \delta^2) \frac{\gamma_1 \sin \theta_w + \gamma_2 \cos \theta_w}{8M_z^2} Y_{f_R} \gamma_1 \sin \theta_w \quad (2.14b)$$

$$A_f^{\tilde{f}} = \frac{(T_{f_L}^3 \alpha \gamma_2 + \frac{1}{2} Y_{f_L} \beta \gamma_1)^2}{2m_{f_L}^2} - \begin{cases} \gamma^2 m_f^2 / 4v_1^2 m_{f_R}^2 ; u, c, t \\ \delta^2 m_f^2 / 4v_2^2 m_{f_R}^2 ; e, \mu, \tau, d, s, b \end{cases} \quad (2.14c)$$

$$B_f^{\tilde{f}} = - \frac{(\frac{1}{2} Y_{f_R} \beta \gamma_1)^2}{2m_{f_R}^2} + \begin{cases} \gamma^2 m_f^2 / 4v_1^2 m_{f_L}^2 ; u, c, t \\ \delta^2 m_f^2 / 4v_2^2 m_{f_L}^2 ; e, \mu, \tau, d, s, b \end{cases} \quad (2.14d)$$

and $\alpha, \beta, \mu, \delta$ are determined by the LSP composition (2.5) and $P_{L,R} = (1 \mp \gamma_5)/2$.

In the non-relativistic limit, $x \ll 1$, (2.13) yields:

$$\langle \sigma v_{rel} \rangle = \tilde{a} + \tilde{b} x, \quad (2.15)$$

where [9]:

$$\tilde{a} = (2\pi)^{-1} \sum_f \theta(m_{LSP} - m_f) m_f^2 \beta_f (A_f - B_f)^2 \quad (2.16a)$$

$$\begin{aligned} \tilde{b} = (2\pi)^{-1} \sum_f \theta(m_{LSP} - m_f) \beta_f \left\{ (A_f^2 + B_f^2) \left(4m_{LSP}^2 - \frac{13}{4} m_f^2 \right) \right. \\ \left. + \frac{21}{2} A_f B_f m_f^2 + (A_f - B_f)^2 \frac{3}{4} \frac{m_f^2}{\beta_f^2} \right\} \end{aligned} \quad (2.16b)$$

and $\beta_f \equiv (1 - m_f^2/m_{LSP}^2)^{\frac{1}{2}}$ and θ is the step function.

We conclude by giving the equation to determine M_0 (or v_1/v_2 for $LSP \approx \tilde{H}$, since $\alpha = \beta = 0$ in that case and m_f/v_1 and $m_f/v_2 \ll 1$, except if the top quark contributes to $(LSP)^2 \rightarrow f\bar{f}$). The evolution of $f(x)$ can be approximated analytically and $f(x)$ integrated from $x = x_f$ down to $x = 0$ to yield the LSP relic density (in units of the closure density) [16]:

$$4\Omega_{\text{LSP}} h^2 = (0.8)(1.25 \times 10^{-10}) \left(\frac{T_{\text{LSP}}}{T_\gamma} \right)^3 \left(\frac{T_\gamma}{2.7^\circ \text{K}} \right)^3 N_F^{1/2} \frac{\text{GeV}^{-2}}{\tilde{a} x_f + \frac{1}{2} \tilde{b} x_f} \quad (2.17)$$

where the scaled freeze-out temperature is given by [18]

$$x_f = \left[\ln(\tilde{a} x_f^{1/2} + \tilde{b} x_f^{3/2}) - \frac{1}{2} \ln \left(\frac{16\pi^6 N_F \epsilon_w}{45 m_{\text{LSP}}^2} \right) \right]^{-1} \quad (2.18)$$

The factor of $(T_{\text{LSP}}/T_\gamma)^3$ accounts for the subsequent reheating of the photons by the annihilations of particles with mass $< x_f m_{\text{LSP}}$. This factor together with N_F were taken from Ref. [22] and are given in Table 2. The fudge factor 0.8 corrects for the fact that the analytic approach overestimates Ω_{LSP} by $\sim 25\%$. Assuming $\Omega_{\text{LSP}} = 1$, $h = \frac{1}{2}$, we use (2.17) to determine M_0 for a given v_1/v_2 , M_2 and ϵ .

2.3 LSP Elastic Scattering off Matter

Having determined the squark masses that ensure $\Omega_{\text{LSP}} = 1$, we now turn to the evaluation of the scattering cross-section for the LSP elastic scattering off of matter.

The simplest assumption is that all of the (non-dissipative) dark matter in the Universe is of the same composition. While it appears that the mass density of the Universe in baryons cannot exceed $\Omega_b \sim 0.2$ without upsetting the successful predictions of standard nucleosynthesis [23] (and inhomogeneous nucleosynthesis cannot accommodate $\Omega_b = 1$ without overproducing ^7Li [24]), it might be that the dark matter in galactic halos is ordinary matter hidden in black holes or little Jupiters (for $M_{\text{halo}}/M_{\text{gal}} \sim 10$, $\Omega_{\text{gal}} \sim 0.2$ in agreement with determinations on small scales and the nucleosynthesis constraints), although this is not an attractive possibility and has problems of its own. We shall assume, therefore, that the LSP makes up the halo of dark matter around our own galaxy. Thus, the LSP will have an approximately isothermal distribution (although it certainly cannot be an exact isothermal distribution) with velocity dispersion $\langle u^2 \rangle \sim \frac{3}{2} v_\odot^2$, where $v_\odot \sim 220 \text{ kms}^{-1}$ is the circular velocity at the solar radius; thus $(\langle u^2 \rangle)^{1/2} \sim 300 \text{ kms}^{-1}$ and the LSP is highly non-relativistic. In order to illustrate the general computation we shall consider first the scattering of photinos off of matter. The invariant amplitude, assuming $m_{\tilde{q}_L} = m_{\tilde{q}_R} = m_{\tilde{q}}$ for simplicity, for $\tilde{\gamma} q \rightarrow \tilde{\gamma} q$ is [16]:

$$\mathcal{M} = \frac{e^2 Q_q^2}{m_{\tilde{q}}^2} \bar{\tilde{\gamma}} \gamma_\mu \gamma_5 \tilde{\gamma} \bar{q} \gamma^\mu \gamma_5 q \quad (2.19)$$

In the non-relativistic limit $|\vec{p}_{\tilde{\gamma}}|/m_{\tilde{\gamma}} \rightarrow 0$, the time component of the photino current vanishes and $\bar{\tilde{\gamma}} \vec{\gamma} \gamma_5 \tilde{\gamma}$ reduces to $2\vec{S}_{\tilde{\gamma}}$, where $\vec{S}_{\tilde{\gamma}}$ is the photino spin operator. Thus [25]:

$$\eta = \frac{4e^2 Q_q^2}{m_{\tilde{q}}^2} \vec{S}_{\tilde{\gamma}} \cdot \vec{S}_q \quad (2.20)$$

In order to evaluate $\sigma(\tilde{\gamma}N \rightarrow \tilde{\gamma}N)$ we need to evaluate the expectation value of η in the nuclear state $|N\rangle$. Provided that the nuclear density is not too high so that nucleons retain their individual identity, we can first evaluate

$$\langle \text{proton} | \vec{S}_q | \text{proton} \rangle = \vec{S}_p \cdot \Delta q \quad (2.21)$$

where $\vec{S}_p$ is the proton (or neutron) spin and Δq is the fraction of the spin carried by the quark q . This factor is discussed in detail in Section 3. Since the $\tilde{\gamma}$'s scatter coherently off of the nucleus N , we must add over all quarks in the proton

$$\langle \text{proton} | \eta | \text{proton} \rangle = \frac{4e^2}{m_{\tilde{q}}^2} \vec{S}_{\tilde{\gamma}} \cdot \vec{S}_p \sum_q Q_q^2 \Delta q \quad (2.22)$$

Next, to evaluate $\langle N | \eta | N \rangle$ we need $\langle N | \vec{S}_p | N \rangle$. The Wigner-Eckardt theorem implies

$$\langle N | \vec{S}_p | N \rangle = \lambda \langle N | \vec{J} | N \rangle \quad (2.23)$$

where $\vec{J}$ is the total nuclear spin. In the single-particle shell model of nuclei that we discuss in Section 4, λ is given by the Landé formula^{*)} [26]

$$\lambda = \frac{1}{2} \left(1 + \frac{s(s+1) - l(l+1)}{J(J+1)} \right) \quad (2.24)$$

where $\vec{s}$ and $\vec{l}$ are the particle's spin and orbital angular momentum, $\vec{J} = \vec{l} + \vec{s}$. Thus,

$$\eta(\tilde{\gamma}N \rightarrow \tilde{\gamma}N) = \frac{4e^2}{m_{\tilde{q}}^2} \cdot \lambda \cdot \sum_q Q_q^2 \Delta q \cdot \vec{S}_{\tilde{\gamma}} \cdot \vec{J} \quad (2.25)$$

and the total cross-section (averaging over initial and summing over final spin of the $\tilde{\gamma}$) is given by [25]

$$\sigma = \frac{4 m_{\tilde{\gamma}}^2 m_N^2}{\pi (m_{\tilde{\gamma}} + m_N)^2} \left(\frac{e^2}{m_{\tilde{q}}^2} \right)^2 \lambda^2 J(J+1) \left(\sum_q Q_q^2 \Delta q \right)^2 \quad (2.26)$$

^{*)} We thank Bruce Campbell for pointing out to us the origin of this formula.

If all left-squarks are degenerate but equally split from degenerate right-squarks, then there is a small additional contribution to (2.27) due to the vector current $\bar{q}\gamma^\mu q$. The expectation value of its time component,

$$\langle N | \sum_q Q_q^2 (\bar{q} \gamma^0 q) | N \rangle = Z + \frac{2}{3} N \equiv \tilde{N} \quad (2.27)$$

where $Z(N)$ is the number of protons (neutrons) in the nucleus, contributes (to leading order in $\tilde{N}$) an additional factor $\sigma \propto \tilde{N}^2$. The final result is then:

$$\begin{aligned} \sigma_{TOT} = & \frac{4 m_{\tilde{g}}^2 m_N^2}{\pi (m_{\tilde{g}} + m_N)^2} e^2 (A-B)^2 \left\{ \lambda^2 J(J+1) \left(\sum_q Q_q^2 A_q \right)^2 \right. \\ & \left. + \left(\frac{|\vec{p}_{\tilde{g}}|}{m_{\tilde{g}}} \right)^2 \left(\frac{A+B}{A-B} \right)^2 \frac{\tilde{N}^2}{8} \right\} \end{aligned} \quad (2.28)$$

where $A = \frac{1}{2} m_{q_L}^2$ and $B = -\frac{1}{2} m_{q_R}^2$. Since $|\vec{p}_{\tilde{g}}|/m_{\tilde{g}} \sim [(3/2)(v_\Theta/c)^2]^{\frac{1}{2}} \sim 10^{-3}$, $(\vec{p}_{\tilde{g}}^2/m_{\tilde{g}}^2)(\tilde{N}^2/8) \sim 0.3 \times 10^{-2}$ for a nucleus as heavy as $^{207}_{82}\text{Pb}$. Since $\lambda^2 J(J+1) (\sum_q Q_q^2 A_q)^2$ is $\sim$ an order of magnitude larger, we shall neglect this term hereafter.

3. - ESTIMATES OF HADRONIC MATRIX ELEMENTS

We now turn to the calculation of the matrix elements of the interactions (2.13) between proton and neutron states. The matrix elements of the quark vector current operators $\bar{q}\gamma_\mu q$ are simply given by the flavour quantum numbers of the proton and neutron, and make in any case very small contributions to the scattering of non-relativistic Majorana spin- $\frac{1}{2}$ LSPs, as we discussed in Section 2. The dominant contributions to the low-energy scattering are from the quark axial vector current operators $\bar{q}\gamma_\mu \gamma_5 q$, whose estimation is more delicate.

The matrix elements of the $SU(3)$ flavour-octet axial currents can be related by $SU(3)$ to the matrix elements of other flavour-octet axial currents $\bar{u}\gamma_\mu \gamma_5 d$ and $\bar{u}\gamma_\mu \gamma_5 s$ which are measured in neutron decay and hyperon β -decay respectively. Flavour $SU(3)$ is already known to inter-relate these very successfully [27] and all the neutron and hyperon data are well described ($\chi^2/\text{DOF} = 8.8/6$) by unbroken $SU(3)$ with the following ratio of the two independent reduced matrix elements [27]:

$$\frac{F}{D} = 0.63 \pm 0.02 \quad (3.1)$$

This value is not far from, but not equal to, the naïve quark model value $F/D = 2/3$.

More problematic is the estimation of the $SU(3)$ flavour-singlet matrix element

$$\langle p | \sqrt{\frac{2}{3}} (\bar{u} \gamma^\mu \gamma_5 u + \bar{d} \gamma^\mu \gamma_5 d + \bar{s} \gamma^\mu \gamma_5 s) | p \rangle = S^\mu a^0 \quad (3.2)$$

Until recently, this was not measured directly, and a dynamical assumption going beyond flavour $SU(3)$ was needed to relate it to flavour-octet matrix elements. Until recently, the canonical assumption has been that [11,25,28]

$$\langle p | \bar{s} \gamma^\mu \gamma_5 s | p \rangle = 0 \quad (3.3)$$

as suggested by the naïve quark model. To explore the consequences of this assumption, we first introduce the corresponding flavour-octet matrix elements

$$\langle p | \bar{u} \gamma^\mu \gamma_5 u - \bar{d} \gamma^\mu \gamma_5 d | p \rangle = S^\mu a^3 \quad (3.4a)$$

$$\langle p | \sqrt{\frac{1}{3}} (\bar{u} \gamma^\mu \gamma_5 u + \bar{d} \gamma^\mu \gamma_5 d - 2 \bar{s} \gamma^\mu \gamma_5 s) | p \rangle = S^\mu a^8 \quad (3.4b)$$

We denote the individual $\bar{q} \gamma_\mu \gamma_5 q$ terms in (3.2) and (3.4) by Δq :

$$\begin{aligned} a^3 &= \Delta u - \Delta d, \quad a^8 = \frac{1}{\sqrt{3}} (\Delta u + \Delta d - 2 \Delta s) \\ a^0 &= \sqrt{\frac{2}{3}} (\Delta u + \Delta d + \Delta s) \end{aligned} \quad (3.5)$$

which can be interpreted in the quark parton model as the fractions of the proton spin carried by partons of flavour q [29]:

$$\Delta q = \int_0^1 dx (q_\uparrow(x) + \bar{q}_\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)) \quad (3.6)$$

Neutron decay tells us that

$$a^3 = \Delta u - \Delta d = g_A = 1.25 \quad (3.7)$$

whilst the hyperon β -decay data and $SU(3)$ suggest [27]

$$a^8 = \frac{1}{\sqrt{3}} (\Delta u + \Delta d - 2\Delta s) = 0.394 \quad (3.8)$$

If one assumes (3.3), one therefore has

$$NQM: \quad \Delta u = 0.97, \quad \Delta d = -0.28, \quad \Delta s = 0 \quad (3.9)$$

and the low-energy LSP-p (and LSP-n) elastic scattering matrix elements are immediately calculable.

Recently a new way to estimate a^0 and hence the Δq has become available [14]. Measurements of deep inelastic μ scattering off polarized protons have enabled the EMC collaboration to measure the spin-dependent structure function $g_1^p(x, Q^2)$ and estimate its zeroth moment [15]:

$$\int_0^1 dx \, g_1^p(x, Q^2) = 0.113 \pm 0.012 \pm 0.025 \quad (3.10)$$

at $\langle Q^2 \rangle \approx 10 \text{ GeV}^2$. According to QCD and the renormalization group, this moment can be related to the a^i (3.2) and (3.4) [30]

$$\int_0^1 dx \, g_1^p(x, Q^2) = \frac{C(Q^2)}{12} \left[a^3 + \frac{1}{\sqrt{3}} a^8 + 2\sqrt{\frac{2}{3}} a^0(Q^2) \right] \quad (3.11)$$

where the coefficient function [31]

$$C(Q^2) = 1 - \frac{\alpha_s(Q^2)}{\pi} + \dots \quad (3.12)$$

and $a^0(Q^2)$ has an anomalous dimension [32] factor $A(Q^2)$ defined by

$$a^0(Q^2) = \bar{a}^0 A(Q^2) : \quad A(Q^2) = 1 + \frac{2}{3} \frac{\alpha_s(Q^2)}{\pi} + \dots \quad (3.13)$$

for three quark flavours (u,d,s). The anomalous dimension factor $A(Q^2)$ does not appear for the flavour-octet currents because they have soft divergences $\propto \sum_q m_q \bar{q} \gamma_5 q$, whilst the divergence of the singlet current contains a hard piece $\propto \epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} G^{\rho\sigma}$ where $G_{\mu\nu}$ is the gluon field strength. On the right-hand side of (3.11), Q^2 has the interpretation of a renormalization scale which is $Q^2 \approx 10 \text{ GeV}^2$ for the EMC measurement. To estimate the LSP-p scattering matrix elements, we need a^3 , a^8 and a^0 ($Q^2 \approx m_q^2 \sim 10^4 \text{ GeV}^2$). This can easily be estimated using (3.7), (3.8), (3.10) and (3.11):

$$\bar{a}^0 = -0.00 \pm 0.21 \quad (3.14)$$

implying

$$\Delta u = 0.74 \pm 0.08, \Delta d = -0.51 \pm 0.08, \Delta s = -0.23 \pm 0.08 \quad (3.15)$$

with no significant variation between $Q^2 = 10 \text{ GeV}^2$ and 10^4 GeV^2 . The results (3.14) and (3.15) differ slightly from those given in Ref. [17], where the small Q^2 dependence in (3.11) was not included, but the changes are much smaller than the errors in (3.10) and (3.15).

The EMC estimates (3.15) are significantly different from the NQM estimates (3.9), because the data (3.10) indicate that $\Delta s \neq 0$ in conflict with the NQM assumption (3.3). Indeed, the NQM prediction for the zeroth moment of $g^P(x, Q^2)$ on the basis of (3.9) would have been [30]

$$\int_0^1 dx g_1^P(x, Q^2) = 0.20 \quad (3.16)$$

which is three standard deviations away from the preliminary EMC result (3.10). It should be noted that most of the error in (3.10) is systematic and associated with the extrapolation of the measured data to $x = 0$. The reliability of this extrapolation merits further study, and the EMC result (3.10) should be confirmed by further measurements on polarized protons. It is also possible to check (3.10) indirectly by doing deep inelastic μD scattering and extracting the polarized neutron structure function $g_1^n(x, Q^2)$. This is because of the reliable Bjorken sum rule [33]

$$\int_0^1 dx [g_1^P(x, Q^2) - g_1^n(x, Q^2)] = \frac{C(Q^2)}{6} a^3 = \frac{2A}{6} \left(1 - \frac{\alpha_s(Q^2)}{\pi} + \dots\right) \quad (3.17)$$

This experimental check is highly desirable. In the meantime, we regard the values (3.10) and (3.16) of the zeroth moment, and the corresponding range of Δq values (3.15) and (3.9), as bracketing the probable range of uncertainty.

We can now use the alternative NQM (3.9) and EMC (3.15) estimates of the Δq (i.e., of the matrix elements $\langle p | \bar{q} \gamma_\mu \gamma_5 q | p \rangle$) to estimate the low-energy LSP-p and LSP-n scattering matrix elements for any given hypothesis about the squark masses. For orientation, we show in Table 3a the predictions for $\tilde{\gamma}p$ and $\tilde{\gamma}n$ scattering if all the squarks have equal masses. The differences between the NQM and EMC estimates of these amplitudes are easy to understand, since in the limit of equal squark masses the interaction (2.13) gives exactly the same combination

$$\sum_q Q_q^2 \bar{q} \gamma_\mu \gamma_5 q \quad (3.18)$$

as appears in polarized muon scattering. Hence if the LSP is a pure $\tilde{\gamma}$, the $\tilde{\gamma}p$ amplitude is directly proportional to $\int_0^1 dx g_1^p(x, Q^2)$. This explains why the NQM estimate for $\tilde{\gamma}p$ scattering is much larger than the EMC estimate. Correspondingly, because of the Bjorken sum rule (3.17) $\int_0^1 dx g_1^n(x, Q^2)$ should be almost zero in the NQM, whereas it should be almost equal and opposite to $\int_0^1 dx g_1^p(x, Q^2)$ if the EMC estimate (3.10) is used. Hence $\tilde{\gamma}n$ scattering is greatly enhanced if the EMC estimate is used instead of the NQM. These differences in the $\tilde{\gamma}p$ and $\tilde{\gamma}n$ scattering amplitudes have important implications for $\tilde{\gamma}$ -nucleus scattering, as we discuss in Section 4. Table 3b shows the corresponding variations in predictions for low-energy $\tilde{H}p$ and $\tilde{H}n$ scattering, where we see that the differences between the NQM and EMC estimates are much less dramatic.

4. - SINGLE-PARTICLE SHELL MODEL OF NUCLEI

We complete the discussion of LSP scattering off nuclei by describing the nuclear shell model [26] used in the evaluation of λ . Although the evaluation of λ could be quite complicated in general, nuclei exhibit properties characteristic of shell structure that are familiar in atomic physics. For example, when certain "magic numbers" of nucleons occur in the nucleus, the corresponding nucleus exhibits remarkable stability properties (e.g., the binding energy markedly decreases for isotopes with one more nucleon), and ground state properties of nuclei, such as spin J and parity, can be described by those of one extra nucleon interacting with the effective potential of a nuclear "core".

For nuclei whose angular momentum distribution can be described by a single particle of spin s in an unfilled shell of orbital angular momentum ℓ , $\vec{J} = \vec{\ell} + \vec{s}$ (the even number of nucleons that remain have filled lower energy shells in spin pairs $\uparrow\downarrow$ with opposite orbital angular momentum and do not contribute to $\vec{J}$), the evaluation of λ is straightforward. Since $\langle N | (\vec{J} - \vec{s})^2 | N \rangle = \langle N | \vec{\ell}^2 | N \rangle$,

$$\langle N | \vec{J} \cdot \vec{s} | N \rangle = \frac{1}{2} (J(J+1) + s(s+1) - \ell(\ell+1)) \quad (4.1)$$

Thus, λ is given by Eq. (2.24). For nuclei that consist of one nucleon more outside a closed shell (or one nucleon less than required to close a shell)

$$\lambda = \pm (2\ell+1)^{-1} ; \quad J = \ell \pm 1/2 \quad (4.2)$$

Note that λ has the same interpretation as Δq , i.e., it gives the fraction of the angular momentum carried by the nucleon.

The single-particle nuclear shell model is not always a good approximation and in several cases is a poor description of the nucleus. As a means of gauging the quality of the model, we shall calculate for each nucleus of interest the magnetic dipole moment, since this is a vector operator that is related to $\vec{S}_p$, and compare with the experimental value. For a single nucleon in an orbital of angular momentum ℓ , the magnetic dipole moment is defined (in units of the nuclear magneton) as [26]

$$\mu \equiv \langle g \ell_z + \mu_i \sigma_z \rangle_{J_z=J} \quad (4.3)$$

where $g = 1$ and $\mu_i = 2.79$ for protons (0 and -1.91 for neutrons respectively), the expectation value is in the state with $J_z = J$, and $\ell_z (\frac{1}{2}\sigma_z)$ is the z-component of the orbital angular momentum (spin). Expressing $|N\rangle_{J_z=J}$ in terms of eigenstates of ℓ_z, σ_z [26]

$$\mu = \left(J - \frac{1}{2}\right)g + \mu_i \quad ; \quad J = \ell + \frac{1}{2} \quad (4.4a)$$

$$\mu = Jg + \frac{J}{J+1} \left(\frac{1}{2}g - \mu_i\right) ; \quad J = \ell - \frac{1}{2} \quad (4.4b)$$

In Table 4 we give the value of $\lambda^2 J(J+1)$ for various nuclei that are of interest for future dark matter searches in the laboratory of particle candidates with spin-dependent interactions [25,34]. The value of μ_{Th} is calculated using (4.4). For nuclei that have one nucleon missing in a shell, one can attempt a description in terms of one unpaired nucleon in a single vacancy shell or one nucleon hole in a filled shell. Although this makes no difference for the evaluation of μ_{Th} in (4.4), it leads to a different sign of the electric quadrupole moment of the nucleus. Thus, the choice in Table 4 is made according to the experimental sign of the electric quadrupole moment. This choice would be immaterial in (2.29) since $\sigma \propto (\sum Q_q^2)^2$; however, if the LSP is neither pure photino nor pure higgsino, the scattering cross-sections for nucleons and nucleon holes will greatly differ if there is significant mixing.

We now discuss the deuteron 2_1H . The deuteron has $J = 1$ and one would naively expect the ground state to consist of an unpaired neutron and proton in an S-wave with $\vec{J} = \vec{S}_n + \vec{S}_p$; thus the fraction of the angular momentum carried by each is $\lambda = \frac{1}{2}$ and $\lambda^2 J(J+1) = \frac{1}{2}$. The small correction from this naïve value in Table 4 is due to the presence of a D-wave component in the ground state wave function (without which the electric quadrupole moment would vanish); since the strong interactions are parity-conserving, it is reasonable to assume that the ground state only contains D-S mixing. Writing $|{}^2_1H\rangle = a|S\rangle + b|D\rangle$, the magnetic dipole moment turns out to be:

$$\mu_{Th}({}^2_1H) = 0.879 |a|^2 + 0.31 |b|^2 \quad (4.5)$$

Fitting μ_{Th} to the experimental value $\mu_{Ex} = 0.857$ and using the normalization condition $|a|^2 + |b|^2 = 1$ yields

$$|a|^2 = 0.962 \quad ; \quad |b|^2 = 0.038 \quad (4.6)$$

We need to evaluate $\langle {}^2_1H | \vec{S}_p | {}^2_1H \rangle = \langle {}^2_1H | \vec{S}_n | {}^2_1H \rangle = \lambda \langle {}^2_1H | \vec{J} | {}^2_1H \rangle$ in order to determine the LSP cross-section. Writing the S and D wave components in terms of eigenstates of $\vec{L}$ and $\vec{S}$ (the total orbital and spin angular momentum)

$$|S\rangle_{J_z=1} = |L_z=0; S_z=1\rangle \quad (4.7a)$$

$$|D\rangle_{J_z=1} = \sqrt{\frac{3}{5}} |L_z=2; S_z=-1\rangle - \sqrt{\frac{3}{10}} |L_z=1; S_z=0\rangle + \frac{1}{\sqrt{10}} |L_z=0; S_z=1\rangle \quad (4.7b)$$

we obtain:

$$\lambda_S = \langle S | (\vec{S}_p)_z | S \rangle = 1/2 \quad (4.8a)$$

$$\lambda_D = \langle D | (\vec{S}_p)_z | D \rangle = -1/4 \quad (4.8b)$$

Thus:

$$\langle {}^2_1H | \vec{S}_p | {}^2_1H \rangle = \frac{1}{2} (|a|^2 - \frac{1}{2} |b|^2) \langle {}^2_1H | \vec{J} | {}^2_1H \rangle \quad (4.9)$$

and this gives $\lambda^2 J(J+1)$ for the deuteron in Table 4c. (The estimates of $\lambda^2 J(J+1)$ for the other elements of Table 4c were obtained in a similar way.) Therefore, the photino elastic scattering cross-section off deuterium D is given by (assuming degenerate squark masses):

$$\sigma(\tilde{\gamma} D \rightarrow \tilde{\gamma} D) = \frac{4 m_D^2 m_{\tilde{\gamma}}^2}{\pi (m_D + m_{\tilde{\gamma}})^2} \left(\frac{e^2}{m_{\tilde{q}}^2} \right)^2 \frac{0.89}{2} (\alpha' + \beta')^2 \quad (4.10)$$

where the hadronic matrix elements are:

$$\alpha'^2 = F_p \equiv \left(\sum_{\text{proton}} Q_q^2 \Delta q \right)^2 \quad (4.11a)$$

$$\beta'^2 = F_n \equiv \left(\sum_{\text{neutron}} Q_q^2 \Delta q \right)^2 \quad (4.11b)$$

and are given in Table 3a.

For a general LSP composition we write $\alpha'^2 = F_p \cdot G_p$ (and similarly for the neutron $\beta'^2 = F_n \cdot G_n$) with

$$G_p \equiv \left\{ \frac{m_{\tilde{f}}^2}{e^2} \sum_{\text{proton}} (A_q - B_q) \Delta q \right\}^2 / F_p \quad (4.12)$$

here $m_{\tilde{f}}$ is the sfermion mass determined by $\Omega_{\text{LSP}} = 1$ if all sfermions are degenerate and $\text{LSP} = \tilde{\gamma}$ and we determine it numerically from (2.17) (e.g. [11], $m_{\tilde{f}} \approx 57 \text{ GeV} (1 + 0.04(m_{\tilde{\gamma}}/\text{GeV})^2)^{1/4}$ for a pure photino of mass $30 \text{ GeV} \gtrsim m_{\tilde{\gamma}} \gtrsim 4.8 \text{ GeV}$). Finally, A_q and B_q are given by (2.10), (2.14), Table 1 and (2.17). If the LSP becomes pure Higgsino, $(e^2/m_{\tilde{f}}^2)^2 \rightarrow 2G_F^2 \cos^2 2\beta$, where G_F is the Fermi constant and $\tan\beta = v_2/v_1$ is determined by $\Omega_{\text{LSP}} = 1$ instead if we assume $m_{\tilde{f}}/m_{\tilde{\gamma}} \ll 1$. Equation (4.10) then becomes

$$\sigma(\tilde{A}D \rightarrow \tilde{A}D) = \frac{4m_D^2 m_{\tilde{H}}^2}{\pi(m_D + m_{\tilde{H}})^2} \left(\frac{g_2^2}{4M_W^2} \right)^2 \cos^2 2\beta \frac{0.89}{2} (\alpha' + \beta')^2 \quad (4.13)$$

with $\alpha' = \sum_{\text{proton}} T_q^3 \Delta q$ and $\beta' = \sum_{\text{neutron}} T_q^3 \Delta q$ instead. These hadronic matrix elements are given in Table 3b.

Note that the EMC results imply $\sum_{\text{proton}} Q_q^2 \Delta q \sim -\sum_{\text{neutron}} Q_q^2 \Delta q$, while the NQM implies $F_p \gg F_n$. Thus $(\alpha' + \beta')_{\text{EMC}}^2 \ll (\alpha' + \beta')_{\text{NQM}}^2$. Thus, the predicted quality of a nucleus as LSP detector can change dramatically in some cases depending on whether EMC or NQM estimates of the hadronic matrix elements are used. We now turn to our results on the LSP interaction with matter.

5. - DISCUSSION AND CONCLUSIONS

We combine the results discussed in the previous sections to obtain the rate of LSP elastic scattering off nuclei in the laboratory [25]

$$R = \frac{3.6 \text{ events}}{\text{kg} \cdot \text{day}} \left[\frac{100 \text{ GeV}}{m_{\tilde{f}}} \right]^4 F_{p,n} G_{p,n} F_M \left(\frac{\rho}{0.3 \text{ GeV cm}^{-3}} \right) \left(\frac{\langle |\vec{u} - \vec{v}_0| \rangle}{320 \text{ km s}^{-1}} \right) \quad (5.1)$$

where F_p (F_n) and G_p (G_n) should be used if the nucleus has one unpaired proton (neutron) or an appropriate combination such as (4.10) if more than one nucleon is unpaired. The figure of merit F_M is defined as:

$$F_M = (\text{Abundance}) \cdot \frac{4 m_{\text{LSP}} m_N}{(m_{\text{LSP}} + m_N)^2} \frac{4}{3} \lambda^2 J(J+1) \quad (5.2)$$

so that $F_M \leq 1$. The local halo density, ρ , is uncertain by about a factor of 1.5; a plausible range if galaxies form in pre-existing dark matter halos, as it is widely believed, is [35] $0.2 \leq \rho / \text{GeV cm}^{-3} \lesssim 0.45$, and for more likely values of the parameters, $0.3 \lesssim \rho / \text{GeV cm}^{-3} \lesssim 0.45$. Mass models of the galaxy imply similar values [36]. We assume that the halo particles have a Maxwellian distribution of velocities in their rest frame and assume that the local escape velocity is much larger than the halo particle's velocity dispersion, which with these assumptions is given by $\langle u^2 \rangle = 3/2 v_0^2$, where $v_0 = 220 \pm 20 \text{ km s}^{-1}$ is the circular velocity at the solar radius [37]. Neglecting the earth's motion about the sun, the average incident velocity of the halo particles in the earth's rest frame is $\langle |\vec{u} - \vec{v}_0| \rangle \approx 1.2 (\langle u^2 \rangle)^{1/2} \approx 320 \text{ km s}^{-1}$; this is the last factor in (5.1) (including the solar motion relative to the local standard of rest nearly cancels the effect of the escape velocity).

First we discuss our results for $G_{p,n}$ in (5.1) which are presented in Table 5 and Fig. 1 for protons (solid lines) and neutrons (dotted line) as functions of m_{LSP} , for EMC estimates of Δq , $\epsilon = M_W$ and various values of v_1/v_2 (e.g., in minimal supergravity models inspired by the $E_8 \times E_8$ superstring theory compactified on a Calabi-Yau manifold [9], a plausible range is $1.7 \lesssim v_1/v_2 \lesssim 5$, and $\epsilon \gtrsim M_2$). The discontinuity of the various curves at $m_{\text{LSP}} = 4.8 \text{ GeV}$ is due to the step-function approach used to describe the variation of N_F at the deconfinement transition temperature, $T_H = 0.25 \text{ GeV}$ [38] (see Table 2); $T_f = T_H$ for $m_{\text{LSP}} \sim 5 \text{ GeV}$ since $T_f/m_{\text{LSP}} \sim 1/20$. For $v_1 \gtrsim 3v_2$ the sneutrino mass $m_{\tilde{\nu}} < m_{\text{LSP}}$ if the candidate spin- $1/2$ LSP mass is below this threshold (due to the last term in (2.10)) and the $\tilde{\nu}$ becomes the LSP. This is ruled out [13] for $m_{\tilde{\nu}} > m_{\text{ev}}$ ($\sim 3 \text{ GeV}$ [39]), the evaporation mass limit above which the LSP is captured in large numbers by the sun. In this case there would be abundant LSP annihilation and, correspondingly, too large a high-energy neutrino flux from the Sun if $\text{LSP} = \tilde{\nu}$ [13], which is not seen

in underground detectors. The curve for $v_1 = 4v_2$ has been stopped short of the 4.8 GeV limit only for clarity of the figure.

The behaviour of $G_{p,n}$ as a function of m_{LSP} can be readily understood from (2.10). For $v_1 = v_2$, the radiative corrections $\sim C_{f,L,R} M_{\frac{1}{2}}^2$ in (2.10) considerably split squarks and sleptons and the lightest squark has $m_{LSQ} > m_{\tilde{f}}$, thus $G_{p,n} < 1$. As $m_{LSP} \rightarrow 0$ ($M_{\frac{1}{2}} \rightarrow 0$) the radiative corrections become smaller and $G_{p,n} \rightarrow 1$. For $v_1 \neq v_2$, the up squarks can become considerably lighter and $m_{\tilde{u}} < m_{\tilde{f}}$, thus $G_{p,n} > 1$. The downward trend of $G_{p,n}$ for larger m_{LSP} is due to the radiative corrections, since $M_{\frac{1}{2}}$ becomes larger. However, for sufficiently large m_{LSP} , $G_{p,n}$ turns around and increases again due to an increasingly larger higgsino component in the LSP; the presence of this component causes $A_d - B_d$ in (4.12) to become negative (at $m_{LSP} \sim 16$ GeV for $v_1 = 4v_2$) and, since $\Delta d < 0$, to interfere constructively. The presence of the higgsino component causes the rapid increase in $G_{p,n}$ thereafter because $G_{p,n}$ becomes independent of squark masses [(2.14a) and (2.14b) dominate] and tends to the ratio of hadronic matrix elements of higgsinos to photinos, equal to $(\Sigma T_q^3 \Delta q)^2 / (\Sigma Q_q^2 \Delta q)^2 \sim 10$ with EMC estimates of Δq . The solid line that splits off the curve for $v_1 = 2v_2$ corresponds to the value of $G_{p,n}$ for a proton hole; it splits below G_p because the higgsino admixture causes (2.14a) and (2.14b) to reverse sign for a hole, while (2.14c) and (2.14d) do not, and destructive interference results. Finally, we discuss the effect of different values for ϵ ; the vertical bars at $m_{LSP} = 7, 15, 20$ and 30 GeV correspond to the variation in $G_{p,n}$ when ϵ is varied by a factor of 2 about M_W (higher $G_{p,n}$ for lower ϵ , again because of more higgsino component for lower ϵ); we see that for $m_{LSP} \lesssim 20$ GeV, the value of $G_{p,n}$ is quite insensitive to the choice of ϵ . For $-2M_W \lesssim \epsilon \lesssim -0.5 M_W$ a similar behaviour occurs (for $\epsilon \lesssim -2M_W$, though, the LSP drastically changes and becomes mostly zino in the mass range of Fig. 1 if M_2 is large; $G_{p,n}$ is very small in this case).

The factor $G_{p,n}$ in (5.1) alters previous predictions of the high energy solar neutrino flux that results from LSP annihilation in the Sun if $m_{LSP} > m_{ev}$, although the radiative corrections that yield $G < 1$ for $v_1 = v_2$ were already included in one of the models discussed in Ref. [14]. While a small range of masses below 10 GeV might be excluded for values of v_1/v_2 that result in $G \gtrsim 3$ (see Fig. 1 in [14]), due to null detection of this neutrino flux by large underground detectors, we see that $G < 1$ quite generally for $m_{LSP} \sim 10$ -20 GeV and $v_1 \lesssim 4v_2$. Thus, the LSP will be much harder to see either in the laboratory or indirectly in underground detectors.

We now turn to the figure of merit F_M . Our results are plotted in Fig. 2, for the abundances and values of $\lambda^2 J(J+1)$ given in Table 4, and for $m_{LSP} = 5, 20$ and 50 GeV. For different LSP masses the points move to the right (left) parallel to the solid line, which represents the value of F_M for a nucleus of abundance = 100%

and largest $\lambda^2 J(J+1)$; the arrows indicate elements with $F_M < 10^{-2}$. It is thus quite easy to determine the elements with the highest rates from Fig. 2. Note, however, that considerably better performance could be obtained from certain elements if commercially available, enriched samples that we are not aware of could be used.

We have combined our results for F_M and G to give the highest rates ($F_M > 0.3$) for $m_{LSP} = 4, 8, 15, 20$ GeV in Table 6, both for EMC and NQM estimates of Δq . We see that considerably different rates, and correspondingly different strategies to detect the LSP, result for these elements. Note that while $R_{EMC} \gg R_{NQM}$ for an element with an unpaired neutron only, $R_{EMC} \ll R_{NQM}$ if an additional unpaired proton occurs.

In conclusion, we have presented detailed predictions for spin- $\frac{1}{2}$ LSP scattering off matter in models we believe to contain many realistic features, such as unequal squark masses and Higgs v.e.v.s, that have not been considered in this context before. We have assumed $\Omega_{LSP} = 1$, but we note that current constraints from underground detectors already [14] constrain Ω_{LSP} to be quite high if the LSP makes up the halo of our galaxy. Our main conclusions are:

- The LSP elastic scattering cross-section is considerably enhanced above the value implied by the assumption of degenerate sfermions by unequal Higgs v.e.v.s for $m_{LSP} \lesssim 10$ GeV, while it is quite generally suppressed for $m_{LSP} \sim 10$ -20 GeV. The predictions become very model-dependent for $m_{LSP} \gtrsim 20$ GeV.

- The strategy for choosing elements for LSP detection in the laboratory may be greatly altered by the recent EMC result on the spin-dependent structure function of the proton. The EMC result implies generally smaller rates than previously thought, but while it makes some even-A elements (with one unpaired proton and neutron) poor LSP detectors, it makes odd-A elements with unpaired neutrons competitive LSP detectors.

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Table 1

Values of the renormalization constants $C_{f,L,R}$ [17] for the sleptons ($\tilde{\ell}$) and squarks ($\tilde{q}$) together with corresponding hypercharges Y and third component of weak isospin T^3 .

$C_{\ell,L}$	0.496	$Y_{\ell,L}$	-1	$T_{\ell,L}^3$	$\pm 1/2$
$C_{\ell,R}$	0.149	$Y_{\ell,R}$	-2	$T_{\ell,R}^3$	0
$C_{q,L}$	5.290	$Y_{q,L}$	+1/3	$T_{q,L}^3$	$\pm 1/2$
$C_{u,R}$	4.893	$Y_{u,R}$	+4/3	$T_{u,R}^3$	0
$C_{d,R}$	4.843	$Y_{d,R}$	-2/3	$T_{q,R}^3$	0

Table 2

Ratio of the photon to the LSP temperature, after all particles with masses $m < T_f$ have frozen out, for T_f in the range indicated. N_F is the effective number of relativistic degrees of freedom at $T = T_f$ and $T_H = 0.25$ GeV [38].

T_f	$(T_\gamma/T_{LSP})^3$	N_F
$m_\pi - T_H$	4.41	69/8
$T_H - m_c$	15.8	247/8

Table 3

Squared expectation values of hadronic matrix elements for (a) photino and (b) higgsino elastic scattering off protons (p) and neutrons (n) using EMC and NQM estimates of Δq and assuming degenerate squarks for (a) and $m_{\tilde{t}}/m_{\tilde{b}} \ll 1$ for (b).

Table 3a

$ \langle Q_q^2 \Delta q \rangle ^2$	EMC	NQM
p	0.050	0.16
n	0.036	0.00028

Table 3b

$ \langle T_q^3 \Delta q \rangle ^2$	EMC	NQM
p	0.55	0.39
n	0.26	0.39

Table 4

Nuclear shell model predictions for $\lambda^2 J(J+1)$ for nuclei of interest in dark matter searches in the laboratory [25,34]. The first column identifies the element by chemical symbol and total nucleon (proton) number A(Z). The second column gives the experimentally determined total angular momentum J. The third column is the natural abundance of the given isotope, except for those indicated by a superscript e for which a commercially available enriched sample is assumed. The fourth column gives the shell model description of the nucleus in standard notation [26]. Columns 5 and 6 contain respectively the shell model prediction and the measured value of the magnetic dipole moment. Table 4a, 4b, 4c contain the nuclei which are described by one unpaired proton (neutron, proton and neutron). The data of columns 2 and 6 were taken from "Table of Isotopes", eds. C.M. Lederer and V.S. Shirley (Wiley, New York, 1978).

Table 4a

A Z Isotope	J	Abundance	Shell Model	μ_{Th}	μ_{Ex}	$\lambda^2 J(J+1)$
1_1H	1/2	100	$S_{1/2}$ proton	2.79	2.79	3/4
7_3Li	3/2	100 ^e	$P_{3/2}$ proton	3.79	3.256	5/12
$^{11}_5B$	3/2	100 ^e	$P_{3/2}$ proton hole	3.79	2.689	5/12
$^{15}_7N$	1/2	0.37	$P_{1/2}$ proton hole	-0.26	-0.283	1/12
$^{19}_9F$	1/2	100	$S_{1/2}$ proton	2.79	2.629	3/4
$^{27}_{13}Al$	5/2	100	$D_{5/2}$ proton hole	4.79	3.642	7/20
$^{35}_{17}Cl$	3/2	100 ^e	$D_{3/2}$ proton	0.13	0.822	3/20
$^{51}_{23}V$	7/2	99.8	$F_{7/2}$ prpton	5.79	5.151	9/28
$^{69}_{31}Ga, ^{71}_{31}Ga$	3/2	60.1, 39.9	$P_{3/2}$ proton hole	3.79	2.017 2.562	5/12
$^{75}_{33}As$	3/2	100	$P_{3/2}$ proton hole	3.79	1.439	5/12
$^{79}_{35}Br, ^{81}_{35}Br$	3/2	50.7, 49.3	$P_{3/2}$ proton hole	3.79	2.106 2.271	5/12
$^{93}_{41}Nb$	9/2	100	$G_{9/2}$ proton	6.79	6.171	11/36
$^{107}_{47}Ag, ^{109}_{47}Ag$	1/2	57.8, 48.2	$P_{1/2}$ proton	-0.26	-0.114 -0.131	1/12
$^{121}_{51}Sb, ^{123}_{51}Sb$	5/2, 7/2	57.3, 42.7	$G_{7/2}$ proton	1.71	3.359 2.547	7/36
$^{122}_{53}I$	5/2	100	$D_{5/2}$ proton	5.79	2.808	5/20
$^{133}_{55}Cs$	7/2	100	$G_{7/2}$ proton	1.71	2.579	7/36
$^{139}_{57}La$	7/2	99.9	$G_{7/2}$ proton hole	1.71	0.50	7/36
$^{203}_{81}Tl, ^{205}_{81}Tl$	1/2	29.5, 70.5	$S_{1/2}$ proton	2.79	1.622 1.638	3/4

Table 4b

$\frac{A}{Z}$ Isotope	J	Abundance	Shell Model	μ_{Th}	μ_{Ex}	$\lambda^2 J(J+1)$
^3_2He	1/2	100 ^e	S _{1/2} neutron	-1.91	-2.128	3/4
^9_4Be	3/2	100	P _{3/2} neutron hole	-1.91	-1.178	5/12
$^{17}_8\text{O}$	5/2	0.04	D _{5/2} neutron	-1.91	-1.890	7/20
$^{29}_{14}\text{Si}$	1/2	4.7	S _{1/2} neutron	-1.91	-0.555	3/4
$^{47}_{22}\text{Ti}, ^{49}_{22}\text{Ti}$	5/2, 7/2	7.3, 5.5	F _{7/2} neutron hole	-1.91	-0.789 -1.104	9/28
$^{67}_{30}\text{Zn}$	5/2	4.11	F _{5/2} neutron hole	1.36	0.874	5/28
$^{73}_{32}\text{Ge}$	9/2	7.8	G _{9/2} neutron	-1.72	-0.879	11/36
$^{91}_{40}\text{Zr}$	5/2	11.3	D _{5/2} neutron	-1.91	-1.304	7/20
$^{111}_{48}\text{Cd}, ^{113}_{48}\text{Cd}$	1/2	12.8, 12.2	S _{1/2} neutron	-1.91	-0.594 -0.622	3/4
$^{115}_{50}\text{Sn}, ^{117}_{50}\text{Sn}$	1/2	0.4, 7.7	S _{1/2} neutron	-1.91	-0.918 -1.000	3/4
$^{199}_{80}\text{Hg}$	1/2	17.0	P _{1/2} neutron	0.64	0.506	1/12
$^{207}_{82}\text{Pb}$	1/2	22.1	P _{1/2} neutron hole	0.64	0.593	1/12

Table 4c

$\frac{A}{Z}$ Isotope	J	Abundance	Shell Model	μ_{Th}	μ_{Ex}	$\lambda^2 J(J+1)$
^2_1H	1	100 ^e	S ₁ +D ₁	fit	0.857	(1/2)(0.89)
^6_3Li	1	7	S ₁ +D ₁	fit	0.822	(1/2)(0.72)
$^{10}_5\text{B}$	3	100 ^e	D ₃	1.88	1.801	1/3
$^{14}_7\text{N}$	1	99.6	S ₁ +D ₁	fit	0.404	(1/2)(0.06)

Table 5

Correction factor G for several LSP masses and for various values of v_1/v_2 . We have taken $\epsilon = M_W$ here and used current algebra masses throughout.

$\begin{array}{c} G_p \\ M_{LSP} \end{array}$	v/v 1 2		
GeV	1	2	4
3	0.90	3.76	$m_{\tilde{\nu}} < m_{LSP}$
4	0.85	3.43	
6	0.82	2.12	2.94
8	0.74	1.64	2.29
10	0.67	1.31	1.78
15	0.52	0.83	1.14
20	0.40	0.62	1.14
30	0.24	0.61	3.51

$\begin{array}{c} G_n \\ M_{LSP} \end{array}$	v/v 1 2		
GeV	1	2	4
3	0.90	3.29	$m_{\tilde{\nu}} < m_{LSP}$
4	0.84	3.02	
6	0.81	1.93	2.61
8	0.73	1.52	2.05
10	0.66	1.22	1.60
15	0.51	0.77	1.00
20	0.39	0.57	0.92
30	0.23	0.50	2.34

Table 6

LSP interaction rate in the laboratory for nuclei with $F_M > 0.3$ in the LSP mass range $4 \leq m_{\text{LSP}}/\text{GeV} \leq 20$, and for $v_1 = v_2$ and $\varepsilon = M_W$. The upper (lower) value assumes EMC (NQM) values of the Δq 's. For hydrogen an abundance of 100% was assumed, although one would need the fraction of it in dielectric materials if used as target material; see the article of L. Gonzalez-Mestres and D. Perret-Gallix in Ref. [34].

m_{LSP} Isotope	[GeV]	[GeV]	[GeV]	[GeV]
^1_1H	0.29 0.89	0.0085 0.026	0.0013 0.0037	0.00046 0.0013
^2_1H	0.022 2.8	0.0029 0.33	0.00050 0.051	0.00019 0.025
^3_2He	1.4 0.017	0.18 0.0030	0.032 0.00071	0.012 0.00033
^7_3Li	1.0 3.1	0.18 0.55	0.039 0.12	0.016 0.048
^9_4Be	0.66 0.0086	0.14 0.0021	0.030 0.00068	0.013 0.00035
$^{10}_5\text{B}$	0.016 2.0	0.0035 0.40	0.00090 0.092	0.00039 0.052
$^{11}_5\text{B}$	0.84 2.6	0.18 0.54	0.044 0.13	0.019 0.055
$^{19}_9\text{F}$	1.2 3.5	0.29 0.87	0.083 0.25	0.038 0.12
$^{27}_{13}\text{Al}$	0.42 1.3	0.11 0.33	0.036 0.11	0.018 0.051
$^{51}_{23}\text{V}$	0.23 0.71	0.068 0.21	0.026 0.076	0.014 0.040
$^{69}_{31}\text{Ga} + ^{71}_{31}\text{Ga}$	0.23 0.70	0.070 0.21	0.028 0.082	0.015 0.045
$^{75}_{33}\text{As}$	0.21 0.66	0.066 0.20	0.027 0.079	0.015 0.043
$^{79}_{35}\text{Br} + ^{81}_{35}\text{Br}$	0.20 0.62	0.063 0.19	0.026 0.075	0.014 0.041
$^{203}_{81}\text{Tl} + ^{205}_{81}\text{Tl}$	0.15 0.46	0.050 0.15	0.022 0.066	0.013 0.038

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FIGURE CAPTIONS

- Fig. 1 : Correction factors $G_{p,n}$, for protons and neutrons, as a functions of LSP mass for $\epsilon = M_W$ and various values of v_1/v_2 . The vertical bars indicate the variation of G for $\frac{1}{2}M_W \leq \epsilon \leq 2M_W$. See text for explanations.
- Fig. 2 : Figure of merit, F_M , for the various nuclei of Table 4 and (a) $m_{LSP} = 5$ GeV, (b) $m_{LSP} = 20$ GeV and (c) $m_{LSP} = 50$ GeV (where necessary the nuclei are distinguished by the total number of nucleons $A = Z+N$). See text for explanations.

Fig. 1

Fig. 2a

Fig. 2b

Fig. 2c