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GHOST VERTICES FOR THE BOSONIC STRING USING THE GROUP THEORETIC APPROACH TO STRING THEORY

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ABSTRACT

The N-string tree-level scattering vertices for the bosonic string are extended to include anticommuting (ghost) oscillators. These vertices behave correctly under the action of the BRST charge Q and reproduce the known results for the scattering of physical states. This work is an application of the group-theoretic approach to string theory.

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In several recent papers an extremely efficient method for the calculation of string S-matrix elements has been developed.^[1–4] At tree level the scattering of N physical states is given by an expression of the form

$$\int d\mu(z_i) V(z_1, \dots, z_N) |\psi_1\rangle \cdots |\psi_N\rangle, \quad (1)$$

where $|\psi_i\rangle$ are on-shell states, $V(z_1, \dots, z_N)$ is an N -point vertex depending on N Koba-Nielsen variables z_i , and $d\mu(z_i)$ is an integration measure. The vertex V is determined by requiring that it satisfy certain overlap identities for operators of definite conformal weight. The integration measure is fixed by the requirement that zero-norm physical states should decouple from the S-matrix; this condition leads to a set of first-order differential equations which determine the measure.

In order to write down the overlap identities satisfied by V let us first introduce a complex coordinate ξ_i in a neighbourhood U_i of each Koba-Nielsen point, with the property that ξ_i vanishes at z_i . In the coordinate patch U_i we now introduce an operator $R^{(i)}(\xi_i)$ of conformal weight d acting in the Hilbert space of the i th external string. The identity that we wish to impose on the vertex simply states that, in an overlap region where $R^{(i)}(\xi_i)$ and $R^{(j)}(\xi_j)$ are both defined, each operator gives the same result when referred to the same coordinate system. Under a coordinate transformation from ξ_j to ξ_i , $R^{(j)}(\xi_j)$ transforms as

$$R^{(j)}(\xi_j) \rightarrow \left[\frac{\xi_i}{\xi_j} \frac{d\xi_j}{d\xi_i} \right]^d R^{(j)}(\xi_j(\xi_i)); \quad (2)$$

this is just the definition of an operator of conformal weight d . We therefore require that the vertex satisfy the condition^[1–5]

$$V R^{(i)}(\xi_i) = V \left[\frac{\xi_i}{\xi_j} \frac{d\xi_j}{d\xi_i} \right]^d R^{(j)}(\xi_j(\xi_i)). \quad (3)$$

Here we have considered the interpretation of this identity from the point of view of coordinate transformations on the Riemann surface – this is a passive viewpoint. An equivalent interpretation from an active viewpoint is given in ref.[4].

Rather than work with the overlap identity as given in eq.(3) it is often convenient to use an integrated form of the overlap. To derive this we note from the conformal transformation

properties of an operator $R^{(i)}(\xi_i)$ of weight d that $(\xi_i)^{-d} R^{(i)}(\xi_i)$ transforms as a tensor of rank d . This means that under a change of variables $\xi_i \rightarrow \xi'_i(\xi_i)$ we have

$$\frac{R^{(i)}(\xi_i)}{(\xi_i)^d} \rightarrow \frac{R^{(i)}(\xi'_i)}{(\xi'_i)^d} = \left(\frac{d\xi_i}{d\xi'_i} \right)^d \frac{R^{(i)}(\xi_i)}{(\xi_i)^d}. \quad (4)$$

Thus for d positive $(\xi_i)^{-d} R^{(i)}(\xi_i) (d\xi_i)^d$ is invariant under conformal transformations, while for d negative $(\xi_i)^{-d} R^{(i)}(\xi_i) (\partial/\partial \xi_i)^{-d}$ is similarly invariant. If now $\phi(\xi_i)$ is a tensor of rank $1-d$ we can construct the 1-form

$$d\xi_i \phi(\xi_i) \frac{R^{(i)}(\xi_i)}{(\xi_i)^d}. \quad (5)$$

Let us consider the expression

$$V \oint_{\xi_i=0} d\xi_i \phi(\xi_i) \frac{R^{(i)}(\xi_i)}{(\xi_i)^d}. \quad (6)$$

We wish to deform the integration contour around the Riemann surface until it surrounds all other Koba-Nielsen points. As we do this we find that $VR^{(i)}(\xi_i)$ diverges as we approach each Koba Nielsen point $\xi_j = 0$. It is then necessary to use the overlap eq. (3) to replace $VR^{(i)}(\xi_i)$ by $VR^{(j)}(\xi_j)$, which leads to a convergent expression. In this way we obtain the integrated identity^[1-6] *

$$V \sum_{j=1}^N \oint_{\xi_j=0} d\xi_j \phi(\xi_i) \left[\frac{d\xi_i}{d\xi_j} \right]^{1-d} \frac{R^{(j)}(\xi_j)}{(\xi_j)^d} = 0. \quad (7)$$

Here we have assumed that $\phi(\xi_i) [d\xi_i/d\xi_j]^{1-d}$ has poles only at the Koba-Nielsen points. This means that ϕ considered as a tensor field should be meromorphic with poles only at these points.

Let us consider the implications of this for tree level scattering. If $\phi \sim (\xi_i)^n$ as $\xi_i \rightarrow 0$, the behaviour of ϕ as $\xi_i \rightarrow \infty$ is given by transforming to the other coordinate patch on the

* Similar identities have also been obtained subsequently from a different point of view by Vafa^[7] and Witten.^[8]

sphere, with coordinate $\zeta = (\xi_i)^{-1}$. Since ϕ is a tensor of rank $1-d$ it behaves like $\zeta^{-n-2(1-d)}$ as $\zeta \rightarrow 0$, so that in order for ϕ to be regular as $\xi_i \rightarrow \infty$ we must have $n \leq 2(d-1)$. We note in particular that there are no analytic tensors on the sphere for $d \leq 0$, while for $d \geq 1$ there are $2d-1$ such tensors.

So far we have not been at all specific about the way in which the coordinates ξ_i are to be chosen. The vertex will depend upon the transition functions between different coordinate patches, but as far as the scattering of on-shell physical states is concerned all choices lead to the same results. In this paper we shall be considering tree level scattering, so that the appropriate Riemann surface to consider is the sphere. Each coordinate patch will be chosen to cover the whole of the Riemann sphere apart from one point, and different coordinates will be related by $Sl(2, C)$ transformations. Here we shall take the transition functions to have the particular form

$$\xi_i = V_i^{-1}(V_j(\xi_j)), \quad (8)$$

with the V_i in $Sl(2, C)$. The V_i are known as cycling transformations, and the theory behind them is discussed in ref.[4]. There it is shown that V_i^{-1} can be any analytic function with a simple zero at z_i , and that in particular one can use the simple choice

$$(V_i)^{-1}(z) = z - z_i. \quad (9)$$

This leads to significant calculational simplifications, and we shall use this form extensively here.

In this paper we shall use the methods outlined above to determine the contribution of the ghost oscillators to the vertex, thereby obtaining a vertex which is invariant under BRST transformations. Although the inclusion of ghosts has so far not been necessary in this group theoretic approach to on-shell string scattering, one might nevertheless expect that certain advantages will follow from including them. The categorization of physical and zero-norm states takes a particularly elegant form in the BRST approach,^[9,10] and since the decoupling of zero-norm physical states is used to determine the measure this calculation should be simpler in the BRST formalism, particularly in the case of loop diagrams.

In reference [6] the Gaudin-Schwimmer-Veneziano vertex^[11] was extended to include the contribution of ghost oscillators by the above method, where it was shown that this

vertex V was BRST invariant,

$$V \sum_1^3 Q^{(i)} = 0, \quad (10)$$

and that it was related to the gauge-covariant three-vertex^[6,12] by a conformal transformation. Further discussions of the three-point vertex were given later in ref.[13]. Subsequently, using a path integral argument, a generalization of this result to an N -point vertex has been proposed.^[14] Unfortunately, however, a ghost number counting argument shows that for $N > 3$ this vertex gives zero on physical states. It is readily seen that if

$$N_{gh} = \sum_1^N \left(\sum_{n=-1}^{\infty} c_{-n}^{(i)} b_n^{(i)} - \sum_{n=2}^{\infty} b_{-n}^{(i)} c_n^{(i)} \right) \quad (11)$$

is the total ghost number operator^{*} this vertex has ghost number 3. Physical states, however, are identified with non-trivial cohomology classes of the BRST charge^[10,16] we take all states annihilated by Q and then identify those states which differ by a state of the form $Q|\psi\rangle$, which are referred to as zero-norm physical states. One takes physical states to be those cohomology classes of ghost number one, in which case we see that any amplitude involving more than three physical states must vanish. For convenience we point out here that the notation of this paper is related to that of ref.[6] by

$$\beta_n = c_n, \quad \bar{\beta}_n = b_n. \quad (12)$$

The purpose of this paper is to show how to construct a vertex which describes correctly the scattering of physical states in the BRST formalism and which has appropriate BRST transformation properties so as to ensure the decoupling of physical states. A straightforward application of the overlap identities for the ghost C , with conformal weight $d = -1$, and the antighost B , with $d = 2$, yields trivially the vertex V found earlier for a particular choice of cycling for the 3-point and N -point cases. Furthermore, the overlap identities determine completely the BRST transformation properties and the ghost number of the

* Here and elsewhere in this paper we normal order with respect to the $Sl(2, C)$ vacuum,^[15] so that the ghost number operator differs by $3/2$ from the more standard definition.

vertex. As an example let us consider the BRST current^[10,15]

$$j_{BRST}(z) =: \left\{ \frac{1}{2} C(z \partial X)^2 - BC(z \partial C) - \frac{3}{2} (z \partial)^2 C - \frac{3}{2} z \partial C \right\}: \quad (13)$$

This has conformal weight 1, so we would expect it to satisfy eq.(3) with $d = 1$. This can be verified by considering products of the identities for ∂X , B and C . In calculating such products one needs to subtract singular terms from the operator product expansions, but once this is done we find that j_{BRST} does indeed satisfy the expected identity. We omit the details here; the calculation is straightforward. Integrating this overlap then gives $V \sum Q^{(i)} = 0$. For the ghost number current $j =: CB$, on the other hand, an anomalous contribution arises from subtracting the divergence in the operator product of B and C .^[15] It is easy to see that in this case the vertex satisfies

$$V(N_{gh} - 3) = 0. \quad (14)$$

As we have discussed, this vertex is therefore not satisfactory for $N > 3$. From another point of view, the BRST transformation properties of V are not quite what one might expect. In the group theoretic method zero-norm physical states decouple only after integrating over the Koba-Nielsen variables, and in fact this decoupling uniquely determines the measure.^[1-4] The BRST transformation properties of V are such, however, that if one were to use it as a physical vertex zero-norm physical states would decouple without any such integration. This would leave the measure undetermined, in contradiction with the above. We shall show how a different vertex U can be constructed which has the required ghost number and which provides the correct answer, including the measure, for the scattering of physical states. The decoupling of zero-norm physical states requires a knowledge of how to change the moduli of the vertex,^[1-4] and we shall see that this implies corresponding BRST transformation properties of the vertex U .

Let us begin by showing how the integrated ghost and antighost overlap identities determine the vertex V . We will eventually give the vertex for an arbitrary cycling transformation, but for the present we work with the simple choice $(V_i)^{-1}(z) = z - z_i$. In this case the integrated overlap for the antighost B is

$$V \sum_{j=1}^N \oint_{\xi_j=0} d\xi_j \phi(\xi_j) \frac{B^{(j)}(\xi_j)}{\xi_j^2} = 0. \quad (15)$$

ϕ is a tensor of rank -1 , in other words a vector field, and must be analytic everywhere except at the Koba-Nielsen points. It follows from the discussion of convergence given earlier that we can take

$$\phi(\xi_i) = \xi_i^{-n}, \quad n \geq -2. \quad (16)$$

By taking the linear combinations $\phi = 1$, $\phi = \xi_i + z_i$, $\phi = \xi_i^2 + 2\xi_i z_i + z_i^2$ we obtain immediately the three identities

$$\begin{aligned} V \sum_{j=1}^N b_{-1}^{(j)} &= 0 \\ V \sum_{j=1}^N (b_0^{(j)} + z_j b_{-1}^{(j)}) &= 0 \\ V \sum_{j=1}^N (b_1^{(j)} + 2z_j b_0^{(j)} + z_j^2 b_{-1}^{(j)}) &= 0 \end{aligned} \quad (17)$$

which we denote by

$$VR^s = 0, \quad s = 1, 2, 3. \quad (18)$$

The remaining b identities are obtained by taking eq.(16) for $n \geq 1$,

$$\begin{aligned} \phi(\xi_i) = \xi_i^{-n} &= \frac{1}{(\xi_j + z_j - z_i)^n} \\ &= \sum_{p=0}^{\infty} \binom{-n}{p} \frac{\xi_i^p}{(z_j - z_i)^{n+p}} \end{aligned} \quad (19)$$

which leads to

$$V \left\{ b_{-n-1}^{(i)} + \sum_{\substack{i,j=1 \\ i \neq j}}^N \sum_{p=0}^{\infty} \binom{-n}{p} \frac{1}{(z_j - z_i)^{n+p}} b_{p-1}^{(j)} \right\} = 0, \quad n \geq 1. \quad (20)$$

For the ghost C , on the other hand, the integrated overlap is

$$V \sum_{j=1}^N \oint_{\xi_j=0} d\xi_j \phi(\xi_i) C^{(j)}(\xi_j) \xi_j^2 = 0. \quad (21)$$

This time ϕ is a tensor field of rank 2 and the criterion for convergence implies that the

allowed ϕ 's can be written in the form

$$\phi(\xi_i) = \xi_i^{-n}, \quad n \geq 4, \quad (22)$$

thus giving the identities

$$V \left\{ c_{-n+2}^{(i)} + \sum_{\substack{i,j=1 \\ i \neq j}}^N \sum_{p=0}^{\infty} \binom{-n}{p} \frac{1}{(z_j - z_i)^{n+p}} c_{p+2}^{(j)} \right\} = 0, \quad n \geq 4. \quad (23)$$

We note that these identities do not involve $c_{\pm 1}$ or c_0 .

In order to solve for the vertex let us introduce the vacuum $\langle 0|$ which satisfies

$$\begin{aligned} \langle 0|b_n &= 0, & n \leq -2 \\ \langle 0|c_n &= 0, & n \leq 1 \end{aligned} \quad (24)$$

Using the B identities the vertex is given in terms of this vacuum by

$$V = \left\{ \prod_1^N \langle 0| \right\} \exp \left\{ \sum_{\substack{i,j=1 \\ i \neq j}}^N \sum_{m=2}^{\infty} \sum_{p=0}^{\infty} c_m^{(i)} \frac{(-1)^p (p+m-2)!}{(m-2)! p! (z_j - z_i)^{m+p-1}} b_{p-1}^{(j)} \right\} \prod_1^3 R^s. \quad (25)$$

It is now easy to verify that this satisfies the integrated C identity as well as the unintegrated B and C identities

$$\begin{aligned} V \{ (z - z_i) C^{(i)}(z - z_i) - (z - z_j) C^{(j)}(z - z_j) \} &= 0 \\ V \{ (z - z_i)^{-2} B^{(i)}(z - z_i) - (z - z_j)^{-2} B^{(j)}(z - z_j) \} &= 0. \end{aligned} \quad (26)$$

As we have already discussed at some length, the vertex V is not in general the vertex which should be used to determine the scattering of physical states. In order to understand how to construct a more satisfactory vertex, let us consider the way in which V changes if we move a Koba-Nielsen point. In general V depends on the Koba-Nielsen variables through the cycling transformations V_i , and from the discussion given in refs[1-4] we can see that

under a change $V_i \rightarrow \hat{V}_i$ we have

$$V(\hat{V}_i) = V(V_i) \prod_{i=1}^N [V_i^{-1} \hat{V}_i]^{(i)}. \quad (27)$$

For the particular case $V_i^{-1}(z) = z - z_i$, $\hat{V}_i^{-1} = z - \hat{z}_i \equiv z - z_i - \delta_{ik}\epsilon$ it follows that

$$\frac{\partial V}{\partial z_k} = V L_{-1}^{(k)}. \quad (28)$$

It is precisely this equation, or its analogue for a more general cycling transformation, which is used in refs[1-4] to show the decoupling of zero-norm physical states. This decoupling and subsequent determination of the measure is a physical requirement that should be satisfied regardless of the particular formalism being used. With this in mind we motivate the construction of U by considering the form of eq.(28) when written in terms of the BRST charge. The total Virasoro generators are given by the anticommutator of Q with the antighost, so

$$\frac{\partial V}{\partial z_k} = V \{b_{-1}^{(k)}, \sum Q^{(i)}\}. \quad (29)$$

We have shown that $\sum Q^{(i)}$ annihilates V and thus

$$\frac{\partial V}{\partial z_k} = V b_{-1}^{(k)} \sum Q^{(i)}. \quad (30)$$

It follows that for each Koba-Nielsen variable z_k which is to be integrated over we can modify the vertex by including a term $b_{-1}^{(k)}$ without losing the property that spurious states decouple after integration. Conformal invariance indicates that we should integrate over only $N - 3$ of the Koba-Nielsen variables, so that we can include up to $N - 3$ factors $b_{-1}^{(i)}$. This, however, is precisely what is required in order to obtain a non-zero answer for the scattering of physical states; if we define

$$U = V \prod_{i=1}^N{}' b_{-1}^{(i)}, \quad (31)$$

where $\prod'$ means that three fixed values of i are to be omitted from the product, then U satisfies

$$U(N_{gh} - N) = 0. \quad (32)$$

Thus for physical states which have ghost number 1 we have the possibility of obtaining a non-zero answer. The above arguments strongly suggest that the actual physical vertex

which should be used to calculate scattering amplitudes is

$$W = \int \prod' dz_i U. \quad (33)$$

Eliminating the ghost and antighost oscillators leads to the vertex given in ref.[4], which for physical states yields scattering amplitudes which are in agreement with the results of Lovelace^[17] and Olive.^[18] One may also write the vertex in terms of the vacuum $\langle + | \equiv \langle 0 | b_{-1}$; one finds that U is just an exponential of oscillators on this vacuum.

We note that although U satisfies the integrated and unintegrated B identity and the integrated C identity, it does not satisfy the unintegrated C identity of eq.(26). In essence V is the part of U which is insensitive to the choice of which $N - 3$ legs are to be integrated over.

Let us now extend the previous results to the case of a general cycling transformation $\hat{V}_i$. Consider first the B identities. The three analytic vector fields may be chosen to be $\phi = 1$, $\phi = \hat{V}_i(\xi_i)$ and $\phi = (\hat{V}_i(\xi_i))^2$. These lead to three identities analagous to those of eq.(18) which we write as

$$\hat{V} \hat{R}^s = 0, \quad s = 1, 2, 3. \quad (34)$$

The remaining identities are found by taking $\phi = (\xi_i)^{-n}$ for $n \geq 1$. We define

$$(\xi_i)^{-n} \frac{d\xi_j}{d\xi_i} = \sum_{p=0}^{\infty} C_{np}^{ij} (\xi_j)^p, \quad n \geq 1, \quad j \neq i, \quad (35)$$

so that the B identity can be written as

$$\hat{V} \left\{ b_{-n-1}^{(i)} + \sum_{\substack{i,j=1 \\ i \neq j}}^N \sum_{p=0}^{\infty} C_{np}^{ij} b_{p-1}^{(j)} \right\} = 0, \quad n \geq 1. \quad (36)$$

The vertex is then given by

$$V = \left\{ \prod_1^N \langle 0 | \right\} \exp \left\{ \sum_{\substack{i,j=1 \\ i \neq j}}^N \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} c_{m+1}^{(i)} C_{mp}^{ij} b_{p-1}^{(j)} \right\} \prod_1^3 \hat{R}^s. \quad (37)$$

For the cycling transformation that leads to the Lovelace-Olive α -vertex^[17,18] we find the ghost extension of refs[6] for $N = 3$ and of ref.[14] for $N > 3$.

Rather than solve the overlap identities again to find the new vertex, however, we could simply have found it for the new choice of cycling transformation by using

$$\hat{V} \equiv V(\hat{V}_i) = V(V_i) \prod_1^N [V_i^{-1} \hat{V}_i]^{(i)}. \quad (38)$$

It is perhaps worth remarking that the overlap identities determine the vertex only up to a multiplicative c-number factor, and once we have chosen this factor for the original cycling transformation eq.(38) fixes it for any other choice. The fact that $V(\hat{V})$ satisfies all the required overlap identities as well as $V(\hat{V}) \sum Q^{(i)} = 0$ follows trivially from eq.(38).

A consequence of eq.(38) is that under a change in the Koba-Nielsen variables, $\hat{V}(z'_i) = \hat{V}(z_i) \prod [\hat{V}_i(z_i)^{-1} \hat{V}_i(z'_i)]$, and so in particular we have

$$\frac{\partial \hat{V}}{\partial z_k} = \hat{V} \sum_{j=1}^N \sum_n e_{ij}^n L_n^{(j)} \equiv \hat{V} L_{\phi_i}. \quad (39)$$

Given a specific $\hat{V}_i$ the coefficients e_{ij}^n are easily calculated and are discussed further in refs[1-4]. The line of argument given earlier suggests that we should take the physical vertex to be

$$\hat{U} = \hat{V} \prod_i' b_{\phi_i}, \quad (40)$$

where $b_{\phi_i} = \sum e_{ij}^n b_n^{(j)}$ and 3 values of i are omitted from the product. The explicit form of the product $\prod_i' b_{\phi_i}$ will in general be far more complicated than for the case of the simple cycling transformation given earlier. Acting on $\hat{U}$ with the BRST charge then gives

$$\hat{U} \sum Q^{(i)} = \sum_j' (-1)^{N-3-j} \frac{\partial}{\partial z_j} (\hat{V} \prod_{i \neq j}' b_{\phi_i}) + \sum_{k < j}' (\pm) \hat{V} \left\{ \frac{\partial b_{\phi_k}}{\partial z_j} - \frac{\partial b_{\phi_j}}{\partial z_k} + [L_{\phi_j}, b_{\phi_k}] \right\} \prod_{i \neq j, k}' b_{\phi_i}. \quad (41)$$

where the $+$ ($-$) sign occurs when the number of terms b_{ϕ_i} between b_{ϕ_j} and b_{ϕ_k} in the original product $\prod_i' b_{\phi_i}$ is odd (even). The first term vanishes when we integrate over Koba-Nielsen variables and we can see that the second gives zero by considering the following integrability condition resulting from eq.(39):

$$0 = \frac{\partial^2 \hat{V}}{\partial z_i \partial z_j} - (i \leftrightarrow j) = \hat{V} \left\{ \frac{\partial L_{\phi_k}}{\partial z_j} - \frac{\partial L_{\phi_j}}{\partial z_k} + [L_{\phi_j}, L_{\phi_k}] \right\} \quad (42)$$

Since L_n and b_n both have conformal weight two and satisfy the same identities when acting

on V we may conclude that

$$0 = \hat{V} \left\{ \frac{\partial b_{\phi_k}}{\partial z_j} - \frac{\partial b_{\phi_j}}{\partial z_k} + [L_{\phi_j}, b_{\phi_k}] \right\}. \quad (43)$$

Thus the result

$$\hat{W} \Sigma Q^{(j)} \equiv \int \Pi' dz_i \hat{V} \Pi' b_{\phi_i} \Sigma Q^{(j)} = 0 \quad (44)$$

follows. Applying BRST physical states to this vertex we recover the measure and scattering amplitude given in refs[1-4,17,18].

The vertex V seems more aesthetic than U . The vertex U , however, also has a well-defined mathematical meaning, which will be discussed elsewhere.

The extension of this work to loop graphs is given in a paper by one of the present authors.^[19]

Note added: While this work was in progress we received a paper^[20] which also noted that the vertices in ref.[14] gave zero on physical states.

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