CERN-TH.4947/88

MASSLESS PARTICLE BREMSSTRAHLUNG THEOREMS
FOR HIGH ENERGY HADRON INTERACTIONS

L.N. Lipatov^{*)}
CERN - Geneva

ABSTRACT

A generalization of the Low theorem for massless gauge boson and graviton emission in high energy hadron collisions is constructed for small transverse momenta of the produced particles.

^{*)} Permanent address: High Energy Theor.
Dept., Leningrad Nuclear Physics Inst.,
188 350 Gatchina, U.S.S.R.

1. - INTRODUCTION

It is well known that the experimental study of deep inelastic lepton-hadron interactions provides unique information on the quark structure of hadrons [1] if we use the QCD evolution equations for the quark-gluon distributions [2]. On the other hand, because of the Low theorem [3], the electromagnetic and weak processes in low-energy hadron collisions allow us to determine very important static hadronic characteristics such as electric charges, magnetic moments, electromagnetic radii and so on. With growing energy $\sqrt{s}$, the region of applicability of the classical formulas for the soft photon bremsstrahlung amplitudes (see Fig. 1):

$$A_{2 \rightarrow 2+\gamma} = e e_\mu \left(\frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} \right) A_{2 \rightarrow 2}(s, t), \quad S = 2p_1 p_2, \quad t = (p_1 - p_2)^2 \quad (1)$$

increases, as was shown several years ago by V.N. Gribov [4]. More exactly, for the charge exchange processes of the type $\pi^- p \rightarrow \pi^0 n$ in the Regge limit $s \gg m_{\text{char}}^2$ and fixed $t \sim m_{\text{char}}^2$ Eq. (1) turns out to be true in a wide region where the emitted photon has small relative energies and transverse components $K_\perp$ of its momentum

$$\beta \ll 1, \quad \alpha \ll 1, \quad |K_\perp| \ll m_{\text{char}}, \quad K = \beta p_1 + \alpha p_2 + K_\perp. \quad (2)$$

Here m_{char} is a characteristic value for the momentum transfer q in the elastic process.

In this paper we generalize Eq. (1) to the case of non-Abelian boson and graviton emission and calculate corrections $\sim K_\perp / m_{\text{char}}$ to it analogous to those in the Low theorem. In the multi-Regge kinematics, we suppose the Regge behaviour of the inelastic amplitude $A_{2 \rightarrow 3}$ to have correct analytic properties in the final state energies $\sqrt{s_1}, \sqrt{s_2}$ (see Fig. 1). For the quasi-elastic kinematics corresponding to the massless particle emission in the fragmentation region, more detailed information on the hadronic mass spectrum is necessary. We use the string model as a guide in this yet uninvestigated field. Massless particle production in the dual resonance model was studied more carefully in Ref. [5].

2. - THE LOW THEOREM FOR YANG-MILLS BOSON AND GRAVITON PRODUCTION

In this section we consider the cases when the soft particle produced in hadron-hadron collisions are non-Abelian gauge bosons or gravitons. The Low formulae for photon bremsstrahlung can be obtained from our final expressions by

the substitution $g\vec{T}_i \rightarrow e_i$ where $\vec{T}_i$ is the gauge group generator in the corresponding representation, e_i is the electric charge of the external hadron i , and g is the Yang-Mills coupling constant. If we identify these massless gauge particles with the Weinberg-Salam Z and W bosons before symmetry breaking, then the generalized Low theorem can be used for the calculation of probabilities for weak interaction soft lepton production.

The inelastic amplitude $A_{2 \rightarrow 3}$ depends on five invariants. We choose the following independent set of them

$$\begin{aligned} S_1 &\equiv 2 p_1 k, \quad S_2 \equiv 2 p_2 k, \quad S \equiv 2 p_1 p_2, \\ t_1 &= (p_1 - p_{1'})^2, \quad t_2 = (p_2 - p_2')^2. \end{aligned} \quad (3)$$

For simplicity the masses of all hadrons are considered to be equal

$$p_1^2 = p_{1'}^2 = p_2^2 = p_{2'}^2 = m^2, \quad k^2 = 0. \quad (4)$$

All other invariants can be expressed in terms of selected ones

$$\begin{aligned} S_{1'} &\equiv 2 p_{1'} k = S_1 - t_1 + t_2, \quad S_{2'} \equiv 2 p_{2'} k = S_2 - t_2 + t_1, \\ 2 p_1 p_{2'} &= S - S_1 + t_1, \quad 2 p_2 p_{1'} = S - S_2 + t_2, \quad 2 p_{1'} p_{2'} = S - S_1 - S_2. \end{aligned} \quad (5)$$

We begin with the production of a non-Abelian gauge boson. There are four Feynman diagrams corresponding to its emission from initial and final hadrons, and one diagram with the boson produced in virtual processes (see Fig. 2). For simplicity, we consider the scattering of spinless particles. In this case, the contribution of the anomalous magnetic moment to the Low formula is absent. The elastic amplitudes, corresponding to the blobs in Fig. 2, generally depend on the virtualities of the external particles. However, following Ref. [3] we consider the contributions of the diagrams of Fig. 2 as a dispersion representation in variables s_1, s_2 . In this case the hadrons are supposed to lie on their mass shell and the elastic amplitudes depend only on two Mandelstam invariants which are different for the four diagrams of Fig. 2. At high energies $\sqrt{s} \gg m_{\text{char}}$, we can neglect a small difference in s for the various blobs in Fig. 1 and write down the inelastic amplitude in the form (see Figs. 1 and 2):

$$\begin{aligned} \vec{A}_{2 \rightarrow 2+g} = g \ell_\mu \left[\left(\frac{2P_{2'}}{S_{2'}} \vec{T}_{2'} - \frac{2P_2}{S_2} \vec{T}_2 \right)^\mu A(s, t_2) + \right. \\ \left. + \left(\frac{2P_{2'}}{S_{2'}} \vec{T}_{2'} - \frac{2P_2}{S_2} \vec{T}_2 \right)^\mu A(s, t_1) \right] + \vec{\Delta A}(s, s_1, s_2, t_1, t_2). \end{aligned} \quad (6)$$

Here ΔA is the contribution of excited intermediate states in the dispersion relations in the variables $s_i, s_{i'}$. The generators T_i in Eq. (6) act on the unitary spin indices of the i^{th} particle. Due to the invariance of the elastic amplitude with respect to gauge group transformations we have

$$(\vec{T}_{2'} + \vec{T}_{2'} - \vec{T}_1 - \vec{T}_2) A(s, t) = 0. \quad (7)$$

Furthermore, we can extract from ΔA a subtraction-like term regular at $t_1 = t_2$ due to Eq. (7), which also has no singularity in the variables s_1, s_2 and rewrite Eq. (6) in the form

$$\begin{aligned} \vec{A}_{2 \rightarrow 2+g} = g \ell_\mu \frac{2}{t_1 - t_2} \left[(B_{2'} \vec{T}_{2'} - B_2 \vec{T}_2)^\mu A(s, t_2) + \right. \\ \left. + (B_{2'} \vec{T}_{2'} - B_2 \vec{T}_2)^\mu A(s, t_1) \right] + \vec{\Delta A}_{2 \rightarrow 2+g}, \end{aligned} \quad (8)$$

where

$$\begin{aligned} B_i = \frac{(t_1 - t_2) P_i}{S_i} - \frac{q_1 + q_2}{2}, \\ B_{i'} = \frac{(t_1 - t_2) P_{i'}}{S_{i'}} - \frac{q_1 + q_2}{2} \end{aligned} \quad (9)$$

are gauge invariant effective vertices satisfying the condition $kB = 0$, and

$$q_1 = P_1 - P_{1'}, \quad q_2 = P_{2'} - P_2, \quad k = q_1 - q_2, \quad q_i^2 = t_i. \quad (10)$$

One can verify that for a sufficiently soft boson with $s_1, s_2 \ll m_{\text{char}}^2$, the contri-

bution of the last term $\tilde{\Delta}\tilde{A}$ in Eq. (8) is of the relative order of $\omega^2/m_{\text{char}}^2 \ll 1$ in comparison with the other terms; here ω is the boson frequency in the c.m. system. Therefore $\tilde{\Delta}\tilde{A}$ can be neglected in the Low approximation:

$$\begin{aligned} \vec{A}_{\ell \rightarrow \ell+g} \Big|_{\omega \rightarrow 0} = & g \ell_{\mu} \left[\left(\frac{2P_{1'}}{S_{1'}} \vec{T}_{1'} - \frac{2P_1}{S_1} \vec{T}_1 + \frac{2P_{2'}}{S_{2'}} \vec{T}_{2'} - \frac{2P_2}{S_2} \vec{T}_2 \right) \hat{A}(s,t) \right. \\ & \left. - \left(B_{1'} \vec{T}_{1'} - B_1 \vec{T}_1 - B_{2'} \vec{T}_{2'} + B_2 \vec{T}_2 \right) \hat{\frac{\partial}{\partial t}} A(s,t) \right]. \end{aligned} \quad (11)$$

The second term in the sum (11) is an order ω^0 correction to the first term which has the order ω^{-1} . For non-zero spin particles, Eq. (11) contains an additional term $\sim \omega^0$ related to their magnetic moments.

Now we consider graviton emission in hadronic interactions. Again the external particles are assumed for simplicity to be spinless. In this case they interact with the graviton through their stress tensor $\theta_{\mu\nu}$ with the following vertex:

$$V = \kappa \ell_{\mu\nu} \left[P_1^{\mu} P_2^{\nu} + P_1^{\nu} P_2^{\mu} - \frac{\delta^{\mu\nu}}{2} 2(P_1 P_2 - m^2) \right]. \quad (12)$$

Here κ is the gravitational coupling constant connected with the Newtonian gravity constant G by the formula

$$G = \frac{\kappa^2}{8\pi}. \quad (13)$$

It can be verified that the expressions

$$\begin{aligned} b &= \frac{1}{(t_1 - t_2)^2} \left[(S_{1'} - S_1) A(s, t_2) + (S_{2'} - S_2) A(s, t_1) \right], \\ b_{\mu} &= \frac{\ell_{\mu\nu}}{(t_1 - t_2)} \left[(k_{1'} - k_1)^{\nu} A(s, t_2) + (k_{2'} - k_2)^{\nu} A(s, t_1) \right] \end{aligned} \quad (14)$$

due to relations

$$\begin{aligned} S_{1'} - S_1 + S_{2'} - S_2 &= 0; \quad K_{1'} - K_1 + K_{2'} - K_2 = K; \\ S_{1'} - S_1 - S_{2'} + S_2 &= -2(t_1 - t_2); \quad \ell_{\mu\nu} K^\nu = \ell_{\mu\nu} K^\mu = 0 \end{aligned} \quad (15)$$

have no singularity at $t_1 = t_2$. Therefore, we can add to the contribution of the first four diagrams of Fig. 2 the subtraction-like terms proportional to expressions (14) and write the graviton production amplitude in the form [cf. (8)]:

$$\begin{aligned} A_{2 \rightarrow 2+G} &= 2 \frac{\ell_{\mu\nu}}{(t_1 - t_2)^2} \left[(S_{1'} B_{1'}^\mu B_{1'}^\nu - S_1 B_1^\mu B_1^\nu) A(S, t_2) + \right. \\ &\quad \left. + (S_{2'} B_{2'}^\mu B_{2'}^\nu - S_2 B_2^\mu B_2^\nu) A(S, t_1) \right] + \widetilde{\Delta} A_{2 \rightarrow 2+G}, \end{aligned} \quad (16)$$

where vectors B_i are defined by Eq. (9). When obtaining (16) we omitted in Eq. (12) the terms $\sim \delta_{\mu\nu}$ having in mind the constraint $\delta_{\mu\nu} \ell^{\mu\nu} = 0$ on the graviton polarization tensor $\ell^{\mu\nu}$. Note that the expression (16) is invariant under on-shell gauge transformations

$$\ell_{\mu\nu} \rightarrow \ell_{\mu\nu} + C_\mu K_\nu + K_\mu C_\nu. \quad (17)$$

The last term in Eq. (16) represents the contribution of the excited intermediate states in the dispersion relations in the variables s_i . It is small ($\sim \omega$) and therefore can be neglected in the Low limit

$$\begin{aligned} A_{2 \rightarrow 2+G} \Big|_{\omega \rightarrow 0} &= 2 \ell_{\mu\nu} \left\{ \left(\frac{P_{1'}^\mu P_{1'}^\nu}{S_{1'}} - \frac{P_1^\mu P_1^\nu}{S_1} + \frac{P_{2'}^\mu P_{2'}^\nu}{S_{2'}} - \frac{P_2^\mu P_2^\nu}{S_2} \right) A(S, t) + \right. \\ &\quad \left. + \int \frac{t_1 - t_2}{2} \left(-\frac{P_{1'}^\mu P_{1'}^\nu}{S_{1'}} + \frac{P_1^\mu P_1^\nu}{S_1} + \frac{P_{2'}^\mu P_{2'}^\nu}{S_{2'}} - \frac{P_2^\mu P_2^\nu}{S_2} \right) - \frac{(q_1 + q_2)^\mu (q_1 + q_2)^\nu}{2} \frac{\partial}{\partial t} A(S, t) \right\}. \end{aligned} \quad (18)$$

Here the term proportional to the amplitude $A(s, t)$ is of order ω^{-1} and the second term is a correction to it of order ω^0 .

3. - MASSLESS PARTICLE PRODUCTION IN THE MULTI-REGGE KINEMATICS

In this section we consider the production of massless particles with small transverse momenta (2) in the multi-Regge region

$$S \gg s_1 \gg m_{\text{char}}^2, \quad S \gg s_2 \gg m_{\text{char}}^2, \quad \frac{s_1 s_2}{S} = \vec{k}_\perp^2. \quad (19)$$

It was proved by V.N. Gribov [4] that the leading term (1) of the Low formula describes correctly the photon bremsstrahlung amplitude in the region (19) and (2). For the case of emission of a non-Abelian gauge boson or graviton, it can be shown by the same method that the region of applicability of the expressions corresponding to the first terms in Eqs. (11) and (18) can be extended to the bounds (19) and (2). These terms describe the pole approximation in the dispersion representation in the variables s_1 and s_2 . The contributions from excited intermediate states are small ($\sim k_\perp/m_{\text{char}}$) due to gauge invariance (cf. [4]). However, as we shall show later, they turn out to be of the same order as the second terms in formulae (11) and (18). The necessity of the inelastic contributions follows from the analyticity conditions for the scattering amplitudes.

To simplify our analysis, we postulate here that the elastic and inelastic amplitudes in the kinematic region (19) have the Regge asymptotic behaviour [6]

$$\begin{aligned} f(s, t) \Big|_{S \rightarrow \infty} &= \left[\left(-\frac{S}{m^2} \right)^{\alpha(t)} + \tau \left(\frac{S}{m^2} \right)^{\alpha(t)} \right] \beta(t), \quad m^2 = m_{\text{char}}^2, \\ A_{2 \rightarrow 3} \Big|_{s, s_i \rightarrow \infty} &= \left[\left(-\frac{S}{m^2} \right)^{\alpha_2(t_2)} + \tau_2 \left(\frac{S}{m^2} \right)^{\alpha_2(t_2)} \right] \left[\left(-\frac{S_1}{m^2} \right)^{\alpha_1(t_1) - \alpha_2(t_2)} + \tau_1 \tau_2 \left(\frac{S_1}{m^2} \right)^{\alpha_1(t_1) - \alpha_2(t_2)} \right] f(\lambda, t_1, t_2) \\ &+ \left[\left(-\frac{S}{m^2} \right)^{\alpha_2(t_1)} + \tau_1 \left(\frac{S}{m^2} \right)^{\alpha_2(t_1)} \right] \left[\left(-\frac{S_2}{m^2} \right)^{\alpha_2(t_2) - \alpha_1(t_1)} + \tau_1 \tau_2 \left(\frac{S_2}{m^2} \right)^{\alpha_2(t_2) - \alpha_1(t_1)} \right] f(\lambda, t_1, t_2) \end{aligned} \quad (20)$$

where $\alpha_i(t_i)$, $\tau_i = \pm 1$ are Regge pole trajectories and their signatures in the t_1 and t_2 channels. We suppose further that they coincide for $i = 1$ and 2 :

$$\alpha_1(t) = \alpha_2(t) = \alpha(t), \quad \tau_1 = \tau_2 = \tau. \quad (21)$$

The first term in Eq. (20) can be represented as a sum of more simple terms corresponding to the four diagrams of Fig. 3 in which dotted lines denote possible discontinuities in the variables s and s_1 . Analogously the second term in Eq. (20) has singularities in the invariants s and s_2 . Note that simultaneous discontinuities in s_1 and s_2 , forbidden from general considerations [6], are absent in Eq. (20).

If the produced particle has a small transverse momentum $K_\perp \ll m_{\text{char}}$ (2) the invariants t_1 and t_2 almost coincide

$$\begin{aligned} t_1 &\simeq t + K_\perp q, \\ t_2 &\simeq t - K_\perp q, \quad q = \frac{1}{2}(q_1 + q_2). \end{aligned} \quad (22)$$

Furthermore, for massless particles we have also

$$\Lambda = \frac{s m_{\text{char}}^2}{s_1 s_2} = \frac{m_{\text{char}}^2}{K_\perp^2} \rightarrow \infty. \quad (23)$$

By expanding (20) in a series in $K_\perp$ and using (19), (21), (22) and (23) we obtain

$$\begin{aligned} A_{2 \rightarrow 3} \Big|_{\substack{s_i \rightarrow \infty \\ K_\perp \rightarrow 0}} &= f \int_{t_{1,2} \rightarrow t}^{\infty} \left[2 + (t_2 - t_1) \frac{\partial}{\partial t} + (t_1 - t_2) \alpha'(t) \ln \frac{(-s_1) s_2}{m^2 m^2} \right] \left[\left(\frac{s}{m^2} \right)^{\alpha(t)} + \left(\frac{s}{m^2} \right)^{\alpha(t)} \right] + \\ &+ f \int_{t_{1,2} \rightarrow t}^{\infty} \left[2 + (t_1 - t_2) \frac{\partial}{\partial t} + (t_2 - t_1) \alpha'(t) \ln \frac{(-s_2) s_1}{m^2 m^2} \right] \left[\left(\frac{s}{m^2} \right)^{\alpha(t)} + \left(\frac{s}{m^2} \right)^{\alpha(t)} \right]. \end{aligned} \quad (24)$$

Now let us compare this expression with Eqs. (11) and (18) for gauge boson and graviton production amplitudes in the Low limit $\omega \rightarrow 0$.

In the multi-Regge kinematics (19) we have the approximate equalities

$$p_1 - p_{2'} \simeq q_1^\perp, \quad p_{2'} - p_2 \simeq q_2^\perp, \quad s_{i'} \simeq s_i, \quad B_{i'} \simeq B_i. \quad (25)$$

Further, at high energies the elastic amplitude $A(s, t)$ in Eq. (11) factorizes in its unitary spin indices

$$A(s, t)_{a_1 b_1}^{a_1' b_1'} = \sum_c \gamma_{a_1}^{a_1'}(c) \gamma_{b_1}^{b_1'}(c) \beta(t) \left[\left(-\frac{s}{m^2} \right)^{\alpha(t)} + \gamma \left(\frac{s}{m^2} \right)^{\alpha(t)} \right] \quad (26)$$

according to a definite t-channel group representation. Therefore, using relations

$$(\mathcal{T}_1 - \mathcal{T}_{1'}) \gamma(c) = \mathcal{T} \gamma(c) \quad ; \quad \gamma(c) (\mathcal{T}_2 - \mathcal{T}_{2'}) = - \gamma(c) \mathcal{T}, \quad (27)$$

where the generator $\vec{T}$ acts on the unitary indices of the reggeon, we can rewrite expression (11) for the gauge boson production amplitude in the form

$$\vec{A}_{l \rightarrow l+g} \Big|_{\substack{s_i \rightarrow \infty \\ k_1 \rightarrow 0}} = g \ell_\mu \left[2 \left(\frac{p_2}{s_2} - \frac{p_1}{s_1} \right)_\mu + (B_1 + B_2)_\mu \frac{\partial}{\partial t} \right]. \quad (28)$$

$$\gamma(c_2) \vec{T}_{c_2 c_1} \gamma(c_1) \beta(t) \left[\left(-\frac{s}{m^2} \right)^{\alpha(t)} + \gamma \left(\frac{s}{m^2} \right)^{\alpha(t)} \right] + \vec{\Delta} \vec{A}.$$

Here we re-established the additional term $\vec{\Delta} \vec{A}$ [see Eq. (18)] corresponding to possible dispersion contributions from excited intermediate states in the s_i channels. Comparison of Eqs. (24) and (28) allows us to conclude that this term is vital for combining the Low theorem with the analytic properties of the inelastic amplitude in multi-Regge kinematics (19). With an appropriate choice of $\vec{\Delta} \vec{A}$, expression (28) can be written finally in the form of Eq. (24)

$$\begin{aligned} \vec{A}_{l \rightarrow l+g} \Big|_{\substack{s_i \rightarrow \infty \\ k_1 \rightarrow 0}} &= g \left\{ (B_1 \ell) \left[\frac{2}{t_2 - t_1} + \frac{\partial}{\partial t} - \alpha' \left(\ln \frac{-s_1}{m^2} + \ln \frac{s_1}{m^2} + c \right) \right] + \right. \\ &+ (B_2 \ell) \left[\frac{2}{t_1 - t_2} + \frac{\partial}{\partial t} - \alpha' \left(\ln \frac{-s_2}{m^2} + \ln \frac{s_2}{m^2} + c \right) \right] \Big\} \cdot \\ &\cdot \gamma(c_2) \vec{T}_{c_2 c_1} \gamma(c_1) \beta(t) \left[\left(-\frac{s}{m^2} \right)^{\alpha(t)} + \gamma \left(\frac{s}{m^2} \right)^{\alpha(t)} \right], \end{aligned} \quad (29)$$

where c is a constant which cannot be obtained from general considerations. For example, in string theory massless gluon production in the tachyon-tachyon scattering is described by Eq. (29) where $m^2 = 1/\alpha'$ and c is given below [see Eq. (53) of Ref. [5]]:

$$\mathcal{C} = -2 \psi(1) = 2C_E, \quad (30)$$

$$\psi(x) = \Gamma'(x)/\Gamma(x), \quad (31)$$

where $C_E \approx 0.577$ is the Euler constant. To interpret Eqs. (29) and (30) we use the well-known mathematical relation

$$\lim_{S_1 \rightarrow \infty} \left(\ln \frac{S_1}{m^2} + C_E \right) = \lim_{S_1 \rightarrow \infty} \sum_{n=1}^{\infty} \frac{1}{n} = \lim_{S_1 \rightarrow \infty} S_1 \sum_{n=1}^{\infty} \frac{1/n}{(S_1 + n m^2)}. \quad (32)$$

Inspecting (32), we see that Eq. (29) can be considered as a dispersion representation in the invariants s_i with a subtraction at $s_i = 0$. Indeed, in the next section the study of massless particle production in the fragmentation region supports this interpretation.

Now we pass to graviton production in the multi-Regge kinematics. Again, to combine the Low formula (18) with Eq. (24) we have to add to Eq. (18) the term $\tilde{\Delta} \tilde{A}_{2 \rightarrow 2+G}$ [see Eq. (16)] corresponding to the excited intermediate states in the dispersion relations in invariants s_i :

$$A_{2 \rightarrow 2+G} \Big|_{\substack{s_i \rightarrow \infty \\ k_1 \rightarrow 0}} = 2 \int B_1^\mu B_1^\nu l_{\mu\nu} \left[\frac{2}{t_2 - t_1} + \frac{\partial}{\partial t} - \alpha'_p \left(\ln \frac{-s_1}{m^2} + \ln \frac{s_1}{m^2} + \tilde{C} \right) \right] + \quad (33)$$

$$+ B_2^\mu B_2^\nu l_{\mu\nu} \left[\frac{2}{t_1 - t_2} + \frac{\partial}{\partial t} - \alpha'_p \left(\ln \frac{-s_2}{m^2} + \ln \frac{s_2}{m^2} + \tilde{C} \right) \right] \int_0^1 A_{2 \rightarrow 2}(s, t),$$

$$\alpha_p(t) = \alpha_p(0) + \alpha'_p t, \quad (34)$$

where $A_{2 \rightarrow 2}(s, t)$ is the elastic amplitude and the constant $\tilde{C}$ for the case of graviton production in the lowest mass closed string scattering is equal [see Eq. (54) of Ref. [5]] to

$$\tilde{C} = 2C_E - 2 \ln 2. \quad (35)$$

Analogously to Eq. (29) we interpret (33) as a dispersion relation in the variables s_i (see the next section).

4. - MASSLESS PARTICLE PRODUCTION IN THE FRAGMENTATION REGION

There are two fragmentation regions for the particles produced in high energy hadron collisions [cf. (19)]

$$\begin{aligned} a) \quad & S_1 \sim S_{1'} \sim m_{\text{char}}^2, \quad \vec{k}_1^2 \sim m_{\text{char}}^2, \quad S_2 \sim S \gg m_{\text{char}}^2, \\ b) \quad & S_2 \sim S_{2'} \sim m_{\text{char}}^2, \quad \vec{k}_1^2 \sim m_{\text{char}}^2, \quad S_1 \sim S \gg m_{\text{char}}^2. \end{aligned} \quad (36)$$

For definiteness consider the region b). Here the amplitude $A_{2 \rightarrow 3}$ has rather complicated analytic properties including many poles and cuts in the channels s_2 , $s_{2'}$, t . Nevertheless, for small transverse momentum (2) of the produced massless particle in the leading approximation a simple factorized formula (1) holds. In this section we are going to study its region of applicability in the dual resonance model (cf. Ref. [5]).

We write down the following expression for the gluon production amplitude in the scattering of the lowest mass excitation of the open string - the tachyons [7]:

$$\begin{aligned} A(s, s_1, s_2, t_1, t_2) &\sim \int \prod_{i=1}^5 dx_i |x_{15} x_{23}|^{-2} \ell_\mu q^\mu(x) \cdot \\ &\cdot \left| \frac{x_{12} x_{35}}{x_{13} x_{25}} \right|^{-2's} \left| \frac{x_{13} x_{45}}{x_{14} x_{35}} \right|^{-2's_1} \left| \frac{x_{25} x_{34}}{x_{24} x_{35}} \right|^{-2's_2} \\ &\cdot \left| \frac{x_{15} x_{34}}{x_{13} x_{45}} \right|^{-2't_1} \left| \frac{x_{23} x_{45}}{x_{25} x_{34}} \right|^{-2't_2}, \quad \alpha(t) = 1 + 2't, \end{aligned} \quad (37)$$

where x_1, x_2, x_3, x_4, x_5 are string co-ordinates for external particles with corresponding momenta $p_1, p_2, p_{2'}, k, p_1$. The effective gluon emission vertex

$a_\mu(x)$ equals

$$a(x) = \frac{p_1}{x_1 - x_4} + \frac{p_2}{x_2 - x_4} - \frac{p_1'}{x_5 - x_4} - \frac{p_2'}{x_3 - x_4} \quad (38)$$

Note that Eq. (37) is invariant under on mass-shell gauge transformations

$$l_\mu \rightarrow l_\mu + c k_\mu \quad (39)$$

and under the Möbius group transformations

$$x_i \rightarrow \frac{a x_i + b}{c x_i + d}, \quad ad - bc = 1, \quad (40)$$

where a, b, c, d are real parameters.

We can use Eq. (40) to fix the following choice of integration variables

$$x_1 = 0, \quad x_2 = 1, \quad x_3 = x, \quad x_4 = y, \quad x_5 = \infty \quad (41)$$

and rewrite Eq. (37) in the form

$$A(s, s_1, s_2, t_1, t_2) \sim \int \frac{dx dy}{(1-x)^2} (\ell a(x, y)).$$

$$\cdot \left| \frac{1}{x} \right|^{-2's} \left| \frac{x}{y} \right|^{-2's_1} \left| \frac{y-x}{y-1} \right|^{-2's_2} \left| \frac{y-x}{x} \right|^{-2't_1} \left| \frac{x-y}{x-1} \right|^{2't_2} \quad (42)$$

In the kinematic region (36b) the essential region of integration is

$$1-x \equiv \varepsilon \sim 1/2's, \quad (43)$$

$$\frac{x-y}{1-y} \equiv z \sim 1$$

and Eq. (42) takes the form

$$A(s, s_1, s_2, t_1, t_2) \Big|_{\alpha' s \sim \alpha' s_1 \rightarrow \infty} \sim \int \frac{d\varepsilon}{|\varepsilon|^2} \frac{dz}{|1-z|^2} \left(l b(\varepsilon, z) \right) \cdot \exp \left(-\alpha' s \varepsilon - \alpha' s_1 \varepsilon \frac{z}{1-z} \right) \left| \frac{z}{1-z} \varepsilon \right|^{-\alpha' t_1} \left| \frac{1-z}{z} \right|^{-\alpha' t_2} |z|^{-\alpha' s_2}, \quad (44)$$

where

$$b(\varepsilon, z) = \varepsilon a(x) \simeq -p_1 \varepsilon - \frac{(1-z)^2}{z} p_2 - \frac{1-z}{z} \cdot \frac{q_1 + q_2}{z}. \quad (45)$$

In the Gribov asymptotics (2) we have in particular

$$s_1 \ll s. \quad (46)$$

In this limit two integration regions in Eq. (44) are essential

$$\begin{aligned} 1) \quad \alpha' s_1 \varepsilon \frac{z}{1-z} &\equiv x_2 \sim 1, \\ 2) \quad \frac{z}{1-z} &\sim 1. \end{aligned} \quad (47)$$

They give respectively two contributions A_1 and A_2 with different analytic properties

$$A = A_1 + A_2, \quad (48)$$

where after re-establishing the absolute normalization of A_1 we obtain (see Fig. 4)

$$A_1(s, s_1, s_2, t_1, t_2) \Big|_{\alpha' s \sim \alpha' s_1 \rightarrow \infty} =$$

$$\begin{aligned}
 &= g^3 \left[c_{i_1 i_2 i_3 i_4 i_5} (-g^2 s)^{\alpha(t_2)} (-g^2 s_1)^{\alpha(t_1) - \alpha(t_2)} + \right. \\
 &+ c_{i_1 i_4 i_2 i_3 i_5} (-g^2 s)^{\alpha(t_2)} (g^2 s_1)^{\alpha(t_1) - \alpha(t_2)} + c_{i_1 i_3 i_2 i_4 i_5} (g^2 s)^{\alpha(t_2)} (-g^2 s_1)^{\alpha(t_1) - \alpha(t_2)} + \\
 &\left. + c_{i_1 i_4 i_3 i_2 i_5} (g^2 s)^{\alpha(t_2)} (g^2 s_1)^{\alpha(t_1) - \alpha(t_2)} \right] J_1\left(\frac{s_1}{s}, t_1, t_2, s_2\right)
 \end{aligned} \quad (49)$$

and (see Fig. 5)

$$\begin{aligned}
 A_2(s, s_1, s_2, t_1, t_2) \Big|_{g^2 s \sim g^2 s_1 \rightarrow \infty} &= g^3 \cdot \\
 &\left\{ \left[c_{i_1 i_2 i_3 i_4 i_5} (-g^2 s)^{\alpha(t_2)} + c_{i_4 i_3 i_2 i_5 i_1} (g^2 s)^{\alpha(t_1)} \right] \phi_1\left(\frac{s_1}{s}, t_1, t_2, s_2\right) + \right. \\
 &+ \left[c_{i_1 i_4 i_2 i_3 i_5} (-g^2 s)^{\alpha(t_1)} + c_{i_5 i_4 i_2 i_3 i_1} (g^2 s)^{\alpha(t_1)} \right] \phi_2\left(\frac{s_1}{s}, t_1, t_2, -s_2\right) + \\
 &\left. + \left[c_{i_1 i_2 i_4 i_3 i_5} (-g^2 s)^{\alpha(t_1)} + c_{i_5 i_2 i_4 i_3 i_1} (g^2 s)^{\alpha(t_1)} \right] \phi_3\left(\frac{s_1}{s}, t_1, s_2, -s_2\right) \right\}
 \end{aligned} \quad (50)$$

In Eqs. (49) and (50) g is the Yang-Mills coupling constant, $c_{i_1, \dots, i_n}$ are the Chan-Paton colour factors [7]

$$c_{i_1, i_2, \dots, i_n} = \frac{1}{2} \text{tr}(\lambda_{i_1} \lambda_{i_2} \dots \lambda_{i_n}), \quad \frac{1}{2} \text{tr} \lambda_{i_1} \lambda_{i_2} = \delta_{i_1 i_2}, \quad (51)$$

where λ_i are the Gell-Mann matrices. Functions J_1 and ϕ_i are regular at small s_1/s and equal in the limit $s_1/s \rightarrow 0$:

$$\begin{aligned}
 J_1\left(\frac{s_1}{s}, t_1, t_2, s_2\right) \Big|_{\frac{s_1}{s} \rightarrow 0} &= \int_0^\infty dx_1 x_1^{-2-\alpha'(t_2)} e^{-x_1} \\
 \int_0^\infty dx_2 x_2^{-1+\alpha'(t_2-t_1)} e^{-x_2} \left(1 - \frac{s_1}{s} \frac{x_1}{x_2}\right)^{\alpha' s_2} \frac{x_2 s}{x_1 s_1} (\ell b) \Big|_{\frac{s_1}{s} \rightarrow 0} &= \\
 &= \Gamma(-\alpha(t_2)) \Gamma(\alpha'(t_2-t_1)) (B^1 \ell), \tag{52}
 \end{aligned}$$

$$\begin{aligned}
 \phi_1\left(\frac{s_1}{s}, t_1, t_2, s_2\right) \Big|_{\frac{s_1}{s} \rightarrow 0} &= \int_0^\infty \frac{dx_1}{x_1^{2+\alpha'(t_1)}} e^{-x_1} \int_0^1 \frac{dz}{(1-z)^2} z^{-2-\alpha' s_2} (\ell b) \left(\frac{z}{1-z}\right)^{\alpha'(t_2-t_1)} \Big|_{\frac{s_1}{s} \rightarrow 0} = \\
 &= \Gamma(-\alpha(t_1)) \frac{\Gamma(1-\alpha' s_2) \Gamma(\alpha(t_1)-\alpha(t_2))}{\Gamma(1-\alpha' s_2)} (B^2 \ell),
 \end{aligned}$$

$$\begin{aligned}
 \phi_2\left(\frac{s_1}{s}, t_1, t_2, -s_2\right) \Big|_{\frac{s_1}{s} \rightarrow 0} &= - \int_0^\infty \frac{dx_1}{x_1^{2+\alpha'(t_1)}} e^{-x_1} \int_1^\infty \frac{dz}{(z-1)^2} z^{-2-\alpha' s_2} (\ell b) \left(\frac{z}{z-1}\right)^{\alpha'(t_2-t_1)} \Big|_{\frac{s_1}{s} \rightarrow 0} = \\
 &= \Gamma(-\alpha(t_1)) \frac{\Gamma(1+\alpha' s_2) \Gamma(\alpha(t_1)-\alpha(t_2))}{\Gamma(1+\alpha' s_2)} (B^2 \ell),
 \end{aligned}$$

$$\begin{aligned}
 \phi_3\left(\frac{s_1}{s}, t_1, s_2, -s_2\right) \Big|_{\frac{s_1}{s} \rightarrow 0} &= - \int_0^\infty \frac{dx_1}{x_1^{2+\alpha'(t_1)}} e^{-x_1} \int_{-\infty}^0 \frac{dz}{(1-z)^2} (-z)^{-2-\alpha' s_2} (\ell b) \left(\frac{z}{1-z}\right)^{\alpha'(t_2-t_1)} \Big|_{\frac{s_1}{s} \rightarrow 0} = \\
 &= \Gamma(-\alpha(t_1)) \frac{\Gamma(1-\alpha' s_2) \Gamma(1+\alpha' s_2)}{\Gamma(1+\alpha'(t_2-t_1))} \frac{1}{\alpha'} \left(\frac{p_2}{s_1} - \frac{p_2}{s_2}, \ell\right), \tag{53}
 \end{aligned}$$

where the four vectors B^1, B^2 were given earlier [see (9)].

In the Gribov asymptotics (2) we have also $t_2 \rightarrow t_1$ [see (22)] so Eqs. (52) and (53) can be simplified

$$J_1 \Big|_{\substack{\frac{s_1}{s} \rightarrow 0 \\ t_{1,2} \rightarrow t}} = \frac{\Gamma(-\alpha(t))}{\alpha'(t_2-t_1)} (B^1 \ell) [1 + \alpha'(t_2-t_1) \chi(t)],$$

$$\chi(t) = -\frac{1}{2} \psi(-\alpha(t)) + \psi(1), \quad \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}, \quad (54)$$

$$\begin{aligned} \phi_1 \Big|_{\substack{s_1 \rightarrow 0 \\ t_{1,2} \rightarrow t}} &= \frac{\Gamma'(-\alpha(t))}{\alpha'(t_1 - t_2)} (B^2 \ell) \left[1 + \alpha'(t_1 - t_2) (\chi(t) - \psi(1 - \alpha' s_2)) \right], \\ \phi_2 \Big|_{\substack{s_1 \rightarrow 0 \\ t_{1,2} \rightarrow t}} &= \frac{\Gamma'(-\alpha(t))}{\alpha'(t_1 - t_2)} (B^2 \ell) \left[1 + \alpha'(t_1 - t_2) (\chi(t) - \psi(1 + \alpha' s_2)) \right], \end{aligned} \quad (55)$$

$$\phi_3 \Big|_{\substack{s_1 \rightarrow 0 \\ t_{1,2} \rightarrow t}} = \Gamma'(-\alpha(t)) \Gamma'(1 - \alpha' s_2) \Gamma'(1 + \alpha' s_2) \frac{1}{\alpha'} \left(\frac{p_2}{s_2} - \frac{p_2}{s_2'}, \ell \right) \frac{s_2}{s_2'} (B^2 \ell) = B^2 \ell.$$

Substituting (54) and (55) into Eqs. (48)-(50), we finally obtain for the gluon production amplitude in the fragmentation region (34b) in the Gribov limit (2):

$$\begin{aligned} A(s, s_1, s_2, t_1, t_2) \Big|_{\substack{s_1 \rightarrow 0 \\ k_1 \rightarrow 0}} &= g \ell_n \left[2 \left(\frac{p_2}{s_2} - \frac{p_2}{s_2'} \right)^\mu + (B^1 + B^2) \frac{\mu}{\alpha'} \right] g^2 \Gamma'(-\alpha(t)) \\ &\quad \left[\frac{1}{2} (C_{i_1 i_2 i_3 i_4 i_5} + C_{i_1 i_4 i_2 i_3 i_5}) (-\alpha' s)^{\alpha(t)} + \frac{1}{2} (C_{i_1 i_3 i_2 i_4 i_5} + C_{i_1 i_4 i_3 i_2 i_5}) (\alpha' s)^{\alpha(t)} \right] + \\ &\quad + \alpha' \Gamma'(-\alpha(t)) g^3 \ell (B_2, \ell) \left[-\psi(1 - \alpha' s_2) + \psi(1) \right] \left[C_{i_1 i_2 i_3 i_4 i_5} (-\alpha' s)^{\alpha(t)} + C_{i_1 i_4 i_3 i_2 i_5} (\alpha' s)^{\alpha(t)} \right] + \\ &\quad + (B_2 \ell) \left[-\psi(1 + \alpha' s_2) + \psi(1) \right] \left[C_{i_1 i_4 i_2 i_3 i_5} (-\alpha' s)^{\alpha(t)} + C_{i_1 i_3 i_2 i_4 i_5} (\alpha' s)^{\alpha(t)} \right] + \\ &\quad + \frac{\pi s_2}{\sin(\pi \alpha' s_2)} \left(\frac{p_2}{s_2} - \frac{p_2}{s_2'}, \ell \right) \left[C_{i_1 i_2 i_3 i_4 i_5} (-\alpha' s)^{\alpha(t)} + C_{i_1 i_4 i_3 i_2 i_5} (\alpha' s)^{\alpha(t)} \right] + \\ &\quad + (B_1 \ell) \left[\left(\ln \frac{1}{-\alpha' s_1} + \psi(1) \right) \left[C_{i_1 i_2 i_3 i_4 i_5} (-\alpha' s)^{\alpha(t)} + C_{i_1 i_3 i_2 i_4 i_5} (\alpha' s)^{\alpha(t)} \right] + \right. \\ &\quad \left. + \left(\ln \frac{1}{\alpha' s_1} + \psi(1) \right) \left[C_{i_1 i_4 i_2 i_3 i_5} (-\alpha' s)^{\alpha(t)} + C_{i_1 i_4 i_3 i_2 i_5} (\alpha' s)^{\alpha(t)} \right] \right]. \end{aligned} \quad (56)$$

Due to the relation

$$\psi(x) = -\sum_{n=0}^{\infty} \left(\frac{1}{x+n} - \frac{1}{1+n} \right) + \psi(1) \quad (57)$$

it is obvious that the inelastic amplitude in the fragmentation region (56) has infinitely many poles at positive and negative values of s_2 . In the limit $s_2 \rightarrow \infty$, we return to Eq. (29) for each of the t-channel invariant amplitudes.

An analogous calculation of graviton production amplitudes at high energy (closed string) tachyon scattering in fragmentation region gives the result:

$$\begin{aligned} A(s, s_1, s_2, t_1, t_2) \Big|_{\substack{s_1 \rightarrow 0 \\ k_1 \rightarrow 0}} &= 2 l^{\mu\nu} x^3 \left[(-\alpha'_p s)^{\alpha_p(t_2)} + (\alpha'_p s)^{\alpha_p(t_2)} \right] \left[(-\alpha'_p s_2)^{\alpha_p(t_1)} + (\alpha'_p s_2)^{\alpha_p(t_1)} \right] \\ &\cdot B_\mu^1 B_\nu^1 \frac{\Gamma(-\alpha_p(t_2)) \Gamma(\frac{\alpha_p(t_2)+1}{2}) \Gamma(\alpha_p(t_2)-\alpha_p(t_1)) \Gamma(\frac{\alpha_p(t_1)-\alpha_p(t_2)+1}{2})}{\Gamma(\frac{1}{2}) \Gamma(\frac{\alpha_p(t_2)+1}{2}) \Gamma(\frac{1}{2}) \Gamma(\frac{\alpha_p(t_2)-\alpha_p(t_1)+1}{2})} + \\ &+ B_\mu^2 B_\nu^2 \left[(-\alpha'_p s)^{\alpha_p(t_1)} + (\alpha'_p s)^{\alpha_p(t_1)} \right] \frac{\Gamma(-\alpha_p(t_1)) \Gamma(\frac{\alpha_p(t_1)+1}{2}) \Gamma(\frac{\alpha'_p s_2}{2}) \Gamma(-\frac{\alpha'_p s_2}{2}) \Gamma(\frac{\alpha_p(t_1)-\alpha_p(t_2)}{2})}{\Gamma(\frac{1}{2}) \Gamma(\frac{\alpha_p(t_1)+1}{2}) \Gamma(\frac{\alpha'_p s_2}{2}) \Gamma(-\frac{\alpha'_p s_2}{2}) \Gamma(\frac{\alpha_p(t_1)-\alpha_p(t_2)}{2})} \\ &= l^{\mu\nu} x \left\{ B_\mu^1 B_\nu^1 \left[\frac{2}{t_2-t_1} + \frac{\partial}{\partial t} - \alpha'_p \left(\ln(-\alpha'_p s_2) + \ln(\alpha'_p s_2) + 2C_E - 2\ln 2 \right) \right] + \right. \\ &+ B_\mu^2 B_\nu^2 \left[\frac{2}{t_2-t_2} + \frac{\partial}{\partial t} - \alpha'_p \left(\psi(-\frac{\alpha'_p s_2}{2}) + \psi(\frac{\alpha'_p s_2}{2}) - 2\psi(1) \right) \right] \Big\} \cdot \\ &\cdot \frac{2}{\alpha'_p} x^2 \left[(-\alpha'_p s)^{\alpha_p(t)} + (\alpha'_p s)^{\alpha_p(t)} \right] \frac{\Gamma(-\alpha_p(t)) \Gamma(\frac{1+\alpha_p(t)}{2})}{\Gamma(\frac{1}{2}) \Gamma(1+\frac{\alpha_p(t)}{2})}, \alpha_p(t) = 2 + \alpha'_p t. \end{aligned} \quad (58)$$

Again in the limit $s_2 \rightarrow \infty$ we obtain Eq. (32).

ACKNOWLEDGEMENTS

The author is grateful to V.N. Grivov, E.A. Antonov, A.P. Bukhvostov, R. Kirschner, V.A. Kudryavtsev and E.A. Kuraev for their stimulating remarks in the initial stage of the work and to D. Amati, G. Veneziano, L. Lukashuk and other participants in the theoretical seminar of CERN for interesting discussions.

REFERENCES

- [1] R.P. Feynman, Photon-hadron interactions (Benjamin, 1972).
- [2] L.N. Lipatov, Yad. Fiz. 20 (1974) 181;
G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.
- [3] F.E. Low, Phys. Rev 110 (1958) 974.
- [4] V.N. Gribov, Sov. J. of Nucl. Phys. 5 (1967) 399.
- [5] L.N. Lipatov, preprint LNPI-1356 (1987).
- [6] R.C. Brower, C.E. de Tar and J.H. Weis, Physics Reports 14 (1974) 257-367.
- [7] J.H. Schwarz, Physics Reports 89 (1982) 233.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5