POLARIZATION OF STORED ANTIPROTONS BY THE STERN-GERLACH EFFECT

T.O. Niinikoski

CERN

Geneva, Switzerland

We discuss the polarization of stored antiproton beams by spatially separating particles with opposite spin directions, using the Stern-Gerlach effect in alternating quadrupole fields.

INTRODUCTION

It has been recently proposed¹⁻⁵ that particle beams could be self-polarized in storage rings or colliders. This self-polarization is based on the Stern-Gerlach effect: in an inhomogeneous magnetic field, the particles with spins aligned parallel or antiparallel to the field are deflected in opposite directions and become spatially separated. We may immediately note the well-known fact that the Stern-Gerlach experiment, with a single pass through a quadrupole field, only works for neutral or ion beams having electronic magnetic moment, because the uncertainty principle together with the Lorentz force causes proton or electron beams to smear much more than their splitting. The uncertainty principle, however, becomes insignificant in our case where the beam particles receive repetitive kicks along their otherwise stable orbits.

A conventional storage ring has quadrupole magnets with inhomogeneous fields. Considering one single magnet, particles with different spin directions are deflected by different angles in the quadrupole field. Since this difference in the deflection angle is extremely weak, the next quadrupole with opposite polarity would almost exactly cancel the effect of the first. However, the bending magnets between the quadrupoles may rotate the spin in such a way that the kicks obtained in the quadrupoles add up, always following the variation of the vertical deflection angle. For a FODO cell, therefore, we must require that the vertical betatron phase advance from the first to the second quadrupole approximately equals the spin phase advance minus π over complete precession cycles in the dipole magnet between the quadrupoles. A more general additional requirement is that the spin and vertical betatron phase advances, modulo 2π , in a complete machine cycle are equal. This can be expressed as

$$a\gamma - n = \nu_y - m,$$

where $a = (g - 2)/2$ ($= 1.793$ for protons), $\gamma = (1 - \beta^2)^{-1/2}$ is the Lorentz factor, n is the nearest integer below $a\gamma$ (the number of spin cycles in a machine turn), ν_y is the number of vertical betatron oscillations in a machine turn, and m is the nearest integer below ν_y . A mathematically rigorous treatment⁴ leads to the condition

$$a\gamma \pm \nu_y = kP, \quad (1)$$

where k is an integer and P is the periodicity of the machine lattice. This corresponds to an intrinsic spin resonance, which quickly destroys any vertical polarization by dephasing the vertical spin component. The horizontal spin components, however, will remain synchronized by the resonant horizontal fields, and there is no horizontal depolarization in first order.

This situation can be visualized in analogy with magnetic resonance in solids, where transformation to rotating frame allows the main time dependence of the spin Hamiltonian to be removed. It is known experimentally that in solids no depolarization will occur as long as the oscillating transverse field is maintained, in spite of large randomly fluctuating dipolar fields.

The above arguments are valid for a perfectly linear machine and for the case where the synchrotron oscillations average out the differences in the energy dependences of the spin and betatron tunes. For a real machine a complicated spin matching must be performed. This paper focuses on the first-order beam-splitting speed; the problems for real machines are only briefly discussed.

SEPARATION OF PARTICLES WITH DIFFERENT SPIN DIRECTIONS

The field of a quadrupole magnet is described in first order by

$$B_x = -by,$$

$$B_y = bx,$$

in a coordinate system where x and y are the transverse horizontal and vertical axes, respectively, and z is tangent to the design orbit. The force on the magnetic dipole $\vec{\mu}$ is⁶

$$\delta \vec{F} = \nabla (\vec{\mu} \cdot \vec{B}) = b \nabla (-\mu_x y + \mu_y x) = b(\hat{i} \mu_y - \hat{j} \mu_x), \quad (2)$$

and the additional deflections of the particle in the x - z and y - z planes are, respectively,

$$\delta x' = -b \mu_y \Delta L / E,$$

$$\delta y' = -b \mu_x \Delta L / E,$$

where μ_x and μ_y are the projections of the magnetic moment along the x and y directions, ΔL is the length of the quadrupole, and $E = \gamma m$ is the beam energy. We note that when the spin points in the direction of the beam, there is no deflection; when it points along x , the particle is deflected vertically; when it points up, the deflection is horizontal. The radial field gradient b is positive for quadrupoles focusing in the vertical plane and negative for those focusing in the horizontal plane.

Between the quadrupoles there are bending magnets with homogeneous field. The motion of the spin in an homogeneous field is described by the BMT equation:⁷

$$d\vec{\sigma}/dt = (q/m\gamma)\vec{\sigma} \times [(1+a)\vec{B}_{\parallel} + (1+a\gamma)\vec{B}_{\perp}].$$

Here γ and a are as defined earlier, $\vec{\sigma}$ is a unit vector describing the spin direction in the laboratory frame, q is the charge of the particle, and m is the rest mass; $\vec{B}_{\parallel}$ and $\vec{B}_{\perp}$ are laboratory frame fields parallel and perpendicular to the motion of the particle. The required integrated field strength for a 90° spin rotation for protons is $2.7 \text{ T} \cdot \text{m}$, independent of energy in the relativistic limit.

The vertical betatron motion can be described by an homogeneous differential equation

$$y'' + \omega_y^2(s)y = 0, \quad (3)$$

with the well-known solutions $y_1 = \sqrt{\beta_y(s)} \cos [\psi_y(s)]$ and $y_2 = \sqrt{\beta_y(s)} \sin [\psi_y(s)]$; here $\beta_y(s) = [d\psi(s)/ds]^{-1}$, and s is the distance along the design orbit. When the betatron motion is excited by the Stern-Gerlach effect, Eq. (3) becomes

$$y'' + \omega_y^2(s)y = -\mu/\gamma m b(s) e^{-i\Phi_x(s)}. \quad (4)$$

Here $b(s)$ is the gradient of the field and $e^{-i\Phi_x(s)}$ is the projection of the spin on the x-axis.

The solution of this inhomogeneous differential equation has been presented⁴; we shall quote here the resulting vertical splitting rate of the beam halves with opposite μ_x :

$$\frac{dy}{dt} = \frac{\mu}{\gamma m 4\pi \nu_y f} \int_{s_0}^{s_0+L} \beta^{1/2}(s) e^{-i\psi_y(s)} b(s) e^{-i\Phi_x(s)} ds. \quad (5)$$

By inserting values of b/γ , ν_y , and f that are appropriate for existing or planned large colliders such as the CERN Super Proton Synchrotron (SPS), the Fermilab Tevatron, and the Superconducting Super Collider (SSC), Eq. (5) gives separation speeds below 1 mm/day, which makes the method somewhat impractical. It is interesting, however, to extrapolate toward a purpose-built machine, which is optimized for spin separation. Quadrupole magnets can be built with gradient $b = 100 \text{ T} \cdot \text{m}^{-1}$. If we assume that alternating gradient magnets have this strength all over the ring with $f = 50 \text{ kHz}$ (such as the CERN SPS), and if we operate at proton energy $\gamma m_p = 100 \text{ GeV}/c^2$ and $\nu_y \approx 10$, we find

$$(\Delta y/\Delta t)_{\max} = 4.5 \text{ mm/h}.$$

This rate of separation of the beam into polarized halves would seem to be much faster than the rate of blow-up or loss.

POLARIZING LOW-ENERGY ANTIPROTONS

Examining Eq. (5) we note that the splitting rate is inversely proportional to the beam energy γ . Accelerators usually operate at constant b/γ ; however, if a machine lattice could be designed which had a very high value of b while conserving a low tune ν_y , a substantial increase in the splitting rate could be observed. This requirement could be achieved by placing the strong quadrupole pairs as close together as possible while maintaining a spin rotation by π in the interleaved dipole. Also, the machine should have a low cycling frequency f , which means a large perimeter. Summarizing, the machine would look like a large weak-focusing stretcher ring with very strong closely spaced quadrupole pairs.

The superperiods of the machine could also include reverse bends or vertical bends, in order to achieve a sufficient length of quadrupoles while maintaining a large diameter and low tune.

A recent proposal⁵ explores the use of a solenoid spin rotator for the same purpose as a dipole magnet between the quadrupoles.

DISCUSSION

At least the following problems must be solved before our suggested scheme may be considered as a serious alternative for polarized beams:

1) Beam energy spread makes a spread both in the spin tune $a\gamma$ and the betatron tune ν_y . Can these be compensated so that all spins will maintain phase coherence with the betatron oscillations?

The energy dependence of the betatron tune is suppressed to a large extent by the averaging due to synchrotron oscillations at low energies. The spin motion associated with the acceleration

and deceleration due to synchrotron oscillations is adiabatic and thus conserves the beam polarization in the horizontal plane. The adiabatic spin manipulation has been proven experimentally during acceleration and deceleration.⁸

The acceleration or deceleration far from the exact resonance $a\gamma = kP \pm \nu_y$ would turn the rotating horizontal polarization adiabatically in the vertical orientation, conserving the opposite alignments in the beam halves as long as the betatron frequency spread allows vertical coherence to be maintained.

2) The spin and betatron oscillations must be phase-locked over extended periods of time. Is this possible with existing beam-diagnostic devices?

The harmonics of the betatron frequency can be measured by specially built pick-ups and low-noise amplifiers. The additional motion due to the spin [Eq. (4)] might be observable as a sideband to a suitable high harmonic of the betatron frequency,⁹ and allow the phase locking of the two oscillations. This technique might also allow the monitoring of the separation build-up, and hence polarization.

3) Higher-order effects may destroy (dephase) the rotating horizontal polarization. One such mechanism may be the coupling of the vertical and horizontal betatron oscillations due to uncompensated axial fields, for example.

Without attempting to find definite answers to this question, we first would like to point out that there are several techniques to make the spin tune energy-independent. These are generally based on spin rotation about the longitudinal or the transverse horizontal axis.¹⁰ This can be accomplished with so-called 'Siberian Snakes',¹¹ which rotate the spin through an angle independent of energy. Other spin manipulation techniques might involve vertical-bend spin rotators and closed-orbit distortions, which need to be optimized only at the energy of storage if further acceleration or deceleration of the polarized beam is not required.

Bypassing all the complexities of spin resonances in accelerators, we would like to draw some analogies with spin resonances in solids in order to hint at a way towards complete spin matching for achieving efficient self-polarization. The snake spin-rotator in an accelerator is exactly analogous with the spin-echo scheme, where incoherence resulting from Larmor frequency spread can be almost indefinitely repaired by 180° spin-reversing pulses. Much more complex schemes do exist in solid-state NMR, which remove the effects of spin diffusion^{12,13} and dipolar broadening,¹⁴⁻¹⁶ their analogies could probably be easily implemented in accelerators by the introduction of various snakes, vertical bends, and orbit distortions.

If we suppose that the technical difficulties in tuning a collider accurately to a spin resonance during extended periods can be overcome, it may be worth while to discuss the possible ways by which experiments could use the polarization in the beam halves. Firstly, each beam bunch has zero average polarization in spite of the fact that the particles with opposite horizontal spin components are spatially separated. The obvious solution is to steer the bunches so that beam halves with definite spin states enter in collision.

Secondly, during the beam separation at intrinsic resonance $a\gamma \pm \nu_y = kP$, the spin phase cannot be fixed in the machine lattice, because this would require running at a strong instability condition $\nu_y = \text{integer}$. In all experiments, however, spin reversal is necessary, and this could now be accomplished simply by correlating the spin phase with the scattering data. This would require the tagging of the events with the vertical betatron phases of the colliding particles. A possible way of doing this is to determine the vertical momentum of the individual events with sufficiently high accuracy, so that the sum of the vertical momenta of the beam particles, and therefore the difference of their polarization vectors can be determined.

Thirdly, selective slow extraction from the polarized beam halves should lead to a polarized extracted beam, again because the betatron and spin phases are correlated. This, however, would

require the extraction to take place in a vertical plane, and the remaining circulating beam would not acquire overall polarization because of the spread of the vertical betatron and horizontal spin phases.

In order to get an overall polarization in the circulating beam, the upper or lower half of the beam would have to be extracted or destroyed in one machine turn, and the polarization in the remaining beam would have to be turned adiabatically into vertical alignment by accelerating or decelerating immediately (in a short time compared with the coherence time of vertical betatron oscillations) off the intrinsic spin resonance at which the beam separation is done. Alternatively, the machine tune could be adiabatically shifted off the resonance by changing the quadrupole magnet currents immediately after the fast extraction of one half of the beam.

Finally, if the polarization can be adiabatically turned into the vertical position, as discussed above, the experiments could operate with beams in pure spin states of arbitrary alignment by using spin rotators.

We conclude that self-polarization of protons in existing colliders and storage rings might be impractically slow. However, a purpose-built machine could provide a sufficiently fast polarization speed, so that it might be worth examining the outstanding problems related with the method.

Acknowledgements

The author would like to thank J. Bell, Y. Onel, A. Penzo and R. Rossmanith for many stimulating discussions on the self-polarization of antiprotons.

REFERENCES

1. T.O. Niinikoski and R. Rossmanith, preprint CERN-EP/85-46 (1985).
2. T.O. Niinikoski and R. Rossmanith, Proc. 6th Int. Symp. on Polarization Phenomena in Nuclear Physics, Osaka, 1985 [Suppl. J. Phys. Soc. Japan 55:1134 (1986)].
3. T.O. Niinikoski and R. Rossmanith, Proc. Int. Workshops on Polarized Beams at the SSC, Ann Arbor, 1985, and on Polarized Antiprotons, Bodega Bay, 1985, eds. A.D. Krisch, A.M.T. Lin and O. Chamberlain (AIP Conf. Proc. No. 145, New York, 1986), p. 244.
4. T.O. Niinikoski and R. Rossmanith, Self-polarization of protons in storage rings, Brookhaven Nat. Lab. Accelerator Division Internal Report 86-4 (1986), to be published in Nuclear Instruments and Methods in Physics Research.
5. Y. Onel, A. Penzo and R. Rossmanith, Polarized antiprotons at LEAR, Proc. Symp. on High-Energy Spin Physics, Protvino, 1986 (to be published); A. Penzo, these Proceedings.
6. R. Leighton, Principles of modern physics (McGraw-Hill, Inc., New York, 1959).
7. V. Bargmann, L. Michel and V.L. Telegdi, Phys. Rev. Lett. 2:435 (1959).
8. E. Gyorud et al., Proc. Int. Symp. on High-Energy Spin Physics, Brookhaven, 1982 (AIP Conf. Proc. No. 95, New York, 1983), p. 407.
9. This suggestion is due to S. van der Meer (private communication).
10. E. Courant, J. Phys. 46:C2-713 (1985).
11. Ya.S. Derbenev and A. Kondratenko, Proc. Int. Symp. on High-Energy Physics with Polarized Beams and Targets, Argonne, 1978 (AIP Conf. Proc. No. 51, New York, 1979), p. 292.
12. H.Y. Carr and E.M. Purcell, Phys. Rev. 94:630 (1954).
13. S. Meiboom and D. Gill, Rev. Sci. Instrum. 29:6881 (1958).
14. E.D. Ostroff and J.S. Waugh, Phys. Rev. Lett. 16:1097 (1966).
15. P. Mansfield and D. Ware, Phys. Rev. 168:318 (1968).
16. L.M. Stacey, R.W. Vaughan and D.D. Elleman, Phys. Rev. Lett. 26:1153 (1971).