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EXTRA SPACE-TIME DIMENSIONS: TOWARDS A SOLUTION TO THE STRONG CP-PROBLEM

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ABSTRACT

We discuss the possibility that the strong CP-problem is solved by raising the number of spatial dimensions à la Kaluza-Klein. We construct higher dimensional manifolds which do not have the structure $M_4 \times (\text{compact space})$ where the θ -problem is absent, while locally the results of the standard dimensional reduction are reproduced.

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Quantum chromodynamics is the basis of our current understanding of the strong interactions of hadrons. One of the parameters characterizing it plays a specific rôle. This is the θ -angle which does not appear in the original Lagrangian and defines the choice of the vacuum state among the possible physically distinct vacua [1]. The complicated vacuum structure results in the prediction of P and T violation in strong interactions, unless $\theta = 0$. Experimentally, $\theta < 10^{-9}$ (see, e.g., Ref. [2] and references therein). So there arises the naturalness problem: how should this extremely small value be explained?

The significance of the θ -problem was realized long ago [1,3]. The known mechanisms which could lead naturally to the physical equivalence of different θ -worlds are based on the global $U(1)$ Peccei-Quinn symmetry [3]. It implies the existence of a light pseudoscalar particle (axion) [4] or a massless strongly-interacting fermion. However, neither fermions of this type nor axions have ever been observed. So the search for another explanation of the precise strong P and T conservation should be continued.

The purpose of this letter is to find out whether the introduction of extra compactified space dimensions can solve the θ -puzzle. First, let us recall the formulation of the problem in 3+1 dimensions for $SU(2)$ gauge theory [1]. The natural intention is to consider our three-dimensional space as a compact manifold, say, the three-sphere S_3 . Any vacuum gauge field has the form $A_i = g \partial_i g^{-1}$, where g is a continuous gauge transformation defining a map from S_3 to $SU(2)$. As is well known, these fall into homotopy classes, labelled by the integers $n = 0, \pm 1, \dots$. Due to the quantum tunnelling [5] between the classical vacua with different topological numbers $|n\rangle$, the quantum ground state has the form

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta} |n\rangle, \quad (1)$$

where θ is the vacuum angle. Equivalently, after the gauge fixing [which forbids "local" $SU(2)$ gauge transformations of the form $\exp(i\tau^a \alpha^a(x))$ with $\alpha^a(x)$ being continuous functions on S_3], there survives a discrete set of "large" gauge transformations, giving rise to periodicity of the Hamiltonian H ,

$$[SU(2)_{\text{whole}}] / [SU(2)_{\text{local}}] = \mathbb{Z},$$

$$[H, \mathbb{Z}] = 0. \quad (2)$$

As a consequence of this periodicity, the quantum gauge theory acquires an additional parameter θ , distinguishing between the different representations of the discrete group $\mathbb{Z}$. So the very existence of the θ -problem is based on two facts. The first is expressed by Eq. (2) and relies on the homotopical classification of

admissible vacuum gauge configurations. The second is the possibility of tunnelling between the different $|n\rangle$ -vacua. The Peccei-Quinn solution to the strong CP-problem implies the forbidding of the tunnelling due to an anomaly in the chiral fermionic current. This ensures the unobservability of the θ -angle while $|n\rangle$ -vacua still exist. We shall see that higher dimensions could provide another solution connected with the absence of $|n\rangle$ -vacua structure.

The idea is due to the observation that the existence of θ depends strongly on the global geometry of space. This may be modified in the presence of extra dimensions.

Consider high dimensional gauge theory with gauge group G coupled to gravity. Usually one looks for a solution corresponding to compactification of the type $M = M^4 \times K$, where M^4 is Minkowski space-time and K is a compact manifold (K could be a coset space G/H ; see Ref. [6] and, for a review, Ref. [7]). To keep the whole space component compact, one identifies the points at the three-dimensional space infinity, so that $M^4 = \text{time} \times S^3$, S^3 being a sphere of very large radius. Clearly, the direct product structure of M leaves the homotopy classification of gauge mappings intact, so that θ persists. However, this is not the case if we alter slightly the compactification scheme^{*)} and consider the space-time of the type

$$M = \text{time} \times B, \quad (3)$$

where the global geometry of B is chosen in such a way that the set of mappings $B \rightarrow G$ is homotopically trivial. For instance, one may take B to have a topology of the four-sphere S^4 . The unobservability of the fifth co-ordinate is provided by the highly anisotropic form of the manifold. One can imagine it in the form of a four-dimensional ellipsoid with one radius being small (compactified dimension) and the three others being large (the observed dimensions). To gain a more detailed understanding, it is convenient to turn to the simplest $U(1)$ model in two dimensions which exhibits many of the main features of non-Abelian gauge theories in four dimensions [9,1].

Consider first Higgs electrodynamics in 1+1 dimensions:

$$S^{(2)} = \int d^2x \mathcal{L}_2, \quad \mathcal{L}_2 = -\frac{1}{4} F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\varphi|^2 - \lambda(\varphi^*\varphi - C^2)^2_{(4)}$$

For definiteness, the Higgs v.e.v. is assumed to be large, $C \gg 1$, thus providing for the weak-coupling regime. When formulated on the ring, i.e., with the periodic

*) For other non-standard compactifications, see Ref. [8].

boundary conditions on all fields, the theory possesses explicit θ -dependence. The large gauge transformations making a non-trivial mapping $S^1 \rightarrow U(1)$ in the $A_0 = 0$ gauge have the form

$$\begin{aligned} A_1 &\rightarrow A_1 + \frac{2\pi}{eL} n, \\ \psi &\rightarrow \exp\left(\frac{2\pi i n x}{L}\right) \psi, \end{aligned} \quad n = 0, \pm 1, \dots \quad (5)$$

L being the length of the ring. Now include the extra space dimension. One possibility is to form a direct product $S^1 \times S^1_{\text{comp}}$ which is globally equivalent to the torus sketched in Fig. 1a. The effective two-dimensional Lagrangian is

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} e^2 \psi^* \psi \chi^2, \quad (6)$$

where the scalar field χ originates from the third component of the vector potential. Another possibility is to consider a sausage-like manifold globally equivalent to the two-sphere (Fig. 1b). In the first case, the non-trivial mapping structure is intact. Moreover, now there are two types of mappings, namely $S^1 \rightarrow U(1)$ and $S^1_{\text{comp}} \rightarrow U(1)$, hence two θ -angles arise. However, at low energies we expect the one related to $S^1_{\text{comp}} \rightarrow U(1)$ to be unobservable. The situation is quite different on the "sausage". All the mappings $S_2 \rightarrow U(1)$ are trivial. The transformations (5) are related to the trivial one by homotopy ($0 \leq \alpha \leq n$):

$$\begin{aligned} A_1 &\rightarrow A_1 + \frac{2\pi \alpha}{eL}, \\ \psi &\rightarrow \exp\left(\frac{2\pi i \alpha x}{L}\right) \psi. \end{aligned} \quad (7)$$

This means that all the gauge freedom is removed by the gauge fixing and no $|n\rangle$ vacua arise. Therefore, no θ parameter appears and the whole system is T-invariant^{*}). Contrary to case 1a, the global two-dimensional Lagrangian does not exist, due to the strong mixing of different modes near the points $x = 0$, $x = L$. The action for the β -independent excitations (β is an angular co-ordinate; we consider the manifold B to be cylindrically symmetric, see Fig. 1b) has the form

^{*}) There is a small violation of P symmetry due to the dependence of the size of the compactified dimension $R(x)$ on the co-ordinate x . It is proportional to $dR(x)/dx \sim R(x)/L$ and small far from the "end points" $x = 0$, $x = L$ (see Fig. 1b).

$$S_{eff} = \int d^2x R(x) \mathcal{L}_{eff}, \quad (8)$$

where $\mathcal{L}_{eff}$ is defined in Eq. (6), $R(x) = 0$ for $x = 0, L$. So for x lying far from the end points, the theory is equivalent to 1+1 electrodynamics but with θ automatically equal to zero.

Exactly the same things happen when one goes to the 3+1 case. The popular compactification scheme uses $M^4 \times G/H$ as a model of our world [6,7]. To solve the strong CP-problem, one should consider the non-singular manifold $\text{time} \times B$, where B has the structure G/H at every three-dimensional spatial point x , while the size $R(x)$ of compactified dimensions depends on x and equals zero on the large two-dimensional sphere S_2 . [Compare with the $U(1)$ model in 2+1 dimensions where $R(x) = 0$ for $x = 0, L$.] Hence, far from the surface S_2 , we will have a theory which reveals all the attractive features of $M^4 \times G/H$ compactification, but with $\theta = 0$.

So far we have considered only higher dimensional gauge theories. What about the θ puzzle in pure gravitational theories à la Kaluza-Klein, where gauge fields originate from the metric? Leaving aside the question whether the manifolds of the type (3) satisfy the equations of motion, we see that the standard compactification scheme does result in the θ -dependence, but the proposed one does not.

The reader may wonder whether the local measurements of the θ -parameter can tell us about the global geometry of the space. Suppose one finds experimentally $\theta \neq 0^{*)}$. Is it possible to argue that the space does not have the form (3)? It is clear that such a conclusion is in contradiction with causality. Let us explain the resolution of this problem using the example of 2+1 $U(1)$ theory. Consider for simplicity the case without spontaneous symmetry breaking. In 1+1 dimensions, the θ -angle is connected with the existence of a sourceless electric field [10]

$$E = \frac{\theta e}{\pi}. \quad (9)$$

On the manifold of the type Fig. 1a, the constant electric field without charges does not exist (this is just the other reason for the absence of the θ -angle). Nevertheless, one can consider the excited state in which the classical scalar field is non-zero and carries out fractional electric charges $\pm q$ which are localized near $x = 0, L$. For such a state, the observer in the middle of the manifold

*) $\theta = 0$ could correspond to three-dimensional space as well as higher dimensional spaces with different types of compactification.

($x \approx L/2$) will find a non-zero θ -angle $\theta = \pi q/e$ (recall that the Coulomb potential rises linearly in one-dimensional theory and on a manifold with one compactified dimension). So $\theta = 0$ cannot unambiguously fix the topology of the space manifold. However, the formation of the state with widely-separated fractional charges during the evolution of the Universe seems to be highly improbable, if not excluded absolutely. Moreover, this state is unstable and after the time $t \sim L$, the θ -angle will disappear everywhere. The same line of reasoning is applicable also to the realistic case of the observed three-dimensional world.

To conclude this letter, we wish to emphasize that there are a lot of questions to be answered before this solution of the strong CP-problem can be considered seriously. They are mainly connected with the possibility of the dynamical origin of the manifold with desired properties. The size of additional dimensions must be sufficiently small and stable at the present period of the evolution of the Universe (the standard assumption is $R \sim 1/M_{\text{planck}}$), while the size of the three-dimensional Universe should increase, giving rise to the Hubble expansion. The difference between the standard compactification scheme and the proposed one is that the latter predicts the existence of the end of the three-space "hard sky", where the curvature of the three-space becomes comparable with the curvature in additional dimensions. It is clear from observations that the distance to the end of the three-space has to be larger than the size of the visible part of the Universe.

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(a)

(b)

Fig. 1