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MIXING COEFFICIENTS OF THE LATTICE
WEAK HAMILTONIAN WITH DIMENSION FIVE OPERATORS^(*)

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Abst

We compute the mixing coefficients of the four fermion operators which appear in the non-leptonic $\Delta S=1$, octet Hamiltonian with operators of dimension five. This calculation completes the construction of the perturbative part of the weak Hamiltonian, to be used in Monte Carlo simulations of lattice QCD with Wilson fermions.

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Composite operators in lattice QCD with Wilson fermions^[1] require appropriate subtractions to recover the prescribed chiral behaviour in the continuum limit. This is true, in particular, for the four fermion operators of the effective non-leptonic weak Hamiltonian. According to ref.[2], operators of the highest dimension appear in the subtraction with coefficients which can be computed reliably in perturbation theory. This is because these coefficients vanish in the continuum limit $\epsilon_0 \rightarrow 0$ (or $\beta \rightarrow \infty$) due to the asymptotic freedom of QCD. In contrast, lower dimension operators enter with power-divergent coefficients and have to be subtracted in a non-perturbative way, by enforcing the appropriate soft-pion theorems.

In the case of the $\Delta I=1/2$ weak non leptonic Hamiltonian, it has been shown^[3] that one can use perturbation theory not only to compute the mixing coefficients of the dimension six operators, but also those of the operators of dimension five, provided that the lattice spacing is sufficiently small so that the GIM cancellation is operative:

$$a^{-1} \gg m_c \quad (1)$$

where m_c is the charmed quark mass.

The mixing coefficients of the weak Hamiltonian with the dimension six operators have been already computed in refs.[4]. In this note we present the lowest order calculation of the coefficients of the operators with dimension five. This calculation completes the construction of the perturbative part of the weak non leptonic Hamiltonian^[5], to be used in Monte Carlo simulations of lattice QCD.

We consider the octet, $\Delta I=1/2$, bare operators

$$\phi^{(-)} = [\bar{s}_L \gamma_\mu d_L \bar{u}_L \gamma_\mu u_L - \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma_\mu d_L] - (u \bar{u} \rightarrow c \bar{c}) \quad (2)$$

$$\begin{aligned} \phi^{(+)} = & 1/5 [\bar{s}_L \gamma_\mu d_L \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma_\mu d_L \\ & + 2 \bar{s}_L \gamma_\mu d_L \bar{d}_L \gamma_\mu u_L + 2 \bar{s}_L \gamma_\mu u_L \bar{s}_L \gamma_\mu d_L] \end{aligned} \quad (3)$$

$$- [\bar{s}_L \gamma_\mu d_L \bar{c}_L \gamma_\mu c_L + \bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma_\mu d_L]$$

For simplicity, we shall restrict to the chiral limit: $m_u = m_d = m_s = 0$, $m_c \neq 0$.

In this limit, the operators (2) and (3) are the only ones that require the subtraction of operators of dimension less than six. The other terms of the weak Hamiltonian transform according to the 27 - representation of SU(3) and can mix only with four fermion operators.

There are two, independent gauge invariant, operators of dimension five, namely:

$$\phi_s^{(1)} = i g_0 \bar{s} \sigma_{\mu\nu} F^{\mu\nu} d \quad (4)$$

$$\phi_s^{(2)} = \bar{s} D_\mu D^\mu d \quad (5)$$

where D_μ is the QCD covariant derivative:

$$D_\mu = \partial_\mu + i g_0 \lambda^a/2 A_\mu^a$$

g_0 the bare coupling constant and $F_{\mu\nu} = \lambda^a/2 F_{\mu\nu}^a$ the gluon field-strength. We can express the corresponding subtraction to $\phi^{(\pm)}$, according to:

$$\delta_s \phi^{(\pm)} = g_0^2 m_c^2 (A^{(\pm)} \phi_s^{(1)} + B^{(\pm)} \phi_s^{(2)}) \quad (6)$$

The factor m_c is the remnant of the GIM suppression factor $m_c - m_u$, in the chiral limit. Using the identity^(f):

$$\bar{s} D_\mu D^\mu d = \bar{s} \not{D} \not{D} d + \frac{1}{2} g_0 \bar{s} \sigma_{\mu\nu} F^{\mu\nu} d \quad (7)$$

and the equation of motion we can in fact eliminate $\phi_s^{(2)}$ and express the subtraction according to:

$$\delta_s \phi^{(\pm)} = g_0^2 m_c d_s^{(\pm)} (i g_0 \bar{s} \sigma_{\mu\nu} F^{\mu\nu} d) \quad (8)$$

(f) We use hermitian γ -matrices, with $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ and $\sigma_{\mu\nu} = 1/2 [\gamma_\mu, \gamma_\nu]$.

with

$$d_s^{(\pm)} = A^{(\pm)} + \frac{1}{2} B^{(\pm)} \quad (9)$$

Finally, the renormalized operators are^(f):

$$\phi^{(\pm)} = Z_{LAT}^{(\pm)} (\mu a) [\phi^{(\pm)} + \delta_6 \phi^{(\pm)} + \delta_s \phi^{(\pm)} + c^{(\pm)} s d] \quad (10)$$

with μ the subtraction point and Z_{LAT} the normalization constant computed in ref.[4]. As discussed in ref.[3] and verified explicitly in this paper, the coefficients A and B are free from logarithmic singularities of the form $\ln(a m_c)$.

We now describe the explicit calculation of $A^{(\pm)}$ and $B^{(\pm)}$. The coefficients B can be obtained from the insertion of the bare operators (2) and (3) and of the counterterms $\delta_6 \phi^{(\pm)}$ into the s-d propagator. To one loop, the insertion of $\phi^{(\pm)}$ gives a vanishing contribution and so does the one loop insertion of $\delta_6 \phi^{(\pm)}$. The lowest order result is obtained from the two-loop diagrams shown in Fig.1. We expand the corresponding amplitude, $\Gamma^{(\pm)}(p)$, in powers of the external momentum:

$$\Gamma^{(\pm)}(p) = g_0^2 [\gamma_0^{(\pm)} + \gamma^{(\pm)} p^2 + \dots] \quad (11)$$

From eqs.(5), (6) and (10) we obtain:

$$B^{(\pm)} = - \frac{\partial}{\partial m_c} \gamma^{(\pm)} \Big|_{m=0} \quad (12)$$

σ , equivalently:

$$B^{(\pm)} = - \frac{1}{8} g^{\mu\nu} \frac{\partial^3 \Gamma^{(\pm)}(p)}{\partial m_c \partial p_\mu \partial p_\nu} \Big|_{p=0, m_c=0} \quad (13)$$

(f) In the exact SU(3) limit, due to CP invariance no pseudoscalar counterterm is present[3,6].

The computation of $A^{(t)}$ requires the insertion of four fermion operators in the s - d -gluon vertex. Again, the one loop insertion of $\phi^{(t)}$ gives a vanishing contribution. The lowest order result is obtained from:

- i) the one loop insertion of the counterterms $\delta_g \phi^{(t)}$, Fig.2;
- ii) the insertion of $\phi^{(t)}$ in the two loop diagrams of Fig.3.

Keeping only linear terms in the external momenta, the result can be written as:

$$\Gamma_\mu^{A^{(t)}}(p, k) = \bar{s}(p - \frac{k}{2}) \epsilon_0^3 \frac{\lambda^A}{2} [2\alpha^{(t)} \sigma_{\mu\nu} k^\nu + 2\beta^{(t)} p_\mu] d(p + \frac{k}{2}) \quad (14)$$

Comparing with eqs.(4), (6) and (10), one obtains:

$$A^{(t)} = - \frac{\partial \alpha^{(t)}}{\partial m_c} \Big|_{m_c=0} \quad (15)$$

Due to gauge invariance, one can reobtain $B^{(t)}$ from $\Gamma_\mu^{(t)}$, according to:

$$B^{(t)} = - \frac{\partial \beta^{(t)}}{\partial m_c} \Big|_{m_c=0} \quad (16)$$

The equality of the results in eqs.(12) and (16) provides a powerful check of the numerical calculation.

A delicate point is the question of the infrared divergences. As stated previously, one expects both $A^{(t)}$ and $B^{(t)}$ to be free from infrared divergences, proportional to $\ln(a m_c)$. Of course, this is true only after the insertion of the bare operators is added to the insertion of the counterterms. The latter give a vanishing contribution to $B^{(t)}$, so that $\gamma^{(t)}$ and $B^{(t)}$ must be free from infrared singularities.

The situation for $A^{(t)}$ is more complicated. The diagrams in Fig.2 do give rise to an infrared singular contribution to $A^{(t)}$, which has to be cancelled by contributions from the diagrams in Fig.3. Of the latter, only those in Fig.3a and Fig.3b have subintegrations corresponding to the diagrams which define the dimension six counterterms, $\delta_g \phi^{(t)}$. Thus the diagrams in Figs.3c to 3e must be free from the infrared singularity.

The above pattern of infrared singularities has been verified by explicit analytic calculation.

The presence of infrared singularities makes it inconvenient to compute separately the contributions to $A^{(t)}$ from the diagrams of Fig.2 and Fig.3. We have proceeded as follows. Consider first the diagram in Fig.3a (to simplify the notation we refer explicitly to the case of $\phi^{(*)}$). The corresponding amplitude can be written as:

$$\Gamma_\mu^{A^{(*)}}(3a) = \bar{s}(p - \frac{k}{2})(-ie_0) \int \frac{d^4 l}{(2\pi)^4}.$$

$$K_\mu^{bd}(p - \frac{k}{2}, p + \frac{k}{2}, l, k, l, m_j).$$

$$[S_1(1-k) v_\mu^A(l-k, l) S_1(l) \tilde{f}_0^b] d_0(p + \frac{k}{2}) \quad (17)$$

$$K_\mu^{bd}(p - \frac{k}{2}, p + \frac{k}{2}, l, k, l, m_j) = (-ie_0)^2 \int \frac{d^4 q}{(2\pi)^4}.$$

$$\cdot (-[\gamma_\rho (\frac{1-\gamma_5}{2}) S_0(p + \frac{k}{2} - q) v_\nu^B(p + \frac{k}{2} - q, p + \frac{k}{2})]_\mu^b.$$

$$\cdot [\gamma_\rho (\frac{1-\gamma_5}{2}) S_1(l-k+q) v_\nu^B(l-k+q, l-k)]_\mu^b$$

$$- [a \leftrightarrow c] + \text{crossed terms} \quad (18)$$

where $S_1(S_0)$ is the internal (external) quark propagator and v_μ^A the q - q -gluon vertex on the lattice, the indices a, b etc. run over colour and Dirac indices, and we have to subtract the charm from the up quark contribution.

Crossed terms correspond to the second diagram in Fig.3a, where the internal gluon attaches to the s -quark line. We can isolate, in the contributions of Fig.2, those where the diagrams defining $\delta_g \phi^{(*)}$ are precisely the same which appear as a subintegration in

Fig.3a. The sum of the two contributions is obtained by the substitution:

$$K_{ac}^{bd}(\vec{p} - \frac{\vec{k}}{2}; \vec{p} + \frac{\vec{k}}{2} | -\vec{k}, l, m) \rightarrow K_{ac}^{bd}(\vec{p} - \frac{\vec{k}}{2}; \vec{p} + \frac{\vec{k}}{2} | l - \vec{k}, l, m) - K_{ac}^{bd}(0,0,0,0) \quad (19)$$

Once this is done, and the derivative with respect to m_c is performed inside the integrals, the integrand is free from non-integrable singularities in the limit $m_c=0$, and is suitable for numerical calculation. The same procedure can be applied to the diagrams 3b. In this case, however, the subtraction is not necessary: the term to be subtracted has a vanishing projection over the $q_{\mu\nu} k^\nu$ from factor.

The algebraic manipulations needed to project the form factors $A(\pm)$ and $B(\pm)$ have been performed with SCHOONSCHIP. Integrals have been approximated by discrete sums over a regular lattice. For each component of l and q we define a discrete variable k such that e.g.:

$$l_x = -\pi + (k_x - 1) \frac{2\pi}{L} \quad k_x = 1, \dots, L \quad (20)$$

and:

$$\frac{1}{L} \sum_{k_x} \rightarrow \int \frac{d^4 l}{(2\pi)} \quad (21)$$

To avoid singularities from massless quark and gluon propagators, the points corresponding to a vanishing denominator have been eliminated from the sum. Calculations with $L=4,6,8,10$ have been performed on the CRAY XMP at CINECA (Bologna). The calculation with $L=10$ has required about 30 hours of CPU time.

Our results are summarized in Table 2 and Table 3. The contributions to $A(\pm)$ and $B(\pm)$ of each pair of diagrams in Fig.3(a) to Fig.3(e) are decomposed according to:

$$A_i^{(\pm)} = f_i^{(\pm)} V_A^{(0)} \quad (22)$$

where i labels the diagram, $f_i^{(\pm)}$ are the color factors reported in Table 1, and a similar expression holds for $B_i^{(\pm)}$. The values found for $V_A^{(0)}$ and $V_B^{(0)}$ are reported in Tables 2 and 3, as functions of L . A linear extrapolation in $1/L$ of the last two points is used to estimate the value of the integrals. The latter values are also reported in the Tables (last row). Combining the extrapolated values, we obtain^(f), for $N=3$:

$$\begin{aligned} A^{(+)} &= -1.10 \cdot 10^{-4} \\ A^{(-)} &= +1.98 \cdot 10^{-4} \\ B^{(+)} &= -B^{(-)} = 9.56 \cdot 10^{-4} \end{aligned} \quad (23)$$

As stated previously, an important check to the numerical calculation is provided by the consistency of eqs.(13) and (16), which is a consequence of gauge invariance. Using the color factors of Table 1, one writes:

$$\begin{aligned} B^{(\pm)} &= \pm \frac{N^2-1}{2N} [V_B^{(3a)} + V_B^{(3b)} + V_B^{(3c)} + V_B^{(3d)}] + \frac{1}{2} [V_B^{(3a)} + V_B^{(3b)}] + \\ &\quad \mp \frac{N}{2} [V_B^{(3a)} + V_B^{(3c)} + \frac{1}{2} V_B^{(3a)} + V_B^{(3b)}] \end{aligned} \quad (24)$$

On the other hand, the color factor of $\Gamma^{(\pm)}$ is immediately seen to be:

$$f_\Gamma^{(\pm)} = \pm \frac{N^2-1}{2N} \quad (25)$$

^(f) The equality $B^{(+)} + B^{(-)} = 0$ follows trivially from the fact that $V_B^{(3a)} = -V_B^{(3b)}$ while all other diagrams have opposite color factors for $B^{(\pm)}$.

So that, if we denote by $B_{\Gamma}^{(z)}$ the value obtained from eq.(13), and write:

$$B_{\Gamma}^{(z)} = \epsilon_{\Gamma}^{(z)} V_{\Gamma} \quad (26)$$

gauge invariance implies the three equations:

$$V_B^{(3a)} + V_B^{(3b)} + V_B^{(3c)} + V_B^{(3d)} = V_{\Gamma} \quad (27)$$

$$V_B^{(3a)} + V_B^{(3b)} = 0 \quad (28)$$

$$V_B^{(3a)} + V_B^{(3c)} + \frac{1}{2} V_B^{(3d)} + V_B^{(3e)} = \Delta = 0 \quad (29)$$

By inspection, of the relevant diagrams, one sees that eqs.(27) and (28) are trivially satisfied, but eq.(29) is non trivial and it provides a check of the numerical precision of our calculation. From the values given in Table 3, we find:

$$\Delta = (-0.47; 0.18; 0.14; 0.09) \times 10^{-4} \quad (30)$$

for $L=4,6,8,10$, respectively, with an extrapolated value:

$$\Delta = -0.15 \times 10^{-4} \quad (31)$$

We take $|\Delta_{\infty}|$ as a measure of the error in our estimate of the extrapolated values of $A^{(\pm)}$ and $B^{(\pm)}$. This agrees approximately with the spread which results from different extrapolation methods, e.g. by including a quadratic dependence in $1/L$, in the extrapolation. Combining the previous results we obtain:

$$d_5^{(+)} = A^{(+)} + \frac{1}{2} B^{(+)} = (3.68 \pm 0.15) \times 10^{-4} \quad (32)$$

$$d_5^{(-)} = A^{(-)} + \frac{1}{2} B^{(-)} = (-2.80 \pm 0.15) \times 10^{-4}$$

We have stressed that the coefficient of the dimension three operator in eq.(10) can be reliably computed only by a non perturbative method. It is nonetheless interesting to record its value in lowest order perturbation theory. To order g_0^2 , $c^{(z)}$ is obtained from $\gamma_0^{(z)}$, defined in eq.(11), and adding the counterterm contribution to $\Gamma^{(z)}$:

$$c_{\text{pert}}^{(z)} = g_0^2 \frac{\partial}{\partial \eta_z} [\gamma_0^{(z)} + \delta_z \gamma_0^{(z)}]_{m_z=0} \quad (33)$$

With the same method described above, we find:

$$-\frac{\partial}{\partial m_z} (\gamma_0^{(z)})_{m_z=0} = \mp \frac{N^2-1}{2N} (1.46; 2.09; 2.27; 2.36) \times 10^{-3} \quad (34)$$

for $L=4,6,8,10$, respectively. Extrapolating linearly in $1/L$ to $L=\infty$, we obtain:

$$-\frac{\partial}{\partial m_z} \gamma_0^{(\pm)} = \mp \left(\frac{N^2-1}{2N} \right) 2.69 \cdot 10^{-3} g_0^2 m_z \quad (35)$$

Furthermore, a simple, one loop calculation, gives:

$$-\frac{\partial}{\partial m_z} [\delta_z \gamma_0^{(z)}] = \mp 5.55 \times 10^{-3} \quad (36)$$

So that, finally:

$$c_{\text{par}}^{(\pm)} = \pm g_0^2 m_c (3.59 + 5.55) 10^{-3} = \pm g_0^2 m_c 0.913 \cdot 10^{-2} \quad (37)$$

The numerical calculations have been performed with the CRAY XMP of CINECA, Bologna. We thank the Theory Committee of INFN for providing the needed financial support.

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Tab. 1

Diagram	3a	3b	3c	3d	3e
$f^{(\pm)}$	$\pm \frac{N^2-1}{2N} + \frac{1}{2} \mp \frac{N}{2}$	$\pm \frac{N^2-1}{2N} + \frac{1}{2}$	$\pm \frac{N^2-1}{2N} + \frac{N}{2}$	$\pm \frac{N^2-1}{2N} + \frac{N}{4}$	$\mp \frac{N}{2}$

Tab. 2

L	3a-subtraction	3b	3c	3d	3e
$V_A \times 10^4$					
4	-0.48	1.79	0.19	0.00012	1.15
6	-0.65	1.40	0.36	-0.024	1.61
8	-0.71	1.42	0.47	-0.023	1.85
10	-0.75	1.49	0.54	-0.017	2.00
∞ (extrapolated)	-0.88	1.76	0.85	-0.010	2.60

Tab. 3

L	3a=-(3b)	3c	3d	3e	
$V_B \times 10^4$	4	-2.56	1.50	2.15	-0.48
	6	-2.43	1.72	3.34	-0.78
	8	-2.57	1.73	3.77	-0.91
	10	-2.74	1.80	4.00	-0.97
	∞ (extrapolated)	-3.50	2.10	4.90	-1.2

Table Captions

Tab.1 Color factors of the diagrams shown in fig.(3a)-(3e). The $(\pm)$ refers to the insertion of the operator $\phi^{(+)}$ or $\phi^{(-)}$ in the vertex.

Tab.2 The contribution of the diagrams (3a)-(3e) to V_A is reported for different value of the lattice size L . The last row is a linear extrapolation of the last two points to $1/L=0$. The first column includes the contribution of the counterterms, Fig.2.

Tab.3 Same as in Tab.2, for V_B .

Figure Captions

Fig.1 Lowest order diagrams for the calculation of $B(\pm)$. The open dot represents the insertion of the bare operators, $\phi(\pm)$.

Fig.2 One loop contribution to $A(\pm)$ from the insertion of $\delta_6 \phi(\pm)$.

Fig.3 Two loop diagrams for the calculation of $A(\pm)$.

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fig. 2

fig. 3

fig. 1