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NEUTRAL PION DECAYS PROBE NEW PHYSICS BEYOND THE STANDARD MODEL

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ABSTRACT

We study the decay $\pi^0 \rightarrow \gamma + \text{unobservable particles}$. The observation of such a process would give unambiguous evidence for the existence of a new light gauge boson of the type predicted by a number of current models. It is shown that the decay rate of the production of such a boson can be experimentally interesting. The contribution of the background processes ($\pi^0 \rightarrow \gamma + \text{scalars}$, $\pi^0 \rightarrow \gamma + \text{fermions}$) is proved to be negligibly small.

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1. - INTRODUCTION

Nowadays, there are a lot of models in high energy physics, the experimental status of which is not well established. First of all, this concerns various enlargements of the standard $SU(3) \times SU(2) \times U(1)$ model: GUTs, supergravity and supersymmetric theories, etc. The Higgs sector of the standard model still suffers from the high level of arbitrariness. Some data point out that possibly there should be a new gauge boson ("the fifth force"). The observation of new particles predicted by the models would be a decisive step in the experimental confirmation of various models. For these purposes, one should build new, more and more powerful accelerators. (J.E. Wheeler compared this process with the building of pyramids in ancient Egypt.) However, some of the predicted particles (photino, right-handed neutrino, scalar particles in models with non-minimal Higgs sector, etc.) could, in principle, be light enough to be produced in rare decays of the light hadrons ($K, \pi, \dots$) and leptons. An attractive feature of using low-energy experiments for a search for new particles consists of the possibility of having intensive beams of initial particles (e.g., at meson factories).

The decay $K^+ \rightarrow \pi^+ + \text{"nothing"}$ ("nothing" means neutral, weakly-interacting particles which are not detected) is one of the well-known examples of such processes.

It was shown in Refs. [1,2] that it is rather important to search for the decay $\pi^0 \rightarrow \text{"nothing"}$. This process is strictly forbidden in the framework of the standard model, and its observation would testify to the existence of new physics beyond the standard model.

Invisible neutrals produced in the decay can be light photinos [1] (provided there is considerable mixture between the left and right scalar quarks), neutrinos [2] (if it possesses the right-handed partner) or scalar particles. It is worth reviewing these possibilities.

$$(i) \pi^0 \rightarrow \nu \bar{\nu}$$

a) ν_R does not interact with any particles. In this case the decay $\pi^0 \rightarrow \nu \bar{\nu}$ takes place due to Z^0 -boson exchange and can only occur provided the neutrino is massive. The decay rate is [2]

$$\begin{aligned} BR_z^{\max}(\pi^0 \rightarrow \nu \bar{\nu} ; m_\nu = 55 \text{ MeV}) &= 2.8 \times 10^{-9} \\ BR_z^{\max}(\pi^0 \rightarrow \nu \bar{\nu} ; m_\nu = 100 \text{ eV}) &\simeq 4 \times 10^{-20} \end{aligned}$$

One should compare it with experiment:

$$BR_{exp}(\pi^0 \rightarrow \text{"nothing"}) \lesssim 2.4 \times 10^{-5}$$

b) ν_R has a Yukawa coupling with some heavy scalar (the case of light scalars is reviewed below). From the neutral current experiments one obtains [2]:

$$BR_{\substack{\text{heavy} \\ \text{scalar}}}(\pi^0 \rightarrow \nu \bar{\nu}) \lesssim 5 \times 10^{-6}$$

ii) $\pi^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$

The limit on the scalar quark mixture can be obtained from the experimental bound on the scalar quark mass [3]. So, one gets [1]:

$$BR(\pi^0 \rightarrow \tilde{\gamma}\tilde{\gamma}) \lesssim 5 \times 10^{-8}$$

iii) $\pi^0 \rightarrow \text{scalars}$

The contribution to this process of the "known" scalars (i.e., those present in the existing models) is negligibly small. For instance, the production of the axions is strongly suppressed because it is forbidden by P-parity. The π^0 decay rate into the scalar neutrinos is equal to zero at the tree level. But the production of some new light scalars (introduced ad hoc) could, in principle, saturate the experimental limit.

The characteristic feature of the experiments in the search for processes like $K^+ \rightarrow \pi^+ + \text{"nothing"}$, $\pi^0 \rightarrow \text{"nothing"}$, etc., is the ambiguity of the interpretation of the positive result because various hypothetic particles can play the role of "nothing".

As will be shown in this paper, the radiative decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$ is essentially different in this respect. Having observed it, one can be certain that one has found just new gauge bosons and not anything else. A negative result of such an experiment would give us new non-trivial constraints on the properties of a hypothetic gauge boson.

Our paper is organized as follows: in Section 2 we will estimate the contribution of scalar particles to the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$. In Section 3, the contribution of fermions is calculated. In Section 4 the production of a new gauge boson is discussed. The conclusions are given in Section 5.

2. - CONTRIBUTION OF THE SCALAR PARTICLES TO THE DECAY $\pi^0 \rightarrow \gamma + \text{"NOTHING"}$

First, we note that decays of the type $A \rightarrow \gamma + B$, where A and B are scalar particles (i.e., spinless, properties of A and B with respect to P-reflection can be arbitrary), are strictly forbidden by angular momentum conservation. Indeed, let us consider this process in the rest frame of the particle A. Due to the momentum conservation, the photon and the particle B produced in the decay move along the same line. The projection of their orbital momentum on this line vanishes, whereas the projection of the photon spin is ± 1 . So, the decay $A \rightarrow \gamma + B$ is strictly forbidden.

Therefore, only those processes can contribute to the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$, in which two or more scalar particles are produced. Now we shall show that the estimate of the rate of producing two scalar particles allows one to neglect their contribution to the process $\pi^0 \rightarrow \gamma + \text{"nothing"}$. It is worth making a reservation here: although the following arguments are not completely rigorous, from our point of view, they are sufficiently convincing.

These new light scalars have not been observed in the experiments yet, so they should interact with matter very weakly. In particular, they must be electrically neutral. So their interaction with matter can take place either due to Yukawa coupling with fermions (quarks in our case) or by means of W and Z boson exchange. Generally speaking, new scalars can exchange with already known particles with a new hypothetical boson. However, the contribution of such diagrams is strongly suppressed. Moreover, in the case of the scalar neutrino, one has the possibility of its interaction with quarks by means of gaugino exchange.

Let us examine the case of the Yukawa coupling of a new scalar particle B in more detail. Possible couplings are

$$\bar{q}qB \quad ; \quad \bar{q}i\gamma_5 qB$$

Let us introduce the operator of "partial" charge parity $\tilde{C}$ which acts only on quarks, photons and gluons:

$$\tilde{C}: \quad q \longrightarrow q^c = C\bar{q}^T \quad , \quad \gamma \longrightarrow -\gamma$$

All the other particles are not transformed by C. Then:

$$\tilde{C}(\pi^0) = +1 \quad , \quad \tilde{C}(\gamma) = -1 \quad , \quad \tilde{C}(B_1 \dots B_n) = +1$$

Therefore the decay $\pi^0 \rightarrow \gamma + \text{scalars}$ is forbidden by $\tilde{C}$ -invariance and can take place only due to weak interactions. (In principle, the interaction of the quarks with a new gauge boson can violate $\tilde{C}$ -parity as well. But, as we have noticed, the contribution of such processes is negligibly small.)

Consequently, the decay $\pi^0 \rightarrow \gamma + BB$ is described by three types of diagrams (Fig. 1). In the case of the scalar neutrino, there is one more type of diagram (Fig. 2). Let us obtain an estimate of their contribution to BR ($\pi^0 \rightarrow \gamma + BB$):

$$\begin{aligned} 1a) \quad BR &\sim \frac{\alpha_Y^2}{\alpha} G_F^2 m_\pi^4 \phi \\ 1b) \quad BR &\sim \alpha G_F^2 m_\pi^4 \phi \\ 1c) \quad BR &\sim \frac{1}{\alpha} G_F^2 m_\pi^4 \phi \\ 2) \quad BR &\sim \alpha^3 \frac{m_\pi^4}{m_q^4} \phi \end{aligned}$$

Here $\alpha_Y = f_Y^2/4\pi$ is the Yukawa coupling constant. Since new scalar particles should be invisible, one should have $\alpha_Y < \alpha$. ϕ is the phase-space-suppressing factor of the decay into three particles relative to the decay into two particles, $\phi \sim 10^{-2}$. The scalar quark mass has to be large: $m_q \gtrsim 70$ GeV [3].

One can see that the main contribution to the decay rate is given by the diagrams of type 1c). That is why it seems worth calculating the decay rate just in this case. Because the new scalar particles are electrically neutral, the interaction Lagrangian reads:

$$\begin{aligned} \mathcal{L}_{int} = & i Z_\mu \frac{Y_B}{2} \frac{g}{\cos \theta_w} (\partial_\mu B^\dagger B - B^\dagger \partial_\mu B) + \\ & + \sum_{q=u,d} Z_\mu \frac{g}{\cos \theta_w} \bar{q} \gamma_\mu \left[\frac{1-\gamma_5}{2} T_{3q} - e_q \sin^2 \theta_w \right] q \end{aligned}$$

The effective low-energy Lagrangian is

$$\begin{aligned} \mathcal{L}_{eff} = & \sum_{q=u,d} i Y_B G_F \sqrt{2} (\partial_\mu B^\dagger B - B^\dagger \partial_\mu B) \cdot \\ & \cdot \bar{q} \gamma_\mu \left[(1-\gamma_5) T_{3q} - 2 e_q \sin^2 \theta_w \right] q \end{aligned}$$

The reasoning involving $\tilde{C}$ -parity shows that the axial quark current does not contribute to the decay. So, the Lagrangian describing the process $\pi^0(p) \rightarrow \gamma(q) + B(K_1) + \bar{B}(K_2)$ is

$$\mathcal{L} = \sum_{q=u,d} \left\{ i\sqrt{2} Y_B G_F \bar{q} \gamma_\mu (T_{3q} - 2e_q \sin^2 \theta_w) q \cdot (\partial_\mu B^\dagger B - B^\dagger \partial_\mu B) + e e_q \bar{q} \gamma_\mu q A_\mu \right\}$$

Let us calculate the matrix element of the decay at the first non-vanishing order of perturbation theory:

$$\begin{aligned} M &= \frac{i^2}{2} \int d^4x d^4y \langle B \bar{B} \gamma | T \mathcal{L}(x) \mathcal{L}(y) | \pi^0 \rangle = \\ &= \sqrt{2} Y_B G_F (k_2 - k_1)_\mu \epsilon_\nu^*(q) e \sum_q e_q (T_{3q} - 2e_q \sin^2 \theta_w) \cdot \\ &\cdot \int d^4x d^4y e^{i(k_1+k_2)x + i q y} \langle 0 | T J_\mu(x) J_\nu(y) | \pi^0(p) \rangle \end{aligned}$$

where $J_\mu = \bar{q} \gamma_\mu q$.

The expression of the matrix element of the T-product of two quark vector currents between the vacuum and neutral pion states is known from the $\pi^0 \rightarrow \gamma\gamma$ analysis. After summation over colour in the PCAC limit one gets:

$$\begin{aligned} &3 \sum_q e_q (T_{3q} - 2e_q \sin^2 \theta_w) \int d^4x d^4y e^{i k x + i q y} \cdot \\ &\cdot \langle 0 | T J_\mu(x) J_\nu(y) | \pi^0(p) \rangle = \frac{3}{4 \pi^2} \frac{1}{f_\pi} \left(\frac{1}{6} - \frac{2}{3} \sin^2 \theta_w \right) \cdot \\ &\cdot \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma (2\pi)^4 \delta^{(4)}(p - k - q) \end{aligned}$$

Therefore:

$$M = -\frac{3}{\sqrt{2} \pi^2} \frac{e Y_B G_F}{f_\pi} \left(\frac{1}{6} - \frac{2}{3} \sin^2 \theta_w \right) \epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\rho \epsilon^{*\nu} q^\sigma$$

Integrating over scalar particle momenta and assuming for simplicity that $m_B = 0$ ($m_B \ll m_\pi$), we have:

$$W = \int \frac{d^3 k_1}{2 k_{10}} \frac{d^3 k_2}{2 k_{20}} \delta^{(4)}(P - k_1 - k_2) |M|^2 =$$

$$= \frac{g}{2\pi^4} \frac{Y_B^2 G_F^2 e^2}{g_\pi^2} \left(\frac{1}{6} - \frac{2}{3} \sin^2 \theta_w \right)^2 \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{\alpha\beta\gamma\delta} \varepsilon^{*\nu} \varepsilon^\beta q^\sigma q^\delta \cdot$$

$$\cdot \frac{\pi}{120} \left[2 P^\rho P^\alpha P^\gamma P^\mu - \frac{3}{2} P^2 (P^\gamma P^\rho g^{\mu\alpha} + P^\mu P^\alpha g^{\gamma\rho}) + \right.$$

$$+ P^2 (P^\gamma P^\mu g^{\rho\alpha} + P^\rho P^\alpha g^{\mu\gamma} + P^\rho P^\mu g^{\alpha\gamma} + P^\alpha P^\gamma g^{\rho\mu}) +$$

$$\left. + \frac{1}{4} P^4 (g^{\rho\alpha} g^{\mu\gamma} + g^{\rho\gamma} g^{\mu\alpha} + g^{\rho\mu} g^{\alpha\gamma}) \right]$$

$$P = p - q$$

The antisymmetry of $\varepsilon_{\mu\nu\rho\sigma}$ and the fact that $q^2 = 0$, $(q\varepsilon) = 0$, $\varepsilon^2 = -1$ give us:

$$w = \frac{3}{4\pi^2} Y_B^2 G_F^2 \alpha \frac{1}{g_\pi^2} \left(\frac{1}{6} - \frac{2}{3} \sin^2 \theta_w \right)^2 (p-q)^2 (pq)^2$$

It is worth noting that this expression does not depend on the photon polarization. So all polarizations of the outgoing photons have equal probabilities and summing over them reduces to multiplying by two. In the rest frame of the π^0 the kinematics of the decay gives us:

$$(p-q)^2 = m_\pi^2 - 2m_\pi q_0, \quad pq = m_\pi q_0$$

The energy spectrum of photons after summing over their polarizations reads:

$$d\Gamma = \frac{3}{(2\pi)^6} Y_B^2 G_F^2 \alpha \left(\frac{1}{6} - \frac{2}{3} \sin^2 \theta_w \right)^2 \frac{m_\pi^2}{g_\pi^2} q_0^3 (m_\pi - 2q_0) dq_0$$

The maximum of the spectrum corresponds to the photons with the energy $q_0 = 3/8m = 50.6$ MeV. The existence of the photons with such an energy could be the signature for this process, because the photons, produced in the background decay $\pi^0 \rightarrow \gamma\gamma$, have energy $q_0 = \frac{1}{2}m = 67.5$ MeV.

The full width proves to be:

$$\Gamma(\pi^0 \rightarrow \gamma + B\bar{B}) = \frac{3}{320} \frac{Y_B^2 G_F^2 \alpha}{(2\pi)^6} \left(\frac{1}{6} - \frac{2}{3} \sin^2 \theta_w \right)^2 \frac{m_\pi^7}{g_\pi^2} \quad (1)$$

Then:

$$BR(\pi^0 \rightarrow \gamma + B\bar{B}) \cong 10^{-18} Y_B^2$$

So we have obtained that the contribution of the (pseudo) scalar particles to the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$ is negligibly small

$$\text{BR}(\pi^0 \rightarrow \gamma + \text{scalars}) \lesssim 10^{-18}$$

3. - PRODUCTION OF PAIRS OF LIGHT NEUTRAL FERMIONS IN THE DECAY $\pi^0 \rightarrow \gamma + \text{"NOTHING"}$

The decay $\pi^0 \rightarrow \gamma + \nu\bar{\nu}$ is the only cold process of the type $\pi^0 \rightarrow \gamma + \text{"nothing"}$, allowed in the standard $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ model. (From the data on the search for $K^+ \rightarrow \pi^+ + \text{"nothing"}$, the Higgs boson must be heavier than 350 MeV. In the case of the non-minimal Higgs sector, there could be light Higgs bosons. This possibility is examined in the preceding section.) So we have to estimate the background created by this process. For generality we assume that the neutrino possesses the right-handed partner and a small mass.

First let us consider the Higgs sector. The most general effective quark-neutrino interaction Lagrangian for this sector reads [2]:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{G_F}{\sqrt{2}} \left[g_{\text{PP}} \bar{\nu} i \gamma^5 \nu + g_{\text{SP}} \bar{\nu} \nu \right] q i \gamma_5 q + \\ & + \frac{G_F}{\sqrt{2}} \left[g_{\text{PS}} \bar{\nu} i \gamma^5 \nu + g_{\text{SS}} \bar{\nu} \nu \right] \bar{q} q \end{aligned}$$

The decay $\pi^0 \rightarrow \gamma \nu\bar{\nu}$, due to the Higgs exchange, is described by the Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{eff}} + e e_q \bar{q} \gamma_\mu q A_\mu \quad (2)$$

In the standard model:

$$\begin{aligned} g_{\text{PP}} = g_{\text{SP}} &= 2m_\nu \frac{m_q}{m_\pi^2 - m_H^2} \lll 1 \\ g_{\text{PS}} = g_{\text{SS}} &= 0 \end{aligned}$$

The Lagrangian (2) respects $\tilde{C}$ -parity, so the process $\pi^0 \rightarrow \gamma \nu\bar{\nu}$, through the Higgs exchange, can take place only in higher orders in G_F , even if there exists ν_R and $m_\nu \neq 0$.

Now let us consider the decays which take place due to Z-boson exchange. By $\tilde{C}$ -invariance one gets that the axial quark current does not contribute to the decay rate. So, the effective quark-neutrino Lagrangian is:

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu \sum_{q=u,d} (\tau_{3q} - 2 e_q \sin^2 \theta_w) \bar{q} \gamma_\mu q$$

It is worthwhile recalling that the right-handed neutrino, even if it exists, is an SU(2) singlet and has hypercharge $Y = 0$ and, therefore, does not interact with W^- and Z -bosons. Discussion like that in the case $\pi^0 \rightarrow \gamma + BB$ leads to the matrix element of the decay $\pi^0(p) \rightarrow \gamma(q) \nu(k_1) \bar{\nu}(k_2)$:

$$M = \frac{e G_F}{\sqrt{2}} \bar{u}(k_1) \gamma_\mu (1 - \gamma_5) u(k_2) \epsilon_\nu^*(q) \sum_q e_q (\tau_{3q} - 2 e_q \sin^2 \theta_w) \cdot \int d^4x d^4y e^{i(k_1+k_2)x + i q y} \langle 0 | T J_\mu(x) J_\nu(y) | \pi^0(p) \rangle$$

Here, as above, $J_\mu = \bar{q} \gamma_\mu q$. For simplicity we put $m_\nu = 0$. Then

$$\Gamma(\pi^0 \rightarrow \gamma + \nu \bar{\nu}) = \frac{3}{320} \frac{\alpha G_F^2}{(2\pi)^6} \left(\frac{1}{6} - \frac{2}{3} \sin^2 \theta_w \right)^2 \frac{m_\pi^7}{f_\pi^2} \quad (3)$$

$$\text{BR}(\pi^0 \rightarrow \gamma + \nu \bar{\nu}) \simeq 8 \cdot 10^{-19}$$

This estimate is in agreement with that of Ref. [6]. Photinos are another possible candidate for the role of light neutral fermions [1,5]. The most general effective low-energy quark-photino interaction Lagrangian reads:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \sum_q e^2 e_q^2 \frac{1}{\tilde{m}_L^2 \tilde{m}_R^2 - \mu^2} \left\{ \tilde{m}^2 \bar{\tilde{\gamma}} \gamma_5 \gamma_\mu \tilde{\gamma} \bar{q} \gamma_\mu \gamma_5 q - \right. \\ \left. - \frac{1}{2} \Delta \tilde{m}^2 \bar{\tilde{\gamma}} \gamma_5 \gamma_\mu \tilde{\gamma} \bar{q} \gamma_\mu q + \mu^2 [\bar{\tilde{\gamma}} \tilde{\gamma} \bar{q} q + \bar{\tilde{\gamma}} \gamma_5 \tilde{\gamma} \bar{q} \gamma_5 q] \right\} \quad (4)$$

Here $\tilde{m}^2 = \frac{1}{2}(\tilde{m}_L^2 + \tilde{m}_R^2)$, $\Delta \tilde{m}^2 = \tilde{m}_R^2 - \tilde{m}_L^2$, where $\tilde{m}_L$ ($\tilde{m}_R$) is the mass of the left (right) scalar quark and μ is the parameter describing the mixing of scalar quarks. The term in the effective Lagrangian (4), which is proportional to $\Delta \tilde{m}^2$, violates $\tilde{C}$ -parity. Therefore, if $\tilde{m}_L \neq \tilde{m}_R$ the process $\pi^0 \rightarrow \gamma \tilde{\gamma} \tilde{\gamma}$ can take place without invoking weak corrections, and hence, its rate may be non-negligible.

So, the decay $\pi^0 \rightarrow \gamma \tilde{\gamma} \tilde{\gamma}$ is described by the Lagrangian

$$\mathcal{L} = \sum_{q=u,d} \left\{ C_q \bar{\psi}_{5\mu} J_\mu + e e_q J_\mu A_\mu \right\}$$

where

$$C_q = - \frac{e^2 e_q^2}{4} \frac{\Delta \tilde{m}^2}{\tilde{m}_L^2 \tilde{m}_R^2 - \mu^2}$$

the photino current is $\Gamma_{5\mu} = \bar{\tilde{\gamma}} \gamma_5 \tilde{\gamma}_\mu \tilde{\gamma}$, and the quark current is $J_\mu = \bar{q} \gamma_\mu q$.

The matrix element of the process $\pi^0(p) \rightarrow \gamma(q) \tilde{\gamma}(k_1) \tilde{\gamma}(k_2)$ to the lowest order of perturbation theory is:

$$\begin{aligned} M &= \frac{i^2}{2} \int d^4x d^4y \langle \gamma \tilde{\gamma} \tilde{\gamma} | T \mathcal{L}(x) \mathcal{L}(y) | \pi^0 \rangle = \\ &= - \sum_{q=u,d} e e_q C_q \epsilon_\nu^*(q) [\bar{u}(k_1) \gamma_5 \gamma_\mu u(k_2) - \bar{u}(k_2) \gamma_5 \gamma_\mu u(k_1)] \times \\ &\quad \times \int d^4x d^4y e^{i(k_1+k_2)x + i q y} \langle 0 | T J_\mu(x) J_\nu(y) | \pi^0(p) \rangle \end{aligned}$$

Arguing as above, one obtains

$$\Gamma(\pi^0 \rightarrow \gamma \tilde{\gamma} \tilde{\gamma}) = \frac{1}{960} \frac{\alpha^3}{(2\pi)^4} \left(\frac{\Delta \tilde{m}^2}{\tilde{m}_L^2 \tilde{m}_R^2 - \mu^2} \right)^2 \frac{m_\pi^7}{f_\pi^2}$$

The energy spectrum of the outgoing photons turns out to be the same as in the decays $\pi^0 \rightarrow \gamma B \bar{B}$, $\gamma \nu \bar{\nu}$. Let us obtain the upper bound on the decay rate $\pi^0 \rightarrow \gamma \tilde{\gamma} \tilde{\gamma}$ using the experimental data. The accelerator data imply that the scalar quark should be heavier than 70 GeV [3]. Hence:

$$\tilde{m}_{sq}^2 = \tilde{m}^2 \pm \sqrt{\mu^4 + \frac{1}{4}(\Delta \tilde{m}^2)^2} \geq \tilde{m}_0^2 = (70 \text{ GeV})^2$$

The experiments on P-violation in the hadron interaction impose the following constraints on the squark mass splittings [6]:

$$\left| \frac{\Delta \tilde{m}^2}{\tilde{m}^4} \right| \lesssim \left(\frac{1}{140 \text{ GeV}} \right)^2 \frac{1}{\ln x - 2.56}, \quad \text{if } x \gg 1$$

$$\left| \frac{\Delta \tilde{m}^2}{\tilde{m}^4} \right| \lesssim \left(\frac{1}{105 \text{ GeV}} \right)^2, \quad \text{if} \quad \alpha = 1$$

where $X = (\text{squark mass}/\text{gluino mass})^2$. One can see that the upper bound on the decay rate follows from the limit on the scalar quark mass^{*}:

$$\Gamma(\pi^0 \rightarrow \gamma \tilde{\gamma} \tilde{\gamma}) \lesssim \frac{1}{960} \frac{\alpha^3}{(2\pi)^4} \frac{m_\pi^7}{f_\pi^2} \frac{1}{\tilde{m}_0^4}$$

$$\text{BR}(\pi^0 \rightarrow \gamma \tilde{\gamma} \tilde{\gamma}) \lesssim 1.4 \cdot 10^{-16}$$

Hence, the photino production in the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$ cannot create any observable background. Generally speaking, a Higgsino and a Goldstino can also be produced in the decay. But we know from the mass spectrum of the observed particles that the supersymmetry breaking scale has to be much greater than 1 TeV. So the contribution of the Goldstino is rather small.

The Higgsinos can, in principle, violate $\tilde{C}$ -parity (if there is scalar quark mass splitting). But due to the large squark mass their contribution is negligible [as we have mentioned, $(m_\pi/m_{\tilde{q}})^4 < 10^{-12}$].

Let us summarize. The contribution of the neutrinos to the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$ is comparable with that of scalars and is equal to $\text{BR}(\pi^0 \rightarrow \gamma \nu \bar{\nu}) \approx 10^{-18}$. The background due to Goldstinos cannot exceed this quantity. The contribution of the photinos and Higgsinos can be larger, but all the same, it is negligibly small, less than 10^{-16} .

^{*}) Here we assume that the photino is massless, $M_{\tilde{\gamma}} = 0$. This case is of particular interest because by cosmological considerations [4], one obtains that $M_{\tilde{\gamma}} \lesssim 0(100 \text{ eV})$ or $M_{\tilde{\gamma}} > 4 \text{ GeV}$.

4. - PRODUCTION OF A NEW LIGHT GAUGE BOSON IN THE DECAY $\pi^0 \rightarrow \gamma + \text{"NOTHING"}$

Many modern models predict an enlargement of the standard $SU(3) \times SU(2) \times U(1)$ group by an extra $U(1)$ factor. This possibility is realized, for example, in GUTs [8], SUSY models [9], and superstring theories [10]. Recently the interest for such models has increased also in connection with possible indications of the existence of a new long-range interaction ("the fifth force") [11].

The possibility of the existence of a vector boson, which interacts with the baryon current, was suggested long ago by Lee and Yang [12]. If this boson is massless, then the results of the experiments on the check of the equivalence principle imply a strong constraint on its coupling constant [13]: $\alpha_{LY} \lesssim 10^{-47}$. If, however, this boson is massive, so that its Compton wavelength has a microscopic value, this constraint is no longer valid.

So, we shall look for the decay of the neutral pion into a photon and a massive vector boson X . The case when the mass of the new boson X is considerably different from zero is of particular interest from the experimental point of view, because in the rest frame the photons produced in the decay $\pi^0 \rightarrow \gamma X$ will then have an energy smaller than $m_\pi/2$, i.e., the energy of photons in the background decay $\pi^0 \rightarrow \gamma\gamma$.

The interaction Lagrangian of the new hypothetical gauge boson X with the vector quark current reads:

$$\mathcal{L} = \sum_q g_q \bar{q} \gamma_\mu q X_\mu$$

By $\tilde{C}$ -parity arguments, one can conclude that a coupling of X to the axial quark current gives a negligible small contribution to the decay $\pi^0 \rightarrow \gamma X$.

The amplitude of the decay $\pi^0 \rightarrow \gamma X$ can be obtained, as above, from that of $\pi^0 \rightarrow \gamma\gamma$. One gets:

$$\Gamma(\pi^0 \rightarrow \gamma X) = \frac{2\alpha}{(4\pi)^4} (2g_u + g_d)^2 \frac{m_\pi^3}{f_\pi^2} \left(1 - \frac{m_X^2}{m_\pi^2}\right)^3 \quad (8)$$

where $f_\pi = 93$ MeV and m_X is the mass of the X boson. To get the feeling of the numbers, let us consider the $\tilde{U}(1)$ model of Fayet [9] with the spontaneous SUSY breaking by a D term. The extra gauge boson of this model (usually called the U -boson) interacts with the vector quark current $\bar{q} \gamma_\mu q$ with the coupling constant^{*)}:

^{*)} The U -boson has also an axial coupling which is unimportant for us.

$$g_q = 2 \cdot 10^{-3} m_u (\text{GeV}) r \cos \varphi \quad (9)$$

where ($0 < r \leq 1$) and ϕ are the parameters which are fixed by the Higgs sector of the model and the scale of SUSY breaking respectively. The analysis of the possible effects of the U-boson in various experiments (VN-scattering, $K^+ \rightarrow \pi^+ U$ decays, "beam-dump" experiments, e^+e^- annihilation, ϕ , T-decays, etc. [14]) gives the conservative allowed region of the U-boson's masses^{*}: $100 \text{ MeV} < m_U < 120 \text{ MeV}$.

From the experimental data on atomic P-violation [15], one can get the constraint on the parameters r and ϕ :

$$\text{for } m_U = 100\text{--}120 \text{ MeV } r \cos \phi < 0.15.$$

From (8) and (9) one obtains for $m_U = 100 \text{ MeV}$:

$$\text{BR}(\pi^0 \rightarrow \gamma U) \text{BR}(U \rightarrow \nu \bar{\nu}) \leq 6.5 \cdot 10^{-8} \cos \varphi \quad (10)$$

In the derivation of (10) we have taken into account that the branching ratio of U-boson decay into invisible channels is $\text{BR}(U \rightarrow \nu \bar{\nu}) = 0.6$. In an analogous way, one can estimate the rate of the production of a new gauge X-boson in the decay $\pi^0 \rightarrow \gamma X$ in the framework of other models, which contain an extra $U(1)$ -factor. Comparing (10) with (1), (3) and (7), one can see that the background due to the production of any invisible particles, except the gauge boson, is quite insignificant in the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$. This fact turns the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$ into the unique instrument of searching for new light gauge bosons.

5. - CONCLUSIONS AND OUTLOOK

From the preceding discussion, one can conclude that the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$ is quite a convenient tool for investigating the properties of possible new light (with mass less than m_π) gauge bosons. The background due to the production of other light neutral particles (axion, neutrino, photino and so on) proves to be negligibly small in this process. This makes the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$ unique in the sense that its observation would allow one to describe properties of this "nothing" unambiguously.

^{*}) The question whether the region of masses $m_U < 100 \text{ MeV}$ is excluded or not by $K^+ \rightarrow \pi^+ U$ data requires special investigation because the theoretical calculation of the decay rate involves considerable uncertainties.

Hence the analysis of the experimental possibilities of the search for the decay $\pi^0 \rightarrow \gamma + \text{"nothing"}$ seems to be of great importance in the investigation of theories which contain an extra light gauge boson [8-10]. At present the possibility of such an experiment at the level of 10^{-6} - 10^{-8} is being discussed at the Institute for Nuclear Research of the USSR Academy of Sciences.

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FIGURE CAPTIONS

Fig. 1 : Type of diagrams for the decay $\pi^0 \rightarrow \gamma + \text{scalars}$.

Fig. 2 : Diagrams for the decay $\pi^0 \rightarrow \gamma + \tilde{\nu}\tilde{\nu}$.

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 2