CM-P00062410

Ref. TH.4939/88

Archives

THE EFFECTS OF PION EXCHANGE CORRECTIONS ON THE

2ν $\beta\beta$ -DECAY NUCLEAR MATRIX ELEMENTS

M. Ericson

CERN - Geneva
and
Institut de Physique Nucléaire
69622 Villeurbanne

and

J.D. Vergados

Physics Department
University of Ioannina, GR45332

A B S T R A C T

We examine the rôle of pion exchange diagrams in the nuclear matrix elements which enter the 2ν $\beta\beta$ -decay. The corresponding effective operators are constructed using PCAC and soft-pion theorems. The matrix elements of these operators are evaluated in the case of the $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ decay. We find that these effects are small and cannot possibly account for the discrepancy between the shell-model results and the experimental values.

CERN-TH.4939/88

January 1988

DOUBLE β DECAY

Double β decay has been the subject of an intense activity, both experimentally and theoretically. It can occur via second-order weak interactions, e.g.,

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \tilde{\nu}_e + \tilde{\nu}_e \quad 2\nu \beta\beta \text{ decay} \quad (1)$$

In the presence of lepton-violating interaction, another process is possible:

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- \quad 0\nu \beta\beta \text{ decay} \quad (2)$$

In geochemical experiments [1] which measure the relative abundance of daughter nuclei it is not possible to distinguish between processes 1 and 2. This is instead feasible in laboratory experiments with the measurement of the total energy of the two electrons [2-5]. The best limit on $0\nu \beta\beta$ decay comes from the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ decay which gives [2] $T_{1/2} > 5.0 \times 10^{23}$ y. This limit has improved by an order of magnitude in a decade [3-5].

The detection of the 0ν process would signal not only the violation of lepton quantum number conservation but also the fact that neutrinos are massive Majorana particles. Interest has therefore focused on this process.

However, $2\nu \beta\beta$ decay is also interesting for the following reasons:

- (i) by the challenge raised by the measurement of lifetimes larger than 10^{20} years; from this point of view, the recent measurement [6] of $T_{1/2}(2\nu)$ between 0.8 and 1.1 times 10^{20} y for the $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ is a remarkable result;
- (ii) the width $\Gamma_{2\nu}$ of this process must be known for the interpretation of the geochemical experiments [1] in order to extract the width $\Gamma_{0\nu}$ for the neutrinoless process from the total lifetime;
- (iii) it can serve as a test of our ability to evaluate matrix elements of two-body operators in systems with a complex structure, such as those entering double β -decay. Thus, even though the more interesting operator entering $0\nu \beta\beta$ -decay is not identical to that associated with the 2ν process, one can gain a certain amount of confidence in the prediction of the 0ν rate which is required in order to extract information about the neutrino properties from the $0\nu \beta\beta$ -decay data.

The value of the 2ν nuclear matrix elements to the energetically allowed states is still under debate as evidenced by the large discrepancies between the results of the various calculations. The general consensus [6-11] is that these large deviations are due to the fact that these matrix elements are small and they can be affected by some detailed aspects of the wave function which are not fully

under control. The smallness of these nuclear matrix elements is also consistent with the available experimental data. In fact the geochemical rates [7] of the $^{180}\text{T}_e \rightarrow ^{130}\text{Xe}$ decay are about 200 times slower than the theoretical predictions [8] based on large shell-model calculations. From this point of view, it is not surprising that the evaluations of these tiny matrix elements come under periodic revision. For instance, it has recently been pointed out [12-14] that a suppression mechanism required by the data is present in the quasi-particle random phase approximation (QRPA) when both the particle-particle and particle-hole interactions are consistently incorporated.

In view of the smallness of the matrix element for 2ν double β decay, one may wonder about the rôle played by the pion exchange effects which are ignored in present calculations: the $e\nu$ pair is produced together with a pion which is reabsorbed by another neutron, leading to the mission of the second $e\nu$ pair. The two-body operators for such processes are different from the usual one, since they involve the pion propagator. They are not automatically submitted to the same cancellations as the main term. In fact, meson exchange corrections have a significant influence on many nuclear processes, in particular when cancellations occur in the impulse approximation matrix element. It is interesting to explore whether the same occurs in 2ν double β decay, which is the aim of the present article. The specific case of ^{48}Ca is considered.

THE 2ν $\beta\beta$ DECAY PROCESS

A second-order weak interaction proceeds via the diagram shown in Fig. 1a. In the standard evaluation, the virtual intermediate states are purely nuclear. The amplitude then takes the form [15-17]

$$M = \left(\frac{G_F}{\sqrt{2}}\right)^2 \sum_n \frac{\langle \bar{e} \bar{e} \tilde{\nu}_e \tilde{\nu}_e | H_w | \tilde{\nu}_e \bar{e} n \rangle \langle \tilde{\nu}_e \bar{e} n | H_w | i \rangle}{E_n - E_i + E_e + E_{\tilde{\nu}_e}} \quad (3)$$

where H_w is the weak interaction Hamiltonian.

In principle one has to introduce labels for the leptons and ensure antisymmetrization for identical leptons. These subtle points, which are not made explicit here, are included in the actual calculations. The energy denominator depending on the lepton energies, one cannot separate in principle the nuclear part from the phase space integral. To achieve this separation, we replace $E_e + E_{\tilde{\nu}_e}$ by an average value [15-17]

$$E_e + E_{\tilde{\nu}_e} \rightarrow \langle E_e + E_{\tilde{\nu}_e} \rangle = \frac{1}{2} T_0 + m_e c^2$$

where $T_0 = M(Z) - M(Z-2)$ is the available energy. Then in the allowed approximation of the standard V-A theory, the amplitude for $0^+ \rightarrow 0^+$ decays can be cast in the form [15]

$$M = \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{g_A^2}{m_e} \left\{ \vec{p}_1 \cdot \vec{p}_2 \frac{M_{GT}}{\mu_0} + \vec{p}_1^0 \vec{p}_2^0 \left(\frac{g_V}{g_A}\right)^2 \frac{M_F}{\tilde{\mu}_0} \right\} \quad (4)$$

where λ_i^λ are the relevant leptonic currents defined by

$$P_i^\lambda = \bar{u}(p_{ei}) \gamma^\lambda (1 - \gamma_5) v(p_{\nu i}) \quad i = 1, 2$$

and M_{GT} and M_F are the Gamow-Teller and Fermi matrix elements defined by

$$M_{GT} = \langle 0_f^+ | \vec{\gamma} \cdot \vec{Y} | 0_i^+ \rangle, \quad \vec{Y} = \sum_i \tau^+(i) \vec{\sigma}(i) \quad (5a)$$

$$M_F = \langle 0_f^+ | T_+ T_+ | 0_i^+ \rangle, \quad T_+ = \sum_i \tau_+(i) \quad (5b)$$

The Fermi matrix element M_F has been included only for completeness of the discussion since it vanishes for ground-state to ground-state transitions due to isospin conservation. The quantity μ_0 is defined by

$$\frac{M_{GT}}{\mu_0} = \sum_n \frac{\langle 0_f^+ | \vec{Y} | 1^+ n \rangle \cdot \langle 1^+ n | \vec{Y} | 0_i^+ \rangle}{\mu_n} \quad (6a)$$

where

$$\mu_n = 1 + [E_x(1^+ n) - E_i + T_0/2] / m_e c^2 \quad (6b)$$

At this point we should stress that if the quantity μ is defined via Eq. (6a), the amplitude of Eq. (4) involves no approximation. In practice, however, most shell-model calculations replace μ_n in Eq. (6a) by an average value associated with some average excitation energy $\langle E_x(1^+) \rangle$ (closure approximation). The recent calculations [12-14] done in the context of QRPA have the advantage that the summation over the intermediate states is explicitly performed.

Concerning the meson exchange mechanisms for 2ν emission, one possibility would be the $\Delta(3/2, 3/2)$ resonance contribution shown in Fig. 1b. The amplitude associated with the process of Fig. 1b is proportional to $\sqrt{P_\Delta}$, where P_Δ is the

probability of finding the $\Delta(3/2, 3/2)$ resonance in the nucleus [18]. To this order, however, the $\Delta(3/2, 3/2)$ resonance gives no contribution for $0^+ \rightarrow 0^+$ decays due to angular momentum conservation selection rules. Thus one is left with meson exchange contributions.

The relevant meson exchange amplitudes are shown in Figs. 2a-2d for the vector part and Figs. 3a-3d for the axial part. The vertices appearing in Fig. 2 are obtained from CVC. In this way the two-body operators built from the graphs of Fig. 2 are similar to those entering into the meson exchange corrections to the Compton amplitude [19].

Similarly, PCAC provides the vertices of Fig. 3 which are thus derived from the πN scattering amplitude. The graphs of Fig. 3 therefore have a connection to pion double-charge exchange. For the vertex $\pi \rightarrow \pi + 2e^- + 2\tilde{\nu}_e$ entering in Fig. 3b we have used a quark model of the pion which gives $2\lambda^0 \lambda^0$.

Ignoring the small energy of the exchanged pion, we obtain the following expressions, valid for $0^+ \rightarrow 0^+$ decays, for the two-body operators in momentum space. For the vector part,

$$V_1 = \left(\frac{G_F}{\sqrt{2}}\right)^2 \left[\frac{2}{3} \left(\frac{g_A}{f_\pi}\right)^2 \vec{p}_1 \cdot \vec{p}_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{1}{q^2 + m_\pi^2} \right] \epsilon_1^+ \epsilon_2^+ \quad (7a)$$

$$V_2 = \frac{G_F}{\sqrt{2}} \left[\frac{2}{3} \left(\frac{g_A}{f_\pi}\right)^2 \vec{p}_1 \cdot \vec{p}_2 (-4) \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{(q^2 + m_\pi^2)^2} \right] \epsilon_1^+ \epsilon_2^+ \quad (7b)$$

$$V_3 = \left(\frac{G_F}{\sqrt{2}}\right)^2 \left[\frac{2}{3} \left(\frac{g_A}{f_\pi}\right)^2 \vec{p}_1 \cdot \vec{p}_2 (4) \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) q^2}{(q^2 + m_\pi^2)^3} \right] \epsilon_1^+ \epsilon_2^+ \quad (7c)$$

$$V_4 = \left(\frac{G_F}{\sqrt{2}}\right)^2 \left[\frac{2}{3} \left(\frac{g_A}{f_\pi}\right)^2 \vec{p}_1 \cdot \vec{p}_2 (-3) \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{(q^2 + m_\pi^2)^2} \right] \epsilon_1^+ \epsilon_2^+ = \frac{3}{4} V_2 \quad (7d)$$

where $g_A = 1.24$ and f_π is the charged pion decay constant ($f_\pi = 130$ MeV). We make the standard decomposition

$$3(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) = q^2 [\vec{\sigma}_1 \cdot \vec{\sigma}_2 + T(\vec{\sigma}_1, \vec{\sigma}_2, \vec{q})] \quad (8)$$

where T is the usual tensor: $T(\vec{\sigma}_1, \vec{\sigma}_2, \vec{q}) = 3(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$. In the chiral limit $m_\pi \rightarrow 0$, a cancellation occurs, similar to that of Compton scattering [19]. Namely, the central pieces of V_1, V_2, V_3, V_4 add to zero, and the tensor parts of V_1, V_2, V_3 also add to zero, so that $V = V_1 + V_2 + V_3 + V_4 \rightarrow (V_4)_{\text{tensor}}$.

Coming now to the axial part, we obtain

$$A_1 = \left(\frac{G_F}{\sqrt{2}}\right)^2 \left\{ -\frac{2}{f_\pi^2} \vec{p}_1 \cdot \vec{p}_2 + (f_\pi 4\pi d_1)^2 \vec{p}_1 \cdot \vec{p}_2 \left(-\frac{1}{3}\right) (\vec{\sigma}_1 \times \vec{q}) (\vec{\sigma}_2 \times \vec{q}) \right\} \frac{\tau_1^+ \tau_2^+}{q^2 + m_\pi^2} \quad (9a)$$

The first part corresponds to the rescattering of s-wave pions and the second part to that of p waves. In expression (9a) d_1 is the spin-dependent part of the charge exchange p wave πN scattering volume, extrapolated for soft pions: $d_1 \approx 0.07 \text{ m}_\pi^{-3}$.

In the πN amplitudes which appear at the vertices of Fig. 3, we have, as usual in the meson exchange technique, retained only the non-Born parts (d_1 for instance is dominated by the intermediate virtual Δ excitation). It is now a straightforward matter to perform a Fourier transform of the amplitudes of Eqs. (7a)-(7d) and (9a) and get the corresponding effective operators in co-ordinate space. Thus Eq. (4) becomes

$$\mathcal{M} = \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{g_A^2}{m_\pi^2} \left\{ \vec{p}_1 \cdot \vec{p}_2 \frac{M_{GT}}{\mu_0} + K_1 [\vec{p}_1 \cdot \vec{p}_2 (V + \xi_1 A) - \vec{p}_1 \cdot \vec{p}_2 \xi_0 A_0] \right\} \quad (11)$$

where

$$K_1 = \frac{2}{3} \frac{m_\pi m_\pi}{f_\pi^2} \frac{1}{4\pi} = 2.2 \times 10^{-4}, \quad \xi_0 = 3/g_A^2 = 2, \quad \xi_1 = \frac{1}{3} (4\pi d_1 m_\pi^3)^2 \frac{f_\pi}{m_\pi} = 0.49$$

The operators entering the matrix elements V , A and A_0 are respectively

$$\Omega_V = \sum_{i \neq j} \tau^+(i) \tau^+(j) \left\{ \vec{\sigma}_i \cdot \vec{\sigma}_j \left[u_1^s + \frac{4}{3} (u_2^s + u_3^s) + u_4^s \right] + T(\vec{\sigma}_i, \vec{\sigma}_j, \hat{x}) \left[\frac{4}{3} u_2^T + u_3^T + u_4^T \right] \right\} \quad (12)$$

$$\Omega_A = \sum_{i \neq j} \tau^+(i) \tau^+(j) \left\{ \frac{2}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j [u_1^s(x) - 4\pi \delta(\vec{x})] - \frac{1}{3} T(\vec{\sigma}_i, \vec{\sigma}_j, \hat{x}) u_4^T(x) \right\} \quad (13)$$

$$\Omega_0 = \sum_{i \neq j} \tau^+(i) \tau^+(j) \gamma_0(x) \quad (14)$$

where

$$x = m_\pi r_{ij}, \quad \gamma_0(x) = e^{-x}/x \quad \text{and}$$

$$\begin{aligned}
 u_1^s(x) &= y_0(x) = e^{-x}/x, & u_1^T(x) &= x^2 y_0(x) = (1+3x+3x^2) y_0(x) \\
 u_4^s(x) &= u_2^s(x) = \frac{1}{2}(x-2)y_0(x), & u_4^T(x) &= u_2^T(x) = \frac{1}{2}(x+1)y_0(x) \\
 u_3^s(x) &= -\frac{1}{2}(x-2)y_0(x) + \frac{1}{8}(x-3)e^{-x}, & u_3^T(x) &= -\frac{1}{2}(x+1)y_0(x) + \frac{1}{8}xe^{-x}
 \end{aligned} \tag{15}$$

Note that the amplitude (10) has been scaled by the electron mass m_e , a standard practice in 2ν $\beta\beta$ -decay calculations. The relative importance of the exchange current contribution is set by the value of K_1 compared to $1/\mu_0$. We note that K_1 is of order 10^{-4} while $1/\mu_0$ is larger due to the smallness of the energy denominators. For the $A = 132$ system for instance the Los Alamos group [16] estimates $1/\mu_0 \approx 4 \times 10^{-2}$. Even if this must be decreased by an order of magnitude to bring the theoretical prediction into agreement with experiment, $1/\mu_0$ remains larger than K_1 . The meson exchange terms could be significant only if M_{GT} is greatly suppressed.

We will now calculate the relevant nuclear matrix elements entering the $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}(\text{g.s.})$ decay. This transition is particularly suited for our purposes since:

- (i) the nuclei involved have simple nuclear structure; in fact they can adequately be described in terms of the $0f_{7/2}$ shell, i.e., both protons and neutrons are restricted to occupy the $0f_{7/2}$ shell;
- (ii) the ground state transition matrix element M_{GT} is known to be suppressed [7,11,20,21];
- (iii) The quantity μ_0 can be calculated exactly. In fact, within the $0f_{7/2}$ shell there is only one intermediate 1^+ state of ^{48}Sc which is of the form $[f_{7/2}(n)f_{7/2}(p)]1^+$. Thus, Eq. (3) is reduced to Eq. (4) without approximation with $\mu_0 = \mu(1^+)$.

In order to give a scale to the amount of cancellation occurring in M_{GT} , we can therefore compare the quantity M_{GT} to the sum rule involving the double Gamow-Teller operator [22] $Y_+ = \sum_i \vec{\sigma}_i \tau_i^+$

$$S_{GT} = \sum_f \langle 0_f^+ | Y_+ Y_+ | 0 \rangle \approx 6(N-Z)(N-Z+1) \tag{16}$$

This result only holds for Pauli blocked nuclei, which ^{48}Ca is not. But its order of magnitude still gives an element of comparison for the double GT strength to the ground state. We shall see below that the matrix element M_{GT} for ground state transition is 0.24. It thus exhausts only $(0.24)^2/S_{GT} \approx (0.24)^2/6 \times 72 = 1.3 \times 10^{-4}$ of the sum rule, evidentiating the smallness of the quantity M_{GT} .

The initial wave function in the above model is unique, i.e., of the form of $0f_{7/2}(n)$. The final state wave function is of the form [7,20]

$${}^{48}\text{Ti}(0^+) = \sum_j C_j \left\{ 0 f_{7/2}^6(n); 0 f_{7/2}^2(p); 0^+ \right\} \quad (17)$$

where $J = 0, 2, 4, 6$. The coefficients C_j depend, of course, on the two-body interaction. In this work we use the Kuo-Brown renormalized G-matrix elements. Then one finds [15-20] that the g.s. wave function corresponds to

$$C_j = 0.908, -0.418, -0.043, 0.004 \quad \text{for} \quad J = 0, 2, 4, 6 \quad (18)$$

Furthermore the quantity μ_0 is in this case

$$\mu_0 = 11$$

The nuclear matrix element is

$$M_{6T} = -0.238 \quad M_{2v} = \frac{M_{6T}}{\mu_0} = -0.0216 \quad (19)$$

The smallness of M_{6T} can be traced back to the well-known cancellation between the $J = 0$ and $J = 2$ components of the ${}^{48}\text{Ti}$ wave function.

The exchange current matrix elements, associated with the operators of Eqs. (12)-(14), including the factors ξ_0 and ξ_1 but not the scale K_1 , are given in the Table. The tensor contribution and, to a lesser extent, the scalar contribution also suffer from the above-mentioned cancellation. The resulting contribution to the 2ν $\beta\beta$ -decay is very small mainly due to the smallness of the parameter K_1 , i.e.,

$$K_1(V + \xi_1 A) = 2.2 \times 10^{-4} \times (0.046 + 0.49 \times 0.098) = 0.76 \times 10^{-5} \quad (20a)$$

$$-K_1 \xi_0 A_0 = -2.2 \times 10^{-4} \times 2 \times (-0.095) = 0.41 \times 10^{-4} \quad (20b)$$

Thus the dominant time-component term is three orders of magnitude smaller than the usual matrix elements M_{2v} .

The above exchange current matrix elements may be somewhat uncertain due to the short-range two-nucleon correlation. In order to test ^{*)} this we employed two nucleon-wave functions of the type [15]

^{*)} This is the reason why no cut-off factors were employed in deriving Eqs. (12)-(19).

$$\Psi_{\omega}(\vec{x}_1, \vec{x}_2) = \left\{ 1 - c(|\vec{x}_1 - \vec{x}_2|) \right\} \Psi_{sm}(\vec{x}_1, \vec{x}_2) \quad (21)$$

where Ψ_{sm} is the usual shell model wave function and

$$c(r) = e^{-\alpha r^2} (1 - b r^2) \quad , \quad \alpha = 1.1 \text{ fm}^{-2} \quad b = 0.68 \text{ fm}^{-2} \quad (22)$$

The results thus obtained also appear in the Table. From these we obtain

$$K_1(V + \xi_1 A) = 0.62 \times 10^{-5} \quad , \quad -K_1 \xi_1 A_0 = 0.37 \times 10^{-4}$$

i.e., a small modification.

The above results indicate that, even though there exist some cancellations nuclear in origin in the matrix elements of the exchange current operators, the main suppression comes from the smallness of K_1 compared to $1/\mu_0$. This is expected to be a general result since in the exchange mechanism there is no small energy (mass) scale to compete with the small energy denominators μ_n . A large μ_0 can in principle arise even if the energy denominators μ_n are all small compared to m_π , if it so happens that the various terms in the sum of Eq. (6a) contribute destructively. But even then the value M_{GT}/μ_0 must become much smaller than that dictated by experiment before the exchange current mechanism becomes effective.

It thus appears that the smallness of the $M_{2\nu}$ matrix element must be explained in terms of traditional nuclear physics. The QPRPA [12-14] calculations are in this direction.

To summarize: we have investigated the possible rôle played by the meson exchange process in the neutrinoless double β -decay. We have seen that such contributions are much smaller than those associated with the usual process even though the latter exhausts only a small fraction of the double Gamow-Teller sum rule. They are not, therefore, expected to play a crucial rôle if only a reduction of about an order of magnitude of the usual theoretical nuclear matrix elements are needed to bring them into agreement with those extracted from experiment.

ACKNOWLEDGEMENTS

One of us (J.D.V.) would like to thank the Theoretical Physics Division of CERN and the Greek CERN Committee for providing support for a visit at CERN where most of this work was performed.

Operator	No correlation		Gaussian correlation	
	Scalar	Tensor	Scalar	Tensor
Ω_{V_1}	0.118	-	0.079	-
Ω_{V_2}	-0.067	-0.015	-0.031	-0.015
Ω_{V_3}	0.044	0.002	0.011	0.006
Ω_A	0.078	-0.020	0.052	-0.014
Ω_{A_0}	-0.095	-	-0.086	-
$V_1 + V_2 + V_3 + V_4 + \xi_1 A = 0.035 \quad V_1 + V_2 + V_3 + V_4 + \xi_1 A = 0.028$ $-\xi_0 A_0 = 0.186 \quad -\xi_0 A_0 = 0.168$				

Table The various exchange current matrix elements entering the $2\nu \beta\beta$ -decay $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ in dimensionless units [here in units of K_1 (see Eq. (11))]. The matrix element Ω_{V_1} , which does not interfere with the usual nuclear matrix element, is dominant. The matrix element V_4 is not given since $V_4 = 3/4 V_2$.

REFERENCES

- [1] T. Kirsten, H. Richter and E. Jessberger - Phys.Rev.Lett. 50 (1983) 474;
Z.Phys. C16 (1983) 189.
- [2] D.O. Caldwell et al. - Phys.Rev. D33 (1986) 2737;
D.O. Caldwell et al. - to be published.
- [3] T. Eijiri et al. - Nucl.Phys. A448 (1986) 27.
- [4] F. Avignone et al. - Phys.Rev.Lett. 50 (1983) 721; Phys.Rev. C34 (1986)
666.
- [5] F. Bellotti et al. - Lett.Nuovo Cimento 33 (1982) 273.
- [6] S.R. Elliott, A.A. Hahn and M.K. Moe - Phys.Rev.Lett. 59 (1987) 2020.
- [7] J.D. Vergados - Phys.Rev. C13 (1976) 865; C24 (1980) 641.
- [8] W.C. Haxton, G.J. Stephenson and D. Strottman - Phys.Rev.Lett. 47 (1981)
153; Phys.Rev. D25 (1982) 2360.
- [9] K. Grotz, H.V. Klapdor and J. Metzinger - J.Phys. G9 (1983) 169.
H.V. Klapdor and K. Grotz - Phys.Lett. 142B (1984) 323.
- [10] T. Tomoda, A. Faessler, K.W. Schmid and F. Grümmer - Nucl.Phys. A452 (1986)
591.
- [11] L. Zamick and N. Auerbach - Phys.Rev. C26 (1982) 2185.
- [12] P. Vogel and M.R. Zirnbauer - Phys.Rev.Lett. 57 (1986) 3148.
- [13] O. Civitarese, A. Faessler and T. Tomoda - Phys.Lett. 194 (1987) 11.
- [14] J. Engel, P. Vogel and M.R. Zirnbauer - To appear in Phys.Rev. C
- [15] J.D. Vergados - Physics Reports 133 (1986) 1.
- [16] W.C. Haxton and G. Stephenson - Progr.Part.Nucl.Phys. 19 (1984) 409.
- [17] M. Doi, T. Kotani and E. Takasugi - Progr.Theor.Phys. (Suppl) 83 (1985) 1.
- [18] J.D. Vergados and M. Ericson - Nucl.Phys. B195 (1982) 262.
- [19] M. Ericson and M. Rosa-Clot - Phys.Lett. 188B (1987) 11.
- [20] L.D. Skouras and J.D. Vergados - Phys.Rev. C28 (1983) 2122.
- [21] W.C. Haxton, S.P. Rosen and G.J. Stephenson - Phys.Rev. D26 (1982) 1805.
- [22] P. Vogel, M. Ericson and J.D. Vergados - in preparation.

FIGURE CAPTIONS

Fig. 1a Diagramatic representation of 2ν $\beta\beta$ -decay which occurs as a second-order process in the ordinary weak interaction.

Fig. 1b 2ν $\beta\beta$ -decay induced by the direct decay of the $\Delta(3/2,3/2)$ resonance present in the nuclear medium. This process does not contribute to $0^+ \rightarrow 0^+$ decays.

Fig. 2 Pion exchange diagrams for the vector part.

Fig. 3 Pion exchange diagrams for the axial part.

a

b

- Figure 1 -

- Figure 2 -

- Figure 3 -