TWO-PHOTON ANNIHILATION INTO PROTON-ANTIPROTON IN A QUARK-DIQUARK SCHEME^{*)}

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A B S T R A C T

We consider $\gamma\gamma \rightarrow p\bar{p}$ in the framework of the Brodsky-Farrar-Lepage scheme allowing for (scalar) diquarks as quasi-elementary constituents. We compute the unpolarized cross-section using different proton wave functions. Their normalization is fixed by computing the electromagnetic form factor of the proton in the same framework. Good agreement with the experimental data can be obtained. Two-photon annihilation into other baryons is also discussed.

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1. Introduction

The correct description of exclusive processes in terms of perturbative QCD or, in general, in the framework of constituent models still presents many challenging problems. Most experimental data are available only at moderately large values of energy, in a region where many non-perturbative effects may still play a crucial role. In some previous work[1,2] it has been shown that introducing diquarks as quasi-elementary constituents inside baryons leads to a much better understanding of observed phenomena: vector diquarks prove to be an essential tool in describing many spin effects[1], otherwise inexplicable in perturbative QCD pure quark models; hyperon-antihyperon production in $p\bar{p}$ annihilation[2] is also easily understood in terms of a quark-diquark scheme. Diquarks, as underlined in Ref.[1], have also received many other experimental and theoretical supports.

In Ref.[1] the idea of diquarks has been applied to large angle nucleon-nucleon elastic scattering in the framework of the end-point model; it was stressed, however, that the most general and natural scheme in which one should use diquarks as constituents would be that of Brodsky-Farrar-Lepage (BFL)[3]. In such a scheme, in fact, the introduction of diquarks, while not changing some correct qualitative results, would lead both to helicity flips and relative phases, crucial to spin effects.

In this paper we start working along these lines by computing $\gamma\gamma \rightarrow p\bar{p}$ annihilation in the scheme of Ref.[3] and allowing for diquarks as quasi-elementary constituents. We will not consider, for such a process, spin effects, for which no data are available. As we learned from nucleon-nucleon elastic scattering[1], while vector diquarks are essential when dealing with spin effects, most of the contribution to the unpolarized cross section comes from scalar ones. As a consequence we consider here mainly scalar diquark contributions, that is, we look at the proton as a bound state of a quark and a scalar diquark. One should not underestimate, however, the role of vector diquarks when trying to compute other quantities rather than unpolarized cross-sections. Possible corrections due to vector diquarks will be discussed at the end of the paper.

The plan of the paper is as follows. In Sect.2 we define our formalism and explicitly compute, in our quark-diquark scheme, all elementary amplitudes contributing to $\gamma\gamma \rightarrow p\bar{p}$. In Sect. 3 we discuss, in general, quark-diquark proton wave functions and derive those corresponding to the ones recently introduced by Chernyak and Zhitnitsky[4] and King and Sachrajda[5]. Most of the details of this computation are collected in an appendix. In Sect.4 we compute the electromagnetic form factor of the nucleon in order to fix the normalization of the various proton wave functions introduced in Sect. 3; we are thus able to predict the unpolarized cross sections

for $\gamma\gamma \rightarrow p\bar{p}$ and some hyperon channels in a parameter free scheme and give numerical results. At the end of Sect.4 we compute some contributions due to vector diquarks and discuss how they might change our results. We end this paper with some comments and our conclusions (Sect.5).

2. Elementary amplitudes

Along the same lines as in Ref.[3] the $\gamma\gamma \rightarrow p\bar{p}$ helicity amplitudes are given, in terms of quark (q) and scalar diquark (S) constituents by

$$M_{\lambda_1\lambda_2;\eta_1\eta_2}(s,t) = \int dx_1 dx_2 \phi_P(x_1)\phi_{\bar{P}}(x_2) T_{\lambda_1\lambda_2;\eta_1\eta_2}(x_1,x_2;s,t) \quad (2.1)$$

where $\phi_P(x)$ is the proton wave function which determines the distribution of the fraction x of the longitudinal momentum of the proton carried by the quark; T is the amplitude for the elementary process $\gamma\gamma \rightarrow q\bar{q}S\bar{S}$; in our case, in which we consider only scalar (ud) diquarks the $q(\bar{q})$ is always an up (anti-up) quark and has the same helicity as the $p(\bar{p})$. Other kinematical quantities and helicities $(\eta_1, \eta_2, \lambda_1, \lambda_2)$ are defined in Fig.1. We neglect the QCD evolution of the proton wave function as it would be significant only over a much larger energy range than that we will consider here.

The diagrams contributing to T are given in Fig. 2. The blobs appearing at the $gS, \gamma gS$ and $\gamma\gamma gS$ vertices are respectively 3,4 and 5 point functions which describe the couplings of photons and gluons to the scalar diquarks. In the moderate energy region we will consider ($\sqrt{s} = 2 - 4\text{GeV}$) the diquarks can be thought of as almost elementary objects so that we take for these various functions their Born approximations, see Fig. 3. The composite nature of the diquark is taken care of by phenomenological vertex functions. Due to current conservation only one such function is allowed for each n-point blob. The pointlike couplings of photons and gluons to scalar diquarks are summarized in Table 1. Inserting the diagrams of Fig. 3 into Fig. 2 we get a total amount of 16 diagrams contributing to T . In Table 2 we give the explicit results for this whole set of diagrams, $T^{(i)}(x_1, x_2, s, t)$ ($i = 1, 2, \dots, 16$) which must be summed up to give the full elementary amplitude

$$T_{\lambda_1\lambda_2;\eta_1\eta_2}(x_1, x_2; s, t) = \sum_i T_{\lambda_1\lambda_2;\eta_1\eta_2}^{(i)}(x_1, x_2; s, t) \quad (2.2)$$

The $\gamma\gamma \rightarrow p\bar{p}$ amplitudes, as obtained from Table 2, eq. (2.2) and eq. (2.1) have

still to be properly symmetrized:

$$M_{\lambda_1\lambda_2;\eta_1\eta_2}^s = \frac{1}{\sqrt{2}} \left[M_{\lambda_1\lambda_2;\eta_1\eta_2}(s,t) + (-1)^{\lambda_1-\lambda_2} M_{\lambda_1\lambda_2;\eta_2\eta_1}(s,u) \right] \quad (2.3)$$

This amounts to take into account also the whole set of diagrams which can be obtained from those of Table 2 by interchanging γ_1 and γ_2 .

The normalization of the $\gamma\gamma \rightarrow p\bar{p}$ amplitudes is such that the unpolarized differential cross-section is given by

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} \frac{1}{s^2} \frac{1}{4} \sum_{\lambda_1\lambda_2\eta_1\eta_2} |M_{\lambda_1\lambda_2\eta_1\eta_2}^s(s,t)|^2 \quad (2.4)$$

Inspection of Table 2 reveals that the fixed angle behaviour of the differential cross-section is

$$\frac{d\sigma}{dt} \sim \frac{1}{s^4} |F(g^2)|^2 \quad (2.5)$$

where $|F|^2$ stays for a bilinear combination of the diquark form factors and vertex functions. This combination may still depend on s and t . Therefore, we do not expect scaling to hold in the intermediate energy region where we believe diquarks to be important. Asymptotically, however, $F \longrightarrow 1/s f(\theta)$ (see also Sect.4) and the usual scaling behaviour[3,6]

$$\frac{d\sigma}{dt} \sim \frac{1}{s^6} \quad (2.6)$$

is recovered. According to the parametrization of the diquark form factor successfully exploited in previous work[1,2] this scaling behaviour should be visible when $s \gg Q_0^2 \simeq 3\text{GeV}^2$.

Due to our simplification of just considering scalar diquarks the (anti) proton carries the helicity of the single (anti) quark. When neglecting quark masses, quark and antiquark and hence proton and antiproton have to be in opposite helicity states. This is the same result as in the pure quark picture[7] (helicity sum rule). However, taking also into account vector diquarks we would get non-zero amplitudes for equal helicities of proton and antiproton.

3. Proton wave functions in the q-Q scheme

Having computed, in the previous Section, the elementary amplitudes we now turn to discuss the proton wave functions, in our quark-diquark scheme, which have to be inserted into eqs. (2.1) and (2.3) in order to be able to actually compute the $\gamma\gamma \rightarrow p\bar{p}$ observables.

To be definite let us start with some definitions. In obvious generalization of the ansatz made by Brodsky and Lepage[3], we describe the quark-diquark Fock state in a certain baryon by a wave function

$$|B\rangle = \frac{u_\lambda(xp)}{\sqrt{x}} \psi_S(x, k_\perp) \quad (3.1)$$

for scalar diquarks and similarly for a vector diquark (with an additional polarization vector). $\vec{k}_\perp$ is the intrinsic transverse momentum of the quark with respect to the baryon momentum $\vec{p}$. The wave function is normalized such that

$$\int dx \int \frac{d^2 k_\perp}{16\pi^3} |\psi_S(x, k_\perp)|^2 = \delta \leq 1 \quad (3.2)$$

leaving room for other Fock states. It is plausible to expect the wave function to be strongly peaked at low $k_\perp$. Therefore, the leading behaviour of an observable in the hard scattering region is obtained by neglecting $k_\perp$ in the calculation of the hard scattering amplitudes - as we did in Sect. 2 - and convoluting these amplitudes with wave functions integrated over $k_\perp \leq \tilde{Q}$, with $\tilde{Q} = \min_i x_i Q$, the so-called distribution amplitudes

$$\tilde{\phi}_S(x) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_S(x, k_\perp) \Theta(\tilde{Q}^2 - 2k_\perp^2) \quad (3.3)$$

The mild Q^2 dependence generated by the limit of integration as well as that raised by the QCD evolution is neglected in this work because we can compare our results with data only in a rather limited range of Q^2 . Since the $k_\perp$ dependence of ψ is not known it is convenient to write

$$\tilde{\phi}_S(x) = f_S \phi_S(x) \quad (3.4)$$

with

$$\int dx \phi_S(x) = 1 \quad (3.5)$$

ϕ_S is loosely referred to as the longitudinal momentum wave function. The effect of the $k_\perp$ dependence is then contained in f which acts as a kind of normalization constant of the wave function $\tilde{\phi}$. Our ignorance of the exact $k_\perp$ dependence results thus in an imperfect knowledge of f .

For the spin, flavour and colour dependence of the wave functions we use $SU(6)$ type functions which for a given baryon B with helicity λ we write as

$$\begin{aligned} |B, \lambda\rangle &= \frac{1}{\sqrt{3}} \sum_{ij\mu\mu'\alpha\bar{\alpha}} C_{ij}^{\mu\mu'}(B, \lambda) \delta_{\alpha\bar{\alpha}} V_{i\mu\alpha} q_{j\mu'\bar{\alpha}} f_V \phi_V(x) \\ &+ \frac{1}{\sqrt{3}} \sum_{ij\alpha\bar{\alpha}} C_{ij}^{0\lambda}(B, \lambda) \delta_{\alpha\bar{\alpha}} S_{i0\alpha} q_{j\lambda\bar{\alpha}} f_S \phi_S(x) \end{aligned} \quad (3.6)$$

where μ and $\mu' = \lambda - \mu$ label helicities, α and $\bar{\alpha}$ colours, i and j flavours. The C 's are Clebsch-Gordan coefficients and ϕ_S, ϕ_V are the scalar and vector wave functions according to eqs. (3.4) and (3.5). Although we mainly need ϕ_S for $\gamma\gamma \rightarrow p\bar{p}$ we will make use of the other piece in the discussion of the Chernyak-Zhitnitsky (CZ) wave function. We note that for $SU(6)$ type wave functions the scalar (ud) diquark is also an isospin scalar and the vector (uu,ud,dd) diquark an isovector. This remark completes the definition of the quark-diquark wave function used by us. Let us now turn to the discussion of specific examples of which we will make use when computing observables.

a) Symmetric wave function

Guided by the meson wave function - as another example of a hadron made of two constituents - one may try

$$\phi_S(x) = 6x(1-x) \quad (3.7)$$

b) Non-relativistic wave function

This one should be strongly peaked at $x_0 \simeq 1/3$. As a suitable parametrization we use

$$\phi_S = \delta(x - x_o) \quad (3.8)$$

with x_o being a free parameter.

c) Asymptotic wave function

Starting from the asymptotic pure quark wave function

$$\varphi_{as} = 120x_1x_2x_3 \quad (3.9)$$

one obtains a quark-diquark function by integrating over one of the fractional momenta, e.g.

$$\phi_S(x_1) = \int dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \varphi_{as} \quad (3.10)$$

leading to

$$\phi_{as}(x) = 20x(1 - x)^3 \quad (3.11)$$

d) CZ wave function

One may also try to turn the very interesting CZ wave function into a quark-diquark one. This has been worked out in the Appendix, where details are given. We derive there from the CZ pure quark wave function the quark-diquark CZ wave function

$$\phi_S = 2.87\phi_{as}[1 - 5.94x + 9.3x^2] \quad (3.12)$$

In order to get an idea on the magnitude of f_S we assume $\psi_S(x, k_\perp)$ to factorize in $k_\perp$ and x and make the plausible ansatz, supported by many observations in inclusive reactions, that large intrinsic transverse momenta are strongly suppressed by an exponential cut-off

$$\sim \exp(-k_\perp^2/2a^2) \quad (3.13)$$

Interpreting the transverse momentum as the Fermi motion of the constituents inside baryons (with radius $\simeq 1fm$), one estimates a to be about $300MeV$. Proper normalization of ψ_S leads to

$$f_S = \frac{a}{4\pi} c \sqrt{\delta} \quad (3.14)$$

where c depends on the x dependence of the wave function. For the examples discussed above and with $a = 300 \text{ MeV}$ and $\delta \simeq 1$ we find for f_S values of about 40 MeV .¹

4. Comparison with Experiment

Due to our ignorance of the wave functions and the constant f_S arising from the integration of the wave functions over the intrinsic momentum - we only have an estimate of its magnitude which according to the discussion in Sect. 3 should be of the order of 40 MeV - a genuine prediction of the $\gamma\gamma \rightarrow p\bar{p}$ cross section is not possible; rather we may learn about the wave function by comparison with experiment. However, in order to get non-trivial results one has to discuss within the same scheme as many reactions as possible. At the present stage of the quark-diquark model this means that we have to treat jointly $\gamma\gamma \rightarrow p\bar{p}$ and the electromagnetic form factor G_M of the proton. We adopt the following fitting strategy. For each of the four wave functions discussed in the previous section we try to find a best fit to G_M by varying f_S . In doing so we are attempting to improve our first study of the electromagnetic form factor[1] in which we only used the non-relativistic function (3.8); for details we refer to this work. In the second step we compute the differential and integrated ($|\cos \delta| < 0.6$) cross-section for $\gamma\gamma \rightarrow p\bar{p}$ keeping everything fixed.

To be specific let us comment on conventions used in our scheme.

- For the strong coupling we take as usual

$$\alpha_s(Q^2) = \frac{12\pi}{25 \ln(Q^2/\Lambda^2)} \quad (4.1)$$

with $\Lambda = 200 \text{ MeV}$. For Q^2 we take in each of the amplitudes $T^{(i)}$ the modulus of the momentum g squared carried by the associated gluon. Following the recipe applied by other authors[7] we restrict α_s to be smaller than 0.5.

- Quark masses are neglected; for the S diquark mass we take the value 250 MeV as used in Ref.1. Our results turn out to be rather stable against the actual value of

¹Under the same assumptions this value of f_S is valid for the π wave function as well. On the other hand, according to Brodsky-Lepage [3] f_S is related to the π -decay constant by $f_S = f_\pi/2\sqrt{3} = 26.85 \text{ MeV}$ which is in reasonable agreement with our estimate. Applying this idea of an exponential cut-off to the CZ pure quark wave function of the nucleon, we obtain $f_N = 4.5 \cdot 10^{-3} \text{ GeV}^2$ which is to be compared with $(5.2 \pm 0.3) \cdot 10^{-3} \text{ GeV}^2$ as found by Chernyak-Zhitnitsky[4] exploiting the QCD sum rules. Again the agreement is good.

the mass. So, for example, using 580MeV instead (the value of the vector diquark mass favoured by Ref.1) the cross-sections are reduced by about 15%.

- The diquark form factor is parametrized as in Ref.1

$$F_S(Q^2) = \kappa \frac{Q_0^2}{Q^2 + Q_0^2} \quad (4.2)$$

$$\kappa = \begin{cases} \alpha_S(Q^2)/\alpha_S(Q_0^2) & Q^2 \geq Q_0^2 \\ 1 & Q^2 < Q_0^2 \end{cases}$$

with $Q_0^2 = 3.22\text{GeV}^2$. This parametrization is well in line with our understanding of diquarks. At medium Q^2 a diquark acts as a quasi-elementary constituent bound by non-perturbative forces whereas at large Q^2 it resembles more and more two quarks kept collinear through the exchange of a hard gluon.

- As we said in Sect. 2, the four and five point functions are approximated by the Born order. The composite nature of the diquark is taken care of by the vertex functions

$$F_S^{(4)}(Q^2) = F_S^{(5)}(Q^2) = F_S(Q^2) \quad (4.3)$$

Q is identified with the momentum of the gluon entering the vertex function. Eq.(4.3) guarantees the correct asymptotic behaviour of the elementary amplitudes, each diquark vertex contains only one hard gluon for $Q^2 \rightarrow \infty$.

With this our model is fully specified and we can turn to its comparison with data. In Fig. 4 the electromagnetic form factor of the proton is displayed ² and compared to the diquark model for the 4 wave functions. The fitted values of f_S are listed in Table 3. Except for the symmetric wave function with which the Q dependence of G_M is not accounted for properly, we obtain good descriptions of the data.³ The values found for f_S are reasonable. In terms of the exponential cut-off in the intrinsic transverse momentum, the parameter a seems to be rather 600MeV instead of 300MeV . This large value has been also found by the EMC[9] in a study of the transverse momentum in semi-inclusive deep inelastic μp scattering. Taking a literally as the mean Fermi momentum, one is led to conclude that the valence quarks of the proton are confined to a sphere with a radius of about 0.5 fm rather than 1 fm . This “hard core” of the proton is surrounded by a cloud of sea quarks and gluons which resemble the meson

²The original form factor data[8] have been obtained under the assumption $G_E = G_M/\mu_p$ which at large Q^2 is in disagreement with any hard scattering model. Therefore, these data have been reanalyzed in Ref. 1 under the assumption $G_E = G_M$ for $Q^2 > 3\text{GeV}^2$, a relation which to a good approximation is predicted by the diquark model.

³As has been discussed in Ref. 1 the form factor is dominated by the qS amplitude in which the photon is absorbed by the quark. Vector diquarks contribute only through the Vq amplitude implying a four-point function. Due to the vector diquark form factors this contribution is small at medium values of Q^2 . In our calculation we, therefore, neglect this term except for the CZ wave function where a small contribution from it slightly improves the Q dependence of G_M .

cloud more and more with increasing distance. The value of 141MeV we find for the non-relativistic wave function seems to be uncomfortable high for this type of interpretation.

In Fig. 5 we now compare the predictions for $\gamma\gamma \rightarrow p\bar{p}$ with the present data[10-12] for the integrated cross section σ . Since the symmetric wave function (3.7) is already ruled out by the electromagnetic form factor we have not used it for computing $\gamma\gamma \rightarrow p\bar{p}$. The other 3 examples differ only by about a factor of 3 in σ , whereby the asymptotic wave function gives the best result. It is very encouraging that the diquark model is in touch with the data for the highest measured energy. This is a clear improvement as compared to the pure quark scheme[7]⁴ in which the predicted cross sections are smaller by about an order of magnitude as quoted in ref. 12. With the diquarks, we have taken into account non-perturbative effects allowing us to apply this model at medium energies where the pure quark scheme fails. Note that the predicted energy dependence of σ is flatter than that of the quark model (cf.(2.5)). Only for very large s when the diquarks are dissolved into quarks do we obtain the same energy dependence. Below $\sqrt{s} = 2.4\text{GeV}$ our predictions are definitely below the data. It is obvious that the model cannot be applied seriously at such low values of energy close to the reaction threshold. The proton mass is not negligible (resulting in errors of $O(2m_p^2/s)$), one has to consider resonance production, the photon structure function (vector meson dominance) and, last but not least, higher orders of perturbation theory have likely to be taken into account as well.

The differential cross section at the highest energy is compared to our predictions in Fig. 6. Since the statistics of the data is so limited no definite conclusion can be drawn. However, there is a slight tendency that although the shape of the predicted cross section seems to be in fair agreement and although the integrated cross section is predicted correctly, our results at large angles seem to be a bit small. Nevertheless, this effect is of the order of 1 standard deviation. Taking into account as well the systematic errors quoted by the experimentalists this effect is even smaller. We note again that our cross-sections are much larger than those in the pure quark scheme.

In contrast to other work[7] our results are stable against different choices of α_s . Changes in the magnitude of α_s are absorbed in f_S (note, em. form factor $\sim \alpha_s f_S^2$, $d\sigma/dt(\gamma\gamma \rightarrow p\bar{p}) \sim \alpha_s^2 f_S^4$) and only minor effects from the Q^2 dependence - well within the errors of the data - remain. For instance, choosing $\Lambda = 100\text{MeV}$ in (4.1) one obtains an decrease of the $\gamma\gamma$ cross-section by about 7% ; the use of the parametrization of α_s exploited in Refs.[7,13] (with $c_1 = 0.016$) increases σ by about 15% . This is consistent with neglecting the QCD evolution of the wave function since it has the same source as the running coupling. From a strong dependence on

⁴The results presented by Ref. 7 have been confirmed by Miller and Gunion[13]. Therefore, we do not compare with those found by Damgaard[14] previously which are at variance with other results.

the choice of the α_S one should expect a similarly strong effect from the evolution.

With the diquark model being fully specified by now (parameters, wave functions) we are able to also predict the $\gamma\gamma$ annihilation into hyperon- antihyperon, utilizing the SU(6) type wave function (3.6). In Fig. 7 we show predictions for ratios of $Y\bar{Y}$ cross sections over the $p\bar{p}$ one. The strange quark mass is taken to be $150MeV$ and the strange diquark mass $400MeV$ correspondingly. Note that in the phase space factor the hadron masses are taken into account.

Our results are quite different from those presented by Farrar et al[7]. Particularly striking are the large cross sections for $\Sigma^-\bar{\Sigma}^-$ and $\Xi^-\bar{\Xi}^-$. This phenomena can be traced back to the large contributions from the diagrams in which the two photons are attached to the diquark line. In these diagrams the gluon propagator ($g^2 = sx_1x_2$) has particular small values for the asymptotic and the CZ wave functions (maxima at $x = 0.25$ and 0.15 resp., see Fig. 8). The charges of the diquarks favour then, for instance, the $\Sigma^-\bar{\Sigma}^-$ channel over the $p\bar{p}$ one.

Data on the $Y\bar{Y}$ channels would be very interesting since the ratios of cross sections are insensitive to details of the model. This is not only true for our model but also for the pure quark one. With the exception of an upper limit on $\sigma(\Lambda\bar{\Lambda})/\sigma(p\bar{p})$ [15] - which is compatible with our results - no experimental information is available as yet.

As we said repeatedly we have ignored vector diquarks since we expect them to give only negligible contributions to unpolarized cross sections in which we are solely interested here. Our expectation is based on studies of the electromagnetic form factors and , within the limited framework of the end-point model, several hadronic reactions [1,2]. However, in order to have at least a partial check of it in the process under investigation we have computed as a representative all those diagrams in which the photons are attached to the quark line (for VgV couplings we refer to ref.1). Varying the vector diquark form factor parameter Q_1^2 between 1 and $3.2GeV^2$, we obtain an increase of the $\gamma\gamma$ cross section between 5 and 15 % well inside the uncertainties of our model.

We can also utilize these elementary vector diquark amplitudes to get a rough estimate of the magnitude of the $\gamma\gamma \rightarrow \Delta^{++}\bar{\Delta}^{++}$ cross section. Taking for the wave function

$$|\Delta^{++}; 3/2 > = \frac{1}{\sqrt{3}} \sum_{\alpha, \bar{\alpha}} \delta_{\alpha, \bar{\alpha}} V_{1\alpha}(uu) u_{+\bar{\alpha}} f_S \phi_{as}(x) \quad (4.4)$$

$$|\Delta^{++}; 1/2\rangle = \frac{1}{3} \sum_{\alpha\bar{\alpha}} \delta_{\alpha\bar{\alpha}} [\sqrt{2}V_{0\alpha}(uu)u_{+\bar{\alpha}} + V_{1\alpha}(uu)u_{-\bar{\alpha}}] f_S \phi_{aS}(x)$$

we obtain for the ratio $\sigma(\Delta\bar{\Delta})/\sigma(p\bar{p})$ a value of 0.1 at $\sqrt{s} = 3\text{GeV}$. If the set of diagrams we consider does not mislead us, the diquark model yields a ratio in accord with the experimental upper limit of 2 [15]. This result has to be contrasted with the large ratio of about 50 found in the quark model.

We have not attempted a complete calculation of the contribution from the vector diquarks since their wave functions and form factors are not really fixed at the present stage of our investigations. This can only be done in reactions to which the vector diquarks provide substantial contributions and not only small corrections as in the case of two photon annihilation reactions. Such processes will be investigated by us in forthcoming papers.

5. Comments and conclusions

We have applied the idea of diquarks as constituents, in the framework of the BFL scheme, to compute $\gamma\gamma \rightarrow p\bar{p}$ cross-sections. In this first application, and in the absence of spin data, we have considered only scalar diquarks, which is expected to be a reasonable approximation. After computing the elementary $\gamma\gamma \rightarrow q\bar{q}$ amplitudes we have convoluted them with different proton wave functions, thus obtaining the helicity amplitudes for the full process $\gamma\gamma \rightarrow p\bar{p}$. Among the examples of wave functions there is one which we have derived from the highly debated CZ pure quark wave function. We also have given an interpretation of the constant f_S in terms of integrated Fermi-motion.

To compare our model with data we have adopted the following strategy. Firstly, we have fitted f_S to the electromagnetic form factor of the proton using various wave functions; one of our examples, namely the symmetric wave function (3.7), turned out to fail with the Q dependence of G_M . In a second step we have computed the $\gamma\gamma \rightarrow p\bar{p}$ cross-sections for the remaining three wave functions. In all cases we are able to get reasonable agreement with the experimental data on both the angular and energy dependence of the cross section for $\sqrt{s} \geq 2.4\text{GeV}$. Although our predictions are consistent with the data they still seem to be a bit low. This is particularly true for the non-relativistic wave function (3.9). The results for the other examples are very similar. Whereas the asymptotic wave function (3.11) provides the larger cross-section, the CZ wave function (3.12) requires the smaller value for f_S which is easier to interpret in terms of Fermi-motion.

We have not attempted to optimize our results by choosing proper values for the diquark form factor parameter Q_0 . We have simply taken the value obtained in Ref.[1] although there it has been determined within the framework of the end-point model which may not necessarily lead to the same value as in the scheme exploited here. In order to give an impression on the effect of the choice of Q_0 we have repeated our calculation with $Q_0^2 = 2.5 \text{ GeV}^2$ instead of 3.22 GeV^2 . We have found results of similar quality as those presented above but with slightly larger $\gamma\gamma$ cross-sections (asymptotic wave function - 30% increase with $f_S = 109 \text{ MeV}$; CZ wave function - 10% increase with $f_S = 81 \text{ MeV}$).

When comparing with the analogous results obtained in the pure quark case[7] our model is superior. It predicts cross sections for $\gamma\gamma \rightarrow p\bar{p}$ about an order of magnitude larger than those obtained by Farrar et al. Moreover, the diquark model is much less sensitive to details of the calculation as for instance the choice of α_S or the wave function than the other one. This stability is related to the fact that in the diquark model a much smaller number of diagrams contributes and no strong cancellations occur between their amplitudes.

We, therefore, conclude that the diquark model is a reasonable attempt to describe exclusive reactions involving baryons at intermediate values of Q^2 . For this class of reactions we expect the pure quark picture to emerge only at very large values of Q^2 . For other exclusive reactions not involving baryons, our model does not give any modification and the asymptotic picture may hold down to rather low values of Q^2 . Indeed, for $\gamma\gamma \rightarrow M\bar{M}$ this seems to be the case[16].

The data on $\gamma\gamma \rightarrow p\bar{p}$ at our disposal do not constitute a crucial check of the diquark model regarding their limited statistics and the low energies at which they have been measured. Data at higher energies which may be obtained at future accelerators like the Super LEAR or a bottom factory would be extremely interesting. One may see in such data, for example, the transition of the energy dependence of the fixed angle cross-section from $\simeq s^{-4}$ to s^{-6} , the asymptotic behaviour.

We have not yet attempted to compute the decay $\Psi \rightarrow p\bar{p}$. In our model it is not at the same level of sophistication as G_M or $\gamma\gamma \rightarrow p\bar{p}$ since, in contrast to these cases, it involves Feynman diagrams with a loop. We will tackle this problem in a forthcoming paper. We also will consider the application of our model to other processes, like $\gamma p \rightarrow \gamma p$ and $pp \rightarrow pp$. The full contribution of vector diquarks has also to be taken into account. Contrary to the electromagnetic form factor and the two photon annihilation reactions, one has to face propagator singularities for this reaction. Thus, the computational effort will be considerably larger. The $\eta_c \rightarrow B\bar{B}$ decay, one of the simplest processes involving only vector diquarks, is under investigation [17].

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6. Appendix

Chernyak and Zhitnitsky make the following ansatz for the proton wave function instead of eq.(3.6) (colour indices omitted for convenience)

$$\begin{aligned}
 \varphi_{p\lambda=\pm 1/2}(x_1, x_2, x_3) = & \pm \frac{1}{\sqrt{18}} \left\{ [\varphi(1, 2, 3)u_{\pm}u_{\mp} + \varphi(2, 1, 3)u_{\mp}u_{\pm}]d_{\pm} + \right. \\
 & + [\varphi(1, 3, 2)u_{\pm}d_{\pm} + \varphi(2, 3, 1)d_{\pm}u_{\pm}]u_{\mp} + [\varphi(3, 1, 2)u_{\mp}d_{\pm} + \varphi(3, 2, 1)d_{\pm}u_{\mp}]u_{\pm} - \\
 & - [\varphi(2, 1, 3) + \varphi(3, 1, 2)]d_{\mp}u_{\pm}u_{\pm} - [\varphi(3, 2, 1) + \varphi(1, 2, 3)]u_{\pm}d_{\mp}u_{\pm} + \\
 & \left. - [\varphi(1, 3, 2) + \varphi(2, 3, 1)]u_{\pm}u_{\pm}d_{\mp} \right\} \quad (6.1)
 \end{aligned}$$

where $\varphi(1, 2, 3)$ is a shorthand notation for $\varphi(x_1, x_2, x_3)$. It is, as the quark-diquark function, cf. eqs.(3.4,5), normalized to 1 with a constant $f_N/4\sqrt{6}$. The 3 quarks carry, always in the order in which they appear in eq.(6.1), the fractions x_1, x_2 and x_3 of the proton longitudinal momentum. In the case that φ is a function totally symmetric in x , as for example the asymptotic wave function (3.9), (6.1) reduces to the familiar $SU(6)$ wave function.

In order to convert this pure quark wave function into a quark-diquark one we firstly have to couple pairs of quarks into states of total spin and isospin (diquarks)

$$\begin{aligned}
 u_{\pm}d_{\mp} &= \frac{1}{2} [V_0^1(ud) + V_0^0(ud) \pm S^1(ud) \pm S^0(ud)] \\
 d_{\pm}u_{\mp} &= \frac{1}{2} [V_0^1(ud) - V_0^0(ud) \pm S^1(ud) \mp S^0(ud)] \\
 u_{\pm}u_{\mp} &= \frac{1}{\sqrt{2}} [V_0^1(uu) \pm S^1(uu)] \quad (6.2)
 \end{aligned}$$

$$\begin{aligned}
u_{\pm}d_{\pm} &= \frac{1}{\sqrt{2}}[V_{\pm 1}^1(ud) + V_{\pm 1}^0(ud)] \\
d_{\pm}u_{\pm} &= \frac{1}{\sqrt{2}}[V_{\pm 1}^1(ud) - V_{\pm 1}^0(ud)] \\
u_{\pm}u_{\pm} &= V_{\pm 1}^1(uu)
\end{aligned}$$

where the lower indices are helicities as before and the upper ones are 3rd components of the isospin. Secondly, we have to integrate φ over one variable, as we did in Sect. 3c

$$\phi_i(x_i) = \int dx_j dx_k \delta(1 - x_1 - x_2 - x_3) \varphi(x_1, x_2, x_3) \quad (6.3)$$

($i, j, k = (1, 2, 3)$ pairwise different). Insertion of (6.2) and (6.3) into (6.1) finishes the conversion

$$\begin{aligned}
\phi_{p\lambda=\pm 1/2}(x) &= \pm \frac{1}{\sqrt{18}} \left\{ \phi_2(x) [-\sqrt{2}V_{\pm 1}^1(ud)u_{\mp} + 2V_{\pm 1}^1(uu)d_{\mp}] + \right. \\
&\quad \pm \phi_3(x) [-\sqrt{2}V_0^1(uu)d_{\pm} + V_0^1(ud)u_{\pm}] + \\
&\quad \left. \pm [2\phi_1(x) + \phi_3(x)] S^0(ud)u_{\pm} \right\} \quad (6.4)
\end{aligned}$$

Comparison with the wave function (3.6) reveals

$$\phi_{V1}(x) = \phi_2(x); \quad \phi_{V0}(x) = \phi_3(x); \quad \phi_S(x) = \frac{1}{3}[2\phi_1(x) + \phi_3(x)] \quad (6.5)$$

There is an interesting point to note. The CZ ansatz (6.1) contains non- $SU(6)$ diquarks, namely isovector S and isoscalar V diquarks (cf.(6.2)). However, due to internal symmetries, the contribution from these diquarks drop out in the quark-diquark wave function (6.4).

Taking the QCD sum rules constrained wave function $\varphi(x_1, x_2, x_3)$ of CZ, in fact we take the more accurate function given by Ref.[5], we can compute the quark-diquark wave functions from eqs.(6.3) and (6.5) and find

$$\begin{aligned}
\phi_S &= 2.87\phi_{as}[1 - 5.94x + 9.3x^2] \\
\phi_{V_1} &= 5.58\phi_{as}[1 - 4.75x + 5.33x^2] \\
\phi_{V_0} &= 4.81\phi_{as}[1 - 4.72x + 5.47x^2]
\end{aligned} \tag{6.6}$$

Let us notice finally that the moments of the quark-diquark wave function

$$(m, n)_i \equiv \int dx x^m (1-x)^n \phi_i(x) \quad i = 1, 2, 3 \tag{6.7}$$

are related to the moments of the pure quark one

$$(i, j, k) \equiv \int dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) x_1^i x_2^j x_3^k \varphi(x_1, x_2, x_3) \tag{6.8}$$

by

$$\begin{aligned}
(m, n)_1 &= \sum_{l=0}^n (-1)^l \binom{n}{l} (m+l, 0, 0) \\
(m, n)_2 &= \sum_{l=0}^n (-1)^l \binom{n}{l} (0, m+l, 0) \\
(m, n)_3 &= \sum_{l=0}^n (-1)^l \binom{n}{l} (0, 0, m+l)
\end{aligned} \tag{6.9}$$

The wave functions proposed in Refs. (4) and (5) are constrained by the values of a few moments derived from QCD sum rules. The same procedure can then be applied directly to the quark-diquark wave function by writing it in the general form

$$\phi_i(x) = \phi_{as}(x) \left(C_0^{(i)} + C_1^{(i)} x + C_2^{(i)} x^2 \right) \tag{6.10}$$

and by fixing the coefficients such that they reproduce the QCD sum rule values of the lower moments. Although a first glance to eq.(6.9) may lead to the impression that only the diagonal pure quark moments are used, a closer inspection reveals that

this is not so since there are many linear relations from momentum conservation between these moments. For example one has the relation

$$(n, 0, 0) - (n + 1, 0, 0) = (n, 1, 0) + (n, 0, 1). \quad (6.11)$$

The diagonal elements form a sufficient set of independent moments. In Table 4 the moments given by King and Sachrajda are compared with those following from the wave functions (6.6). The agreement between the two sets of moments tells us that the physical content of the QCD sum rules is fully taken care of by the quark-diquark wave function within the errors.

In Fig.8 we plot the CZ wave functions (6.6) as well as the asymptotic one (3.11). The latter one is very smooth with just a maximum at $x \simeq 0.25$ a value which is very plausible considering that this wave function was obtained from the symmetric pure quark wave function (3.9). A diquark carries then in general a larger fraction of momentum than a quark. The CZ functions are very asymmetric as is the original pure quark one. All three of them show two pronounced maxima telling us that the single quark has most likely either a rather large value of x or a low one, medium values are suppressed. The main difference between ϕ_S and ϕ_{V_μ} is that for the first one a large value of x has a higher probability (about 65%) than a small value whereas for ϕ_V the situation is reversed and even more unbalanced. The probability for a large value of x amounts only to about 20 % in the case of vector diquarks. The two vector diquark functions are very similar to each other, their differences are hardly significant considering the uncertainties in the process of their determination. For applications one should use $\phi_{V_0} = \phi_{V_1}$. Our double-humped quark-diquark wave functions bear some resemblance to the π meson wave function obtained from QCD sum rules[18] and from lattice gauge theory [19].

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$-ie_0e_Q(Q - \bar{Q})^\mu$
$-ig_sT_{ij}^a(Q - \bar{Q})^\mu$
$2ie_0^2e_Q^2g^{\mu\nu}$
$2ig_se_0e_QT_{ij}^ag^{\mu\nu}$

Table 1: Pointlike couplings of photons and gluons to scalar diquarks. a, i, j colour indices, e_Q diquark charge in units of the elementary charge e_0 , T^a Gell-Mann colour matrices.

Table 2. Elementary helicity amplitudes for $\gamma\gamma \rightarrow p\bar{p}$ to Born order. All masses are neglected except that of the scalar diquark (m_s) in propagators. The common factor is $C = 4ig_s^2 e_1 e_2 / 3$; g is the gluon momentum for each diagram; $4/3$ is the colour factor; e_1 and e_2 are the electric charges seen by photons 1 and 2 respectively.

$$T^{(1)} = \frac{CF(g^2)}{4x_1x_2(1-x_1)(1-x_2)} \frac{\sqrt{tu}}{su^2} \delta_{\lambda_1, -\lambda_2} \times \\ \{2(\eta_1 + \eta_2)(x_1 - x_2)u + 4(\eta_2 - \eta_1)(1 - x_1)(1 - x_2)t + \\ 2\lambda_2\eta_1\eta_2[(1 - \eta_1\eta_2)t - (1 + \eta_1\eta_2)u] \times \\ [(1 - \eta_1\eta_2)(2 - x_1 - x_2) + 2(2x_1x_2 - x_1 - x_2)]\}$$

$$T^{(2)} = \frac{CF(g^2)}{8x_1(1-x_1)^2(1-x_2)} \frac{\sqrt{tu}}{us^2} \delta_{\lambda_1, -\lambda_2} \times \\ \{-(\eta_1 + \eta_2)s[4(1-x_1)^2(1-x_2) + x_1^2(2-x_1-x_2)] + \\ 4(2\lambda_2)(1 + \eta_1\eta_2)(1-x_1)(1-x_2)s + \\ x_1[(\eta_2 - \eta_1)(t+u) + 4\lambda_2\eta_1\eta_2(t-u)] \times \\ [2(1-x_1)(2-x_2) + (x_1-x_2)(2-x_1)]\}$$

$$T^{(3)} = -T^{(2)}(x_1 \leftrightarrow x_2; \lambda_1 \leftrightarrow \lambda_2; \eta_1 \leftrightarrow \eta_2)$$

$$T^{(4)} = \frac{C(1-x_2)F^{(4)}(g^2)}{x_1(x_1x_2s + x_1t + x_2u)[(1-x_2)t - m_s^2]} \frac{\sqrt{tu}}{st} \eta_2 \delta_{\lambda_1, -\lambda_2} \times \\ \{(1 + 2\lambda_2\eta_1)x_1(1-x_2)t^2 + (1 - 2\lambda_2\eta_1) \times \\ [x_2(1-x_1)u^2 + (x_1+x_2)tu] - 2x_1x_2tu\}$$

$$T^{(5)} = \frac{C(1-x_1)F^{(4)}(g^2)}{x_1(x_1x_2s + x_1t + x_2u)[(1-x_1)u - m_s^2]} \frac{\sqrt{tu}}{st} \eta_2 \delta_{\lambda_1, -\lambda_2} \times \\ \{(1 + 2\lambda_2\eta_1)x_1(1-x_2)t^2 - (1 - 2\lambda_2\eta_1) \times \\ [(1-x_1)(2-x_2)u^2 + (2+x_1-x_2)tu] + 2x_1(2-x_2)tu\}$$

$$T^{(6)} = - \frac{CF^{(4)}(g^2)}{x_1(x_1x_2s + x_1t + x_2u)} \frac{\sqrt{tu}}{st} \delta_{\lambda_1, -\lambda_2} \times \\ [4\lambda_2\eta_1\eta_2x_1(t-u) - 2\lambda_2(1+\eta_1\eta_2)s - (\eta_1 - \eta_2)(1-2x_1)s]$$

$$T^{(7)} = T^{(5)}(x_1 \leftrightarrow x_2; \lambda_1 \leftrightarrow \lambda_2; t \leftrightarrow u)$$

$$T^{(8)} = T^{(4)}(x_1 \leftrightarrow x_2; \lambda_1 \leftrightarrow \lambda_2; t \leftrightarrow u)$$

$$T^{(9)} = T^{(6)}(x_1 \leftrightarrow x_2; \lambda_1 \leftrightarrow \lambda_2; t \leftrightarrow u)$$

$$T^{(10)} = \frac{4CF^{(5)}(g^2)tu}{x_1x_2[(1-x_1)u - m_s^2][(1-x_2)u - m_s^2]} \frac{\sqrt{tu}}{s^2} \delta_{\lambda_1, -\lambda_2} \times \\ 2\lambda_2\eta_1\eta_2(1-x_1)(1-x_2)$$

$$T^{(11)} = - \frac{2CF^{(5)}(g^2)}{x_1x_2[(1-x_1)u - m_s^2]} \frac{\sqrt{tu}}{s^2} \delta_{\lambda_1, -\lambda_2} \eta_2(1-x_1) \times \\ [(1+2\lambda_2\eta_1)t + (1-2\lambda_2\eta_1)u]$$

$$T^{(12)} = - T^{(11)}(x_1 \leftrightarrow x_2; \lambda_1 \leftrightarrow \lambda_2; \eta_1 \leftrightarrow \eta_2)$$

$$T^{(i)} = 0, \quad i = 13, 14, 15, 16.$$

wave fct. eq.	remarks	$f_S[MeV]$
(3.7)	symmetric	48.8
(3.9)	non-relativistic $x_0 = 0.40$	141.2
(3.11)	asymptotic	96.4
(3.12)	CZ	75.6

Table 3: The constant f_S as obtained from fits to the electromagnetic form factor for various wave functions.

(ijk)	KS [5]	(6.6)
100	0.46 - 0.59	0.520
010	0.18 - 0.21	0.199
001	0.22 - 0.26	0.239
200	0.27 - 0.37	0.302
020	0.08 - 0.09	0.085
002	0.10 - 0.12	0.109
110	0.08 - 0.10	0.103
101	0.09 - 0.11	0.117
011	—	0.013

Table 4: Moments of the pure quark wave function as obtained from QCD sum rules [6] compared with those obtained from the quark-diquark wave functions (6.6).

Figure captions

- Fig. 1: $\gamma\gamma \rightarrow p\bar{p}$ in the center of mass system.
- Fig. 2: Elementary diagrams contributing to $\gamma\gamma \rightarrow q\bar{q}S\bar{S}$.
- Fig. 3: Born approximations to photon and gluon couplings to scalar diquarks.
- Fig. 4: The electromagnetic form factor of the proton, $Q^4 G_M/\mu_p$ vs. Q^2 . μ_p is the magnetic moment of the proton. The lines correspond to the symmetric (dash-dot-dotted), non-relativistic with $x_0 = 0.40$ (dash-dotted), asymptotic (solid) and CZ (dashed) wave functions. Data taken from Ref.[8] using $G_E = G_M(\bullet)$ and, for comparison, $G_E = G_M/\mu_p(\circ)$.
- Fig. 5: The integrated cross section for $\gamma\gamma \rightarrow p\bar{p}$ vs. $\sqrt{s}(|\cos\Theta| < 0.6)$. Lines as in Fig.4. Data taken from TASSO[10] ($\blacksquare$), JADE[11] (+) and TPC[12] ($\circ$). The bar at the solid line indicates the uncertainty of our prediction.
- Fig. 6: The differential cross section $d\sigma/d\cos\Theta$ vs. $\cos\Theta$ at $2.4 < \sqrt{s} < 2.8\text{GeV}$. Lines as in Fig.4. Data taken from TASSO[10] ($\blacksquare$) and TPC[12] ($\circ$).
- Fig. 7: Predictions for $\gamma\gamma$ annihilation into hyperon-antihyperon using the asymptotic wave function (solid line) and the CZ one (dashed line). Shown are ratios of integrated cross sections $\sigma_{\gamma\gamma \rightarrow Y\bar{Y}}/\sigma_{\gamma\gamma \rightarrow p\bar{p}}$ vs. $\sqrt{s}$.
- Fig. 8: The asymptotic and the CZ quark-diquark wave functions vs. the momentum fraction x the quark carries.

Fig. 1

Fig. 2

$$\begin{aligned}
& \text{Diagram 1: } \text{Double line with a loop and a wavy line } q \text{ attached to the top.} = \left(\text{Diagram 1.1: Double line with a vertical wavy line } q \text{ attached to the top.} \right) \times F_s(q^2) \\
& \text{Diagram 2: } \text{Double line with a loop and two wavy lines } q \text{ attached to the top.} = \left(\text{Diagram 2.1: Double line with two vertical wavy lines } q \text{ attached to the top.} + \text{Diagram 2.2: Double line with one vertical wavy line } q \text{ and one vertical line with an arrow attached to the top.} + \text{Diagram 2.3: Double line with one wavy line } q \text{ and one line with an arrow attached to the top.} \right) \times F_s^{(4)}(q^2) \\
& \text{Diagram 3: } \text{Double line with a loop and three wavy lines } q \text{ attached to the top.} = \left(\text{Diagram 3.1: Double line with three vertical wavy lines } q \text{ attached to the top.} + \text{Diagram 3.2: Double line with one vertical wavy line } q \text{ and two vertical lines with arrows attached to the top.} + \text{Diagram 3.3: Double line with two wavy lines } q \text{ and one vertical line with an arrow attached to the top.} + \text{Diagram 3.4: Double line with one wavy line } q \text{ and two lines with arrows attached to the top.} + \text{Diagram 3.5: Double line with two wavy lines } q \text{ and one line with an arrow attached to the top.} + \text{Diagram 3.6: Double line with one wavy line } q \text{ and two lines with arrows attached to the top.} \right) \times F_s^{(5)}(q^2)
\end{aligned}$$

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8