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**LATTICE QCD CALCULATION OF $K \rightarrow \pi$ NON LEPTONIC WEAK
TRANSITION AND OTHER HADRONIC WEAK PARAMETERS**

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Presented by L.Maiani

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LATTICE QCD CALCULATION OF $K \rightarrow \pi$ NON LEPTONIC WEAK TRANSITION AND OTHER HADRONIC WEAK PARAMETERS*

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1. INTRODUCTION

In this talk, I would like to present very preliminary results obtained by our collaboration on the $K \rightarrow \pi$ matrix elements of the weak non-leptonic hamiltonian, computed in lattice QCD with Wilson fermions and in the quenched approximation. Independent results on the subject have been presented by the UCLA group⁽¹⁾, who is considering the $K \rightarrow \pi\pi$ matrix elements.

It is by now well established⁽²⁾ that the calculation of non-leptonic transitions requires a lattice that allows propagation of a charmed quark, i.e. a quark much heavier than the other (u,d,s) quarks. Therefore an obvious, but interesting, by-product of our work is the calculation of the axial coupling and of the B-parameter of $c\bar{q}$ mesons. Preliminary data on the last issues have been presented at Uppsala⁽³⁾ and I will illustrate a more advanced version of the results in Sect.3.

2. SOME TECHNICAL DETAILS

Our results have been obtained with two distinct lattices, worked out at the CRAY XMP of CINECA (Bologna) and at the CRAY II of the Ecole Polytechnique (Paris). The relevant parameters of the two lattices are reported in Table 1. Part of the work concerning the B-parameter for light mesons has been done, in collaboration with C.Sachrajda, on a third lattice identical to the Bologna one, at the CRAY XMP of the Rutherford Laboratory.

The value of a^{-1} is estimated from the vector meson masses. The values of the hopping parameter K are chosen so that the corresponding pseudoscalar mesons lie in the same mass range (0.5+0.8 GeV) in both lattices. The charmed quark bare mass, in physical units, is:

$$m_c \sim 0.9 \text{ GeV (Bologna)}$$

$$m_c \sim 1.2 \text{ GeV (Paris)}$$

corresponding to a D mass of about 1.5 and 1.7 GeV, respectively.

* Presented by L.Maiani

The calculations presented here have required about 400 and 1500 hours of CPU time for the Bologna and Paris calculations, respectively, and have been supported by INFN in Italy and CNRS in France.

3. AXIAL COUPLING AND B-PARAMETER OF HEAVY MESONS

The axial coupling is defined, as usual, according to:

$$Z_A <0| \partial^\mu A_\mu |\pi^+(q)> = i f_\pi m_\pi^2 \quad (3.1)$$

where A_μ is the local axial current^[*] ($A_\mu = \bar{q} \gamma_\mu \gamma_5 q$) and Z_A the associated normalization constant⁽⁴⁾ which can be taken either from perturbation theory calculations ($Z_A = 0.87$ at $\beta=6.0$, ref.(5)) or from current-algebra relations⁽⁶⁾ ($Z_A = 0.85$ at $\beta=6.0$). Earlier determinations of f_π have been obtained in refs.(6,7). Using the perturbation theory value of Z_A , we obtain:

$$f_\pi / \sqrt{2} = \begin{cases} 110 \pm 20 \text{ MeV (Bologna, } Z_A = 0.85) \\ 115 \pm 23 \text{ MeV (Paris, } Z_A = 0.88) \end{cases}$$

which are quite consistent among themselves and with the experimental value: $(f_\pi)_{\text{exp}} \approx 131 \text{ MeV}$.

The results for heavier mesons, including the kaon, are summarized in Table 2 and Table 3. Uncertainties associated with the value of a^{-1} and of Z_A drop out in the ratio f_M/f_π , which is reported in the first column. In the second column we give the value of f_M obtained by rescaling the ratio with the experimental value of f_π . The central values of f_D and f_F decrease consistently between the Bologna and Paris calculations, but the variation in m_c between the two cases is too small to allow any test of the m_c dependence. If we use the non-relativistic quark model scaling law:

$$f_M \sqrt{M} = \text{const} \quad (3.2)$$

we estimate, in both cases:

$$f_B \approx 110 \text{ MeV} \quad (3.3)$$

considerably on the low-side, with respect to QCD sum rules determinations⁽⁹⁾.

The B-parameter is defined, as usual, according to

$$2M <\bar{M}_0| [\bar{Q} \gamma_\mu (1-\gamma_5) q]^2 | M_0> = \left(\frac{8}{3} f_M^2 M^2 \right) B \quad (3.4)$$

where the operator is meant to include the perturbative lattice corrections. We find:

$$B = 0.70 \pm 0.17 \text{ (Bologna, } a^{-1} \approx 2 \text{ GeV, } m_c \approx 0.9 \text{ GeV)}$$

$$B = 1.00 \pm 0.15 \text{ (Paris, } a^{-1} \approx 3 \text{ GeV, } m_c \approx 1.2 \text{ GeV)}$$

(3.5)

4. THE NON-LEPTONIC LATTICE HAMILTONIAN

The Standard model effective hamiltonian for $\Delta S=1$ transitions, computed at a subtraction scale $\mu \gg m_c$ and neglecting third family contributions, is:⁽¹⁰⁾

$$H_{\text{eff}} = \sqrt{2} G_F [c^{(-)}(\mu) \bar{\psi}^{(-)}(\mu) + c^{(+)}(\mu) \bar{\psi}^{(+)}(\mu)] \quad (4.1)$$

with:

$$\bar{\psi}^{(\pm)} = (\bar{s}_L \gamma^\mu d_L \bar{u}_L \gamma^\mu u_L \pm \bar{s}_L \gamma^\mu u_L \bar{u}_L \gamma^\mu d_L) - (u \rightarrow c)$$

and:

$$\begin{aligned} c^{(+)}(\mu \sim 2 \text{ GeV}) &= 0.7 \pm 0.8 \\ c^{(-)}(\mu \sim 2 \text{ GeV}) &= - (1.8 \pm 1.4) \end{aligned} \quad (4.2)$$

At large values of μ , asymptotic freedom is supposed to be valid and the calculation of $c^{(\pm)}$ reliable. The matrix elements of $\bar{\psi}^{(\pm)}$, on the other hand, are quite difficult to estimate with usual methods, e.g. the valence quark approximation, because μ is still far from the typical scale of the hadronic bound states, which is of the order of $\Lambda_{\text{QCD}} \ll m_c$. The break-down of the valence (or vacuum saturation) approximation is illustrated by the fact that, as we let μ to go below the charm threshold a new operator

[*] Hermitean γ -matrices, with $\{\gamma_\mu, \gamma_\nu\} = 2 \delta_{\mu\nu}$, are used, so that A_μ is purely imaginary, as indicated in eq.(3.1).

Table 1
Parameters of the lattice calculations

	Lattice volume	β	a^{-1}	K	Confs
Bologna	$10^2 \times 20 \times 40$	6.0	2 GeV	0.1515 0.1530 light 0.1545 0.1350 heavy	15 (periodic boundary conditions)
Paris (Ecole Polytechnique)	$16^3 \times 48$	6.2	3 GeV	0.1500 0.1510 light 0.1520 0.1350 heavy	same

Table 2
Values of the axial coupling of heavy pseudoscalar mesons, at $\beta = 6.2$ (Paris).

M	$(f_M/f_\pi)_{\text{Lat}}$	$(f_M/f_\pi)_{\text{Lat}} \times (f_\pi)_{\text{expt}}$	expt
K	1.20 ± 0.04	$158 \pm 5 \text{ MeV}$	165 MeV
D ($M_D = 1.72 \text{ GeV}$)	1.37 ± 0.20	180 ± 26	< 300 MeV ref. (8)
F ($M_F = 1.81 \text{ GeV}$)	1.66 ± 0.25	218 ± 33	-

Table 3
Values of the axial coupling of pseudoscalar mesons at $\beta = 6.0$ (Bologna)

M	$(f_M/f_\pi)_{\text{lat}}$	$(f_M/f_\pi)_{\text{lat}} \times (f_\pi)_{\text{expt}}$
K	1.13 ± 0.07	$149 \pm 9 \text{ MeV}$
D ($M_D = 1.55 \text{ GeV}$)	1.49 ± 0.10	196 ± 13
F ($M_F = 1.67 \text{ GeV}$)	1.64 ± 0.14	215 ± 18

structure for H_{eff} emerges⁽¹¹⁾ even in perturbation theory due to the so-called penguin diagrams. This effect is widely held to be responsible for the large $\Delta I=1/2$ enhancement observed in K-decays, in addition to the small enhancement due to the short distance effects, eq.(4.2).

Lattice QCD offers an ideal tool to compute the non leptonic amplitudes. We keep μ large, where eq.(4.1) is reliable, and let the lattice work out for us the fully non perturbative matrix elements. This approach has been initiated by the works of ref.(12), where $K \rightarrow \pi$ matrix elements of four fermion operators have been computed for the first time, and later followed by our group and by the UCLA group^(1,13) using the Wilson formulation for fermion fields on the lattice. Investigation of the same problem with staggered fermions has been carried out by Sharpe and coworkers, ref.(14).

Before going to the practical calculation, however, a theoretical problem had to be solved. If we take a four fermion operator on the lattice, we see quite easily that, due to the Wilson term in the fermionic action:

- i. its matrix elements are badly divergent as the lattice spacing, a , tends to zero;
- ii. its matrix elements do not respect the chiral symmetry rules.

A closer inspection^(4,2) reveals that the two problems are related and that one can cure both by subtracting from the bare operator other operators of equal or lower dimension, with coefficients such that the chiral rules are obeyed.

The structure of the (renormalized) hamiltonian is different for $\Delta I=3/2$, where only four fermion operators can appear, and $\Delta I=1/2$, where lower dimension operators are allowed. Schematically, one has⁽²⁾:

$$(\hat{\phi}_{3/2})_{\text{ren}} = (\hat{\phi}_{3/2})_{\text{bare}} + \delta_6 \hat{\phi}_{3/2} \quad (4.3)$$

$$\begin{aligned} (\hat{\phi}_{1/2})_{\text{ren}} = & (\hat{\phi}_{1/2})_{\text{bare}} + \delta_6 \hat{\phi}_{1/2} + \\ & + d_5 (m_c - m_u) \bar{s} \sigma_{\mu\nu} F^{\mu\nu} d + \\ & + c(\bar{s} d) + c_5 (m_s - m_d)(\bar{s} \gamma_5 d) \end{aligned} \quad (4.4)$$

where δ_6 $\hat{\phi}$ are dimension six (four fermion) counterterms, which appear with finite coefficients. The GIM cancellation makes so that the coefficient d_5 is also finite⁽²⁾ as $a \rightarrow 0$, while c and c_5 , although proportional to $(m_c - m_u)$, are expected to be badly divergent. As a consequence, in the limit $\beta \rightarrow \infty$ the perturbation theory values of δ_6 $\hat{\phi}$ and d_5 can be used, while c and c_5 have to be determined, in any case, with a non perturbative method.

A determination of δ_6 $\hat{\phi}$ has been given time ago⁽¹⁵⁾ and we have just completed⁽¹⁶⁾ the calculation of the coefficient d_5 in eq.(4.4).

To fix the coefficient c , and to evaluate the $K \rightarrow \pi\pi$ transitions, we adopt the following strategy. We can compute, on the lattice, the matrix elements:

$$\langle K(k)^+ | \hat{\phi}_{\text{pert}} | \pi^+(q) \rangle \quad (4.5)$$

$$\langle K(k)^+ | \bar{s} d | \pi^+(q) \rangle \quad (4.6)$$

where $\hat{\phi}_{\text{pert}}$ is $\hat{\phi}_{\text{bare}}$ plus all the perturbative corrections in eq.(4.4), and the matrix elements are given as functions of the meson mass (we work in the SU(3) limit, $M_K = M_\pi$) and of the external momenta k and q . For the renormalized operator, chiral perturbation theory implies^(17,2):

$$\langle K^+(k) | \hat{\phi}_{\text{ren}} | \pi^+(q) \rangle = \delta M^2 + \gamma(kq) + \dots \quad (4.7)$$

dots representing higher order terms in M^2 and/or external momenta. Due to its divergent part, the matrix element in (4.5) is not going to vanish at $M^2 = (kq) = 0$, but we may take out the divergent term precisely enforcing eq.(4.7) i.e. defining:

$$\hat{\phi}_{\text{ren}} = \hat{\phi}_{\text{pert}} + c(\bar{s} d) \quad (4.8)$$

with:

$$c = \frac{\langle K^+ | \hat{\phi}_{\text{pert}} | \pi^+ \rangle}{\langle K^+ | \bar{s} d | \pi^+ \rangle} \Big|_{M^2 = kq = 0} \quad (4.9)$$

With this definition, not only $\hat{\phi}_{\text{ren}}$ is expected to be finite, but it should also satisfy the soft pion theorem:

$$\begin{aligned} \langle K^+ | \hat{\phi}_{\text{ren}} | \pi^+ \pi^+ \rangle = & \frac{i}{f_\pi} (m_K^2 - m_\pi^2) \gamma + \\ & + (\text{higher order}) \end{aligned} \quad (4.10)$$

Thus, after the subtraction, we need to determine the slope

in (kq) , i.e. γ , to get the $K \rightarrow \pi\pi$ amplitude. For any M , this requires to determine the matrix elements for at least two values of (kq) , which is, of course, an unpleasant complication. We can obtain that either by giving one of the two mesons a space-like momentum q or, as discussed below, by comparing the matrix elements:

$$\langle K | \bar{\psi} | \pi \rangle, \langle 0 | \bar{\psi} | \bar{K} \pi \rangle \quad (4.11)$$

for any of the three operators $\bar{\psi}_{\text{pert}}, \bar{\psi}_d, \bar{\psi}_{\text{ren}}$.

The subtraction procedure just described is the price to pay if we want to restrict the lattice calculation to three point functions, i.e. to $K \rightarrow \pi$ matrix elements. Can we avoid making it?

One possibility is to work with staggered fermions⁽¹⁴⁾, where divergences are absent and chiral rules are obeyed, as in the continuum. The problem here is that the vector and axial currents are also free from the Adler-Bell-Jackiw anomalies, while we know, that the physical weak currents must have them, weak currents being related by the CVC hypothesis to the e.m. current which, in turn, is anomalous, least $\pi^0 \rightarrow \gamma\gamma$ would not occur. Whether this problem can be cured without bringing back the divergences and their related subtractions, I do not know, but I doubt.

The other possibility is the one followed by the UCLA group⁽¹⁾ and relies on the fact that all parity violating counterterms in eqs.(4.3) and (4.4) do vanish in the limit $m_s = m_d$, because of combined CP and SU(3) symmetry. Thus, the parity violating part of $\bar{\psi}_{\text{bare}}$ is (multiplicatively) renormalizable and no subtraction is needed to compute directly $K \rightarrow \pi\pi$ on the lattice. Of course, one must compute a four-point function in this case, which will introduce other sources of uncertainty, with the lattice volumes and quark masses presently obtainable. In the long run, both $K \rightarrow \pi$ and $K \rightarrow \pi\pi$ should be computed as a test of the Standard Theory and of chiral symmetry.

5. THREE POINT FUNCTIONS ON THE LATTICE

The lattice diagrams for the matrix elements of four fermion operators and of quark bilinears are displayed in Fig.1. While the so called "eight" diagram involves only quark propagators stemming from the origin, the "eye"

FIGURE 1

Lattice diagrams for the matrix elements of the Weak Hamiltonian: (a) figure-eight diagram; (b) eye diagram; (c) insertion of the $\bar{3}d$ counterterm.

diagram^(*) and the quark-bilinear insertion (needed for the counterterm, eqs.(4.8) and (4.9)) involve a propagator between two arbitrary points. To compute this for all values of x and y would take too much CPU time, but, fortunately, we do not need so much information. Following the method discussed in refs.(6) and (13), we fix the time component of x , t_x , at a value sufficiently large so that one-pion propagation from the origin to x dominates, and compute directly the propagator:

$$S_1(y,0) \equiv \sum_{\vec{x}} S_0(y,x) \gamma_5 S_0(x,0) \quad (5.1)$$

S_1 corresponds to one quark line from the origin to y , with one pseudoscalar density with vanishing space momentum inserted at t_x . S_0 is obtained by solving numerically an equation of the form:

$$\sum_z K(y,z) S_0(z,0) = \delta_{y,0} \quad (5.2)$$

(*) This pictorial naming has been introduced in ref.(13).

and S_1 is obtained by solving the same equation with S_0 as a source:

$$\sum_y K(y,z) S_1(z,0) = \sum_{\vec{x}} \delta_{y,x} \gamma_5 S_0(x,0) \quad (5.3)$$

Thus computing S_1 , for a given t_x , takes about the same time as computing S_0 .

After all this is done, we end up with the three-point correlation function:

$$G_\phi(t_y, t_x, \vec{q}) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{q} \cdot \vec{y}} \langle \bar{q} \gamma_5 q(\vec{y}, t_y) \phi(0) \bar{q} \gamma_5 q(\vec{x}, t_x) \rangle \quad (5.4)$$

with t_x fixed ($t_x=6$ or 12 for Bologna and Paris lattices, respectively) and for all values of t_y . Until now, we have analyzed the data at $\vec{q}=0$ only, from which we can obtain the two matrix elements eq.(4.11) as follows. If lowest-lying states dominate in the propagation (i.e. for t_y, t_x large and far apart) we expect:

$$G_\phi(t_y, t_x) \sim Z_5 e^{-M_p t_y} e^{-M_p t_x} \langle K | \phi | \pi \rangle (t_y \ll t_x) \quad (5.5)$$

$$G_\phi(t_y, t_x) \sim Z_5 e^{-M_p t_y} e^{-M_p t_x} \langle 0 | \phi | \bar{K} \pi \rangle (0 \ll t_y, t_x) \quad (5.6)$$

for both p-s meson at rest, and:

$$P_5(t) = \sum_{\vec{x}} \langle \bar{q} \gamma_5 q(\vec{x}, t) q \gamma_5 q(0) \rangle \sim Z_5 e^{-M_p t} (t > 0) \quad (5.7)$$

Thus, the ratio:

$$R(t_y) = \frac{G_\phi(t_y, t_x)}{P_5(t_y) P_5(t_x)} \quad (5.8)$$

as function of t_y should exhibit two flat regions, in correspondence of the two cases in eqs.(5.5) and (5.6). The values of R in these two regions give directly the matrix elements in (4.11), up to the normalization constant, Z_5 .

The existence of two different plateaus is clearly seen in Fig.2, where we plot the ratio eq.(5.8) for the insertion of the quark bilinear, $\bar{s} d$, and the three values of the Wilson parameter, K . Similar plateaus are seen in the eye diagram correlation functions of $\phi^{(\pm)}$ per Figs.5,6.

With three values of K , we can determine six independent matrix elements for each operator, three at $(P_K \cdot P_\pi) = +M^2$ and three at $(P_K \cdot P_\pi) = -M^2$, which we fit with a linear expression:

$$\begin{aligned} \langle \phi \rangle &= A + B M^2 + C (P_K \cdot P_\pi) \\ &= A + \frac{C+B}{2} [(P_K \cdot P_\pi) + M^2] + \\ &\quad + \frac{C-B}{2} [(P_K \cdot P_\pi) - M^2] \end{aligned} \quad (5.9)$$

The value of C is the (unrenormalized) slope in $(P_K \cdot P_\pi)$ which we need in the end. Fig.3 shows a plot of the matrix elements of sd versus M^2 (for $\langle K | sd | \pi \rangle$) or versus $-M^2$ (for $\langle 0 | sd | \bar{K} \pi \rangle$). As seen from eq.(5.9) a linear dependence means to fit the two parts with independent slopes, requiring the two lines to meet at $M^2=0$. The values we obtain for A and C , in the case of $\bar{s}d$, are listed in Table 4. Errors are obtained by computing each matrix element for three clusters of five configurations each and taking the dispersion of the three values, see ref.(6). One can prove the identity:

$$A = \frac{\partial M_p^2}{\partial 1/K} \quad (5.10)$$

The values of the r.h.s. of eq.(5.10), obtained from the two-point function eq.(5.7) are also listed in Table 4. They agree well with the corresponding A , and provide a nice check that we can indeed measure the matrix elements with the plateau method, at least in this case.

Table 4
Coefficients of the linear fit of the matrix elements of $\bar{s}d$. In the last column, the slope of M_p^2 vs K^{-1} is given, which should coincide with A , see eq.(5.10) of text.

	A	C	$\partial M_p^2 / \partial (1/k)$
Bologna	1.55 ± 0.27	-6.55 ± 0.54	1.23
Paris	0.62 ± 0.14	-7.9 ± 3.8	0.89

FIGURE 2

The ratio $R(t_v)$ vs time, eq.(5.8) of text, for the insertion of $\bar{s}d$ ($\beta=6.0$, Bologna calculation). The operator is located at $t=1$, and one pion is fixed at $t=7$; K is the hopping parameter, and the quark mass decreases from the upper to the lowest figure.

FIGURE 3

The matrix element of $\bar{s}d$ vs $(P_K \cdot P_\pi)(\beta=6.0$, Bologna calculation). The linear fit corresponds to the parameters in Table 4.

FIGURE 4

The ratio $R_{K\bar{K}}$ vs $\xi_{\pm}$, eqs.(6.1) and (6.2) of text ($\beta=6.2$, Paris calculation).

6. RESULTS

6.1 $\Delta I=3/2$ Transition. It can be obtained from the value of the B-parameter, eq.(3.4), for light mesons, i.e. extrapolated to the chiral limit. To reduce statistical and systematic errors, we plot the plateau values of the ratio:

$$R_{K_0\bar{K}_0} = \frac{\langle (\bar{s} \gamma_5 d)(t_y) \vartheta_{K_0\bar{K}_0}(0) \bar{s} \gamma_5 d(t_x) \rangle}{P_5(t_y) P_5(t_x)} \quad (6.1)$$

versus the quantity:

$$\xi_{\pm} = \pm \frac{8}{3} \left[\frac{\langle A_0(t) \bar{q} \gamma_5 q(0) \rangle}{P_5(t)} \right]^2 \quad (6.2)$$

(which is quite constant in t , for $t > 10$, as it should in the one-particle exchange approximation). Points in this plot are nicely aligned, fig.4, and the slope gives directly the B-parameter, up to a sign and a factor $(Z_A)^2$. We fit the points according to:

$$R_{K_0\bar{K}_0} = A - (B Z_A^2) \xi + C \xi^2 \quad (6.3)$$

and obtain ($Z_A=0.86$):

$$\begin{aligned} B &= 1.10 \pm 0.45 \quad (\text{Bologna, } a^{-1} \sim 2 \text{ GeV}) \\ B &= 0.60 \pm 0.20 \quad (\text{Paris, } a^{-1} \sim 3 \text{ GeV}) \end{aligned} \quad (6.4)$$

The B-factor is close to unity and remarkably constant for light and heavy mesons. A value of B between 0.6 to unity gives the $K^+ \rightarrow \pi^+ \pi^0$ of a factor 2+3 too large⁽¹⁸⁾. This, however, neglects the genuine electromagnetic corrections which could contaminate quite consistently the $\Delta I=3/2$ amplitude. For instance the effect of the electropenguin diagrams^(*) could reduce by as much as 30+40% the pure $\Delta I=3/2$ amplitude⁽²⁰⁾.

In a chiral world, the constant A in eq.(6.3) should vanish, and the plot in Fig.4 should cross the origin. This is definitely not so, as first observed in ref.(21) and shown by the data in Fig.4. The combined points at $\xi < 0$, however, suggest that the $K_0\bar{K}_0$ matrix element is anyway smooth in the kaon momentum. The non-vanishing of A may be due to the inaccuracy of perturbation theory to reproduce the counterterms needed to restore completely

(*) These diagrams have been first considered in the calculation of CP violating amplitudes, see e.g. ref.(19).

the chiral behaviour at the present values of β . Indeed, similar discrepancies have been seen in the values of Z_V and Z_A measured from current algebra⁽⁶⁾, with respect to the values given by perturbation theory. What error such effects introduce on the value of B is hard to tell. It is quite interesting that, within errors, the determination of the $K \rightarrow \pi\pi$, $\Delta I=3/2$ amplitude by the UCLA group⁽¹⁾, where no counterterms are needed, agrees with the value obtained with the present method.

6.2 $\Delta I=1/2$ transition (very preliminary). Plateaus are clearly seen in the eye diagram correlation functions of $\vartheta_{\text{pert}}^{(\pm)}$, Figs.5 and 6, especially for the two largest values of the quark mass. This is, of course, not surprising: $\vartheta_{\text{pert}}^{(\pm)}$ have still a divergent term proportional to $\bar{s}d$, so that, to some extent, the structure of Figs.5 and 6 reflects that of Fig.2. To subtract the divergent part, we have followed two methods, with similar results. Figs.7 and 8 show the correlation function of:

$$\vartheta_{\text{pert}}^{(\pm)} = \vartheta_{\text{pert}}^{(\pm)} - c \bar{s}d$$

where c is so adjusted, as to give average value of zero in a control region (which is $30 \leq t_y \leq 35$ in the Bologna case shown in the figures). A plateau for $t_y \sim 10$ is then seen in the result, indicating a non vanishing matrix element for $\vartheta_{\text{ren}}^{(\pm)}$ between vacuum and $\bar{K} \pi$ states. Alternatively, with very close results, we have fitted the matrix elements of $\vartheta_{\text{pert}}^{(\pm)}$ with a linear expression similar to eq.(5.9):

$$\langle \vartheta_{\text{pert}}^{(\pm)} \rangle = A^{\pm} + B^{\pm} M^2 + C^{\pm} (P_K \cdot P_{\pi}) \quad (6.5)$$

Defining:

$$\vartheta_{\text{ren}}^{(\pm)} = \vartheta_{\text{pert}}^{(\pm)} - \left(\frac{A^{(\pm)}}{A} \right) (\bar{s}d) \quad (6.6)$$

the slope $(P_K \cdot P_{\pi})$ of $\vartheta_{\text{ren}}^{(\pm)}$ is therefore:

$$\gamma_{\text{eye}}^{(\pm)} = C^{(\pm)} - \left(\frac{A^{(\pm)}}{A} \right) C \quad (6.7)$$

FIGURE 5
Same as un Fig.2, for the operator $\vartheta_{\text{per}}^{(-)}$

FIGURE 6
Same as in Fig.2, for the operator $\vartheta_{\text{per}}^{(+)}$

FIGURE 7

Same as in Fig.2, for the operator: $\hat{\phi}_{\text{per}}^{(-)} - c(\hat{\phi})$ with c defined so that the ratio vanishes in $30 \leq t_y \leq 35$. The plateau around $t_y = 10$ corresponds to the matrix element of the renormalized operator.

FIGURE 8

Same as in Fig.7, for $\hat{\phi}_{\text{per}}^{(+)}$

where A and C are given in Table 4. The matrix elements of $\vartheta^{(\pm)}_{\text{pert}}$ and $\vartheta^{(\pm)}_{\text{ren}}$ obtained in this way are plotted versus $(P_K \cdot P_\pi)$ in Figs.9 and 10. The result of $\gamma^{(\pm)}_{\text{eye}}$ is given in Table 5, together with the slopes of the eight diagram contributions, which we denote by $\gamma^{(\pm)}_{\text{eight}}$:

$$\langle \vartheta^{(\pm)}_{\text{pert}} \rangle \Big|_{\text{eight diagram}} = \text{const} + \gamma^{(\pm)}_{\text{eight}} (P_K \cdot P_\pi)$$

Also in this case, we find a non vanishing constant, indicating a small violation of chiral symmetry, similar to that displayed in Fig.4. The errors in $\gamma^{(\pm)}$ are assigned as before, by computing the dispersion of the values of $\gamma^{(\pm)}$ over three clusters of five configurations each. It is now a simple matter to compute the ratio of $K_L \rightarrow \pi^+ \pi^-$ to $K^+ \rightarrow \pi^+ \pi^0$ amplitudes. Neglecting phase-space differences, we have:

$$\begin{aligned} A(K_L \rightarrow \pi^+ \pi^-)_{\Delta I=1/2} \sim & \\ & i \left(\frac{m_K^2 - m_\pi^2}{f_\pi} \right) \sqrt{2} c^{(-)}(a^{-1}) [\gamma^{(-)}_{\text{eight}} + \gamma^{(-)}_{\text{eye}}] + \\ & + c^{(+)}(a^{-1}) [\gamma^{(+)}_{\text{eye}} + \frac{1}{3} \gamma^{(+)}_{\text{eight}}] \end{aligned} \quad (6.8)$$

$$A(K^+ \rightarrow \pi^+ \pi^0) \sim i \left(\frac{m_K^2 - m_\pi^2}{f_\pi} \right) \sqrt{2} c^{(+)}(\mu) \gamma^{(+)}_{\text{eight}} \quad (6.9)$$

With $c^{(+)} \sim 0.75$, $c^{(-)} \sim 1.7$, we find:

$$R = \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K^+ \rightarrow \pi^+ \pi^0)} = \begin{cases} -16 \pm 24 & \text{(Bologna)} \\ -14 \pm 9 & \text{(Paris)} \end{cases} \quad (6.10)$$

Errors are estimated from the dispersion over the three clusters of five configurations. This procedure is somewhat misleading for the Bologna data. The large dispersion is in fact due to a cluster with a very small $\Delta I=3/2$ amplitude which makes the ratio very large in

FIGURE 9
Matrix elements of $\vartheta^{(-)}_{\text{pert}}$, (a) and $\vartheta^{(-)}_{\text{ren}}$, (b) vs $(P_K \cdot P_\pi)$. Lines give the best fit to eq.(6.5) of text.

FIGURE 10
Same as in Fig 9, for $\vartheta^{(+)}_{\text{pert}}$, (a) and $\vartheta^{(+)}_{\text{ren}}$, (b).

magnitude. All ratios have the same sign. As for the Paris data, one of the clusters gives a result with the opposite sign as the average. Combining in quadrature, we obtain, finally:

$$R = -15 \pm 9 \quad (6.11)$$

The ratio is large in magnitude, as desired, but it has a puzzling sign. The experimental ratio:

$$\frac{\Gamma(K_L \rightarrow \pi^+ \pi^-)}{\Gamma(K_L \rightarrow \pi^0 \pi^0)} > 2 \quad (6.12)$$

requires constructive interference between the $\Delta I=1/2$ and $3/2$ amplitudes. If the minus sign that we find is confirmed, it could be reconciled with (6.12) by a large contamination of the e.m. corrections in the $\Delta I=3/2$ channel. We cannot however exclude, at present, that this minus sign is produced by an error in the programs (but this is rather unlikely) or it is an artifact of the subtraction procedure or it is due to insufficient smoothness of the amplitude. Work to clarify the situation is in progress.

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Table 5
The slope in $(P_K P_\pi)$ of the matrix element of the renormalized operators $\hat{\phi}^{(\pm)}$, for eye and eight diagrams, in lattice units, at $\beta=6.0$ and $\beta=6.2$

	$\gamma_{eye}^{(-)}$	$\gamma_{eight}^{(-)}$	$\gamma_{eye}^{(+)}$	$\gamma_{eight}^{(+)}$
Bologna, $\beta=6.0$	0.026(3)	-0.0035(18)	-0.024(5)	0.005(2)
Paris, $\beta=6.2$	0.0062(55)	-0.0021(8)	-0.014(7)	0.0015(3)

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