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**BOSE-EINSTEIN CONDENSATION
IN MANY-PARTICLE GAUGE
THEORIES AND EXTERNAL CHARGE**

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BOSE-EINSTEIN CONDENSATION IN MANY-PARTICLE GAUGE THEORIES AND EXTERNAL CHARGE

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SUMMARY: The realization of the charged Bose-Einstein condensation in gauge theories with finite particle density is investigated. Examples are found for a situation when two charged Bose-Einstein condensates coexist in equilibrium in systems described by Abelian gauge theories. Also Abelian and non-Abelian systems are constructed for which a charged Bose-Einstein condensate is balanced by the charge of a fermionic subsystem. A limiting transition to infinitely heavy fermions with infinite chemical potential is pointed out, which turns the fermionic subsystem into an external charge in an Abelian theory. On the contrary, the non-Abelian gauge theory with external charge cannot be obtained by the same dynamical limiting process since it requires a special quantization scheme. It is realized for the Yang-Mills-Higgs model with a constant external charge. Eigenfrequencies of the system are found, which include an additional quantum degree of freedom induced by the external charge. The contribution of the eigenfrequencies into the thermodynamical potential is found.

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1. INTRODUCTION

The study of many-particle systems in the context of gauge theory and lattice gauge theory has become recently an active field of investigation [1], [2]. Its close relation with so attractive problems as cosmology and ion collision [3] justifies its importance.

In this context the phenomenon of the Bose-Einstein condensation (B-E-c) has called the attention of some investigators [2], [4], [5]. In parallel to the description given by Bogoliubov [6] for the B-E-c in the non-relativistic dominion, where a system of bosons with mass m and chemical potential which depends on the temperature, $\mu(T)$, is described by the Hamiltonian density

$$\mathcal{H} = \frac{1}{2m} |\vec{\nabla}\psi|^2 + \frac{\lambda}{4!m} (\psi^\dagger\psi)^2 - M(T)\psi^\dagger\psi \quad (1.1)$$

$$M(T) = \mu(T) - m$$

in such a way that there exists a critical temperature T_c responsible for a second order phase transition (i.e. for $\mu(T > T_c) < m$ the symmetry is exact, but for $\mu(T < T_c) > m$ the symmetry is broken and a B-E condensed state appears), these authors have studied the connection between the symmetry breaking mechanism at finite density and the B-E-c in relativistic systems.

Let us now describe a simple relativistic system in which this mechanism acts (the relativistic ideal Bose gas) in order to demonstrate how this phenomenon occurs in the field theory at finite temperature and density.

This model has a Lagrangian density

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi \quad (1.2)$$

which due to the U(1) global symmetry, has the conserved number of particles N with the density given by

$$N = i (\phi^* \partial_0 \phi - \phi \partial_0 \phi^*) \quad (1.3)$$

The partition function of the Grand canonical ensemble is

$$\mathcal{Z} = \text{Tr} \exp [-\beta \mathcal{L}(\mu)] \quad (1.4)$$

where β is the inverse of the absolute temperature, μ the chemical potential and $\mathcal{L}(\mu) = \mathcal{L} - \mu N$ is the many-particle Hamiltonian density.

Calculating the thermodynamic potential $\Omega = -\frac{1}{\beta} \ln \mathcal{Z}$ within the one-loop approximation one obtains

$$\Omega = V \int \frac{d^3 k}{(2\pi)^3} \left\{ E + \frac{1}{\beta} \ln [1 - e^{-\beta(E-\mu)}] + \frac{1}{\beta} \ln [1 - e^{-\beta(E+\mu)}] \right\}$$

$$E = [k^2 + m^2]^{1/2} \quad (1.5)$$

(here V is the volume of the system) and then the charge density of the system can be found by the thermodynamical relation

$$\rho = -\frac{1}{V} \left(\frac{\partial \Omega}{\partial \mu} \right)_{T,V} = \int \frac{d^3 k}{(2\pi)^3} (n_k - \bar{n}_k) \quad (1.6)$$

where $n_k = 1 / \exp[\beta(E_k - \mu) - 1]$ and $\bar{n}_k = n_k(\mu \rightarrow -\mu)$, as it is known, represent the number of positively and negatively charged bosons in the theory, respectively.

Then, the B-E-c of this gas appears as a consequence of the

following mechanism. In order to maintain the requirement that the number density n_k be non-negative the chemical potential changes under the restriction $|\mu| \leq m$. For T above some critical temperature T_c , there exists $\mu < m$ which substituted in (1.6) gives a good correction to the total charge density of the system, but for T below T_c no such μ can be found, and then (1.6) with $\mu = m$ is interpreted as an expression for the charge density of the excited state, while the remnant part of the conserved charge is condensed in the ground state $K=0$.

The formal description of the condensation state is gained following the Bogoliubov's replacement

$$\phi = \xi + \frac{1}{2\beta} \sum_n' \int \frac{d^3k}{(2\pi)^3} \left[\psi_n(\vec{k}) \exp(i\vec{k}_n \cdot \vec{x}) + \psi_n^*(\vec{k}) \exp(-i\vec{k}_n \cdot \vec{x}) \right] \quad (1.7)$$

where the prime prescribes to omit the term $K=0$. This substitution implies that the ground state expectation value is non-zero

$\langle 0 | \phi | 0 \rangle = \xi$ and there is a net density of charge stored in the ground state, the value of which is derived from (1.4) to be $\rho_0 = \mu \xi^2$.

The peculiarity of this so simple a model is that although the particles are charged their interaction is absent. Now, once the gauge field is introduced it displays the interaction between the charges. So, the charged system becomes unstable, and one is left with the following alternative: either to consider a neutral dynamical system, or to include an equilibrator in the form of an external (nondynamical) distributed charged background J_0 . The latter should interact with the gauge field A_0 through the term $A_0 J_0$ appearing in the Lagrangian density. In the present paper we are studying this alternative. It is common in the literature

[2,4,7] to deal with the external neutralizer whose existence admits the dynamical subsystem to have charge in the form of a gas and/or of a charged B-E condensate. The introduction of the external charged background, however, demands a nonconventional quantization procedure [8], as far as non-Abelian gauge theories are concerned, since the term $A_0 J_0$ violates the gauge invariance.

By this reason we start with the consideration of a neutral dynamical system. Such system obeys the neutrality condition in the form $\partial \Omega / \partial \mu = 0$, that is equivalent, in the formal context, to certain asymptotical conditions controlling the speed of decrease of the path integration variables which play the role of the "electric" field in the Gauss law constraints of the system (eqs.(2.11), (2.12)). When dealing with such neutral dynamical system we pursue the purpose of studying the possibility of a B-E condensation of its charged subsystems. This possibility is demonstrated both in Abelian and non-Abelian systems, in the latter case it is achieved by the inclusion of fermions (cf.[4]). On the other hand by making some of the charged particles infinitely heavy, $M \rightarrow \infty$, one may try to obtain the classical background charge in a dynamical way. This program succeeds for Abelian theories, but fails for non-Abelian ones. The reason for this failure is recognized as due to the alteration in the number of degrees of freedom following from the gauge invariance violation by the external charge [9]. Such alteration cannot be reproduced by a continuous limiting process.

This circumstance makes us to treat the case of external neutralizer for the non-Abelian case on a separate basis, taking into account the correct number of degrees of freedom. This treatment completely differs from those in the literature [2].

The plan of the paper is as follows. In Secs.2,3 we investigate the realization of charged B-E-c in subsystems of neutral dynamical systems which contain only interaction between bosons. In Sec.2 this program is carried out for the Abelian case, and in Sec.3 it is shown that for the non-Abelian case the same mechanism of charge compensation does not work for the condensate and the B-E-c is not allowed.

In Sec.4 we include fermions and find (as in ref./4/) that the charged B-E-c of a subsystem takes place in the Abelian as well as in the non-Abelian case, once there exists a non-zero density of fermions $\mu' \neq 0$ (μ' will be called leptonic chemical potential). Then, in Sec.5 we search a prescription which leads to a dynamical limit where the effect of the fermions in the system can be reduced to the required interacting background charge contribution. This prescription implies that the chemical potential of fermions μ' tend to infinity together with their mass (eq.(5.1)). We obtain in that section that, whereas for the Abelian case the mechanism works in a satisfactory way, for the non-Abelian one a pathological behaviour appears which hinders an analogous result.

Finally, in Sec.6 we show that precisely the non-Abelian theory which includes a constant external charge is drastically different, in what concerns the quantization procedure, from the conventional gauge theory, giving rise to new thermodynamical consequences due to the contribution of the extra degrees of freedom. For the Yang-Mills-Higgs model with external current and chemical potential the one-loop effective potential is presented as a sum over the spectra of bosonic excitations which have been calculated, including the extra degree of freedom.

In conclusion (Sec.7) we summarize the main results of the paper. We also explain that as long as the equilibrium requirement is disregarded, both the Abelian and non-Abelian theories might be treated on equal grounds without going into contradiction with the gauge-invariance, by assuming other asymptotical conditions for the path-integration variables. The latter are equivalent to the nonvanishing of the surface term in the Gauss law and induce the traditional non-equilibrium equation

$\partial \Omega / \partial \mu = J_c$, like (1.6). However, it remains unclear under what conditions the equilibrium requirement can be disregarded, indeed, to make this alternative approach meaningful. Perhaps, it can be applicable to processes fast as compared to the characteristic instability time of the system with a non-compensated charge.

2. ABELIAN MODELS OF BOSONS

Let us investigate a system with two different scalar charged fields in interaction, given by the Lagrangian density

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \\ & - (D_\mu \chi)^* (D^\mu \chi) - M^2 \chi^* \chi - \lambda' (\chi^* \chi)^2 \end{aligned} \quad (2.1)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D_\mu = \partial_\mu - ie A_\mu$$

where m , λ and M , λ' are the masses and selfcoupling constants of the complex fields ϕ and χ , respectively.

As a consequence of the Noether theorem for the infinitesimal global gauge symmetries $\delta \phi = -i\phi\omega$, $\delta \chi = -i\chi\omega$

this system has a conserved particle density

$$\mathcal{N} = -i[(D_0\phi)^*\phi - (D_0\phi)\phi^* + (D_0\chi)^*\chi - (D_0\chi)\chi^*] \quad (2.2)$$

If we now pass to the Hamiltonian formalism, where the canonical momenta are given by

$$\begin{aligned} \pi_\mu &= \frac{\delta \mathcal{L}}{\delta \partial^\mu \phi} = F_{0\mu} \\ p &= \frac{\delta \mathcal{L}}{\delta \partial^0 \phi} = (D_0\phi)^* \quad , \quad p^* = \frac{\delta \mathcal{L}}{\delta \partial^0 \phi^*} = (D_0\phi) \\ q &= \frac{\delta \mathcal{L}}{\delta \partial^0 \chi} = (D_0\chi)^* \quad \quad q^* = \frac{\delta \mathcal{L}}{\delta \partial^0 \chi^*} = (D_0\chi) \end{aligned} \quad (2.3)$$

the Hamiltonian density becomes

$$\begin{aligned} \mathcal{H} &= \frac{1}{2}(\pi^2 + B^2) + p p^* + q q^* + (D_i\phi)^*(D_i\phi) \\ &\quad + (D_i\chi)^*(D_i\chi) - A_i \partial^i \phi + V(\phi) + V'(\chi) \end{aligned} \quad (2.4)$$

where

$$\begin{aligned} \partial &= \partial_i \pi_i + ie(\phi p - \phi^* p^*) + ie(\chi q - \chi^* q^*) \\ V(\phi) &= m^2 \phi^2 + \lambda \phi^4 \\ V'(\chi) &= M^2 \chi^2 + \lambda' \chi^4 \end{aligned} \quad (2.5)$$

and the particle density is

$$\mathcal{N} = \mathcal{N}(\phi) + \mathcal{N}(\chi) \quad (2.6)$$

where

$$\mathcal{N}(\phi) = -i(\phi p - \phi^* p^*) \quad (2.7a)$$

$$N(x) = -i(x\dot{q} - \dot{x}^*q^*) \quad (2.7b)$$

It is easy to demonstrate that in this model not only the total number of particles N is conserved, but also the corresponding number for each field, because

$$[H, N(\phi)] = [H, N(x)] = 0 \quad (2.8)$$

This result gives the possibility to introduce independently the number of particles for each field in the theory, and therefore we are going to investigate the most general case where the partition function of the Grand canonical ensemble (1.4) is determined by the following many-particle Hamiltonian density

$$\mathcal{H}(\mu) = \mathcal{H} - \mu_1 N(\phi) - \mu_2 N(x) \quad (2.9)$$

As it is known, the partition function (1.4) can be written as a functional integral and its extension to gauge theories is straightforward [10]. Thus, we can write

$$\tilde{Z} = \int e^{iS} \delta(\pi^0) \delta(A^0) \delta(\sigma) \delta(F) \det\left(\frac{\delta F}{\delta \omega}\right) [dA_\mu][d\pi_\mu] \\ [d\phi][d\rho][d\phi^*][d\rho^*][dx][dq][dx^*][dq^*] \quad (2.10a)$$

with

$$S = \int_0^\beta d\tau \int d^3x \left[\pi_i \dot{A}_i + p \dot{\phi} + p^* \dot{\phi}^* + q \dot{x} + q^* \dot{x}^* - \mathcal{H}(\mu, \mu') \right] \quad (2.10b)$$

Here the many-particle Hamiltonian density can be explicitly written as

$$\begin{aligned} \mathcal{L}(\mu, \mu') = & \frac{1}{2} (\pi^2 + \beta^2) + p p^* + q q^* + (D_i \phi)^* (D_i \phi) \\ & + (D_i \chi)^* (D_i \chi) - (A_0 + \mu/2) \gamma + \mu \partial_i \pi_i - \mu' \mathcal{N}(\chi) \end{aligned} \quad (2.10c)$$

which is equivalent to (2.9) with the following redefinition for the chemical potentials: $\mu = \mu_1$, $\mu' = \mu_2 - \mu_1$. In (2.10a) $\delta(A^0)$, $\delta(F)$ represent, as usual, the gauge conditions needed in order to fix a field configuration for every equivalence class, $\delta(\pi^0)$ and $\delta(\gamma)$ are the corresponding constraint conditions which generate the gauge symmetry, and ω parametrize possible gauge transformations.

Henceforth, we are going to consider the asymptotical condition

$$\pi_i \xrightarrow[r \rightarrow \infty]{} \frac{1}{r^{2+\varepsilon}}, \quad \varepsilon > 0 \quad (2.11)$$

or in other terms, to assume that the surface term $\oint_{r \rightarrow \infty} \vec{\pi} \cdot \vec{n} ds$ is equal to zero.

If we now take into account that the system is constrained by the condition¹ $\gamma \simeq 0$ which can be found straightforwardly by the application of the generalized Hamiltonian method of Dirac [11], the abovementioned condition (2.11) implies

$$Q = e \int \mathcal{N} d^3x \simeq \oint_{r \rightarrow \infty} \vec{\pi} \cdot \vec{n} ds = 0 \quad (2.12)$$

Thus, the total charge Q is equal to zero, and the neutrality of the system is guaranteed.

¹ Here ($\simeq 0$) is the Dirac's symbol of weak equality, which means that the constraint γ may have nonvanishing Poisson brackets with some canonical variables.

In order to perform the functional integration in the momenta it is needed to transform the term $\int A_0 \partial_i \pi_i d^3x$, that is involved in the $A_0 \gamma$ -term in (2.10c), into $\int \pi_i \partial_i A_0 d^3x$ using integration by parts. It follows from the definition $\pi_i = \partial_i A_0 - \partial_0 A_i$ that the following asymptotic behaviours of A_0

$$A_0 \xrightarrow{r \rightarrow \infty} \text{const.} + \frac{1}{r^{1+\epsilon/2}} \quad (2.13)$$

may agree with (2.11). This is sufficient for nullifying the surface term

$$\oint A_0 \vec{\pi} \cdot \vec{n} dS = 0 \quad (2.14)$$

Now the functional integration in the momenta subject to the conditions (2.11), (2.13) results in

$$\begin{aligned} \mathcal{Z} &= \int e^{i \int_0^T dx_0 \int d^3x \mathcal{L}_{\text{eff}}} \\ &\quad \delta(F) \det\left(\frac{\partial F}{\partial w}\right) [dA_\mu] [d\phi] [d\phi^*] [dx] [dx^*] \\ \mathcal{L}_{\text{eff}} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (\tilde{D}_\mu \phi)^* (\tilde{D}^\mu \phi) - V(\phi) \\ &\quad - (\tilde{D}_\mu \chi)^* (\tilde{D}^\mu \chi) - V'(\chi) \\ \tilde{D}_\nu &= [\partial_\nu + ie (A_\nu + \mu_1/e \delta_{\nu 0})] \\ \tilde{Q}_\nu &= [\partial_\nu + ie (A_\nu + \mu_2/e \delta_{\nu 0})] \end{aligned} \quad (2.15)$$

The classical equations of motion resulting from this effective Lagrangian are given now as follows

$$\frac{\delta \mathcal{L}_{\text{eff}}}{\delta A^\nu} \Big|_S = -2e(\mu_1 \bar{\phi}^2 + \mu_2 \bar{\chi}^2) g_{\nu 0} = 0 \quad (2.16a)$$

$$\frac{\delta \mathcal{L}_{eff}}{\delta \phi} \Big|_s = (\mu_1^2 - m^2 - 2\lambda \bar{\phi}^2) \bar{\phi} = 0 \quad (2.16b)$$

$$\frac{\delta \mathcal{L}_{eff}}{\delta \chi} \Big|_s = (\mu_2^2 - M^2 - 2\lambda' \bar{\chi}^2) \bar{\chi} = 0 \quad (2.16c)$$

where we confined ourselves to solutions of the form

$$s = \{ \bar{A}_p = 0, \bar{\phi}, \bar{\chi} = \text{real constants} \} \quad (2.17)$$

Then from (2.16) the following possible solutions are obtained

$$\bar{\phi} = \bar{\chi} = 0 \quad (2.18a)$$

$$\begin{aligned} \bar{\phi}^2 &= \frac{\mu_1^2 - m^2}{2\lambda} & \bar{\chi}^2 &= \frac{\mu_2^2 - M^2}{2\lambda'} \\ \mu_1 \frac{\mu_1^2 - m^2}{2\lambda} &= -\mu_2 \frac{\mu_2^2 - M^2}{2\lambda'} \\ \text{if } |\mu_1| > m, |\mu_2| > M \end{aligned} \quad (2.18b)$$

Hence, the system admits a charged symmetry-breaking solution if the introduced chemical potentials μ_1 and μ_2 satisfy the relations (2.18b). This solution arises as a consequence of the neutrality $Q=0$ (2.12) that can be reached through the compensation of the two independent conserved charges. This is easy to show if we take into account that the conjugate momenta associated to the fields ϕ and χ in $\mathcal{L}_{eff}$ are given by $p=(D_0\phi)^*$ and $q=(D_0\chi)^*$ so that in the stationary solution they take the values

$$\bar{p} = -i\mu_1 \bar{\phi}, \quad \bar{q} = -i\mu_2 \bar{\chi} \quad (2.19)$$

which when substituted in the general expression for the conserved number of particles (2.7) give rise to $N(\bar{\phi}) = -2\mu_1 \bar{\phi}^2$

and $\mathcal{N}(\bar{\chi}) = -2\mu_2 \bar{\chi}^2$. Then the classical conditions (2.16a) can be interpreted as the relation between μ_1 and μ_2 needed in order to guarantee the neutrality of the ground state. The Bose-Einstein phase consists in this case of two oppositely charged condensates, the total charge being still zero.

Moreover, if we consider the thermodynamic corrections, the equation analogous to (2.16a), with the equation $\langle \gamma \rangle = 0$ kept in mind¹, takes the form

$$\frac{1}{V} \left[\frac{\partial \Omega}{\partial \mu} \right]_{T,V} = \frac{1}{e} \langle \oint \vec{\pi} \cdot \vec{n} ds \rangle \quad (2.20a)$$

which, in view of the neutrality condition (2.12), transforms into

$$\frac{1}{V} \left[\frac{\partial \Omega}{\partial \mu} \right]_{T,V} = 0 \quad (2.20b)$$

The charged B-E solution may be induced in this situation by taking

$$\frac{1}{V} \left[\frac{\partial \Omega}{\partial \mu} \right]_{T,V} = \text{const} \quad (2.21)$$

and preserving the neutrality of the system (eq.(2.20b)), which now can be written as

$$\langle \mathcal{N}(\phi) \rangle - \langle \mathcal{N}(\chi) \rangle = 0 \quad (2.22)$$

¹ It is convenient to underline that once the quantum level is present, only for non-covariant gauges the average value of the constraint γ (taken as usual from the starting classical theory) vanishes. When a covariant gauge is taken, the Gauss Law is modified depending now of additional fields. But the statistical average of this new Gauss Law is also zero and then our analysis remains valid for covariant gauges, where Q contains the charge of the additional charged fields (for example, the ghost fields in the Lorentz gauge).

On the other hand, if we consider the Higgs model

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (D_\mu \phi)^* (D^\mu \phi) + m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \quad (2.23)$$

in place of the system (2.1), and introduce the chemical potential μ associated to the conserved charge of the system (2.7a) to obtain after the momenta integration, with the condition (2.11) preserved, the following effective Lagrangian

$$\begin{aligned} \mathcal{L}_{eff} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (\tilde{D}_\mu \phi)^* (\tilde{D}^\mu \phi) + V(\phi) \\ \tilde{D}_\nu &= \partial_\nu - ie(A_\nu + \mu/e \delta_{\nu 0}) \\ V(\phi) &= m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \end{aligned} \quad (2.24)$$

with the classical equation of motion analogous to (2.16) are given by

$$\frac{\delta \mathcal{L}_{eff}}{\delta \phi} = -\tilde{D}^\mu \tilde{D}_\mu^* \phi^* + m^2 \phi^* - 2\lambda \phi \phi^* = 0 \quad (2.25)$$

$$\frac{\delta \mathcal{L}_{eff}}{\delta A^\mu} = \partial^\mu F_{\mu\nu} - ie[\phi (\tilde{D}_\nu \phi)^* - \phi^* \tilde{D}_\nu \phi] = 0 \quad (2.26)$$

we look for a stationary solution of the type $A_\mu = 0$, $\phi = \xi$ (ξ is a real constant), which is translationally invariant, we find

$$\left. \frac{\delta \mathcal{L}_{eff}}{\delta \phi} \right|_{\substack{\phi=\xi \\ A_\mu=0}} = (\mu^2 + m^2 - 2\lambda \xi^2) \xi = 0 \quad (2.27)$$

$$\left. \frac{\delta \mathcal{L}}{\delta A^\mu} \right|_{\substack{\phi=\xi \\ A_\mu=0}} = -2e\mu \xi^2 = 0 \quad (2.28)$$

Hence, from (2.28) a solution with both μ and ξ different from zero is forbidden, and we obtain as the more favourable energetically solution $\mu = 0$, $\xi = m^2/2\lambda \neq 0$. The charge (2.7a) of this Bose-condensed state is zero owing to the first equation (2.19) with $\mu = \mu_1 = 0$. This formal result reflects the evident fact that it is impossible to condensate interacting charged particles in a ground charged state within the model of one charged scalar field interacting with a neutral Abelian vector field with no external charge, and what is more, in this case the chemical potential μ , associated to the conserved charge, is zero.

3. NON-ABELIAN MODELS OF BOSONS

In this section we are going to follow an analogous approach for the non-Abelian case to the one presented above for the Abelian model of bosons.

We begin studying the simple model of interaction between the Yang-Mills field and only one Higgs boson in which the scalar Bose field has several components that could in principle condensate.

3.1. YANG-MILLS-HIGGS

Let us consider the Yang-Mills-Higgs system whose Lagrangian density is given by

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a - \frac{1}{2} (D_\mu \phi)^\dagger (D^\mu \phi) + V(\phi) \\ F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g \epsilon^{abc} A_\mu^b A_\nu^c \\ D_\mu &= \partial_\mu - g A_\mu^b \tau^b\end{aligned}\tag{3.1}$$

$$V(\phi) = \frac{m^2}{2} \phi^2 - \frac{\lambda}{4} \phi^4$$

We are going to consider the adjoint representation for the SU(2) algebra, where the generators of the group τ_b^a and the structure constants C_{abc} coincide with the completely antisymmetric tensor ϵ^{abc} .

The conserved particle density related through the Noether Theorem with the infinitesimal gauge transformations $\delta\phi = \tau^a \phi \omega^a$,
 $\delta A_\nu^a = -g \epsilon^{abc} A_\nu^b \omega^c$ is

$$N^a = \frac{1}{2} [(\mathcal{D}_0 \phi)^t \tau^a \phi - \phi^t \tau^a (\mathcal{D}_0 \phi)] + \epsilon^{abc} A_\nu^b F_{0\nu}^c \quad (3.2)$$

Different isotopic components $a=1,2,3$ of the number of particles do not Poisson commute between themselves, but satisfy the commutation relation characteristic of SU(2) algebra

$$[N^a, N^b] = \epsilon^{abc} N^c \quad (3.3)$$

Therefore, in order to construct the many-particle Hamiltonian of the theory we can introduce a conserved number of particles only along one of the isotopic axes, for instance, along the third-axis, then

$$H(\mu) = H + \mu N^3 \quad (3.4)$$

After the functional integration in the momenta where the condition (2.11) have been used the partition function can be written as follows

$$\tilde{\mathcal{L}} = \int \exp i \left[\int_0^{\beta} d\tau \int d^3x \tilde{\mathcal{L}}_{\text{eff}} \right] \delta(F^a) \det \left(\frac{\partial F^e}{\partial w^e} \right) [dA_\mu^a] [d\phi] \quad (3.5)$$

$$\tilde{\mathcal{L}}_{\text{eff}} = -\frac{1}{4} \tilde{F}_{\mu\nu}^a \tilde{F}_a^{\mu\nu} - \frac{1}{2} (\tilde{D}_\mu \phi)^t (\tilde{D}^\mu \phi) + V(\phi)$$

$$\tilde{F}_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g \epsilon_{abc} (A_\mu^b + \frac{\mu}{g} \delta^{b3} \delta_{\mu 0}) (A_\nu^c + \frac{\mu}{g} \delta^{c3} \delta_{\nu 0})$$

$$\tilde{D}_\nu = \partial_\nu - g (A_\nu^a + \frac{\mu}{g} \delta_{\nu 0} \delta^{a3}) \tau^a$$

and, as usual, $\delta(F^a)$, $a=1,2,3$ are the gauge fixing conditions, the superscript t stands for transposition.

The analysis of the stationary solution in this case leads to the equations

$$\frac{\delta \tilde{\mathcal{L}}_{\text{eff}}}{\delta A_\nu^e} = \tilde{D}_\mu^{ec} \tilde{F}_c^{\mu\nu} - \frac{g}{2} [\phi^t \tau^e (\tilde{D}^\nu \phi) - (\tilde{D}^\nu \phi)^t \tau^e \phi] = 0 \quad (3.6)$$

$$\frac{\delta \tilde{\mathcal{L}}_{\text{eff}}}{\delta \phi} = -\frac{1}{2} \tilde{D}^\mu (\tilde{D}_\mu \phi)^t + \frac{m^2}{2} \phi^t - \frac{\lambda}{2} \phi^2 \phi^t = 0 \quad (3.7)$$

where

$$\tilde{F}_\mu^{bc} = \partial_\mu \delta^{bc} - g \epsilon^{bac} (A_\mu^a + \frac{\mu}{g} \delta_{\mu 0} \delta^{a3}) \quad (3.8)$$

Taking $S = \{ \tilde{A}_\mu^a = 0, \tilde{\phi} = \bar{\xi}, \text{ (with } \bar{\xi} \text{ -being a real constant)} \}$ as stationary solution we obtain for (3.6)

$$\left. \frac{\delta \tilde{\mathcal{L}}_{\text{eff}}}{\delta A_\nu^e} \right|_S = \frac{g\mu}{2} \left[\frac{\bar{\xi}}{r} (\delta_{r3} \delta_{e5} + \delta_{re} \delta_{35} - 2\delta_{rs} \delta_{e3}) \bar{\xi}_s \right] g^{\nu 0} = 0 \quad (3.9)$$

which can be splitted in the components (as long as $\mu \neq 0$)

$$e=1 \quad \bar{\xi}_1, \bar{\xi}_3 = 0 \quad (3.10a)$$

$$e=2 \quad \xi_2 \xi_3 = 0 \quad (3.10b)$$

$$e=3 \quad \xi_1^2 + \xi_2^2 = 0 \quad (3.10c)$$

and the equation (3.7) turns into

$$\left. \frac{\delta \mathcal{L}_{eff}}{\delta \phi} \right|_S = \frac{1}{2} [m^2 - \lambda \xi^2] \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}^t + \frac{1}{2} \mu^2 \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}^t = 0 \quad (3.11)$$

From (3.10) it is evident that the only symmetry breaking solution in this problem is $\xi_1 = \xi_2 = 0$, $\xi_3 \neq 0$. Taking into account (3.11) the complete solution may be written in the form

$$\xi = \begin{pmatrix} 0 \\ 0 \\ \xi_3 \end{pmatrix}, \quad \xi_3^2 = m^2/\lambda \quad (3.12)$$

which is independent of the chemical potential.

It is interesting moreover to note that once we have introduced the chemical potential in the theory, different component of the Boson field ϕ have different nature, the first two components being charged and the third neutral. Hence, the symmetry breaking solution (3.12) does not produce a charged condensed state. On the other hand, we have that in contrast to the Abelian case the chemical potential is not determined by the tree equations.

Let us calculate now the quantum correction to the charge in order to see how the neutrality condition works at this level. Consider the loop-expansion induced by the shift $\phi \rightarrow \phi + \xi$ in $\mathcal{L}_{eff}$ /12/ taking the following relations as gauge fixing conditions

$$F^+ = (\partial_\nu - \mu \delta_{\nu 4}) A_\nu^+ - g \phi_f^+ \xi_{f5}^+ \xi_{f3}^+ - f^+(\bar{x}, z) = 0 \quad (3.13a)$$

$$F^- = (\partial_\nu + \mu \delta_{\mu\nu}) A_\nu^- - g \phi_r \tau_{rs}^- \xi_s^- - f^-(\vec{x}, \tau) = 0 \quad (3.13b)$$

$$F^3 = \partial_\mu A_\mu^3 - f^3(\vec{x}, \tau) = 0 \quad (3.13c)$$

Here $f^\pm(\vec{x}, \tau)$ and $f^3(\vec{x}, \tau)$ are, as usual, arbitrary functions, and the following change of variables has been performed

$$A_\mu^\pm = \frac{A_\mu^1 \mp i A_\mu^2}{\sqrt{2}}, \quad \phi^\pm = \frac{\phi_1 \mp i \phi_2}{\sqrt{2}}, \quad \tau^\pm = \frac{\tau^1 \mp i \tau^2}{\sqrt{2}} \quad (3.14)$$

We obtain the one-loop approximation for the partition function

$$\mathcal{Z}^{(1)} = \int \exp \left\{ \frac{1}{\beta} \sum_{K_4} \int \frac{d^3 K}{(2\pi)^3} [\mathcal{L}_Q + \mathcal{L}_g] \right\} \det \left(\frac{\partial F^a}{\partial w^c} \right) [dA_\mu^a] [d\phi] \quad (3.15)$$

where $K_4 = (2\pi n/\beta)$, $n = 0, \pm 1, \pm 2, \dots$

$$\mathcal{L}_Q + \mathcal{L}_g = -\frac{1}{2} A_\mu^c \Delta_{\mu\nu}^{-1cc'} A_\nu^{c'} - \frac{1}{2} \phi^c \mathcal{Q}_{cc'}^{-1} \phi^{c'}$$

$$\Delta_{\mu\nu}^{-1cc'} = \begin{pmatrix} 0 & \Delta_{\mu\nu}^{-1+-} & 0 \\ \Delta_{\mu\nu}^{-1+-} & 0 & 0 \\ 0 & 0 & \Delta_{\mu\nu}^{-133} \end{pmatrix}, \quad \mathcal{Q}_{cc'}^{-1} = \begin{pmatrix} 0 & \mathcal{Q}_{+-}^{-1} & 0 \\ \mathcal{Q}_{+-}^{-1} & 0 & 0 \\ 0 & 0 & \mathcal{Q}_{33}^{-1} \end{pmatrix} \quad (3.16)$$

$$\Delta_{\mu\nu}^{-1+-} = (K_\lambda^- K_\lambda^- + g^2 \xi_3^2) \delta_{\mu\nu}$$

$$\Delta_{\mu\nu}^{-133} = (K_\lambda^+ K_\lambda^+ + g^2 \xi_3^2) \delta_{\mu\nu}$$

$$\Delta_{\mu\nu}^{-1,33} = K^2 \delta_{\mu\nu}$$

$$\mathcal{Q}_{+-}^{-1} = K_{\lambda}^{-} K_{\lambda}^{-} + g^2 \xi_3^2 \quad \mathcal{Q}_{-+}^{-1} = K_{\lambda}^{+} K_{\lambda}^{+} + g^2 \xi_3^2$$

$$\mathcal{Q}_{33}^{-1} = K^2 + 2\lambda \xi_3^2$$

and

$$K_{\lambda}^{\pm} = K_{\lambda} \pm i\mu \delta_{\lambda 4} \quad (3.17)$$

$\omega^{1,2,3}$

parametrize possible gauge transformations. Now ta-

king into account that $\mathcal{L}_{\text{eff}}$ is invariant under the local SU(2) transformations

$$\begin{aligned} \delta A_{\nu}^{\pm} &= (\partial_{\nu} \mp i\mu \delta_{\nu 4}) \omega^{\pm} \mp ig (A_{\nu}^{\pm} \omega^3 - \omega^{\pm} A_{\nu}^3) \\ \delta A_{\nu}^3 &= \partial_{\nu} \omega^3 + g (A_{\nu}^1 \omega^2 - A_{\nu}^2 \omega^1) \\ \delta \phi^{\pm} &= \mp ig (\omega^{\pm} \phi_3 - \phi^{\pm} \omega^3) \\ \delta \phi^3 &= g (\omega^1 \phi_2 - \omega^2 \phi_1) \end{aligned} \quad (3.18)$$

with

$$\omega^{\pm} = \frac{\omega^1 \mp i\omega^2}{\sqrt{2}}$$

we find that the ghost determinant is given by

$$\det(\partial F^a / \partial \omega^c) = \det \begin{bmatrix} \partial F^+ / \partial \omega^+ & & \\ & \partial F^- / \partial \omega^- & \\ & & \partial F^3 / \partial \omega^3 \end{bmatrix}$$

$$= \det [-(K_{\lambda}^{+} K_{\lambda}^{+} + g^2 \xi_3^2) (K_{\lambda}^{-} K_{\lambda}^{-} + g^2 \xi_3^2) K^2] \quad (3.19)$$

where $\text{Det } A \equiv \exp [\text{Tr } \ln A]$ and the trace Tr includes only the operation $\frac{i}{\beta} \sum_{K_4} \int d^3 K / (2\pi)^3$. The final expression for the one-loop partition function (3.15) is

$$\Omega^{(1)} = \text{Det}^{-1/2} \left\{ (K_1^+ K_1^+ + g^2 \xi_3^2)^3 (K_1^- K_1^- + g^2 \xi_3^2)^3 (K^2)^2 (K^2 + 2\lambda \xi_3^2) \right\} \quad (3.20)$$

The particle spectrum of the problem is summarized in Table 1. As it is seen from Table 1 the only difference with the ordinary Higgs model is in different dispersion relations for the charged components $A^\pm$ of the gauge field.

The one-loop thermodynamic potential may be written as

$$\Omega^{(1)} = - \frac{1}{2\beta} \sum_{K_4} \int \frac{d^3 K}{(2\pi)^3} \left[\sum_{i=1}^4 n_i \ln \mathfrak{F}_i \right] \quad (3.21)$$

where $\mathfrak{F}_i$ stand for the factors included in the functional determinant (3.20), and n_i for their multiplicities.

Taking into account that the partition function (3.5) is constrained by the Gauss Law $\delta(\gamma^a)$ the asymptotical condition (2.11) gives rise to the thermodynamic relation

$$\langle N^3 \rangle = - \frac{1}{V} \left[\frac{\partial \Omega}{\partial \mu} \right]_{T, V} = 0 \quad (3.22)$$

which tells us that the average charge of the system, defined as usual in thermodynamics, is equal to zero. The expression (3.22) is the analog to (2.12) for the thermodynamic average. Indeed, once the asymptotical condition (2.11) is assumed, μ appears in the exponential of the partition function, in the form

$$\frac{\mu}{g} \gamma^3 = \frac{\mu}{g} (\partial_i \Pi_i^3 - g N^3) \quad . \text{ Then eq.(3.22) is equivalent to}$$

$$\frac{1}{V} \left[\frac{\partial \Omega}{\partial \mu} \right]_{T,V} = \frac{1}{g} \langle \oint \vec{\Pi}^3 \cdot \vec{n} dS \rangle - \langle N^3 \rangle = 0 \quad (3.23)$$

where $\langle \oint \vec{\Pi} \cdot \vec{n} dS \rangle = 0$. Thus, for the one-loop approximation we obtain after the summation over K_4

$$\langle N^3 \rangle^{(1)} = \frac{1}{V} \left[\frac{\partial \Omega^{(1)}}{\partial \mu} \right]_{T,V} = \int \frac{3 d^3 K}{(2\pi)^3} [n_K - \bar{n}_K] = 0 \quad (3.24)$$

where the particle number density is

$$n_K = \frac{1}{e^{\beta(\epsilon - \mu)} - 1}, \quad \epsilon^2 = \vec{K}^2 + g^2 \xi_3^2$$

$$\bar{n}_K = n_K (\mu \rightarrow -\mu) \quad (3.25)$$

Thus, as in the Abelian case we see from (3.24) that not only the charged components of the Higgs field $\xi_{1,2}$ are equal to zero, but the chemical potential must be zero, because $\langle N^3 \rangle^{(1)}$ is a monotonous increasing function of μ and the solution $\mu = 0$, $\langle N^3 \rangle^{(1)} = 0$ exists.

3.2. YANG-MILLS HIGGS WITH TWO SCALAR FIELDS

In the same way as in the Abelian case it makes sense to investigate if with the introduction of an extra boson field the charged B-E-c can be realized. With this purpose we are going to study briefly the following model which is the non-Abelian analog to the one studied in Sec.2.

Let the Lagrangian density be

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{2} (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{m^2}{2} \phi^\dagger \phi - \\ & - \frac{\lambda}{4} (\phi^\dagger \phi)^2 - \frac{1}{2} (D_\mu \chi)^\dagger (D^\mu \chi) - \frac{M^2}{2} \chi^\dagger \chi - \frac{\lambda'}{4} (\chi^\dagger \chi)^2 \end{aligned} \quad (3.26)$$

The following conserved number of particles density is associated with the global SU(2) symmetry

$$N^a = \epsilon_{abc} A_\nu^b F_{0\nu}^c + \frac{1}{2} [(D_0 \phi)^t \tau^a \phi - \phi^t \tau^a (D_0 \phi)] + \frac{1}{2} [(D_0 \chi)^t \tau^a \chi - \chi^t \tau^a (D_0 \chi)] \quad (3.27)$$

In the phase space where $\pi_\nu^a = F_{0\nu}^a$, $p = D_0 \phi$, $q = D_0 \chi$ are the conjugate momenta to the fields A_ν^a , ϕ and χ respectively, the conserved number of particles density is expressed as

$$N^a = N^a(A_\nu^b) + N^a(\phi) + N^a(\chi) \quad (3.28)$$

with

$$N^a(A_\nu^b) = \epsilon_{abc} A_\nu^b \pi_\nu^c \quad (3.29a)$$

$$N^a(\phi) = \frac{1}{2} [p^t \tau^a \phi - \phi^t \tau^a p] \quad (3.29b)$$

$$N^a(\chi) = \frac{1}{2} [q^t \tau^a \chi - \chi^t \tau^a q] \quad (3.29c)$$

and contrary to the Abelian case now we have that only N^a commute with the Hamiltonian of the system, but not $N^a(A_\nu^b)$, $N^a(\phi)$ and $N^a(\chi)$ separately. This limitation hinders the possibility to introduce different chemical potentials and to gain a neutral condensed solution of the form (2.18b) which would permit the charged B-E-c of each sort of scalar particles ϕ and χ .

The reason for this lack of conservation is given by the presence of the interaction mediated by charged gauge field A_ν^a , which gives rise to processes where the charges of different sorts of particles are not conserved separately (Fig.1).

4. THE INCLUSION OF FERMIONS

4.1. ABELIAN MODEL

Now we are going to analyze briefly how the charged B-E-c at zero temperature can be obtained for an Abelian gauge system which includes fermions, where the density of charged fermions which occupies at this temperature the Fermi sphere, must act as neutralizing factor of the charged boson condensate.

Thus, let us complement the Lagrangian (2.23) with the fermionic part

$$\mathcal{L}_\psi = \bar{\psi}(i\gamma^\mu D_\mu - M)\psi \quad (4.1)$$

where M is the mass of the fermions, and define the many-particle Hamiltonian density

$$\mathcal{H}(\mu, \mu') = \mathcal{H} - \mu N_0 - \mu' \mathcal{Y} \quad (4.2)$$

with

$$N_0 = -i[(D_0\phi)^*\phi - (D_0\phi)\phi^*] + \bar{\psi}\gamma_0\psi \quad (4.3)$$

$$\mathcal{Y} = \bar{\psi}\gamma_0\psi \quad (4.4)$$

Here N_0 is the conserved density of charged particles and $\mathcal{Y}$ the density of fermions, that we shall refer to as "leptonic" charge density. Its conservation is obtained from the additional symmetry of the Lagrangian under the transformation $\delta\psi = -i\alpha\psi$, $\delta\bar{\psi} = i\alpha\bar{\psi}$ with α constant.

Then, the fermionic contribution to the thermodynamic potential in the one-loop approximation is known to be

$$\Omega_F^{(1)} = \frac{-4}{\beta} \int \frac{d^3 p}{(2\pi)^3} \left[\ln(1 + e^{-\beta[\omega - (\mu + \mu')]}) + \ln(1 + e^{-\beta[\omega + (\mu + \mu')]}) \right]$$

$$\omega = \sqrt{\vec{p}^2 + M^2} \quad (4.5)$$

and the equation of "electric" neutrality for the statistical boson-fermion system can be written as

$$\frac{\partial \Omega_B^{(0)}}{\partial \mu} + \frac{\partial \Omega_F^{(1)}}{\partial \mu} = 0 \quad (4.6)$$

where $\Omega_B^{(0)}$ is the thermodynamic potential in the tree-approximation for the boson sector while $\Omega_F^{(1)}$ is the fermionic one-loop approximation (4.5)¹. If we now take the zero temperature limit of equation (4.6) we obtain provided that $\mu' > 0$; $|\mu'| > |\mu|$

$$\lim_{\beta \rightarrow \infty} \left[\frac{\partial \Omega_B^{(0)}}{\partial \mu} + \frac{\partial \Omega_F^{(1)}}{\partial \mu} \right] = \mu \xi^2 + \frac{1}{3\pi^2} [(\mu + \mu')^2 - M^2]^{3/2} = 0 \quad (4.7)$$

and the charged B-E condensed solution ($\xi \neq 0$, $\mu \neq 0$) is guaranteed if the following relations between the chemical potentials: $\mu < 0$, and $|\mu + \mu'| > M$, are fulfilled.

It is interesting to note that in this model the leptonic charge coincides with the fermionic "electric" charge. Hence, to introduce the leptonic chemical potential μ' is the same as to consider the conservation of bosonic and fermionic electric charges separately and to introduce a different chemical potential to each one, in the same way as we did for the system with two boson fields of Sec.2.

¹ We exclude the one-loop term for the bosons in (4.6) considering a very high density of fermions.

4.2. NON-ABELIAN MODEL

In Sec.3.2 we found that, contrary to the Abelian case, the charged B-E-c for a non-Abelian many-particle theory of bosons is not realizable without external charge at least at the classical level.

Nevertheless, the desirable fact to have two chemical potentials in a non-Abelian system can be obtained by the inclusion of fermions, because this field has an extra symmetry which gives rise to the leptonic charge. Thus we are going to consider a non-Abelian system formed by the boson sector of the Weinberg-Salam model complemented with fermion fields whose masses are given at hand, and then we are going to search for the possibility to obtain a charged B-E condensate in this case.

We start from the Lagrangian

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^2 + |D_\mu \varphi|^2 + V(\varphi) + \\ & + \bar{\psi} \gamma_\mu (i \partial_\mu + g/2 \tau^a W_\mu^a - g'/2 B_\mu) \psi - M \bar{\psi} \psi \end{aligned} \quad (4.8)$$

where

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu = i \partial_\mu + g/2 \tau^a W_\mu^a + g'/2 B_\mu$$

$$V(\varphi) = m^2 \varphi^2 - \lambda \varphi^4$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \varphi = \begin{pmatrix} \frac{\varphi_1 + i \varphi_2}{\sqrt{2}} \\ \frac{\varphi_3 + i \varphi_4}{\sqrt{2}} \end{pmatrix}$$

The conserved particle densities are given by the Noether theorem to be

$$N^a = -\epsilon^{abc} W_\mu^b F^{\mu c} - \frac{i}{2} [(D^0 \varphi)^\dagger \tau^a \varphi - \varphi^\dagger \tau^a (D^0 \varphi)] + \quad (4.9a)$$

$$+ \frac{1}{2} \bar{\psi} \gamma^0 \tau^a \psi$$

$$\eta^0 = -\frac{i}{2} [(D^0 \varphi)^\dagger \varphi - \varphi^\dagger (D^0 \varphi)] - \frac{1}{2} \bar{\psi} \gamma^0 \psi \quad (4.9b)$$

$$\mathcal{Y}^0 = \bar{\psi} \gamma^0 \psi \quad (4.10)$$

where the following infinitesimal global gauge transformations have been considered

$$\delta \omega_\alpha^a = g \epsilon^{abc} W_\alpha^b \omega^c \quad (4.11a)$$

$$\delta \varphi = \frac{i}{2} [g \tau^a \omega^a + g' \omega^0] \varphi \quad (4.11b)$$

$$\delta \psi = \frac{i}{2} [g \tau^a \omega^a - g' \omega^0] \psi \quad (4.11c)$$

ω^a , $a = 1, 2, 3$ and ω^0 denote the independent infinitesimal parameters of a transformation.

Now, let us introduce the many-particle Hamiltonian density with two chemical potentials

$$\mathcal{H}(\mu, \mu') = \mathcal{H} - \mu (N^0 - \eta^0) - \mu' \mathcal{Y}^0 \quad (4.12)$$

Working in a similar way as in the above cases, and taking into account the asymptotical condition (2.11), we arrive at the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{4} \tilde{F}_{\mu\nu}^a \tilde{F}^{\mu\nu a} - \frac{1}{4} B_{\mu\nu}^2 + V(\varphi) + |(i\partial_\mu + \frac{g}{2} \tau^a \tilde{W}_\mu^a + \frac{g'}{2} \tilde{B}_\mu) \varphi|^2 \\ & + \bar{\psi} \gamma^\mu (i\partial_\mu + \frac{g}{2} \tau^a \tilde{W}_\mu^a - \frac{g'}{2} \tilde{B}_\mu - \mu' \delta_{\mu 0}) \psi - M \bar{\psi} \psi \end{aligned} \quad (4.13)$$

where

$$\tilde{W}_\nu^a = W_\nu^a - \frac{\mu}{g} \delta^{a3} \delta_{\nu 0} \quad (4.14a)$$

$$\tilde{B}_\nu = B_\nu + \frac{\mu}{g} \delta_{\nu 0} \quad (4.14b)$$

and $\tilde{F}_{\mu\nu}$, $B_{\mu\nu}$ have the same definitions as in (4.8) but now expressed in terms of $\tilde{W}_\nu^a$ and $\tilde{B}_\nu$.

From this effective Lagrangian we can obtain the thermodynamic potential $\Omega(\mu, \mu', \beta)$ whose fermionic statistical part in the one-loop approximation is given by

$$\begin{aligned} \Omega_F^{(1)} = & -\frac{2}{\beta} \int \frac{d^3 p}{(2\pi)^3} \left[\ln(1 + e^{-\beta(\omega - \mu')}) + \ln(1 + e^{-\beta(\omega + \mu')}) \right. \\ & \left. + \ln(1 + e^{-\beta[\omega - (\mu + \mu')]}) + \ln(1 + e^{-\beta[\omega + (\mu + \mu')]}) \right] \\ \omega = & \sqrt{\vec{p}^2 + M^2} \end{aligned} \quad (4.15)$$

and the equation of neutrality for this statistical system, in a similar way as in the Abelian case (eq.(4.6)), can be written as

$$\frac{\partial \Omega_B^{(0)}}{\partial \mu} + \frac{\partial \Omega_F^{(1)}}{\partial \mu} = 0 \quad (4.16)$$

where $\Omega_B^{(0)}$ is the thermodynamic potential in the tree approximation for the boson sector, while $\Omega_F^{(1)}$ is the one-loop approximation (4.15) for the fermion sector¹. Thus, from (4.15) and (4.16) we obtain

$$\mu \xi^2 + 2 \int \frac{d^3 p}{(2\pi)^3} \left[\frac{1}{1 + e^{\beta[\omega - (\mu + \mu')]} } - \frac{1}{1 + e^{\beta[\omega + (\mu + \mu')]} } \right] = 0 \quad (4.17)$$

which in the zero temperature limit transforms into

¹ Here we take a reservation similar to the one taken in (4.6).

$$\lim_{\beta \rightarrow \infty} \left[\frac{\partial \Omega_B^{(0)}}{\partial \mu} + \frac{\partial \Omega_F^{(1)}}{\partial \mu} \right] = \mu \xi^2 + \frac{1}{6\pi^2} [(\mu + \mu')^2 - M^2]^{\frac{3}{2}} = 0 \quad (4.18)$$

where a charged B-E condensed solution ($\mu \neq 0$, $\xi \neq 0$) is allowed if the relation $\mu < 0$, $\mu' > 0$, $|\mu| < |\mu'|$, $|\mu' + \mu| > M$ is fulfilled. Then, it is clear from these conditions and the relations (4.17) and (4.18) that only through the introduction of the leptonic chemical potential $\mu' \neq 0$, the charged B-E-c is reached in this model; this is equivalent to the well known fact that an increase of an external lepton charge leads to an increase of symmetry breaking /13/ with the chemical potential μ playing here the role of the average values of the gauge fields $\langle W_4^3 \rangle$ and $\langle B_4 \rangle$ in such a case through the relations $\langle W_4^3 \rangle = -i \frac{\mu}{g}$, $\langle B_4 \rangle = i \frac{\mu'}{g'}$. But it is interesting to note that contrary to the Abelian case, the leptonic chemical potential μ' cannot be associated in this case to the fermionic component of the $N_0^3 - n_0$ charge, as it is obvious from (4.9) and (4.10). This fact is strictly related with the non-separate conservation of charge for models with charged gauge fields as it is underlined in Sec.3.2.

5. THE EXTERNAL CHARGE AS A DYNAMICAL LIMIT IN MODELS WITH FERMIONS

In this section we are going to investigate the possibility to obtain an external charge starting from a statistical system of particles in interaction, for which the separation of charge is allowed, and taking the limit $M \rightarrow \infty$ for the mass of one sort of them. In the previous sections we described examples of charge separation. The example of Sec.4 that includes separation of charge between bosons and fermions serves universally

in Abelian and non-Abelian cases. For this reason we shall try to use fermions for preparing the external charge.

5.1. ABELIAN CASE

We start with the Abelian model of interaction between bosons and fermions of Sec.4.1.

From equation (4.7) we see that if we can obtain the limiting situation under $M \rightarrow \infty$, in which the fermionic contribution $\frac{1}{3\pi^2} [(\mu + \mu')^2 - M^2]^{3/2}$ can transform into a constant density C , then it is reasonable to hope that the effect of the fermions can be completely reduced to a complementary constant term in equation (2.28) to be interpreted as an external charge.

Taking into account that the chemical potential μ is related with both the bosons and fermions, the only parameter which can grow with the mass M without affecting the boson sector is the leptonic chemical potential μ' , hence the limit

$$\mu' \xrightarrow{M \rightarrow \infty} \mu + M + \frac{C}{2M} \quad (5.1)$$

gives the desirable result.

Now that we have the prescription needed to obtain a constant density of heavy fermions in the zero temperature limit (4.18), we can try to attain, in a formal way, the external charge starting from the partition function of the complete system, and applying such a prescription to the fermion contribution.

Now, the functional contribution of the fermions to the partition function can be expanded in power series of the gauge field near the zero value. This series contains only one-loop

diagrams as indicated in Fig.2.

The first term, the thermodynamic potential diagram at $A_p=0$, does not depend on the field and can be absorbed in the normalizing factor of the partition function, then we are going to begin with the analysis of the limiting behaviour of the second diagram (the one-loop tadpole) whose analytical expression is given by

$$V_{\text{tad}} = \text{Tr} \int d^4x \delta_p G_c(0) A_p(z) \quad (5.2)$$

with

$$G_c(0) = \frac{i}{\beta} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{i p_\mu \gamma_\mu - (\mu + \mu') \gamma_4 - M} \quad (5.3)$$

After summing over $p_4 = (2n+1)\frac{\pi}{\beta}$, $n=0, \pm 1, \pm 2, \dots$ we obtain for

$$V_{\text{tad}} = 2e \int d^4x \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{1 + e^{\beta[E - (\mu + \mu')]}]} - \frac{1}{1 + e^{\beta[E + (\mu + \mu')]}]} \right] A_4(z) \quad (5.4)$$

$$E^2 = \vec{p}^2 + M^2$$

which in the zero temperature limit reduces to

$$\lim_{\beta \rightarrow \infty} V_{\text{tad}} = e \int d^4x \left\{ \frac{2}{3\pi^2} [(\mu + \mu')^2 - M^2]^{3/2} \right\} A_4(z) \quad (5.5)$$

Finally, if we apply the prescription (5.1) we obtain

$$\lim_{\beta \rightarrow \infty} V_{\text{tad}} = e \int d^4x \left(\frac{2}{3\pi^2} C^{3/2} \right) A_4(z) = \int d^4x J_1 A_4(z) \quad (5.6)$$

$\mu' \rightarrow M \rightarrow \infty$

which after the identification $J_1 = e \frac{2}{3\pi^2} C^{3/2}$ is the interaction term between the gauge field and the external charge that usually appears in the Lagrangian /2,7/.

The next coefficient in the expansion of the thermodynamical potential corresponds to the one-loop polarization operator which in the zero temperature limit has the renormalized expression (the divergent part of the polarization operator is absorbed by the renormalization of the $(F_{\mu\nu} F^{\mu\nu})$ -term of the Lagrangian (2.1))

$$\Pi_{\mu\nu}(K) = (\Pi_{\mu\nu}^{(0)}(K))^R + \Pi_{\mu\nu}^{(c)}(K) \quad (5.7)$$

where

$$(\Pi_{\mu\nu}^{(0)})^R = \left(\delta_{\mu\nu} - \frac{K_\mu K_\nu}{K^2} \right) \Pi^R(K) \quad (5.8)$$

$$\Pi^R(K) = \frac{e^2}{12\pi^2} (K^2 + K_4^2)^2 \int_0^\infty \frac{dz^2 (1 + 2 \frac{M^2}{z^2}) (1 - \frac{4M^2}{z^2})^{1/2}}{4M^2 z^2 (z^2 + K^2 + K_4^2)} \quad (5.9)$$

$$K_4 = 2\pi\bar{\eta}/\beta$$

And /14/

$$J_{i4}^{(c)}(K) = \left(\delta_{ij} - \frac{K_i K_j}{K^2} \right) A + \frac{K_i K_j}{K^2} \frac{K_4^2}{K^2} J_{44}^{(c)} \quad (5.10a)$$

$$J_{i4}^{(c)}(K) = J_{4i}^{(c)} = - \frac{K_4 K_i}{K^2} J_{44}^{(c)} \quad (5.10b)$$

$$J_{44}^{(c)}(K) = - \frac{e^2}{\pi^2} \int_0^1 \frac{p^2 dp}{\mathcal{E}_p} \left\{ 1 + \frac{K^2 + K_4^2 - 4\mathcal{E}_p^2}{8pK} \ln a - \frac{ik_4 \mathcal{E}_p}{2pK} \ln b \right\} \quad (5.10c)$$

$$A(K) = - \frac{e^2}{2\pi^2} \int_0^1 \frac{p^2 dp}{\mathcal{E}_p} \left\{ 1 - \frac{K_4^2}{K^2} + \frac{-K_4^2 + K^2 + 4\mathcal{E}_p^2 K_4^2 + 4p^2 K^2}{8pK^3} \ln a + \right. \\ \left. + i \frac{K_4 \mathcal{E}_p}{2pK^3} (K^2 + K_4^2) \ln b \right\} \quad (5.11)$$

with

$$Q = \frac{(K^2 - 2pK + K_4^2)^2 + 4K_4^2 \mathcal{E}_p^2}{(K^2 + 2pK + K_4^2)^2 + 4K_4^2 \mathcal{E}_p^2} \quad (5.12a)$$

$$b = \frac{(K^2 + K_4^2)^2 - 4(p \cdot K + i K_4 \epsilon_p)^2}{(K^2 + K_4^2)^2 - 4(p \cdot K - i K_4 \epsilon_p)^2} \quad (5.12b)$$

here we have used $\epsilon_p^2 = \vec{p}^2 + M^2$, $K^2 = \vec{K}^2$, $\bar{\mu} = (\mu + \mu')$

In the limit of very heavy fermions (5.1) we have

$$a \rightarrow 1, \quad b \rightarrow 1, \quad [\bar{\mu}^2 - M^2]^{1/2} \rightarrow C^{1/2} \quad (5.13)$$

hence

$$\pi_{44}^{(c)} \rightarrow A \rightarrow 0 \quad (5.14a)$$

$$\pi^R(K) \rightarrow \frac{1}{M^2} \frac{e^2}{12\pi^2} (K^2 + K_4^2)^2 \int_0^\infty \frac{d\xi}{\xi^2} (1 + \frac{3}{\xi})(1 - \frac{4}{\xi})^{1/2} \rightarrow 0 \quad (5.14b)$$

Therefore this term does not contribute.

The fourth term is the photon splitting vertex $\overline{\pi}_{\mu_1 \mu_2 \mu_3}$ in the one-loop approximation. It has /15/ the following Lorentz-covariant tensor decomposition (in any inertial frame)

$$\overline{\pi}_{\mu_1 \mu_2 \mu_3}(K^{(1)}, K^{(2)}, K^{(3)}) = \delta^{(4)}(K^{(1)} + K^{(2)} + K^{(3)}) \cdot \quad (5.15)$$

$$\cdot \sum_{p, q, s=1,2} f_{\mu_1}^{(p,1)} f_{\mu_2}^{(q,2)} f_{\mu_3}^{(s,3)} \pi_{(p,q,s)} + \sum_{q=1,2} \sum_{i,j,K=1} d_{\mu_1}^{(i)} d_{\mu_2}^{(j)} f_{\mu_3 K}^{(q,K)} \pi_q(K)$$

where $f_{\mu_K}^{(1,K)} = a_{\mu_K}^{(K)}$, $f_{\mu_K}^{(2,K)} = b_{\mu_K}^{(K)}$

$$a_{\mu}^{(1)} = (u K^{(1)}) K_{\mu}^{(1)} - (K^{(1)})^2 u_{\mu} \quad (5.16a)$$

$$b_{\mu}^{(1)} = \{ (u K^{(2)}) (K^{(1)})^2 - (u K^{(1)}) (K^{(1)} K^{(2)}) \} u_{\mu} + \{ K^{(1)} K^{(2)} - (u K^{(1)}) (u K^{(2)}) \} K_{\mu}^{(1)} + \{ (u K^{(1)})^2 - (K^{(1)})^2 \} K_{\mu}^{(2)} \quad (5.16b)$$

$$d_{\mu}^{(1)} = - \epsilon_{\mu\nu\lambda\rho} K_{\nu}^{(2)} u_{\lambda} K_{\rho}^{(1)} \quad (5.16c)$$

u_μ is the four-velocity of the medium and $\epsilon_{\mu\nu\lambda\rho}$ the completely antisymmetric unit tensor. The other basic vectors $a_\mu^{(2)}$, $b_\mu^{(2)}$, $d_\mu^{(2)}$ are obtained by the interchange $K_\mu^{(1)} \leftrightarrow K_\mu^{(2)}$ and $a_\mu^{(3)}$, $b_\mu^{(3)}$, $d_\mu^{(3)}$ by the substitution $K_\mu^{(1)} \rightarrow K_\mu^{(3)}$ in (5.16).

The scalar coefficients $\pi_{(p,q,s)}$ and $\pi_q^{(\kappa)}$ from (5.15) are given as

$$\pi_{(p,q,s)} = \pi_{\mu_1\mu_2\mu_3} f_{\mu_1}^{(p,1)} f_{\mu_2}^{(q,2)} f_{\mu_3}^{(s,3)} / (f^{(p,1)})^2 (f^{(q,2)})^2 (f^{(s,3)})^2 \quad (5.17a)$$

$$p, q, s = 1, 2$$

$$\pi_q^{(\kappa)} = \frac{\pi_{\mu_1\mu_2\mu_3} f_{\mu_\kappa}^{(q,\kappa)} d_{\mu_i}^{(i)} d_{\mu_j}^{(j)}}{(f^{(q,\kappa)})^2 (d^{(i)})^2 (d^{(j)})^2} \quad \begin{matrix} i \neq \kappa, j > i, j \neq \kappa \\ i, j, \kappa = 1, 2, 3 \end{matrix} \quad (5.17b)$$

The four-dimensional transversality conditions,

$$K_{\mu_i}^{(i)} \pi_{\mu_1\mu_2\mu_3} = 0, \quad i = 1, 2, 3$$

leave only the spatial components $\pi_{ij\kappa}$ independent. They are enough for the complete set of scalar coefficients (5.17) to be found. The needed components $\pi_{ij\kappa}$ at finite temperature and in the rest frame ($u^2 = 0$, $u_4^2 = -1$) were calculated in /16/, and in the limit of zero temperature they are

$$\pi_{ij\kappa} = \frac{2e^3}{(2\pi)^3} \int_0^{\sqrt{\mu^2 + M^2}} d\vec{p} \operatorname{Re} \left\{ H_1 [(\vec{p} \cdot \vec{K}^{(2)} + i\epsilon K_4^{(2)}) \right. \quad (5.18)$$

$$\begin{aligned} & (K_r^{(1)} \delta_j - K_j^{(1)} \delta_r + K_i^{(1)} \delta_j + 2p_i \delta_r) + (\vec{p} \cdot \vec{K}^{(1)} + i\epsilon K_4^{(1)}) \\ & (K_r^{(2)} \delta_j + K_j^{(2)} \delta_r - K_i^{(2)} \delta_j - 2p_i \delta_r) + 4p_i p_r + 2p_j p_r K_i^{(1)} - \\ & - 2p_i p_r K_j^{(2)} + 2(K_r^{(1)} - K_r^{(2)}) p_i p_j + p_r (K_i^{(2)} K_j^{(1)} - K_i^{(1)} K_j^{(2)}) - \end{aligned}$$

$$\begin{aligned}
 & - p_i (K_j^{(1)} K_r^{(2)} + K_r^{(1)} K_j^{(2)}) - p_j (K_i^{(2)} K_r^{(1)} + K_i^{(1)} K_r^{(2)}) - \\
 & - (\vec{K}^{(1)} \vec{K}^{(2)} + K_4^{(1)} K_4^{(2)}) (p_r \delta_{ij} - p_i \delta_{jl} - p_j \delta_{ir}) \} + \\
 & + \int_0^{\sqrt{\vec{\mu}^2 - M^2} - K^{(1)}} d\vec{p} \, R_2 \{ H_2 [(2(\vec{p} \cdot \vec{K}^{(1)} + i \varepsilon_i K_4) + (K^{(1)})^2 - K_4^{(1)} (K_4^{(1)} + K_4^{(2)})) \\
 & + (\delta_{ij} (p_r + K_r^{(1)}) + \delta_{jr} (p_i + K_i^{(1)}) - \delta_{ir} (p_j + K_j^{(1)})) + (\vec{p} \cdot \vec{K}^{(2)} + i \varepsilon_j K_4^{(2)}) \\
 & (K_r^{(1)} \delta_{ij} - K_j^{(1)} \delta_{ir} + K_i^{(1)} \delta_{jr} + 2 p_i \delta_{jr}) - (\vec{p} \cdot \vec{K}^{(1)} + i \varepsilon_i K_4^{(1)}) + \\
 & + (\vec{K}^{(1)})^2) (K_i^{(2)} \delta_{ij} - K_i^{(2)} \delta_{jr} + K_j^{(2)} \delta_{ir} - 2 p_j \delta_{ir}) - \\
 & - 2 p_i p_r K_j^{(2)} - (\vec{K}^{(1)} \cdot \vec{K}^{(2)}) (p_r \delta_{ij} - p_i \delta_{jr} - p_j \delta_{ir}) + \\
 & + 4 p_i p_j p_r + 2 p_i p_j (K_i^{(1)} - K_r^{(2)}) + 2 p_j p_i K_i^{(1)} + \\
 & + p_r (K_i^{(2)} K_j^{(1)} - K_i^{(1)} K_j^{(2)}) - p_i (K_j^{(1)} K_r^{(2)} + K_r^{(1)} K_j^{(2)}) - \\
 & - p_j (K_i^{(2)} K_r^{(1)} + K_i^{(1)} K_r^{(2)}) \} + [K^{(1)} \leftrightarrow - K^{(2)} \quad i \leftrightarrow j]
 \end{aligned}$$

in the last term].

Here

$$\begin{aligned}
 H_1 &= [2 i \varepsilon_i \{ 2 (\vec{p} \cdot \vec{K}^{(1)} + i \varepsilon_i K_4^{(1)}) + (\vec{K}^{(1)})^2 + (K_4^{(1)})^2 \} - 2 (\vec{p} \cdot \vec{K}^{(2)} + \\
 & + i \varepsilon_i K_4^{(2)}) + (\vec{K}^{(2)})^2 + (K_4^{(2)})^2 \}]^{-1} \\
 H_2 &= [2 i \varepsilon_i \{ - 2 (\vec{p} \cdot \vec{K}^{(1)} + i \varepsilon_i K_4^{(1)}) - (\vec{K}^{(1)})^2 + (K_4^{(1)})^2 \} - 2 (\vec{p} \cdot (\vec{K}^{(1)} \cdot \vec{K}^{(2)}) \\
 & + i \varepsilon_i (K_4^{(1)} + K_4^{(2)})) + (\vec{K}^{(2)})^2 \cdot (\vec{K}^{(1)})^2 + (K_4^{(1)} + K_4^{(2)})^2 \}]^{-1} \\
 \varepsilon_0 &= [\vec{p}^2 + M^2]^{1/2}, \quad \varepsilon_i = [(\vec{p} + \vec{K}^{(1)})^2 + M^2]^{1/2}
 \end{aligned}$$

In the limit of very heavy fermions (5.1), taking into account that $\sqrt{\vec{\mu}^2 - M^2} \rightarrow C^{1/2}$, all the integrals in (5.18) continue to be defined within finite limits and the integrands go to zero as $O(M^{-1})$. Therefore, in the required limit all the

scalar coefficients in (5.15) go to zero, and there is no contribution of this diagram into the thermodynamic potential either.

For the rest of the diagrams the following analysis may be carried out. First recall that at zero temperature limit the summation over p_4 is replaced by an integral

$$\beta^{-1} \sum_{p_4} \rightarrow (2\pi)^{-1} \int_{-\infty}^{\infty} d p_4 \quad (5.19)$$

We can propose the following change in the integrand of the Green function (5.3)

$$I(\vec{p}, k) = \frac{1}{(\vec{p}' + \vec{p}_M - \frac{\vec{p}}{M} \delta_4 - 1)M} \quad (5.20)$$

where $\vec{p}' = M^{-1} \vec{p}_M$, and express the n-point polarization operator at zero temperature as¹

$$\lim_{\beta \rightarrow \infty} \Pi_{\mu_1, \dots, \mu_n}(\vec{k}, \vec{p}, M) = M^{4-n} \Pi_{\mu_1, \dots, \mu_n}^{\infty}(\vec{k}/M, \vec{p}/M, 1) \quad (5.21)$$

Here the superscript ∞ denotes that the replacing (5.19) takes place. From (5.21) we have that for $n > 4$ in the limit of very heavy fermions the n-point polarization operator tends to zero faster than M^{4-n} , because $\Pi_{\mu_1, \dots, \mu_n}^{\infty}(\vec{k}/M, \vec{p}/M, 1)$ tends to zero in the limit $\vec{k}/M \rightarrow 0$ owing to the transversality.

The only case left for separate consideration is $n=4$. Let $\Pi_{\mu_1, \mu_2, \mu_3, \mu_4}^{Reg}(\vec{k}, \vec{p}, M)$ be the four-point one-loop polarization operator regularized by the gauge-invariant Pauli-

¹ Similar results are reached at finite temperature in the limit $M \rightarrow \infty$, where an expression analogous to (5.19) is found $\beta^{-1} \sum_{p_4} \rightarrow \frac{M}{2\pi} \int_{-\infty}^{\infty} d p_4$

Villars regularization (with the common regularizer for the Fermion loop, the so-called cycle regularization). This regularization may be removed without any singularity left. To see this let us present here the covariant decomposition of the transverse polarization operator $\Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{Reg}$. This is /15/.

$$\Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{Reg} (K^i, \bar{\mu}, M) \delta^{(4)} \left(\sum_{i=1}^4 K^{(i)} \right) = \sum_{n_1, \dots, n_4=1}^4 C_{\mu_1}^{(n_1)} \dots C_{\mu_4}^{(n_4)} \Pi_{(n_1 \dots n_4)} \delta^{(4)} \left(\sum_{i=1}^4 K^{(i)} \right) \quad (5.22)$$

with $C_{\mu_i}^{(1)} = a_{\mu_i}^{(1)}$, $C_{\mu_i}^{(2)} = b_{\mu_i}^{(1)}$, $C_{\mu_i}^{(3)} = h_{\mu_i}^{(1)}$ and $a_{\mu_i}^{(i)}$, $b_{\mu_i}^{(i)}$, $h_{\mu_i}^{(i)}$ are formed from the transverse Lorentz-vectors

$$a_{\mu}^{(1)} = (\mathcal{U} K^{(1)}) K_{\mu}^{(1)} - (K_{\mu}^{(1)})^2 \mathcal{U}_{\mu} \quad (5.23a)$$

$$\begin{aligned} b_{\mu}^{(1)} = & \mathcal{U}_{\mu} \{ (\mathcal{U} K^{(2)}) (K^{(1)})^2 - (\mathcal{U} K^{(1)}) (K^{(1)} K^{(2)}) \} \\ & + K_{\mu}^{(1)} \{ (K^{(1)} K^{(2)}) - (\mathcal{U} K^{(1)}) (\mathcal{U} K^{(2)}) \} \\ & + K_{\mu}^{(2)} \{ (\mathcal{U} K^{(1)})^2 - (K^{(1)})^2 \} \end{aligned} \quad (5.23b)$$

as follows: $a_{\mu_i}^{(i)}$, $i=1,2,3,4$ is obtained from (5.23a) by the replacement $K^{(1)} \rightarrow K^{(i)}$, $b_{\mu_2}^{(2)}$ is obtained from (5.23b) by the substitution $K^{(1)} \rightarrow K^{(2)}$, $K^{(2)} \rightarrow K^{(1)}$, $b_{\mu_i}^{(i)}$, $i=1,3,4$ is obtained from (5.23b) by the replacement $K^{(1)} \rightarrow K^{(i)}$. And the third set of transverse vectors $h_{\mu_i}^{(i)}$ is given by: $h_{\mu_1}^{(1)}$ is formed from $b_{\mu_1}^{(1)}$ by the substitution $K^{(2)} \rightarrow K^{(3)}$; $h_{\mu_i}^{(i)}$, $i=2,4$, are obtained from $h_{\mu_1}^{(1)}$ by the substitution $K^{(1)} \rightarrow K^{(i)}$; finally, the vector $h_{\mu_3}^{(3)}$ is obtained from $h_{\mu_2}^{(2)}$ by the interchange $K^{(2)} \rightarrow K^{(3)}$, $K^{(3)} \rightarrow K^{(2)}$.

Although at first glance $\Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{Reg} (K^i, \bar{\mu}, 1)$ seems logarithmically divergent, it is as a matter of fact, known to be

strongly convergent. The latter occurs because, as it is evident from the covariant decomposition (5.22), the four-photon diagram is proportional to a factor involving the product of four vectors taken from the set $K_{\mu}^{(1)}$, $K_{\mu}^{(2)}$, $K_{\mu}^{(3)}$, u_{μ} . This permits to conclude that the first four terms of the Taylor expansion in powers of K disappear and the power of the electron momentum in the integrand of the Feynman diagram associated to $\Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{\omega}$ diminishes by four units, thus the superficial divergence reduces from $D=0$ to $D=-4$.

Then, taking into account the transversality condition for the regularized polarization operator (5.22), we have after the change of variables (5.20)

$$K_{\mu_i}^{(i)} \Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{Reg} \left(\frac{K^i}{\bar{M}}, \frac{\bar{u}}{\bar{M}}, 1 \right) = 0, \quad i=1,2,3,4 \quad (5.24)$$

Therefore, in the limit (5.1) it is obtained that

$$K_{\mu_i}^{(i)} \Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{Reg} (0, 1, 1) = 0 \quad (5.25)$$

Hence

$$\Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{Reg} (0, 1, 1) = 0 \quad (5.26)$$

because in this limit the four-vertex polarization operator

$\Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{Reg} (0, 1, 1) = 0$ does not contain the $K_{\mu_i}^{(i)}$ vectors required in order to construct the transvers structure (5.22). The removal of the regularization states this result already for $\Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^{\omega}$.

Note, that by handling the zero temperature statistical part $\Pi_{\mu\nu}^{(c)}(K, M, \mu)$ of the two-point polarization operator in the same manner as above we would obtain from the transvers-

salinity

$$K_{\mu} \pi_{\mu\nu}^{(c)}(K, M, \mu) = 0 \quad (5.27)$$

after the substitution (5.20)

$$M^2 K_{\mu} \pi_{\mu\nu}^{(c)}\left(\frac{K}{M}, 1, \frac{\mu}{M}\right) = 0 \quad (5.28)$$

that in the limit (5.1) the tensor $\pi_{\mu\nu}^{(c)}\left(\frac{K}{M}, 1, \frac{\mu}{M}\right)$ goes to zero faster than $1/M^2$. Therefore,

$$\pi_{\mu\nu}^{(c)}(0, 1, 1) = 0 \quad (5.29)$$

This is not in contradiction with the well-known fact /14/ that $\pi_{\mu\nu}^{(c)}$ does not disappear in the zero-momentum limit $K_{\mu} = 0$, $|\vec{K}| \rightarrow 0$, responsible for the Debye screening

$$\pi_{\mu\nu}^{(c)}(0, M, \mu) \neq 0 \quad (5.30)$$

What disappears, is $\pi_{\mu\nu}^{(c)}(0, M, M)$, in agreement with (5.29). This is confirmed by eqs.(5.10c), (5.11).

Hence, as it was hoped, in the given limit the contribution of the fermions to the partition function enters only through the term (5.6) interpreted as the interaction with an external source.

5.2. NON-ABELIAN CASE

Now it is interesting to see for the non-Abelian model developed in Sec.4.2 what is the behaviour of the fermionic contribution to the partition function in the limit of very heavy fermions, where the prescription $(\mu + \mu')^2 \rightarrow M^2 + (6\pi^2 J_0)^{2/3}$ must be taken at the same time in order to guarantee that the distribution of fermions in the Fermi sphere be different from zero.

In this limit equation (4.18) takes the form

$$\lim_{\substack{\beta \rightarrow \infty \\ M \rightarrow \infty}} \left[\frac{\partial \Omega_{13}^{(0)}}{\partial \mu} + \frac{\partial \Omega^{(1)}}{\partial \mu} \right] = \mu \bar{\xi}^2 + J_0 = 0 \quad (5.31)$$

and then it seems natural that in this limit the role of the fermions can be reduced to the appearing of an external charge in the same way as in the Abelian case studied in the previous section.

Therefore, in order to corroborate this assumption we are going to follow the same method that was used above in the Abelian case. This is, to work with the contribution of the fermions to the partition function of the system

$$\int e^{\int d^4x \bar{\psi} [i\gamma^\mu \tilde{D}_\mu - M] \psi} \quad d\bar{\psi} d\psi = \exp \left\{ \int d^4x \ln [i\gamma^\mu \tilde{D}_\mu - M] \right\} \quad (5.32)$$

$$\tilde{D}_\mu = i\partial_\mu + g(W_\mu^a - \frac{\mu}{g} S^{a3} \delta_{\mu 0}) \frac{\tau^a}{2} - g'(B_\mu + \frac{\mu}{g'} \delta_{\mu 0}) \frac{I}{2} - \mu' \delta_{\mu 0}$$

and to expand the functional logarithm in powers of the gauge fields W_μ^a and B_μ around their zero values, that is

$$\begin{aligned} \ln [i\gamma^\mu \tilde{D}_\mu - M] &= \ln \left[i\gamma^\nu (i\partial_\nu - \mu \frac{\tau^3 I}{2} \delta_{\nu 0} - \mu' \delta_{\nu 0}) - M \right] + \\ &+ ig \int d^4y \frac{\tau^a}{2} \frac{\delta(x-y) d^4y}{i\gamma^\nu (i\partial_\nu - \mu \frac{\tau^3 I}{2} \delta_{\nu 0} - \mu' \delta_{\nu 0}) - M} W_\mu^a(y) \\ &- ig' \int d^4y \frac{I}{2} \frac{\delta(x-y) d^4y}{i\gamma^\nu (i\partial_\nu - \mu \frac{\tau^3 I}{2} \delta_{\nu 0} - \mu' \delta_{\nu 0}) - M} B_\mu(y) \\ &+ \dots \end{aligned} \quad (5.33)$$

Because the first term in these series does not depend on the

fields and will be only a renormalizing factor, we are going to focus our attention in the first-order terms.

Then the prescription henceforth is to obtain the zero temperature limit for these terms and to analyse their very heavy fermion limit.

Taking into account the functional trace the first term that we must calculate is

$$Tr \int d^4x \frac{\tau^6}{2} \delta^f G_0(z) W_f^b(z) \quad (5.34)$$

where

$$G_0 = \frac{1}{\beta} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{i p_0 \gamma_0 - \mu \frac{\tau^3 + I}{2} \gamma_4 - \mu' \gamma_4 - M} \quad (5.35)$$

After the summation over p_4 , following the same method as before, and after the matrix trace operation we arrive to

$$\begin{aligned} Tr \int d^4x \frac{\tau^6}{2} \delta^f G_0 W_f^b(z) &= \int d^4x \int \frac{d^3p}{(2\pi)^3} \cdot W_4^3(z) \\ &\left[\frac{1}{1 + e^{\beta[E - (\mu + \mu')]} } - \frac{1}{1 + e^{\beta[E + (\mu + \mu')]} } \right. \\ &\left. - \frac{1}{1 + e^{\beta[E - \mu']]} } + \frac{1}{1 + e^{\beta[E + \mu']]} } \right] \quad (5.36) \end{aligned}$$

which in the zero temperature limit turns into

$$\begin{aligned} \lim_{\beta \rightarrow \infty} \left\{ Tr \int d^4x \frac{\tau^6}{2} \delta^f G_0 W_f^b(z) \right\} &= \\ &= \frac{1}{6\pi^2} \int d^4x \left\{ [(\mu + \mu')^2 - M^2]^{3/2} [\mu'^2 - M^2]^{3/2} \right\} W_4^3(z) \end{aligned} \quad (5.37)$$

In the same way for the second first-order term

we obtain

$$\int d^4z \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{1 + e^{\beta[\varepsilon - (\mu + \mu')]} } - \frac{1}{1 + e^{\beta[\varepsilon + (\mu + \mu')]} } \right. \\ \left. + \frac{1}{1 + e^{\beta[\varepsilon - \mu']}} - \frac{1}{1 + e^{\beta[\varepsilon + \mu']}} \right] \quad (5.39)$$

Then the zero temperature limit of the latter expression is given by

$$\lim_{\beta \rightarrow \infty} \left\{ T_F \int d^4z \frac{I}{2} \gamma_F G_0 B_f(z) \right\} = \\ = \frac{1}{4\pi^2} \int d^4z \left\{ [(\mu + \mu')^2 - M^2]^{3/2} + [\mu'^2 - M^2]^{3/2} \right\} B_4(z) \quad (5.40)$$

With (5.37) and (5.40) the first-order terms in (5.33) for the zero temperature limit can be arranged in the following way

$$\frac{1}{4\pi^2} \int d^4z \left\{ [\mu'^2 - M^2]^{3/2} [g W_4^3(z) + g' B_4(z)] - \right. \\ \left. - [(\mu + \mu')^2 - M^2]^{3/2} [g W_4^3(z) - g' B_4(z)] \right\} \quad (5.41)$$

Taking the limit of very heavy fermions used in (5.31) the coefficient of the field $[g W_4^3 - g' B_4]$ corresponding to the non-zero conserved charge behaves like an external charge in correspondence with the result (5.31), but at the same time the coefficient of the other combination $[g W_4^3 + g' B_4]$ goes to infinity because in this limit we have $(\mu'^2 - M^2) \rightarrow 1/\mu M$

Therefore, although the charged B-E-c is obtained for this model owing to the inclusion of fermions, a static situation where the role of the fermions would be reduced to an interacting constant external charge cannot be reached, because as we saw some pathology appears in the system.

The origin of this pathology is related with the fact that the number of charged fermions is not conserved independently,

but only the complete number of fermions is, as it is quoted by the equation

$$\frac{\partial \Omega_F}{\partial \mu'} = \text{const} \quad (5.42)$$

which in the one-loop approximation and zero temperature limit can be obtained from (4.15)

$$\lim_{\beta \rightarrow \infty} \frac{\partial \Omega_F^{(1)}}{\partial \mu'} = \frac{1}{6\pi^2} \left\{ [(\mu + \mu')^2 - M^2]^{3/2} + [\mu'^2 - M^2]^{3/2} \right\} = \text{const.} \quad (5.43)$$

Once we want to keep the number of charged particles

$\frac{1}{6\pi^2} [(\mu + \mu')^2 - M^2]^{3/2}$ equal to a constant external parameter following the prescription used in (5.31) the equation of conservation of the number of fermions (5.43) cannot be fulfilled.

Finally, from (5.41) we can corroborate that only for $\mu = 0$ the sought external charge in the limit of very heavy fermions $(\mu')^2 \rightarrow M^2 + (3\pi^2 J_0)^{2/3}$ can be reached, but then this external charge only is going to affect the Abelian sector of the analyzed model, i.e. it will appear as an interaction term $J_0 B_0$ in the Lagrangian, while the contribution of the non-Abelian field W_μ^a to (5.41) is cancelled. Therefore, this result does not differ from the one analysed in Sec.5.1. In general the same result can be obtained by introducing only a chemical potential μ to the charge (4.96) in (4.12), so that the many-particle Hamiltonian density be

$$\mathcal{H}(\mu, \mu') = \mathcal{H} - \mu \eta^0 - \mu' \mathcal{Y}^0 \quad (5.44)$$

Then (5.34) would be zero, and as can be noted from (5.33) in this case the role of μ in (5.38) is to rescale the leptonic chemical potential μ' .

The more general case where two independent chemical potentials μ_1 and μ_2 are introduced in the partition function through the many-particle Hamiltonian density

$$\mathcal{H}(\mu_1, \mu_2, \mu') = \mathcal{H} - \mu_1 N_0^3 - \mu_2 \eta_0 - \mu' \frac{1}{2} \quad (5.45)$$

gives the same result in relation with the impossibility to obtain the constant external charge.

6. YANG-MILLS-HIGGS MANY-PARTICLE THEORY WITH EXTERNAL CHARGE

Consider the tree approximation of the thermodynamic potential associated to the effective Lagrangian (3.5)

$$\Omega^{(0)} = -V \mathcal{L}_{eff}^{(0)} = \frac{1}{2} \bar{\xi}^t [-\mu^2 (\bar{\chi}^3)^t \bar{\chi}^3 - m^2 + \frac{1}{2} \xi^2] \xi \quad (6.1)$$

One can show that the state $\bar{\xi}^t = (\bar{\xi}_1, \bar{\xi}_2, 0)$ is energetically more favourable than $\bar{\xi}^t = (0, 0, \bar{\xi}_3)$. Then, introducing an external static source $J_\mu^a = J_0 \delta^{a3} \delta_{\mu 0}$ in (3.6)

$$\frac{\delta \mathcal{L}_{eff}}{\delta A_\rho^b} = \tilde{V}_\rho^{bc} \tilde{F}_0^{\rho\nu} = \frac{g}{2} [\phi^t \bar{\chi}^b (\bar{D}^\nu \phi) - (\bar{D}^\nu \phi) \bar{\chi}^b \phi] = \quad (6.2)$$

$$= g J_0 \delta^{\nu 0} \delta^{b3}$$

we arrive to the charged solution

$$\bar{\xi}_3 = 0, \quad \bar{\xi}_1 = \bar{\xi}_2 = \bar{\xi}, \quad \bar{\xi} = \sqrt{\frac{m^2 + \mu^2}{2\lambda}} \quad (6.3)$$

In this context the charged B-E-c has been considered for non-Abelian models /2,4/. Nevertheless, as we have shown in the previous section, such an external charge is not associated in non-Abelian models with a limiting stage of the fermionic contribution of a dynamical system, which is able to guarantee the

charged B-E-c before the limiting process. This becomes the reason for what we are going to show now, namely, that the theory that includes this external charge is drastically different from the conventional gauge theory, because the external charge should be not only taken into account in relation with the change of the stationary solution, but also must be considered in relation with the quantization procedure which is really modified in the cases of non-Abelian models with external source /8/.

As has been pointed out by Kiskis /9/, the pure Yang-Mills field Lagrangian reduces its symmetry when a term of interaction with external charge is added. The reduction of the symmetry takes place not only in the pure Yang-Mills system, but in any non-Abelian gauge model with external charge and, of course it has to be considered in order to quantize the theory. Then the quantization of systems with external charge should start with the Hamiltonian formulation of the theory, including the modification produced by the charge, in the spirit of Ref./8/.

Henceforth, differently from Kiskis /9/, who proposed a Hamiltonian expressed in terms of unconstrained pairs of canonical variables as the first step towards the quantization of the theory, we shall consider the classical Hamiltonian of Yang-Mills-Higgs fields with constant external charge as a function of all the phase space variables, which are subjected to constraints of first and second class, following in this sense the generalized Hamiltonian method of Dirac /11/. Thanks to this way to attack the problem we can use, in a straightforward form the results of Fradkin et al. /17/ to quantize the theory by the path integral method, taking into account both the first- and second-class constraints. This method has the advantage that it permits

in a realizable way the quantization of the system, and also presents the theory in a covariant and gauge invariant form when there is an external source which does not break completely the gauge symmetry of the system.

Let us start with the Hamiltonian density of the Yang-Mills-Higgs system¹ in presence of an Abelian static source

$$\mathcal{H} = \frac{1}{2} (\vec{\pi}^a \cdot \vec{\pi}^a + \vec{B}^a \cdot \vec{B}^a) + \frac{1}{2} \rho^2 + \frac{1}{2} (D_i \phi)^2 - \frac{m^2}{2} \phi^2 + \frac{1}{4} \phi^4 - A_0^a \delta^a + g \int_0 A_0^a \delta^a z \quad (6.4)$$

where the conjugate momenta are defined in the known way (as in Sec.3.1) and

$$B_i^a = \frac{1}{2} \epsilon_{ijk} F_{jk}^a \quad (6.5a)$$

$$\delta^a = \vec{\partial} \cdot \vec{\pi}^a - g \epsilon_{abc} \vec{A}^b \cdot \vec{\pi}^c - g \rho^2 \phi \quad (6.5b)$$

Considering the consistency condition for the constraints which uses the total Hamiltonian $H_T = H + \mathcal{U}^a \pi_0^a$ where $\mathcal{U}^a$ are the Lagrange multipliers and $\pi_0^a \approx 0$ are the primary constraints of the theory we obtain

$$\dot{\pi}_0^a = \{ \pi_0^a, H_T \} = \tilde{\delta}^a \approx 0 \quad (6.6)$$

$$\tilde{\delta}^a = \vec{\partial} \cdot \vec{\pi}^a - g \epsilon_{abc} \vec{A}^b \cdot \vec{\pi}^c - g \rho^2 \phi - g \int_0 \delta^a z \quad (6.7)$$

Now

$$\dot{\tilde{\delta}}^a = \{ \tilde{\delta}^a, H_T \} = g \epsilon_{abc} A_c^b \delta^c \quad (6.8)$$

which taking into account (6.7) and (6.5b) transforms into

¹ Results analogous to the ones that will be obtained here hold for SU(2)xU(1) models /18/.

$$\chi^b = \epsilon_{3bc} A_0^c J_0 \simeq 0 \quad (6.9)$$

and finally

$$\dot{\chi}^b = \{ \chi^b, H_T \} = 0 \quad (6.10)$$

Taking all the Poisson brackets between the constraints we can classify them in the pattern given in table II.

Thus, considering the number of gauge generators that we have obtained, the symmetry group of the theory is now $U(1)$ as a subgroup of $SU(2)$, instead of the original $SU(2)$ for $J_0 = 0$. Therefore in the quantification procedure we have to consider, first, the modifications in the functional integral owing to the existence of second-class constraints /17/, and second, once we have discarded the primary constraints π_c^3 , as it is customary, by the gauge condition $A_0^3 = 0$, the remainder gauge freedom must be eliminated by imposing only one condition $F = 0$. The situation is well different from the one with $J_0 = 0$, where one has to fix three gauge conditions $F^a = 0$ ($a=1,2,3$) as we can see in (3.13).

The partition function of such a kind of system must be written in the following form

$$\mathcal{Z} = \int e^{iS} \sqrt{\det \{ \psi_n, \bar{\psi}_m \}} \delta(\theta) [d\varphi] [dp] \quad (6.11)$$

where $\{ \psi_n, \bar{\psi}_m \}$ represents the Poisson brackets among all the constraints, including the gauge conditions, and $\delta(\theta)$ are the Dirac's δ 's of each constraint and gauge condition.

The determinant in (6.11) takes the form

$$\det \{ \psi_n, \bar{\psi}_m \} = \det \{ \eta^\alpha, \eta^\beta \} \det^2 \{ F^a, \delta^a \} \quad (6.12)$$

$$= \det \{ \eta^\alpha, \eta^\beta \} \det^2 (\partial F / \partial \omega^c)$$

where η^α are the second-class constraints, γ^a the first-class constraints and F^a the gauge conditions. Then in this case the partition function is given by

$$\begin{aligned} \mathcal{Z} = & \int e^{iS} \delta(\pi_0^a) \delta(\tilde{\gamma}^a) \delta(\chi^a) \delta(A_0^3) \delta(F) \cdot \\ & \cdot \sqrt{\det \{ \eta^\alpha, \eta^\beta \}} \cdot \det \left(\frac{\partial F}{\partial \omega} \right) [d\pi_0^a] [dA_0^a] [d\phi] [d\phi] \quad (6.13) \end{aligned}$$

We have considered only two gauge conditions $\delta(A_0^3)$ and $\delta(F)$ which correspond to the gauge generators π_0^3 and $\tilde{\gamma}^3$ respectively. A corroboration of the necessity of such a restricted number of gauge conditions will be given in Appendix A. Moreover, we are going to choose specifically

$$F_\eta = \partial_\nu A_\nu^3 + \partial/\partial \eta \phi^t \tau^3 \xi - f(\vec{x}, t) = 0 \quad (6.14)$$

with $f(\vec{x}, t)$ being an arbitrary function, and $\eta \rightarrow 1$, which is a convenient gauge for systems with broken symmetry [19]. The parameter ξ which appears in this expression is the symmetry breaking solution of the minimum equations (6.3).

Integrating the functional integral (6.13) over the canonical variables π_0^a , A_c^a and taking into account that

$$S = \int_0^\beta d\tau \int d^3x [p \dot{\phi} + \vec{\pi}^a \cdot \vec{A}^a - \mathcal{H} + \mu N^3] \quad (6.15)$$

where

$$N^3 = \epsilon^{3bc} \vec{A}^b \cdot \vec{\pi}^c + p \tau^3 \phi \quad (6.16)$$

we obtain (after having written $\delta(\tilde{\gamma}^a)$ in the form $\delta(\tilde{\gamma}^a) = \int e^{\int \lambda^a \gamma^a d^4x} d\lambda^a$ and defined $A_0^a \equiv \lambda^a$).

$$\begin{aligned} \mathcal{Z} = & \int \exp \left[i \int_0^{\beta} d\tau \int d^3x \left\{ p \partial_0 \phi + \vec{\pi}^a \cdot \partial_0 \vec{A}^a - \right. \right. \\ & - \frac{1}{2} (\vec{\pi}^a \cdot \vec{\pi}^a + \vec{B}^a \cdot \vec{B}^a) - \frac{1}{2} (D_i \phi)^2 + \frac{m^2}{2} \phi^2 \phi - \\ & \left. \left. - \frac{\lambda}{4} (\phi^2 \phi)^2 + (A_0^a + \frac{\kappa}{g} \delta^{a3}) \tilde{Y}^a - g J_0 (A_0^a + \frac{\kappa}{g} \delta^{a3}) \right\} \right] \\ & \sqrt{\det \{ \eta_i^\alpha, \eta_j^\beta \}} \delta(F) \det \left(\frac{\partial F}{\partial \omega} \right) [d\pi_i^a] [dA_\mu^a] [d\phi] [d\phi] \end{aligned} \quad (6.17)$$

Here we have, moreover, utilized a neutrality condition of the (3.10) type for π_i^3 . Now we can perform the integrations in the momenta π_i^a , p , taking into account that neither the determinant of the second-class constraints nor the ghost determinant depend on these momenta. The obtained result is

$$\mathcal{Z} = \int \exp \left[i \int_0^{\beta} d\tau \int d^3x \mathcal{L}_{\text{eff}} \right] \sqrt{\det \{ \eta_i^\alpha, \eta_j^\beta \}} \delta(F) \det \left(\frac{\partial F}{\partial \omega} \right) [dA_\mu^a] [d\phi] \quad (6.18)$$

where

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} \tilde{F}_{\mu\nu}^a \tilde{F}_{\mu\nu}^a - \frac{1}{2} (\tilde{D}_\mu \phi)^2 (D_\mu \phi) + \frac{m^2}{2} \phi^2 - \frac{\lambda}{4} \phi^4 - g J_4 A_4^a \delta^{a3} \quad (6.19)$$

The covariant derivative and the Yang-Mills field strength tensor are defined in the same way as in (3.5).

Shifting the Higgs field by the stationary solution (6.3) and taking the quadratic part in the fields, we obtain the quadratic contribution which enters in the partition function to be

$$\begin{aligned} \mathcal{L}_Q = & -\frac{1}{2} A_\mu^a \Delta_{\mu\nu}^{-1ab} A_\nu^b - \frac{1}{2} \phi \mathcal{O}_{rs}^{-1} \phi + \phi_r M_{rs}^b A_\nu^b \\ \Delta_{\mu\nu}^{-1ab} = & K_{\lambda}^{ac} K_{\lambda}^{cb} \delta_{\mu\nu} - K_{\mu}^{ac} K_{\nu}^{cb} - \xi \tau_{cd}^a \tau_{de}^b \xi_c + \eta K_{\mu} K_{\nu} \delta^{a3} \delta^{b3} \\ \mathcal{O}_{rs}^{-1} = & K_{rs}^2 \delta_{rs} - 2\mu K_4 \tau_{rs}^3 + \mu^2 \delta_{rs} \delta_{s3} + 2\lambda \xi_{rs} \xi - \frac{g^2}{\eta} \tau_{rs}^3 \xi \xi \tau_{rs}^3 \end{aligned} \quad (6.20)$$

$$M_{r\nu}^a = ig\bar{\xi} \left[(K_\nu^+ \delta^{a2} - K_\nu^- \delta^{a1}) \delta_{r3} - 2\mu \delta_{\nu 4} \delta^{a3} (\delta_{r1} + \delta_{r2}) \right]$$

where

$$K_\lambda^{ab} = K_\lambda \delta^{ab} - \mu \tilde{\tau}_{ab}^3 \delta_{\lambda 4}, \quad K_\lambda^\pm = K_\lambda \pm \mu \delta_{\lambda 4} \quad (6.21)$$

The quadratic contribution of the gauge condition (6.14) has been included in (6.20). After functional integration in the fields we find

$$\tilde{Z} = \text{Det}^{-1/2}(\mathcal{Q}_{rs}^{-1}) \cdot \text{Det}^{-1/2}(\Delta_{\mu\nu}^{-1ab} - N_{\mu\nu}^{ab}) \text{Det}^{-1/2} \{ \bar{\eta}_\alpha \eta_\beta \} \text{Det} \left(\frac{\partial F}{\partial \omega} \right) \quad (6.22)$$

with $N_{\mu\nu}^{ab} = M_{\mu r}^a(-k) \mathcal{Q}_{rs}(k) M_{s\nu}^b(k)$, where $\mathcal{Q}_{rs}(k)$ is the inverse of $\mathcal{Q}_{rs}^{-1}(k)$, defined by $\int \mathcal{Q}_{rp}^{-1}(k) \mathcal{Q}_{ps}(k) dk = \delta_{rs}$. The ghost and the second-class constraints determinants are given by respectively,

$$\text{Det} \left(\frac{\partial F}{\partial \omega} \right) = -(K^2 + g^2 \tilde{\xi}^2 / \eta^2) \quad (6.23)$$

$$\text{Det}^{-1/2} \{ \bar{\eta}_\alpha, \eta_\beta \} = g^2 J_A \quad (6.24)$$

here we have used the following transformation for the global

(1) subgroup of the SU(2) gauge group

$$\delta A_\mu^{1,2} = \mp g A_\mu^{2,1} \omega \quad (6.25a)$$

$$\delta A_\mu^3 = 0 \quad (6.25b)$$

$$\delta \phi_r = g \omega \tilde{\tau}_{rs}^3 \phi_s \quad (6.25c)$$

The main difficulty now is in obtaining the eigenmodes of the system, that is, in calculating the determinant of the propagators. Among them, the most cumbersome is $\text{Det}^{-1/2} [\Delta_{\mu\nu}^{-1ab} - N_{\mu\nu}^{ab}]$, because it is a (12x12) nondiagonal matrix in the

direct product of isospin space and Lorentz space. However, the following change in the field variables

$$A_{\mu}^1 = \frac{1}{\sqrt{2}} [A_{\mu}^+ - A_{\mu}^-], \quad A_{\mu}^2 = \frac{i}{\sqrt{2}} [A_{\mu}^+ + A_{\mu}^-] \quad (6.26)$$

which is equivalent to a similarity transformation in the isospin space, makes it possible to write the different propagator components in a more simple form and permits us to fulfill the task of evaluating the determinant and to find the particle spectrum (the frequencies ω_i are given in Table III). There appears as many eigenmodes as there are degrees of freedoms in the theory, namely 10, to be confronted with the standard number of degrees of freedom that is 9 for $J_0=0$.

The additional degree of freedom is not an unimportant matter, but enters in the statistical description of the system /18,20/ through the thermodynamic potential, which in the one-loop approximation is given by

$$\Omega^{(1)} = \Omega_0^{(1)} + \Omega_{\beta}^{(1)} \quad (6.27a)$$

$$\Omega_0^{(1)} = \int \frac{d^3K}{(2\pi)^3} \sum_{i=1}^7 \frac{n_i \omega_i}{2} \quad (6.27b)$$

$$\Omega_{\beta}^{(1)} = \int \frac{d^3K}{(2\pi)^3} \sum_{i=1}^7 \frac{n_i}{\beta} \ln(1 - e^{-\beta \omega_i}) \quad (6.27c)$$

where $\Omega_0^{(1)}$ and $\Omega_{\beta}^{(1)}$ are the vacuum and temperature one-loop approximation of the thermodynamic potential and n_i are the corresponding multiplicity to each mode (these appear summarized in Table III).

Now it is interesting to analyze what the behaviour of this theory is like when we switch off the source. From the minimum equation (6.3) we have the following relation for J_0

$$J_0 = \mu \xi^2 \quad (6.28)$$

and then the way to switch off the source is considering the limit $\mu \rightarrow 0$. In this limit, the eigenmodes of the problem are summarized in Table IV.

Taking into account the multiplicity of each mode and the fact that from (6.3) the limiting value of ξ is given by

$$\lim_{\mu \rightarrow 0} \xi = \sqrt{m^2/2\lambda} \quad (6.29)$$

we see that the system regains in this limit the spectrum of the usual Yang-Mills-Higgs gauge-invariant system plus an extra zero mode $\omega_2'^2 = 0$. The reason why the gauge-invariant configuration is not completely reproduced in the limit is in the lack of gauge-fixing conditions imposed (remember that their number is greater for $J_0=0$).

7. CONCLUDING REMARKS

The aim of this work is to deepen in the understanding of the charged B-E-c phenomenon for gauge models, where, as we showed starting from the Gauss' law, the neutrality of the condensed ground state plays a crucial role.

In the developement of this paper we established a parallel between the behaviour of the Abelian and non-Abelian gauge systems in relation to the charged B-E-c phenomenon, projecting their main differences. We have accomplished this task in two stages.

First we investigate the possibility to obtain the charge B-E-c in gauge systems which only include dynamical fields. We found that in spite of the results obtained for the Abelian

models, for the studied non-Abelian gauge system, which only included bosons, the charged B-E-c is not permitted at the classical level. This result is linked to the fact that in the non-Abelian gauge model the charges associated to the different fields are not conserved separately as it is the case in the Abelian model. So, we obtain the charged B-E-c for the non-Abelian model through the inclusion of fermions /4/, where the leptonic conserved charge plays a crucial role.

Second, starting from the results obtained for the dynamical models which include fermions, we analyzed the possibility to simulate a neutralizing constant external charge as the limit of very heavy fermions ($M \rightarrow \infty$) with growing "leptonic" chemical potential ($\mu' \rightarrow \infty$) for the fermion contribution to the partition function, and we found that while this mechanism works in a good way for the Abelian model, for the non-Abelian one some pathologies appear, which we interpret as a consequence of the impossibility to alter the gauge symmetry of the theory by a dynamical limiting process. Finally, we study a non-Abelian many-particle theory in presence of a constant external charge, and show that the effect of this external source on the boson sector of the theory is completely different from the one produced by an additional dynamical field. For this system with external charge the gauge symmetry is lost and an extra degree of freedom appears which is related with an extra eigenmode in the frequency spectrum of the system.

In conclusion, we would like to indicate that the charged Abelian and non-Abelian systems can be treated on equal footing without any conflict with gauge invariance, provided that we do not care about equilibrating their charge by introducing the in-

interacting charged background.

To see this we can, instead of considering the condition (2.11) prescribe the asymptotic behaviour to the path-integration variable in the partition function, like e.g. in (2.10a)

$$\pi_i \xrightarrow{r \rightarrow \infty} \rho/r^2 \quad (7.1)$$

where ρ is a real constant. Now in place of (2.13) one should write

$$A_0 \xrightarrow{r \rightarrow \infty} \alpha + \frac{1}{r} \quad (7.2)$$

The constant α in this asymptotic behaviour is not essential since after the shift of integration variable $A_0 \rightarrow A_0 - \alpha$ in (2.15) it appears only multiplied by $\int \gamma d^4x$, which is zero due to the Gauss constraint condition. With the remaining term $A_0 \sim 1/r$ and condition (7.1) the condition

$$\oint A_0 \vec{\pi} \cdot \vec{n} ds \quad (7.3)$$

needed to carry out in the usual way the functional integration in the momenta is guaranteed. At the same time it follows from eq.(2.20a) that

$$\frac{1}{V} \left[\frac{\partial \Omega}{\partial \mu} \right]_{T,V} = \frac{1}{e} \langle \oint \vec{\pi} \cdot \vec{n} ds \rangle = \frac{\rho}{e} \quad (7.4)$$

In this case the average of the total charge of the system is not zero, as it is evident from

$$\begin{aligned} \langle \gamma \rangle &= \langle \oint \vec{\pi} \cdot \vec{n} ds \rangle - e \langle N \rangle = 0 \\ e \langle N \rangle &= \rho \end{aligned} \quad (7.5)$$

After the integration over momenta we have the same representa-

tion for the Grand partition function as usual, say (2.15), but μ in it is determined by equation (7.4), and not by (2.20b). Besides, it is easy to show that the term $\mu \oint \vec{\pi} \cdot \vec{n} ds$ in the Hamiltonian does not break either Abelian or non-Abelian gauge symmetry in the sense that it does not modify the constraints, like γ in (2.10c), and does not violate the Poisson commutation $[\int \mathcal{H}(\mu) d^3x, \gamma] = 0$ either /21/. Nevertheless, the required equilibrium in the statistical description of the theory is lost, and it requires a further study to specify under what conditions the present situation may be realized.

We want to point out here that it would be interesting to investigate in the further two problems which are related with the spirit of the results obtained in this paper. These are, the possibility to obtain a charges B-E-c for a richer non-Abelian model, such as the SU(5) unified model, taking into account only the contribution of bosons at the classical level, and the possibility to obtain a covariant external charge as some limit in a dynamical interacting non-Abelian gauge model.

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APPENDIX A

As a corroboration of the fact that only one gauge fixing condition is needed in (6.18), i.e. $\delta(F)$, we have that, if one does not fix any gauge condition, the only singular part in (6.22) is

$$\det[\Delta_{\mu\nu}^{ab} - N_{\mu\nu}^{ab}] = \begin{vmatrix} \Delta_{\mu\nu}^{++} - N_{\mu\nu}^{++} & \Delta_{\mu\nu}^{+-} - N_{\mu\nu}^{+-} \\ \Delta_{\mu\nu}^{-+} - N_{\mu\nu}^{-+} & \Delta_{\mu\nu}^{--} - N_{\mu\nu}^{--} \\ & & \Delta_{\mu\nu}^{33} - N_{\mu\nu}^{33} \end{vmatrix} \quad (\text{A.1})$$

$$= \begin{vmatrix} \Delta_{\mu\nu}^{++} - N_{\mu\nu}^{++} & \Delta_{\mu\nu}^{+-} - N_{\mu\nu}^{+-} \\ \Delta_{\mu\nu}^{-+} - N_{\mu\nu}^{-+} & \Delta_{\mu\nu}^{--} - N_{\mu\nu}^{--} \end{vmatrix} \cdot \det[\Delta_{\mu\nu}^{33} - N_{\mu\nu}^{33}]$$

The first factor here is nonzero

$$\begin{vmatrix} \Delta_{\mu\nu}^{++} - N_{\mu\nu}^{++} & \Delta_{\mu\nu}^{+-} - N_{\mu\nu}^{+-} \\ \Delta_{\mu\nu}^{-+} - N_{\mu\nu}^{-+} & \Delta_{\mu\nu}^{--} - N_{\mu\nu}^{--} \end{vmatrix} \quad (\text{A.2})$$

whereas the second is calculated to be

$$\det[\Delta_{\mu\nu}^{33} - N_{\mu\nu}^{33}] = \frac{g^2 \xi^2}{D_1} [-(2\lambda \xi^2 + K^2)K^2 - 4\mu^2 K_4^2 + D_1] \left[\frac{g^2 \xi^2 (1 + 4\mu^2 K_4^2)}{D_1} + K^2 \right] \quad (\text{A.3})$$

where

$$D_1 = (2\lambda \xi^2 + K^2)K^2 + 4\mu^2 K_4^2 \quad (\text{A.4})$$

Eq.(A.3) is identically zero.

Hence, with a U(1) type gauge condition (6.14) all the singularity of the problem may be overcome.

From the effective Lagrangians (3.5) and (6.19) it is evident that the different behaviours of their corresponding quadratic operators ($\Delta_{\mu\nu}^{ab} - N_{\mu\nu}^{ab}$) are strictly related to the stationary solutions that are used in each case in order to develop the stationary phase method (i.e. (3.12) and (6.3) respectively).

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Fig.1. Although the net charge is conserved in each of these processes, it can be transferred from one field (ϕ^+ , for example) to another (A_M^+ in (a), or χ^+ in (b)).

$$\int d^4x [\bar{\psi}(i\gamma^\mu \tilde{D}_\mu - M)\psi] = e \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \dots \right]$$

Fig.2. Diagrammatic development of the fermion contribution to the partition function near the zero value of the A_μ field (Here solid lines refer to spinor fields and wavy lines to gauge fields).

TABLE 1

FIELD	DISPERSION RELATION	MULTIPLICITY
A_μ^+	$K_\lambda^+ K_\lambda^+ + g^2 \xi_3^2 = 0$	3 - fold
A_μ^-	$K_\lambda^- K_\lambda^- + g^2 \xi_3^2 = 0$	3 - fold
A_μ^3	$K^2 = 0$	2 - fold
ϕ_3	$K^2 + 2 m^2 = 0$	1 - fold

TABLE 2

FIRST-CLASS CONSTRAINTS	SECOND-CLASS CONSTRAINTS
π_0^3	$\pi_0^{1,2}$
$\tilde{\gamma}^3$	$\tilde{\gamma}^{1,2}$
	$\chi^1 = A_0^1$
	$\chi^2 = A_0^2$

TABLE 3

EIGENMODES	MULTIPLICITY
$\omega_{1,2}^2 = \frac{1}{2} \left[(K_3^2 + 2\mu^2 + g^2 \xi^2) \pm \sqrt{K_3^4 + 2g^2 \xi^2 K_3^2 + g^2 \xi^2 (8\mu^2 + g^2 \xi^2)} \right]$	1 - fold
$\omega_{3,4}^2 = \frac{1}{2} \left[(2K_3^2 + 4\mu^2 + (g^2 + 2\lambda) \xi^2) \pm \sqrt{[4\mu^2 + (2\lambda - g^2) \xi^2]^2 + 16\mu^2 K_3^2} \right]$	1 - fold
$\omega_{5,6}^2 = \frac{1}{2} \left[(2K_3^2 + 2\mu^2 + g^2 \xi^2) \pm \sqrt{g^4 \xi^4 + 16\mu^2 K_3^2} \right]$	2 - fold
$\omega_7^2 = K_3^2 + g^2 \xi^2$	2 - fold

TABLE 4

$\mu \neq 0$	$\mu = 0$	MULTIPLICITY
$\omega_{1,2}^2$	$(\omega_1')^2 = K_3^2 + g^2 \xi'^2$	1 - fold
	$(\omega_2')^2 = 0$	1 - fold
$\omega_{3,4}^2$	$(\omega_3')^2 = K_3^2 + 2\lambda \xi'^2$	1 - fold
	$(\omega_4')^2 = K_3^2 + g^2 \xi'^2$	1 - fold
$\omega_{5,6}^2$	$(\omega_5')^2 = K_3^2$	2 - fold
	$(\omega_6')^2 = K_3^2 + g^2 \xi'^2$	2 - fold
ω_7^2	$(\omega_7')^2 = K_3^2 + g^2 \xi'^2$	2 - fold

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