Н 87-264

cc

LIBRARY
PHYSICAL INSTITUTE

P.N. LEBEDEV PHYSICAL INSTITUTE
USSR ACADEMY OF SCIENCES

91

15 MAR 1988

PREPRINT

264

E.S. FRADKIN, M.A. VASILIEV

**SUPERALGEBRA OF HIGHER
SPINS AND AUXILIARY FIELDS**

Moscow-1987

Препринты Физического института имени П.Н. Лебедева АН СССР являются самостоятельными научными публикациями и издаются по следующим направлениям исследований Института:

- **физика высоких энергий и космических лучей**
- **оптика и спектроскопия**
- **квантовая радиофизика**
- **физика твердого тела**
- **физика космоса**
- **физика плазмы**

В библиографических ссылках на препринты Физического института имени П.Н. Лебедева мы рекомендуем указывать: инициалы и фамилию автора, номер препринта, место издания, сокращенное наименование Института-издателя, год издания.

Пример библиографической ссылки:

И.И. Иванов. Препринт 125, Москва, ФИАН, 1986.

Preprints of the P.N. Lebedev Physical Institute of the Academy of Sciences of the USSR are its independent publications and are issued in the Institute's following fields of research:

- **high energy and cosmic ray Physics**
- **optics and spectroscopy**
- **quantum Radiophysics**
- **solid state Physics**
- **cosmophysics**
- **plasma Physics**

In bibliographical references to the P.N. Lebedev Physical Institute's preprints we recommend to indicate: the author's initials and name, preprint number, place of the publication, abbreviation of the Institute-publisher, year of the publication:

Example of a bibliographical reference:

I.I. Ivanov. Preprint 125, Moscow, FIAN, 1986.

I.E.Tamm Department of Theoretical Physics

Physics of high energies and cosmic rays

Preprint N 264

E.S.Fradkin and M.A.Vasiliev

SUPERALGEBRA OF HIGHER SPINS AND AUXILIARY FIELDS

Moscow - 1987

ABSTRACT

An infinite-dimensional non-abelian superalgebra is constructed denoted as $shsa(1)$ which gives rise at the linearized level to linearized curvatures of both massless and auxiliary fields, suggested previously by one of us (M.V.). Various properties of $shsa(1)$ are analysed in detail. Specifically, subalgebras of $shsa(1)$ are found which pretend themselves for the role of independent superalgebras of higher spins and auxiliary fields. A simple operator realization of $shsa(1)$ is presented. P-reversal automorphisms are constructed. N=2 de Sitter superalgebra $osp(2,4)$ is shown to be a maximal finite-dimensional subalgebra of $shsa(1)/o(2)$. It is observed that the even (boson) subalgebra of $shsa(1)$ decomposes into the direct sum of two infinite-dimensional Lie algebras, each giving rise to massless fields of all spins. Possible physical implications of this fact are discussed briefly.

1. Introduction

In this paper, we continue the investigation of the higher-spin problem, the problem of introducing a consistent gravitational interaction for massless higher-spin fields ($s > 2$), which was initiated in refs./1-4/. In refs./1,2/, the new form of description of free massless higher-spin fields on the flat /1/ and anti-de Sitter /2/ backgrounds was suggested by one of us, based on appropriate gauge fields similar to those of the vierbein formulation of gravity and supergravity (the same formulation for higher-spin massless fermions in the flat space was independently obtained by Aragone and Deser /5/). Specifically, the linearized higher-spin curvatures on the de Sitter background were found in ref./2/. These served in ref./3/ as the starting point for determination of the non-abelian infinite-dimensional higher-spin superalgebras $shs_{\rho}(1)$ ($\rho=0$ or 1) generalizing $N=1$ de Sitter superalgebra $osp(1,4)$. It was argued in refs./2,3/ that some extension of the systems of higher-spin fields, used in these references, is desirable incorporating auxiliary fields which do not possess non-trivial dynamical degrees of freedom. Motivated by this conclusion, one of us suggested in ref./4/ new linearized curvatures different from those of ref./2/. It was shown in ref./4/, that these new linearized curvatures give rise to a class of gauge auxiliary fields in (anti-) de Sitter (or flat) space.

Here we construct the non-abelian infinite-dimensional superalgebra, $shsa(1)$ (super-higher-spin-auxiliary at $N=1$), which generates at the linearized level both the linearized higher-spin curvatures of ref./2/ and the linearized auxiliary curvatures of ref./4/, and contains the non-abelian higher-spin superalgebra $shs_{\rho}(1)$ of ref./3/ as subalgebra.

We hope that the superalgebra $shsa(1)$ will enable one to overcome some difficulties emerged in attempts to construct consistent higher-spin theories in the framework of the higher-spin superalgebras $shs_{\rho}(1)$ of ref./3/ (the most evident one among these difficulties is the "extra field problem" pointed out in refs./2,3/).

In addition, the auxiliary fields, which are present among the gauge fields originating from $\text{shsa}(1)$, seem to be relevant to the standard problem of closing the gauge algebra in the related higher-spin theories. Although a sequence of superalgebras, $\text{shsa}(N)$, exists corresponding to N -extended supersymmetry, only the simplest case, $N=1$, will be analyzed in detail in the present paper.

An important and somewhat unexpected feature of $\text{shsa}(1)$ consists in the fact that any field of spin $s \geq 1$ occurs twice among the gauge fields originating from this superalgebra. Specifically, two massless spin-2 fields are present (this property takes place not only for $\text{shsa}(1)$ but also for those its subalgebras which pretend themselves for the role of superalgebras of higher spins and auxiliary fields). This can lead to important cosmological implications if the spin-2 massless fields will serve both as gravitational fields in two worlds whose mutual interaction will be suppressed by powers of the gravitational constant.

The rest part of the paper is organized as follows.

In Section 2, we collect the main results of the paper. Namely, explicit expressions for curvatures of $\text{shsa}(1)$, hermiticity conditions, P -transformation laws and a bilinear invariant form of $\text{shsa}(1)$ are described here.

In Section 3, we show that de Sitter subalgebra $\text{sp}_{d,s}(4)$ of $\text{shsa}(1)$ as well as its $N=1$ supersymmetric extension, $\text{osp}_{d,s}(1,4)$ can be chosen uniquely (up to automorphisms) under some reasonable conditions. We also demonstrate here that linearization of $\text{shsa}(1)$ leads either to linearized higher-spin curvatures of ref./2/ or to linearized auxiliary curvatures of ref./4/.

In Section 4, we analyse most general reality (hermiticity) conditions and P -reversal automorphisms which are in accordance with the Lorentz structure (i.e. interchanging dotted and undotted spinorial indices). It is shown that only two inequivalent real forms of complex superalgebra of higher spins and auxiliary fields exist related to such reality conditions. In addition a class of Lorentz covariant automorphisms of $\text{shsa}(1)$ and its complexification, $\text{shsa}(1, \mathbb{C})$, is constructed here which do not modify the types of spinorial indices.

A simple operator realization of $shsa(1)$ is presented in Section 5. In addition, some consequences of commutation relations for fermion higher-spin generators are derived here assuming that full higher-spin subalgebras of superalgebras of higher spins and auxiliary fields are realized in a positive definite Hilbert space (the full higher-spin subalgebras are supposed to serve as global symmetry algebras of physical excitations over some symmetric vacuum, and physical states should form in this case some unitary representations of these subalgebras). The consequences above reduce to some inequalities on boson generators similar to the well-known energy non-negativity condition in ordinary supersymmetry. It is shown that these inequalities are consistent only for one of the two real forms found in Section 4. It is this real form which is denoted as $shaa(1)$.

In Section 6, various finite-and infinite-dimensional subalgebras of $shsa(1)$ are considered. Specifically, we analyse in detail infinite-dimensional subalgebras denoted as $shsa(1|a,b)$ (a and b take on arbitrary values 0 or 1), which can serve themselves as independent superalgebras of higher spins and auxiliary fields. $N=2$ de Sitter superalgebra $osp(2,4)$ is shown here to be a subalgebra of $shsa(1)$.

In Section 7, the superalgebra $shsa(1)$ is described in a new basis in which its even subalgebra reduces explicitly to the direct sum of two infinite-dimensional Lie algebras, each giving rise to all boson higher-spin and auxiliary fields. Possible physical implications of this fact are discussed briefly.

In Section 8, we summarize the main results of the paper once again.

In Appendix, the main steps are discussed briefly which led us to the construction of $shsa(1)$.

2. Non-Abelian Superalgebra of Higher Spins and Auxiliary Fields

The main result of the paper as a whole consists in constructing a non-abelian superalgebra which gives rise at the linearized level to both linearized higher-

spin curvatures suggested in ref./2/ and linearized curvatures of auxiliary fields suggested in ref./4/. In this section we formulate the final results. Various properties of the superalgebra proposed, as well as the main assumptions used in its derivation, will be discussed in more detail in subsequent sections and Appendix.

In the present paper, we use the conventions and notations introduced in refs./2,3/. In particular, given Lie superalgebra g with the generators (basis elements) T_i obeying the relations

$$[T_i, T_j] = \mathcal{U}_{ij}^k T_k, \quad (2.1)$$

we introduce the parameters $\tilde{\mathcal{Z}}_i^k$ which are supposed to be even (odd) elements of some Grassmann algebra if T_i are even (odd) elements of g . The commutator

$$[\tilde{\mathcal{Z}}_1^i T_i, \tilde{\mathcal{Z}}_2^j T_j] \text{ is of the form } [\tilde{\mathcal{Z}}_1, \tilde{\mathcal{Z}}_2]^i T_i \text{ where, by definition,} \\ [\tilde{\mathcal{Z}}_1, \tilde{\mathcal{Z}}_2]^k = \tilde{\mathcal{Z}}_1^i \tilde{\mathcal{Z}}_2^j \mathcal{U}_{ij}^k. \quad (2.2)$$

The composition law (2.1) corresponds to some Lie superalgebra if and only if the following relations hold for arbitrary $\tilde{\mathcal{Z}}_{1,2,3}$:

$$[\tilde{\mathcal{Z}}_1, \tilde{\mathcal{Z}}_2] = -[\tilde{\mathcal{Z}}_2, \tilde{\mathcal{Z}}_1], \quad (2.3)$$

$$[\tilde{\mathcal{Z}}_1, [\tilde{\mathcal{Z}}_2, \tilde{\mathcal{Z}}_3]] + [\tilde{\mathcal{Z}}_2, [\tilde{\mathcal{Z}}_3, \tilde{\mathcal{Z}}_1]] + [\tilde{\mathcal{Z}}_3, [\tilde{\mathcal{Z}}_1, \tilde{\mathcal{Z}}_2]] = 0 \quad (2.4)$$

(indices are suppressed here).

The gauge fields (connection one-forms) ω_ν^i , related to g , have the same Grassmann properties as $\tilde{\mathcal{Z}}^i$. Within the notations above, the infinitesimal gauge transformations and curvatures take the form

$$\delta \omega_\nu^i = \partial_\nu \tilde{\mathcal{Z}}^i + [\omega_\nu, \tilde{\mathcal{Z}}]^i, \quad (2.5)$$

$$R_{\nu\mu}^i = \partial_\nu \omega_\mu^i - \partial_\mu \omega_\nu^i + [\omega_\nu, \omega_\mu]^i. \quad (2.6)$$

It follows from the Jacobi identities (2.4) that the curvatures transform homogeneously,

$$\delta R_{\nu\mu}^i = [R_{\nu\mu}, \tilde{\mathcal{Z}}]^i \quad (2.7)$$

and obey the Bianchi identities,

$$\epsilon^{\nu\mu\rho\sigma} (\partial_\nu R_{\mu\rho}^i + [\omega_\nu, R_{\mu\rho}]^i) = 0. \quad (2.8)$$

Let us first consider the complexification, $\text{shsa}(1; \mathbb{C})$, of the real superalgebra of higher spins and auxiliary fields. Its basis is formed by the set of supergenerators $T_{AB}^{\alpha(n), \beta(m)}$ where $\alpha, \beta, \dots, \dot{\alpha}, \dot{\beta}, \dots = 1, 2$ are spinorial indices while the indices $A, B, C, \dots$ take on values 0 or 1 and enumerate different multispinors.* The ranks of multispinors, n and m , run over all non-negative integer values. In accordance with the normal relation between spin and statistics, the (super)generators $T_{AB}^{\alpha(n), \beta(m)}$ are assumed to be even (odd) elements of $\text{shsa}(1, \mathbb{C})$ if $n+m$ is even (odd).

In terms of the composition law (2.2), the superalgebra $\text{shsa}(1, \mathbb{C})$ can be defined by the relation

$$\begin{aligned} \left[\tilde{\mathfrak{z}}_1, \tilde{\mathfrak{z}}_2 \right]_{\alpha(n), \beta(m)}^{FG} &= \frac{1}{2} \sum_{p,q,s,k,l,t} i^{s+t-1} \frac{n!m!}{p!q!s!k!l!t!} \times \\ &\times \delta(|F+A+C|_2) \delta(|G+B+D|_2) \delta(n-p-q) \delta(m-k-l) \times \\ &\times \left[(-1)^{C(p+s)+D(k+t)} (-1)^{A(q+s)+B(t+l)+s+t+(p+k+s+t)(q+l+s+t)} \right] \times \\ &\times \sum_1^{AB} d(p) \gamma(s), \beta(k) \dot{\delta}(t) \sum_2^{CD} d(q) \gamma(s), \beta(l) \dot{\delta}(t) \end{aligned} \quad (2.9)$$

where the summation over the integer-valued variables p, q, s, k, l, t is carried out from 0 to ∞ while the summation over A, B, C, D is carried out from 0 to 1. The following notations are also currently used through the text

$$\delta(n) = \begin{cases} 1 & \text{at } n=0 \\ 0 & \text{at } n \neq 0 \end{cases}, \quad |n|_2 = n - \left[\frac{n}{2} \right] \tau \quad (2.10)$$

where $\left[\frac{n}{2} \right]$ is the integer part of $\frac{n}{2}$ (let us stress that $|n|_2$ takes on va-

* The same as in refs./2-4/, we use the convention that a number of spinorial indices is indicated in parentheses (except for the case of a single index). Indices denoted by one and the same letter are supposed to be symmetrized separately over lower and upper indices. After that, the maximal possible number of upper and lower indices denoted by the same letter should be contracted.

lues from 0 to $r-1$, the convention which will become essential later).

Using eq.(2.6), one can readily construct the curvatures of $\text{shsa}(1;\mathbb{C})$. With the aid of a simple identity,

$$(-1)^n - (-1)^m = 2(-1)^n \delta(|n+m+1|_2) \quad (2.11)$$

these curvatures take the form

$$\begin{aligned} R_{\nu\mu, \alpha(n), \beta(m)}^{FG} &= \partial_\nu \omega_{\mu, \alpha(n), \beta(m)}^{FG} - \partial_\mu \omega_{\nu, \alpha(n), \beta(m)}^{FG} \\ &+ \sum_{p, q, s, k, \ell, t; A, B, C, D} \delta(|F+A+C|_2) \delta(|G+B+D|_2) \delta(n-p-q) \delta(m-k-\ell) \times \\ &\times i^{s+t-1} (-1)^{C(p+s)+D(t+k)} \times \frac{n!m!}{p!q!s!k!\ell!t!} \times \\ &\times \delta(|C(p+s)+D(k+t)+A(q+s)+B(\ell+t)+s+t+(p+k+s+t)(q+\ell+s+t)+1|_2) \times \\ &\times \omega_{\nu, \alpha(p)\gamma(s), \beta(k)\delta(t)}^{AB} \omega_{\mu, \alpha(q), \beta(\ell)}^{CD} \gamma(s) \delta(t) \end{aligned} \quad (2.12)$$

It is not hard to verify that eq.(2.9) defines some Lie superalgebra, acting along the lines of ref./3/ (see also Appendix). Moreover, one can easily see that the higher-spin superalgebra $\text{shs}(1;\mathbb{C})$, proposed in ref./3/, coincides with the subalgebra of $\text{shsa}(1;\mathbb{C})$ spanned by the generators $T_{00}^{\alpha(n), \beta(m)}$ which give rise to the parameters $\xi_{\alpha(n), \beta(m)}^{00}$ and gauge fields $\omega_{\nu, \alpha(n), \beta(m)}^{00}$.

For physical applications, real forms of $\text{shsa}(1;\mathbb{C})$ are of importance which contain the Lorentz subalgebra $O_4(3,1)$. (We identify generators of $O_4(3,1)$ with the generators $T_{00}^{\alpha(2), \beta(0)}$ and $T_{00}^{\alpha(0), \beta(2)}$ related to each other by appropriate reality conditions). In terms of gauge fields, the real form $\text{shsa}(1)$ of $\text{shsa}(1;\mathbb{C})$, which obeys this condition and has a number of other nice properties, can be extracted by the following "hermiticity" conditions:

$$(\omega_{\nu, \alpha(m), \beta(n)})^\dagger = (-1)^{An+Bm} \omega_{\nu, \beta(m), \alpha(n)}^{BA} \quad (2.13)$$

(various reasons leading to the specific choice (2.13) will be discussed in more

detail in Sections 4,5; for more detail on the relation between real forms of complex Lie superalgebras and hermiticity conditions imposed on their gauge fields see e.g. refs./6,10/.

Apart from the hermiticity conditions, the P-transformation properties of gauge fields are of physical interest. The action of relevant P-reversal automorphisms is described by the relation

$$P(\omega_{\gamma, \alpha(n), \beta(m)}^{AB}) = (-1)^{\eta(A+B) + \mathcal{L}(n+m) + \Delta(A \cdot n + B \cdot m)} \times \\ \times i^{\Delta(n+m+|n+m|_2-2)} \omega_{\gamma, \beta(m), \alpha(n)}^{BA}, \quad (2.14)$$

where η , $\mathcal{L}$ and Δ are arbitrary parameters taking values 0 or 1, while $\omega_{\gamma \dots} = (\omega_0, \omega_1, -\omega_2, \omega_3)$ if $\omega_{\gamma} = (\omega_0, \omega_1, \omega_2, \omega_3)$, that is the P-transformation under consideration corresponds to the reflection of the x_2 -axis (let us also remind the reader that, in contrast to hermitian conjugation, the operation P does not change a sign of the imaginary unit, i , and does not reverse an order of product factors).

Superalgebra $shsa(1; \mathbb{C})$ possesses a non-degenerate bilinear form

$$(\overline{\mathfrak{Z}}_1, \overline{\mathfrak{Z}}_2) = \sum_{nmAB} i^{n+m} (-1)^{An+Bm} \frac{1}{n!m!} \overline{\mathfrak{Z}}_1^{AB, \alpha(n), \beta(m)} \overline{\mathfrak{Z}}_2^{AB, \alpha(n), \beta(m)}, \quad (2.15)$$

which is symmetric,

$$(\overline{\mathfrak{Z}}_1, \overline{\mathfrak{Z}}_2) = (\overline{\mathfrak{Z}}_2, \overline{\mathfrak{Z}}_1) \quad (2.16)$$

and invariant,

$$([\overline{\mathfrak{Z}}, \overline{\mathfrak{Z}}_1], \overline{\mathfrak{Z}}_2) + (\overline{\mathfrak{Z}}_1, [\overline{\mathfrak{Z}}, \overline{\mathfrak{Z}}_2]) = 0, \quad (2.17)$$

as can readily be seen with the aid of eq.(2.9). The same expression (2.15) provides simultaneously the invariant form of the real superalgebra $shsa(1)$ whose parameters are restricted by the hermiticity conditions of the form (2.13). In this case, the form $(\overline{\mathfrak{Z}}_1, \overline{\mathfrak{Z}}_2)$ turns out to be hermitian,

$$(\overline{\mathfrak{Z}}_1, \overline{\mathfrak{Z}}_2)^{\dagger} = (\overline{\mathfrak{Z}}_1, \overline{\mathfrak{Z}}_2), \quad (2.18)$$

where the operation $\dagger$ should be understood as some involution in the Grassmann algebra where the parameters $\overline{\mathfrak{Z}}_{1,2}$ and the form $(\overline{\mathfrak{Z}}_1, \overline{\mathfrak{Z}}_2)$ itself take on their values. Note also that the restriction of the form (2.15) to the parameters

$\sum^{\infty} d(n), \dot{\beta}(m)$ gives rise to the invariant form of the higher-spin superalgebras of ref./3/.

To conclude this section, we note that the superalgebras $\text{shsa}(1; \mathbb{C})$ and $\text{shsa}(1)$ are not simple and have the structure $\text{shsa}'(1; \mathbb{C}) \oplus \mathfrak{o}(2; \mathbb{C})$ and $\text{shsa}'(1) \oplus \mathfrak{o}(2)$, respectively, where the abelian subalgebras $\mathfrak{o}(2; \mathbb{C})$ and $\mathfrak{o}(2)$ originate from the generator $T^{\infty}_{d(0), \dot{\beta}(0)}$, while the subalgebras $\text{shsa}'(1; \mathbb{C})$ and $\text{shsa}'(1)$ are simple.

3. De Sitter Subalgebras and Linearization

Among various subalgebras of $\text{shs}(1)$, de Sitter subalgebra $\text{sp}_{dS}(4)$ and $N=1$ de Sitter subalgebra $\text{osp}_{dS}(1, 4)$ are of great importance. Their gauge fields are identified with the gravitational vierbein and Lorentz connection and the fields of $N=1$ supergravity. Moreover, in order to analyse a particle spectrum of higher-spin theories based on $\text{shsa}(1)$, one has to define some linearization procedure. The simplest possibility for such a linearization is to assume that the gravitational fields corresponding to $\text{sp}_{dS}(4)$ have zero-order background components describing a vacuum de Sitter space, while all other gauge fields of $\text{shsa}(1)$ are of the first order. In fact, this linearization corresponds to the contraction of $\text{shsa}(1)$ with respect to its subalgebra $\text{sp}_{dS}(4)$ (for more detail see ref./3/).

There was a unique possibility for choosing (super)generators $T_{d(2)}, T_{\dot{\beta}(2)}, T_{d\dot{\beta}}$ (and $T_d, T_{\dot{\beta}}$) of $\text{sp}_{dS}(4)$ ($\text{osp}_{dS}(1, 4)$) in the framework of the higher-spin superalgebras $\text{shs}_p(4)$ proposed in ref./3/. For the first sight, the situation for $\text{shsa}(1)$ is more complicated since this superalgebra possesses a number of generators realizing the appropriate representations of the Lorentz group. However, it turns out that without loss of generality one can identify the generators of $\text{sp}_{dS}(4)$ ($\text{osp}_{dS}(1, 4)$) with the generators $T^{\infty}_{d(2), \dot{\beta}(0)}, T^{\infty}_{d(0), \dot{\beta}(2)}, T^{\infty}_{d(1), \dot{\beta}(1)}$ (and $T^{\infty}_{d(0), \dot{\beta}(1)}, T^{\infty}_{d(1), \dot{\beta}(0)}$) of $\text{shsa}(1)$. In other words, we can assume that $\text{sp}_{dS}(4) \subset \text{osp}_{dS}(1, 4) \subset \text{shs}_0(1) \subset \text{shsa}(1)$ where the superalgebra $\text{shs}_0(1)$ is identified with the subalgebra of $\text{shsa}(1)$ spanned by the genera-

tors $T^{00}_{\alpha(m),\beta(m)}$.

In order to prove that this is really the case, let us consider first the complex superalgebra $\text{shsa}(1;\mathbb{C})$. The complexified Lorentz subalgebra is of the structure $\mathfrak{sp}(2;\mathbb{C}) \oplus \mathfrak{sp}(2;\mathbb{C}) \subset \mathfrak{sp}_{dS}(4;\mathbb{C})$. Its generators, $t_{1\alpha(2)}$ and $t_{2\beta(2)}$, are assumed to possess the standard commutation relations with all generators of $\text{shsa}(1;\mathbb{C})$. For example,

$$[t_{1\gamma(2)}, T^{AB}_{\alpha(m),\beta(m)}] = \eta_{\gamma\alpha} T^{AB}_{\alpha(n-1)\gamma,\beta(m)} \quad (3.1)$$

($T \neq t$). One can make sure that only the generators $T^{00}_{\alpha(2),\beta(0)}$ and $T^{00}_{\alpha(0),\beta(2)}$ have these properties. Therefore, these generators are identified from now on with the Lorentz generators $t_{1\alpha(2)}$ and $t_{2\beta(2)}$.

As for the generators of de Sitter translations, $t_{\alpha\beta}$, these have to obey the relations

$$[t_{\alpha\beta}, t_{\gamma\delta}] = \epsilon_{\alpha\gamma} t_{2\beta\delta} + \epsilon_{\beta\delta} t_{1\alpha\gamma} \quad (3.2)$$

which, in common with those containing the Lorentz generators, define complex de Sitter superalgebra $\text{sp}_{dS}(4;\mathbb{C})$. In the framework of $\text{shsa}(1;\mathbb{C})$, the most general form for the generator $t_{\alpha\beta}$ is

$$t_{\alpha\beta} = \sum_{AB} \alpha_{AB} T^{AB}_{\alpha(1),\beta(1)} \quad (3.3)$$

Eq.(3.2) imposes some restrictions on the coefficients α_{AB} but it does not fix them completely leaving a two-parameter arbitrariness. However, the following proposition is true.

Proposition 1. Given generators $t_{\alpha\beta}$ (3.3) satisfying eq.(3.2), there exists a basis $T^{AB}_{\alpha(n),\beta(m)}$ of $\text{shsa}(1;\mathbb{C})$ in which the composition law coincides in its form with that for the initial generators $T^{AB}_{\alpha(m),\beta(m)}$, and $t_{\alpha\beta} = T^{00}_{\alpha(2),\beta(2)} T^{00}_{\alpha(0),\beta(0)} = T^{00}_{\alpha(0),\beta(2)} T^{00}_{\alpha(2),\beta(0)}$. In other words, any two de Sitter subalgebras $\text{sp}_{dS}(4;\mathbb{C}) \subset \text{shsa}(1;\mathbb{C})$ are related by some automorphism of $\text{shsa}(1;\mathbb{C})$ if their Lorentz generators coincide with $T^{00}_{\alpha(2),\beta(0)}$ and $T^{00}_{\alpha(0),\beta(2)}$.

We omit the explicit proof of Proposition 1 as it is straightforward and simple enough. Note only that any two de Sitter subalgebras of the above type turn out to

be related one to another by some inner automorphism of $\text{shsa}(1; \mathbb{C})$ originating from the generators $T^{10}_{\alpha(0), \beta(0)}$ and $T^{01}_{\alpha(0), \beta(0)}$.

In fact, the similar statement is true for de Sitter subalgebras $\text{osp}(1, 4; \mathbb{C})$.

Proposition 2. Any two de Sitter superalgebras $\text{osp}(1, 4; \mathbb{C}) \subset \text{shsa}(1; \mathbb{C})$, formed by the generators $T^{00}_{\alpha(2), \beta(0)}$, $T^{00}_{\alpha(0), \beta(0)}$, $t_{\alpha\beta}$ (3.3) and supergenerators of the form

$$t_{\alpha} = \sum_{AB} b_{AB} T^{AB}_{\alpha(1), \beta(0)}, \quad \hat{t}_{\beta} = \sum_{AB} c_{AB} T^{AB}_{\alpha(0), \beta(1)} \quad (3.4)$$

are related one to another by some automorphism $\tilde{\mathcal{T}}$ of $\text{shsa}(1) (\text{osp}(1, 4; \mathbb{C}) = \mathcal{G} \circ \text{osp}(1, 4; \mathbb{C}) \circ \tilde{\mathcal{T}}^{-1})$.

In order to prove Proposition 2, one has to use inner automorphisms of $\text{shsa}(1; \mathbb{C})$ originating from the generators $T^{AB}_{\alpha(0), \beta(0)}$ with $A=1$ or $/$ and $B=1$.

Thus, we identify the generators of $\text{sp}_{dS}(4; \mathbb{C}) (\text{osp}_{dS}(1, 4; \mathbb{C}))$ with $T^{00}_{\alpha(n), \beta(m)}$ at $n+m=2$ ($1 \leq n+m \leq 2$). The related gauge fields will be identified with the (super)gravitational fields. In fact, one has to consider real forms, $\text{osp}_{dS}(1, 4)$ and $\text{sp}_{dS}(4)$, of the complex superalgebras above. The corresponding reality conditions will be discussed in more detail in the subsequent section.

When the gravitational sector is fixed, one can easily analyse the linearized curvatures originating from the full curvatures (2.12). It will be assumed that the gravitational gauge fields related to $\text{sp}_{dS}(4)$ are decomposed as follows:

$$\omega^{00}_{\gamma, \alpha(1), \beta(1)} = h_{\gamma\alpha\beta} + \omega^{100}_{\gamma, \alpha(1), \beta(1)}, \quad (3.5)$$

$$\omega^{00}_{\gamma, \alpha(2), \beta(0)} = w_{\gamma\alpha(2)} + \omega^{100}_{\gamma, \alpha(2), \beta(0)}, \quad (3.6)$$

$$\omega^{00}_{\gamma, \alpha(0), \beta(2)} = \bar{w}_{\gamma\beta(2)} + \omega^{100}_{\gamma, \alpha(0), \beta(2)}, \quad (3.7)$$

where the fields h , w and $\bar{w}$ describe the vierbein and connection of the background de Sitter space, and are of zero order in some (implicit) expansion parameter, while the fields ω^{100} (3.5)-(3.7) and all other fields of $\text{shsa}(1)$ are of the first order. It is also assumed that all background curvatures vanish for chosen background (zero-order) gauge fields (for more detail see ref./2/).

After simple transformations, one finds from eq.(2.12) that this linearization leads to the following linearized curvatures

$$R_{\nu\mu, \alpha(n), \beta(m)}^{FG} = D_{\nu}^L \omega_{\mu, \alpha(n), \beta(m)}^{FG} + \sum_{p,q,s,k,\ell,t} \delta(n-p-q) \delta(m-k-\ell) \delta(p+s-1) \delta(k+t-1) \times \\ \times i^{s+t-1} (-1)^{F+G} \frac{n!m!}{q!\ell!} \delta(|F+G+s+t+1|_2) \times \\ \times h_{\nu\alpha(p)\gamma(s), \beta(k)\delta(\ell)} \omega_{\mu, \alpha(q)\gamma(s), \beta(\ell)\delta(t)}^{FG} - (\nu \leftrightarrow \mu)^{(3.8)}$$

where primes are omitted for the deviations (3.5)-(3.7) of the gravitational fields, and D_{ν}^L is the background Lorentz covariant derivative,

$$D_{\nu}^L \omega_{\mu, \alpha(n), \beta(m)} = \partial_{\nu} \omega_{\mu, \alpha(n), \beta(m)} + n \bar{W}_{\nu\alpha} \gamma \omega_{\mu, \alpha(n-1), \beta(m)} + m \bar{W}_{\nu\beta} \delta \omega_{\mu, \alpha(n), \beta(m-1)} \quad (3.9)$$

As is seen from eq.(3.8), there are two essentially distinguishing cases, $F=G$ and $F+G=1$. At $F=G$, the linearized curvatures (3.8) coincide with the linearized higher-spin curvatures proposed in ref./2/, while at $F+G=1$ the curvatures (3.8) coincide with the linearized curvatures of auxiliary fields proposed in ref./4/. Thus, the fields $\omega_{\nu, \alpha(n), \beta(m)}^{FG}$ at $F=G$ should be identified with the higher-spin fields, while those at $F+G=1$ should be identified with the auxiliary fields of ref./4/.

As a result, any higher-spin or auxiliary field appears twice in the framework of shsa(1). The two copies of fields of any type are completely equivalent on the linearized level. However, they distinguish between on the non-linear level. In fact, it is this difference that leads to the fact that the conjugation (2.13) transforms $\omega_{\nu, \alpha(n), \beta(m)}^{AB}$ into $\omega_{\nu, \beta(m), \alpha(n)}^{BA}$ (we had not succeeded in constructing a non-trivial superalgebra giving rise to a single set of self-conjugated auxiliary fields - for more detail see Appendix)

4. Hermitian Conjugation and P-Transformations

We have already presented in Section 2 the final results for the involution extracting the real form $\text{shsa}(1)$ of $\text{shsa}(1; \mathbb{C})$ and for the P-transformation automorphisms. Here we discuss the reasons that led us to the specific laws (2.13), (2.14).

Most general hermiticity conditions on the gauge fields, which are in accordance with the Lorentz structure, are of the form

$$(\omega_{\nu, \alpha(n), \beta(m)}^{AB})^\dagger = \sum_{\tilde{c}\tilde{d}} \beta_{\tilde{c}\tilde{d}}^{AB}(n, m) \omega_{\nu, \beta(m), \alpha(n)}^{\tilde{c}\tilde{d}}, \quad (4.1)$$

where the matrices $\beta_{\tilde{c}\tilde{d}}^{AB}(n, m)$ (with respect to pairs of indices (AB) and $(\tilde{c}\tilde{d})$) are assumed to be non-degenerate for all n and m . The well-known fact, that hermitian conjugation interchanges dotted and undotted spinorial indices, originates from the requirement that the conjugation under consideration should extract Lorentz real form, $O_L(3, 1)$, out the complex Lie algebra $\text{sp}(2; \mathbb{C}) \oplus \text{sp}(2; \mathbb{C})$.

The hermiticity conditions (4.1) will be supposed to leave invariant the higher-spin superalgebra $\text{shs}(1; \mathbb{C}) \subset \text{shsa}(1; \mathbb{C})$ spanned by the generators $T^{00}{}_{\alpha(n), \beta(m)}$ i.e. it will be assumed that

$$\beta_{\tilde{c}\tilde{d}}^{00}(n, m) = \delta_{\tilde{c}}^0 \delta_{\tilde{d}}^0 \tilde{\beta}(n, m). \quad (4.2)$$

Requiring the curvatures (2.12) to obey the hermiticity conditions analogous to eq. (4.1),

$$(R_{\nu\mu, \alpha(n), \beta(m)}^{AB})^\dagger = \sum_{\tilde{c}\tilde{d}} \beta_{\tilde{c}\tilde{d}}^{AB}(n, m) R_{\nu\mu, \beta(m), \alpha(n)}^{\tilde{c}\tilde{d}}, \quad (4.3)$$

one finds that the following restrictions on the coefficients $\beta_{\tilde{c}\tilde{d}}^{AB}(n, m)$ must hold

$$\begin{aligned} & \sum_{ABC\tilde{d}} \delta(I|F+A+C|_2) \delta(I|G+B+D|_2) (-1)^{A(q+s)+B(t+\ell)} \times \\ & \times \delta(I|C(p+s)+D(t+k)+A(q+s)+B(t+\ell)+s+t+(p+k+s+t)(q+\ell+s+t)+1|_2) \times \\ & \times \beta_{s\tau}^{AB}(p+s, k+t) \beta_{\ell v}^{\tilde{c}\tilde{d}}(q+s, \ell+t) = \\ & = (-1)^{2(k+t)+V(p+s)} \beta_{\ell v}^{FG}(\tau+v)(p+q, k+t) \times \\ & \delta(I|S(\ell+t)+T(q+s)+2(k+t)+V(p+s)+s+t+(p+k+s+t)(q+\ell+s+t)+1|_2). \end{aligned} \quad (4.4)$$

After some tedious calculations similar to those for the higher-spin superalgebras $shs(1; \mathbb{C})$, described in Section 4 of ref. /3/, we found that the general solution of eqs. (4.2), (4.4) is of the form

$$\beta_{\mathcal{C}\mathcal{D}}^{AB}(n, m) = \delta_{\mathcal{D}}^A \delta_{\mathcal{C}}^B (-1)^{\nu n + \mu m + \alpha A + \beta B + (\rho+1)(An+Bm)} \times \\ \times i^{\rho(n+m+|n+m|/2-2)} \quad (4.5)$$

where ν, μ, α, β and ρ are arbitrary parameters taking on values 0 or 1.

Note that this arbitrariness is due to the fact that $shs(1; \mathbb{C})$ possesses involutive automorphisms which are described by the following linear changes of variables:

$$\omega_{\nu, \alpha(n), \beta(m)}^{AB} = (-1)^{\alpha A + \beta B + \gamma n + d m + e(An+Bm)} \times \\ \times i^{e(n+m+|n+m|/2-2)} \omega_{\nu, \alpha(n), \beta(m)}^{AB} \quad (4.6)$$

where a, b, c, d and e can take on arbitrary values 0 or 1. Indeed, one can easily make sure that the curvatures R' , arising from R (2.12) after the substitution (4.6), have the same structure as the curvatures R themselves. It is the important property of the automorphisms (4.6) that they leave the subalgebra $osp_{\mathcal{S}}(1, 4; \mathbb{C})$ invariant, i.e. the fields of $N=1$ supergravity transform among themselves under the transformations (4.6).

Requiring the operation $\dagger$ to be involutive, $(\omega \dagger)^\dagger = \omega$, one finds the following restrictions on the parameters in eq. (4.5):

$$\nu = \mu, \quad \alpha = \beta. \quad (4.7)$$

Taking also into account that one can always find such fields ω' (4.6) that the parameters ν' and α' will vanish in the related coefficients β' (4.1), (4.5), we set

$$\nu = \mu = \alpha = \beta = 0, \quad (4.8)$$

that does not therefore lead to loss of generality.

Thus, there are only two inequivalent hermiticity conditions,

$$(\omega_{\nu, \alpha(n), \beta(m)}^{AB})^\dagger = (-1)^{(\rho+1)(An+Bm)} i^{\rho(n+m+|n+m|/2-2)} \times \\ \times \omega_{\nu, \beta(m), \alpha(n)}^{BA}, \quad (4.9)$$

$\rho = 0$ or 1. These extend the hermiticity conditions for $\text{shs}(1; \mathbb{C})$ found in ref./3/, which can themselves be obtained from eq.(4.9) taken at $A=B=0$. In the framework of $\text{shs}(1; \mathbb{C})$ we had not a criterion indicating any preference of some specific choice $\rho = 0$ or 1 and, therefore, we were enforced to consider the corresponding real forms $\text{shs}_\rho(1)$ both. However, the situation differs drastically for $\text{shsa}(1; \mathbb{C})$ as compared to that for $\text{shs}(1; \mathbb{C})$. Namely, it will be demonstrated in the next section that only the real superalgebra related to the conditions (4.9) at $\rho=0$ does not lead to physical ghosts. Therefore, we confine ourselves to consideration of this real form only. It is this real form corresponding to the conditions (2.13) which is denoted as $\text{shsa}(1)$.

Let us note that following to the method suggested in ref./8/ one can also use eqs.(4.1), (4.5) in the case when eq.(4.7) does not hold in order to construct formally consistent conjugation conditions containing explicitly counterparts of the Klein operator for the involutive automorphisms (4.6). As shown in refs./6-8/, such a procedure leads in fact to some generalized $(^*Z_2)^n$ -graded Lie superalgebras (in the case under consideration, involutive automorphisms (4.6) enable one to construct generalized $(^*Z_2)^n$ -graded Lie superalgebras with $2 \leq n \leq 5$). However, as demonstrated in refs./6,7/, such generalized superalgebras lead to ghosts in non-trivial cases, i.e. relevant representations are not unitary. Therefore, we shall not discuss here these possibilities in more detail.

As for P-transformation laws, they can be found quite similarly to that as done above for hermiticity conditions. The most general P-transformation law for gauge fields, which is in accordance with the Lorentz structure and corresponds to reversing X_2 -axis, is of the form

$$P(\omega_{\nu, \dot{\alpha}(n), \dot{\beta}(m)}^{AB}) = \sum_{c\dot{d}} \gamma_{c\dot{d}}^{AB}(n, m) \omega_{\dot{\nu}, \beta(m), \dot{\alpha}(n)}^{c\dot{d}} \quad (4.10)$$

where $\gamma_{c\dot{d}}^{AB}(n, m)$ are some non-degenerate matrices for all n and m . As mentioned in Section 2, dotted indices indicate changing signs of X_2 -components of vectors (forms). The fact that the P-transformation interchanges dotted and undotted spinorial indices originates from the invariance of the flat vierbein, $\hbar_{\nu \dot{\alpha} \dot{\beta}}$, with

respect to such transformations (for more detail see refs./2,3/). Let us also recall that, in contrast to hermitian conjugation, the operation P does not change an order of product factors and a sign of imaginary unit i .

Requiring the subalgebra $\text{shs}(1; \mathbb{C})$ to be invariant with respect to P-transformations and the curvatures of $\text{shsa}(1; \mathbb{C})$ to transform as a whole, one obtains some restrictions on the coefficients $\gamma_{CD}^{AB}(n,m)$ similar to eqs.(4.2), (4.4). Their general solution turns out to be

$$\gamma_{CD}^{AB}(n,m) = \delta_D^A \delta_C^B (-1)^{2n+2m+2A+2B} + \Delta(A+B) \times \\ \times i^{1/2}(n+m+|n+m|_2-2) \quad (4.11)$$

where the arbitrary parameters $\eta, \beta, \mathcal{L}, \delta$ and Δ take on values 0 or 1 and originate from the automorphisms (4.6) as before.

In fact, eqs.(4.10), (4.11) correspond to the complex case of $\text{shsa}(1; \mathbb{C})$. In order to construct P-transformations corresponding to $\text{shsa}(1)$, one has to require the P-operation (4.10), (4.11) to conserve the hermiticity conditions (2.13). This imposes the additional restrictions

$$\eta = \beta, \quad \mathcal{L} = \delta \quad (4.12)$$

and leads to the P-transformation law (2.14).

Let us note that the parameters $\eta, \mathcal{L}$ and Δ , surviving in eq.(2.14), cannot be restricted further in the framework of $\text{shsa}(1)$ itself. They correspond to $(^*Z_2)^3$ -grading of $\text{shsa}(1)$ originating from mutually commuting involutive automorphisms provided by eq.(4.6) taken at $a=b=1, c=d=e=0$; $a=b=e=0, c=d=1$ and $a=b=c=d=0, e=1$ (to avoid misunderstandings, we stress that it is $(^*Z_2)^3$ -grading of ordinary Lie superalgebra $\text{shsa}(1)$, which has nothing to do with generalized $(^*Z_2)^n$ -graded Lie superalgebras mentioned above). From the physical point of view, this $(^*Z_2)^3$ -grading implies existence of superselection rules with respect to the numbers $|A+B|_2$, $|n+m|_2$ and $\frac{1}{2}|n+m+|n+m|_2-2|_4$. The superselection rule related to $|n+m|_2$ distinguishes between bosons and fermions. That, related to $|A+B|_2$, distinguishes between higher-spin and auxiliary fields. The boson-fermion superselection rule should take place in an arbitrary relativistic theory, while the rest ones may be

broken in a dynamical theory (an action may be non-invariant under the corresponding transformations (4.6)).

5. Operator Realization and Inequalities on Boson Generators

When superalgebra shsa(1) had been constructed, we discovered that it admitted a natural operator realization generalizing that found by one of us in ref. /10/ for higher-spin superalgebras. Namely, in addition to spinorial operators $\hat{q}_\alpha$ and $\hat{z}_\beta$ introduced in ref./10/ which obey Heisenberg commutation relations,

$$[\hat{q}_\alpha, \hat{q}_\beta] = 2i\epsilon_{\alpha\beta}, [\hat{z}_\alpha, \hat{z}_\beta] = 2i\epsilon_{\alpha\beta}, [\hat{q}_\alpha, \hat{z}_\beta] = 0 \quad (5.1)$$

($\epsilon_{12} = 1$), operators $\hat{Q}$ and $\hat{R}$ are to be introduced possessing the properties $\{\hat{Q}, \hat{q}_\alpha\} = 0$, $[\hat{Q}, \hat{z}_\beta] = 0$, $[\hat{Q}, \hat{R}] = 0$, $\hat{Q}^2 = 1$; (5.2)

$$[\hat{R}, \hat{q}_\alpha] = 0, \{\hat{R}, \hat{z}_\beta\} = 0, \hat{R}^2 = 1 \quad (5.3)$$

($\{, \}$ and $[,]$ denote anticommutator and commutator respectively). Consider an associative algebra of all polynomials constructed from the operators $\hat{q}_\alpha, \hat{z}_\beta, \hat{Q}$ and $\hat{R}$. An arbitrary element of this algebra is of the form

$$\mathcal{P}(\hat{q}, \hat{z}, \hat{Q}, \hat{R}) = \sum_{n,m; A,B} \mathfrak{Z}^{AB} d(n) \beta(m) \frac{1}{n!m!} (\hat{Q})^A (\hat{R})^B \underbrace{\hat{q}_\alpha \dots \hat{q}_\alpha}_n \underbrace{\hat{z}_\beta \dots \hat{z}_\beta}_m \quad (5.4)$$

where the Weyl (symmetric) ordering for $\hat{q}_\alpha$ and $\hat{z}_\beta$ is implied, and the summation parameters n, m and A, B take on their values from 0 to ∞ and from 0 to 1 respectively ($(\hat{Q})^2 = (\hat{R})^2 = 1$). Starting from this associative algebra, one defines as usual a Lie superalgebra by fixing an automorphism of boson-fermion parity, f , as follows:

$$f(\mathcal{P}(\hat{q}, \hat{z}, \hat{Q}, \hat{R})) = \mathcal{P}(-\hat{q}, -\hat{z}, \hat{Q}, \hat{R}) \quad (5.5)$$

(for more detail see e.g. ref./10/). In fact, this implies that the coefficients $\mathfrak{Z}^{AB} d(n) \beta(m)$ in eq.(5.4) are assumed to be even (odd) elements of some Grassmann algebra if the number of spinorial indices, $n+m$, is even (odd), and that the product law (2.9) in the Lie superalgebra constructed in such a way is identical

fied with the ordinary commutator for elements of the form (5.4).

Acting in the same fashion as in ref./10/ for higher-spin superalgebras, one can make sure that the complex superalgebra arising as a result coincides with the superalgebra $\text{shsa}(1; \mathbb{C})$ as defined in Section 2, while the reality conditions (2.13) now read

$$(\hat{\mathcal{Q}}_a)^+ = \hat{\mathcal{Z}}_a, \quad (\hat{\mathcal{Z}}_a)^+ = \hat{\mathcal{Q}}_a, \quad \hat{\mathcal{Q}}^+ = \hat{R}, \quad \hat{R}^+ = \hat{\mathcal{Q}}. \quad (5.6)$$

Moreover, extended superalgebras of higher spins and auxiliary fields, $\text{shsa}(N)$, can now easily be constructed by introducing Clifford generating elements $(\{\hat{\psi}_i, \hat{\psi}_j\} = 2\delta_{ij}, \psi_i^+ = \psi_i)$ as in ref./10/ for higher-spin superalgebras (in fact, as emphasized in ref./10/, it is convenient to handle with superalgebras formed by polynomials of even overall powers in $\hat{\psi}$, $\hat{\mathcal{Q}}$ and $\hat{\mathcal{Z}}$).

Let us note that four of basis automorphisms described by eq.(4.6) originate from the following automorphisms of the initial associative algebra:

$$\mathcal{T}_{\mathcal{Q}}(\mathcal{P}(\hat{\mathcal{Q}}, \hat{\mathcal{Z}}, \hat{\mathcal{Q}}, \hat{R})) = \mathcal{P}(-\hat{\mathcal{Q}}, \hat{\mathcal{Z}}, \hat{\mathcal{Q}}, \hat{R}), \quad (5.7)$$

$$\mathcal{T}_{\mathcal{Z}}(\mathcal{P}(\hat{\mathcal{Q}}, \hat{\mathcal{Z}}, \hat{\mathcal{Q}}, \hat{R})) = \mathcal{P}(\hat{\mathcal{Q}}, -\hat{\mathcal{Z}}, \hat{\mathcal{Q}}, \hat{R}), \quad (5.8)$$

$$\mathcal{T}_{\hat{\mathcal{Q}}}(\mathcal{P}(\hat{\mathcal{Q}}, \hat{\mathcal{Z}}, \hat{\mathcal{Q}}, \hat{R})) = \mathcal{P}(\hat{\mathcal{Q}}, \hat{\mathcal{Z}}, -\hat{\mathcal{Q}}, \hat{R}), \quad (5.9)$$

$$\mathcal{T}_{\hat{R}}(\mathcal{P}(\hat{\mathcal{Q}}, \hat{\mathcal{Z}}, \hat{\mathcal{Q}}, \hat{R})) = \mathcal{P}(\hat{\mathcal{Q}}, \hat{\mathcal{Z}}, \hat{\mathcal{Q}}, -\hat{R}), \quad (5.10)$$

while the fifth ($a=b=c=d=0, e=1$) originates in the same fashion as discussed in ref./10/ for higher-spin superalgebras from the following antiautomorphism, $\mathcal{Q}$, of the associative algebra:

$$\mathcal{Q}(\hat{\mathcal{Q}}_a) = i\hat{\mathcal{Q}}_a, \quad \mathcal{Q}(\hat{\mathcal{Z}}_a) = i\hat{\mathcal{Z}}_a, \quad \mathcal{Q}(\hat{\mathcal{Q}}) = \hat{\mathcal{Q}}, \quad \mathcal{Q}(\hat{R}) = \hat{R}. \quad (5.11)$$

It is also worth mentioning that the same as for higher-spin superalgebras in ref./10/, the bilinear form (2.5) is nothing but $t\mathcal{Z}(\mathcal{P}_1, \mathcal{P}_2)$ where, by definition, $t\mathcal{Z}(\mathcal{P}) = \mathcal{Z}^{00}, \mathcal{Z}^{(0)\hat{R}(0)}$ for an arbitrary element $\mathcal{P}$ of the form (5.4) (the Weyl ordering is essential here). This definition of the trace operation is consistent, $t\mathcal{Z}([\mathcal{P}_1, \mathcal{P}_2]) = 0$, because constants (i.e. polynomials independent on $\hat{\mathcal{Q}}, \hat{\mathcal{Z}}, \hat{\mathcal{Q}}$ and $\hat{R}$) form Abelian component $\mathfrak{o}(2)$ in the decomposition

$$\text{shsa}(1) = \text{shsa}'(1) \oplus o(2).$$

It is important that the operator realization above does not lead to unitary representations of $\text{shsa}(1)$. To see this one can introduce^{*} creation and annihilation operators $\alpha_i^\dagger$ and α_i ($i=1,2$) constructed from $\hat{\psi}_\alpha$ and $\hat{\psi}_\beta = (\hat{\psi}_\alpha)^\dagger$, define corresponding Fock space F and identify a representation space V with $F \oplus (\hat{Q}|0\rangle \otimes F) \oplus (\hat{R}|0\rangle \otimes F) \oplus (\hat{Q}\hat{R}|0\rangle \otimes F)$ where $|0\rangle$ is a Fock vacuum. Then, by using eqs.(5.1)-(5.3),(5.6), one observes that V turns out to be non-positive definite whatever values are chosen for $\langle 0|\hat{Q}|0\rangle = \langle 0|\hat{R}|0\rangle$ and $\langle 0|\hat{Q}\hat{R}|0\rangle$ (the same is true for tensorial products $V \otimes \dots \otimes V$ which can be described by introducing additional species for all operators and considering polynomials with no terms mixing different species).

Perhaps, this implies that $\text{shsa}(1)$ admits no unitary representations at all and one may wonder whether any consistent field theory can exist in this case, based on $\text{shsa}(1)$. For the first sight, it cannot because it seems that $\text{shsa}(1)$ should serve as a global symmetry of a symmetric physical vacuum state, and physical states must form some unitary representation of $\text{shsa}(1)$ if physical ghosts are assumed to be absent. However, a more careful analysis is needed here because there are auxiliary fields among the gauge fields related to $\text{shsa}(1)$, the fields which were shown in ref./4/ to possess no physical degrees of freedom after complete gauge fixing. In fact, it is likely that $\text{shsa}(1)$ will be deformed in such a way after appropriate localization in a related physical theory that only its higher-spin subalgebra, $\text{shs}^F(1)$, will lead to nontrivial global symmetry acting on physical states, the subalgebra which gives rise to massless fields $\mathcal{W}_\nu^{AB}, \alpha(n), \beta(m)$ with $A=B$ (evidently, $\text{shs}^F(1)$ coincides with the subalgebra of $\text{shsa}(1)$ formed by polynomials constructed from $\hat{q}, \hat{r}$ and the Klein operator $\hat{K} = \hat{Q}\hat{R}$). Indeed, after complete gauge fixing auxiliary fields, $\mathcal{W}_\nu^{AB}, \alpha(n), \beta(m)$ with $A+B=1$, cannot form nontrivial representations at all while non-unitarity of pure gauge degrees of freedom is both well-known and non-dangerous.

^{*} We are grateful to S.E.Konahtein for illuminating discussion of this point.

Thus, it is only necessary to require the full higher-spin subalgebra of $\text{shsa}(1)$ to possess unitary representations. Remarkably, $\text{shs}^f(1)$ does indeed possess unitary representations. For example, this can easily be seen by identifying a representation space^{*} V of $\text{shs}^f(1)$ with $F \oplus (\hat{\mathcal{K}}|0\rangle \oplus F)$ (here $\hat{\mathcal{K}} = \hat{Q}\hat{R}$) and by setting $\langle 0|\hat{\mathcal{K}}|0\rangle = 0$ (note that for $\text{shsa}(1)$ itself, in which $\hat{Q}$ and $\hat{R}$ act separately, the latter condition leads to ghosts because it implies that $\hat{Q}|0\rangle$ and $\hat{R}|0\rangle$ are zero-norm states). Let us however analyse this point from somewhat different point of view. In particular, the subsequent analysis will contain a proof of the fact that eq.(2.13) (or, equivalently, eq. (5.6)) describes the only admissible reality conditions if $\text{shs}^f(1)$ is required to possess unitary representations.

As is well-known, inequalities on spectra of boson operators can often be obtained in unitary representations of Lie superalgebras as consequences of anticommutation relations for conjugate fermion operators. A famous example of such inequalities is provided by the non-negativity condition for energy operator in ordinary supersymmetry. Indeed, if it follows from structure relations of some superalgebra g that an operator A can be represented in the form

$$A = \alpha \sum_i \{\psi_i, \psi_i^+\} \quad (5.12)$$

where ψ_i are some odd elements of g then A is semi-positive (negative) definite in any unitary representation of g if α is positive (negative). Moreover, if A admits simultaneously a representation

$$A = \beta \sum_i \{\varphi_i, \varphi_i^+\} \quad (5.13)$$

with $\alpha\beta < 0$ then g admits no unitary representations at all with A non-vanishing (both finite- and infinite-dimensional). In physical terms, it follows

^{*} Obviously, the higher-spin superalgebra of ref./3/, $\text{shs}_0(1)$, which is formed /10/ by the operators $\hat{Q}$ and $\hat{R}$, admits unitary representations as well (to see this, one can set $V=F$ in this case).

from eqs.(5.12),(5.13) at $\alpha \cdot \beta < 0$ that ghosts (negative norm states) are inevitable.

Let us analyse from this point of view the real forms of $\text{shsa}(1; \mathcal{C})$ extracted by the conditions of the form (4.9). It follows from the antihermiticity conditions for the quantities $\mathcal{W}_{\nu}^{AB}, \alpha(n), \beta(m), T_{AB}^{\alpha(n), \beta(m)}$ combined with eq.(4.9) that the generators $T_{AB}^{\alpha(n), \beta(m)}$ should obey the relations

$$(T_{AB}^{\alpha(n), \beta(m)})^\dagger = -(-1)^{(\rho+1)(A+Bm)} i^{\rho(n+m+1)(n+m)_2-2} \times \\ \times T_{BA}^{\beta(m), \alpha(n)} \quad (5.14)$$

(for definiteness, we have assumed here that the generators T commute with the gauge fields $\mathcal{W}$, but one can equally well assume that fermion generators anticommute with fermion gauge fields). On the other hand, it follows from eqs.(2.1), (2.2), (2.9) that

$$\{T_{AB}^{\alpha(n), \beta(m)}, T_{\mathcal{C}\mathcal{D}}^{\gamma(u), \delta(v)}\} = \\ = \frac{1}{2} \sum_{\rho\gamma\beta\kappa\ell t} \delta(n-\rho-\beta) \delta(m-\kappa-\ell) \delta(u-\gamma-\beta) \delta(v-\ell-t) \times \\ i^{3+t-1} \frac{(\rho+\gamma)!(\kappa+\ell)!}{\rho!\gamma!\beta!\kappa!\ell!t!} \underbrace{\epsilon^{\alpha\gamma} \dots \epsilon^{\alpha\gamma}}_s \underbrace{\epsilon^{\beta\delta} \dots \epsilon^{\beta\delta}}_t \quad (5.15) \\ \times [(-1)^{Cn+2m} - (-1)^{Au+Bv+3+t+(n+m)(u+v)}] T_{A+C\ B+D}^{\alpha(\rho)\gamma(\gamma)\beta(\kappa)\delta(\ell)}$$

We are interested in those relations contained in eq.(5.15) whose left hand sides contain anticommutators of conjugate fermion operators. By setting in eq.

(5.15) $u=n, v=n, n+m=1$ and using eq.(5.14), one finds

$$\{T_{AB}^{\alpha(n), \beta(m)}, (T_{\mathcal{C}\mathcal{D}}^{\gamma(n), \delta(m)})^\dagger\} = \\ = \frac{1}{2} \sum_{\rho\gamma\beta\kappa\ell t} \delta(n-\rho-\beta) \delta(m-\kappa-\ell) \delta(m-\gamma-\beta) \delta(n-\ell-t) \times \\ \times i^{3+t+1} \frac{(\rho+\gamma)!(\kappa+\ell)!}{\rho!\gamma!\beta!\kappa!\ell!t!} \underbrace{\epsilon^{\alpha\gamma} \dots \epsilon^{\alpha\gamma}}_s \underbrace{\epsilon^{\beta\delta} \dots \epsilon^{\beta\delta}}_t \times \quad (5.16)$$

$$\times i^{\rho(n+m-1)} (-1)^{(\rho+1)(2n+Cm)} [(-1)^{Cn+2m} + (-1)^{Am+Bn+3+t}] \times \\ \times T_{A+C, B+D}^{\alpha(p)\gamma(q), \beta(k)\delta(e)}$$

at $n+m$ odd. Then, one finds from eq.(5.16) taken at $A=D$, $B=C$ and symmetrized over the indices α, γ and β, δ that

$$\{T_{AB}^{\alpha(n), \beta(m)}, (T_{AB}^{\beta(n), \alpha(m)})^\dagger\} = i^{\rho(n+m-1)+1} \frac{((n+m)!)^2}{(n!)^2 (m!)^2} \times \\ \times (-1)^{(\rho+1)(An+Bm)+Bn+Am} T_{A+BA, B}^{\alpha(n+m), \beta(n+m)} \quad (5.17)$$

at $n+m$ odd.

Let us introduce some positive definite hermitian matrix $C_{\alpha\beta}$ (e.g.

$C_{\alpha\beta} = \delta_{\alpha\beta}$). As is easily seen, the quantity

$$\underbrace{C_{\alpha\beta} \dots C_{\alpha\beta}}_{n+m} \{T_{AB}^{\alpha(n), \beta(m)}, (T_{AB}^{\beta(n), \alpha(m)})^\dagger\} \quad (5.18)$$

can be represented in the form $\sum_{i,j} dij \{\psi_i, \psi_j^\dagger\}$ where dij is some positive definite hermitian matrix. Taking into account that only the higher-spin subalgebra with the generators T_{AA} is required to possess unitary representations, we fix $A=B$ in eq.(5.17). As a result, one derives from eq.(5.17) that the following inequalities should take place in unitary representations of $shs^f(1)$:

$$(-1)^A i^{\rho(n+m-1)} \rho(n+m, C) \geq 0 \quad (5.19)$$

at $|n+m|_2=1$

where

$$\rho(n, C) = i \underbrace{C_{\alpha\beta} \dots C_{\alpha\beta}}_n T_{00}^{\alpha(n), \beta(n)} \quad (5.20)$$

(note that all inequalities of the type (5.19) should be understood as the inequalities on averages of operators over arbitrary states in a Hilbert space under consideration). Obviously, inequalities (5.19) are consistent for $A=0$ and 1 if and only if $\rho = 0$ (the trivial case $P(n, C) \equiv 0$ is disregarded).

Thus it is shown that only higher-spin subalgebra $shs^f(1)$ of the real superalgebra $shsa(1)$, originating from the hermiticity conditions (2.13), can admit unitary representations. Consequently, we confine ourselves to consideration of this real form of $shsa(1; \mathbb{C})$ only, i.e. we set $\rho = 0$ everywhere.

The inequalities (5.19) take now the following simple form

$$P(n, C) \geq 0 \quad \text{at } |n|_2 = 1 \quad (5.21)$$

with the matrix $C_{\alpha\beta}$ in eq.(5.20) assumed to be hermitian and positive definite. These inequalities serve as a direct generalization of the energy non-negativity condition in ordinary supersymmetry (both flat and anti-de Sitter) which is itself a particular case of eq.(5.21) at $n=1, A=0$,

$$P(1, C) = i C_{\alpha\beta} T_{00}^{\alpha(1)\beta(1)} \geq 0. \quad (5.22)$$

Here $C_{\alpha\beta}$ describes some time-like direction on which the four-momentum vector is projected.

In tensorial terms, eq.(5.21) contains some components of the operators $T_{00}^{\alpha(n)}$ (at n odd) which are symmetric traceless tensors with respect to vectorial indices $a = 0-3$. As an example, for $C_{\alpha\beta} = \delta_{\alpha\beta}$, eq.(5.21) reduces to the following

$$i T_{00}^{\overbrace{\alpha\alpha\alpha\alpha}^n} \geq 0 \quad \text{at } n \text{ odd}. \quad (5.23)$$

Formally, one can derive analogous inequalities on boson generators of $shsa(1)$ as a whole assuming that it is realized in a positive definite Hilbert space. It

seems natural to expect that if $\text{shsa}(1)$ admits no unitary representations, then this fact will manifest itself in some inconsistencies of the type (5.12), (5.13) at $\alpha\beta < 0$. However, we had found no such inconsistencies (in the case $\rho=0$) when analyzed a broad class of inequalities which should hold in unitary representations of $\text{shsa}(1)$. Perhaps, this indicates that although the operator realization described at the beginning of this section does not lead to unitary representations of $\text{shsa}(1)$, some unitary representations of $\text{shsa}(1)$ exist which are not related directly to this realization. It seems interesting in itself to solve the question definitely whether unitary representations of $\text{shsa}(1)$ exist or not (although their physical relevance is not completely clear as discussed at the beginning of this section).

We conclude this section by presenting some inequalities which should hold in unitary representations of $\text{shsa}(1)$ (if they exist), and can be derived in the same fashion as before in derivation of eq.(5.21) for $\text{shsf}(1)$. Namely, one can easily show that the matrix $R_{AB}(n, C)$ defined by the relation

$$R_{AB}(n, C) = i(-1)^B T_{AB}^{\alpha(n) \beta(m)} \underbrace{C_{\alpha\beta} \dots C_{\alpha\beta}}_n \quad (5.27)$$

should form some (semi)positive definite hermitian matrix (when averaged over an arbitrary state in a representation Hilbert space) for all odd n and any positive definite hermitian matrix $C_{\alpha\beta}$. Obviously, this condition includes eq.(5.21) as a particular case. Note, however, that the matrix $R_{AB}(n, C)$ is not necessarily semi-positive definite in non-unitary representations of $\text{shsa}(1)$ including those based on the operator realization above.

It is also worth mentioning that proceeding in the same fashion as discussed above for $\text{shs}^f(1)$, one can derive analogous inequalities for the higher-spin superalgebras of ref./3/ without referring to $\text{shsa}(1)$. Specifically, one finds that the inequalities $\rho(n, C) > 0$ hold at n odd for $\text{shs}_0(1)$ were $\rho(n, C)$ is defined by eq.(5.20) and T_{00}'''' are identified with generators of $\text{shs}_0(1)$. For $\text{shs}_2(1)$, corresponding inequalities will have the form of eq.(5.19) at $A=0$ and $\rho=1$, but no inconsistencies emerge in the framework of $\text{shs}_2(1)$ itself (note however that some different arguments are presented in ref./10/ for extended higher-spin superalgebras which rule out the case $\rho=1$).

6. Subalgebras of $\text{shsa}(1)$ and Alternative Possibilities for Superalgebras of Higher Spins and Auxiliary Fields

In this Section, we consider subalgebras of $\text{shsa}(1)$ paying main attention to those of them which can serve themselves as superalgebras of higher spins and auxiliary fields.

To begin with, we make the following general remark. If $\mathcal{T}_i$ ($i=1, 2, \dots$) are some automorphisms of an arbitrary algebra A , then the elements $a \in A$, obeying the conditions

$$\mathcal{T}_i(a) = a, \quad (6.1)$$

form some subalgebra of A .

This general property enables one to construct subalgebras of $\text{shsa}(1)$ with the aid of the automorphisms (4.6). Note that eq.(4.6) describes the automorphisms of $\text{shsa}(1; \mathbb{C})$. Hence, in order to construct subalgebras of the real algebra $\text{shsa}(1)$ starting from the automorphisms (4.6), one must require the conditions of the type (6.1) to be invariant under the conjugation (involution) extracting $\text{shsa}(1)$ out of $\text{shsa}(1; \mathbb{C})$. It is worth mentioning that a class of subalgebras arising as a result is broader than that constructed by use only automorphisms of $\text{shsa}(1)$ itself.

Thus, any set of gauge fields $\mathcal{W}$ (equivalently, parameters $\mathfrak{F}$), obeying the system of equations of the form

$$\begin{aligned} \mathcal{W}_{\gamma, \alpha(n), \beta(m)}^{AB} = & (-1)^{\alpha_j A + \beta_j B + c_j n + d_j m + e_j (A+Bm)} \times \\ & \times i e_j (n+m + |n+m|_2 - 2) \mathcal{W}_{\gamma, \alpha(n), \beta(m)}^{AB} \end{aligned} \quad (6.2)$$

($j=1, 2, \dots$), corresponds to some subalgebra of $\text{shsa}(1)$ if these equations are invariant under the interchange $(A, n) \leftrightarrow (B, m)$. Let us give a few examples of subalgebras of $\text{shsa}(1)$ which can be obtained in such a way.

The higher-spin subalgebra $\text{shs}_0(1)$, spanned by the generators $T_{00}^{\alpha(n), \beta(m)}$, arises from the conditions

$$\mathcal{W}_{\gamma, \alpha(n), \beta(m)}^{AB} = (-1)^A \mathcal{W}_{\gamma, \alpha(n), \beta(m)}^{AB}, \quad (6.3a)$$

$$\mathcal{W}_{\gamma, \alpha(n), \beta(m)}^{AB} = (-1)^B \mathcal{W}_{\gamma, \alpha(n), \beta(m)}^{AB}. \quad (6.3b)$$

The full higher-spin subalgebra, ^{$\text{shs}^{\mathbb{F}(1)}$} spanned by the generators $T_{AA}^{\alpha(n), \beta(m)}$, with both $A=0$ and $A=1$, originates from the condition

$$\mathcal{W}_{\gamma, \alpha(n), \beta(m)}^{AB} = (-1)^{A+B} \mathcal{W}_{\gamma, \alpha(n), \beta(m)}^{AB} \quad (6.4)$$

The subalgebra, spanned by the generators $T_{AB}^{d(n), \beta(m)}$ with even n and m , arises from the equations

$$\mathcal{W}_{\nu, d(n), \beta(m)}^{AB} = (-1)^n \mathcal{W}_{\nu, d(n), \beta(m)}^{AB}, \quad (6.5a)$$

$$\mathcal{W}_{\nu, d(n), \beta(m)}^{AB} = (-1)^m \mathcal{W}_{\nu, d(n), \beta(m)}^{AB}. \quad (6.5b)$$

The even (boson) subalgebra of $\text{shsa}(1)$ can be extracted by the condition

$$\mathcal{W}_{\nu, d(n), \beta(m)}^{AB} = (-1)^{n+m} \mathcal{W}_{\nu, d(n), \beta(m)}^{AB}. \quad (6.6)$$

In principle, it is not difficult to describe all subalgebras of $\text{shsa}(1)$ which can be extracted by virtue of the conditions (6.2). We consider in more detail those subalgebras of $\text{shsa}(1)$ which are of greatest interest as candidates for the role of independent superalgebras of higher spins and auxiliary fields. These subalgebras, denoted as $\text{shsa}(1|a,b)$, are extracted by the conditions

$$\mathcal{W}_{\nu, d(n), \beta(m)}^{AB} = (-1)^{\alpha(A+B) + \beta(n+m) + A n + B m} i^{n+m+|n+m|_2 - 2} \times \mathcal{W}_{\nu, d(n), \beta(m)}^{AB}. \quad (6.7)$$

It is not difficult to make sure that subalgebras $\text{shsa}(1|a,b)$ give rise to the following non-zero gauge fields (all other gauge fields of $\text{shsa}(1|a,b)$ vanish):

(i) Boson higher-spin fields of even spins, i.e. the fields $\mathcal{W}_{\nu, d(n), \beta(m)}^{AA}$ with $A = 0, 1$ and $n+m = 2(s-1)$ at $s = 2, 4, 6 \dots$

(ii) Fermion higher-spin fields $\mathcal{W}_{\nu, d(n), \beta(m)}^{AA}$ with $(n+m) = 2(s-1)$ at $s = \frac{3}{2} + 2k + |b + A|_2$, $k = 0, 1, 2 \dots$

(iii) Boson auxiliary fields $\mathcal{W}_{\nu, d(n), \beta(m)}^{A+1}$ with an arbitrary integer parameter $t = \frac{1}{2}(n-m)$ obeying the condition $|t|_2 = |1 + \alpha|_2$.

(iii) Fermion auxiliary fields $\mathcal{W}_{\nu, d(n), \beta(m)}^{A+1}$ with an arbitrary integer parameter $t = \frac{1}{2}(n-m-1)$ obeying the condition $|t|_2 = |\alpha + \beta + A|_2$.

Note that all subalgebras $\text{shsa}(1|a,b)$ contain twice boson higher-spin fields of each even spin, but do not contain boson higher-spin fields of odd spins. On the other hand, all these subalgebras contain once fermion higher-spin fields of each spin $s > \frac{3}{2}$.

Similarly, subalgebras $\text{shsa}(1|a,b)$ contain twice boson auxiliary fields with

the parameter t possessing an appropriate parity related to a and do not contain boson auxiliary fields of the opposite parity of t . Fermion auxiliary fields emerge once for each representation of the Lorentz group.

The subalgebras $shsa(1|a,b)$ deserve great attention as independent candidates for the role of superalgebras of higher spins and auxiliary fields. Indeed, the same as $shsa(1)$ itself, they contain complete systems of gauge fields which were related in refs./2,4/ to free higher-spin and auxiliary fields, and, therefore, may describe non-trivial interactions of these fields. Specifically, superalgebras $shsa(1|a,b)$ contain de Sitter subalgebra $Sp_dS(4)$ giving rise ^{the} to gravitational spin-2 field. It can well turn out that consistent theories exist based on both $shsa(1)$ and $shs(1|a,b)$.

However, let us stress once again that both $shsa(1)$ and $shsa(1|a,b)$ contain twice boson higher-spin fields of any even spin. Specifically, they contain twice spin-two fields that may lead to important physical consequences if these spin-two fields may serve both as gravitational fields in two worlds whose mutual interaction is suppressed enough (for more detail see next section).

One can construct a number of other infinite-dimensional subalgebras of $shsa(1)$ by imposing further restrictions of the type (6.2). All these subalgebras contain the Lorentz subalgebra $O_4(3,1)$ spanned by the generators $T_{00}^{a(2)\dot{a}(0)}$, and $T_{00}^{a(0)\dot{a}(2)}$. In fact, all those subalgebras of $shsa(1)$, which can be obtained in such a way and contain themselves $Sp_dS(4)$ as subalgebra, may serve as independent superalgebras of higher-spins and auxiliary fields (if the latter survive after imposing conditions (6.2)). The rest ones, which do not contain $Sp_dS(4)$, are of interest as possible counterparts of the Lorentz subalgebra in (super)gravity, the subalgebra which remains undeformed after transition to dynamical local symmetries (for more detail see ref./3/). Another class of Lorentz-like subalgebras of $shsa(1)$ is formed by those spanned by the generators $T_{AB}^{a(0)\dot{a}(2k)}$, and $T_{AB}^{a(2k)\dot{a}(0)}$ and their further intersections with the subalgebras above. Now let us discuss finite-dimensional subalgebras of $shsa(1)$.

One of most interesting subalgebras of $shsa(1)$ is that spanned by the genera-

tors $T_{00}^{\alpha(n), \beta(m)}$ with $0 \leq n+m \leq 2$ and $T_{11}^{\alpha(n), \beta(m)}$ with $0 \leq n+m \leq 1$.

Indeed, one can readily make sure that these generators form some subalgebra of $\text{shsa}(1)$. In fact, it is easily verified that this subalgebra is isomorphic to the direct sum of $N=2$ de Sitter superalgebra $osp_{dS}(2, 4)$ and the trivial $o(2)$ subalgebra originating from the generator $T_{00}^{\alpha(0), \beta(0)}$ (let us stress that this $o(2)$ subalgebra emerges in the decomposition $\text{shsa}(1) = \text{shsa}'(1) \oplus o(2)$ in which the subalgebra $\text{shsa}'(1)$ is simple). The generators $T_{00}^{\alpha(n), \beta(m)}$ with $1 \leq n+m \leq 2$ form the subalgebra $osp_{dS}(1, 4)$ while the supergenerators $T_{11}^{\alpha(k), \beta(l)}$ with $k+l = 1$ combined with the $o(2)$ -generator $T_{11}^{\alpha(0), \beta(0)}$ extend $osp_{dS}(1, 4)$ to $osp_{dS}(2, 4)$. The superalgebra $osp_{dS}(2, 4) \oplus o(2)$ is a maximal finite-dimensional subalgebra of $\text{shsa}(1)$, i.e. it does not belong to any other finite-dimensional subalgebra of $\text{shsa}(1)$. Note that one can construct a family of subalgebras $osp(2, 4) \subset \text{shsa}(1)$ by virtue of inner automorphisms induced by the generators $T_{AB}^{\alpha(0), \beta(0)}$. These subalgebras $osp(2, 4)$ are spanned by some specific combinations of the generators $T_{AB}^{\alpha(n), \beta(m)}$ with $0 \leq n+m \leq 2$.

Perhaps, the fact that $\text{shsa}'(1)$ contains $osp_{dS}(2, 4)$ as its maximal finite-dimensional subalgebra indicates that the interpretation of $\text{shsa}'(1)$ as $N=1$ superalgebra of higher spins and auxiliary fields is somewhat inadequate, and one should regard $\text{shsa}(1)$ as $N=2$ superalgebra. The fact is also indicating in favour of this point of view that the superalgebras $\text{shsa}(1|a, b)$ contain $osp_{dS}(1, 4)$ as their maximal finite-dimensional subalgebras and, therefore, pretend themselves for the role of $N=1$ superalgebras of higher spins and auxiliary fields.

Note also that it is the choice of $\rho = 0$ for the involution (4.9) which ensures the subalgebra above to coincide with $osp(2, 4)$. The opposite choice, $\rho = 1$, leads to superalgebra $osp(1, 1, 4)$ which is known to possess no nontrivial unitary representations [9, 6].

In addition to $osp_{dS}(2, 4) \oplus o(2)$, we mention the following maximal finite-dimensional subalgebras of $\text{shsa}(1)$, which are Lie algebras.

(1) The subalgebra $sp(4) \oplus sp(4) \oplus o(2) \oplus o(2)$ spanned by the generators

$T_{AA}^{\alpha(n), \beta(m)}$ with $A=0$ or 1 and $n+m=0$ or 2 .

(ii) The subalgebra $\mathfrak{o}(3,1) \oplus \mathfrak{o}(3,1) \oplus (\mathfrak{o}(2))^4$ spanned by the generators $T_{AA}^{\alpha(n)} \hat{A}^{\beta(m)}$ with $n=0, m=2$ and $n=2, m=0$ ($A=0$ and 1) combined with the generators $T_{AB}^{\alpha(0)} \hat{A}^{\beta(0)}$ with arbitrary A and B .

Note that the fact, that the subalgebras (i) and (ii) contain each their component at least twice, follows from the reducibility of the boson sector as discussed in the next section.

Let us also mention that a great number of contractions of $\mathfrak{shsa}(1)$ can be constructed by virtue of its subalgebras as explained in ref./3/. However, it seems that all such contractions (both flat and non-flat) can hardly be used for constructing non-trivial interacting theories of higher spins and auxiliary fields. On the other hand, linearized curvatures of higher-spin fields and auxiliary fields, found respectively in ref./2/ and ref./4/, correspond to the contraction of $\mathfrak{shsa}(1)$ with respect to $Sp_{dS}(4)$ (for more detail see ref./3/). Note also that although we found in ref./3/ a contraction $\mathfrak{shs}_{\rho}^*(1)$ of $\mathfrak{shs}_{\rho}(1)$, which could not be obtained by virtue of any subalgebra of $\mathfrak{shs}_{\rho}(1)$, we did not succeed in constructing any its extension to some contraction of $\mathfrak{shsa}(1)$. In fact, $\mathfrak{shs}_{\rho}^*(1)$ was shown in ref./10/ to coincide with the Poisson bracket contraction of $\mathfrak{shs}_{\rho}(1)$, and these are the operators $\hat{Q}$ and $\hat{R}$ which prevent from extending this contraction to $\mathfrak{shsa}(1)$ as a whole.

7. Reducibility of the Boson Sector of $\mathfrak{shsa}(1)$

The fact that even (boson) subalgebra of $\mathfrak{shsa}(1)$ splits into a direct sum of two subalgebras is quite clear from the operator realization of $\mathfrak{shsa}(1)$ described in Section 5. Indeed, the Klein operator $\hat{K} = \hat{Q}\hat{R}$ commutes with all bosonic generators of $\mathfrak{shsa}(1)$. Since $\hat{K}$ is hermitian and $\hat{K}^2 = 1$, one simply projects out two subalgebras mentioned above by means of projectors $\Pi_{\pm} = \frac{1}{\sqrt{2}}(1 \pm \hat{K})$. In the component notations one proceeds as follows.

Let us consider superalgebra shsa(1) in a new basis by introducing new gauge fields W as follows:

$$W_{\nu, \alpha(n), \beta(m)}^{A\Omega} = \sum_{B, C=0,1} (-1)^{\Omega C} \delta(|A+B+C|_2) (W_{\nu, \alpha(n), \beta(m)})^{BC} \quad (7.1)$$

where the index Ω , as well as A, B and C, takes on values 0 or 1. By using the simple relation

$$\sum_{\Omega=0,1} (-1)^{\Omega \Omega} = 2 \delta(|\Omega|_2), \quad (7.2)$$

one can easily see that

$$(W_{\nu, \alpha(n), \beta(m)})^{AB} = \frac{1}{2} \sum_{\Omega=0,1} (-1)^{\Omega B} W_{\nu, \alpha(n), \beta(m)}^{A+B\Omega}. \quad (7.3)$$

Comparing eqs.(2.13) and (7.1), one finds that the fields W obey the following hermiticity conditions:

$$(W_{\nu, \alpha(n), \beta(m)})^{A\Omega} = (-1)^{A(m+\Omega)} W_{\nu, \beta(m), \alpha(n)}^{A|\Omega+n+m|_2}. \quad (7.4)$$

In addition, one finds by comparing eqs.(2.9), (2.12), (7.1), (7.3) that the curvatures in the new basis take the form

$$\begin{aligned} \hat{R}_{\nu\mu, \alpha(n), \beta(m)}^{A\Omega} &= \partial_{\nu} W_{\mu, \alpha(n), \beta(m)}^{A\Omega} - \partial_{\mu} W_{\nu, \alpha(n), \beta(m)}^{A\Omega} \\ &+ \frac{1}{2} \sum_{B,C,p,q,s,k,\ell,t} \delta(|A+B+C|_2) \delta(n-p-q) \delta(m-k-\ell) \times \\ &\times i^{s+t-1} \frac{n!m!}{p!q!s!k!\ell!t} \times \\ &\times [(-1)^{C(p+s)} W_{\nu, \alpha(p)\gamma(s), \beta(k)\delta(t)}^{B\Omega} W_{\mu, \alpha(q)\gamma(\ell), \beta(e)}^{C|\Omega+p+s+t+k|_2} \gamma(s) \delta(t) - \\ &- (-1)^{B(q+s)+s+t+(p+k+s+t)(q+\ell+s+t)} \times \\ &\times W_{\nu, \alpha(p)\gamma(s), \beta(k)\delta(t)}^{B|\Omega+t+\ell+q+s|_2} W_{\mu, \alpha(q)\gamma(\ell), \beta(e)}^{C\Omega} \gamma(s) \delta(t)] . \end{aligned} \quad (7.5)$$

In the pure bosonic case, all gauge fields carry even numbers of spinorial indices. By setting $|p+s+k+t|_2 = |q+s+l+t|_2 = 0$ in eq.(7.5), one reduces curvatures $\delta \hat{R}$ of the even subalgebra of shsa(1) to the form

$$\begin{aligned} \delta \hat{R}_{\mu\nu}^{A\Omega}, \alpha(n), \beta(m) &= \partial_\nu \delta W_{\mu}^{A\Omega}, \alpha(n), \beta(m) - \partial_\mu \delta W_{\nu}^{A\Omega}, \alpha(n), \beta(m) \\ &+ \sum_{BCpqsk\ell t} \delta(1A+B+C|_2) \delta(n-p-q) \delta(m-k-\ell) \delta(1p+s+k+t|_2) \times \\ &\times \delta(1q+s+\ell+t|_2) \delta(1C(p+s)+B(q+s)+s+t+1|_2) \times \\ &\times i^{s+t-1} (-1)^{C(p+s)} \frac{n!m!}{p!q!s!k!\ell!t!} \times \\ &\delta W_{\nu}^{B\Omega}, \alpha(p)\gamma(s), \beta(k) \delta(t) \delta W_{\mu}^{C\Omega}, \alpha(q)\gamma(s), \beta(\ell) \delta(t), \end{aligned} \quad (7.6)$$

and now it is obvious that the even subalgebra of shsa(1) is of the structure

$\mathcal{b}_0 \oplus \mathcal{b}_1$ where the Lie algebras $\mathcal{b}_0$ and $\mathcal{b}_1$ correspond to the gauge fields $\delta W_{\mu\nu}^{A\Omega}, \alpha(n), \beta(m)$ with $\Omega = 0$ and 1 respectively.

Although curvatures (7.6) are identical at $\Omega = 0$ and $\Omega = 1$, real Lie algebras $\mathcal{b}_0$ and $\mathcal{b}_1$ are non-isomorphic since the hermitian conjugation (7.4) distinguishes between $\Omega = 0$ and $\Omega = 1$. Thus, $\mathcal{b}_0$ and $\mathcal{b}_1$ form two inequivalent real forms of one and the same complex Lie algebra $\mathcal{b}(\mathbb{C})$. Let us stress that, as follows from eq.(7.4), the difference between $\mathcal{b}_0$ and $\mathcal{b}_1$ is only related to the auxiliary sector of gauge fields $\delta W_{\mu\nu}^{A\Omega}, \alpha(n), \beta(m)$, while the higher-spin subalgebras of $\mathcal{b}_0$ and $\mathcal{b}_1$ are isomorphic. This fact is very important because auxiliary fields do not possess non-trivial physical degrees of freedom and, therefore, the auxiliary sector is expected to be trivial in some sense.

In other words, the structure of $\mathcal{b}_0$ and $\mathcal{b}_1$ indicates that the parts of an action related to $\mathcal{b}_0$ and $\mathcal{b}_1$ will have opposite signs in the sector of auxiliary fields, but this is not dangerous because auxiliary fields cannot lead to ghosts.

On the other hand, it is remarkable that the higher-spin subalgebras of $\mathcal{b}_1$ and $\mathcal{b}_0$ are identical, and, therefore, any two species of higher-spin fields will most likely have coinciding signs in the action. An additional argument in favour of this conclusion comes from the fact that self-conjugate higher-spin parameters give con-

tributions of coinciding signs into the invariant form (2.15).

Let us also note that the boson sectors of the superalgebras $\text{shsa}(1|a,b)$ of previous section reduce to a direct sum of two infinite-dimensional subalgebras too. This explains the boson field doubling for these superalgebras, observed in Section 6.

Algebras $\mathcal{b}_\Omega$ ($\Omega = 0$ or 1) contain both de Sitter subalgebras $\text{sp}_\Omega(4)$ with the gauge fields $\mathcal{W}_{\nu, d(n), \dot{\beta}(m)}^{\Omega\Omega}$ at $n+m=2$. One can linearize each algebra $\mathcal{b}_\Omega$ with respect to its de Sitter subalgebra $\text{sp}_\Omega(4)$. For both $\Omega = 0$ and $\Omega = 1$, corresponding linearized curvatures will coincide with linearized (bosonic) higher-spin curvatures of ref./2/ at $A=0$ and with linearized curvatures of bosonic auxiliary fields of ref./4/ at $A=1$.

The results above put on an idea that a hypothetical higher-spin theory related to $\text{shsa}(1)$ may describe two boson worlds, each containing its own gravitational field, which interact only through fermion fields. It should be stressed that this fermion interaction is inevitable since $\text{shsa}(1)$ as a whole does not decompose into a direct sum of two subalgebras (the superalgebra $\text{shsa}'(1) = \text{shsa}(1)/\mathfrak{o}(2)$ is simple). Moreover, fermion fields will look unusual enough in the framework of each of these worlds, because these fields realize some infinite-dimensional representations of corresponding Lorentz subalgebras $\mathcal{O}_\Omega(3,1) \subset \mathfrak{sp}_\Omega(4)$. Perhaps, this circumstance can prevent from the interpretation above enforcing one to conclude that only some specific combination of the "gravitational" fields related to $\mathcal{b}_\Omega$ can serve as the true gravitational field (e.g. $\mathcal{W}_{\nu, d(n), \dot{\beta}(m)}^{\Omega\Omega}$ with $n+m=2$).

Nevertheless the possibility exists that a fermion interaction between the two bosonic worlds will be weak enough (e.g. suppressed by powers of ^{the} gravitational constant), while significant low-energy dynamical effects will only be caused by lower-spin fermions which are not connected directly to $\text{shsa}(1)$ itself and, in principle, may have the standard gravitational interactions within the worlds both.

An opposite and, perhaps, more probable possibility is that only some specific combination of the spin-2 fields (say, $\mathcal{W}_{\nu, d(1), \dot{\beta}(1)}^{\Omega\Omega}$) will remain massless in a physically relevant phase with spontaneously broken higher-spin symmetries,

and this combination will be the only candidate for the role of the gravitational field. However a more detailed discussion of these intriguing possibilities requires for concrete information on a dynamics of higher-spin and lower-spin fields, the question lying beyond the scope of this paper.

8. Conclusion

The main result of the present paper consists in constructing infinite-dimensional non-abelian superalgebra $shsa(1)$ which gives rise at the linearized level to both linearized higher-spin curvatures proposed in ref./2/ and linearized curvatures of auxiliary fields proposed in ref./4/. The full (non-abelian) higher-spin superalgebra $shs_{\infty}(1)$ constructed in ref./3/ is incorporated naturally in $shsa(1)$ as its subalgebra.

The subalgebras $shsa(1 | a, b) \subset shsa(1)$ ($a, b = 0$ or 1) are extracted which themselves pretend for the role of non-trivial superalgebras of higher spins and auxiliary fields. In contrast to $shsa(1)$ giving rise to two species of any higher-spin field as well as any auxiliary one, the subalgebras $shsa(1 | a, b)$ give rise to some parts of the higher spins and auxiliary fields. Specifically, all the superalgebras $shsa(1 | a, b)$ do not generate higher-spin fields of odd spins. On the other hand, they generate twice higher-spin fields of even spins.

The doubling of higher-spin fields in the framework of $shsa(1)$ (or $shsa(1 | a, b)$) may have important physical implications. One possibility, discussed briefly in Section 7, is that two worlds may exist whose mutual interaction is suppressed by powers of the gravitational constant while each of these worlds possesses its own gravitation.

The superalgebra $shsa(1)$ has been specialized as one of two inequivalent real forms of the complex superalgebra $shsa(1; \mathbb{C})$. The requirement that

the purely higher-spin subalgebra of $\text{shsa}(1)$ should possess non-trivial unitary representations has served as a criterion for this specific choice. Namely, it follows from the anticommutation relations for supergenerators of real forms of the higher-spin subalgebra that, for its unitary representations (that is in a positive definite Hilbert space), some inequalities on bosonic generators should hold. It has been shown that only for the real form $\text{shsa}(1)$ of $\text{shsa}(1; \mathbb{C})$ these inequalities turn out to be consistent. It should be mentioned that although the criterion above does not work for the real higher-spin superalgebras, $\text{shs}_\rho(1)$ ($\rho = 0$ or 1), found in ref./3/, only one of them, $\text{shs}_0(1)$, admits a consistent extension to $\text{shsa}(1)$. A sequence of inequalities on boson generators of higher-spin subalgebra of $\text{shsa}(1)$ has been derived in Section 5, which should take place in unitary representations and generalize the well-known energy non-negativity condition in ordinary supersymmetry.

In addition, we have described a simple operator realization of $\text{shsa}(1)$, have constructed the P-reversal automorphism of $\text{shsa}(1)$, have discussed various subalgebras of $\text{shsa}(1)$ and have shown explicitly that the even subalgebra of $\text{shsa}(1)$ decomposes into the direct sum of two infinite-dimensional subalgebras, each generating all bosonic higher-spin and auxiliary fields.

We were arrived at superalgebra $\text{shsa}(1)$ in our investigation of the higher-spin problem, the problem of introducing a consistent gravitational interaction for massless higher-spin fields. As pointed out in refs./2,3/, auxiliary fields may be useful for overcoming some technical difficulties arising due to the presence of "extra fields" in the framework of the higher-spin superalgebras suggested in these references for the description of a higher-spin-gravitational interaction. We believe that the superalgebra $\text{shsa}(1)$, found in the present paper, will actually ensure an adequate basis for overcoming these difficulties (this question is now under investigation). Moreover, this superalgebra may simultaneously enable one to solve the standard problem of closing the gauge algebra in related higher-spin theories. This point of view is strongly supported by the fact that auxiliary

fields related to $\text{shsa}(1)$ include naturally /4/ the Schrius-West auxiliary fields which are known to provide closure of the algebra in $N=1$ supergravity /11/.

The gauge fields and curvatures originating from $\text{shsa}(1)$ are hoped to serve as basic geometric objects for constructing a consistent higher-spin theory. However, determination of the superalgebra $\text{shsa}(1)$ is not sufficient in itself to solve the higher-spin problem. In order to construct the consistent higher-spin theory, one needs to know an adequate local deformation of $\text{shsa}(1)$. In fact, this problem, which is now under investigation, is highly nontrivial and has essentially dynamical origin. Nevertheless, the author's opinion is that superalgebra $\text{shsa}(1)$ is an important and inevitable point on the long road towards the solution of the higher-spin problem.

Appendix. Main Steps Towards shsa(1)

In this Appendix we discuss the main assumptions used in constructing shsa(1).

First of all, it was assumed from the very beginning that a non-abelian superalgebra of higher-spins and auxiliary fields, shsa(1), should contain one of higher-spin superalgebras $shs_p(1)$ of ref./3/ as its subalgebra described by the parameters $\tilde{\mathcal{J}}_{\alpha(n),\beta(m)}$. It was initially assumed that it should also contain the set of auxiliary generators $\tilde{\mathcal{T}}_{\alpha(n),\beta(m)}$ with the corresponding parameters $\xi_{\alpha(n),\beta(m)}$ for all $n \geq 0, m \geq 0$. The linearized curvatures for these generators were assumed to coincide with those constructed by one of us in ref./4/. Notice that we identify the linearization under consideration with the contraction of shsa(1) with respect to its de Sitter subalgebra, $sp_{dS}(4) \subset shs_p(1) \subset shsa(1)$ (for more detail about constructing contractions via subalgebras see ref./3/).

Since one of superalgebras $shs_p(1)$ is assumed to be a subalgebra of shsa(1), the contraction of shsa(1) should exist generated by this subalgebra. In practice, this implies that the set of generators $\tilde{\mathcal{T}}_{\alpha(n),\beta(m)}$ (equivalently, parameters $\xi_{\alpha(n),\beta(m)}$) should form some representation of $shs_p(1)$. Therefore, the first essential step consists in determination of this representation. In fact, it is more convenient to begin with the determination of the corresponding complex representation of $shs(1;\mathbb{C})$, the complexification of $shs_p(1)$, and investigate existence of appropriate reality conditions afterwards.

It is convenient to describe the representation of $shs(1;\mathbb{C})$ under investigation in the form of some composition law as follows:

$$[\mathcal{E}, \mathfrak{Z}]_{\mathcal{E} \alpha(n), \beta(m)} = \sum_{p, q, s, k, \ell, t} i^{s+t-1} \frac{n! m!}{p! q! s! k! \ell! t!} \delta(n-p-q) \delta(m-k-\ell) \times \\ \times \beta(p, q, s, k, \ell, t) \mathcal{E}_{\alpha(p) \gamma(s), \beta(k) \delta(t)} \mathfrak{Z}_{\alpha(q) \gamma(s), \beta(\ell) \delta(t)} \quad (\text{A.1})$$

where $\beta(p, q, s, k, \ell, t)$ is some unknown function, while the symbol $[\mathcal{E}, \mathfrak{Z}]_{\mathcal{E}}$ denotes the result of the action of some element of $\text{shs}(1; \mathbb{C})$ with the parameters $\mathfrak{Z}$ on the element of the representation with the parameters $\mathcal{E}$. The requirement, that eq.(A.1) describes some representation of $\text{shs}(1; \mathbb{C})$, manifests itself in the Jacobi identities

$$[[\mathcal{E}, \mathfrak{Z}_1]_{\mathcal{E}}, \mathfrak{Z}_2]_{\mathcal{E}} - [[\mathcal{E}, \mathfrak{Z}_2]_{\mathcal{E}}, \mathfrak{Z}_1]_{\mathcal{E}} = [\mathcal{E}, [\mathfrak{Z}_1, \mathfrak{Z}_2]_{\mathfrak{Z}}]_{\mathcal{E}} \quad (\text{A.2})$$

where the composition law $[\mathfrak{Z}_1, \mathfrak{Z}_2]_{\mathfrak{Z}}$,

$$[\mathfrak{Z}_1, \mathfrak{Z}_2]_{\mathfrak{Z} \alpha(n), \beta(m)} = \sum_{p, q, s, k, \ell, t} i^{s+t-1} \frac{n! m!}{p! q! s! k! \ell! t!} \delta(n-p-q) \delta(m-k-\ell) \times \\ \times \alpha(p, q, s, k, \ell, t) \mathfrak{Z}_1 \alpha(p) \gamma(s), \beta(k) \delta(t) \mathfrak{Z}_2 \alpha(q) \gamma(s), \beta(\ell) \delta(t) \quad (\text{A.3})$$

is that of $\text{shs}(1; \mathbb{C})$, and the coefficients $\alpha(p, q, s, k, \ell, t)$ are of the form [3]

$$\alpha(p, q, s, k, \ell, t) = \delta(|(p+k)(q+\ell) + (p+k)(s+t) + (q+\ell)(s+t) + 1|_2). \quad (\text{A.4})$$

In full analogy to the higher-spin case considered in ref.[3], one can make sure that eq.(A.2) imposes the following restrictions on the function β :

$$\beta(p+s, q+v, u, k+t, \ell+g, f) \beta(p+q, r, s+v, k+\ell, h, t+g) - \\ - (-1)^{v+g+(q+v+u+\ell+g+f)(r+s+v+h+t+g)} \times \\ \times \beta(p+u, r+v, s, k+f, h+g, t) \beta(p+r, q, u+v, k+h, \ell, f+g) - \\ - \beta(p, q+r, u+s, k, \ell+h, f+t) \alpha(q+u, r+s, v, \ell+f, h+t, g) = 0. \quad (\text{A.5})$$

which should hold for arbitrary non-negative integer variables $p, q, s, \dots$ (the same as in ref.[3], we disregard the two-componentness of spinorial indices in the derivation of eq.(A.5)).

We searched for a solution of eq.(A.5) obeying the "initial data" originating from the structure of linearized curvatures of auxiliary fields found in ref.[4].

These are of the form

$$\beta(p, 0, 0, k, 0, 2) = \beta(p, 0, 0, k, 2, 0) = 0, \quad (A.6)$$

$$\beta(p, 0, 2, k, 0, 0) = \beta(p, 2, 0, k, 0, 0) = 0, \quad (A.7)$$

$$\beta(p, 0, 0, k, 1, 1) \neq 0, \beta(p, 1, 1, k, 0, 0) \neq 0, \quad (A.8)$$

$$\beta(p, 0, 1, k, 1, 0) = \beta(p, 1, 0, k, 0, 1) = 0, \quad (A.9)$$

$$\beta(p, 0, 1, k, 0, 1) \neq 0, \beta(p, 1, 0, k, 1, 0) \neq 0. \quad (A.10)$$

Eqs. (A.6)-(A.8) express the requirement that the parameters $\beta_{\alpha(n), \dot{\beta}(m)}$ have to form the multispinorial representations of the (complexified) Lorentz subalgebra of $\text{shs}(1; \mathbb{C})$. Eqs. (A.9), (A.10) ensure that the terms, containing the vierbein $h_{\nu \alpha \dot{\beta}}$ in the corresponding linearized curvatures, coincide with those which emerge in the linearized auxiliary curvatures of ref./4/.

Eq. (A.5) combined with the conditions (A.6)-(A.10) can be solved analogously to that as the similar equation for $\text{shs}(1; \mathbb{C})$ itself was solved in ref./3/. After some tedious calculations, we found that there were the following two inequivalent solutions only:

$$\beta_1(p, q, s, k, \ell, t) = \delta(|(p+k+s+t)(q+\ell+s+t)+q+t+1|_2), \quad (A.11)$$

$$\beta_2(p, q, s, k, \ell, t) = \delta(|(p+k+s+t)(q+\ell+s+t)+s+\ell+1|_2). \quad (A.12)$$

Comparison of eqs. (A.11), (A.12) with eq. (A.1) shows that the representations related to β_1 and β_2 coincide when the parameters $\beta_{\alpha(n), \dot{\beta}(m)}$ are even ($|q+s+1+t|_2 = 0$) but distinguish between when the parameters $\beta_{\alpha(n), \dot{\beta}(m)}$ are odd ($|q+s+1+t|_2 = 1$). Moreover, one can readily see that any of the solutions (A.11), (A.12) is non-symmetric with respect to the interchange of dotted and undotted spinorial indices which instead transforms the solution (A.11) into (A.12) and vice versa. Thus, it is obvious that using only the solution (A.11) or only the solution (A.12), one cannot construct a real superalgebra with the reality conditions interchanging dotted and undotted spinorial indices.

The simplest possibility is to suppose that superalgebra $\text{shsa}(1)$ contains the two auxiliary sectors in such a way that the conjugation transforms one into another and vice versa. Within the conventions used in the main text of the paper, these two sectors are described by the parameters $\beta_{\alpha(n), \dot{\beta}(m)}^{AB}$ with $A+B = 1$. Indeed,

eq.(2.9), taken at $C=D=0$ and $A=1, B=0$, corresponds to eq.(A.11), while eq.(2.9), taken at $C=D=0$ and $A=0, B=1$, corresponds to eq.(A.12) (recall that the higher-spin superalgebra $\text{shs}(1;\mathbb{C})$ is identified with the sector described by the parameters $\sum^{\infty} \alpha(m), \beta(m)$).

When the representations (A.11), (A.12) were found, we attempted to construct such a simple superalgebra (disregarding the trivial $\mathfrak{o}(2)$ -component in $\text{shs}(1;\mathbb{C})$) that its contraction with respect to $\text{shs}(1;\mathbb{C})$ would lead to these representations. However, we could not succeed in these attempts until we used the single higher-spin sector identified with $\text{shs}(1;\mathbb{C})$. On the other hand, the doubling of the auxiliary sector putted on an idea that the higher-spin sector should be doubled too (at least in presence of fermions). Assuming that this doubling really took place, we easily guessed the solution (2.9).

The question on the uniqueness of this solution was not investigated systematically enough. Nevertheless, we believe that the proposed superalgebra $\text{shsa}(1)$ is most natural while other possible $N=1$ superalgebras of higher spins and auxiliary fields are exhausted by some its subalgebras and contractions (most promising candidates are $\text{shsa}(1|a,b)$ of Section 6).

References

- /1/. M.A.Vasiliev, Sov. Yad. Phys. 32 (1980) 855.
- /2/. M.A.Vasiliev, Lebedev Institute Preprint N 233 (1986); to appear in Fort. der Phys. (1987).
- /3/. E.S.Fradkin and M.A.Vasiliev, Lebedev Institute Preprints NN 257, 258 (1986); Ann. of Phys. (N.Y.) (1987), 172, 63.
- /4/. M.A.Vasiliev, Lebedev Institute Preprint N 246 (1987) (submitted to Nucl. Phys. B).
- /5/. C.Aragone and S.Deser, Nucl. Phys. B170 [RS1] (1980) 329.
- /6/. M.A.Vasiliev, Lebedev Institute Preprint N 181 (1985) (to appear in "Quantum Field Theory and Quantum Statistics", eds. I.A.Batalin, C.J.Isham and G.A.Vilkovisky, Adam Hilger LTD, Bristol).
- /7/. M.A.Vasiliev, Class. Quant. Grav. 2 (1985) 645.
- /8/. M.A.Vasiliev, Pis'ma Zh. Eksp. Teor. Fiz. 40 (1984) 437.
- /9/. K.Pilch, P. van Nieuwenhuizen and M.F.Sohnius, Commun. Math. Phys. 98 (1985) 105.
- /10/. M.A.Vasiliev, Lebedev Institute Preprint N 290 (1986); to appear in Fort. der Phys. (1987).
- /11/. M.F.Sohnius and P.C.West, Phys. Lett. 105B (1981) 353.

Препринты Физического института имени П.Н. Лебедева АН СССР рассылаются научным организациям на основе взаимного обмена.

Наш адрес: 117924, Москва В-333, Ленинский проспект, 53

Preprints of the P.N. Lebedev Physical Institute of the Academy of Sciences of the USSR are distributed by scientific organizations on the basis of mutual exchange.

Our address is: USSR, 117924, Moscow B-333, Leninsky prospect, 53

Т – 04845. Подписано в печать 4. 02. 1987 г.
Заказ № 549. Тираж 100 экз. П.л. 2,7.

Отпечатано в Отделе научно-технической информации ФИАН СССР
Москва, В-333, Ленинский проспект, 53