AGS - THE ISR COMPUTER PROGRAM FOR SYNCHROTRON DESIGN,
ORBIT ANALYSIS AND INSERTION MATCHING

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"AGS - The ISR Computer System for Synchrotron
Design and Orbit Analysis", by E. Keil and P. Strolin)

G E N E V A
1975

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ABSTRACT

This is a detailed guide to the use of the current version of a FORTRAN program for carrying out computations required in the design or modification of alternating-gradient synchrotrons and storage rings. The program, which runs on the CDC 7600 computer at CERN, computes linear transformation matrices of a ring structure, its betatron and dispersion functions, and modifications of parameters to achieve specified properties; it tracks sets of particle trajectories, finds closed orbits when elements of the structure are displaced, computes the equilibrium orbit, designs closed-orbit bumps, tracks betatron functions through the structure and matches insertions in the structure to specified betatron and dispersion functions. The report supersedes CERN 69-5 (AGS - The ISR computer system for synchrotron design and orbit analysis, by E. Keil and P. Strolin).
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1. INTRODUCTION

AGS is a package of FORTRAN programs which performs the following tasks frequently occurring in the design of Alternating Gradient Synchrotrons.

i) It computes the linear transformation matrices of a string of elements - the "structure".

ii) It calculates the betatron and dispersion functions of the structure.

iii) It modifies the parameters of the structure such as to achieve e.g. specified Q-values, to match insertions into the structure, etc.

iv) It tracks sets of particle trajectories through the structure or parts of it, both in the horizontal and vertical plane. In this mode closed orbit bumps may be included.

v) It finds the closed orbit, when elements are displaced, either by specified amounts or in various random fashions.

vi) It computes the co-ordinates of the equilibrium orbit.

vii) It designs closed orbit bumps, both in the horizontal and vertical plane.

viii) It tracks the betatron functions through the structure or part of it.

ix) It matches "insertions" to specified betatron and dispersion functions.

Certain non-linear effects may also be calculated with the program. Output may be printed on paper or written onto tape for use by subsequent programs (plotting programs for example).

2. DATA STRUCTURE

The operation of the program is governed by a sequence of control cards, each of which initiates a specified function of the program, and of data cards belonging to that function as schematically indicated below:

. .

control card No. N
data cards for control card No. N
control cards No. (N + 1)
data cards for control card No. (N + 1)
. .
### Table I
Control cards of the AGS program

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of following data cards</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHN</td>
<td>n *)</td>
<td>Design bump in the horizontal plane</td>
</tr>
<tr>
<td>BV</td>
<td>n</td>
<td>Design bump in the vertical plane</td>
</tr>
<tr>
<td>CL</td>
<td>n</td>
<td>Search for a closed orbit</td>
</tr>
<tr>
<td>CO</td>
<td>0</td>
<td>Punch a compressed structure deck</td>
</tr>
<tr>
<td>Eb **)</td>
<td>n</td>
<td>End of structure reading, read and store momentum error cards</td>
</tr>
<tr>
<td>EF</td>
<td>0</td>
<td>Switch the tape flag off, write an end-of-file and backspace</td>
</tr>
<tr>
<td>GE</td>
<td>0</td>
<td>Geometry calculations</td>
</tr>
<tr>
<td>GO</td>
<td>0</td>
<td>Active structure calculations after parameter reading</td>
</tr>
<tr>
<td>MA</td>
<td>1</td>
<td>Match parameters</td>
</tr>
<tr>
<td>MB</td>
<td>n</td>
<td>Match characteristics functions of a beam transfer channel</td>
</tr>
<tr>
<td>MI</td>
<td>n</td>
<td>Closed orbit calculations in machines with misaligned elements</td>
</tr>
<tr>
<td>MX</td>
<td>0</td>
<td>Print the current matrices</td>
</tr>
<tr>
<td>NO</td>
<td>0</td>
<td>Suppress structure calculations after parameter reading</td>
</tr>
<tr>
<td>PU</td>
<td>0</td>
<td>Punch parameters</td>
</tr>
<tr>
<td>RI</td>
<td>1</td>
<td>Read and store a random number</td>
</tr>
<tr>
<td>RΦ</td>
<td>0</td>
<td>Print a random number</td>
</tr>
<tr>
<td>S1</td>
<td>n</td>
<td>Read and store a structure in expanded form</td>
</tr>
<tr>
<td>S2</td>
<td>n</td>
<td>Read and store a structure in compressed form</td>
</tr>
<tr>
<td>S3</td>
<td>n</td>
<td>Read and store a structure in normal form</td>
</tr>
<tr>
<td>ST</td>
<td>0</td>
<td>Stop the operation of AGS</td>
</tr>
<tr>
<td>TA</td>
<td>0</td>
<td>Switch the tape flag on, subsequent output will also be written onto tape</td>
</tr>
<tr>
<td>TB</td>
<td>3</td>
<td>Track characteristics functions of a beam transfer channel</td>
</tr>
<tr>
<td>TE</td>
<td>1</td>
<td>Read and store a text as heading</td>
</tr>
<tr>
<td>TH</td>
<td>n</td>
<td>Track in the horizontal plane</td>
</tr>
<tr>
<td>TS</td>
<td>n</td>
<td>Alternative tracking command for large horizontal orbit displacement</td>
</tr>
<tr>
<td>TV</td>
<td>n</td>
<td>Track in the vertical plane</td>
</tr>
<tr>
<td>bb</td>
<td>n</td>
<td>Read and store parameters, perform structure calculations, print results</td>
</tr>
</tbody>
</table>

*) n is a positive integer determined by the control card and data cards.  
**) b is a BCD blank
The permissible set of control cards is shown in Table I together with a brief description of their function. With one exception the control cards have a mnemonic code punched in columns 1 and 2. A general flow diagram of the AGS program is shown in Fig. 1.

The program contains flags which check the proper sequence of control cards. The structure flag IEL is set after a structure has been read in. The parameter flag ILIST is set when parameters have been read in; it is turned off each time a new structure is encountered. It is safest to start the data deck in the following fashion:

TE control card
heading
S1 or S2 or S3 control card
structure
Eb control card
energy error cards
bb control cards
parameter cards

After this sequence all control cards may occur, including new TE, S1, S2, S3, Eb and bb control cards.

When in the place of a control card a card is found which does not contain one of the codes shown in Table I, the message

ILLEGAL CONTROL CARD

is printed and the program skips through the data until another control card is found. During this search, cards with blanks in the first two columns are not considered as control cards.

3. REPRESENTATION OF A SYNCHROTRON

A synchrotron is composed of a number of elements (IEL). Elements may be either magnets or drift spaces. The properties of each element are defined by a two-step procedure.

In the first step each element is identified by

i) a length index LD
ii) an angle index LAN
iii) a gradient index LK
iv) a name NAME which is optional

The convention of the program is that elements with the same value of LD(LAN, LK) are automatically given the same length (angle, gradient) in the second step. Hence, the repetition of identical elements in the layout of the structure is coded into the indices LD, LAN, LK. We say that the arrays LD(I), LAN(I), LK(I), I = 1, IEL define the "structure" of the synchrotron.
In the second step of the program actual values of length, angle and gradient are assigned to each of the indices LD, LAN and LK used, i.e. a given value n for one of the indices is understood to mean a certain value x for the parameter in question. In this way, all "parameters" of the structure are defined. If an index takes the value 0 the associated parameter is permanently considered to be 0.0.

<table>
<thead>
<tr>
<th>Element No.</th>
<th>LD</th>
<th>LAN</th>
<th>LK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

An example is shown in Table II. The elements No. 2 and 5 are straight sections since their bending angle and gradient vanish. Furthermore they have the same lengths because LD(2) = LD(5) = 2. All other elements have the same lengths, and the elements No. 1 and 3, and 4 and 6 are equal. Interpreting LK = 5 as a focusing and LK = 6 as a defocusing gradient, the example becomes a period of a typical F0F0DO structure.

4. **READING OF A STRUCTURE**

Three different methods exist to read a structure into the program. They are selected by the control cards S1, S2 or S3. The indices describing LAN and LK must be in the range 0 to 200 inclusive and those describing LD must be in the range 1 to 200. If LAN = 0 or LK = 0, the corresponding angle or gradient is taken to be 0. The index 99 is used as a straight/wedge magnet flag (see 6.4); it should not be assigned to an element. If columns 5, 4, 5 of the S1, S2, S3 cards are zero or blank, a flag WHOLE is set true, otherwise it is set false.

If WHOLE is true, the program assumes that the structure represents a super-period of a complete machine, or the complete machine, it checks that the total bending angle is equal to $2\pi \pm 0.02$ and it performs the structure calculations described in 7., unless they are explicitly suppressed by a previously read NO control card (see 17.10).

If WHOLE is false, the program assumes that the structure just represents an arbitrary piece of beam transport channel, e.g. an "insertion" and just prints a list of the elements. Checks of the total bending angle and structure calculations are not performed.
4.1 Reading an expanded structure (S1)

Each element is described by a set of three cards which contain LD, NAME, LAN, LK in the format (IS, llX, A4/IS/IS). The sets of cards describing the elements must occur in the order in which the elements are traversed by the beam.

A sequence of elements which occurs \( n \) times in immediate succession, may be specified by the following sequence of cards:

\[
\begin{align*}
\text{n} & \quad \text{in format (IS)} \\
( & \quad \text{in column 1} \\
\text{cards describing the sequence} & \\
) & \quad \text{in column 1}
\end{align*}
\]

When the pair of brackets includes the whole sequence, this sequence is interpreted as a super-period of a synchrotron with \( n \) super-periods (ISUP = n). \( \text{IEL} \) is the number of elements in one super-period. All computations will be done for one super-period only. Brackets may be nested to any depth.

The structure input is terminated by the E control card. Since this control card also initiates reading the momentum errors, the sequence of the control functions SI or S3 and E is fixed (see 5).

4.2 Reading of a compressed structure (S2)

The list of variables:

ISUP, IEL, (LD(I), LAN(I), LK(I), NAME(I), I = 1, IEL) is presented on data cards in the compact format (8X, 214, 10X, 4(313, A4)/(6(313, A4))). Because of the compactness and because this input option does not allow modifications of the elements apart from substitution, these cards are usually punched by the program via the CO control card from a structure which was read in, in a more convenient way (see 17.5).

4.3 Read a normal structure (S3)

Each element is described by one card which contains LD, LAN, LK, NAME in that order in format (3IS, 1X, A4). With this exception this input option is identical to S1. In particular, repeated sequences may be included between brackets and the structure input is terminated by the E control card (see 5).

4.4 Error conditions on structure reading

The following messages are generated by S1 and S3:

i) SPURIOUS CHARACTER CH

where CH are the characters in error, occurs whenever characters other than bb or (b or)b or Eb occurs in the first two columns, or when (b or)b or Eb occurs in the wrong place (b is a BCD blank).
ii) TOO MANY ELEMENTS IN SEQUENCE
The dimension of the working space (see 18) is insufficient for reading the structure.

iii) NON-MATCHING BRACKETS
The number of open brackets differs from that of the closed ones.

iv) TOO MANY ELEMENTS
The working space is insufficient for storing the structure.

v) INDEX OUT OF RANGE
An index is outside the range 0 to 200.

All error conditions on structure reading inhibit all structure computations up to the next S1 or S2 or S3 control card.

5. READING OF MOMENTUM ERRORS (E)

The E control card is followed by a card in format (3I5) which contains IP, IPRINT, ITER in that order. IP is the number of momentum errors to be included in the calculation (see 7.1.13), IPRINT controls the output of the results of these calculations (see 7.5.2.), and ITER determines the maximum number of iterations used in finding the closed orbit of the off-momentum particles (see 7.3.).

The convention of the program is that the case for Δp/p = 0 is always calculated and printed. Hence IPRINT and ITER are only used for IP > 0.

If IP > 0, the momentum errors Δp/p to be studied are punched on the following IP cards with format (F10.5). The momentum errors are given as plain numbers e.g. for Δp/p = 1% the program expects to find a card with the number 0.01.

If IP > 50 the error message

TOO MANY MOMENTUM VALUES

is printed, and the program skips through the data cards until it finds another control card which has no blank in columns 1 and 2.

At the beginning of execution of the program, IP = 0, IPRINT = 0, ITER = 0. Hence, if structures are read by the S2 option (4.2.) and if no momentum errors are to be studied, an E control card and its associated data card(s) need not be in the data.

6. READING OF PARAMETERS

A control card with blanks in the first two columns is interpreted as a card containing the number n of parameter cards to follow, in format I3.

Whenever a structure is read, all parameters are reset to zero. Hence, when reading parameters for the first time, only those different from zero need be specified explicitly.
Within a structure, parameter cards are cumulative: only parameters which should be changed between calculations need to be specified.

Each of these parameter cards contains I, A, B, C in format (I5, 3F10.5). From now on, whenever an index takes the value I, the associated parameters will be A, B and C. When the reading of the parameters is successfully terminated, the program automatically enters the structure computations, unless the N0 option is active (see 17.10.).

6.1 Length of elements

If I is one of the length indices LD, A is the effective length L for the magnetic field gradient, given in metres. B describes \( \frac{dL}{dx} \) in radians and C describes \( \frac{d^2L}{dx^2} \) in m\(^{-1}\), the first and second derivative of L at the upstream end of the element with respect to the distance x from the equilibrium orbit, taken positive away from the machine centre (\( \frac{dL}{dx} \) and \( \frac{d^2L}{dx^2} \) are positive if the element becomes longer for increasing x). The length variation at the downstream end of the n'th element is taken to be equal to the negative upstream length variation of the \( (n+1) \)th element. The sign conventions are shown in Fig. 2. For more detailed considerations on wedge and straight magnets, on effective lengths for bending and gradient, and on edge focusing, see 7.1.2.

6.2 Angle of elements

If I is an angle index LAN, then A gives the deflection angle \( \phi \) in radians for a particle with \( \Delta p/p = 0 \) on the equilibrium orbit. Bending towards the machine centre means a positive bending angle. Reversed bending magnets with \( \phi < 0 \) are permitted. B and C are ignored.

6.3 Gradient of elements

If I is a gradient index LK, then A = \( K_V \) in m\(^{-2}\), B = \( \frac{dK_V}{dx} \) in m\(^{-3}\) and C = \( \frac{d^2K_V}{dx^2} \) in m\(^{-4}\). \( K_V \) is our gradient parameter defined by \(^{1}\)

\[
K_V = -\frac{1}{B_0} \frac{dB_z}{dx}
\]  \hspace{1cm} (1)

where \( B_0 \) is the magnetic rigidity of a particle with \( \Delta p/p = 0 \) and \( \frac{dB_z}{dx} \) is the magnetic field gradient. From (1) follows the sign convention \( K_V > 0 \) for horizontally defocusing magnets.

6.4 Straight magnets and wedge magnets

A parameter card with I = 99 acts as a switch between wedge magnets with edges perpendicular to the equilibrium orbit and straight magnets with parallel end faces and having equal entrance and exit angles. If A = 1.0 straight magnets and otherwise wedge magnets are used.
6.5 An example

As an example let us assume that we want to study the structure given in Table II with the parameters given in Table III. They correspond roughly to a CPS period. This can be accomplished with the set of parameter cards given in Table IV.

<table>
<thead>
<tr>
<th>Element No.</th>
<th>Length [m]</th>
<th>Angle [rad]</th>
<th>K [m⁻²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>0.031</td>
<td>-0.06</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>0.031</td>
<td>-0.06</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
<td>0.032</td>
<td>+0.06</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.5</td>
<td>0.032</td>
<td>+0.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>col.</th>
<th>5</th>
<th>6</th>
<th>15</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.031</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.032</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table IV
Example of parameter cards

6.6 Error conditions

i) WRONG PARAMETER I
A parameter number I is outside the range 1 to 200.

ii) SUM OF ANGLES φ
The sum of angles φ differs from 2π by more than 0.02 radians for the whole machine, that is ISUP times the sum of angles explicitly specified. This test is only done if an entire machine is specified (see 4.).

These errors inhibit all calculations on the current structure except reading new parameters.
7. **STRUCTURE COMPUTATIONS**

After successfully reading parameters, the program automatically enters the structure computations, unless the NO option is specified (see 17.10).

7.1 **Magnet matrices**

7.1.1 **Magnet matrices for the equilibrium orbit**

We first consider how the magnet matrices are constructed for a trajectory with \( \Delta p/p = 0 \) and no horizontal displacement \( x \) from the equilibrium orbit. The length \( D \), the angle \( \phi \) and the gradient \( K_v \) are known. The matrices are constructed as follows:

Vertical plane \( K_v > 0 \):

\[
\begin{pmatrix}
\cos(D \sqrt{K_v}) & \frac{1}{\sqrt{K_v}} \sin(D \sqrt{K_v}) \\
-\frac{1}{\sqrt{K_v}} \sin(D \sqrt{K_v}) & \cos(D \sqrt{K_v})
\end{pmatrix}
\]  
(2)

Vertical plane \( K_v = 0 \):

\[
\begin{pmatrix}
1 & D \\
0 & 1
\end{pmatrix}
\]  
(3)

Vertical plane \( K_v < 0 \):

\[
\begin{pmatrix}
\cosh(D \sqrt{-K_v}) & \frac{1}{\sqrt{-K_v}} \sinh(D \sqrt{-K_v}) \\
-\frac{1}{\sqrt{-K_v}} \sinh(D \sqrt{-K_v}) & \cosh(D \sqrt{-K_v})
\end{pmatrix}
\]  
(4)

In the horizontal plane we find for the gradient parameter \( K_h \)

\[
K_h = -K_v + \left(\frac{\phi}{D}\right)^2
\]  
(5)

Horizontal plane \( K_h < 0 \):

\[
\begin{pmatrix}
\cosh(D \sqrt{-K_h}) & \frac{1}{\sqrt{-K_h}} \sinh(D \sqrt{-K_h}) & \frac{\phi}{D \sqrt{-K_h}} \left(\cosh(D \sqrt{-K_h}) - 1\right) \\
\sqrt{-K_h} \sinh(D \sqrt{-K_h}) & \cosh(D \sqrt{-K_h}) & \frac{\phi}{D \sqrt{-K_h}} \sinh(D \sqrt{-K_h}) \\
0 & 0 & 1
\end{pmatrix}
\]  
(6)
Horizontal plane $K_h = 0$:

\[
\begin{pmatrix}
1 & D & \frac{\phi D}{2} \\
0 & 1 & \phi \\
0 & 0 & 1
\end{pmatrix}
\]  

(7)

Horizontal plane $K_h > 0$:

\[
\begin{pmatrix}
\cos(D \sqrt{K_h}) & \frac{1}{\sqrt{K_h}} \sin(D \sqrt{K_h}) & \frac{\phi}{D \sqrt{K_h}} [1 - \cos(D \sqrt{K_h})] \\
-\frac{1}{\sqrt{K_h}} \sin(D \sqrt{K_h}) & \cos(D \sqrt{K_h}) & \frac{\phi}{D \sqrt{K_h}} \sin(D \sqrt{K_h}) \\
0 & 0 & 1
\end{pmatrix}
\]

(8)

7.1.2 **Effective lengths, edge focusing.**

**Wedge magnets and straight magnets**

All these expressions are related to the description of a real magnet with smooth field and gradient at its ends by a model magnet with abrupt edges for field and gradient. The effective length for field may be defined as follows:

\[
L_B(x) = \frac{1}{B_z(x,0)} \int_{-s_m}^{+s_m} B_z(x,s) \, ds
\]

(9)

where $B_z(x,0)$ is the field in the middle of the magnets and the integral extends to values of $s_m$ where the field essentially vanishes. We shall not consider the case where two magnets are close together such that their stray fields overlap.

If we find that $L_B(x)$ changes with the radial co-ordinate $x$, we may proceed in one of two ways:

i) We say that the magnet has edge focusing. In this case we continue to take $L_B$ as the effective length of the magnet, and we take the variation of $L_B$ into account by including edge focusing matrices in the matrix of the whole magnet.

ii) We say that the magnet has an effective length for gradient $L_G$ which is different from $L_B$, we use $L_G$ in the magnet matrices, and we do not include edge focusing matrices. $L_G$ is defined as follows:

\[
L_G(x) = \frac{1}{(dB_z/dx)_{x,0}} \int_{-s_m}^{+s_m} (dB_z/dx)_{x,s} \, ds
\]

(10)
It can be shown that $L_B$ and $L_G$ are related in the following way \(2\):

$$L_G(x) - L_B(x) = \frac{B_G(x, o)}{dB_G/dx}_{x, o} \left(\frac{dL_B}{dx}\right)_x$$  \(11\)

It can also be shown that the two descriptions i) and ii) are equivalent, i.e. that a quadrupole with length \(11\) and gradient \(K\), and a magnet edge with an angle \(\phi = \arctan(dL_B/dx)\) have the same matrix to the first order.

The program adopts the second attitude and, hence, does not allow the user specifying edge focusing with arbitrary angles explicitly.

Usually, \(L_B(x)\) and \(L_G(x)\) are determined by measurements on magnet models. The terms wedge magnet and straight magnet are related to the way in which these measurements are taken. The two possible ways are schematically shown in Fig. 3.

In the wedge magnet model the coils are moved at right angles to the equilibrium orbit, on radii joining the magnet edges to its centre of curvature. \(L_G\)-denotes the correction to be applied to the wedge magnet picture in order to describe the magnet correctly.

In the straight magnet model the coils are moved parallel to each other and not at right angles to the equilibrium orbit. In this case \(L_G\) gives the correction to be applied to the straight magnet picture.

Although it is, in principle, possible to transform experimental data on \(L_G\) such that they suit the computational model, it is far easier to adopt the same model for the measurements and the computation. To this end, the program accepts both wedge magnets and straight magnets. However, all magnets are assumed to be of the same kind. When straight magnets are chosen, the matrices \((2)\) to \((4)\) and \((6)\) to \((8)\) are enclosed between the following matrices

$$
\begin{pmatrix}
1 & 0 & 0 \\
\pm \frac{\phi}{\beta} & \tan \phi & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

\(12\)

to obtain the transformation matrix of an element. The upper sign is valid for the vertical matrices \((2)\) to \((4)\) in which case the top left 2 x 2 sub-matrix of \((12)\) is used. The lower sign is valid for the horizontal matrices \((6)\) to \((8)\).

7.1.3 Magnet matrices for $\Delta p \neq 0$ and orbit displacements $x \neq 0$

In cases where either the momentum error $\Delta p/p$ or the distance of the trajectory from the equilibrium orbit $x$ do not vanish, certain correction terms are applied to the element length and gradient before they are inserted into the magnet matrices.
7.1.3.1 Length corrections

Length corrections take into account two effects, the length variations \( \frac{dL_i}{dx} \) and \( \frac{d^2 L_i}{dx^2} \) and - for wedge magnets only - the fact that outside the equilibrium orbit (for \( x > 0 \)) the trajectory in the magnet is longer.

Hence we take for the \( i \)'th element

\[
D_i = L_i + x_i \frac{dL_i}{dx} - x_{i+1} \frac{dL_{i+1}}{dx} + \frac{x_i^2}{2} \frac{d^2 L_i}{dx^2} - \frac{x_{i+1}^2}{2} \frac{d^2 L_{i+1}}{dx^2} \tag{13}
\]

If the \( i \)'th element is a wedge magnet the following correction \( d_i \) is added to the length

\[
d_i = \frac{\phi_i}{2} (x_i + x_{i+1}) \tag{14}
\]

Here \( (i+1) \) is to be understood as a cyclic index.

7.1.3.2 Gradient corrections

The gradient parameter \( K_y \) requires corrections to take into account, on one hand the momentum error \( \frac{dp}{d\phi} \), since the definition adopted (1) involves the magnetic rigidity, and on the other hand radial variations of the gradient.

If the particle follows a straight trajectory inside the \( i \)'th element, entering it with a radial displacement \( x_i \), and leaving it with displacement \( x_{i+1} \), and if the gradient \( K_y \) has the radial dependence

\[
K_y = g + \frac{dg}{dx} + \frac{x_i^2}{2} \frac{d^2 g}{dx^2} \tag{15}
\]

then the average gradient seen by the particle is in the \( i \)'th element

\[
K_{yi} = g_i + \frac{1}{2}(x_i + x_{i+1}) + \frac{D_i}{12}(x_i - x_{i+1})
+ \frac{1}{2D_i} \frac{d^2 g_i}{dx^2} \frac{F_2}{2}
+ \frac{1}{2D_i} \frac{d^2 g_i}{dx^2} \frac{F_3}{2} \tag{16}
\]

where

\[
\overline{x_i} = \frac{1}{2}(x_i + x_{i+1}) + \frac{D_i}{12}(x_i - x_{i+1})
\]

and

\[
\overline{x_{i+1}} = \frac{1}{20} \left[ 78(x_i^2 + x_{i+1}^2) + 54x_i x_{i+1} + 22D_i (x_i \overline{x_i} - x_{i+1} \overline{x_{i+1}}) + 13D_i (x_i \overline{x_{i+1}} - x_{i+1} \overline{x_i} + 1) + 2D_i^2 (x_{i+1}^2 + x_i^2) + 3D_i^2 \overline{x_i} \overline{x_{i+1}} \right]
\]

The factors \( F_2 \) and \( F_3 \) take care of the following complication. If one is looking for the shape of a trajectory, which is a straight line when the field in an element vanishes, one must use \( F_2 = \frac{1}{2} \) and \( F_3 = \frac{1}{3} \) in the above equation. If, on the other hand, one is looking at the focusing in the vicinity of a known trajectory, one must use \( F_2 = F_3 = 1 \) in order to obtain an element which behaves like a driftspace when the gradient vanishes.
The momentum error $\Delta p/p$ is then taken into account by multiplying (16) by a factor $(1 + \Delta p/p)^{-1}$

$$K_{vi} \rightarrow \frac{K_{vi}}{1 + \Delta p/p} \quad (17)$$

7.1.3.3 Parameters for the magnet matrices

The length $D_1$ calculated according to (13) and (14), and the gradient calculated from (16) and (17) are then inserted into the magnet matrices (2) to (4), (6) to (8) and (12) and yield the transformation matrix of the $i$'th element.

7.1.4 Error condition in matrix computation

If the program finds that a $D_1$ is zero or negative, it prints the message

**LENGTH k IS ZERO OR NEGATIVE**

where $k$ is the index for which $A(k) = 0$ and inhibits further structure computations.

7.2 Betatron functions

Knowing the matrices $M_i$ for all IEL elements the program may now calculate the matrix product

$$\Pi_{IEL,1} = M_1 M_2 \cdots M_2 M_1 \quad (18)$$

Since the structure represented by the IEL elements is a super-period of the synchrotron, its top left $2 \times 2$ sub-matrix may be written in the form

$$\Pi_{IEL,1} = \begin{pmatrix} \cos \mu + \alpha_1 \sin \mu & \beta_1 \sin \mu \\ -\gamma_1 \sin \mu & \cos \mu - \alpha_1 \sin \mu \end{pmatrix} \quad (19)$$

where $\alpha_1$, $\beta_1$ and $\gamma_1$ have their usual meaning 1) and $\mu = 2\pi Q/ISUP$. Since the matrix product (18) starts with $M_1$ on the right, $\alpha_1$, $\beta_1$ and $\gamma_1$ are in fact the betatron functions at the entrance of the first element. $\mu$ is real and the betatron motion is stable if the absolute value of the trace of (19) is less than 2. If this is not the case, the $Q$ value and all $\alpha$'s, $\beta$'s and phase advances in the unstable plane are set to zero, and none of the subsequent calculations of betatron functions are performed.

The product matrix $\Pi_{IEL+i,i+1}$ from the $(i+1)$'th element to the $(IEL+i)'$th element - the indices are taken modulo IEL - is obtained from the preceding product by a similarity transformation

$$\Pi_{IEL+i,i+1} = M_i^{-1} \Pi_{IEL+i-1,i} M_i^{-1} \quad (20)$$
The betatron functions at the entrance of the \((i+1)\)'th element may then be found in the same way from (20) as those for the first element from (19). The phase advance \(\mu_i\) in the \(i\)'th element can be calculated from \(\alpha_i, \beta_i\) and the elements \(m_{jk}^i\) of the transformations matrix \(M_i\) using the formula

\[
\mu_i = \arctg \frac{m_{12}^i}{\beta_i m_{11}^i - \alpha_i m_{12}^i}
\]  

(21)

The phase advance \(\mu_k\) up to the end of the \(k\)'th element is given by

\[
\mu_k = \sum_{i=1}^{k} \mu_i
\]

(22)

7.3 Momentum compaction function and determination of the closed orbit

The momentum compaction function \(\alpha_p^i\) and its azimuthal derivative \(\alpha'_p^i = \frac{d\alpha_p^i}{ds}\) at the entrance of the \(i\)'th element are solutions to the equation

\[
\begin{pmatrix}
\alpha_p^i \\
\alpha'_p^i \\
1
\end{pmatrix} = \Pi_{IET+i-1,i} \begin{pmatrix}
\alpha_p^i \\
\alpha'_p^i \\
1
\end{pmatrix}
\]

(23)

which can be written explicitly:

\[
\alpha'_p^i = \frac{\pi_{13}^i \pi_{21}^i + (1 - \pi_{11}^i) \pi_{23}^i}{(1 - \pi_{11}^i)(1 - \pi_{22}^i) - \pi_{21}^i \pi_{12}^i}
\]

(24)

and

\[
\alpha_p^i = \frac{\pi_{12}^i \alpha'_p^i \pi_{13}^i}{1 - \pi_{11}^i}
\]

(25)

where the \(m_{jk}^i\) are the elements of the matrix \(\Pi_{IET+i-1,i}\). In principle, \(\alpha_p^i\) and \(\alpha'_p^i\) can be calculated in this way for all \(i\). However, it is more straightforward to calculate only \(\alpha_p^i\) and \(\alpha'_p^i\) in this way and all the others by matrix multiplication.

The same procedure with the vector in (23) replaced by

\[
\begin{pmatrix}
x_i \\
x'_i \\
\Delta p/p
\end{pmatrix}
\]

(26)
can be used to determine the closed orbit of a particle with momentum error $\Delta p/p$. Since the matrices themselves depend on $\Delta p/p$ an iterative procedure is used in which the program alternates between calculating matrices and finding the closed orbit according to (23) and (26).

The maximum number of searches to find the closed orbit is $\text{ITER} + 10$. $\text{ITER}$ is the third variable on the data card following the $E$ control card (see 5). For each search the program makes three iterations, calculates $\text{XFACT} = \frac{x_2 - x_2}{x_1 - x_2}$ and $\text{XXFACT} = \frac{x_3 - x_2}{x_2 - x_1}$. If $\text{XFACT} \cdot \text{XXFACT}$ is greater than 1, the following message is printed:

$$\text{ABS(XFACT} \cdot \text{XXFACT)}.\text{GT.1.0}$$

The program stops the search and jumps to the next momentum error. Otherwise the program checks the value of $R = \left| \frac{x_2 - x_2}{x_2 - x_3} \right|$; if it is less than $10^{-4}$ the closed orbit is considered as satisfactory, the program computes the matrices and then continues with the next momentum error. If $R$ is greater than $10^{-4}$ the program continues the search of closed orbit. If after $\text{ITER} + 10$ iterations, $R$ is still greater than $10^{-4}$ the program prints the message

NO CONVERGENCE

stops the search and jumps to the next momentum error.

Knowing the shape of the momentum compaction function or of the closed orbit we can calculate the ratio $\gamma_t$ of the transition energy to the rest energy to first order. In general

$$\gamma_t = \frac{\Delta C}{2} \frac{C}{\Delta p/p} \quad (27)$$

where $C$ is the circumference of the machine. To the first order the circumference changes are only due to the displacement of the closed orbit inside the magnets with bending angle $\phi_i$. The contribution of the $i$'th magnet $\Delta C_i$ is given by

$$\Delta C_i = \frac{\phi}{2} (x_i + x_{i+1}) \quad (28)$$

The program sums all these contributions and then uses (27) to calculate $\gamma_t$. For machines with imaginary transition energy in which the radicand of (27) is negative, its absolute value is taken and $\gamma_t$ is given a negative sign.

7.4 Chromaticity

The chromaticity $Q' = dQ/dp/p$ in the two planes is given by the following expression 7)

$$Q'_{x} = - \frac{1}{4\pi} \int_{M=Q} (K_{x} - \alpha_{p} \frac{dK_{x}}{dx}) \beta_{x} \, ds$$

$$= - \frac{1}{2\pi p} \int_{M} (K_{x} \alpha_{p} \beta_{x} + \frac{d\gamma_{p}}{ds} \alpha_{x} - \frac{1}{2} \alpha_{p} \gamma_{x}) \, ds$$
\[ Q'_z = -\frac{1}{4\pi} \int_{M+Q} \left( K_v - \alpha_p \frac{dK_v}{dx} \right) \beta z \, ds \]
\[ -\frac{1}{4\pi \alpha_p} \int_M (K_v \alpha_p \beta z + \alpha_p \gamma z) \, ds \]

The integrals marked M+Q are done for bending magnets and quadrupoles, the ones marked M are done for the bending magnets only. At the moment, the program only includes the following terms:

\[ Q'_x = -\frac{1}{4\pi} \int_{M+Q} K_h \beta_x \, ds \]
\[ Q'_z = -\frac{1}{4\pi} \int_{M+Q} K_v \beta_z \, ds \]

This allows at least to calculate the chromaticity for separated function lattices without sextupole correction. The above integrals can be evaluated in closed form for any magnet. The contribution of the \( i \)'th element to the chromaticity can be expressed in terms of the values of \( \alpha, \beta, \gamma \) at the entrance of the element, and its matrix elements as follows

\[ -\frac{K}{4\pi} \left[ \frac{1}{2}(\beta + \frac{\gamma}{K}) + \frac{1}{2}(\beta - \frac{\gamma}{K}) m_{11} m_{12} - \alpha m_{12}^2 \right] \]

7.5 Output from the structure computations

7.5.1 Minimum output

The structure computations for \( \Delta p/p = 0 \) are always made and always generate output which contains headings, a table containing all elements, one per line, and some summary information about the betatron functions. The sample, shown in Figs. 4 and 5, gives the case explained in Tables II and III. Fig. 4 shows the input deck and Fig. 5 the printout. The printout should be largely self-explanatory. A few comments only should be adequate:

i) angles are printed in milliradians although they are given in radians
ii) the columns headed by \( X \) and \( XX \) give \( \alpha_p \) in metres and \( \alpha'_p \) in radians respectively
iii) \( \alpha, \beta, \alpha_p, \alpha'_p \) are given for the entrance of the element, the accumulated phase advance \( \mu \) is given for the exit of the elements. \( \beta \) is given in metres and \( \mu/2\pi \) is given in radians/2\pi. Hence one revolution in the phase plane corresponds to \( \mu/2\pi = 1 \).

7.5.2 Output for \( \Delta p/p \neq 0 \)

The amount of output for each of the IP cases with \( \Delta p/p \neq 0 \) is controlled by IPRINT, the second variable on the data card following the \( E \) control card (see 5).
If IPRINT = 0 each case with Δp/p ≠ 0 generates a heading, a table of one line per element giving all elements and a line of summary information about the betatron functions. The contents of the table is as described in 7.5.1, except that the columns headed by X and XX now contain the closed orbit position and slope for that momentum error, expressed in m and rad, respectively.

If IPRINT ≠ 0 only the summary line about the betatron functions is printed.

7.5.3 Binary output

If the tape flag is set by a previously read TA control card (see 17.6) which is still active, the tables of elements which are printed are also written onto a file with logical number 3 in the form of IEL binary records - one record per element - containing the following information

IEL, NO, NAME, L, dL/dx, d²L/dx², φ, Kν, dKν/dx, d³Kν/dx³,

αp, α'p, θp, βν, αν, ων/2π, ων/2π

The last record of a structure differs from all the others by having the element number NO equal to IEL which is easily detected by any program reading this information. The tape flag is switched off when an EF control card is read (see 17.7). No binary output is generated for subsequent structure computations. The standard option initially loaded with the AGS program is "tape flag off".

8. MATCHING OF PARAMETERS (MA)

Matching of parameters is selected by an MA control card. It is followed by a single data card containing

(IMAT(I), I = 1,6) and (XMAT(I), I = 1,5)

in format (6I5, 5F10.5). The type of matching is determined by IMAT(I), the meaning of the other variables depends on the procedure selected. If IMAT(I) is not between 1 and 7 an error message

MISSING ROUTINE j

occurs where j is IMAT(I).

After all matching operations the program automatically enters the structure computations.

8.1 Matching procedures Nos. 1 to 3

The matching procedures Nos. 1, 2, 3 are all related to adjusting parameters such as to obtain certain characteristics of the phase advance μ.
The parameters have the following meaning:

| IMAT(1) | 1 or 2 or 3 |
| IMAT(2) | variable gradient index |
| IMAT(3) | variable gradient index |
| IMAT(4) | number of iterations - 1 |
| IMAT(5) | first element number of the sequence to be matched, IMAT(5) = 1, if zero or blank |
| IMAT(6) | last element number of the sequence to be matched, IMAT(6) = IEL if zero or blank |

Let $\mu_h(\delta)$ and $\mu_v(\delta)$ be the phase advance for the momentum error $\delta$ in the horizontal and vertical plane, respectively, from the beginning of the IMAT(5)'th element to the end of the IMAT(6)'th element. The matching routines then try to fulfill the equations shown in Table V.

**Table V

Matching equations**

<table>
<thead>
<tr>
<th>IMAT(1)</th>
<th>Type</th>
<th>Parameter</th>
<th>Equations</th>
</tr>
</thead>
</table>
| 1       | quadrupole | $K_y$     | $\mu_h(0)/2\pi = XMAT(1)/ISUP$
|         |         |           | $\mu_v(0)/2\pi = XMAT(2)/ISUP$
| 2       | sextupole | $\frac{dK_y}{dx}$ | $[\mu_h(\delta) - \mu_h(-\delta)]/2\pi = XMAT(1)/ISUP$
|         |         |           | $[\mu_v(\delta) - \mu_v(-\delta)]/2\pi = XMAT(2)/ISUP$
| 3       | octupole | $\frac{d^2K_y}{dx^2}$ | $[\mu_h(\delta) + \mu_h(-\delta) - 2\mu_h(0)]/2\pi = XMAT(1)/ISUP$
|         |         |           | $[\mu_v(\delta) + \mu_v(-\delta) - 2\mu_v(0)]/2\pi = XMAT(2)/ISUP$

$XMAT(3)$ is the relative step-width in the parameter trials, $XMAT(3) = 0.01$ if zero or blank.

$XMAT(4) = \delta$, the momentum error $\Delta p/p$ for which the matching operations 2 and 3 are performed. $XMAT(4) = 0.01$ if zero or blank.

$XMAT(5)$ is not used.

If IMAT(5) = 1 and IMAT(6) = IEL we have, for quadrupole matching, $XMAT(1) = Q_h \text{ and } XMAT(2) = Q_v$. This is the reason for the factors $1/ISUP$ in the matching equations.

For quadrupole matching, adjustments of the phase advance in one direction (radial or vertical) are also admitted. In this case, either IMAT(2) or IMAT(3), and either $XMAT(1)$ or $XMAT(2)$ must be zero. The non-vanishing $XMAT$ determines the direction.

If the data do not make sense, or if 10 attempts to obtain stable sets of gradients fail, matching is abandoned and one of the following error messages is generated:

NO MATCH
ZERO PAR
8.2 Matching procedure No. 4

The matching procedure No. 4 simultaneously adjusts the phase advance in a piece of structure and the betatron functions at its beginning. The routine was originally written for matching Collins straight sections 3) into a synchrotron structure. However, the coding is more general and should allow for more complicated cases as well. The parameters have the following meaning:

IMAT(1) 4
IMAT(2) variable gradient index. The second variable gradient index is assumed to be IMAT(2)+1
IMAT(3) variable length index. The second variable length index is assumed to be IMAT(3)+1
IMAT(4) number of iterations - 1
IMAT(5) first element of insertion to be matched
IMAT(6) last element of insertion to be matched
XMAT(1) \((\mu/2\pi) \times \text{ISUP}, \) assumed to be equal in both planes
XMAT(2) relative step-width in gradient and length parameters,
XMAT(2) = 0.01 if zero or blank
XMAT(3) wanted horizontal \( \beta \) at beginning of insertion
XMAT(4) wanted horizontal \( \alpha \) at beginning of insertion
XMAT(5) not used

The conventions for IMAT(2) and IMAT(3) have to be taken into account when writing structure cards for this matching operation. If the data do not make sense the message ERROR IN DATA is generated and matching is abandoned. Since this matching procedure uses itself the matching procedure No. 1, those errors may occur too. More general betatron matching routines have now been incorporated in the program (see 14).

8.3 Matching procedure No. 5

This procedure tries to achieve given values of \( \alpha_p \) and of \( \alpha'_p \) at the entrance of a specified element by adjusting two free parameters \( \sigma \) and \( \tau \). Following the method chosen for the ISR 4) we start with three gradients \( K_i, K_j \) and \( K_k \) for the horizontally focusing and for the two kinds of horizontally defocusing quadrupole, respectively, which obey the following relation:

\[ K_i + K_j + K_k = 0 \]  \hspace{1cm} (29)

The matching routine replaces them by new gradients

\[ K_i' = \sigma K_i \]
\[ K_j' = \sigma(K_j + \tau(K_j + K_k)) \]
\[ K_k' = \sigma(K_k - \tau(K_j + K_k)) \]

such that the specified values of \( \alpha_p \) and \( \alpha'_p \) are achieved. It may be seen that (29) also holds for the primed \( K \)'s.
The parameters have the following meaning:

- **IMAT(1)**: 5
- **IMAT(2)**: variable gradient index \( i \)
- **IMAT(3)**: variable gradient index \( j \)
- **IMAT(4)**: variable gradient index \( k \)
- **IMAT(5)**: number of iterations - 1
- **IMAT(6)**: number of the element at whose entrance matching is to be achieved
- **XMAT(1)**: value of \( \alpha_p \) to be achieved
- **XMAT(2)**: value of \( \alpha'_p \) to be achieved
- **XMAT(3)**: relative step-width in gradient variations,
  \( XMAT(3) = 0.01 \) if zero or blank
- **XMAT(4)**: not used
- **XMAT(5)**: not used

### 8.4 Matching procedure No. 6

It was found in the ISR that the maximum value of the vertical \( \beta \) functions could be minimized by varying the length of the long straight sections in the inner arc \(^4\).

This procedure systematically changes two coupled parameters. In each step it simultaneously varies the two parameters by their respective step-width. It continues in a given direction until it finds that the last \( \beta \) value is bigger than the previous one. In this case, it reverses the direction of the scan and takes half the step-widths. It continues this operation until \( |XMAT(1)| \) has become smaller than \( XMAT(3) \). (Two coupled parameters and appropriate step-widths are necessary when varying straight section lengths in order to keep the circumference constant).

The parameters are:

- **IMAT(1)**: 6
- **IMAT(2)**: variable parameter index
- **IMAT(3)**: variable parameter index
- **IMAT(4)**: = 1, vertical \( \beta \)'s are minimized
  = 2, horizontal \( \beta \)'s are minimized
- **IMAT(5)**: not used
- **IMAT(6)**: not used
- **XMAT(1)**: step in parameter given by \( IMAT(2) \)
- **XMAT(2)**: step in parameter given by \( IMAT(3) \)
- **XMAT(3)**: minimum step to be taken in the parameter variation

The program abandones matching if the data do not make sense (ERROR IN DATA) or if the particle motion becomes unstable (ONE BETA IS ZERO).
8.5 Matching procedure No. 7

This procedure is a simplified version of the procedure No. 5. It tries to achieve a given value of $\alpha_p$ at the entrance of a given element by variation of the gradients in up to 3 elements without changing their gradient ratio. Hence, there is just one free parameter to be adjusted. The data for this procedure are:

| IMAT(1) | 7 |
| IMAT(2) | variable gradient index |
| IMAT(3) | variable gradient index |
| IMAT(4) | variable gradient index |
| IMAT(5) | number of iterations - 1 |
| IMAT(6) | number of the element at whose entrance matching is to be achieved |
| XMAT(1) | value of $\alpha_p$ to be achieved |
| XMAT(2) | not used |
| XMAT(3) | relative step-width in gradient variations, $\text{XMAT}(3) = 0.01$ if zero or blank |
| XMAT(4) | not used |
| XMAT(5) | not used |

9. CLOSED ORBIT CALCULATIONS IN MACHINES WITH MISALIGNED ELEMENTS (MI)

The $3 \times 3$ matrix formalism used in the horizontal plane can be adapted to the calculation of the distorted orbit in a machine with misaligned elements. To this end the $m_1$ and $m_2$, matrix elements must become functions of the misalignment and the matrices must operate on column vectors of the form 5):

$$\begin{pmatrix} x \\ x' \\ 1 \end{pmatrix}$$

This is done in the misalignment subroutines of the AGS program. Their operation is determined by the contents of a data card containing

$$(\text{IMIS}(I), I = 1,6) \ , \ (\text{XMIS}(I), I = 1,5)$$

in format (6I5, 5F10.5). These variables have the following meaning:

| IMIS(1)   | misalignment type |
| IMIS(2)   | number of machines studied in a run |
| IMIS(3)   | number of elements which are displaced as a whole |
| IMIS(4)   | not used |
| IMIS(5)   | not used |
| IMIS(6)   | not used |
XMIS(1) peak or rms displacement
XMIS(2) not used
XMIS(3) not used
XMIS(4) not used
XMIS(5) histogram bin width, must be $\neq 0$

9.1 Misalignment types

The misalignment types available in the program are shown in Table VI. Sketches of the various types are shown in Fig. 6.

<table>
<thead>
<tr>
<th>IMIS(1)</th>
<th>Distribution of displacements</th>
<th>Type of displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>individual</td>
<td>displacement of IMIS(3) elements</td>
</tr>
<tr>
<td>11</td>
<td>uniform</td>
<td>parallel displacement of IMIS(3) elements</td>
</tr>
<tr>
<td>21</td>
<td>gaussian</td>
<td>idem</td>
</tr>
<tr>
<td>12</td>
<td>uniform</td>
<td>tilt of single elements</td>
</tr>
<tr>
<td>22</td>
<td>gaussian</td>
<td>idem</td>
</tr>
<tr>
<td>13</td>
<td>uniform</td>
<td>independent displacement of entrance and exit of single elements</td>
</tr>
<tr>
<td>23</td>
<td>gaussian</td>
<td>idem</td>
</tr>
<tr>
<td>14</td>
<td>uniform</td>
<td>independent displacement of beginning and end of groups of IMIS(3) $\geq 2$ elements</td>
</tr>
<tr>
<td>24</td>
<td>gaussian</td>
<td>idem</td>
</tr>
<tr>
<td>15</td>
<td>uniform</td>
<td>hinge type displacement of IMIS(3) $\geq 2$ elements together</td>
</tr>
<tr>
<td>25</td>
<td>gaussian</td>
<td>idem</td>
</tr>
<tr>
<td>16</td>
<td>uniform</td>
<td>hinge type displacement of single elements</td>
</tr>
<tr>
<td>26</td>
<td>gaussian</td>
<td>idem</td>
</tr>
</tbody>
</table>

For IMIS(1) = 1 the serial numbers and entrance and exit displacements are given on IMIS(3) data cards in format (IS, 25X, 2F10.5).

For all uniform distributions the element displacements are uniformly chosen at random between $-$ XMIS(1) and $+$ XMIS(1). For all gaussian distributions XMIS(1) is the rms width.

When an element or a group of elements is tilted its beginning and end are displaced by equal amounts in opposite direction. It is this displacement which is given by XMIS(1).

When element displacements are of the hinge type the end of one element and the beginning of the next have equal displacements.
In all cases where several elements are displaced together, they are all on a straight line whose positions at the beginning and end of the group are determined by the misalignment type.

9.2 Output from misalignment routines

The amount of output is controlled by the value of IPRINT, the second variable on the data card following the E control card (see 5).

If IPRINT = 0 the whole table of elements with one line per element is printed for each of the IMIS(2) machines in the columns X and XX the distorted closed orbit and its derivative, and in the columns BETAX and BETAZ the entrance and exit displacements, respectively. If the tape flag is on (see 17.6) the same information is also written onto tape.

If IPRINT ≠ 0 none of this output is produced.

The results of the misalignment computations are then summarized in:

i) a table showing the average peak distortion (average over the IMIS(2) peak distortions in each machine), its variance, the worst distortion and the rms distortion (the rms average being taken over the IEL distortions in each of the IMIS(2) machine),

ii) a histogram showing the distribution of the peak distortion over up to 100 channels of width XMIS(5),

iii) unordered and ordered listings of the peak distortions.

If IMIS(2) = 1 – one machine only is computed – the program also produces a listing of the entrance and exit displacements of each element.

9.3 Error condition

If IMIS(2) > IDIM, the size of the working space, (see 18) an error exit occurs (IRUN.....IS TOO HIGH).

10. GEOMETRY CALCULATIONS (GE)

Since all the information necessary for the calculation of the shape of a synchrotron or storage ring is available inside the AGS program, these calculations are easily done. They are initiated by a GE control card. No data cards are required.

10.1 Calculation of exit co-ordinates of all elements

The convention of the program is to start with the co-ordinates (x, y) of the entrance of the first element being (0, 0) and with the equilibrium orbit along the x axis, as shown in Fig. 7 (x = 0).
Let the co-ordinate of the exit of the j'th element be \((x_j, y_j)\) and the slope \(\chi_j\).
The co-ordinates of the \((j+1)\)'th element and the slope \(\chi_{j+1}\) of the equilibrium orbit are
then calculated by one of the following procedures, depending on the type of the element.

i) In a wedge magnet with length \(d_{j+1}\) and bending angle \(\phi_{j+1} \neq 0\) we add an arc of
a circle with radius \(\rho = d_{j+1} / \phi_{j+1}\):

\[
\begin{align*}
    x_{j+1} &= x_j + \rho (\sin \chi_j + \phi_j - \sin \chi_j) \\
    y_{j+1} &= y_j + \rho (\cos \chi_j - \cos (\chi_j + \phi_{j+1}) \\
    \chi_{j+1} &= \chi_j + \phi_{j+1}
\end{align*}
\]  
(31)

All magnets are assumed to be wedge magnets unless the straight magnet flag
is set (see 6.4).

ii) In an element with \(\phi_{j+1} = 0\) we add a straight line of the appropriate length
\(d_{j+1}\) and direction to the equilibrium orbit

\[
\begin{align*}
    x_{j+1} &= x_j + d_{j+1} \cos \chi_j \\
    y_{j+1} &= y_j + d_{j+1} \sin \chi_j \\
    \chi_{j+1} &= \psi_j
\end{align*}
\]  
(32)

iii) In a straight magnet with length \(d_{j+1}\) and bending angle \(\phi_{j+1} \neq 0\), the trajectory
is extended by a straight line of length \(d_{j+1}\), whose direction is changed by
\(\phi_{j+1}/2\) at the entrance and at the exit:

\[
\begin{align*}
    x_{j+1} &= x_j + d_{j+1} \cos (\chi_j + \phi_{j+1}/2) \\
    y_{j+1} &= y_j + d_{j+1} \sin (\chi_j + \phi_{j+1}/2) \\
    \chi_{j+1} &= \chi_j + \phi_{j+1}
\end{align*}
\]  
(33)

Straight magnets are only assumed when the straight magnet flag is set (see 6.4).

10.2 Calculation of the machine centre and
of maximum and minimum radii

If the structure is a complete super-period (WHOLE is true, see 4), it is known that
either end of it must have the same distance from the machine centre, and that the angle
between the radii joining the machine centre and the two ends of the super-period must be
equal to the total bending angle. These facts are sufficient for the calculations of the
machine centre as shown in Fig. 7. The following relations hold:

\[\text{tg}(x_{\text{in}}/2) = a/2b\]  
(34)
\[ a = \left( x_{\text{IEL}}^2 + y_{\text{IEL}}^2 \right)^{\frac{1}{2}} \]  

(35)

The co-ordinates of the machine centre \((x_c, y_c)\) are then:

\[ x_c = x_{\text{IEL}} / 2 - b \sin \gamma \]

\[ y_c = y_{\text{IEL}} / 2 + b \sin \gamma \]  

(36)

where the angle \(\gamma\) is shown in Fig. 7. This procedure is adopted if the number of super-periods \(ISUP\) (see 4.1) is greater than 1. If \(ISUP = 1\), and if \(WHOLE\) is false, the machine centre is not calculated. Knowing the co-ordinates of the machine centre and of all elements it is easy to calculate the minimum and maximum radii of the machine.

10.3 Output from geometry calculations

The output of the geometry routine consists of a table of one line per element giving the characteristics of the element and the co-ordinates of its end. This table is also written onto a binary file if the tape flag (see 17.6) is set. There is one record containing the following information:

\[ \text{IEL, (NAME(I), LENGTH(I), ANGLE(I), GRADIENT(I))} \]

\[ X(I), Y(I), \text{PSI(I)}, I = 1, \text{IEL}, A(99) \]

The geometry printout is terminated by a list of the centre co-ordinates and the maximum and minimum radii if they are calculated.

11. TRACKING OF A SET OF TRAJECTORIES (TH, TV)

Control cards containing TH or TV initiate tracking of a set of trajectories through the structure or part of it, either in the horizontal plane (TH) or in the vertical plane (TV).

As for finding the closed orbit for particles with a momentum error, the trajectories are found by an iterative procedure in which the trajectory found in one step is used for the calculation of the matrices of the following step.

During tracking closed orbit bumps may be included which are represented by abrupt changes ("kicks") of the slope of the particle trajectory at the entrance of specified elements.

One call of the tracking routines allows to calculate a whole set of trajectories in which one of the initial conditions \((x, x', \text{ or } \Delta p/p)\) is systematically varied.

The control card (TH or TV) is followed by data card containing

\[ (\text{ITR(I), I = 1,6}), (\text{XTR(I), I = 1,7}) \]

in format \((6I5, 4 F10.5, F4.0, 2F5.0)\). The parameters are interpreted as follows:
ITR(1) number of the element at whose entrance tracking starts
ITR(2) number of the element at whose exit tracking stops
ITR(3) number of iteration - 3 in finding the trajectory.
    Usually ITR(3) = 0, i.e. just 3 iterations are sufficient
ITR(4) number of trajectories to be tracked
ITR(5) switch for selecting the variable initial condition:
    if ITR(5) = 1, x is variable; if ITR(5) = 2, x' is variable;
    if ITR(5) = 3, dp/p is variable
ITR(6) switch for selecting the form of output and the presence or
    absence of kicks
XTR(1) starting position x of the first trajectory (mm)
XTR(2) starting slope x' of the first trajectory (mrad)
XTR(3) momentum error dp/p of the first trajectory (per mille)
XTR(4) step in the variable parameter (mm, mrad, per mille)
XTR(5) is the longitudinal scale of the plot in metres per "inch"
XTR(6) is the transverse scale in millimetres per "inch"
XTR(7) is the position in "inches" of the zero of the transverse scale.
    XTR(7) may be negative, positive, or zero. The transverse
    range of the plot will be from -XTR(7) * XTR(6) mm to
    (10-XTR(7)) * XTR(6) mm.

ITR(6) may take all positive values and is interpreted as follows:

ITR(6) = 0 results in the output containing a list of all properties
    of the elements (name, length, angle, gradient and derivatives) and x and x'.
ITR(6) = 1 gives a list of only element name and length, x and x', and of
    the horizontal and vertical transfer matrices from the beginning of the
    trajectory up to the entrance of the element being printed. Not for
    vertical tracking (TH).
ITR(6) > 2 causes the program to include kicks. This option requires
    further data cards. The first of them gives the number of kicks NKICK
    in format (IS), the remaining NKICK data cards contain the number of the
    elements at whose entrance a kick occurs and the kick itself in mrad, in
    format (IS, F10.5). The output is the same as for ITR(6) = 0.

The last 3 floating point parameters specify the output plotted on a CALCOMP plotter.
Plots are generated if XTR(5) ≠ 0. If XTR(5) > 0 a new plot is started with properly
labelled axes etc., if XTR(5) < 0 the trajectories are drawn into an existing plot from
a previous tracking operation. The convention of the program is that all trajectories
of the set are plotted on the same graph. The numbers XTR(5) to XTR(7) give the scale
factors to be applied. The transverse dimension of the graph paper is 15 "inches" the
longitudinal dimension is determined by XTR(5). If a point is outside the transverse
dimension of the plot, the whole job is abandoned by the plotting routines.
If ITR(I) is different from 1, 2 or 3 (for I = 5) or if ITR(I) is not included in the range 1, ..., IEL (for I = 1, 2), the error message
ERROR IN ITR(I)
is printed.

12. DESIGN OF CLOSED ORBIT BUMPS (BH, BV)

Control cards containing BH or BV initiate the design of a closed orbit bump, either in the horizontal (BH) or in the vertical plane (BV).

A closed orbit bump is a trajectory which, at the entrance of an element IBU(1), is situated on the equilibrium orbit for the corresponding momentum, and which, at the entrance of an element IBU(3), has a specified displacement with respect to the central equilibrium orbit and which at the entrance of an element IBU(5) is again on the equilibrium orbit. In the case of four kick bumps, also the slope at the entrance of the element IBU(3) must be specified.

This closed orbit bump is obtained by means of two "kicks" (see 11) upstream to IBU(3) - namely at the entrance of the element IBU(1) and of the element IBU(2) - and two other kicks downstream to IBU(3) - namely at the entrance of the element IBU(4) and of the element IBU(5).

The program tracks, as described in 11, trajectories starting on the equilibrium orbit at the entrance of IBU(1) and computes the magnitude of the kicks such as to satisfy the above requirements.

In the cases where only a given displacement is required at IBU(3), three kicks are sufficient. For practical reasons, the program deals only with the case where one of these three kicks is given upstream to IBU(3) - namely at IBU(1) and the other two downstream. The slope of the orbit at IBU(3) turns out to be automatically determined.

The control card (BH or BV) is followed by a data card containing

(IBU(1), I = 1,6), (XBU(1), I = 1,7)
in format (6I5, 4F10.5, F4.0, 2F3.0). The parameters are interpreted as follows:

IBU(1) number of the element at whose entrance tracking starts and the first kick is given
IBU(2) number of the element at whose entrance the second kick is given. For three kick bumps IBU(2) must be zero
IBU(3) number of the element at whose entrance the wanted displacement and, in the case where IBU(2) = 0, the slope of the trajectory with respect to the \( \Delta p/p = 0 \) equilibrium orbit are specified
IBU(4) number of the element at whose entrance the third kick is given
IBU(5) number of the element at whose entrance tracking stops and 
the fourth kick is given. The numbers IBU(1) to IBU(5) must be 
in ascending order, with the exception of three kick bumps where 
IBU(2) must be zero
IBU(6) see parameter ITR(3), section 11
XBU(1) required displacement of the trajectory at the entrance of 
element IBU(3), expressed in mm
XBU(2) required slope of the trajectory at the entrance of the element 
IBU(3), expressed in mrad
XBU(3) momentum deviation $\Delta p/p$ of the trajectory, expressed in per mille
XBU(4) not used
XBU(5) see parameter XTR(5), section 11
XBU(6) see parameter XTR(6), section 11
XBU(7) see parameter XTR(7), section 11

The output gives the trajectory in the same form as obtained when tracking with 
ITR(6) = 0 (see 11). In addition to this, it contains the required displacement and 
possibly slope at the entrance of the element IBU(3), the displacement and the slope 
(XCO, XXCO) of the closed orbit where the kicks are given and the kicks themselves (DXX), 
expressed in mrad.

If the rules for IBU are violated the error message

`WRONG IBU`

is printed. If the number of iterations is insufficient, the message

`IBU(6) IS TOO SMALL`

is generated.

13. TRACKING OF THE CHARACTERISTIC FUNCTIONS (TB)

The control card containing TB initiates the tracking of the following characteristic 
functions:

- the Twiss parameters $\beta_\theta$, $\beta_\nu$, $\alpha_\theta$, $\alpha_\nu$
- the phase-shift through the magnet structure $\nu_\theta$, $\nu_\nu$
- the momentum compaction function $\alpha_p$ and its derivative $\alpha'_p$

The control card is followed by three cards:

- the first card contains LFILE, KMIN, KMAX in format (3I5)
- the second card contains $\beta_\nu_0$, $\beta_\theta_0$, $\alpha_\nu_0$, $\alpha_\theta_0$ in format (6F10.5)
- the third card contains $\alpha_p_0$, $\alpha'_p$ in format (2F10.5)
The parameters are interpreted as follows:

\[ \begin{align*}
\text{LFILE} & \quad \text{determines the form of output} \\
& \quad \begin{cases}
1 & \text{no output} \\
2 & \text{output to printer} \\
3 & \text{output to printer and tape 3} \\
> 3, \leq 99 & \text{output to tape (LFILE) only}
\end{cases}
\end{align*} \]

\[ \begin{align*}
\text{KMIN} & \quad \text{number of the starting element for tracking} \\
\text{KMAX} & \quad \text{number of the exit element for tracking}
\end{align*} \]

\[ \beta_{V0}, \beta_{H0}, \alpha_{V0}, \alpha_{H0}, \frac{\mu_{V0}}{2\pi}, \frac{\mu_{H0}}{2\pi}, \alpha_{p0}, \alpha'_{p0} \] are the values at entrance of element KMIN of the functions to be tracked.

13.1 Output

The output consists of a heading and the following quantities at the exit of each element:

\[ I \quad L \quad A \quad \beta_{V} \quad \beta_{H} \quad \alpha_{V} \quad \alpha_{H} \quad \frac{\mu_{V}}{2\pi} \quad \frac{\mu_{H}}{2\pi} \quad \alpha_{p} \quad \alpha'_{p} \]

\[ I = \text{number of the element} \]

\[ L = \text{accumulated length} \]

\[ A = \text{accumulated deflection angle} \]

On tape, the output is in binary; the records contain the following information: first record contains the number of elements NEL and is followed by NEL records, one for each element containing \( I \quad L \quad A \quad \beta_{V} \quad \beta_{H} \quad \alpha_{V} \quad \alpha_{H} \quad \frac{\mu_{V}}{2\pi} \quad \frac{\mu_{H}}{2\pi} \quad \alpha_{p} \quad \alpha'_{p} \)

13.2 Error messages

The program abandones tracking if

\[ \text{LFILE} > 99 \quad (\text{ILLEGAL OUTPUT UNIT}) \]

\[ \text{KMIN} < 1 \text{ or } > \text{IEL} \quad (\text{ILLEGAL STARTING ELEMENT}) \]

\[ \text{KMAX} < \text{KMIN} \text{ or } > \text{IEL} \quad (\text{ILLEGAL EXIT ELEMENT}) \]

14. MATCHING OF THE CHARACTERISTIC FUNCTIONS (MG)

The parameters of a beam transport channel or part of a ring machine can be adjusted by the program to obtain specified values of the characteristic functions \( \beta_{V}, \alpha_{V}, \frac{\mu_{V}}{2\pi}, \beta_{H}, \alpha_{H}, \frac{\mu_{H}}{2\pi}, \alpha_{p}, \alpha'_{p} \), at the end of the channel, given the values at the beginning of the channel. Values of these functions at intermediate points (maximum 10) may also be imposed, together with upper and lower bounds on \( \beta_{V}, \beta_{H} \text{ and } |\alpha_{p}| \). For each of the above constraints a weighting factor, which may be zero, can be specified.

The adjustable parameters are the gradients (K) and the positions (P) of quadrupoles, and the lengths (L) of any element. However, the P and L modes cannot be used together in the same matching operation. Thus P mode permits quadrupoles of fixed length and variable
K to move in a channel of fixed total length. In L mode, lengths of any elements (quadrupoles, straight-sections and bending magnets) can be varied according to two options; either variable total channel length or fixed total channel length in which case there must be at least two variable lengths specified. Variable K is permitted in all cases. Upper and lower limits on all parameters can be chosen and must be specified consistently with the current parameter values on entry into the matching procedure.

Gradient bending magnets can be considered as quadrupoles in the matching procedure; the bending angle remains fixed, however.

This matching facility uses the CERN library program MINUIT 6), which minimizes a function FCN of many parameters, here the K and P or L parameters described above. The function FCN is generated from the product of terms of the form

\[ 1 + w_i^2(x_i - x_{0i})^2 \]

where \( x_{0i} \) represents the required value of one of the characteristic functions \( \beta_\nu \) ... etc. at the exit of the channel and \( x_i \) represents the actual value calculated with the current parameters by the tracking routine (T8). The \( w_i \) are the corresponding specified weights. The product of these quadratic terms minus 1 is the FCN value returned to MINUIT; for a perfect match it is zero. A facility exists also to impose further constraints via a user-supplied function CONST which is added to the FCN value above.

This matching is initiated by the control card MB. The control card is followed by

a) One card in format (3X2A1, 3X2A1, 3I5, 1F10.2) containing

<table>
<thead>
<tr>
<th>VARYL</th>
<th>MODE</th>
<th>RESTOR</th>
<th>MODIFY</th>
<th>KMIN</th>
<th>KMAX</th>
<th>NIP</th>
<th>COLL</th>
</tr>
</thead>
<tbody>
<tr>
<td>blank if fixed total length is desired</td>
<td>V if variable total length is desired (default = fixed length)</td>
<td>restored after matching</td>
<td>if non-blank, FCN will call function CONST (supplied by the user) and add CONST to the FCN output function value</td>
<td>number of first element of the transfer channel within which matching is to be performed</td>
<td>number of the last element of the transfer channel</td>
<td>number of intermediate points for matching (≤ 10)</td>
<td>minimum length between elements allowed (in m) (default = COLL = 0.05 m)</td>
</tr>
</tbody>
</table>
b) A sequence of cards in format (IS, 4XAI, 3F10.5) containing a description of the
elements to be adjusted.

NOELT = element number (must be in increasing order)
  L length will be adjusted
NTYPE = P position will be adjusted K gradient will be adjusted
ERROR = approximate error in the quantity to be adjusted
MINVAL = lower bound of the quantity to be adjusted
MAXVAL = upper bound of the quantity to be adjusted

This sequence is terminated with a card containing blanks or zeros in columns 1 - 5.
If in addition there is a C in column 10, the program expects to find at the end of
the MB data a sequence of MINUIT commands terminated by END RETURN.

c) Eight cards in format (3F10.5) for end-point-matching describing $\beta_V, \alpha_V, \frac{\mu_V}{2\pi}, \beta_h,$
$q_h, \frac{\mu_h}{2\pi}, \alpha_p, \alpha_p'.

1st field  value of function at the entrance of first element
2nd field  value desired at exit of last element
3rd field  matching weight

d) Six cards in format (10X, 2F10.5) for the limits describing $\beta_v min, \beta_v max, \beta_h min,$
$\beta_h max, |\alpha_p min|, |\alpha_p max|.$

1st field  specified value
2nd field  matching weight

e) NIP sets of eight cards (a set for each intermediate point) describing $\beta_V, \alpha_V, \frac{\mu_V}{2\pi}, \beta_h, q_h, \frac{\mu_h}{2\pi}, \alpha_p, \alpha_p'$ in format (110, 2F10.5) for $\beta_V$ and in format (10X, 2F10.5)
for the others.

1st field  NOL = element number (for $\beta_V$ only)
2nd field  desired value
3rd field  matching weight

f) MINUIT commands if C was in column 10 of last card of b); if column 10 was blank,
the program uses its standard sequence of MINUIT command cards:

PAGE
PRINTOUT 2
SIMPLEX S00
CALLFCN 3
ENDbRETURN
14.1 Error message

The program abandones matching if NIP (number of intermediate points) is greater than 10. It prints TOO MANY INTERMEDIATE POINTS MATCHING and stops.

If an element number does not lie between KMIN and KMAX, or if the first element is of type P, the program prints ILLEGAL ELEMENT NO n - DATA CARD IGNORED and reads the next data card.

If the total length is fixed (VARYL \n V) and only one element length is adjustable, the program abandons matching, prints ***NO MINIMIZATION ATTEMPTED WITH ONLY ONE VARIABLE LENGTH*** and reads next control card.

If the total length is fixed but is different from the sum of the lengths by more than \(10^{-9}\), the program prints ***CONFUSING INPUT - CHECK PARAMETER INDEPENDENCE*** and stops.

If the variable MODE is different from P or L, the program prints ***ILLEGAL MODE, L MODE ASSUMED*** and goes on with MODE = L.

14.2 Some notes on matching problems

- Difficulties can arise in practice in obtaining solutions to apparently well-defined matching problems, particularly where there are more than 3 or 4 constraints. These difficulties appear to arise mainly from the nature of the problem rather than from significant defects in the matching program or the MINUIT routines. Certain regions of parameter space may contain no real solutions, other regions containing solutions may be separated from the starting point by barriers imposed by bounds on the parameters and, finally, genuine local minima may occur with convergence of MINUIT but non-zero FCN ('mountain lakes'). The following notes may help users to overcome these difficulties.

1) A typical situation requires the matching of \(\beta_v\), \(\alpha_v\), \(\beta_h\), \(\alpha_h\), \(\alpha_p\) and \(\alpha'_p\). At first sight only six parameters are needed to satisfy the six boundary conditions. However, an additional constraint for matching \(\alpha_p\), \(\alpha'_p\) is imposed by the Courant and Snyder invariant

\[
W = \frac{1}{\beta_h} \left[ \alpha_p^2 + (\alpha_h \alpha_p + \beta_h \alpha'_p)^2 \right]
\]

In a channel without dispersion (bending magnets) \(W\) is invariant, and boundary conditions must take account of this if a correct match is to be achieved. In dispersive channels \(W\) can be modified, but a considerable amount of bending is usually necessary to make substantial changes in \(W\) without reaching excessive \(\beta\) values.

One should also note that \(\alpha_p\), \(\alpha'_p\) matching is in general constrained by the horizontal betatron phase shift \(\nu_h\), which in turn is determined by the betatron matching requirements.
ii) In general, more variable parameters should be allowed than the number of constraints.

iii) Weighting factors can only be determined by experience and depend on the nature of the problem. As a guide, however, we have found that, for typical storage-ring lattices and insertions, the following weights generally provide a good starting point

\[
\delta_y, \delta_h \sim 1 \\
\alpha_v, \alpha_h, \alpha_p \sim 10 \\
\alpha_p' \sim 100
\]

With such weighting factors we usually find a "good match" indicated by a final FCN value of \( \sim 10^{-15} \) or less. An FCN of around unity often indicates that the solution is being impeded by a parameter limit. FCN values of \( 10^4 \) to \( 10^5 \) are common on entry; if they remain above \( \sim 10^8 \) at the end of the matching attempt, the starting values of the parameters are probably too far from a solution, or the problem is over-constrained.

iv) The required number of calls to FCN depends on the number of variable parameters. Typically, 1000 to 3000 calls are appropriate for 4 - 8 variable parameters. The computation time depends strongly on the number of channel elements to be tracked at each call to FCN.

v) It is almost always advantageous to choose initial parameter values and constraints from some approximate solution, e.g. thin-lens formulae, or from some rough physical intuition. However, an exact thin-lens solution does not guarantee a corresponding thick-lens solution if \( \beta \)-values in quadrupoles approach the quadrupole length.

vi) In a channel with many variable elements it is advisable to match short sections consecutively, even if it means imposing artificial constraints at intermediate positions. Once a single complete solution for the whole channel is obtained it is then usually easier to force this solution in the required direction by judicious use of parameter limits, intermediate-point matching etc.

vii) When solutions cannot be reached with variable-position (P) mode, or variable-length (L) mode with fixed total channel length, it is often effective to allow the total channel length to change (VL mode). The channel then usually, though not always, tends to become longer! However, it is sometimes possible to coax the solution back towards the desired situation as indicated in vi) above.
viii) In end-point and intermediate-point matching, \( \beta_v \) and \( \beta_h \) must be specified non-zero and non-blank, even if the weighting is zero, otherwise an overflow or indefinite condition will occur.

ix) The current parameter values on entry into the matching procedure must lie inside the specified limits, otherwise MINUIT abandons the minimization.

14.3 Betatron matching and tracking example

Fig. 10 is an annotated example of a complete deck for reading a structure, setting initial parameter values, matching betatron functions and finally tracking. In some of the parameter cards columns 36 upwards have been used for listing comments. Fig. 11 shows some of the output resulting from this run.

15. SEARCH FOR A CLOSED ORBIT FOR OFF-MOMENTUM PARTICLES (CL)

The control card CL initiates a search for a closed orbit for off-momentum particles. This facility is provided in addition to the normal closed orbit search initiated by structure calculations for particles with a momentum error (see 7.3). It is supposed to overcome difficulties in finding the closed orbit for an AGS structure consisting of too many elements with non-linearities in gradient or effective length. With the control card CL the program performs a MINUIT minimization of

\[
\left[ 1 + 10^4 (x_0 - x_1)^2 \right] \left[ 1 + 10^4 (x'_0 - x'_1)^2 \right] - 1
\]

where
- \( x_0 \) = initial horizontal displacement
- \( x'_0 \) = initial derivative of \( x_0 \)
- \( x_1 \) = horizontal displacement tracked through one super-period
- \( x'_1 \) = derivative of \( x_1 \)

The calculation is done for the momentum errors read after the last preceding \( E \) command. The control card CL is followed by

a) A sequence of data cards (one for each momentum error) in format (4F10.5) containing

- \( X \) = best guess of the position of the closed orbit at entrance of first element (in mm)
- \( DX \) = possible error in guess of \( X \)
- \( XX \) = corresponding derivative of \( X \) (in mrad)
- \( DXX \) = possible error in guess of \( XX \)

b) A blank card indicating that program provides its own sequence of MINUIT command cards. This sequence is
or a sequence of MINUIT command cards.

15.1 Error message

The program abandones the search if no momentum error is specified. It prints
***NO MOMENTUM ERRORS*** and reads next control card.

If the momentum error is greater than 0.5 the program prints ***MOMENTUM ERROR
MAGNITUDE nn TOO LARGE*** and jumps to next momentum error.

16. ALTERNATIVE TRACKING COMMAND (TS)

The control card TS initiates tracking for large horizontal orbit displacements. A
fourth order Taylor series integration of the equation of motion is used, taking into
account second order terms (d^2X/dx^2) in the gradient correction and second order terms in
the effective lengths of elements. As the concept of effective lengths is used the magnet
structure is assumed to consist of piece-wise constant gradient elements. A step-size of
20 cm is used in the integration inside magnets whereas the straight sections are taken
in one step.

The input is exactly the same as for the TH or TV command (ITR(3) is not used).

The output gives a short list containing only beam length and trajectory co-ordinates.

16.1 Error message

If ITR(5) is different from 1, 2 or 3, or if ITR(I) is not included in the range 1,
IEL (for I = 1, 2) the error message ERROR IN ITR(I) is printed.

17. INPUT/OUTPUT CONTROL STATEMENTS

17.1 Read and store a text as heading (TE)

A heading of 80 alpha-numeric characters, the date and time are printed above the
output for most calculations. The heading is all blanks until it is set by reading a
heading from the data card in format (8A10) following a TE control card. On this data
card all 80 characters may be used. A heading remains active until it is overwritten by
a new one.
17.2 Read and store a random number (RI)

The random number generator used in the misalignment routines may be started at a specified random number by an RI control card followed by a data card giving the starting number in format (020).

17.3 Print a random number (RO)

The current random number in the random number generator, is printed when an RO control card is encountered.

The two control cards RI and RO together allow to continue the calculation of misalignments in different runs of the program always using fresh random numbers.

17.4 Print the current matrices (MX)

When an MX control card is found in the deck the program computes all element matrices for $\Delta p/p = 0$ on the equilibrium orbit and prints them. Corrected matrices for $\Delta p/p \neq 0$ or for trajectories away from the equilibrium orbit, are not accessible via this control card, see section 11 instead.

17.5 Punch a compressed structure deck (CO)

Cards to be read by the compressed structure input option S2 (see 4.2) are punched when a CO control card is found in the data. This allows to put into compressed form structures which were read in using some more convenient format and which are reasonably stationary.

17.6 Switch tape flag on (TA)

A TA control card sets the tape flag. Output which is generated by structure computations (see 7.4.3) and by geometry calculations (see 10.3) is also written onto a file in binary form. This allows AGS results to be used by other programs. The tape flag is switched off at the beginning of a run.

17.7 Switch the tape flag off (EF)

An EF control card switches the tape flag off, writes an end of file mark and backspaces the file across it. In this way, several sets of binary results may be written into the same file but, hopefully, the last one is always terminated by an end of file mark.

17.8 Print and punch parameters (PU)

When a PU control card is found, the program prepares a parameter list (see 6), prints it and writes it onto TAPE4. The list starts with a card giving the number of cards to follow (format IS) and parameter cards containing I, A, B, C (format IS, 3 fixed point fields of width 10 chosen such that the maximum number of significant digits is included). Only parameters with at least one of the quantities A, B, C non-zero are included.
17.9 **Structure computation and output (GO)**

If the GO control card is active, the program enters the structure computations and prints the output according to the variable IPRINT (see 7.4.2), as soon as the parameters have been read in. At the beginning of the program GO is active.

17.10 **No automatic structure computation and output (NO)**

if the NO control card is active, the program reads the parameters, but does not enter the structure calculations.

17.11 **Stop card (ST)**

A control card with ST in the first two columns terminates the execution of the AGS program.

18. **PROGRAMMING DETAILS**

The information described so far should be sufficient to prepare the data cards for the AGS program. This chapter contains some details about the program which are considered useful for running the program on a computer or modify it. This applies to running the program on the CDC 7600 computer at CERN.

The maximum number of elements IDIM which can be handled by the program is defined in a dummy main program. This is achieved by setting the working space in the following symbolic form

\[
\text{COMMON IDIM, W(IDIM, 14)}
\]

An example with IDIM = 100 is shown in Fig. 8.

All the operations of the program are confined to the subroutine NEWAGS and its own subordinate subroutines. In this way, small structures can be investigated with a fairly small amount of memory. This is also the reason for the segmented structure of the program adopted, which is shown in Fig. 9. An alphabetic list of the error messages generated by the program is given in Table IX. The control cards to run the program are described in Table X.

There exist two versions of the program MINUIT, which differ in the number of parameters which they can handle, as shown below, and in the memory space required.

<table>
<thead>
<tr>
<th>Short version MINUIT</th>
<th>Long version MINUITL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum total number of parameters</td>
<td>30</td>
</tr>
<tr>
<td>Maximum number of variable parameters</td>
<td>15</td>
</tr>
</tbody>
</table>
The maximum permissible memory space for a job, and the loading space required for the program, limit the number of elements which AGS can handle to the values below:

<table>
<thead>
<tr>
<th>Maximum number of elements allowed in AGS</th>
<th>without segmentation</th>
<th>with segmentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MINUITS</td>
<td>1590</td>
<td>2115</td>
</tr>
<tr>
<td>MINUITL</td>
<td>675</td>
<td>1200</td>
</tr>
</tbody>
</table>

This version of AGS has the number 75.02.
<table>
<thead>
<tr>
<th>Error message</th>
<th>Routine</th>
<th>Explained on page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABS(XFACT * XXFACT).GT.1.0</td>
<td>NEWAGS</td>
<td>15</td>
</tr>
<tr>
<td>CONFUSING INPUT - CHECK PARAMETER INDEPENDENCE</td>
<td>FCNEX</td>
<td>32</td>
</tr>
<tr>
<td>ERROR IN DATA</td>
<td>MATCHB, MATCH</td>
<td>20</td>
</tr>
<tr>
<td>ERROR IN ITR(...)</td>
<td>ATRACK, TRACK</td>
<td>27, 35</td>
</tr>
<tr>
<td>IBU(6) IS TOO SMALL</td>
<td>BUMP</td>
<td>28</td>
</tr>
<tr>
<td>ILLEGAL CONTROL CARD</td>
<td>NEWAGS</td>
<td>3</td>
</tr>
<tr>
<td>ILLEGAL ELEMENT NO, .. DATA CARD IGNORED</td>
<td>MATCHB</td>
<td>32</td>
</tr>
<tr>
<td>ILLEGAL EXIT ELEMENT</td>
<td>BETATR</td>
<td>29</td>
</tr>
<tr>
<td>ILLEGAL MODE ..., L MODE ASSUMED</td>
<td>MATCHB</td>
<td>32</td>
</tr>
<tr>
<td>ILLEGAL OUTPUT UNIT</td>
<td>BETATR</td>
<td>29</td>
</tr>
<tr>
<td>ILLEGAL STARTING ELEMENT</td>
<td>BETATR</td>
<td>29</td>
</tr>
<tr>
<td>INDEX OUT OF RANGE</td>
<td>RING</td>
<td>6</td>
</tr>
<tr>
<td>IRUN ... IS TOO HIGH</td>
<td>MISAL</td>
<td>23</td>
</tr>
<tr>
<td>LENGTH ... IS ZERO OR NEGATIVE</td>
<td>MATRIX, NEWAGS</td>
<td>13</td>
</tr>
<tr>
<td>MISSING ROUTINE</td>
<td>ADJUST</td>
<td>17</td>
</tr>
<tr>
<td>MOMENTUM ERROR, MAGNITUDE .. TOO LARGE</td>
<td>CLOSE</td>
<td>35</td>
</tr>
<tr>
<td>NO CONVERGENCE</td>
<td>NEWAGS</td>
<td>15</td>
</tr>
<tr>
<td>NO MATCH</td>
<td>MATCHB, MATCHB, MATCH</td>
<td>18</td>
</tr>
<tr>
<td>NO MINIMIZATION ATTEMPTED WITH ONLY ONE VARIABLE LENGTH</td>
<td>MATCHB</td>
<td>32</td>
</tr>
<tr>
<td>NO MOMENTUM ERRORS</td>
<td>CLOSE</td>
<td>35</td>
</tr>
<tr>
<td>NON-MATCHING BRACKETS</td>
<td>01RDER, 02RDER</td>
<td>6</td>
</tr>
<tr>
<td>ONE BETA IS ZERO</td>
<td>MATCHB</td>
<td>20</td>
</tr>
<tr>
<td>SPURIOUS CHARACTER</td>
<td>01RDER, 02RDER</td>
<td>5</td>
</tr>
<tr>
<td>SUM OF ANGLES</td>
<td>PARAM</td>
<td>8</td>
</tr>
<tr>
<td>TOO MANY ELEMENTS</td>
<td>RING</td>
<td>6</td>
</tr>
<tr>
<td>TOO MANY ELEMENTS IN SEQUENCE</td>
<td>RING, 01RDER, 02RDER</td>
<td>6</td>
</tr>
<tr>
<td>TOO MANY INTERMEDIATE POINTS MATCHING</td>
<td>MATCHB</td>
<td>32</td>
</tr>
<tr>
<td>TOO MANY MOMENTUM VALUES</td>
<td>NEWAGS</td>
<td>6</td>
</tr>
<tr>
<td>WRONG IBU</td>
<td>BUMP</td>
<td>28</td>
</tr>
<tr>
<td>WRONG PARAMETER I</td>
<td>PARAM</td>
<td>8</td>
</tr>
<tr>
<td>ZERO PAR</td>
<td>MATCHB, MATCH</td>
<td>18</td>
</tr>
</tbody>
</table>
Table X
Control cards to run the AGS program

NAME.
ACCOUNT, name, division, account no.
ATTACH, LIB, 7600LIBRARY, ID = PR0GLIB.
LIBRARY, LIB.
FIND, A, AGSMARTI nnn, ID = ISRMARTI, CY = 1.*
FIND, B, AGSMARTIBIN, ID = ISRMARTI, CY = 1.
FIND, C, AGSSEGDIR, ID = ISRMARTI, CY = 1.†
FIND, D, MINCHM, ID = ISRMARTI, CY = 1.†
FIND, MINUITL, 7600MINUITL, ID = PR0GLIB. †
SEGLOAD, I = C.†
LOAD, D.†
LOAD, MINUITL.†
LOAD, A.
B.
e.o.r.
data
e.o.f.

*) nnn can be 100, 400, 1200, 1500 and corresponds to the maximum number of elements IDIM which can be handled by the program. This permanent file contains only the main program (in binary).

The permanent file AGSMARTIBIN contains the binary version of all the AGS routines.

The permanent file AGSSEGDIR contains the directives for the segmentation. The permanent file MINCHM contains a binary version of the dummy MINUIT subroutines MINOS, CONTOUR, HESSE to save space when using MINUIT. The short version of MINUIT is in the 7600 library and the long version is on the permanent file 7500MINUITL.

†) Control cards used only if the segmentation is necessary.

+) Control cards needed for the long version of MINUIT.
REFERENCES

7) G. Gendreau, A. Le Duff, G. Neyret; Spring Study on Accelerator Theory, CERN, April 1972, AMC-1, p. 35.
Fig. 1 General flow diagram of the ACS program
Fig. 2 Sign convention for $\frac{dL}{dx}$
Wedge magnet

Straight magnet

Fig. 3 Measurement of effective length
### Fig. 5 Simple Fuden Structure

**Table: Calculated Values**

<table>
<thead>
<tr>
<th>NIL</th>
<th>LENGTH DL/AX</th>
<th>D2L/DX2</th>
<th>ANGLE</th>
<th>K(V)</th>
<th>DK(V)/DX</th>
<th>DPX/DX2</th>
<th>X</th>
<th>XX</th>
<th>HETAX</th>
<th>BETA</th>
<th>ALPHA</th>
<th>ALPHA</th>
<th>MUX/2PI</th>
<th>MUZ/2PI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5000</td>
<td>-0.003</td>
<td>-0.000</td>
<td>31.0000</td>
<td>-0.000000</td>
<td>-0.000000</td>
<td>-0.000000</td>
<td>1.63495</td>
<td>.24657</td>
<td>13.01</td>
<td>13.75</td>
<td>-2.164</td>
<td>2.183</td>
<td>0.02988</td>
</tr>
<tr>
<td>2</td>
<td>1.5000</td>
<td>-0.000</td>
<td>-0.000</td>
<td>31.0000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>1.95322</td>
<td>0.00000</td>
<td>19.71</td>
<td>8.69</td>
<td>-0.616</td>
<td>-0.017</td>
<td>0.03561</td>
</tr>
<tr>
<td>3</td>
<td>2.5000</td>
<td>-0.000</td>
<td>-0.000</td>
<td>31.0000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>1.87562</td>
<td>0.00000</td>
<td>19.71</td>
<td>8.69</td>
<td>-0.358</td>
<td>-0.087</td>
<td>0.05789</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>-0.000</td>
<td>-0.000</td>
<td>31.0000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>1.63495</td>
<td>0.24657</td>
<td>13.01</td>
<td>13.75</td>
<td>-2.164</td>
<td>2.183</td>
<td>0.02988</td>
</tr>
<tr>
<td>5</td>
<td>1.5000</td>
<td>-0.000</td>
<td>-0.000</td>
<td>31.0000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>1.33494</td>
<td>0.00000</td>
<td>8.59</td>
<td>19.90</td>
<td>-0.084</td>
<td>-0.038</td>
<td>0.12511</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>-0.000</td>
<td>-0.000</td>
<td>32.0000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>1.33494</td>
<td>0.00000</td>
<td>8.59</td>
<td>19.90</td>
<td>-0.084</td>
<td>-0.038</td>
<td>0.12511</td>
</tr>
</tbody>
</table>

**Calculation Details**

- **NP**: \( \cos(NH) \)
- **QC**: \( \sin(QH) \)
- **HETAX**: \( \alpha \)
- **BETA**: \( \beta \)
- **ALPHA**: \( \alpha \)
- **MUX/2PI**: \( \mu_{x}/2\pi \)
- **MUZ/2PI**: \( \mu_{z}/2\pi \)

**Additional Values**

- \( 0.6366 \)
- \( 0.0034 \)
- \( 8.0747 \)
- \( 48.8555 \)
- \( 10.3336 \)
- \( 0.1595 \)
- \( -0.0414 \)
- \( 19.0981 \)
- \( 1.33495 \)
- \( 7.9401 \)

**Time:** 411 Seconds

---

*Fig. 5 Printout of results from data of Fig. 4*
Fig. 6 Types of element misalignment
Fig. 7 Calculation of machine centre
AGS
ILLEG, LINOUL, MATRIX, MPED, NEWAGS
O1RDER, O2RDER, OUTPUT, PARAM, PLOT,
PRINT, RECOV, RESULT, RING, SAVE,
TRACK, TRKMAT.

ADJUST
M1ATCH
M2ATCH
M3ATCH
M4ATCH
M5ATCH
M6ATCH
M7ATCH

MISAL
PREPU
TLOT

BETATR
CLOSE
CONSTR
FCN
MATCHB
MINUITS

Fig. 9 Segmentation structure of AGS program
Fig. 10 Example of matching - data deck
**Matching Betatron Tracking**

**Example Data for Initial Simulation**

**Match Distance (m)**: 2.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

**Match Position (mm)**: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

**Match Gradient (rad)**: -0.0000 -0.0000 -0.0000 -0.0000 -0.0000 -0.0000 -0.0000 -0.0000 -0.0000 -0.0000

**Match Alpha (rad)**: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

**Match Alpha' (rad)**: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

**Initial Element Parameters**

<table>
<thead>
<tr>
<th>ELT. NO.</th>
<th>LENGTH (m)</th>
<th>POSIT. (mm)</th>
<th>GRADIENT (rad)</th>
<th>ALPHA (rad)</th>
<th>ALPHA' (rad)</th>
<th>MIN/2P</th>
<th>MAX/2P</th>
<th>ALPHA0</th>
<th>ALPHA0'</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 S1</td>
<td>2.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2 S2</td>
<td>3.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3 S3</td>
<td>4.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>4 S4</td>
<td>5.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>5 S5</td>
<td>6.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>6 S6</td>
<td>7.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>7 S7</td>
<td>8.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>8 S8</td>
<td>9.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>9 S9</td>
<td>10.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Time** = 4.444 Seconds

**Betatron Matching Results**

**Time** = 4.444 Seconds

**Betatron Function Tracking**

**Match Position**

<table>
<thead>
<tr>
<th>NO ELEMENT</th>
<th>DISTANCE (m)</th>
<th>GRADIENT (rad)</th>
<th>ALPHAV (rad)</th>
<th>ALPHAT (rad)</th>
<th>MIN/2P</th>
<th>MAX/2P</th>
<th>ALPHAT (rad)</th>
<th>ALPHAV (rad)</th>
<th>ALPHAV (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>1 S1</td>
<td>2.0000</td>
<td>-0.0260</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0260</td>
<td>-0.0260</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2 S2</td>
<td>3.0000</td>
<td>-0.0360</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0360</td>
<td>-0.0360</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3 S3</td>
<td>4.0000</td>
<td>-0.0460</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0460</td>
<td>-0.0460</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>4 S4</td>
<td>5.0000</td>
<td>-0.0560</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0560</td>
<td>-0.0560</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>5 S5</td>
<td>6.0000</td>
<td>-0.0660</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0660</td>
<td>-0.0660</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>6 S6</td>
<td>7.0000</td>
<td>-0.0760</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0760</td>
<td>-0.0760</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>7 S7</td>
<td>8.0000</td>
<td>-0.0860</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0860</td>
<td>-0.0860</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>8 S8</td>
<td>9.0000</td>
<td>-0.0960</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0960</td>
<td>-0.0960</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>9 S9</td>
<td>10.0000</td>
<td>-0.1060</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.1060</td>
<td>-0.1060</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>10 S10</td>
<td>11.0000</td>
<td>-0.1160</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.1160</td>
<td>-0.1160</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Time** = 0.413 Seconds

Fig. 11: Printout of results from data of Fig. 10
To take into account the wishes of the main users, to save paper and queue time at printing, a second version (75.03) of the AGS program has been created. This version is on cycle 2 of the permanent file AGSMARTIBIN, ID = ISRMARTI, in binary form.

The main differences with respect to the version 75.02 are described below.

1. **S1, S2, S3 command**

   If columns 3, 4, 5 of the S1, S2, S3 card are zero or blank, a flag WHOLE is set true, otherwise it is set false; if WHOLE is true, the program assumes that the structure represents a super-period of a complete machine (as in version 75.02).

   If WHOLE is false, the structure just represents an arbitrary piece of beam channel; if a "2" is punched in column 5, the list of the elements is printed; if a "1" is punched in column 5, printing is suppressed.

2. **Control of output in structure calculations (PR)**

   This version contains a command PR controlling the output of the results of the structure calculations.

   The PR command is necessary when entering the structure calculations; it follows the last parameter of the structure, unless the NO command is present.

   The PR command has the following format: A2, I3, I5 to read the mnemonic code PR and the two variables IPRINT and N.

   IPRINT selects the mode of printing:

   a) If the calculations are done only for the central orbit ($\Delta p/p = 0$), when
      IPRINT = 0, only a summary line is printed;
      IPRINT = 1, the desired elements as well as the summary line are printed.

   b) If the calculations are done for different orbits ($\Delta p/p = 0$ and $\Delta p/p \neq 0$),
      IPRINT selects the mode of printing according to the following table.

     | IPRINT | $\Delta p/p = 0$  | $\Delta p/p \neq 0$ |
     |--------|-------------------|---------------------|
     | blank or 0 | summary line      | summary line        |
     | 1      | desired elements  | summary line        |
     | 2      | desired elements  | desired elements    |
     | 3      | summary line      | desired elements    |

   $N$ = number of intervals to be printed ($N < 50$). If $N = 0$ or blank (and IPRINT > 0) all elements are printed.
This card is followed by N cards with format IX, I4, IX, I4, giving N1 and N2, where
N1 is the first element of the interval;
N2 is the last element of the interval.
If N2 = 0 or blank, only the element N1 is printed.
To print the last element, one can punch L in column 1 instead of specifying N1.
To print the elements from N1 to the last one, L can be punched in column 6 and
N2 left blank or zero.

Example 1

We have a structure of 98 elements. The first element, elements 20 to 30, 90 to 98
are printed for $\Delta p/p = 0, \pm 0.01$.

```
col.    1      5      10
        T E x a m p l e  f o r  P R  c o m m a n d
        S 3
        ; structure
        A E
        2
        0.01
        -0.01
        ; n parameters
        ;
        PR 2   3
        B 1
        20   30
        90 L
```

Example 2

The structure is not entered before the tracking command if the NO command is used
after the last parameter. After the data for the TV command the command GO is read and
finally the PR command to select the printing option.

Same structure as in Example 1, but only the first and the last element are printed
for the central orbit as well as the summary line for the other orbits.
Example 3

The control command MA initiates matching operations and then automatically enters the structure computations. Therefore the GO command is not necessary, but a PR command must follow the last card of MA data.

Same structure, only the summary line is printed after matching for any orbit.

Error message

If the number of parameters is wrong, or if the PR card is missing, the program writes one of the following messages:

    ILLEGAL DATA IN FIELD

or

    YOU HAVE MORE PARAMETER CARDS THAN THE NUMBER YOU SPECIFIED

and stops.
Input

The control card MB is followed by

a) one card containing:
VARYL, MODE, RESTOR, MODIFY, KMIN, KMAX, NIP, GØLL, NOFIN in format (3X2A1, 3X2A1, 3I5, F10.2, IS) where the first eight variables are the same as in version 75.02 and NOFIN = number of the exit element for \( \Delta u \) (default value of NOFIN = KMAX).

b) A sequence of cards in format (IS, 4X2A1, 3F10.5) containing a description of the elements to be adjusted (as in version 75.02).

c) End-point matching cards; six cards describing \( \beta_y, \alpha_y, \beta_h, \alpha_h, \alpha_p, \alpha'_p \) in format (3F10.5);
1st field: value of function at entrance of element KMIN
2nd field: desired value of function at exit of element KMAX
3rd field: matching weight,
and two cards describing \( \frac{\Delta u_y}{2\pi} \) and \( \frac{\Delta u_h}{2\pi} \) in format (I10, 2F10.5);
1st field: number of the entrance element for \( \Delta u \): INELy
2nd field: desired value of \( \frac{\Delta u}{2\pi} \)h
3rd field: matching weight
default value of INELV = KMIN
default value of INELH = INELV

\[
\left( \frac{\Delta u}{2\pi} \right)_{\text{calc}} = \frac{\mu_{\text{NOFIN}} - \mu_{\text{INEL}}}{2\pi}
\]

d) Six cards for the limits (as in version 75.02).

e) NIP sets of eight cards describing \( \beta_y, \alpha_y, \beta_h, \alpha_h, \alpha_p, \alpha'_p \), \( \frac{\Delta u_y}{2\pi}, \frac{\Delta u_h}{2\pi} \) in format (I10, 2F10.5) for \( \beta_y \), \( \frac{\Delta u_y}{2\pi}, \frac{\Delta u_h}{2\pi} \) in format (10X, 2F10.5) for the others.
1st field = ordinal number of the intermediate point (on \( \beta_y \) card) = NOL
2nd field = ordinal number of entrance element for \( \frac{\Delta u}{2\pi} = \text{INELy} \)
3rd field = matching weight

\[
\frac{\Delta u}{2\pi} = \frac{\mu_{\text{NOL}} - \mu_{\text{INEL}}}{2\pi}
\]

f) MINUIT commands as in version 75.02.

Output

In this version the printing of data is suppressed. After the matching, angles are printed in addition to the lengths and the positions of the elements; the values of the functions are printed at the exit of the elements.
3. **Structure calculations output**

In the structure calculations the functions are printed at the exit of elements, the order of the columns is modified and more decimals are printed (to standardize AGS results). When \( \Delta p/p = 0 \), the following quantities are printed: e1. no, name, l, distance, angle, \( K, \beta_v, \beta_h, \alpha_v, \alpha_h, \frac{\mu_v}{2\pi}, \frac{\mu_h}{2\pi}, \alpha_p, \alpha'_p \).

When \( \Delta p/p \neq 0 \): e1.no, name, \( \frac{dl}{dx}, \frac{d^2l}{dx^2}, \beta_v, \beta_h, \alpha_v, \alpha_h, \frac{\mu_v}{2\pi}, \frac{\mu_h}{2\pi}, \frac{x}{\Delta p/p}, \frac{x'}{\Delta p/p} \), where

\( x = \) position of the particle on the closed orbit;
\( x' = \) its derivative with respect to \( s \).

**Binary output**

If the tape flag is active, the values are written onto tape 3 in binary form at exit of the elements in the following order:

\[ \text{IEL, e1.no, name, l,} \frac{dl}{dx}, \frac{d^2l}{dx^2}, \alpha_v, \alpha_h, \beta_v, \beta_h, \mu_v, \mu_h, A, A', \text{ where} \]

\[ A = \alpha_p \quad \text{ and } \quad A' = \alpha'_p \text{ if } \Delta p/p = 0 \]

\[ A = x/(\Delta p/p) \text{ and } A' = x'/(\Delta p/p) \text{ if } \Delta p/p \neq 0. \]

4. **Geometry calculations (GE)**

In this version it is possible to control the output from geometry calculations.

The GE control card is followed by

1) one card with format IS reading N. N allows the choice of printing.

\[ N = 0 \text{ or blank, only the last element is printed} \]

\[ N < -1 \quad \text{all elements are printed} \]

\[ N > 0 \quad N = \text{number of intervals to be printed} \]

2) if \( N > 0 \), N cards with format 1XI4, 1XI4 reading N1 and N2. N1 and N2 have the same definitions as for the PR command.

**Binary output**

The complete table is written onto a binary file if the tape flag is set.

5. **Matching of the characteristic functions (MB)**

In this version the parameters of a beam transport channel can be adjusted by the program to obtain specified values of the characteristic functions \( \beta_v, \alpha_v, \beta_h, \alpha_h, \alpha_p, \alpha'_p, \frac{\Delta \mu_v}{2\pi}, \frac{\Delta \mu_h}{2\pi} \) at the end of the channel, given the values at the beginning. \( \Delta \mu \) means the difference of phase between the entrance of a specified element and the exit of another specified element.

Values of these functions at intermediate points (maximum 10) may also be imposed.