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HADRON PHYSICS AT ISR ENERGIES

M. Jacob

Lectures given in the
Academic Training Programme of CERN 1974

G E N E V A

1974

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ABSTRACT

These four lectures cover some facets of hadron physics being studied at present at the ISR. Elastic scattering, total cross-section, diffraction excitation and large transverse momentum phenomena are discussed in detail. This work represents a status report on these rapidly developing fields. These notes are written more as a complement to two of the author's previous reviews, his NAL Conference Rapporteur's talk in 1972 and Ebeltoft lecture notes in 1973, than as a self-contained extensive paper on very high energy hadron physics. Emphasis is put on the largely complementary aspect of research at the ISR and at NAL (SPS).

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1. INTRODUCTION

This series of lectures comes three years after research actually started at the CERN Intersecting Storage Rings. As is now well-known, this unique instrument allows experimentation in a huge energy range, where only scanty pieces of information could hitherto be collected from cosmic-ray research. Physicists have been served by a magnificent instrument, faithful to its promises and continuously improving beyond them. As an unforeseen reward, many new phenomena have been discovered and already partly studied. Our views and understanding of hadronic interactions are much enriched and to a large extent different from what they were when the machine started.

Coasting beams at the ISR can span a momentum range extending from 11 to 31 GeV/c. With practically head-on collisions, the corresponding centre-of-mass energy may, therefore, vary from 22 to 62 GeV. Translated into terms of standard machine energy, with a stationary proton target, this corresponds to an energy range extending from 250 to 2000 GeV. There is a sizeable overlap with the NAL (SPS) energy range. One can also go much further!

The machine luminosity is such that 10^5 /sec interactions occur in each intersection region, while the two coasting beams intercept each other. This may be regarded as a small rate, as compared to that available on a standard type of accelerator. Nevertheless, it should be kept in mind that secondary beams, actually used in experimentation, are often down by as much as a factor of 10^6 in intensity from what is available in the primary proton beam. One rather loses on the variety of collisions which can be studied than on useful intensity. This is a serious drawback. Going all the way to 2000 GeV, even if it is with proton on proton only, is however already extremely interesting! The present luminosity is powerful enough for the detailed study of most hadronic effects. It becomes a serious problem when one looks at rare but interesting phenomena, such as those associated with the observation of a secondary particle with very large transverse momentum. It also makes the search for *a priori* rare but interesting events, such as lepton pair formation, more difficult. Extension of the sensitive solid angle and technical ingenuity should partly overcome the corresponding problems. At the same time, the luminosity achieved requires a beam momentum spread which has its obvious drawbacks. In any case, the list of extremely interesting effects for the study of which the machine is highly suitable is already long.

As a matter of fact it is impossible to go through all of these effects in any comprehensive way during this series of four lectures. There are already too many things to describe and discuss. At the same time, however, there is no point in duplicating here what I already said one year ago at the Ebeltoft School. The corresponding notes, with their reported results and tentative conclusions¹⁾, have so far stood the test of time. Even though one may expect their various sections soon to become but over emphasized and poorly balanced introductions to topical research at the ISR, there is little I could change at present²⁾. I have thus considered these lectures to be a complement to those which I gave a year ago, rather than a new and updated presentation of the same topics. Furthermore, I would like to take this as an excuse to cover here actually only a rather small domain of hadron physics as presently studied at the ISR. In particular, I shall not go back to the question of scaling and to that of correlations among secondaries, which correspond, respectively, to Sections 3 and 5 of Ref. 1. Extensive checks of the scaling property of

inclusive distributions correspond to an extremely important set of results at the ISR. This is a very striking behaviour, postulated on very general theoretical grounds, which could be tested on an extensive energy range and thus gain full credibility. There is little which I could add qualitatively to what is already discussed in Ref. 1 (Section 3) and in earlier reviews³⁾. The extensive paper by G. Giacomelli et al. has been widely quoted. The study of correlations among secondary particles has so far mostly dealt with correlations among charged particles at wide angles and evidence for striking short-range correlations, rapidity-wise, is a very important discovery to be put to the credit of research at the ISR. The corresponding energies were indeed needed in order to separate clearly such short-range clustering effects (as they are most conveniently described) from correlations involving the reaction as a whole and globally referred to as long-range effects. In this case also, I shall not go back to the discussion given in Ref. 1 (Section 5), even though the subject could easily justify a longer discussion and in particular that of correlations for fixed multiplicities³⁻⁵⁾. One has probably to wait for correlations involving a heavy particle before our present understanding of clustering effects can significantly improve from what it is now. The overwhelming majority of the charged particles seen at wide angle are pions. A weak point at the ISR, obviously not met at NAL (SPS), is that the detection system cannot include the reaction vertex. Hence spurious effects, and in particular δ -rays, complicate correlation studies. Indeed, work at NAL on correlations has also been extremely useful, even though the over-all energy does not go as high.

I shall come back though to the question of the rising total cross-section. This was already discussed at some length in Ref. 1. Nevertheless, the very recent results from NAL on the behaviour of the slope parameter b and of the ratio of the real to the imaginary parts of the forward elastic amplitude ρ make it worthwhile stressing once more the connections between σ_{tot} , b and ρ . Furthermore, the impact parameter analysis of the corresponding effects was absent from Ref. 1, while it should be part of a general discussion of hadron physics at the ISR⁶⁾. This new review thus covers again the most global aspect of hadron collisions at very high energies⁷⁾, namely the behaviour of σ_{tot} . This corresponds to Section 2 of these notes. After this very general feature, I have selected the two specialized topics which correspond to the presently most fashionable selection of processes in the study of hadron collisions at the ISR, and at NAL, namely single-diffractive excitation and large transverse momentum secondaries. They are the objects of Section 3 and Section 4 of these notes, respectively.

Diffractive excitation has been recognized long ago as a specific mechanism for particle production. This is, in particular, the case for single-diffractive excitation, whereby one incident particle is quasi-elastically scattered, while the other one is excited into a hadronic state which eventually "decays" with pion production. Evidence was long limited to the excitation of a few prominent low-mass resonances. At the same time, the key properties *a priori* associated with a diffractive process could only be verified on a relatively small energy range (by present standards). It, however, became clear that single diffractive excitation had actually a major role in low-multiplicity reactions, the prominent low-mass resonances contributing a fraction only of the corresponding cross-section^{4, 8)}. What was demonstrated by results at the ISR (mainly those of the CERN-Holland-Lancaster-Manchester Collaboration) was that the mass spectrum of the diffractively-excited hadronic states extends up to masses of the order of at least 10 GeV⁹⁾. Furthermore, the over-all

single-diffractive cross-section, as determined from results from the ISR and from NAL, is close in magnitude to the elastic cross-section and shows a very similar energy behaviour at fixed excitation mass. The study of the hadronic state, into which one of the protons is thus excited through such a collision, is very interesting. How one gradually departs from what is most simply associated with resonance formation and decay (with an isotropic decay distribution in the rest frame of the excited hadron), as one considers higher and higher excitation masses, is as yet very poorly known. The trigger is simple and consists of one proton with almost the same energy (ratio-wise) as that of the incident particle¹⁰⁾. At present, it rightfully attracts very great interest. This is discussed in Section 3. In Section 4, we turn to large transverse momentum phenomena.

It is well known that the many secondary particles which are produced in proton collisions at ISR energies are mostly pions with transverse momenta of relatively low values: $\langle p_T \rangle \approx 0.35$ GeV/c. Their p_T distribution (at wide angle say) is to a good approximation exponential, at least from $p_T = 0.1$ to 1 GeV/c. Extrapolating such a behaviour to a value of p_T of 4 GeV/c, too high kinematically for PS energy but easily accessible at ISR energies, one should casually expect only a very negligible yield¹¹⁾. It was again a very important discovery at the ISR to find out that it is far from being the case! The large p_T rates are way above what the naïve extrapolation of the exponential behaviour, which is so well satisfied up to $p_T = 1$ GeV/c, would lead one to assume. These results were first reported at the Batavia Conference. They were obtained by the Saclay-Strasbourg Collaboration, the CERN-Columbia-Rockefeller Collaboration and the British-Scandinavian Collaboration, working independently¹²⁾. Even though the observed yields are much larger than what a simple extrapolation would lead one to expect, the corresponding cross-sections remain essentially small. As one gets beyond $p_T = 1$ GeV/c and meets an obvious departure from the initial exponential fall-off, the inclusive yield is already about 10^3 times smaller than its value at $p_T = 0.35$ GeV/c and continuously keeps falling with increasing p_T , if of course less dramatically so. However, to the extent that peculiar features in the p_T distribution of secondaries, thus observed in a direction not affected by the Lorentz contraction, reflect most generally hadron structure at very small distances, they are of an *a priori* enormous interest. On the one hand, the dominant pion yield, exponentially limited to low values of p_T , globally corresponds to a coherent production involving the overlapping protons as a whole. On the other hand, the large p_T component, with a significantly different dependence on p_T , is most generally associated with production from parts of the protons acting to a large extent incoherently of one another. Furthermore, there is *a priori* an interesting parallel between hadron production at large transverse momentum and deep inelastic electron scattering¹³⁾. They could be two facets of a practically point-like structure within the proton.

As a matter of fact, it was expected that such a connection could exist even before the ISR started¹⁴⁾. However, if a relatively safe prediction could be made on the grounds of electromagnetic production alone, which should certainly become competitive with an exponentially damped hadronic contribution as one reaches $p_T = 5$ GeV/c, associating the corresponding behaviour with a hadronic effect, and expecting it to show off already clearly at $p_T = 2$ GeV/c, was certainly of a more speculative nature¹⁵⁾. The observation of the effect, at the level of a strong interaction is thus a very important discovery. Furthermore, the connection between deep inelastic electron scattering and production of secondaries

with large transverse momentum became much clearer with the development of the parton model^{15,16}). Indeed and as discussed in Section 4, a parton analysis of large p_T phenomena does not run into any contradiction with the data, while it offers a simple interpretation of many of the observed effects.

The study of reactions involving large p_T secondaries has been pursued as efficiently as possible, since the corresponding effects were first reported one and a half years ago. Many results have been obtained, at the ISR and at NAL, and this justifies coming back at some length to the discussion of Ref. 1, even though it already covers the key features now associated with large p_T phenomena. Besides the marked structure in the p_T dependence itself, which was mentioned already, they include a sizeable positive over negative excess, an important heavy-particle component and particularly large associated multiplicities. All this clearly contrasts in a qualitative way the reactions obtained with a large p_T trigger ($p_T \geq 2$ GeV/c say) with most of the observed events, where the large majority (85%) of the particles produced at wide angles are pions, with practically as many π^- as π^+ by the time one reaches the ISR energy range. A typical trigger for such reactions has so far been the observation of a high-energy γ -ray (π^0) in a lead-glass detector positioned at 90° in the centre-of-mass system. This is the only one which has actually been used so far in association with another detector (counters, spark chambers, streamer chambers, etc.). More versatility in triggering would be of a great interest. Section 4 is merely a status report on the study of such fascinating phenomena.

2. THE TOTAL CROSS-SECTION AND THE SHAPE OF THE PROTON

2.1 The rise of σ_{tot}

Present results on the proton-proton total cross-section are displayed in Fig. 1. The expert may notice a few changes, of the order of 1%, from previously reported values¹⁷). In any case, the by now well-known rise of σ_{tot} , which corresponds to a 10% effect across the ISR energy range, stands out as a prominent fact.

The ISR results reported in Fig. 1a have been obtained by the CERN-Rome and the Pisa-Stony Brook Collaborations. The former experiment reaches $d\sigma/dt$ at $t = 0$, through an extrapolation of the differential cross-section and gets the total cross-section using the optical theorem¹⁸). The total cross-section eventually comes out from the observed counting rate as inversely proportional to the square root of the separately-measured machine luminosity L . The latter experiment registers practically anything which can occur during a collision and, through such a global counting, reaches the total cross-section directly¹⁹). It comes out from the observed counting rate as inversely proportional to the machine luminosity. It should, furthermore, be pointed out that for the lowest two energy points, 22 and 30 GeV, the angular distribution can be followed at low enough values of $|t|$ so as to exhibit the Coulomb peak. The cross-section can then be normalized to Coulomb scattering. This confirms results obtained using an independent luminosity measurement (Fig. 1).

Runs with unequal beam momenta have given points at intermediate energies from those singled out in Fig. 1 (Pisa-Stony Brook). The important experimental asymmetry which this implies as compared to runs with equal beam momenta supports confidence in the calculation

Fig. 1a The situation as it was at the time of the Aix en Provence Conference

Fig. 1b The situation as it is now (figure added in proof)

Fig. 1c The behaviour of the K^+p and K^-p total cross-sections as determined up to 200 GeV (figure added in proof)

of corrections. The results indeed confirm the energy behaviour already well enough displayed in Fig. 1. The different dependences on the machine luminosity of the two approaches has led to a new experiment, at present under way¹⁷⁾, in which both groups are now running on the same intersection at the same time, when successful cross checks between the two sets of results have to make do with connections due to the collection of data at two distinct intersections and at different times. It should eventually reduce the error bars to a 2% level and do away with questions which one may still raise in connection with the measurement of L ²⁰⁾.

A rising cross-section came somewhat as a surprise. If a constant total cross-section was not at all required asymptotically, it nevertheless came out as the *a priori* most simple "possibility" in a variety of theoretical approaches. One should point out, of course, that the K^+p total cross-section was known to rise through the Serpukhov energy range. This came out after the K^-p total cross-section had been found to be "surprisingly" constant²¹⁾ and as a predicted effect from the Regge behaviour of the difference of the K^-p and K^+p total cross-sections. Furthermore the rise of the K^+p total cross-section could have been but a transition behaviour towards a not too distant constant asymptotic limit common to both the K^+p and K^-p channels. Up to 70 GeV, the proton-proton cross-section decreases extremely slowly and results from the first NAL bubble chamber experiments could not contradict a generally accepted approach towards an asymptotic constant value (Fig. 1). The ISR results

can be said to have made us face all of a sudden what is but an "elusive asymptotia". Not only does the total cross-section rise, and this over a tremendous energy range, when simple asymptotic consideration would have preferred to see it constant, but it also rises in a way which differs from anything which one could at present associate with a simple asymptotic regime with a rising cross-section. The effect is far too large to be considered as a mere correction to a well understood leading term. It is far too small as compared to what would be allowed if unitarity only were to limit the inelastic cross-section within the finite range of strong interactions, an *a priori* simple and tempting concept at infinite energy.

2.2 The calculation of σ_{tot}

One usually defines an invariant elastic amplitude A_{ij} such that the differential cross-section reads:

$$\frac{d\sigma}{dt} = \frac{1}{64\pi p^2 s} \sum_{ij} |A_{ij}(s, t)|^2. \quad (2.1)$$

The indices $i(j)$ stand for the helicities in the initial (final) state. The sum implies an actual sum over final states but an average over initial spin states. The centre-of-mass momentum and the centre-of-mass energy squared are denoted by p and s , respectively; t is the momentum transfer squared. According to Eq. (2.1) the optical theorem reads,

$$\text{Im } A_{ii}(s, 0) = 2 \left((p_1 \cdot p_2)^2 - m_1^2 m_2^2 \right)^{\frac{1}{2}} \sigma_{\text{tot}}^i, \quad (2.2)$$

where p_i and m_i , respectively, stand for the momentum and the mass of either incident particles. The ISR energy range is such that the approximation $\sqrt{s} \gg m_i$ is certainly justified. We shall then write from now on

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \sum_{ij} |A_{ij}(s, t)|^2 \quad (2.3)$$

$$\text{Im } A_{ii}(s, 0) = s \sigma_{\text{tot}}^i \quad (2.4)$$

and we shall, whenever not required, suppress the helicity indices. The S matrix element between the initial and final states α and β is written as

$$S_{\alpha\beta} = \delta_{\alpha\beta} + i(2\pi)^4 \delta(P_\alpha - P_\beta) T_{\alpha\beta}, \quad (2.5)$$

while the optical theorem now reads:

$$2 \text{Im } T_{\alpha\alpha} = (2\pi)^4 \sum_n \delta(P_\alpha - P_n) |T_{\alpha n}|^2.$$

The over-all energy momentum vector is denoted by P , and the summation is extended to all available final states n . In terms of the invariant scattering amplitude A , defined by Eq. (2.1), this gives

$$\text{Im } A(s,0) = 2\omega_1\omega_2 (2\pi)^4 \sum_n \delta(P_i - P_n) |T_{in}|^2. \quad (2.6)$$

The energy of the incident particles has been denoted by ω_i . The connection between the total cross-section, an explicit sum over all accessible final states and the imaginary part of the forward elastic amplitude may be symbolically written as:

$$\sigma_{\text{tot}} \sim \sum_n \text{diagram with two circles and three lines} \sim \text{diagram with a wavy line} \quad (2.7)$$

This actually stands for Eqs. (2.6) and (2.2) which provide the needed kinematical terms²²).

A model calculation for σ may then be attempted using a model for the inelastic amplitudes. If one has a relatively simple expression for production amplitudes involving any specific multiplicity of secondaries (a simple multiperipheral amplitude, for instance), one can explicitly conduct the sum over all such final states, computing

$$\sum_n \text{diagram with two circles and three lines}$$

and thus reaching a model calculation of $\sum_n$ or σ_{tot} .

What makes such an *a priori* simple procedure difficult to carry through in principle (as well as in practice) is the uncontrollable interferences between the particular class of diagrams which is kept in a particular model calculation and those which are disregarded. To be more specific, the class of diagrams which is kept in a multiperipheral approximation and globally represented as (a) interferes with those usually not considered on the same footing and globally represented as (b). There could be important compensations between the contributions of two such classes of diagrams in Eq. (2.7).

Nevertheless, having reached a model calculation value for $\text{Im } A(s,0)$, represented as $\sum_n$, one may then attempt to calculate a more involved expression which would be more comprehensive in the included sub-amplitude. The graphical representation chosen for $\text{Im } A$ referring of course to a Regge-pole approximation, there are obvious terms to consider next. They correspond, in particular, to multiple-Reggeon exchange terms in the elastic-scattering amplitude or more specifically in its imaginary part. In so doing there are two particularly interesting ways in which more and more complicated terms can be included. The first one focuses on the s channel. One considers a series of terms

$$\text{diagram with wavy line} + \text{diagram with two wavy lines} + \text{diagram with three wavy lines} + \dots$$

each contributing to the full imaginary part of the elastic amplitude. This is generally associated with the eikonal approach, which provides an explicit expression for each term in the series of graphs thus written. As a particularly interesting feature, if the first term behaves as $s^{1+\alpha}$, with $\alpha < 1$ but positive (thus contributing a rising term to the total cross-section which would violate, if alone, the Froissart bound), the full series damps it out to an allowed $\log^2 s$ behaviour corresponding then to the Froissart limit²³⁾. This mechanism will be described in detail later. The tentative asymptotic expression which one can thus write for the total cross-section is:

$$\sigma_{tot} = \sigma_0 + \lambda \log^2 \left(\frac{s}{s_0} \right). \quad (2.8)$$

As already discussed in detail in Ref. 1, but also as evident from Fig. 1, such a simple expression may reproduce surprisingly well the behaviour of σ_{tot} and this over an energy range which spans the Serpukhov, NAL and ISR domains. There is, however, no stringent reason why an asymptotic expression such as (2.8) should work in such an *a priori* satisfactory way. One may choose s_0 such as to eliminate a generally present linear term. Furthermore, the value of λ which a fit to the data of Fig. 1 now imposes is of the order of 0.6, whereas an asymptotic regime, corresponding say to full absorption over the range of strong interactions, would suggest a value 10^2 times larger! The actual range which corresponds typically to the inverse pion mass would allow a much more important rise of σ_{tot} than is actually observed, if unitarity were the only limitation. We can therefore hardly find a satisfactory answer with the type of behaviour suggested by an eikonal calculation, despite the appealing simplicity of the fit. The relative transparency of the proton remains to be understood.

The second approach focuses on the t channel. One considers the one-Reggeon, two-Reggeon, ..., exchange terms in the calculation of the imaginary part of A . This is graphically represented by the series

Looking at it in the t channel, one sees first the one-Pomeron term, then the two-Pomeron term, ..., in the full $p\bar{p}$ reaction amplitude and calculate their respective contribution before crossing to the s channel. The key point is that what is involved is a sum of graphs in a perturbative field theoretical sense. It is not a straightforward unitarity equation (as seen in the s channel) where the second term would be looked at as a contribution to the right-hand side of Eq. (2.6) with a one-Pomeron exchange term for the production amplitude. Starting from a purely imaginary Reggeon exchange term (Pomeron exchange), one gets successive terms with alternating signs. In particular, the second one is to be subtracted from the first one, when it would add were we to look at it as a contribution to the unitarity relation²⁴⁾. If the first one behaves as s (Pomeron exchange), the second one is found to behave as $s(\log s)^{-1}$ ²⁵⁾. The tentative expression which one may thus write in a "weak-coupling" approximation for the total cross-section is now

$$\sigma_{tot} = \sigma_{\infty} \left(1 - \frac{\mu}{\log \left(\frac{s}{s_0} \right)} \right) \quad (2.9)$$

with μ positive.

This approach corresponds to the Reggeon calculus of Gribov²⁶⁾. It was indeed also used to predict a rising proton-proton total cross-section before the phenomenon was observed²⁷⁾. The cross-section now approaches a limiting value from below. The "correction" term decreasing only logarithmically, one should observe a slow and sustained rise. This, however, occurs over a very wide energy range only. This is the more so in the case that residual Regge terms are expected to provide positive contributions²⁸⁾. Even though they are assumed to decrease at least as fast as $s^{-1/2}$, they could for a while even over-compensate for the rise implied by Eq. (2.9). Similar to Eq. (2.8), Eq. (2.9) has some drawbacks when it comes to a fit to the rising cross-section¹⁾. As discussed in Ref. 1, it is again possible, and easily, to reproduce the data of Fig. 1. The value of σ_{∞} which this implies is however as high as 60 mb! We would be again very far from an asymptotic behaviour with a relatively small non-asymptotic correcting term only. A very large fraction of what is observed at ISR energies would still be foreign to what is eventually expected asymptotically.

Nevertheless, it should be put to the credit of both approaches that they were actually used^{23,27)} in advocating for rising cross-sections long before this was an observed effect and when it could be looked at as an unwelcome, and even unneeded complication. Cheng and Wu on the one hand, and Ter Martirosyan and his co-workers on the other were strong advocates of eventually rising cross-sections. Using their favoured expressions at meeting the observed rise we can easily reach satisfactory fits. Nevertheless, we are left with the troublesome conclusion that asymptotia, which plays a basic role in both, providing the asymptotic trend (2.8) or limit (2.9), is but far away still from the ISR energy range. This is an elusive asymptotia if any²⁹⁾!

Following either the eikonal procedure or the Reggeon calculus prescription, one still sums over a limited class of diagrams only. Important cancellations against hitherto neglected terms could occur. It is clear that obtaining σ_{tot} from a model for particle production is but a model-dependent approach. In any case expressions such as (2.8) and (2.9) can be considered as two "extreme", though very useful, behaviours which one can keep in mind while perusing data. Neither one can be disregarded at present as irrelevant or misleading. It seems that cross-section data on a variety of channels at NAL (SPS) energies will, from now on, be more useful at choosing or preferring one approach as against the other, than comparing the merits of fits to the results of Fig. 1 inspired by either of them.

The two approaches which have been thus opposed both correspond to very general features and they may easily cover the more explicit machinery attached to any specific mechanisms. It has been recognized³⁰⁾ that particular processes can contribute a sizeable rising term to the total cross-section over the ISR energy range. The most popular two are single-diffractive excitation and baryon anti-baryon pair formations. In both cases the relevant threshold effects are much belated and smoothed out³¹⁾. This is largely due to the leading-particle effect. It is, however, probably too early to associate the observed rise with any specific dynamical feature.

2.3 Rising cross-section in the eikonal approach (a parenthesis)

We may consider, for the sake of argument, a simple calculation in which we neglect spin effects⁶⁾. We introduce a scattering amplitude f , which is decomposed into partial waves as

$$f(s, \cos \theta) = \frac{1}{p} \sum_{\ell} (2\ell+1) a_{\ell}(s) P_{\ell}(\cos \theta). \quad (2.10)$$

As easily checked, it is related to the invariant scattering amplitude A , defined by Eq. (2.1), as

$$A(s, t) = 8\pi\sqrt{s} f(s, \cos \theta) \quad (2.11)$$

with

$$t = -2p^2(1 - \cos \theta) = -\vec{q}^2$$

$$\sigma_{el} = \int |f|^2 d\Omega = \int \frac{|A|^2}{16\pi s^2} dt.$$

We may next write an approximation for f which should be reliable in the small-angle (large ℓ) limit, relevant to very high energy elastic scattering (the differential cross-section falls exponentially with $|t|$). We also introduce an impact parameter r , writing $\ell = pr$ and replace the sum over ℓ in Eq. (2.10) by an integral over r . This altogether reads:

$$f(s, \cos \theta) \approx p \int a(r, s) J_0(r\sqrt{-t}) 2r dr. \quad (2.12)$$

We have approximated $P_{\ell}(\cos \theta)$ as $J_0(\ell\theta)$. We may next write the Bessel function using its integral form, getting the Fourier transform

$$f(s, \cos \theta) = \frac{1}{\pi} p \int a(r, s) e^{i\vec{q} \cdot \vec{r}} d^2\vec{r} \quad (2.13)$$

or

$$\frac{A(s, t)}{8\pi s} = \frac{1}{2\pi} \int a(r, s) e^{i\vec{q} \cdot \vec{r}} d^2\vec{r}, \quad (2.14)$$

which is inverted as

$$a(r, s) = \frac{1}{2\pi} \int \frac{A(s, t)}{8\pi s} e^{-i\vec{q} \cdot \vec{r}} d^2\vec{q}. \quad (2.15)$$

Unitarity is easily imposed on the partial wave amplitude appearing in Eq. (2.10), writing

$$a_{\ell}(s) = \frac{e^{2i\delta_{\ell}(s)} - 1}{2i} \quad |e^{2i\delta}| < 1. \quad (2.16)$$

It takes the usual form $e^{i\delta} \sin \delta$ in the elastic limit (δ real). For small phase shifts, $a(r, s)$, as defined by Eq. (2.15), is practically equal to the scattering phase δ . There is,

therefore, a specific procedure which would ensure that a model amplitude is unitary. One may start from a tentative expression $A_B(s,t)$, calculate $a_B(r,s)$, as given by Eq. (2.15) and recalculate $A(s,t)$ using formula (2.14), but replacing first a_B by $(e^{2ia_B} - 1)/2i$. The relevant series expansion corresponds to the terms considered in the eikonal series leading to Eq. (2.8). The procedure practically keeps the small partial waves as they are and damps the large ones down to a value compatible with unitarity as if one would have full absorption instead.

One then introduces a Regge approximation for the invariant amplitude, as it appears in Eq. (2.15), writing

$$\begin{aligned} \frac{A(s,t)}{8\pi s} &= -\beta e^{\frac{-i\pi\alpha(t)}{2}} e^{\frac{t}{2}t} s^{\alpha(t)-1} \\ &= -\beta e^{\frac{-i\pi\alpha(0)}{2}} e^{\frac{Bt}{2}} s^{\alpha(0)-1} \end{aligned} \quad (2.17)$$

with

$$\begin{aligned} B &= (b + 2\alpha' \log s - i\pi\alpha') \\ \alpha(t) &= \alpha(0) + \alpha' t \end{aligned} \quad (2.18)$$

or from Eq. (2.15)

$$a_B(r,s) = -\beta e^{\frac{-i\pi\alpha(0)}{2}} s^{\alpha(0)-1} \frac{1}{B} e^{-\frac{r^2}{2B}} \quad (2.19)$$

$a_B(r,s)$ has a positive imaginary part. If our input term has $\alpha(0) > 1$, the modulus of a_B will, furthermore, be large, as long as the last factor on the right-hand side of Eq. (2.18) is not damping it down, which it will do only at large enough values of r . The value of $a(r,s)$ will as a result of Eq. (2.16) practically reduce to the unitarity limit ($i/2$) and this up to a limiting radius R . The cross-section is then eventually proportional to R^2 . From Eq. (2.19) R^2 is proportional to B . From Eq. (2.18) it is in a first approximation proportional to $\log s$. However, if α' is also proportional to $\log s$, as it should be in the case of a Regge pole input, the resulting cross-section will asymptotically increase as $\log^2 s$. This corresponds to the Froissart limit thus obtained with an input amplitude which would violate it in the first place if it were not eventually corrected as to comply with unitarity.

If $\alpha(0) > 1$ the power behaviour over-compensates the $\log s$ term in B [denominator of Eq. (2.18)]. With a Pomeron input [$\alpha(0) = 1$] we get a very special case where all partial wave amplitudes asymptotically decrease as $(\log s)^{-1}$. The proton becomes more and more transparent as it increases in size. In the calculation of the cross-section the $(\log s)^{-2}$ term in a^2 compensates for the $(\log s)^2$ increase associated with the extending radius. The asymptotic cross-section is constant in the eikonal limit as for the Regge pole input. The first correction term, readily obtained from Eq. (2.16), is negative as compared to the leading one and decreases as $(\log s)^{-1}$. This agrees with Eq. (2.9), or more

naïvely speaking with what is expected from absorptive corrections to a leading Pomeron. All approaches thus agree on the energy behaviour of the "correction term". Its value, however, may differ much from one approach to the next.

The eikonal approach which we have followed gives a cross-section rising as $(\log s)^2$ as all partial waves eventually become saturated. The proton thus looks at a black disc surrounded by a grey edge [the Gaussian cut-off in Eq. (2.19)]. The inelasticity parameter $\eta = |e^{2i\delta}|$ in Eq. (2.16) goes to zero up to an ℓ value increasing as $\sqrt{s} \log s$. As a result, the elastic and the inelastic cross-sections become equal since their ratio is as well known given by $|1 - \eta|^2 / (1 - |\eta|^2)$. The elastic scattering cross-section should eventually become equal to half the total cross-section.

In the Reggeon calculus approach the elastic and total cross-sections are asymptotically equal to their bare Pomeron contribution. The elastic cross-section eventually decreases as $(\log s)^{-1}$, while the total cross-section is constant. Their ratio should therefore go to zero asymptotically.

This dramatic difference in asymptotic behaviour is important in view of the fact that the ratio $R = \sigma_{el} / \sigma_{tot}$ seems at present to approach a limiting value of 0.18 when considered on an energy range extending from PS to ISR energies³²⁾. As discussed later, this could be most easily associated with a limiting impact parameter picture [limiting overlap function]³³⁾. This however turns out not to be compatible with the observed rise of σ_{tot} and only confirms that asymptotia is but an elusive entity. The fact remains that better measurements of the diffractive peak at ISR energies would be most welcome. The reasons will become clearer later but we already see that they could disclose a departure from this apparently fixed value, upward (up to 0.5) if σ_{tot} rises indefinitely or downward (down to 0) if it eventually saturates.

2.4 The slope of the diffractive peak

The rise of the total cross-section should have correlated effects on the behaviour of elastic scattering to the extent that at such high energies it is mainly a shadow effect. It turns out that the differential cross-section has a relatively simple shape. At low values of $|t|$ it can be represented as the sum of two exponentials with a "kink" around $t = -0.13 \text{ (GeV/c)}^2$. This is only a parametrization but, as shown by Fig. 2a, which gives results of the Aachen-CERN-Genova-Harvard-Torino Collaboration at $\sqrt{s} = 53 \text{ GeV}$, it is quite faithful. The slopes are equal to 12.4 and 10.8 $(\text{GeV/c})^{-2}$, respectively. The lower $|t|$ structure is better displayed in the NAL results obtained by the US-USSR "gas jet" Collaboration shown in Fig. 2b. One finds a very good exponential behaviour with a slope of 11.6 $(\text{GeV/c})^2$ at $\sqrt{s} = 16 \text{ GeV}$, the spike in the neighbourhood of $t = 0$ being associated with the Coulomb peak. This effect notwithstanding, the differential cross-section at low values of $|t|$ [$|t| < 0.1 \text{ (GeV/c)}^2$] can be represented very well by an exponential in t ,

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt} \right)_{t=0} e^{bt}, \quad (2.20)$$

where b is the slope of the diffractive peak. As obvious from the values already quoted, it increases with increasing energy. The forward peak shrinks, as expected indeed for diffraction from an extending object.

Fig. 2a The "two slope" structure. Differential proton-proton cross-section. This is at present being further studied by experiment 602

Fig. 2b The Coulomb peak. Notice the shrinking of the low $|t|$ peak between 16 and 53 GeV

To a first approximation we may ignore the kink at $t = -0.13 \text{ (GeV/c)}^2$ and neglect the real part of the forward amplitude, thus writing, according to Eq. (2.1),

$$\frac{d\sigma}{dt} = \frac{\sigma_{\text{tot}}^2}{16\pi} e^{bt}$$

$$\sigma_{\text{el}} = \frac{\sigma_{\text{tot}}^2}{16\pi b}$$
(2.21)

The slope parameter should change with energy according to

$$b(s) = \frac{\sigma_{\text{tot}}^2(s)}{16\pi \sigma_{\text{el}}(s)}$$
(2.22)

If σ_{tot} rises indefinitely, b should also rise the same way since σ_{el} is in any case bounded by σ_{tot} . In particular, an asymptotic $\log^2 s$ behaviour for $\sigma(s)$ implies an asymptotic $\log^2 s$ behaviour for $b(s)$. Figure 3 shows the value of b , as now measured over the Serpukhov, NAL and ISR energy range³⁴⁾. The slope increases with energy but not as σ_{tot} does, when considered over the same range. A linear rise of b (as $0.25 \log s$) is compatible with all results. This is still another piece of evidence that the rising cross-section as now seen

Fig. 3 The slope parameter $d\sigma/dt \sim e^{bt}$ $|t| < 0.1 \text{ (GeV/c)}^2$. The situation has hardly changed over one year.

is not an actual asymptotic feature for which σ_{tot} and b should eventually show the same behaviour. In other words, the increase in size, as determined through the increase of b , is somewhat slower than what one may casually expect from the rise of σ_{tot} . The proton increases in size (larger and larger impact parameters become relevant) but also in opacity (it becomes darker at fixed and moderate impact parameter).

Relation (2.22) is rather close to the MacDowell-Martin bound³⁵⁾ which follows from unitarity. It reads (up to terms negligible at very high energies)

$$(\text{Im} A)^{-1} \frac{d}{dt} (\text{Im} A) \Big|_{t=0} \geq \frac{\sigma_{\text{tot}}^2}{36\pi \sigma_{\text{el}}} \quad (2.23)$$

Assuming an imaginary forward amplitude, we would have then³⁶⁾

$$\ell(s) \geq \frac{\sigma_{\text{tot}}^2}{18\pi \sigma_{\text{el}}} \quad (2.24)$$

which is very close to what is already imposed by an exponential diffractive peak. Even though it may thus look as if one is already close to the limiting value experimentally, quite different behaviour for b and σ_{tot} could still apply without leading to any obvious conflict with unitarity. If, for instance, σ_{tot} increases as $\lambda \log^2 s$, as suggested, say, by the data in Fig. 1, and, if b increases as $\log s$ only, as suggested then by the data in Fig. 3, one would have to reach much higher energies, $\sqrt{s} = 2000$ GeV say (letting the ratio $\sigma_{\text{tot}}/\sigma_{\text{el}}$ reach the value $\frac{1}{2}$), before one would meet actual troubles with formula (2.24).

The NAL results in Fig. 3 have been obtained very recently³⁴⁾. They do not suggest the approach of any limiting value as the energy increases. Better data at ISR energy would be very much welcome.

At larger $|t|$ values, the differential cross-section shows a remarkable structure. As seen in Fig. 4, one finds a diffractive dip followed by a second maximum. Results from the ACGHT Collaboration at four different energies are displayed. One should still emphasize though the accuracy of the exponential approximation. The differential cross-section first falls by over 6 orders of magnitude almost exponentially before it rises again. The diffraction pattern is thus far from what is expected from a dark disk with a relatively sharp edge. It is, however, very different from what is known at PS energies, where a kink instead of a sharp dip is found in the neighbourhood of $1.4 (\text{GeV}/c)^2$. The sharpening of the dip is undoubtedly associated with the practical vanishing of the real part of the scattering amplitude as one reaches ISR energies³⁷⁾. At PS energies there are sizeable Regge contributions which contribute to the real part while they cancel themselves out to a large extent in the imaginary part. The real part of the scattering amplitude does not vanish at the same t value as the imaginary part. As a result, any dip structure is blurred. At ISR energies however, the real part is practically gone and the vanishing of the imaginary part produces a diffractive minimum. Whether the slope at larger $|t|$ values $[|t| > 0.1 (\text{GeV}/c)^2]$ and the location of the dip changes with energy is a question which

Fig. 4 Proton-proton differential cross-section at larger $|t|$ values as observed through the ISR energy range. The data are from the ACGHT Collaboration. These data will be improved upon by new results from experiment 604 (ACGHT) and from the CHOV Collaboration working at the SFM.

deserves further studies³⁸⁾. In any case, the shape of the diffractive pattern up to relatively large values of $|t|$ [$|t| = 3 \text{ (GeV/c)}^2$], as already obtained, provides important pieces of information about the shape of the proton.

2.5 The proton shape

What is meant by the proton shape is a matter of definition. We shall follow the optical analogy and define a proton shape as the Fourier transform of the scattering amplitude, calculated from the diffraction peak assuming it to be purely imaginary. The relation being a linear one, we may parametrize the differential cross-section in terms of exponentials in t {Gaussian in $\vec{q}^2$ [Eq. (2.11)]}, thus benefiting from the fact that the Fourier (a Bessel) transform of a Gaussian is a Gaussian. We neglect the spin-flip amplitude and write formula (2.14)

$$\frac{A(s,t)}{8\pi s} = \frac{1}{2\pi} \int a(r,s) e^{i\vec{q}\cdot\vec{r}} d^2\vec{r} = \int a(r,s) J_0(r\sqrt{-t}) r dr$$

$$a(r,s) = \frac{1}{2\pi} \int \frac{A(s,t)}{8\pi s} e^{-i\vec{q}\cdot\vec{r}} d^2\vec{q} = \int \frac{A(s,t)}{8\pi s} J_0(rq) q dq .$$

The amplitude

$$A(s,t) = i s \sum_i \alpha_i e^{\frac{b_i}{2} t} , \quad (2.25)$$

which corresponds to the differential cross-section (2.1)

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} \sum_{ij} \alpha_i \alpha_j e^{\frac{b_i + b_j}{2} t} , \quad (2.26)$$

then implies the partial wave distribution

$$a(r,s) = \frac{i}{8\pi} \sum_i \alpha_i \frac{e^{-\frac{r^2}{2b_i}}}{b_i} . \quad (2.27)$$

The radius associated with an exponential distribution is $r_i = \sqrt{2b_i}$. A typical slope of 10 (GeV/c)^{-2} thus corresponds to a radius of 0.9 f.

In order to parametrize the diffractive minimum, we have to include at least two terms with different slopes and opposite signs. One may use as a first approximation two exponentials with slopes 10 and 2, respectively, and a ratio $\alpha_2/\alpha_1 \approx -0.5 \times 10^{-2}$ ($\alpha_1 \approx \sigma_{\text{tot}}$). The corresponding fit is shown against the data in Fig. 5a. Using Eq. (2.27) we get two Gaussian distributions, the smaller narrower one being subtracted from the larger wider one. This gives basically a Gaussian distribution which is a little flatter on top than an actual Gaussian should be. This corresponds to Fig. 5b. It may be used in defining the shape of the proton, or rather of the overlap of the two colliding protons. We see that the dramatic structure at large $|t|$ has but an *a priori* negligible origin when so described in terms of r . One should, however, keep in mind that the second maximum is 6 orders of magnitude smaller

(a)

(b)

full absorption

$$H(r) = 1$$

(c)

Fig. 5 The shape of the proton. No sharp edge

than the forward peak. The logarithmic scale of Fig. 5a makes it look important. By the same token it would be easy to present the distribution of Fig. 5b in such a way that the deviation from a Gaussian shape would look more dramatic. We have not considered the kink at $|t| = 0.13 \text{ (GeV/c)}^2$. With the approximation (2.26), it would now involve a small deviation from the Gaussian shape in the wing of the distribution of Fig. 5b. The partial wave would not fall quite as fast at large r ³⁹⁾. This, however, immediately raises the question of the energy dependence of $a(r)$, which is discussed in the following section.

The presence of a relatively small real part should not affect much the preceding discussion if $a(r)$ refers to the imaginary part of the partial wave amplitude⁴⁰⁾. The presence of a relatively small helicity-flip amplitude would also not affect it much either. This has to do with the absence of interference terms in Eq. (2.1). As is well known⁴¹⁾ relations (2.14) are easily generalized to an arbitrary helicity flip $\Delta\lambda$

$$\begin{aligned} \frac{A_{\Delta\lambda}}{8\pi s} &= \int a_{\Delta\lambda}(r,s) J_{\Delta\lambda}(rq) r dr \\ a_{\Delta\lambda}(r,s) &= \int \frac{A_{\Delta\lambda}}{8\pi s} J_{\Delta\lambda}(rq) q dq . \end{aligned} \quad (2.28)$$

Accordingly, an approximation to the helicity-flip amplitude of the form

$$A_{\Delta\lambda} = i s \times \left(-\frac{t}{t_0} \right)^{\frac{\Delta\lambda}{2}} e^{-\frac{t}{t_0}} \quad (2.29)$$

which includes explicitly the generally required angular term (we assume no conspiracy), will lead to the following partial wave distribution:

$$a_{\Delta\lambda}(r,s) = \frac{i s}{8\pi} \left(\frac{r^2}{t_0} \right)^{\frac{\Delta\lambda}{2}} e^{-\frac{r^2}{2t_0}} . \quad (2.30)$$

It corresponds to a peripheral profile. The presence of helicity flip will therefore further depress the partial wave distribution around the origin to slightly enhance it at larger r values. In any case a profile such as the one shown in Fig. 5b appears as a good approximation to the proton shape as determined from the diffractive peak. It is again very different from what is expected from a black disk of radius r_0 . In such a case the differential cross-section is proportional to $J_1^2(r_0\sqrt{-t})/r_0^2 t$, integrating Eq. (2.28) with $a = i/2$ for $r < r_0$ and $a = 0$ for $r > r_0$. The dip would now occur at $|t| = 0.8 \text{ (GeV/c)}^2$ (taking $r_0 = 0.9 \text{ f}$) instead of $|t| = 1.4 \text{ (GeV/c)}^2$. Furthermore, the second maximum would be down by a factor 40 only as compared to the forward peak where one finds over a factor of 10^6 !

Taking Eq. (2.27) with a single exponential, as a first (and acceptable) approximation, one gets [using Eq. (2.22)]

$$a(0) \approx \frac{i \sigma_{\text{tot}}}{8\pi b} = \frac{2i}{R} = 0.36i . \quad (2.31)$$

As emphasized by Amaldi⁶⁾, we are yet far away from pure absorption [$a(0) = 0.5i$]. The proton thus remains a relatively transparent object. Nevertheless, the difference between the actual value of $a(0)$ and the unitarity limit is very sensitive to slight variations in opacity. It is for that reason that, instead of $a(r)$, one often considers separately the contribution of the inelastic channels to $a(r)$, or the overlap function only, first subtracting away the specific contribution from elastic scattering. Keeping still the approximation of a purely imaginary amplitude, this can be done in a very simple way.

We first write Eq. (2.16) as

$$a(r,s) = \frac{b(r,s) - 1}{2i}$$

full absorption corresponding to $\eta = 0$. Next, the elastic cross-section is written as a sum of partial wave cross-sections. This is readily obtained from Eq. (2.10). It reads:

$$\sigma_{\text{el}} = 8\pi \int \frac{|b(r,s) - 1|^2}{4} r dr . \quad (2.32)$$

We use our definition

$$\sigma_{\text{el}} = \frac{1}{16\pi s^2} \int |A|^2 dk$$

together with Eq. (2.10) and hence Eq. (2.11).

With the same over-all normalization, one has

$$\sigma_{\text{in}} = 8\pi \int \frac{1 - |b(r,s)|^2}{4} r dr \quad (2.33)$$

and, combining Eqs. (2.32) and (2.33)

$$\sigma_{\text{tot}} = 8\pi \int \frac{1 - \text{Re } b(r,s)}{2} r dr . \quad (2.34)$$

With a pure imaginary (η real) providing $a(r)$ from the differential cross-section (Fig. 5b), one then determines as well the overlap function $1 - |\eta|^2$ or

$$H(r) = \frac{1}{2\pi r} \frac{d\sigma_{\text{in}}}{dr} = 4 |a(r)| (1 - |a(r)|) . \quad (2.35)$$

The black disk limit corresponds to $H(r) = 1$. With $|a(0)| = 0.36$, we now get $H(0) = 0.92$ ⁴²⁾. It may now look as if the proton is quite opaque at the centre, since the inelastic cross-section is close to its maximum value. The difference with the first impression drawn from a stems from the fact that H depends quadratically on η when a depends only linearly on it in the neighbourhood of $\eta = 0$ (full absorption). Reaching full opacity would mean but a modest increase of $\sigma_{in}(0)$, while $\sigma_{el}(0)$ would accordingly rise appreciably. H is closer to the classical optical analogy ⁴³⁾. The difference between a and H is illustrated by Figs. 5b and 5c. The proton is rather opaque in the centre. However, it quickly becomes grey as one moves out and eventually shows a Gaussian tail. This is as expected from a strong but soft interaction as opposed to the weak but hard interactions relevant for deep inelastic electromagnetic scattering.

2.6 The increasing opacity

The change of the differential cross-section with increasing energy can then be readily translated into a change in shape using Eqs. (2.27) and (2.35). Figure 6a gives two profiles obtained at $\sqrt{s} = 31$ GeV and $\sqrt{s} = 53$ GeV, respectively. In each case one shows $d\sigma_{in}/dr$, as defined through Eq. (2.35). The black disk limit is a straight line. One sees again that already at distances of the order of $0.5 f$, the opacity is far below what full absorption would allow. With increasing energy the opacity increases but is nothing as compared to what unitarity would allow. One also sees that the increase of the inelastic cross-section is mainly peripheral. This is clear in Fig. 6b, which gives $\Delta\sigma_{in}(r)$ between 31 and 53 GeV ⁴⁴⁾. If $\frac{2}{3}$ of the inelastic cross-section is to be found at distances below $1 f$, $\frac{2}{3}$ of the increase in cross-section is to be found above $1 f$. The curves of Fig. 6 are from the analysis of H. Miettinen and P. Pirula ⁴⁴⁾. The key point is that the increase in cross-section is mainly peripheral and very modest as compared to what unitarity would allow within the range of strong interactions. The relative transparency of the proton at such high energies appears as the prominent puzzling fact. The rise of σ_{tot} is again seen as a transient and very limited effect. It does not correspond to any simple asymptotic feature. Better data on $d\sigma/dt$ will allow a finer analysis ⁴⁵⁾. Details may change or appear. The key features stressed here should however remain.

2.7 The real part of the forward amplitude

Another important and easily accessible quantity should reflect the rise of σ_{tot} . It is the ratio between the real part and the imaginary part of the forward elastic amplitude

$$\rho(s) = \frac{\text{Re } A(s,0)}{\text{Im } A(s,0)} .$$

It may be calculated through a dispersion relation. Accordingly, variations of σ [or of $\text{Im } A(s,0)$] will imply *a priori* different variations of $\text{Re } A(s,0)$ and hence variations of ρ . There are, furthermore, important conclusions which one may reach from the fact that the scattering amplitude should become eventually even under crossing symmetry.

That the imaginary part of A is dominantly even is imposed by unitarity since some cross-section(s) would otherwise be negative. That it should be asymptotically even follows from the Pomeranchuk theorem. However, unitarity alone is needed if the cross-section rises asymptotically ⁴⁶⁾. An asymptotically even real part is generally imposed through a dispersion

Fig. 6 Variations of the proton size and opacity between 31 and 53 GeV (better data from 604 and the SFM should improve this analysis) (Miettinen and Pirula)

relation, assuming that the imaginary part of the amplitude, which is odd under crossing, increases only as $s^{1-\epsilon}$ (no subtraction required). It is accordingly supposed to be the case in all so-called "reasonable" models. It could, however, not be the case. We will nevertheless assume that $A(s,0)$ becomes eventually even and that it is to a good approximation even at ISR energies.

The scattering amplitude is an analytic function of s in a cut plane and the discontinuity across the cut is equal to twice the imaginary part. The physical amplitude is the limiting value when the right-hand cut is approached from above (P)

Rotating by π counter-clockwise in the s plane, one reaches a point p' at $|s|e^{i\pi}$, where the new value of the amplitude is $A^*(s)$. It is equal to $A(s)$ in p'' . In the ξ plane ($\xi = \log s$), one starts accordingly from a point $\xi + i\epsilon$ to reach a point $\xi + i\pi - i\epsilon$, where the amplitude is $A^*(\xi + i\epsilon)$. It follows that an asymptotic expression for the amplitude, such as $\log^\alpha s$, is not compatible with the required even property under crossing. One would reach $-i s (\log s + i\pi)^\alpha$ instead of $(i s \log^\alpha s)^*$. There is however an easy trick which would enforce the proper crossing property. One should replace "log s " by "log $s - (i\pi/2)$ " whenever it appears in a model amplitude. Under crossing this particular term becomes $\log s + (i\pi/2)$, which is now the complex conjugate of the initial value. This will do whether we consider an asymptotically rising cross-section, such as Eq. (2.8), or a cross-section rising towards an asymptotic limit, such as Eq. (2.9). A model amplitude such as:

$$A(s,0) = i s \left(\log s - \frac{i\pi}{2} \right)^\alpha \quad (2.36)$$

will however now have a real part, the ratio of which to the imaginary part decreases only logarithmically. Furthermore ρ is positive^{4,7)}

$$\rho(s) = \frac{\alpha\pi}{2} \frac{1}{\log s} \quad (2.37)$$

This is even more interesting since, at Serpukhov energy, $\rho(s)$ is known to be negative and decreasing in a way compatible with a Regge behaviour, namely $|\rho(s)| \sim s^{-1/2}$. If the total cross-section rises, ρ should then pass through zero and become positive, eventually decreasing slowly asymptotically. Estimates based on dispersion calculations readily locate the vanishing of ρ to be within the NAL energy range^{4,8)}. This was compatible with ISR data with relatively large error bars, available at the time of the Fisher-Bourrely or Bartel-Diddens numerical analysis¹⁾. This is now beautifully demonstrated by NAL data, which show that ρ indeed vanishes in the neighbourhood of 250 GeV. This is shown in Fig. 7, which puts together Serpukhov, NAL and ISR values now available. This can be regarded as a beautiful

Fig. 7 The ρ parameter

indirect confirmation of the rising cross-section. The only other possible origin for such a rise of ρ , the behaviour of σ_{tot} notwithstanding, could be a very peculiar behaviour of σ_{pp}^- with respect to σ_{pp} , since they both occur in the dispersion relation through which ρ is eventually calculated. In models usually considered the two cross-section curves never cross, even though the Pomeranchuk theorem requires their approaching each other only asymptotically. As a result σ_{pp}^- is expected to exhibit a marked minimum in the NAL energy range, since its value at top Serpukhov energy is equal to σ_{pp} at top ISR energy. Whether it does it or not is a challenging question. The relative variation of $d\sigma/dt$ for pp and $p\bar{p}$ scattering is also very interesting. All these questions should be answered soon at NAL ⁴⁹⁾. If the real part of the pp amplitude has an important Regge contribution when its imaginary part has but a negligible one, as imposed by duality, one should expect things to be very different for $p\bar{p}$ scattering. The two even (odd) Regge contributions to the pp amplitude $-\beta[1 \pm e^{-i\pi\alpha(t)}/\sin \pi\alpha(t)]s^{\alpha(t)}$ give an over-all $-2\beta \cot \pi\alpha(t) s^{\alpha(t)}$ contribution to the real part of the $p\bar{p}$ amplitude. With $\alpha(0) \approx 1/2$ this should be very small as compared to their contribution to the imaginary part, namely $2\beta s^{\alpha(t)}$. The real part should then be found positive at much lower energies than is required for that in the case of pp scattering. Similar effects should also be found for the K^+p and K^-p channels. In the former case, duality is expected to offer a much stronger constraint than for pp scattering. The rise of the total cross-section is already apparent over the Serpukhov energy range. Checking its behaviour over the NAL energy range is of a great interest. The rising proton-proton cross-section as discovered at the ISR poses many exciting questions for NAL.

It is interesting to use the crossing property of the forward elastic amplitude when it is written in terms of the variable $\xi = \log s$. One may connect the two limiting values through a Taylor series⁵⁰⁾. This reads

$$A^*(\xi) = A(\xi + i\pi) = e^{i\pi \frac{\partial}{\partial \xi}} A(\xi). \quad (2.38)$$

Instead of the scattering amplitude A , we may consider the function $F(s) = A/s$, which is now (asymptotically) odd and (asymptotically) equal to the total cross-section. One gets

$$\text{Re } F(s) - i \text{Im } F(s) = - e^{i\pi \frac{\partial}{\partial \log s}} (\text{Re } F(s) + i \text{Im } F(s)) \quad (2.39)$$

or

$$\begin{aligned} \text{Re } F(s) &= \text{tg } \frac{\pi}{2} \frac{\partial}{\partial \log s} \left\{ \text{Im } F(s) \right\} \\ &= \left(\frac{\pi}{2} \frac{\partial}{\partial \log s} + \frac{1}{3} \left(\frac{\pi}{2} \right)^3 \frac{\partial^3}{(\partial \log s)^3} + \dots \right) \text{Im } F(s), \end{aligned} \quad (2.40)$$

which is particularly simple when σ is, as often the case in model calculations, a simple function of $\log s$ ⁵¹⁾. If σ has a positive derivative with respect to $\log s$, $\rho = \text{Re } F / \text{Im } F$ should be positive. This is again the result imposed for an even amplitude under crossing. Such a calculation cannot replace a full-fledged dispersion relation calculation. Its pedagogical interest is however quite remarkable.

The rising cross-section has destroyed expectation for a comforting but eventually dull asymptotia. It has opened up a new and wide field of investigations at ISR, but also to a very large extent at NAL (SPS) energies.

3. DIFFRACTIVE EXCITATION

3.1 Diffractive signature

It has long been recognized that some inelastic processes bear a very strong kinematical resemblance to high-energy elastic scattering. This is particularly the case when one considers the production of resonances with the same quantum numbers as those of one of the initial particles. There are several well-known examples in pp quasi-two-body collisions. More specifically, one may quote the production of the $\frac{1}{2}^+(1405)$ and $\frac{3}{2}^+(1688)$ baryons. The key features of such reaction mechanisms are the following:

- i) Low momentum transfers: The differential cross-section falls sharply with increasing momentum transfers. The reaction process can then be analysed as a typical exchange mechanism. It thus also corresponds to an *a priori* highly coherent scattering process.
- ii) No quantum number exchange: Analysing the reaction as a quasi-two-body process, as reliable from property (i), the two final hadronic states are found to have the same internal quantum numbers as those of the respective initial hadrons.
- iii) Weak energy dependence: Specifying the final state, the reaction cross-section is found to have practically the same energy dependence as that of elastic scattering.

These three properties will be used from now on in order to define diffractive excitation. Whatever preconceived ideas one has about diffraction, they should be expected to hold. In any case, they appear to apply to a sizeable fraction of all observed inelastic processes, thus signalling a particular mechanism worth a special study. At high energy $\frac{1}{5}$ to $\frac{1}{4}$ of the inelastic cross-section at least is found to be diffractive. Of particular importance is single-diffractive excitation whereby one of the initial particles is merely quasi-elastically scattered, while the other one flares into a many-particle hadronic state, often called a hadronic nova. This gives a particularly simple experimental signature (trigger). It is defined by a final particle isolated from any other in phase space with a centre-of-mass momentum very close (ratio-wise) to that of the corresponding initial particle. At high enough energy a quasi-elastically scattered particle may nevertheless suffer an appreciable energy loss in absolute value if not in relative value. This allows rather large excitation masses, much beyond those of the prominent low mass resonances. It was a very important discovery at the ISR to find that the mass spectrum for such diffractively-excited objects extends up to values greater than 10 GeV⁵¹⁾.

One may analyse diffractive excitation following different approaches. There is obviously a close parallel with optics, which can be justified in the very high energy limit,

where the relevant excited states have masses M which are small as compared to the centre-of-mass energy ($S/M^2 \gg 1$). Following the analysis of Good and Walker it calls for eigenstates of the S matrix which are only a linear superposition of asymptotic states with well-defined particle numbers. It then follows that elastic scattering and diffractive excitation cannot be dissociated. One implies the other. It is, however, more useful for calculation purposes to consider diffractive excitation as a specific exchange process treated on the same footing as elastic scattering, Pomeron exchange having then off-diagonal terms as well as diagonal ones. This also gives inelastic amplitudes with similar kinematical (and dynamical) features as those specific to elastic scattering. The three key properties which have been itemized are obviously met. Elastic scattering, single-diffractive excitation and double-diffractive excitation are then represented according to the graphs of Fig. 8, respectively. If one imposes factorization properties, which would hold by construction for a leading Regge pole, and neglect phase space limits when integrating over momentum transfers, one gets the following relation:

$$\sigma_E \cdot \sigma_{DD} = \frac{\sigma_D^2}{4} \quad (3.1)$$

where σ_E , σ_D , σ_{DD} stand for the elastic, single-diffractive and double-diffractive cross-sections, respectively. If one adds the empirical relation:

$$\sigma_E \approx \sigma_D, \quad (3.2)$$

the corresponding estimate for σ_{DD} is $\sigma_E/4$. Indeed, if there is at present ample experimental evidence for a sizeable single-diffractive cross-section, there is as yet nothing comparable for double excitation. Its experimental signature is however far less defined. Double-diffractive excitation should, however, also exist with an appreciable cross-section⁵²).

Reaction amplitudes, such as those corresponding to Fig. 8, interfere with many others in the calculation of the cross-section for any specific final state. Extracting the full diffractive contribution from experimental distributions is therefore not possible in a

Fig. 8

model-independent way. Nevertheless, kinematical configurations can be selected, in order to give a prominent role to diffractive excitation⁵³). This is, in particular, relatively easy for single-diffractive excitation. The leading particle should then have a momentum close enough to the phase space boundary. The missing mass to the leading particle, M is

$$\begin{aligned} M^2 &= (P_1 + P_2 - P)^2 \\ &= s + m^2 - 2E\sqrt{s} \ , \end{aligned}$$

where E is the centre-of-mass energy of the observed leading particles. The leading particle mass m may often be neglected in front of $\sqrt{s}$ or even M . Since E is close to its maximum value $\sqrt{s}/2$, we write

$$E = (P_L^2 + P_T^2 + m^2)^{\frac{1}{2}} \approx P_L + \frac{P_T^2 + m^2}{2P_L}$$

and $P_L = xP_{\max}$, with

$$P_{\max} = \frac{\sqrt{s}}{2} - \frac{P_T^2 + m^2}{\sqrt{s}} \ .$$

Therefore,

$$M^2 = s(1-x) + \left\{ m^2 + 2(P_T^2 + m^2) \left(\frac{x-1}{x} \right) \right\} \ . \quad (3.3)$$

We may then write, to a good approximation

$$1-x = \frac{M^2}{s} \ . \quad (3.4)$$

Analysing the reaction in terms of the graph of Fig. 8b, there are two obvious choices of variables. One may consider x and p_T , focusing on the leading particle, or t and M^2 , t being the momentum transfer squared at the upper vertex, focusing then on the quasi-two-body aspect of the reaction. Neglecting terms of order s^{-1} , one gets

$$t = -\frac{P_T^2}{x} - m^2 \left(\frac{x-1}{x} \right)^2 \ . \quad (3.5)$$

Since x is always close to 1, in order to signal diffractive excitation, t is always close to $-p_T^2$. Using Eqs. (3.3) and (3.5), the inclusive distribution is very simply related to the differential cross-section with respect to M^2 and t . To leading order in s , one has

$$E \frac{d\sigma}{d^3p} = x \frac{d\sigma}{dx dp_T^2} = s \frac{d\sigma}{dM^2 dt} \ . \quad (3.6)$$

The diffractive differential cross-section with respect to M^2 first rises, as more and more hadronic states become accessible, but it should eventually fall with increasing M^2 . Since $d\sigma/dM^2$ is assumed to depend only weakly, if at all, on s one would otherwise get an unacceptable cross-section, the upper limit of M^2 increasing linearly with s ⁵⁴⁾. It therefore follows that the diffractive contribution to the inclusive distribution (3.6), at fixed (and large) s , should show a sharp rise and a fall as x moves away from 1. From Eq. (3.4) the maximum moves closer and closer to 1 as the energy increases (Fig. 9a). This is very different from what is expected from a leading proton which would have actually fragmented into pions. The distribution should then be similar to that found for neutrons at larger x values, that is fairly flat and dropping to zero as x gets close to 1 (Fig. 9b)⁵⁵⁾. The differential cross-section $d\sigma/dM^2$ now rises with M^2 , favouring large missing masses, associated with the presence of pions rather close in phase space to the leading proton, over the lower missing masses which one would favour instead if no fast pions were to be found travelling with the leading proton. As discussed later, the contrast between these two typical behaviours (Figs. 9a and b) comes out in a particularly simple way in the triple-Regge formalism⁵⁶⁾. The main point is that for x close enough to 1, one may kinematically isolate a contribution from single-diffractive excitation. It turns out to stand out in a very conspicuous way, when it could have been hardly visible *a priori*. This is an important discovery at the ISR. This is seen in Fig. 10, which gives results obtained by the CHLM Collaboration at the ISR. The quasi-elastic peak is very clearly seen. It stands out down to $x = 0.92$ which, at $\sqrt{s} = 30$ GeV, corresponds to a missing mass $M = 8.5$ GeV. Diffraction excitation is thus found to be an important mechanism. It involves excitation masses which extend much beyond the prominent resonances (indeed top PS centre-of-mass energy). Furthermore, the inclusive distribution is practically energy independent when it corresponds to its fall on the lower x side of the quasi-elastic peak, or to the high mass tail of the excitation mass distribution. From Eqs. (3.4) and (3.6), this implies that $d\sigma/dM^2$ falls eventually as $1/M^2$, if practically energy independent, as expected for a diffractive cross-section.

Such a behaviour is strongly supported by recent results from NAL⁵⁷⁾. There one may get an efficient trigger studying inelastic proton collisions off a nucleus which is required

Fig. 9

Fig. 10 The quasi-elastic peak at ISR

to remain in its ground state. Properties (i) and (ii) have then to be satisfied in order to satisfy the trigger. The very sharp nucleus form factor practically limits the reaction to small values of the momentum transfer. Results on $p + d \rightarrow p^* + d$ obtained by the US-USSR "gas jet" Collaboration are shown in Fig. 11⁵⁷⁾. One sees an M^{-2} tail following a sharper initial fall-off. The cross-section is practically energy independent. It is, furthermore, quite sizeable with $d\sigma/dM^2 \sim 0.2 (M)^{-2} \text{ mb/GeV}^2$ at large mass⁵⁸⁾. Preliminary information also supports the production of low-mass resonances with cross-section comparable to those found in the 10-20 GeV range. Property (iii) then also appears to be satisfied as well.

The new and very important point is that diffractive excitation is not limited to the first few prominent resonances. It involves a continuum of states extending to very high masses, even way beyond the full centre-of-mass energy accessible on previous machines. The mass distribution is such as to provide a scaling fall-off of the quasi-elastic peak. As imposed by Eqs. (3.4) and (3.6), together with the practically energy independence of $d\sigma/dM^2$, the quasi-elastic peak of the inclusive distribution (Fig. 9a) increases in height as s , as its location moves towards $x = 1$ as s^{-1} . Its outer edge remains however energy independent as higher and higher mass excitations keep contributing at fixed x with increasing energy [Eq. (3.4)].

3.2 The triple-Regge formalism (The reader already familiar with this question should skip this section)

In a kinematical configuration where s/M^2 can be considered as large, the amplitude associated with the graph of Fig. 8b can be approximated in terms of Reggeon exchange

$$A(s, t, M^2) = \sum_i \beta_i(t) \gamma_i(t, M^2) \left(\frac{s}{M^2} \right)^{\alpha_i(t)}, \quad (3.7)$$

the corresponding differential cross-section is [Eq. (2.1)]:

$$\frac{d\sigma}{dt dM^2} = \frac{1}{16\pi s^2} \sum_{i,j} \beta_i(t) \beta_j^*(t) \gamma_i(t, M^2) \gamma_j^*(t, M^2) \left(\frac{s}{M^2} \right)^{\alpha_i(t) + \alpha_j(t)} \quad (3.8)$$

The quantity $\gamma_i(t, M^2)$ can be considered as the amplitude for a Reggeon of mass squared t and a particle (proton in the present case) to yield hadronic states of mass M . All such states are implicitly summed up in Eq. (3.8), with the over-all normalization defined by Eq. (2.5). The Reggeon signature factor and denominator function have been absorbed together with the Reggeon coupling in β . A Regge-pole exchange implies the factorization property explicit in Eq. (3.7) and is indeed used for the separation of a Reggeon particle amplitude. One may first consider the diagonal terms ($i \equiv j$) and sum explicitly over all final states (f) at fixed M^2 . This gives using Eq. (2.6) [with the kinematical approximations justifiable at large M^2]

$$\sum_f |\gamma_i(t, M^2)|^2 = \frac{2}{(2\pi)^4} M^2 \sigma_i(M^2, t), \quad (3.9)$$

Fig. 11 The quasi-elastic peak at NAL

where $\sigma_i(M^2)$ is the total cross-section for the Reggeon particle (proton) collision at centre-of-mass energy M . One may therefore write

$$\frac{d\tau}{dt dM^2} = \frac{G_i(t)}{s^2} M^2 \left(\frac{s}{M^2}\right)^{2\alpha_i(t)} \sigma_i(M^2, t). \quad (3.10)$$

Together with Eq. (3.6), this gives

$$x \frac{d\tau}{dx dp_T^2} \approx \sum_k R_i^{(k)}(t) \left(\frac{s}{M^2}\right)^{2\alpha_i(t)-1} (M^2)^{\alpha_k(0)-1}, \quad (3.11)$$

approximating $\sigma(M^2, t)$ in terms of a series of Regge terms with respective intercept $\alpha_k(0)$. For $x \approx 1$, where the corresponding approximations are being used, $t \approx -p_T^2$ and $M^2/s = 1 - x$. It follows that, if the missing mass is large enough, where Pomeron exchange is a good enough approximation for $\sigma(M^2)$ [$\alpha_k(0) = 1$], the inclusive distribution scales. Conversely, Pomeron exchange implies $i \equiv j$ as here assumed. More generally, however, we have

$$x \frac{d\tau}{dx dp_T^2} \approx \sum_{ijk} R_{ij}^{(k)}(t) s^{\alpha_k(0)-1} \left(\frac{1}{1-x}\right)^{\alpha_i(t) + \alpha_j(t) - \alpha_k(0)}. \quad (3.12)$$

This is now well known as the triple-Regge approximation. As is also well known, it can be summarized in a graphical form as indicated in Fig. 12. The last diagram (Fig. 12c) is not a field theory graph in the usual sense. It indicates how the summation over all final states in Fig. 12c is performed or approximated. If $i \equiv j$ the corresponding term is positive, since the two sides of Fig. 12b are complex conjugates of each other. With cross terms ($i \neq j$) negative contributions can be introduced. There are no reasons to neglect them when secondary Regge terms are considered for α_k . It is obvious that such an approximation may then become too flexible to be of any strong predictive value. This is obviously

Fig. 12

the case if too many terms are introduced. One may first assume that the quasi-elastic peak corresponds to $x > 0.92$ and therefore to a large enough value of s/M^2 as to justify Pomeron exchange alone for α_i or α_j . The missing mass M is also assumed to be high enough that Pomeron exchange $\alpha_p(0) = 1$, together with leading secondary Reggeon exchange $\alpha_R(0) = 1/2$, should be enough. There are then two terms, namely a triple-Pomeron PPP term and a PPR term. The inclusive distribution reads:

$$x \frac{d\sigma}{dx dt} = R_{PP}^{(P)}(t) \left(\frac{1}{1-x} \right)^{2\alpha_p(t)-1} + R_{PP}^{(R)}(t) \frac{1}{\sqrt{s}} \left(\frac{1}{1-x} \right)^{2\alpha_p(t)-\frac{1}{2}} \quad (3.13)$$

From Eq. (3.10) they correspond to differential cross-sections at $t = 0$ [$\alpha_p(t) = 1$] of the type M^{-2} and M^{-3} , respectively. This is summarized through Table 1 below (we integrate over t , neglecting the slope of the Pomeron). This parametrization may be used in

Table 1

$x \frac{d\sigma}{dx}$	$\frac{d\sigma}{dM^2}$	$\alpha_k(0)$
$\frac{1}{1-x}$	$\frac{1}{M^2}$	1
$\frac{1}{\sqrt{s}} \frac{1}{(1-x)^{3/2}}$	$\frac{1}{M^3}$	$\frac{1}{2}$

analysing the data of Figs. 10 and 11, extracting an obvious scaling term (M^{-2} tail in Fig. 11) and a decreasing contribution with increasing energy at fixed x (M^{-3} term in Fig. 10). Eventually the inclusive distribution scales at fixed x , as hadronic states with increasing masses contribute to it⁵³). This is due to the presence of an M^{-2} tail in the mass distribution. Integrating over it, one gets a $\log s$ contribution to the total cross-section, since the end point increases as s ⁵⁴).

If one wishes to extend the applicability of the triple-Regge formalism to lower values of x (or lower values of s/M^2) it is necessary to introduce secondary trajectory contributions next to $\alpha_p(t)$ and in particular crossed terms. Nevertheless, taking only diagonal terms for the mere sake of simplicity, one may consider Table 2 grouping the RRP and RRR terms. The RRP (scaling) term is a convenient way to describe the rather flat inclusive

Table 2

$x \frac{d\sigma}{dx dp_T^2} (p_T = 0)$	$\frac{d\sigma}{dt dM^2}$ $t = -0.6$	$\alpha_k(0)$
C^{st}	M^2/s^2	1
$\frac{1}{\sqrt{s}} \frac{1}{(1-x)^{1/2}}$	M/s^2	$\frac{1}{2}$

distribution found next to the quasi-elastic peak ($x < 0.9$). The RRP term is also a convenient way to approximate the inclusive proton distribution associated with a fragmenting proton as apposed to a quasi-elastically scattered one. One easily recognizes in Tables 1 and 2 the typical behaviours of Figs. 9a and 9b, respectively.

The triple-Regge analysis of the inclusive proton distribution has been extensively developed over the last two years⁵⁵⁾. It leads one to conclude that there is a sizeable scaling (triple-Pomeranchon) contribution, together with a non-scaling (PPR) term which implies a sharpening with increasing energy of the kink (dip) met next to the peak. More accurate predictions are, however, hardly possible in view of the general neglect of crossed terms which, if included, would involve too many parameters for the accuracy of the data. In any case there is no reason to push the analysis too far. Many kinematical and dynamical approximations are involved in the writing of formula (3.12) and one would not know whether one tests them, or rather the fact of our practically limiting the series to a few terms only. The simplicity and pedagogical interest of the approach is the key point.

3.3 The angular dependence of diffractive excitation

In the former standard range of energies (10-20 GeV) much variety was reported according to the type of excited state actually observed. The t dependence can be very steep as compared to what is found for elastic scattering. This is, in particular, the case for the production of the $1405N^*$. It can be broader as is the case for the production of the $1688N^{*8)}$. The t distribution is generally known to slowly become broader with increasing missing mass. The precise shape in the neighbourhood of $t = 0$ is still the object of investigation⁵⁹⁾. In any case most of the cross-section is confined to the relatively low $|t|$ region ($|t| < 0.5$). Indeed, coherent diffractive production on nuclei, which is sharply cut off in $|t|$ by the nucleus form factor, is known to be quite strong. The ISR (spectrometer) data are still limited to relatively large $|t|$ values [$|t| \geq 0.4$ (GeV/c)² say] and the differential cross-section can then be parametrized in terms of an exponential in t . The slope is of the order of 4 (GeV/c)^{-2 60)}. Data at lower $|t|$ values have been obtained at NAL in double chamber and counter experiments⁶¹⁾. Figure 13 combines then ISR spectrometer data at larger $|t|$ values with NAL track chamber data at lower $|t|$ values both obtained at the same energy. The overlap is still minimal but no discrepancy seems to appear. At present one needs both sets of data in order to get an estimate of the corresponding cross-section. Integrating over t one gets $5 < \sigma_D < 7.5$ mb. The t distribution corresponds to an average over the missing-mass spectrum when we should *a priori* expect some mass dependence. In any case, combining ISR and NAL data, it appears that the distribution is definitely steeper at low $|t|$ than at large $|t|$. An exponential parametrization of the lower $|t|$ data would indeed favour a slope of 8, as opposed to a slope of 4 at larger $|t|$. Such a structure in the t distribution is suggested by a compilation of data. This is also illustrated by Fig. 14, which puts together results from different experiments at different energies but with missing masses at around 3 and 6 GeV. One again sees that the t dependence gets definitely broader at larger $|t|$ values. One also sees that most of the cross-section is concentrated at low $|t|$ but that the behaviour in the neighbourhood of $t = 0$ deserves further investigation. One finally sees that there is apparently no dramatic dependence on energy. A more refined analysis of the t dependence and particularly at low $|t|$, as a function of the excitation mass, should attract much attention at NAL (SPS). A precise determination of the t dependence,

Fig. 13 Momentum transfer dependence of the peak cross-section, combining ISR and NAL results. A determination of σ_D is not possible yet from ISR data alone

Fig. 14 A compilation of data. There is a need for an ISR experiment at low $|t|$

at lower $|t|$ values than those so far reached, should also attract much attention at the ISR. One should improve on the overlap with NAL at lower ISR energies. One should also go down to low $|t|$ values (very small angles), even at higher ISR energy. This is needed in order to better determine the values of the diffractive cross-section and then its energy dependence which are still at the level of "educated guesses". This deserves further studies with very small angle spectrometer(s). It can already be said that the t dependence of single-diffractive excitation is quite different from that of elastic scattering. The parallel indicated by Fig. 8 notwithstanding, the discrepancy could be largely due to the shadow aspect of elastic scattering. As a matter of fact, an important difference between the t dependence of elastic scattering and of quasi-elastic scattering can be expected from an impact parameter analysis. If single-diffractive excitation were to be associated with a purely imaginary amplitude with an exponential t dependence, in much the same way as elastic scattering, one could write the corresponding partial wave amplitudes as Eq. (2.27):

$$a_D(r) = \frac{\alpha_D}{8\pi} \frac{e^{-\frac{r^2}{2B}}}{B} . \quad (3.14)$$

The cross-section would then be given by

$$\sigma_D = 2 \frac{\alpha_D^2}{16\pi B} , \quad (3.15)$$

the factor 2 written in front corresponding to the separate excitation of the two incident protons.

This would correspond to a specific contribution to $H(r)$, as defined through Eq. (2.35)

$$H_D(r) = \frac{\sigma_D}{\pi B} e^{-\frac{r^2}{B}} . \quad (3.16)$$

Such a contribution is embarrassingly maximum at $r = 0$, when the success of absorption models, at providing a reasonable description of quasi-two-body processes, would suggest a rather peripheral distribution. Furthermore if σ_D is as large as σ_{el} , subtracting Eq. (3.16) from Eq. (2.35), we obtain a rather peripheral profile (Fig. 15a) for what should be the impact parameter distribution of a complicated remainder, which one would *a priori* rather see flat with a Gaussian tail. This difficulty has been already emphasized by Sakai and White⁶²⁾, who rather propose important helicity-flip contributions which readily lead to a peripheral distribution (2.30). They advocate in favour of them on the grounds that helicity could be conserved in the t channel as opposed to the s channel. If this is the case, the differential cross-section should dip in the neighbourhood of $t = 0$. There is, however, no reason yet to abandon a dominant no-helicity-flip diffractive amplitude and the more so that coherent production on nuclei is strong. A rather peripheral profile, which would, once subtracted from $H(r)$, provide an even flatter remainder at low r values, can also be very simply obtained assuming a negative interference between two Gaussian amplitudes, namely

$$a_D(r) = \frac{\alpha}{8\pi B} e^{-\frac{r^2}{2B}} - \frac{\beta}{8\pi B'} e^{-\frac{r^2}{2B'}} . \quad (3.17)$$

Fig. 15

One chooses $B' < B$ and α/β in such a way that the corresponding profile now dips at $r = 0$ thus showing an expected peripheral shape. Figure 15b gives the corresponding behaviour for $H^D(r)$ together with the associated behaviour of the remainder⁶³). This is an *a priori* likely configuration, most simply parametrized as Eq. (3.17), which can be used as a first guess for further studies. The differential cross-section should, however, now show a dip as one shifts from a steep exponential fall (slope B) to a lower one (slope B'). This is very similar to what is already considered for elastic scattering [Eq. (2.27)]; however requiring a dip at $r = 0$ now calls for terms of rather similar magnitude. As a result, the marked structure in t must occur at a much lower value than that found for elastic scattering, namely $|t| \sim 0.3 \text{ (GeV/c)}^2$ or so, instead of 1.4 (GeV/c)^2 . If such a change in effective slope is indeed compatible with the data of Fig. 14, there is at most a kink and nothing like the dip found in the elastic scattering differential cross-section. However, using Eq. (3.17), one gets with Eq. (2.26):

$$\frac{d\sigma^D}{dt} = \frac{1}{16\pi} \left\{ |\alpha|^2 e^{Bt} + |\beta|^2 e^{B't} - 2|\alpha||\beta| \cos\varphi e^{\frac{B+B'}{2}t} \right\} \quad (3.18)$$

which undoubtedly shows a pronounced dip rather than a kink. There are, however, three ways in which such a dip could be smeared out into a kink in the data presently available. Firstly, the slope parameters B and B' , and hence the location of the dip, could depend on the missing mass when only an averaged effect can be seen at present. Secondly, helicity-flip contributions could "fill" the dip, while remaining modest as a whole when compared to the dominant non-flip cross-section. Finally, the production amplitude could have a small real part which would quench the last term on the right-hand side of Eq. (3.18), while its contribution to the differential cross-section could again "fill" the dip. The third-mentioned possibility turns out to be the most efficient practically. A small real part admixture switches the dip into a kink when this would otherwise require an important helicity-flip contribution⁶³). A relatively small real part, together with a relatively small helicity-flip contribution, would then result in a sizeable polarization in the kink

region, that is at $|t| \sim 0.3 \text{ (GeV/c)}^2$. Progress with polarized targets should allow the observation of such an effect in an inelastic reaction⁶⁴⁾. This should be considered as an interesting question at NAL (SPS). The corresponding single-diffractive signature (slow proton recoil) is particularly simple. The mainly peripheral nature of single-diffractive excitation, if it turns out to be an actual property, is then associated with a much more pronounced structure in t than that seen in the case of elastic scattering. This clearly deserves further studies for which research at ISR and NAL (SPS) have largely complementary information to provide.

3.4 The hadronic nova

The inclusive parameters [x and $p_T(t)$ dependence] clearly allow separating out a quasi-elastically scattered particle. The other proton flares into a multiparticle state which can be most generally referred to as a hadronic nova. At present, the nature of the nova, the formation of which corresponds to $\frac{1}{5}$ to $\frac{1}{4}$ of the inelastic cross-section, raises several interesting questions. One may look at it in a general sense or, focusing on Fig. 8, consider it as being associated with a Pomeranchon-proton reaction amplitude. Low-mass resonances are particular cases of hadronic novae. Nevertheless, we now know that they contribute only a small fraction of the diffractive cross-section. If the relatively low excitation masses can probably be interpreted as superpositions of resonances, this very picture may in turn lose interest as one reaches excitation masses as high as 10 GeV. Following a classical image, one may indeed consider two specific effects. The proton can be excited into discrete hadronic states, which eventually decay with pion emission. At the same time, however, the higher mass hadron as it originates from a decelerating proton may "radiate" pions through a process similar to Bremstrahlung. All pions, however, globally combine with the final nucleon into the observed missing mass⁶⁵⁾. The dynamics of the nova effect is obviously interesting for its connection with proton excitation. One can thus follow hadron excitation past the point where resonance formation and decay probably lose their interest, except perhaps if considered in a dual sense. The nova is a hadronic channel with the quantum numbers of the proton and hence an *a priori* simple one for duality. Its dual properties are, however, not yet clear. The dynamics of the nova effect is also very interesting as an approach to the study of "Pomeranchon-proton" reactions, which opens a wide and new extension of what is usually referred to as diffraction.

The first question which one may tackle experimentally is that of particle multiplicity in nova decay. The mass distribution is wide and, as already mentioned, ISR results show that it extends beyond 10 GeV. The multiplicity may accordingly be large. Figure 16a gives already one-year old results obtained by the ACGHT Collaboration. They show the quasi-elastic (missing-mass) peak as it appears in reactions where two fast protons (often including, as shown by the missing-mass distribution, a quasi-elastically scattered one) are detected in a small-angle double-arm spectrometer set-up and where detectors around the spectrometers register extra charged particles (pions) produced at small angles together with either one of the protons. One sees that a quasi-elastic peak appears even when up to 6 particles are registered on the other side. The quasi-elastic peak shifts towards larger masses accordingly. The over-all (seen) multiplicities are rather low. Nevertheless, one may conclude that diffractive excitation extends to large masses and to relatively large multiplicities. If one has but the incoherent superposition of a few resonances one may

Fig. 16 The associated charged multiplicity with a quasi-elastically scattered proton as a function of the missing mass: a linear rise merging into a logarithmic behaviour

expect the corresponding multiplicity to increase linearly with the excitation mass. A rather reliable picture is to assume that the excited hadron goes through a step-by-step decay with 0.4 to 0.5 GeV released for each pion emission. If bremsstrahlung-type pions start playing an important role, as expected for large missing masses, a multiperipheral chain may, however, gradually become a better picture. The multiplicity should then eventually increase logarithmically only with the observed missing mass. Figure 16b shows the associated multiplicity observed in the NAL-UCLA experiment at 300 GeV. It indicates a sharp (practically linear) rise followed by a much slower increase. Finally, Fig. 16c combines NAL and ISR results. As expected from a diffractive effect the multiplicity distribution should not depend on energy as long as the quasi-elastic events are properly isolated. However, if one may always define a missing mass to a proton its association with a single diffraction object is practically possible for large enough values of x only and hence for low enough values of M^2 . At 300 GeV, $x = 0.95$ (Fig. 10) corresponds to $M^2 = s(1 - x) = 30 \text{ GeV}^2$. It is therefore still difficult to follow the variation of the associated multiplicity beyond the initial sharp rise⁶⁶⁾. This is a question which should be studied in more detail at the ISR⁶⁷⁾. As well known¹⁾ an isotropic decay corresponds to secondaries scattered over two units of rapidity around a mean rapidity $\bar{Y} = \log \sqrt{s}/M$. A gradual spread over a larger rapidity interval, if seen as the missing mass increases, should be analysed in detail in the new round of ISR experiments⁶⁷⁾. Preliminary data do indicate that this is actually the case⁶⁸⁾. The M^{-2} tail, clearly seen in Fig. 11, is indeed incompatible with a linear rise at large masses. To the extent that the diffractive multiplicity should comply with the over-all logarithmic increase, a $\log M$ behaviour of the associated multiplicity, and hence a gradual spread at constant height of its rapidity distribution, should eventually prevail. Together with the gradual spread rapidity-wise of the associated multiplicity, one should also observe a leading-particle effect within the nova particles setting up. The proton is expected to take a relatively large fraction of the over-all momentum on the average. At identical rapidities a proton and a pion have typical momentum ratios of the order of 3. Beyond such a kinematical effect the proton should also have a larger rapidity than many of the pions if they are to be considered as "radiated" particles, likely to be found as well at small centre-of-mass rapidity. The rapidity distribution of the nova particles should both extend and become skewed. The study of such an effect could be performed with a double-arm spectrometer used together with a detector system covering a wide angular range. Part of the leading-particle effect may correspond to an actual quasi-elastic scattering of the nova proton, all pions being then separated rapidity-wise from both protons. To the extent that the leading proton effect, as clear in Fig. 10 for instance, involves obviously the quasi-elastic peak on top of the flat distribution at lower, but still high, x values, a similar separation in the other proton distribution should be looked for. This is connected to a possible double Pomeron contribution, with vacuum excitation. This is a very interesting phenomenon in itself, although its cross-section could be at most of the order of 0.5 mb. This follows from simple factorization arguments.

The associated multiplicity, as previously discussed, is but the integrated value of the rapidity distribution about which information is also becoming available⁶⁸⁾. The first point is the rapidity gap to be found between the quasi-elastically scattered proton and the other secondaries. Figure 17 gives the angular distribution of the associated secondaries

Fig. 17 Particle distribution with a quasi-elastically scattered proton

to a fast proton, as recently measured by the CHLM Collaboration. This improves much quantitatively upon results already obtained a year ago¹⁾. The key feature stays the same. As the proton momentum approaches its maximum value, but still excluding elastic scattering, one sees that the secondary yield at wide angles drops dramatically. This is to some extent a simple kinematical effect. A pion pair with zero centre-of-mass momentum and 0.6 GeV invariant mass say would, if associated with a proton of energy $x'\sqrt{s/2}$ give a missing mass $M^2 = x' 0.6 \sqrt{s}$. This in turn implies a value of $x = 1 - (0.6x'/\sqrt{s})$ for the leading proton. Hence, even at ISR energies, a value of x 1 to 2% below its maximum value, practically imposes the absence of slow centre-of-mass pions. What is, however, important from a dynamical point of view is the actual probability of such a configuration, or the importance of "showcase" single-diffractive excitation. The angular distributions of Fig. 17 can be transformed into associated rapidity distribution, using the η variable instead¹⁾. The main difficulty stems from the fact that the same transformation should not apply to pions and protons alike when skewness in the rapidity distribution is, as previously discussed, an important point to investigate. In any case, the distribution is clearly displaced towards the hemisphere opposite to that defined by the triggering proton. It is the more displaced the nearer x is to its maximum value. This is shown in Fig. 18, which gives another aspect of the results from the CHLM Collaboration -- already displayed in the preceding figure. The study of low-mass excitations is hampered by the limited momentum (and missing-mass) resolution available at the ISR. One should, however, already analyse what are the dominant trends associated with an increase in missing mass.

To that end, one may consider two extreme cases, which correspond to the graphs of Fig. 19, respectively. In the first case one considers the excitation of a hadronic state which results in a single cluster of secondaries (isotropic decay). The rapidity spread of the final particles is limited ($\Delta Y = 2$). The distribution is centred around $\bar{Y} = \log \sqrt{s/M}$. Its multiplicity (height) increases linearly with M . In the second case the Pomeranchon-proton collision gives secondaries which are uniformly distributed in rapidity and extend over a $\log M^2$ interval. The mean rapidity is again $\bar{Y} = \log \sqrt{s/M}$, but the distribution now extends over an interval $\log M$ on both sides. The associated multiplicity increases as $\log M^2$ (the height remains constant as a function of M). In terms of the triple-Regge approach of Section 3.2, the first case corresponds naturally to the PPR terms, while the second one is to be associated with the PPP term. This, of course, corresponds to an oversimplification. In both cases the rapidity distribution could be more complicated. A look at Fig. 18, as well as at Fig. 11, shows that physics benefits from both but that eventually, as M increases, the second case (or the PPP contribution) prevails, as it should if present at all. This is also illustrated by Fig. 20, which collects again results from the CHLM Collaboration. It gives the mean value of the associated rapidity distribution, which should most simply be $\log \sqrt{s/M}$, together with the location of its inner edge which one may expect to change as $\log \sqrt{s} - 2 \log M$ instead. The data is in agreement with such expectations. The leading proton effect notwithstanding, the distribution should accordingly extend up to $\log s$ on the outer edge, hence up to a value independent of M . As a result all high excitation masses contribute the same amount of fast centre-of-mass secondaries. With an M^{-2} distribution, the inclusive spectrum should not scale but rather include a $\log s$ component. The diffractive cross-section increases as $\log s$. The diffractive multiplicity

$$\langle n_D \rangle = \frac{\lambda \int \log M^2 d(\log M^2)}{\int d(\log M^2)}, \text{ asymptotically}$$

Inclusive distributions of the associated multiplicity to a quasi-elastically scattered proton

The distributions are normalized to the proton yield. They depend on M^2 but not on S as expected for a diffractive process.

Fig. 18 Rapidity distribution as measured by CHLM.
x distribution as measured at NAL (figure added in proof)

Fig. 19 Two extreme cases

increases as $(\lambda/2) \log s$ also, where λ stands for the height of the (universal) central plateau. The integrated yield, summed over all excitation masses, has indeed many features in common with what is expected from non-diffractive production. The distribution of Fig. 19 corresponding to the M^2 tail extends from $\log s$ down to $2(\log \sqrt{s} - \log M)$, as we now consider the incident proton, which eventually gives the quasi-elastically scattered one, as defining rapidity zero. The minimum excitation mass contributing at rapidity y is hence $M_m = \sqrt{s} e^{-y/2}$. If we now assume that the M^2 behaviour holds up to a fixed fraction of the centre-of-mass energy $M_m = \gamma \sqrt{s}$ (γ not too different from 1). The expected diffractive yield is accordingly

$$\begin{aligned}
 \rho(y) &= \lambda \int_{\sqrt{s} e^{-y/2}}^{\gamma \sqrt{s}} \frac{d\sigma}{dM} dM \\
 &= \sigma_0 \lambda \left(\log \gamma + \frac{y}{2} \right), \tag{3.19}
 \end{aligned}$$

since

$$\frac{d\sigma}{dM} = \frac{\sigma_0}{M} \quad \text{at large } M.$$

This corresponds to a linear rise from 0 to $\sigma_0(\lambda/2) \log s$ as y increases from $2 \log 1/\gamma$ to $\log s$. One should, however, add the contribution from the excitation of the other proton,

Fig. 20 CHLM Collaboration. The inclusive distribution of the associated secondaries spreads with increasing mass. The measured values coincide with the kinematical ones. Worth checking for are a leading proton effect and the analogy with photoproduction (asymmetric π^- distribution say)

the decrease of which compensates for the corresponding increase since it is a linear function of y . It follows that the central diffractive distribution is flat. The corresponding yield increases logarithmically with s ⁶⁹⁾. One should then not neglect a diffractive component even in the central region at ISR energy. Disentangling what is diffractive from what is not in an inclusive distribution is indeed difficult except in a well-defined region of phase space⁷⁰⁾. There is a logarithmic rising component in the central "plateau" yield as well as in the total cross-section⁷¹⁾.

With a simple M^{-2} distribution one may trivially calculate any integrated correlation coefficient. A p particle distribution is proportional to

$$\langle n^p \rangle \propto (2\lambda)^p \int (\log M)^p d(\log M) = \frac{\lambda^p}{2(p+1)} (\log s)^{p+1}. \quad (3.20)$$

f_2 will accordingly increase as $(\log s)^2$. One may then point out long-range effects. They are, however, somewhat spurious and correspond to our summing up over different masses. Observing a secondary at a particular rapidity is an obvious bias in favour of some specific excitation mass and hence implies information about the over-all secondary distributions¹⁾ (long-range effects). It is as if we would measure an integrated correlation coefficient mixing together data at different energies! We would certainly notice strong "long-range" correlations. Such hardly interesting effects should, however, not undermine the interest of correlation analysis among the nova secondaries. It should, however, be done at well-defined excitation masses. It is only in that case that one may directly probe for clustering effects among such particles, which have so far been neglected when analysing the second case of Fig. 19. They, however, have no reason to be absent. One could thus distinguish an excited hadronic state (proton cluster) from less strongly correlated pions which could nevertheless show short-range correlations among themselves. Rapidity analyses of PS (AGS) energy reactions already show important clustering effects⁷²⁾. The over-all kinematical conditions for high missing masses observed at ISR energies are similar. Whether or not the same type of clustering effects prevail in hadron-hadron and Pomeron-hadron reactions at the same energy is a question which deserves some detailed analysis⁶⁸⁾.

3.5 The Deck effect at the ISR

It is well known that the diffractive production of low-mass resonance states (A_1 , Q , ...) may be a misleading interpretation from what merely corresponds to the diffractive scattering of an off-shell pion. In such a case the dominant amplitude would be represented by the following graph:

As abundantly debated in the past, the contribution from such a process gives $B\pi$ final states with many common features to those expected for diffractively-produced resonances. The particle B usually stands for either a ρ or a K^* . One may, however, also consider either higher mass mesons for B or diffractive excitation of the pion, at the lower vertex, in order to extend the corresponding mechanism to more complicated final states. Inclusive B production shows typical features associated with pion exchange. In particular, the cross-section can be calculated from the total πp cross-section. Exclusive $B\pi$ production shows typical features associated with diffraction excitation. Both processes are hence expected to keep appreciable cross-sections, in an inclusive and exclusive sense, respectively, at ISR energies. In such a case, however, B should stand for a Δ and the $B\pi$ system may be a $\Delta^{++}\pi^-$ state. Evidence for inclusive Δ^{++} production and diffractive $\Delta^{++}\pi^-$ production has now been reported⁷³⁾. Their analysis should offer new and interesting tests for computation methods developed in connection with the Deck effect. Comparison between different channels, $\Delta^{++}\pi^-$ and ΛK^+ in this case, should help disentangle the overdebated Deck-resonance question which often resulted in a deadlock at lower energies.

4. LARGE TRANSVERSE MOMENTUM PHENOMENA

4.1 Inclusive yields at large transverse momentum

It is well known that most of the secondary particles produced in high-energy collisions are pions with small transverse momentum. The populated region of phase space is practically linear. In the central region the transverse momentum distribution is well reproduced by an exponential, viz.

$$\frac{d\sigma}{dp_T^2} = A e^{-B p_T} \quad \left(\begin{array}{l} 0.1 < p_T < 1 \text{ GeV}/c \\ x \approx 0 \end{array} \right) \quad (4.1)$$

with $B = 6 \text{ GeV}^{-1}$ and, accordingly, $\langle p_T \rangle = 0.33 \text{ GeV}/c$. This refers more specifically to secondaries produced at wide angle in the centre-of-mass system ($x \approx 0$). However, it does not change dramatically with x . At PS energy it is hard to probe such a distribution very much further. The first ISR distributions, at $p_T < 1 \text{ GeV}/c$, were indeed compatible with the approach to a scaling limit with no dramatic change in the p_T distribution⁷²⁾. Nevertheless, one may expect that, if only for the presence of electromagnetic interactions, such an exponential behaviour should not continue to very large values of p_T . As already mentioned in Section 1, by $p_T = 5 \text{ GeV}/c$ or so, electromagnetic production, with a much more gentle fall-off in p_T should become competitive with an exponentially-damped hadronic production^{74,15)}. Furthermore, the scaling property of the electromagnetic strength function, together with the fact that hadron-hadron scattering at fixed (wide) angle appears eventually to fall with energy as an inverse power rather than an exponential^{75,76)}, leads one to expect that strong interactions also could produce secondaries with kinematical distributions similar to those expected for electromagnetic production but then with a much higher yield¹⁴⁾. It could thus be expected that large p_T secondaries should not be so infrequent at ISR energies. As the casual extrapolation of Eq. (4.1) would lead one to conclude, the exponential behaviour found at low p_T was then not expected to hold beyond $2 \text{ GeV}/c$ or so¹⁵⁾. Such predictions got strong support from the parton model description of deep inelastic

electron scattering⁷⁷⁾. Parton-parton collisions could be advocated in order to provide similar effects in hadronic interactions⁷⁸⁾. The proton could thus partly behave as an assembly of point-like constituents contributing incoherent scattering at wide angles. A parton-parton collision with simple gluon exchange has a differential cross-section behaving as $1/t^2$ at large momentum transfer. Questions having to do with the parton structure of the incoming particle and the parton recombination into the final particles notwithstanding, the large p_T distribution of secondaries, which is to be traced to such a hard parton-parton collision, should behave accordingly as p_T^{-4} ¹⁾.

More generally speaking, and as already mentioned in Section 1, the observation of a structure in the p_T distribution is very interesting to the extent that it reveals structure in the matter distribution within the colliding protons. This corresponds to a Fourier image in a direction which is not altered by the Lorentz contraction⁷⁹⁾. The exponential fall-off (4.1) is therefore most generally associated with the "soft" structure discussed in detail in Section 2. A definitely wider distribution at large p_T would, on the other hand, stand for some finer structure within the proton.

Experimental evidence for such an effect came at the time of the Batavia Conference^{12, 55)}. Figure 21, which puts together some of the data obtained by the Saclay-Strasbourg Collaboration, illustrates three important points.

A witness to the power and limitations of present-day wide-angle spectrometers, it combines the inclusive positive yield up to $p_T = 3$ GeV/c with the inclusive pion yield at larger p_T . These data correspond to $\sqrt{s} = 53$ GeV. Also shown is the energy dependence of the integrated pion yield at large p_T ($3 < p_T < 5$ GeV/c).

- i) One notices first that the observed rates at large p_T are considerably larger than what one could expect from a naïve extrapolation of the exponential behaviour followed by the low p_T data.
- ii) One also notices that, as one reaches large p_T values, the pions lose the overwhelming majority which they have at low p_T and, therefore, globally. At the same time, other data show that the positive yield, which is within errors compatible with the negative one at low p_T , becomes definitely larger than one (1.4 say) when one reaches large p_T ($p_T = 3$ GeV/c say).
- iii) One finally notices a strong energy dependence of the integrated pion yield at large p_T in contradistinction with its almost energy-independent character at low p_T . It rises by an order of magnitude over the ISR energy range, when the rise met for low p_T secondaries is at the 10% level at most³⁾.

This certainly calls for structure within the matter distribution discussed in a very global sense in Section 2 [property (i)]. It indicates that such inner structures may collect the quantum numbers of the colliding particles [positive excess, relatively important baryon yield -- property (ii)]⁸⁰⁾. The energy dependence could be but a belated threshold effect⁸¹⁾. Production of a 3 GeV/c p_T pion is kinematically harder than that of a $p\bar{p}$ pair. As discussed later, and imposed in a parton model¹⁵⁾, a quite relevant variable could be $x_T = 2p_T/\sqrt{s}$. As a result, a well-defined dependence on x_T implies an energy dependence at fixed p_T . Since the corresponding distribution should be damped in a particular way at large x_T , this induces a rise with increasing energy at fixed (and large) p_T .

Fig. 21 Inclusive distributions at large p_T

The first and third features previously stressed are also very well illustrated by Fig. 22, which shows the large p_T distribution for π^0 observed by the Aachen-CERN-Munich Collaboration^{82, 83}). One also notices the departure from the exponential behaviour and the prominent increase with energy of the measured yields at large p_T , and the more so the larger is p_T . The explored p_T range is now large enough that an inverse power can definitely be preferred over an exponential if one tries to cover the observed distribution from 1 GeV/c onwards. A both simple and satisfactory fit is provided by p_T^{-8} . This does not, of course, exclude other choices.

The same features have been observed in inclusive distributions measured at NAL for π^0 ⁸⁴), but more extensively now for charged particles⁸⁵). In the latter case the p_T range explored at 200-300 and 400 GeV⁸⁶) extends from 1 to 7 GeV/c.

We now come back to the second point stressed in connection with Fig. 21. At present, results from ISR and NAL can be jointly examined as typical of the observed effects. Figure 23 shows the relative amounts of "stable" charged particles as functions of p_T . The data are from the British-Scandinavian Collaboration^{12, 87}). One sees that, as p_T increases, the relative amount of pions quickly drops from its overwhelming value. The relative amount of heavy particles rises accordingly. It was indeed known that fits of the (4.1) type to the K and $p(\bar{p})$ distribution give lower values for B³). The remarkable point is, however, that beyond 1 GeV/c, the relative rates stabilize and this over a rather wide p_T range. The pions barely keep a majority when they contribute 85% of the total yield at low p_T . Also shown, in a complementary way in Fig. 23, are the π^+/π^- , $p/\bar{p}$ and p/π^+ ratios, which are plotted over the same p_T range. The K^+/K^- ratio is very close to the π^+/π^- one. This altogether corresponds to an excess of positives over negatives. It is, however, not only due to the importance of the proton yield while the antiproton one remains a factor two lower. The π^+/π^- ratio is definitely larger than what it is at low p_T . The stability of these ratios indeed qualitatively defines what can be referred to as a large p_T regime. It hints at the fact that production of large p_T secondaries actually reveals mechanisms which are different from those usually required for all global aspects of production processes. Results from NAL confirm the p_T stability of the particle ratios measured at the ISR^{85, 86}). This is shown in Fig. 24, which combines ratios observed at 300 GeV/c at NAL with data from the British Scandinavian Collaboration at ISR, at the same centre-of-mass energy. The $\bar{p}$ behaviour at large p_T notwithstanding, the definitely small variation of the relative yields over a rather wide p_T range is again impressive. Results shown in Fig. 24 are both very interesting and a plea for more. The BS data go up to 3 GeV/c at $\sqrt{s} = 53$ GeV only. When it could overlap in energy with the Chicago-Princeton data at NAL it does not overlap in transverse momentum. It is very important to obtain these ratios up to 3 GeV/c at least at different ISR energies. This is the more important since the p/π^+ ratio, at $p_T = 3$ GeV/c say, appears to fall with energy. One should rather say that the proton yield does not rise as fast as the pion one when they both do [property (iii)]. The ratio drops by a factor 3 when the centre-of-mass energy doubles (Figs. 24 and 25). There are already significant drops of the p/π^+ ratio between 200 and 300, and 300 and 400 GeV/c, respectively^{85, 86}). Whether this ratio appears to go to a limit or not with increasing energy should be clear with further ISR results. In a parton picture it should.

Fig. 22 Inclusive π^0 distributions at large p_T

Fig. 23 Particle ratios

Fig. 24 Combination of ISR and NAL results. No qualitative difference but normalization problems (see Fig. 25 for present situation)

Fig. 25 Particle ratios

It should be stressed that large p_T protons observed at wide angles could have different dynamical origins with different pertinent energy behaviours. They may be associated with hard scattering effects. They could also be associated with very large multiplicity reactions giving many relatively slow particles among which a proton could statistically pick up a large transverse momentum. Such a mechanism should give a decreasing contribution with increasing energy.

This brings up two remarks:

- i) If very important for their bearing on the proton structure, large p_T phenomena are very shy cross-section-wise. The inclusive pion yield has dropped by several orders of magnitude from its value at $p_T = \langle p_T \rangle$, when the new regime sets in. As result many an *a priori* unlikely configuration or process may be called upon to interpret the corresponding effects.
- ii) It is a pity that practically all available large p_T data are at wide angles ($x \approx 0$). For the inclusive yields the actual value of p_T could be the relevant variable even though some y dependence could be of course of some importance. Results from the British-Scandinavian Collaboration indeed show that inclusive rates at 90° and 60° are practically identical⁸⁷). This, however, corresponds to $x \approx 0$ (less than one unit of rapidity). It would be very interesting to have p_T distributions at smaller production angles. This is even more interesting as soon as one discusses correlations.

For straightforward luminosity reasons the p_T distribution is always measured far away only from the kinematical boundary in ISR experiments ($x_T < 0.3$), while NAL experiments can go much closer to it. If there is an overlap in p_T , there is still practically no overlap in x_T ! NAL experiments, once performed on hydrogen, could give very interesting information about the large x_T behaviour, while doubts may always remain when heavy nuclei are used. Improvement in luminosity and solid-angle detection at ISR should allow some x_T overlap with NAL.

At present, there are still normalization discrepancies among different experiments. They should be resolved with time. One should rather focus on the key points, where results obtained in quite different ways agree. They are once more summarized in Table 3. This clearly distinguishes large transverse momentum phenomena as qualitatively different from the dominant ones.

Table 3

Feature at $x \approx 0$	Low p_T $p_T \approx 0.3 \text{ GeV}/c$	Large p_T $p_T \approx 3 \text{ GeV}/c$
$\frac{d\sigma}{dp_T^2}$	e^{-6p_T}	$p_T^{-8} (?)$
Scaling over the ISR energy range	Good within 10%	Wrong (A rise by an order of magnitude)
Positive/negative ratio	Compatible with 1	Definitely larger than 1 (1.4)
Heavy particle/pion ratio	Small (15%)	Of the order of 1

4.2 On the dynamics of large p_T phenomena⁸⁸⁾

There are two extreme attitudes. One is to stress the low value of the relevant cross-section and to consider the production of a secondary particle with large transverse momentum as a very unlikely but yet "normal" event. It is a statistical fluctuation within the framework of a dynamics, the typical aspects of which are displayed by the overwhelmingly dominant low p_T production. Unlikely very heavy clusters could yield unlikely low multiplicities and hence secondaries with large transverse momentum^{5, 89)}.

To the extent that production models insist on short-range correlations rapidity-wise, the transverse momentum is then most generally expected to be balanced rather locally in rapidity (within two units say). A large p_T secondary, observed at forward angles ($\theta \approx 45^\circ$ say), would imply at least one other large (but opposite) p_T secondary at forward angles also. At the same time the local (rapidity-wise) associated multiplicity is most naturally expected to be lower than usual⁹⁰⁾.

It is perhaps more interesting, however, to try to analyse such reactions as another facet of the hard structure effect evidenced through deep inelastic electron scattering. The p_T (inverse power) dependence, together with the observed important charge ratios, hints at drawing a parallel between the two effects and at analysing such kinematical configurations as due in the first place to incoherent scattering among hard constituents within the proton. This is actually how such an effect was expected. One may even assume that scaling properties hold the same way. The inclusive distribution may then be written as

$$E \frac{d\sigma}{d^3p} = \frac{1}{s^2} F\left(\frac{t}{s}, \frac{u}{s}\right), \quad (4.2)$$

where s , t and u are the Mandelstam invariants relevant to the inclusive process: $pp \rightarrow A + x$, A being the observed large p_T secondary¹⁾. The s^{-2} term, in front of the right-hand side term, exhausts the dimensions. The remainder has to be a dimensionless quantity which, within the scaling assumptions made, can depend on the two ratios only. It is straightforward algebra to rewrite Eq. (4.2) as^{1, 15)}

$$E \frac{d\sigma}{d^3p} = \frac{1}{p_T^4} G\left(\frac{2p_T}{\sqrt{s}}, y\right). \quad (4.3)$$

Relations (4.2) and (4.3) are expected to hold in the large s and large p_T limit.

Another convenient, but less general, way to introduce Eq. (4.3) comes directly from the parton picture⁷⁷⁾. The core of the reaction process is then seen as a hard parton of momentum $y\sqrt{s}/2$ scattering at angle θ on a hard parton coming in the opposite direction. The scattered parton eventually yields hard secondary particles all in its global direction of motion and the momentum of which are a fixed fraction of the parton momentum. Scattering at angle θ depends on gluon exchange between the partons and gives eventually an over-all p_T^{-4} factor. The remainder is a dimensionless quantity which can be written as a function of $2p_T/\sqrt{s}$ and θ . At 90° we then expect

$$\left. \frac{d\sigma}{dp_T^2 dy} \right|_{y=0} = \frac{\pi}{p_T^4} G\left(\frac{2p_T}{\sqrt{s}}, 0\right). \quad (4.4)$$

Using factorization arguments⁹¹⁾ one may relate the behaviour of G as $2p_T/\sqrt{s} \rightarrow 0$ to that of the deep inelastic scaling function when $\omega = q^2/2mv \rightarrow 0$. G is then expected to approach a constant. The inclusive distribution eventually scales in the Feynman way, but the more slowly though the larger p_T is¹⁵⁾.

In this picture the scattered parton "flares" into a jet of secondary hadrons. Some may be hard and the observed particle will be among them. Some may be rather soft and they will not distinguish themselves much from other secondary particles. Since there are more positive than negative hard partons within a proton, there should be a positive excess among hard secondaries. In connection with this it would be extremely interesting to have NAL (SPS) data on large p_T secondaries originating from $\pi^\pm p$ collisions as well as pp (pd) collisions. Factorization of charge effects among hard hadrons should be checked.

It could, however, be that, if one requires one particle with large p_T , it corresponds to a very strong bias in favour of a hard parton having given one single hard hadron only. The initial production of a much larger p_T parton, resulting eventually in several large p_T secondaries, may thus become less probable. The factor against a hard scattered parton sticking with another one in order to give a single meson is the same as the one involved in the corresponding calculation of the form factor⁹²⁾. With an asymptotic $t^{-1}(p_T^{-2})$ dependence for a meson form factor, we are led to expect that the inclusive yield for a single hard meson at large p_T is

$$E \frac{d\sigma}{d^3p} = \frac{M^4}{p_T^8} H\left(\frac{2p_T}{\sqrt{s}}, 0\right) \quad \theta = 90^\circ. \quad (4.5)$$

It could be that a lead-glass trigger on a large p_T π^0 may be enough to get Eq. (4.5), as opposed to Eq. (4.3) as the suitable dependence. One would then expect no other hard particle alongside the π^0 direction. A departure from the p_T^{-4} behaviour calls for a specific mass parameter. The one relevant here is associated with the pion form factor, not with any of the external masses.

At present, it is remarkable that the inclusive π^0 distribution of the CCR Collaboration⁸³⁾ is indeed compatible with such a behaviour. Figure 26 gives the distribution of $p_T^N E(d\sigma/d^3p)$ [with $N = 8.26$ as a best fit to the data] as a function of $x_T = 2p_T/\sqrt{s}$ at $y \approx 0$. All data points are compatible with an asymptotic limit with the scaling properties implied by Eq. (4.5). One may, however, not yet conclude that the data practically impose such a behaviour. They, however, follow it in a remarkable way. If this is considered as supporting evidence for hard (parton) scattering, one should expect three things

- i) The typical event for which one large p_T π^0 comes as a trigger should have no associated hard secondaries on the same side. It may, however, show associated soft particles ($p_T \sim 0.3$ GeV/c). On the other side, it should *a priori* show an uncontrived jet of particles with a few hard particles as well as several soft ones, the more particles the larger is the value of the required transverse momentum.
- ii) Eventually one should retrieve Eq. (4.3) either going to larger p_T (and energies) or using a different trigger and in particular the energy received in a particular solid angle irrespectively of the number of particles that this may correspond to.

Fig. 26 Scaling

- iii) If one rotates the centre-of-mass direction of the triggering particle, the over-all distribution should rotate in a similar way with, of course, some smearing associated with the fact that the centre-of-mass of the parton-parton collision is not the over-all centre of mass. This global rotation should be clear for the hard associated secondaries. It should, however, leave the soft ones around rapidity zero, where they belong in any case. To the extent that short-range correlations extend over two units of rapidity, the optimal rotation of the triggering particle should be 90° to 45° for that specific effect.

Before we turn to the analysis of present data on correlations, which at least support point (i), with no test yet of points (ii) and (iii), it should be stressed that it would be very useful to have a more extended test of formula (4.5). Indeed the charged pion data of the Chicago-Princeton experiment would also support such a behaviour, but with a definitely different value of N ($N = 11$)^{85, 86}. What is even more puzzling is that they would then suggest a vanishing limit for the corresponding scaling function, as s increases. They would favour the factorization of an inverse power of s to a scaling function with finite limiting value as $s \rightarrow \infty$. It should be stressed however that, if the ISR and NAL results overlap in p_T , they do not overlap in x_T . The matter is obviously worth more extensive research. At present, one may conclude that conclusions drawn from a parton picture do not run into conflict with data, while providing straightforward qualitative explanations for many observed effects. The data, however, are far from following the most simple asymptotic expressions which the approach implies. The parton picture nevertheless remains a very interesting scheme for the further analysis of the corresponding processes.

4.3 Correlations involving a large p_T particle

The charged multiplicity associated with a large transverse momentum π^0 has been measured by the CCR Collaboration using chambers covering limited solid angles around the direction of the observed π^0 (90°) and opposite to it, respectively⁹³). This is shown in Fig. 27, where the data are normalized to the average charged multiplicity seen at wide angle. On the same side, the associated multiplicity is practically independent of p_T and then similar to that corresponding to the well-known short-range correlations at wide angles¹). It is definitely larger than what it is on an average. On the opposite side it is even stronger and increases (practically linearly) with p_T . Observing a large p_T secondary is therefore a bias in favour of a large associated multiplicity at wide angles. More detailed results have now been obtained by the Pisa-Stony Brook Collaboration^{12, 94}). Positive correlations on the same side (hemisphere) are found to be rather important, practically p_T independent and localized over two units of rapidity in much the same way as those found when triggering on a slow pion at wide angle. Correlations on the opposite side (hemisphere) are also positive, rather localized in rapidity, but now increase sharply with p_T . One further found them to be indeed relatively well localized azimuthally, as shown in Fig. 28. The distribution is suggestive of a jet the more pronounced the larger p_T is. The correlation shows an important coplanarity effect (as imposed in a parton picture), but one has to conclude that the jet is rather wide. If it *a priori* includes large p_T particles produced near to 180° , it also includes many soft ones with a rather isotropic distribution. With a 4 GeV/c trigger such correlations imply an excess of 4 or so extra charged particles from what is expected on the average in the opposite hemisphere. Altogether one may estimate

Fig. 27 Associated multiplicity with a large $p_T \pi^0$

Fig. 28

that each associated particle takes on the average 0.8 GeV/c of p_T . This is more than what is expected from the soft ones, which appear as composing the majority of the associated particles when their relative number is estimated from Fig. 28. It therefore follows that some of them (one or two or maybe three) should pick up a larger fraction of the balancing p_T and hence define the jet structure. This is further supported by very strong positive correlations among large p_T π^0 seen in opposite directions reported by the CCR Collaboration⁸³⁾. A key point though is that these correlations, important as they are⁹⁵⁾, are small as compared to what is expected from a two π^0 in a configuration where they would be balancing out their large p_T among themselves.

One thus gets an "identity kit picture" of a large p_T reaction triggered on a π^0 , as shown in Fig. 29a. Within two units of rapidity around $y = 0$ (corresponding to the observed π^0) one should find on the same side (hemisphere) 4 to 5 soft pions (3 or so charged pions) and this independently of p_T , while on the other side (hemisphere) one would have around 8 particles, 6 or so being soft pions while 2 say would be hard (with $p_T \gg 0.4$ GeV/c) with a p_T π^0 of 4 GeV/c. As p_T increases, the number of soft pions would increase, practically linearly at first, while the probability of finding hard ones would also go up. Outside this short-range interval the associated multiplicity drops from its average value^{83, 94)}, an effect which is probably energy dependent⁹⁶⁾. Correlations involving large p_T secondaries should be energy dependent and slowly rise with $\sqrt{s}$. Indeed the inclusive distribution is rising. The jet picture is not as clear as one may have wished it to be following the preceding discussion. On the other hand, the data do not contradict it. This tentative picture may be useful at probing further the analysis of such reactions.

Rotating the direction of the π^0 used as a trigger, and following the parton picture, one should expect the configuration to change into that represented as in Fig. 29b. The associated soft secondaries stay the same, while the harder few move to the other side. To the extent that correlations affect mainly two units of rapidity a shift by one unit (45° instead of 90° say) would probably be best at checking whether such an effect occurs. A larger one may leave too big a role to mere energy-momentum conservation. In any case it is extremely important to know how p_T is balanced, whether it is balanced locally in rapidity or in a way such that the full momentum is balanced among the two jets (Fig. 29b). A trigger at 90° may provide an answer using unequal energies. With 11 against 31, the extreme case, one reaches however only a shift of 0.6 unit in rapidity.

Next to this question, associated distributions with different types of triggering particles should be very interesting. Even though it is not accessible at present, the study of correlations between particles and antiparticles would be very efficient at distinguishing between models. For instance an analysis of K^+K^- correlations would tell whether the K^+K^- is well localized within one jet (a $\lambda\bar{\lambda}$ extra line hooked on the parton decay chain) or whether it involves a hard K and a soft $\bar{K}$ (λ quark picked up from a $\lambda\bar{\lambda}$ pair among the very small partons) or else again if the K^+ and the K^- belong to the two opposing jets, in contradistinction to what the hard parton picture suggests.

The question of large p_T secondaries is only 1½ years old. It is an interesting new field of research which deserves much attention. Indeed a large p_T particle has become one of the most popular triggers for experimentation at very high energy, not only at ISR but also at NAL (SPS). This is part of an obvious conclusion for these lectures. If research

at the ISR has already met with very important discoveries, it also has made hadron research at NAL and at the SPS the more exciting.

Fig. 29 Idealized picture with rotating trigger direction. Going from configuration (a) to configuration (b) one gets a skewed distribution with an important yield at 90°.

Note added in proof

These notes were written as this series of four lectures was given in March 1974. The data as discussed in the text correspond to what was available then. Some very interesting and highly relevant pieces of data have actually become available since. Extra figures have then been "added in proof" whenever possible in order to bring this review up to date as of June 1974. I would like to thank R. Stroynowski for his comments.

REFERENCES AND FOOTNOTES

- 1) M. Jacob, Proc. 1973 CERN-JINR School of Physics, Ebeltoft, Denmark, CERN 73-12 (1973), p. 1-138. This paper was developed from a previous paper, Physics at the ISR, a review of recent results, preprint CERN TH. 1639 (1973), to be published in Fields and Quanta.

- 2) The past twelve months have mainly brought confirmation of what could be tentatively advanced from the first results, together with, of course, a much more detailed view of what is happening in very high energy reactions. Confirmation came from NAL as well as from ISR. These past twelve months have also been used for setting up many sophisticated experiments with correlation study on all intersections used for physics. This should soon provide a whole set of new data.

A rapid guided tour around the double ring now includes the following experiments. In I1, correlations at wide angle, at low and large transverse momentum using the Saclay double-arm spectrometer in coincidence with the CCR lead-glass detector. In I2, correlations involving fast secondaries using the CHLM small-angle spectrometer in coincidence with a counter array soon to be improved upon by the British barrel detector. In I4, the split field magnet facility with four experiments already collecting data. In I6, production of hadronic resonances studied through a large opening spectrometer (Aachen-CERN-Munich). In I7, the Aachen-CERN-Heidelberg-Munich streamer chamber gives angular distributions in correlation or not with a large transverse momentum π^0 seen through a lead-glass detector at wide angle. Finally in I8, the PSB practically 4π detector is used in association with a lead-glass detector at wide angle, the CERN-Rome forward negative spectrometer and the MIT-Scandinavian wide-angle spectrometer.

This rapid list does not include the continuation of the study of inclusive distributions at wide angle (and large transverse momentum) as being completed by the BS Collaboration in I2; the new study of $d\sigma/dt$ at low and large $|t|$ by the ACGHT Collaboration in I6 and the PSB-CR joint measurement of σ_{tot} in I8. It also does not include particle search such as monopoles in I1 and quarks in I4 (both soon to start) and the major set-ups for lepton pair search now approved, which should be at the data-taking stage in two years only.

- 3) For a review of scaling and correlation results, one should consult L. Foà, Rapporteur's talk, 2nd Int. Conf. on Elementary Particles, Aix en Provence, 1973, J. Phys. Suppl. (France) 34, C 1, 137 (1973); G. Giacomelli et al., to be published in Nuclear Phys. B (1974). Deviations from scaling are reported when one probes beyond the 10% level. In view of the fact that present theoretical motivations for scaling also imply constant cross-sections, this is not surprising. Discussing such deviations is at present difficult, since one does not know how to separate what is physics proper from mere experimental questions, theoretical problems notwithstanding. Even if only an approximate property, scaling is nonetheless a prominent one. It is found to hold at the two-particle level as well as at the single-particle level.

- 4) For a recent and detailed discussion of clustering effects, one should consult R. Slansky, to be published in Phys. Reports (1974). Correlation results from track chamber data (NAL) are reviewed by J. Whitmore, to be published in Phys. Reports (1974).

- 5) Clustering effects are also described in Ref. 1 and in L. Van Hove and S. Pokorsky, CERN TH. 1656 (1973), to be published in Acta Phys. Polon; E. Berger, Erice Lecture Notes (1973); A. Bialas, Invited talk Pavia Conference (1973). The study of clustering effects in fixed multiplicity reactions is now progressing. Preliminary results are consistent with the cluster parameters derived from inclusive two-particle correlations (see Ref. 1). One may have to wait for a few months before definite statements can be made.

- 6) This was reviewed already by U. Amaldi, Rapporteur's talk, Aix en Provence Conf. 1973, op. cit., p. 241 and by J.D. Jackson, Introduction to hadronic interactions at high energies, Scottish Universities Summer School (1973). I shall indeed borrow much from this latter paper when discussing the impact parameter analysis.

- 7) The present discussion develops from M. Jacob, The elusive asymptotia, to be published in Comments on Nuclear and Particle Physics (1973), and Invited talk, APS Chicago meeting (1974), NAL-Conf-74/26-THY.

- 8) For a general discussion of diffraction excitation, see F. Zachariasen, Phys. Reports 2 C, 1 (1971); R.M. Edelstein et al., Phys. Rev. D5, 1073 (1972); W. Kittel et al., Nuclear Phys. B30, 333 (1971); The original reference on diffractive excitation is M.L. Good and W.D. Walker, Phys. Rev. 120, 1857 (1960). The importance of single excitation of heavy clusters was stressed by M. Jacob and R. Slansky, Phys. Letters 37 B, 408 (1971) and Phys. Rev. D5, 1347 (1972). This mechanism is now known to correspond to 0.2 to 0.25 of all inelastic events and is the dominant one for low multiplicities. As emphasized in the latter papers, at previous standard machine energy (10-20 GeV say), medium and large multiplicity configurations are also forced as well into single clusters kinematically. This is, however, not the case at NAL and ISR energies, where one meets dominantly several light clusters rather than a single heavy one (see Ref. 5).
- 9) For a review, see J. Sens, Invited talk, New York Academy of Sciences (1973). For original papers see M.G. Albrow et al., Nuclear Phys. B51, 388 (1973); B54, 6 (1973).
- 10) At NAL (SPS) it also corresponds to a very slow proton. Diffractive excitation off nuclei, with slow nucleus coherent recoil, offers a clearer signature still. This will be discussed in A. Melissinos, to be published in Phys. Reports.
- 11) As a matter of fact, one first looked at large p_T yields to see leptons which could have eventually stood out from a pion background, overwhelming at low p_T , but exponentially damped with p_T ; leptons have now been seen at NAL, but with a yield of a factor of 10^4 down from that of pions, and this over the 1 to 4 GeV/c range. At this level they may partly come from the decay of the well-known vector mesons.
- 12) M. Banner et al., Phys. Letters 44 B, 537 (1973); F.W. Büsser et al., Phys. Letters 46 B, 471 (1973); B. Alper et al., Phys. Letters 44 B, 521, 527 (1973); ISR discussion meetings 2 and 7, CERN internal reports. For reviews, see Ref. 1; M. Jacob, Comments, op. cit. (1973); J.D. Bjorken, Proc. Aix en Provence Conf. 1973, op. cit. p. 385; S.D. Ellis and R. Thun, IXth Rencontre de Monriond (1974).
- 13) For a review of deep inelastic scattering, see F. Gilman, Phys. Reports 3 C, 542 (1972).
- 14) M. Jacob and S. Berman, Phys. Rev. Letters 25, 1683 (1970).
- 15) This is discussed in great detail in S. Berman, J.D. Bjorken and J. Kogut, Phys. Rev. D4, 3388 (1971), where large p_T phenomena are analysed in the framework of the parton model. This is still the most fruitful way to look at things.
- 16) For a review, see J. Kogut and D. Susskind, Phys. Reports 8 C, 324 (1973).
- 17) For a discussion of these questions, see U. Amaldi, Proc. Aix en Provence Conf. 1973, op. cit., p. 241 and G. Bellettini, Invited talk, Stony Brook Conf. (1973). I am grateful to U. Amaldi for a discussion of the experimental situation. In Fig. 1 the NAL track chamber points of 1972 are to a large extent made obsolete by the new NAL points of 1974. The original references for the ISR results are U. Amaldi et al., Phys. Letters 44 B, 112 (1973) and S.R. Amendolia et al., Phys. Letters 44 B, 119 (1973).
- 18) One assumes that the spin-flip amplitude is negligible and that the real part and the imaginary part are proportional, at least at low $|t|$. These assumptions are supported by the observed shape of $d\sigma/dt$ at low $|t|$, including the Coulomb peak. They should, however, be kept in mind.
- 19) Corrections have to be included for loss of final-state configurations with all charged particles in the pipe. This involves a sizeable fraction of the elastic-scattering cross-section.
- 20) The ACGHT Collaboration is also making an independent measurement of $d\sigma/dt$ at low $|t|$.
- 21) The difference between the K^-p and K^+p total cross-section varies at least as expected. It decreases as $s^{-1/2}$. This is now very well confirmed at NAL as shown in Fig. 1c (added in proof). A similar behaviour applies for the difference between the $\bar{p}p$ and pp total cross-sections (Fig. 1). The K^+p total cross-section shows a positive second derivative with respect to $\log s$. This rules out an approach of the $A - B(\log s)^{-1}$ type as being phenomenologically useful.

- 22) The notation follows those of M. Jacob and G.F. Chew, Strong interaction physics (Benjamin, NY, 1964). I am grateful to A. White for a discussion of model calculation of σ_{tot} . A more detailed discussion is available in Zachariassen's paper in Ref. 8. The strong coupling Regge calculus behaviour of diffractive scattering is described in H. Abarbanel, J. Bronzan and A. White, to be published in Phys. Reports. See also A.A. Migdal, A.M. Polyakov and K.A. Ter Martirosyan, ITEP preprint 102 (1973).
- 23) This is the mechanism emphasized by H. Cheng and T.T. Wu in their series of papers on that matter starting with Phys. Rev. Letters 24, 1456 (1970).
- 24) This difference was emphasized in J. Finkelstein and M. Jacob, Nuovo Cimento 56, 681 (1968). It became the "sign" question, much debated since. The negative sign of the second term seems now to prevail among theorists. It is what is expected from absorption models, though they mainly deal with Reggeon-Pomeranchon cuts. The fact that the two-Pomeranchon graph should be looked at as a field theory (perturbative) term, as opposed to a contribution to a unitarity equation, is at the centre of the strong-coupling Reggeon calculus. See Ref. 22.
- 25) P.D.B. Collins and E. Squires, Regge poles in particle physics, Springer Tracts in Modern Physics (ed. G. Höhler) (Springer, Berlin, 1968), vol. 45.
P.D.B. Collins, Phys. Reports 1 C, 103 (1973).
- 26) A.A. Migdal et al., Ref. 22.
V.N. Gribov, Soviet Phys. JETP 53, 654 (1967);
H. Abarbanel et al., Ref. 22.
- 27) K.A. Ter Martirosyan et al., Soviet Phys. JETP 11, 45 (1970).
A. Tavkhelidze, Rapporteur's talk, Proc. 15th Int. Conf. on High-Energy Physics, Kiev (1970).
- 28) The pp channel having exotic quantum numbers, one expects the odd and even Regge contributions to cancel themselves out in the imaginary part of the elastic forward amplitude. For a review of Regge models and duality, see M. Jacob, Brandeis lecture notes (1970). As discussed there, however, duality may indeed not be so stringent a constraint in baryon-baryon scattering. Decreasing Regge contribution may still have to be added to (2.8) or (2.9) and explain the very flat behaviour of σ_{pp} over the Serpukhov energy range (see Ref. 27). As emphasized in Ref. 1, the K^+p channel should be much better duality-wise and the corresponding effects may show up earlier than they do in pp scattering. The new NAL measurement (Fig. 2b) confirms this.
- 29) Strong coupling limits of the Reggeon calculus, as described in Ref. 26 are not discussed here. If very interesting theoretically, they seem at present of little help when it comes to mere phenomenological fits.
- 30) For a review, see L. Caneschi and M. Giafaloni, Proc. Aix en Provence Conf. 1973, op cit., p. 268.
- 31) The belated threshold effects, as most generally due to leading particle effects, has been stressed by M. Jacob, R. Slansky and C.C. Wu, Phys. Rev. D6, 2444 (1972), and M. Jacob, Invited talk, Argonne Conf. on Duality (1972), NAL preprint. It has been discussed in the framework of the multiperipheral model by Chung-I-Tan and C. Tow, Brown Univ., preprint (1974). Even though some contributions may give a rising term, it still remains to be explained why the rest stays the same.
- 32) D. Morrison, Experimental review of strong interactions at high energy, Hawai Topical Conf. in Particle Physics (1973).
- 33) L. Van Hove, Rev. Mod. Phys. 36, 655 (1966).
- 34) A. Melissinos, Invited talk, Chicago APS Meeting (1974).
- 35) S.W. MacDowell and A. Martin, Phys. Rev. 135 B, 960 (1964).
- 36) To get quick estimates one may consider that $1 \text{ mb} \sim 2.5 (\text{GeV}/c)^{-2}$. With $\sigma_{\text{tot}}/\sigma_{\text{el}} = R$, one gets $b = 2R (\text{GeV}/c)^{-2}$.

- 37) If duality imposes a cancellation among Regge contributions in the imaginary part of the elastic amplitude, the opposite applies to the real part. More explicitly, terms such as $\beta[1 \pm e^{i\pi\alpha(t)}/\sin \pi\alpha(t)]/s^{\alpha(t)}$, for the even (odd) part respectively, give altogether a vanishing imaginary part, but a real part $2\beta s^{\alpha(t)}/\sin \pi\alpha(t)$ which may become negligible in front of the mainly imaginary diffractive part only if s is large enough. At first the real part and, together with it, polarization effects should disappear as $s^{-\frac{1}{2}}$. The opposite applies to the $\bar{p}p$ amplitude (β changes sign for the odd term). The Regge contribution to the imaginary part becomes large when its contribution to the real part is small [$\alpha(0) \sim \frac{1}{2}$]. The same diffractive pattern (Fig. 4) should eventually be found for pp and $\bar{p}p$. However, the way one reaches it with increasing energy should be very different and easily traced to the relevant Regge contributions. The same applies to the K^+p (K^-p) channels.
- 38) The measurements of $d\sigma/dt$ should be much improved upon. The behaviour at low $|t|$ [$t < 0.1$ (GeV/c)²] corresponds to the outer part of the proton. The shrinking is associated with the increase in size. The presence or absence of oscillations with respect to the exponential behaviour should be ascertained. The behaviour at large $|t|$ corresponds to the inner part. The value of the second maximum, which may depend on energy, should be measured over the ISR energy range. See V. Tsarev, NAL-THY (1974).
- 39) Impact parameter analyses have been made by U. Amaldi, Ref. 6; R. Henzi and P. Valin, McGill preprint (1973); F.S. Henyey et al., Michigan preprint (1973); H. Miettinen and D. Pirula (1974).
- 40) The real part is known to be small at $t = 0$. The diffraction minimum indicates that it is small at $|t| = 1.4$. The small variation of $d\sigma/dt$ indicates that it cannot be very important in between.
- 41) One uses the relation $d_{\lambda_1\lambda_2}^J(\theta) \sim J_{|\lambda_1-\lambda_2|}(J\theta)$.
- 42) The quoted values are not to be considered as the most accurate ones, but rather as illustrative of the actual situation. See Ref. 39 for accurate fits.
- 43) The transmission coefficient is equal to $1 - H(r)$.
- 44) H. Miettinen, IXth Rencontre de Monriond (1974). See also Ref. 39. A large scale phenomenological fit will be possible when data from NAL are available on a whole set of channels. The key features here mentioned should already be seen at 300 GeV.
- 45) To the extent that the variation of $d\sigma/dt$ is small, an assumption of a small real part can be justified. J. Bronzan, private communication.
- 46) R.J. Eden, Phys. Rev. Letters 16, 39 (1966);
T. Kinoshita, Perspective in modern physics (Interscience, New York, 1966).
- 47) N.N. Khuri and T. Kinoshita, Phys. Rev. 137 B, 720 (1965). For a review see A. Martin, preprint CERN TH. 1650, Proc. Roy. Soc. Meeting (1973).
- 48) J. Fischer and C. Bourrely, preprint CERN TH. 1652 (1973);
J. Diddens and W. Bertel, preprint CERN (1973);
J.D. Jackson in Ref. 6. The parameter ρ is expected to be positive over the ISR energy range. It should show an extremely wide maximum, but its eventual decrease as $(\log s)^{-1}$ should not be seen even at 2000 GeV. Whether one reaches 5, 10 or 15% at 2000 GeV will limit models for the asymptotic behaviour of σ_{pp} . The qualitative prediction of a sustained positive value for ρ is worth an experimental check.
- 49) This will be reviewed by G. Giacomelli, to be published in Phys. Reports.
- 50) We assume (as usual) that there is no peculiar behaviour in the neighbourhood of the unitarity cut. This method, advocated by V. Gribov and then by J. Bronzan is described in detail by J. Jackson in Ref. 6.
- 51) This is particularly simple for a Regge amplitude with $\sigma \sim \exp [\alpha(0)-1] \log s$. The real part and the imaginary part of the amplitude are then proportional. In general one may have to consider several derivatives in order to get a good approximation and calculating from a dispersion relation might turn out to be easier.

- 52) An unambiguous trigger requires fast particles on both sides and the absence of slow centre-of-mass particles. Such configurations should eventually scale in the Feynman sense, the absence of slow secondaries notwithstanding. Experimental confirmation should come from experiment 603 and the present research at the Split Field Magnets (CHOV and Princeton-Pavia).
- 53) This is actually what limits the mass up to which the diffractive excitation spectrum can be traced. The limit depends on x . It increases as $\sqrt{s}$.
- 54) The differential cross-section $d\sigma/dM$ should fall faster than M^{-1} if the total cross-section reaches a limit eventually. If it falls as M^{-1} (triple-Pomeron behaviour) one gets a rising $\log s$ contribution to σ_{tot} .
- 55) For a review of these questions see, for instance, M. Jacob, Rapporteur's talk, Proc. 16th Int. Conf. on High-Energy Physics, Chicago-Batavia (1972). One may look at the proton as either a fragment of an excited proton state or as obtained from the initial one through the exchange of a secondary Regge trajectory with the remainder. As discussed in Ref. 8, they are two dual pictures. A detailed triple-Regge analysis of available data has been carried out by R. Roberts and D.P. Roy, Rutherford preprint (1974).
- 56) H. Abarbanel et al., Phys. Rev. Letters 26, 937 (1971);
R.D. Peccei et al., Phys. Rev. Letters 26, 1076 (1971);
P. Chliapnikov et al., Phys. Letters 35 B, 581 (1971);
K.A. Ter Martirosyan et al., Contribution to the 16th Int. Conf. on High-Energy Physics, Chicago-Batavia (1972).
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S. Humble, preprint CERN TH. 1827 (1974);
R.D.O. Morrison, Rapporteur's talk, Proc. 15th Int. Conf. on High-Energy Physics, Kiev (1970).
- 60) For a review of the CHLM, PSB and ACGHT results, see ISR discussion meetings 3, 4, 5 and 8 (1973-1974), CERN internal reports.
- 61) The data are summarized in Fig. 14. The structure at low $|t|$ is important for two reasons. In some theoretical pictures, even the zero helicity-flip contribution should vanish at $t = 0$, at least asymptotically. In others, large missing mass are expected to imply large helicity-flip contributions with, accordingly, a dip at low $|t|$. One may remark that the semi-classical condition for helicity flip $\sqrt{-t} > r^{-1}$, where r is the range of the interaction, is the opposite to that for coherence. This should make a large helicity-flip contribution unlikely if one considers diffraction excitation as a highly coherent process. The optical analogy is, however, not so evident for large excitation masses.
- 62) N. Sakai and J.N.J. White, Nuclear Phys. B59, 511 (1973); GIFT report (1973). This difficulty was already noticed by L. Michejda, Fortschr. Phys. 16, 707 (1968);
L. Michejda, T. Turnau and A. Białas, Nuovo Cimento 56 A, 241 (1968).
- 63) M. Jacob and R. Stroynowski, the shape of the quasi-elastic peak, CERN preprint (1974). The discussion of the impact parameter picture of diffractive excitation given here was carried out in collaboration with R. Stroynowski.
- 64) Such a phase difference corresponds to the $\cos \phi$ factor in (3.18). This is only a casual parametrization. I am grateful to L. Dick for a discussion of polarization experiments in quasi-elastic scattering.
- 65) This behaviour was stressed in Ref. 8 as the word nova was first used in that context. For low-excitation mass ($M < 3$ GeV) isotropic decay is probably a good approximation. As the mass increases one would expect a leading proton effect to set in. As remarked in Ref. 8, the nova may then be looked at as dual to a multi-Regge chain.

- 66) If the missing mass is too large ($M^2 > 50 \text{ GeV}^2$ say) one cannot any longer consider that a quasi-elastic process has been isolated. The associated multiplicity is then expected to be proportional to $\log M^2$ in much the same way as the total multiplicity is proportional to $\log s$ starting from rather low values of s . This is verified.
- 67) One should however quickly meet limits associated with the observation of the reaction through the vacuum pipe. A detailed study may rather belong to NAL or to the SPS, the energy limitation notwithstanding.
- 68) CHLM Collaboration, ISR discussion meeting 8, CERN internal report; M. Albrow et al., to be published in Phys. Letters. Practical limitations on the excitation mass ($M < 8 \text{ GeV}$) make it impossible to really test any asymptotic features. What may perhaps rather be explored is the analogy with photoproduction or electroproduction, with distributions already well known at $\sqrt{s} < 8 \text{ GeV}$. What is of importance is the photon fragmentation effect, as stressed by M. Jacob, R. Slansky and C. Wu in Phys. Letters 38 B, 85 (1972), with an inclusive pion distribution clearly asymmetric at small x . It should be interesting to see if such a Pomeron fragmentation term shows up, with an asymmetric pion distribution in the nova rest frame. One should find more pions on the inner side.
- 69) K. Kajantie, Helsinki preprint (1973);
H. Harari et al., Weizmann preprint (1973).
- 70) The separation of a diffractive component from a non-diffractive one, as emphasized by K. Wilson, is only a first approximation, if often a very useful one. As emphasized in Ref. 55, they gradually merge into each other. This is illustrated by Fig. 17, where one sees that the angular distribution of the associated secondaries does not exhibit any qualitative change as one leaves the quasi-elastic peak.
- 71) This is a region in which the separation between different incoherent mechanism loses meaning. It remains that even at ISR energies, the diffractive contribution is still small at wide angles. A large fraction of the diffractive cross-section corresponds to low-excitation masses with a rapid fall-off in mass, this before one finds the M^2 behaviour.
- 72) For a detailed discussion of clustering effects and an up-to-date review of production processes, see R. Slansky, to be published in Phys. Reports (1974), and R. Slansky, Yale reports (1973) and (1974).
- 73) Aachen-CERN-UCLA Collaboration, P. Schlein, private communication. A similar effect should be found with the reaction $pp \rightarrow n\pi^+ + X$, $n(\pi^+)$ replacing $\Delta(\pi^-)$ in the Deck effect graph. The interest of the Deck effect in high-energy reactions has been emphasized by E. Berger.
- 74) For a review of theoretical approaches to large transverse momentum phenomena, see S. Brodsky, to be published in Phys. Reports, and J.C. Polkinghorne, Aix en Provence Conf. 1973, op cit., p. 421.
- 75) H. Abarbanel, S. Drell and F. Gilman, Phys. Rev. Letters 20, 280 (1968).
- 76) J.F. Gunion, S.J. Brodsky and R. Blankenbecler, Phys. Rev. D6, 2652 (1972); D8, 287 (1973).
- 77) R.P. Feynman, Photon-hadron interactions, Frontiers in Physics (Benjamin, Reading, Mass., 1972). A detailed calculation is offered by S.D. Ellis and M.B. Kislinger, to be published in Phys. Rev.
- 78) It should be stressed that one is dealing with small cross-sections. As a result several *a priori* peculiar configurations could be called upon in order to explain the observed effects. It cannot be ruled out for instance that heavy fireballs, at rest or almost in the centre-of-mass system and decaying rarely, but sometimes, into an isotropically distributed low-multiplicity final state, could give the observed effects. Such a picture implies however several *ad hoc* parameters. At present, one should keep an open mind. Yet, since a model provides a useful synthesis when presenting the data, we shall here follow a parton picture.

- 79) One should most generally think in terms of the number of parameters needed to reproduce the transverse momentum distribution. The more parameters there are, the more structure one then sees. An exponential distribution (or a superposition of exponentials) translates into a width (or different widths). An inverse power indicates something point-like. The actual large p_T behaviour corresponds to something at least quasi-point-like.
- 80) One may then entertain the idea that these effects are to be connected to the parton-quark structure of the proton.
- 81) Once leading particle effects are acknowledged, the available energy is on the average much less than $\sqrt{s}$.
- 82) The more recent data of the Aachen-CERN-Heidelberg-Munich Collaboration have been substituted for the former ones of the CERN-Columbia-Rockefeller Collaboration. These data are amenable to a fit of the type $p_T^n \exp [-B(p_T/\sqrt{s})]$ with $6.5 < n < 7.5$ and $20 < B < 30$ as the best fit at $\sqrt{s} = 53$ GeV. At the time of the Aix en Provence Conf., the CCR Collaboration gave $7.5 < n < 8.5$, $B \sim 25$ as a good fit. One should keep in mind that it is yet not possible to disentangle n and B in any satisfactory way. I am indebted to P. Darriulat for a discussion on this point.
- 83) B.G. Pope, Aix en Provence Conf. 1973, op cit., p. 407;
L.J. Carroll, Aix en Provence Conf. 1973, op. cit., p. 409.
- 84) D. Theriot, Aix en Provence Conf., 1973, op. cit., p. 415.
- 85) J. Cronin, Aix en Provence Conf., 1973, op. cit., p. 418;
J. Cronin et al., Phys. Rev. Letters 31, 1426 (1973).
- 86) H.J. Frisch, Invited talk, APS Chicago Meeting (1974). I am much indebted to J. Cronin, H.J. Frisch, R. Mermod and P. Pirouè for discussions on their results. A discussion of the discrepancies between the inverse powers preferred at the ISR (~ 8) and at NAL (~ 11) is in order. Both sets of data are amenable to a fit of the type $p_T^n F(x_T)$; however they hardly overlap in p_T and x_T .

However, if the ISR results are compatible with a limiting inverse power behaviour as $s \rightarrow \infty$ (p_T^{-8}), the NAL results would rather suggest a vanishing yield as $s \rightarrow \infty$, the distribution at fixed x_T decreasing as an inverse power of s .

In order to make sure that a fit of the type $p_T^n F(x_T)$ is meaningful, one should see how equal yield curves appear on a $\log p_T \log x_T$ plot. This is emphasized by P. Darriulat. Their spacing at fixed x_T gives the estimate for n . One should find a spacing independent of x_T as $x_T \rightarrow 0$ ($s \rightarrow \infty$). Since all curves have to jam at $x_T = 1$ (phase space limit) they automatically diverge as one moves toward lower x_T . Hence n is typically larger at NAL (larger x_T) than at the ISR (lower x_T). It seems that the present π^0 data show too diverging curves to clearly suggest a limiting power. The value $n = 8$ appears rather as a transition value. This agrees with what one may expect from the study of correlations.
- 87) G. Jarlskog, Marseille Meeting on Hadron Physics at the ISR (1973).
- 88) There has not been any major theoretical developments over the past 6 months. A survey of theoretical approaches to large p_T phenomena is to be found in the reviews of J.D. Bjorken and J.C. Polkinghorne at Aix en Provence. These papers contain references to the original papers. The present discussion is accordingly very short.
- 89) For a recent review, see ISR discussion meeting 7 (1973), CERN internal reports, and Ellis and Thun in Ref. 12.
- 90) One has to keep in mind, however, that one is looking at an unlikely configuration. Hence what is known to apply in general may not apply here. What is described here is naturally expected in a multiperipheral cluster model and already ruled out for large p_T phenomena. One may still consider that the relevant clusters have to be very heavy. The observed configuration is then always to be centred at rapidity zero.

- 91) One describes the reaction the parton way and assumes that the phase-space distribution of the final hadrons is similar to that of the final partons. Partons are assumed not to interact when in very different regions of phase space. This is what should occur for the two hard partons scattered at wide angles. As a result the final distribution should show factorization properties which could remain true among hard secondaries. In particular, observing a hard π^+ (π^-) on one side should have no relevance on the mean charge of the hard particles on the other side. In the heavy low-multiplicity fireball picture one would expect correlations related to an over-all positive cluster.
- 92) This is similar to what is expected in the parton exchange approach of Blankenbecler, Brodsky and Gunion, where one considers directly pion quark scattering as opposed to quark quark scattering, see the discussion of Ellis and Thun in Ref. 12. As x increases, one would expect the corresponding yield to go down as $p_T^{-4}|p|^{-4}$, hence faster than at 90° ; this is, however, slow at appearing due to kinematical effects.
- 93) This is as given by Ellis and Thun in Ref. 12. Also shown are new data from the ACHM Collaboration which check point (iii). The trigger is a $90^\circ \pi^0$, but the beams have unequal momenta. This corresponds to a rapidity shift of 0.5. A similar effect has been reported also by the PSB Collaboration with a π^0 trigger at 18° . This confirms the expected features. More extensive checks are certainly needed.
- 94) ISR discussion meetings 7 (1973) and 10 (1974), CERN internal reports.
- 95) Correlations are defined dividing by the product of the two single-particle yields which are very small.
- 96) Beside these short-range correlation effects, one sees an important long-range effect, the particle yield at large p_L falling with p_T . This is again expected in a parton picture. It is, however, not specific of it. This should be kept in mind as a bias when triggering on large p_T configurations.

Author's Note

I regret the overly strong bias towards personal works which this reference list shows. This was the easier way out and the list is not at all exhaustive. I hope that the frequent references to review papers will compensate for this.