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PREFACE

The 1974 CERN School of Physics was organized together with the Daresbury Nuclear Physics Laboratory, U.K. It was held at the Beech Hill Hotel on Windermere, in North-West England, and it was attended by 55 young physicists from 32 laboratories; 6 of the students came from countries which are not CERN Member States.

The notes of the lecture courses by C. Jarlskog, P.V. Landshoff and D.H. Perkins are reproduced in these Proceedings. I would like to thank the authors for providing their typed notes and also the Scientific Information Services of CERN for their usual excellent work.

The notes are published as photo-offset reproduction of the lecturers' manuscripts without alterations or further proof reading.

The lectures given by J.S. Bell followed very closely the content of REF.TH.1710-CERN: "An Introduction to Renormalizable Models of Weak Interactions and their Experimental Consequences" by C.H. Llewellyn-Smith. The reader is referred to these notes which are not reproduced in the present Proceedings.

Finally, I wish to express my thanks to our British colleagues, for their hospitality and assistance which contributed so much to the success of the 1974 CERN School of Physics.

O. Kofoed-Hansen, Editor
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WEAK INTERACTIONS

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1. INTRODUCTION

In the past, several surprising discoveries of fundamental impact have been made in the area of weak-interaction physics. The discovery of parity violation and later CP violation eradicated two of our deep-rooted prejudices. Another unexpected finding was the existence of the muon neutrino.

Although such discoveries, together with other less spectacular ones, have increased our understanding of the nature of weak interactions, many questions still remain unanswered.

Putting aside fundamental unsolved questions (such as why does the muon exist), we are expecting that experiments in the near future will supply the answers to several of our less ambitious, but nonetheless quite important, questions. We know that the conventional current-current theory of weak interactions, in spite of explaining the low-energy phenomena quite well, can only be approximately correct, and as yet we do not know how the true theory looks. The theoretical possibilities being almost unbounded, it is certainly very important to gain clues from experiments.

Recently there has been a great deal of excitement in weak interactions both theoretically and experimentally. The theoretical excitement has been due to the advent of the so-called unified theories of weak and electromagnetic interactions (gauge theories). The virtue of these theories is that for the first time one can construct models of weak interactions which are reasonable in many ways*). The hope is that nature chooses one of them.

*) Supported by the Swedish Atomic Research Council, under Contract Number 0310-019.

**) For example, these theories give finite results for measureable quantities, such as cross-sections, while in the conventional current-current theory these quantities are infinitely large owing to higher-order weak interactions.
The experimental excitement is due to the fact that a sizeable fraction of neutrino-like interactions without a charged lepton in the final state has been observed at CERN, FNAL, and ANL. These events are most naturally explained by postulating the existence of the so-called neutral currents. Neutral currents are forbidden by the conventional current-current theory, but are desperately needed in a large number of gauge theories. Undoubtedly, the study of the neutral current processes will constitute one of the most exciting fields of research in the near future. It will also be interesting to see whether any of the gauge theories survive the confrontation with experiments.

In a theorists's dream world, the experimentalist focuses his attention first on weak interaction experiments involving only leptons such as neutrino-charged lepton or neutrino-neutrino scattering. For our understanding of weak interactions, processes involving neutrinos are especially appropriate since these particles do not interact electromagnetically or strongly. Then after extracting enough information from leptonic processes, one would go on to consider processes having an increasing degree of complexity, where one or more hadrons also take part. Hopefully, somewhere along this chain of experiments, the weak interactions will be well understood and can then be used as a tool for studying other domains such as hadron physics, nuclear models, astrophysical models, etc.

1.1 Historical notes

Historically, weak interactions have been studied for about eight decades. However, the line of research has been quite different from the one outlined above. The variety of processes in which weak interactions may be studied is very large because every elementary particle, excepting the photon, interacts directly via weak forces. However, for many decades one was confined to the study of the $\beta$-decay of nuclei, from which many important facts such as the existence of the neutrino and the form of the $\beta$ interaction, were derived.

Apart from the $\beta$-decay of nuclei, much of our present knowledge of weak interactions is based on the studies of the decays of elementary particles, e.g. $\mu \rightarrow e + \nu + \bar{\nu}$, $n \rightarrow p + e + \bar{\nu}$, $\pi \rightarrow e\nu$, $\mu\nu$, etc.

Some further information is obtained in lepton capture processes such as $\mu^- + p + n + \nu_\mu$. The shortcoming of these processes is that they all
are low-energy and low-momentum transfer phenomena and give no restriction on the high-energy behaviour of weak interactions where the conventional theory encounters serious difficulties. Thus high-energy weak interaction experiments are extremely important in the search for the correct theory. These experiments can be done with beams of neutrinos, charged leptons, or even hadrons. Clearly, experiments with neutrinos have the advantage that these particles interact only via weak interactions, whereas in experiments with charged lepton or hadron beams one must isolate the weak-interaction effects from the huge background of electromagnetic or strong interaction effects.

The first experiment with neutrinos was made about 20 years ago, where the first neutrinos came from a reactor and had energies of at most a few MeV. The top neutrino energy available nowadays is about 200 GeV and, hopefully, we will learn much in the near future about weak interactions at these fairly high energies.

1.2 Outline of the lectures

These lectures are subdivided into two parts. Part I (Sections 2-10) deals mainly with the well-established (low-energy and low-momentum transfer) weak interactions. In Section 2 the selection rules for leptons are given; Sections 3-5 summarize our present knowledge of purely leptonic processes, especially the muon decay, and give the current-current theory of leptonic weak interactions. Section 6 deals with β-decay processes (for simplicity we consider mainly the neutron decay) where the vector and axial vector nature of β-decay interaction follows from experimental results. We also discuss, at some length, the conserved (iso)vector current hypothesis. In Section 7, the two-body decay modes of pions and kaons, where the current-current nature of weak interaction is tested, are considered. Section 8 gives the selection rules for semileptonic processes; these follow naturally from the Cabibbo Theory, discussed in Section 9, and finally, in Section 10 we summarize the results of Part I and simply list some of the triumphs and shortcomings of the conventional theory.

*) For example, in the decay $n \rightarrow p e^\nu$, the square of the four-momentum transfer between the nucleons lies between $m_e^2$ and $(m_n - m_p)^2$; $m$ denotes the mass.
Recently a considerable amount of experimental effort has gone into the study of neutral-current processes and searches for new particles (some desperately needed in a large number of theoretical models), especially vector bosons and heavy leptons. Undoubtedly, such studies will constitute an active domain of experimental research in the future. Therefore Part II is devoted to the phenomenology of neutral currents and of vector bosons, heavy leptons, and charmed particles, even though it is conceivable that such particles do not exist.

Finally, note that we have not included deep inelastic neutrino processes since they are covered in the lectures by P.V. Landshoff.

I was asked a number of questions by the students. A few of these questions and their answers are given at the end of the relevant sections.

* * *

REFERENCES
(Section 1)

The reader interested in the history and theory of $\beta$-decay, may enjoy reading:


A summary of the status of weak-interaction theory before the discovery of the muon-neutrino and CP violation is given, for example, in:

2) P.K. Kabir (ed.), The development of weak interaction theory (Gordon and Breach, 1963). Note that several of the puzzles discussed in this book could be very simply explained after the discovery of the muon neutrino.

For a review of weak interactions as of 1965–1966, see:

2. SELECTION RULES FOR LEPTONS

In this section we shall consider the four known leptons, $e^-$, $\nu_e$, $\mu^-$, and $\nu_\mu$, and their antiparticles. These particles are all fermions with spin $\frac{1}{2}$. The reader may consult the Particle Data Group tables for data (such as masses, lifetimes, magnetic moments, etc.) on these particles. They are leptons in the sense that according to our present-day knowledge they do not take part in strong interactions. The charged leptons (muons and electrons) have also electromagnetic interactions while neutrinos interact only via weak interactions. Let us repeat some of the known properties of these leptons.

2.1 Lepton number conservation

2.1.1 The separate conservation rule

All experiments are consistent with the separate conservation of the electronic and muonic (lepton) numbers. One defines the electronic lepton number

$$n_e = 1 \quad \text{for } e^-, \nu_e$$

$$n_e = -1 \quad \text{for } e^+, \bar{\nu}_e$$

$$n_e = 0 \quad \text{for all other particles.}$$

The muonic number is defined by

$$n_\mu = 1 \quad \text{for } \mu^-, \nu_\mu$$

$$n_\mu = -1 \quad \text{for } \mu^+, \bar{\nu}_\mu$$

$$n_\mu = 0 \quad \text{for all other particles.}$$

These quantum numbers are additive. The lepton number conservation rule means that in every reaction

$$(n_e)_i = (n_e)_f, \quad (n_\mu)_i = (n_\mu)_f,$$

where $i$ and $f$ refer to the initial and final states of the reaction.
Consider the decay of a $\mu^-$. The only particle in the final state which is detected is an $e^-$. Assuming the above lepton number conservation rule, we would deduce that the decay mode is

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$(n_e)_i = 0, \quad (n_e)_f = 1 - 1 = 0,$$

$$(n_\mu)_i = 1, \quad (n_\mu)_f = 1.$$  

According to the above scheme the charged leptons produced via neutrinos are

$$\nu_\mu + X \rightarrow \mu^- + Y, \quad \nu_e + X \rightarrow e^- + Y,$$

$$\bar{\nu}_\mu + X \rightarrow \mu^+ + Y, \quad \bar{\nu}_e + X \rightarrow e^+ + Y,$$

where $X$ and $Y$ are hadrons. The relevant experimental numbers are$^{1}$  

$$\frac{\sigma(\nu_\mu \rightarrow \mu^+)}{\sigma(\nu_\mu \rightarrow \mu^-)} < 3.8 \times 10^{-3}$$

$$\frac{\sigma(\nu_\mu \rightarrow e^-)}{\sigma(\nu_\mu \rightarrow \mu^-)} < 1\%.$$  

Further evidence for the lepton conservation law above comes from the absence of processes such as (see Section 4)

$$\mu^\pm \rightarrow e^\pm + \gamma,$$

$$\mu^\pm \rightarrow e^\pm + e^+ + e^-, \text{ etc.}$$

2.1.2 The multiplicative rule$^{2}$

Schemes other than the one discussed above can be postulated. One such scheme is the so-called multiplicative rule, where the sum $n_e + n_\mu$
and \((-\)^{n_e}\) are conserved in every reaction. This scheme allows all the processes permitted by the previous scheme, as well as some others, e.g. \(\mu^+ \rightarrow e^+ + \bar{\nu}_e + \nu_\mu\) should occur with the same branching ratio as \(\mu^+ \rightarrow e^+ + e^- + \bar{\nu}_e + \nu_\mu\) because both modes respect the multiplicative rule, namely:

\[
\mu^+ \rightarrow e^+ + \bar{\nu}_e + \nu_\mu : (n_e + n_{\bar{\nu}_e})_i = (n_e + n_{\nu_\mu})_f = -1 ,
\]

\[
(n_e)_i = 0 , (n_e)_f = -2 , (-1)^{n_e} = 1 .
\]

By this scheme

\[
r = \frac{\Gamma(\mu^+ \rightarrow e^+ + \bar{\nu}_e + \nu_\mu)}{\Gamma(\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu) + \Gamma(\mu^+ \rightarrow e^+ + \bar{\nu}_e + \nu_\mu)} = 0.5 .
\]

Here \(\Gamma\) denotes the branching ratio.

Until recently this scheme was in agreement with all experimental data. However, there is now some evidence against the multiplicative rule from a recent CERN (Gargamelle) experiment\(^3\), where the interaction of \(\nu_e\) and \(\bar{\nu}_e\) with nuclei was studied. If \(r \neq 0\), there would be an additional contribution to \(\bar{\nu}_e\) flux from muon decay\(^4\) (which would give \(e^+\) events). The experiment gave

\[
r < 0.25 \quad 90\% \text{ confidence},
\]

i.e. the multiplicative rule is not favoured.

2.1.3 A third scheme for leptons

This is the Konopinski and Mahmoud assignment. The interested reader may find details of this scheme in a paper by these authors\(^5\).

2.2 Two-component neutrino theory

All experiments are consistent with the assumption that every neutrino is always left-handed (i.e. its spin is antiparallel to its direction of motion) and every antineutrino is right-handed \([\text{two-component neutrino theory}^6]\).
Formally, the property above can be ensured if the neutrino field\textsuperscript{*}) $\psi(x)$ occurs always in the form $(1-\gamma_5)\psi(x)$. The operator $1-\gamma_5$ projects out the negative helicity state for the neutrino and the positive helicity state for the antineutrinos (see Appendix A). Pictorially

\[ \begin{array}{cc}
\text{Momentum vector} & \text{neutrino} & \text{antineutrino} \\
\text{Spin vector} & \leftrightarrow & \rightarrow
\end{array} \]

The two-component theory was deduced from the study of parity violation in $\beta$-decay processes where experiments indeed demonstrated that the neutrino field occurs in the form $(1-\gamma_5)\psi(x)$. The helicities of the electron\textsuperscript{7}) and muon\textsuperscript{8}) neutrinos (or antineutrinos) have been measured directly. Within rather large errors the results are consistent with left-handed neutrinos and right-handed antineutrinos.

Finally, the two-component hypothesis requires the neutrino mass to vanish identically. The experimental upper limits are

\[
m_{\nu_e} < 60 \text{ eV} \quad [\text{Bergkvist}\textsuperscript{9})]
\]
and

\[
m_{\nu_\mu} < 1.2 \text{ MeV} \quad [\text{Backenstoss et al.}\textsuperscript{10})]
\]

* * *

EXERCISES

1) Suggest a few reactions which are forbidden by the separate lepton number conservation rule but are allowed by the multiplicative law.

2) Assume that the neutrino field $\psi(x)$ always occurs in the form $(1-\gamma_5)\psi(x)$. Show that by this hypothesis (two-component hypothesis),
   a) parity must be violated; b) neutrinos must be massless; and c) the neutrino is left-handed and the antineutrino is right-handed.

* *

*) Our metric and conventions are given in Appendix A.
REFERENCES
(Section 2)


3. WEAK INTERACTION EXPERIMENTS WITH LEPTONS

So far nature has supplied us with four leptons \((e^-, \nu_e, \mu^-, \text{ and } \nu_\mu)\) and their antiparticles. With these leptons we could study weak interactions in several classes of processes:

i) Decay processes: Only the muon decays, and its observed decay mode is\(^*)\)

\[
\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu.
\]

(3.1)

However, the identity of the neutrinos is not well established (see Section 2.1). The decay modes \(e\gamma, e^+e^-\), and \(e\gamma\gamma\) have been searched for. These modes are forbidden by the multiplicative and separate conservation rules and are suppressed strongly by the Konopinski-Mahmoud selection rule. None of these modes has ever been seen (see Section 4).

ii) With neutrino (or antineutrino) beams one could do neutrino-lepton scattering off atomic electrons:

\[
\begin{align*}
\nu_e + e^- & \rightarrow \nu_e + e^- \\
\bar{\nu}_e + e^- & \rightarrow \bar{\nu}_e + e^- \\
\nu_\mu + e^- & \rightarrow \nu_\mu + e^- \\
\bar{\nu}_\mu + e^- & \rightarrow \bar{\nu}_\mu + e^- 
\end{align*}
\]

(3.2)

(3.3)

or inverse \(\mu\)-decay processes

\[
\begin{align*}
\nu_\mu + e^- & \rightarrow \mu^- + \nu_e \\
\bar{\nu}_e + e^- & \rightarrow \mu^- + \bar{\nu}_\mu
\end{align*}
\]

The processes (3.3) are the so-called leptonic neutral current processes which we shall discuss later on. Note that, in writing down the above scattering processes, we have assumed the separate lepton number conservation rule.

\(^*)\) The radiative muon decay has also been observed (see Section 4).
If the multiplicative rule holds, one would have in addition

\[ \bar{\nu}_\mu + e^- \rightarrow \mu^- + \bar{\nu}_e, \]

and

\[ \nu_e + e^- \rightarrow \mu^- + \nu_\mu. \]

iii) With colliding beams or high-energy lepton beams (e^+ or \mu^+ beams) one could look for processes such as

\[ e^+ e^- \rightarrow \mu^+ \mu^-, \]

\[ e^- e^- \rightarrow \mu^+ \mu^- \]

\[ \mu^+ e^- \rightarrow \mu^- e^+, \text{ etc.} \]

occurring via the weak interactions.

iv) There are two purely leptonic atoms, namely positronium (e^+e^-) and muonium (\mu^+\mu^-). Weak interactions could in principle influence these atoms.

v) Leptonic interactions may be studied, somewhat indirectly, in the presence of the external field (a nucleus). For example, the transition \( \nu_\mu + \mu^+ + \mu^- + \nu_\mu \), forbidden by energy and momentum conservation, is allowed in the Coulomb field of a nucleus (trident production):

\[ \nu_\mu + (Z) \rightarrow \nu_\mu + \mu^+ + \mu^- + (Z), \]

provided the incident neutrino is sufficiently energetic. Denoting the \( \nu_\mu \rightarrow \mu^+ + \mu^- + \nu_\mu \) transition with the diagram

```
  (\nu_\mu)
   \___\___\
    \   \   \n    \   \   \n    \   \   \n    \   \  (Z)
    \   \   \n    \   \   \n    \   \   \n    \   \   \n  (\nu_\mu + \mu^+) + (\nu_\mu + \mu^-)
```

the relevant diagrams for trident production are:
Among, the processes listed above, so far only class (i) has been well studied. In fact, a great deal of our knowledge of weak interactions comes from the study of muon decay, to which we shall devote the next section. From the processes of class (ii) there is some information on $\bar{\nu}_e - e$ scattering\(^1\) from reactor experiments and on $\bar{\nu}_\mu e$ scattering\(^2\) from CERN. Class (iii) is already supplying some limits on the so-called neutral currents\(^3\). These limits will certainly be improved in the not too distant future. The transition muonium-antimuonium (forbidden in the separate lepton number conservation scheme but allowed by the multiplication rule) was looked for but never found\(^4\). However, since the experiment is very difficult, the multiplicative lepton number conservation law could not be excluded. Finally, the trident production\(^5\) should be a very rich source of potential information. It provides the only possibility of studying transitions $\nu_\mu + \mu \rightarrow \nu_\mu + \mu$ and $\bar{\nu}_\mu + \mu \rightarrow \bar{\nu}_\mu + \mu$. The interested reader may consult the articles in Ref. 5.

We shall return to a more detailed discussion of the leptonic processes in Sections 4 and 5 and in Part II.

* * *
REFERENCES

(Section 3)


4. **MUON DECAY AND RELATED PROCESSES**

Muon decay is the only purely leptonic process in weak interactions which has been well studied experimentally. The muons ($\mu^-$ and $\mu^+$) have spin $\frac{1}{2}$ and are leptons. According to the Particle Data Group compilation\(^1\) the muon decay branching ratios are given by

\[
\begin{align*}
BR \ (\mu \to e + \nu + \bar{\nu}) &= 100\% \\
BR \ (\mu \to e\gamma\gamma) &< 1.6 \times 10^{-5} \\
BR \ (\mu \to 3e) &< 6 \times 10^{-9} \\
BR \ (\mu \to e\gamma) &< 2.2 \times 10^{-8}
\end{align*}
\] (4.1)

The last three modes have never been seen.

In addition to these modes, the radiative muon decay

\[\mu \to e + \nu + \bar{\nu} + \gamma\] (4.2)

has also been observed and studied experimentally\(^2\). The data are evidently consistent with the lepton number conservation rules discussed in Section 2, which demand the absence of the modes $e\gamma\gamma$, $3e$ and $e\gamma$.

The muon decay experiments done so far measure the following quantities:

i) the muon lifetime;

ii) the spectrum of the electrons in the decay of unpolarized as well as polarized muons;

iii) polarization of the electrons in the decay of unpolarized muons; and

iv) photon and electron correlation in the radiative mode (4.2).
4.1 Theoretical description of the muon decay

All the known experimental facts\(^*)\) are consistent with the following simple form for the effective interaction Lagrangian density\(^**)\) describing the decay of muons\(^3\)\) (see the exercises at the end of this section):

\[
\mathcal{L}(\mu^- \to e^- + \bar{\nu}_e + \nu_\mu) = \frac{G}{\sqrt{2}} \bar{\psi}_\mu(x) \gamma^\lambda (1-\gamma_5) \psi_\mu(x) \bar{\psi}_e(x) \gamma^\lambda (1-\gamma_5) \psi_e(x),
\]

\[
\mathcal{L}(\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu) = \frac{G}{\sqrt{2}} \bar{\psi}_\mu(x) \gamma^\lambda (1-\gamma_5) \psi_\mu(x) \bar{\psi}_e(x) \gamma^\lambda (1-\gamma_5) \psi_e(x),
\]

where G is the weak interaction coupling constant determined from the muon lifetime.

The \(\sqrt{2}\) in formulae (4.3) has historical origin; it was introduced, after the discovery of parity violation, in order to preserve the (parity) numerical value of the weak interaction coupling constant. The \(\psi\)'s contain the appropriate absorption and emission operators (for further definitions see Appendix A). The essential features of the above interaction Lagrangian are the following:

a) The interaction is described by a point-like (local) four-fermion interaction without any derivatives where the creation and destruction of particles takes place in a single space-time point \(x\). Pictorially

\[\text{Experimental result} \quad \text{V-A prediction}\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Prediction</th>
</tr>
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<tr>
<td>(\rho)</td>
<td>0.752 ± 0.003</td>
<td>3/4</td>
</tr>
<tr>
<td>(\xi)</td>
<td>0.972 ± 0.013</td>
<td>1</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.755 ± 0.009'</td>
<td>3/4</td>
</tr>
<tr>
<td>(\eta)</td>
<td>-0.12 ± 0.21</td>
<td>0</td>
</tr>
<tr>
<td>(h)</td>
<td>1.00 ± 0.13</td>
<td>1</td>
</tr>
<tr>
<td>(\eta')</td>
<td>0.09 ± 0.14</td>
<td>0</td>
</tr>
</tbody>
</table>

The agreement is indeed impressive.

\(^*)\) In the ordinary decay mode of the muon one measures the parameters \(\rho\) (Michel parameter), \(\xi\), \(\delta\), and \(\eta\) (low-energy parameter) and the helicity \(h\) of the decay electron (or positron). In the radiative decay mode, one more parameter \(\eta'\) is measured. One finds (see the compilation by the Particle Data Group)

\(^**)\) See Discussion at the end of this section for what we mean by an effective Lagrangian density.
b) The interaction is of the current-current form with a muon current 
\[ \psi^\dagger (x) \gamma_\mu (1 - \gamma_5) \mu(x) \] multiplied by the Hermitian conjugate of the
electron current. The currents have the V-A form\(^*)\). The neutrinos
are left-handed (ensured by the factors \(1 - \gamma_5\)) and the antineutrinos
are right-handed.

c) The separate conservation of the muon and electron lepton numbers
has been assumed. If the multiplicative rule is assumed, the modes
\( \mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu \) and \( \mu^+ \rightarrow e^+ + \bar{\nu}_e + \nu_\mu \)
would also be possible (see Section 2.1).

Furthermore, the data on radiative muon decay are consistent with
the basic interaction given in formulae (4.3) modified by radiative cor-
rections, namely:

One may ask whether the form (4.3) for \( \mu \) -decay can be uniquely determined
from experiments. The answer is as follows. If one starts with the most
general four-fermion point interaction (including scalar, pseudoscalar,
vector, axial vector, and tensor couplings), one may deduce from experi-
ments, within rather large errors, the form\(^**)\)

\(^*)\) See Appendix A for definition of V (vector) and A (axial) currents.

\(^**)\) The correction terms to Eq. (4.4) are unpleasantly large at present\(^5\),
but, in principle, they may be eliminated by more accurate measurements.
It is interesting that the study of the radiative muon decay could be
useful\(^6\) in setting upper limits on correction terms to Eq. (4.4).
Note also that Eq. (4.4) is not of the conventional current-current
form. In fact, the neutrinos (unobserved particles) occur in one
factor and the charged leptons (observed particles) in the other. By
Fierz transformation it can be written in the conventional current-
current from. However, in addition to V and A terms one then finds
also scalar and pseudoscalar currents.
\[ L = \frac{1}{\sqrt{2}} \bar{\psi}_e(x) \gamma_\lambda (1 - \gamma_5) \psi_\mu(x) \bar{\psi}_\mu(x) \gamma^\lambda (g_V - g_A \gamma_5) \psi_{\nu_e} (x) \]  

(4.4)

for the decay of the \( \mu^- \) (and the Hermitian conjugate for the \( \mu^+ \) decay).

The quantities \( g_V \) and \( g_A \) are constants restricted by the relation of \( g_V^2 + g_A^2 = 2G^2 \). The relative magnitude of \( g_V \) and \( g_A \) can not be deduced from the muon decay (unless one observes the neutrinos). However, if the two-component neutrino theory is assumed, one must have

\[ g_V = g_A. \]

Then by a Fierz transformation (which reorders spinors; see also the discussions at the end of this section) the interaction can be cast in the form (4.3).

Below we shall keep the form (4.4) and apply it to the inverse muon decay processes

\[ \nu_\mu + e^- \rightarrow \mu^- + \nu_e \]  

(4.5)

and

\[ \bar{\nu}_e + e^- \rightarrow \mu^- + \bar{\nu}_\mu \]  

(4.6)

where one could in fact check the two-component neutrino theory \(^\text{7}\). Furthermore, by these simple examples one can easily demonstrate how the ratio of the cross-section for processes (4.5) and (4.6) depends on the choice of the coupling constants, \( g_V \) and \( g_A \). This simple example can be used not only as a pedagogical tool but can also be applied to other reactions such as neutrino -- and antineutrino -- parton \(^\text{1})\) scattering and to obtain bounds on the ratios of cross-sections.

\(^\text{1})\) A parton is a fictitious point-like constituent of the nucleon. The parton model is discussed in the lectures given by P. Landshoff at this School.
4.2 Inverse muon decay

Consider the inverse $\mu$-decay processes

\[ \bar{\nu}_e + e^- \rightarrow \bar{\nu}_\mu + \mu^- \]  
(a)

\[ \nu_\mu + e^- \rightarrow \mu^- + \nu_e \]  
(b)

We can easily calculate the cross-sections\(^*)\) for large centre-of-mass energy where the lepton masses can be safely neglected. We find, for the interaction form (4.4), that the ratio of the cross-sections at the same centre of mass energy is given by\(^7\)

\[ R = \frac{\sigma(\bar{\nu}_e + e^- \rightarrow \bar{\nu}_\mu + \mu^-)}{\sigma(\nu_\mu + e^- \rightarrow \nu_e + \mu^-)} = \frac{2 - \lambda}{2 + \lambda}, \quad (4.7) \]

where\(^**)\)

\[ \lambda = \frac{2g_Vg_A}{g_V^2 + g_A^2}, \quad -1 \leq \lambda \leq 1 \]

(4.8)

and therefore

\[ \frac{1}{3} \leq R \leq 3, \quad (4.9) \]

where $R = 3$ corresponds to $\lambda = -1$, i.e. $g_V = -g_A$ and $R = \frac{1}{3}$ for $g_V = +g_A$.

These results can easily be understood by using helicity arguments. The neutrino part of the interaction can be rewritten as

\[ \bar{\nu}_\mu \gamma^\lambda (g_V - g_A\gamma_5)\psi_e = \bar{\nu}_\mu \gamma^\lambda \left[ \frac{g_V - g_A}{2} (1 + \gamma_5) + \frac{g_V + g_A}{2} (1 - \gamma_5) \right] \psi_e. \]

If $g_V = g_A$, the first term vanishes and the operator $(1 - \gamma_5)$ projects out left-handed neutrinos and right-handed antineutrinos. If, however, $g_V = -g_A$, the operator $1 + \gamma_5$ is effective and it projects out right-handed neutrinos.

\(^*)\) A type-calculation is given in Appendix B.

\(^**)\) We assume time-reversal invariance, i.e. $g_V$ and $g_A$ are taken to be real.
where $g'_i$ are constants and the extra $\gamma_5$ ensures that $\mathcal{L}$ is a pseudo-scalar quantity. Thus our general interaction Lagrangian for the $\mu^-$ decay is

$$\mathcal{L} = \mathcal{L}_{\text{parity cons.}} + \mathcal{L}_{\text{parity viol.}}$$

The above procedure may seem arbitrary, and one may wonder why we did not take $\bar{\psi}_e(x) \gamma_\mu \psi(x)$ as a factor instead, or why did we put the extra $\gamma_5$ in the second factor instead of the first. It can be proven [see, for example, Källén\textsuperscript{11}], by the Fierz reordering theorem, that the order of the fields and the position where $\gamma_5$ is introduced is irrelevant. All the possible point-like four-fermion interactions can be transformed into each other by a redefinition of the coupling constants.

The measurable quantities in muon decay are functions of the coupling constants $g_i$ and $g'_i$. Therefore, by specifying the interaction, the parameters can be predicted and compared with experiment.

**Question:** What is meant by the word "effective"?

**Answer:** Suppose we write down a Lagrangian density (weak interaction coupling constant multiplied with some function of fields $\psi$, etc.) for the process $i \rightarrow f$, where $i =$ initial state and $f =$ final state, $i \neq f$. Given the Lagrangian density, the $S$-matrix $\langle f | S | i \rangle$ is determined. It is given by an expansion in $G$; namely, the first term is linear in $\mathcal{L}(\text{or } G)$, the second term is quadratic in $\mathcal{L}[\mathcal{O}(G^2)]$, etc. In the conventional theory of weak interactions only the first term $[\mathcal{O}(G)]$ gives sensible results (at low energies). The higher-order terms are theoretically infinitely large, although experimentally they do not seem to matter. In the effective Lagrangian philosophy, only the lowest-order term (order $G$) is considered.

The only higher-order weak-interaction effect is believed to be the $K_L - K_S$ mass difference, which is very small.

The virtue of some modern theories (gauge theories) is that they give sensible results to all orders for leptonic interactions. The


5. **CONVENTIONAL THEORY OF LEPTONIC WEAK INTERACTIONS**

As discussed in the previous section, all the experimental data on muon decay are well described by an interaction Lagrangian of the current-current form. One of the currents is the muon V-A current

\[ \bar{\psi}_{\nu_{\mu}} \gamma^{\lambda}(1-\gamma_{5})\psi_{\mu} \quad \text{for } \mu^- \text{ decay} \]

or

\[ \bar{\psi}_{\mu} \gamma^{\lambda}(1-\gamma_{5})\nu_{\mu} \quad \text{for } \mu^+ \text{ decay} , \]

and the other the electronic current

\[ \bar{\psi}_{\nu_{e}} \gamma^{\lambda}(1-\gamma_{5})\psi_{e} \quad \text{for } e^+ \text{ emission} \]

or

\[ \bar{\psi}_{e} \gamma^{\lambda}(1-\gamma_{5})\nu_{e} \quad \text{for } e^- \text{ emission} , \]

depending on whether one considers the \( \mu^- \) or \( \mu^+ \) decay.

From studies of the \( \beta^\pm \)-decay of nuclei

\[ (Z, N) \rightarrow (Z\pm1, N\pm1) + e^\pm + \nu , \]

where \( Z \) and \( N \) denote the number of protons and neutrons, respectively, one knows that even there the interaction has a current-current form, where the leptonic part is exactly the V-A current in formulae (5.2)\(^*\).

Therefore, it was conjectured\(^1\) that all weak interactions are described by a current-current Lagrangian where each current has a hadronic part and a leptonic part, the leptonic part being assumed to consist of the muon and the electron V-A currents.

\(^*\) The 'hadronic current' describing the destruction of the initial nucleus and the creation of the final one is somewhat more complicated (see Section 6), which is not surprising since nuclei have strong interactions.
5.1 Assumptions of the conventional V-A theory for leptonic processes

According to this plausible assumption (conventional theory) the leptonic Lagrangian is given by

$$\mathcal{L}^{\text{leptonic}} = \frac{G}{\sqrt{2}} \left\{ \bar{\psi}_\mu \gamma_\lambda (1-\gamma_5) \psi_\mu + \bar{\psi}_e \gamma_\lambda (1-\gamma_5) \psi_e \right\} \left\{ \bar{\psi}_\mu \gamma_\lambda (1-\gamma_5) \psi_\mu + \bar{\psi}_e \gamma_\lambda (1-\gamma_5) \psi_e \right\}$$

Pictorially

$$\mathcal{L} = \frac{G}{\sqrt{2}} \left\{ \left\langle \psi_\mu \right| \left| \phi_\mu \right\rangle + \left\langle \psi_e \right| \left| \phi_e \right\rangle \right\} \left\{ \left\langle \psi_\mu \right| \left| \phi_\mu \right\rangle + \left\langle \psi_e \right| \left| \phi_e \right\rangle \right\},$$

where the reader is expected to contract the vertices and supply the relevant $\gamma_\lambda (1-\gamma_5)$ factors. Let us summarize some of the essential features of this Lagrangian:

a) It assumes $\mu$-$e$ universality, i.e. the Lagrangian is invariant under exchange of the fields of $(\nu_\mu, \mu)$ and $(\nu_e, e)$.

b) The separate conservation of the muon and electron lepton numbers is assumed.

c) The Lagrangian is of the current-current form; each current is charged (i.e. at each vertex one unit of charge is exchanged), and every lepton is accompanied by its own neutrino.

5.2 Allowed and forbidden processes

Two types of processes are allowed by Eq. (5.3). These are

i) "Diagonal interactions" obtained by multiplying the muonic vertex by itself or the electronic vertex by itself. The diagonal processes are
\[ \nu_e + e^+ \rightarrow \nu_e + e^+ \]
\[ \bar{\nu}_e + \bar{e}^+ \rightarrow \bar{\nu}_e + \bar{e}^+ \]
\[ e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e \text{, etc.} \]

Similar reactions may also be written for muons and muon neutrinos.

ii) Non-diagonal processes, obtained by taking the 'product' of a muonic and an electronic vertices, e.g. $\bar{\psi}_\mu \nu_\mu \ldots \psi_\mu \bar{\nu}_e \ldots \psi_\nu \bar{\nu}_e$ gives rise to the reactions

\[ \mu^- + \nu_\mu + e^- + \bar{\nu}_e \]
\[ \bar{\nu}_\mu + e^+ + \mu^+ + \bar{\nu}_e \]
\[ \mu^- + e^+ + \bar{\nu}_e + \nu_\mu \text{, etc.} \]

Two classes of reactions are strictly forbidden by the interaction Lagrangian (5.3).

i) All processes which violate the separate conservation of the muonic and electronic lepton numbers are strictly forbidden (to all orders in weak interactions).

ii) All reactions which obey this rule, but where a lepton is not accompanied by its own neutrino, are forbidden in the lowest order. Examples of these forbidden processes are

\[ e^+ + e^- \rightarrow e^+ + e^- , \]
\[ e^+ + e^- \rightarrow \mu^+ + \mu^- , \]
\[ \mu^+ + e^- \rightarrow \mu^+ + e^- , \]

via weak interactions as opposed to electromagnetic, and

*) The lepton and its neutrino need not occur on the same side of the reaction.
\[ \nu_\mu + e^- \rightarrow \nu_\mu + e^- \]
\[ \bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^- . \]

These processes are referred to as neutral current reactions, because they would be allowed in a theory where there are neutral (current) vertices of the kind

\[
\begin{array}{cccc}
\langle \nu_e \rangle, & \langle e \rangle, & \langle \mu \rangle, & \langle \bar{\nu}_\mu \rangle.
\end{array}
\]

5.3 Experimental situation

Although the theoretical cross-sections for all the diagonal and non-diagonal processes were predicted long ago from Eq. (5.3), there are hardly any experimental tests. Except for the muon decay, which cannot test the theory, the only other reaction experimentally investigated is \[ \bar{\nu}_e + e^- \rightarrow \nu_e + e^- . \] This experiment is done with low-energy electron antineutrinos from \( \beta \)-decay in a reactor. The theoretical cross-section is very low (see Section 5.4) at low energies, and the backgrounds are severe. The authors give an upper limit \(^2\) for the cross-section in terms of the predicted V-A value

\[ \sigma^-_{\nu_e} < 1.9 \sigma^-_{\bar{\nu}_e} . \]

As far as the forbidden processes are concerned, experimentally \(^3\) there are two good candidates for the process \[ \bar{\nu}_e + \bar{\nu}_e \rightarrow \bar{\nu}_e + \bar{\nu}_e \] from experiments in the Gargamelle bubble chamber at CERN. The analysis of the experiment is not yet completed, and hopefully we shall have better information in the near future.

One immediate implication of the existence of neutral currents is that the conventional theory is incomplete or incorrect \(^**\). We shall return to the question of neutral currents in Part II.

*) See also Section 3.

**) Note that a large number of gauge theories require the existence of neutral current processes.
5.4 Difficulties of the conventional theory

Consider any leptonic scattering process, e.g. $\nu_e + e \rightarrow \nu_e + e$, at high energies where the lepton masses can be neglected. The cross-section for such a process involves $|<f|U|i>|^2$, where $f$ and $i$ refer to the initial and final states, respectively. Thus $\sigma \sim G^2$, where $G \sim 10^{-5}/(M_P)^2$. However, any cross-section must have the dimension (length)$^2$ which is equivalent to (energy)$^{-2}$ in our units $\hbar = c = 1$. The only invariant at our disposal is $s$, defined by

$$s = (q_e + q_{\nu_e})^2 \approx 2 m_e E_\nu, \quad E_\nu \gg m_e,$$

where $E_\nu$ is the incoming neutrino energy in the laboratory. Therefore, by dimensional arguments

$$\sigma = \text{(numerical factor)} \, G^2 s \sim G^2 m_e E_\nu,$$

i.e. the theoretical cross-section increases indefinitely with increasing neutrino energy! Although this result may describe the data quite well at low energies, it must be wrong because it violates unitarity at high energies ($\sqrt{s} \sim 300$ GeV).

A very important feature of the above formula for the cross-section is the proportionality with the target mass. Because of this fact, leptonic cross-sections are small and therefore hard to study.

Another difficulty, closely related to the one above, is that higher-order weak interactions give infinities which cannot be defined away, i.e. the theory is not renormalizable. For instance, consider the process $\nu_e + e \rightarrow \nu_e + e$.

In the second-order weak interaction, the process would go via the diagram
with a virtual e, $\nu_e$ intermediate state (other diagrams are also possible). In this case one must do an integration over the loop momenta $d^4k$. For large loop momenta each propagator introduces one power of $k$ in the denominator, and one ends up with a quadratic divergent integral $\int (d^4k/k^2) \to \infty$. Therefore, the second-order weak correction to $\nu_e$ scattering is infinitely large! The inclusion of the other diagrams does not change the result. This is a general defect of the theory which shows up in all processes. These infinities cannot be removed satisfactorily. Therefore at high energies the true theory must differ substantially from the conventional theory. However, the low-energy limit of the theory must coincide with that of the conventional theory as far as the non-diagonal interactions (the muon-decay terms) are concerned. Unfortunately the diagonal pieces of the conventional theory have not been tested well enough to give any real restriction on the true theory.

**REFERENCES**

(Section 5)

6. NEUTRON $\beta$-DECAY AND THE CONSERVED ISOVECTOR CURRENT HYPOTHESIS

As mentioned in the Introduction, a great deal of our knowledge of weak interactions is based on studies of $\beta^+$-decay of nuclei. The simplest such decay is the neutron $\beta$-decay

$$n \rightarrow p + e^- + \bar{\nu}_e.$$

The types of experiments one may perform are rather similar to the ones mentioned earlier for the $\mu$ decay -- although, here one can do more since there are two detectable particles in the final state. For example, in neutron $\beta$-decay, one measures the neutron lifetime and the $p_e \cdot p_\nu$, $s_n \cdot \bar{p}_e$, $s_n \cdot \bar{p}_\nu$, and $s_n \cdot (\bar{p}_e \times \bar{p}_\nu)$ correlations, where $p_e (p_\nu)$ and $s_n$ refer to the momentum of the electron (antineutrino) and the spin of the neutron. The antineutrino momentum is obtained from the proton (recoil) and the electron momenta.

In a theoretical description of nuclear $\beta$-decay, historically one used to consider the most general four-fermion-type-interaction including scalar, pseudoscalar, vector, axial and tensor interactions $^{**}$). The experimental data were used to reduce the number of theoretical possibilities. In fact, after a period of great confusion, it was shown that the $\beta$-decay interaction is given by vector and axial vector transitions among the nucleons coupled to the $V-A$ leptonic current for the leptons, e.g. the neutron $\beta$-decay amplitude can be written as

$$M(n \rightarrow p + e^- + \bar{\nu}_e) = \frac{G_F}{\sqrt{2}} \langle p | V_\lambda - A_\lambda | n \rangle \bar{u}_e \gamma^\lambda (1 - \gamma_5) \nu_{\bar{e}}.$$

$^{*})$ The decay of a free proton is forbidden by baryon number and energy-momentum conservation rules. The experimental lower limit$^1$ on the lifetime of the proton is $\tau_p > 2 \times 10^{36}$ years. In some gauge theories$^2$ the baryon number conservation is not an exact law of nature. Then the proton may decay (e.g. into leptons), with a lifetime which cannot be reliably calculated. Very rough estimates give $\tau_p \geq 10^{26}$ years.

$^{**}$ See discussions at the end of Section 4.
where the factor $G\beta/\sqrt{2}$ has been introduced for convenience and we have dropped the uninteresting normalization factors for leptons; $V_\lambda$ and $A_\lambda$ are vector and axial vector operators.

6.1 Matrix elements of the vector and axial currents $V_\lambda$ and $A_\lambda$

If neutrons and protons were point-like objects, in analogy with $\mu$-decay, we could try to express the operators $V_\lambda$ and $A_\lambda$ in terms of fields $(V_\lambda \sim \bar{\psi}_p \gamma_\lambda \psi_n, A_\lambda \sim \bar{\psi}_p \gamma_\lambda \gamma_5 \psi_n)$. However, even if the currents (in the absence of strong interactions, and neglecting electromagnetic effects) had such simple forms, in the real world, owing to strong interactions*, they could acquire much more complicated forms. In fact, we do not know how to write the operators $V_\lambda$ and $A_\lambda$ in terms of the basic fields $\bar{\psi}_p$ and $\psi_n$. However, we can write the most general form of the matrix elements**, $\langle p|V_\lambda|n \rangle$ and $\langle p|A_\lambda|n \rangle$, in terms of (as yet unknown) form factors which contain the dynamics. Conventionally, one writes***

$$
\langle p|V_\lambda|n \rangle = \frac{\bar{u}_p}{\sqrt{4p_0n_0}} \left\{ \gamma_\lambda f_{1}(t) - i \frac{\sigma_{\lambda\nu}}{2M} f_{2}(t) + \frac{q_\lambda}{2M} f_{3}(t) \right\} u_n \quad (6.1)
$$

and

$$
\langle p|A_\lambda|n \rangle = \frac{\bar{u}_p}{\sqrt{4p_0n_0}} \left\{ \gamma_\lambda g_{1}(t) - i \frac{\sigma_{\lambda\nu}}{2M} g_{2}(t) + \frac{q_\lambda}{2M} g_{3}(t) \right\} \gamma_5 u_n \quad (6.2)
$$

where $M$ is the nucleon mass, introduced on dimensional grounds.

Further

$$
q_\lambda = p_\lambda - n_\lambda, \quad t = q^2, \quad \sigma_{\lambda\nu} = \frac{i}{2} (\gamma_\lambda \gamma_\nu - \gamma_\nu \gamma_\lambda).
$$

*) For example, owing to diagrams where pions go across the point-like vertices, such as

***) How this is done is explained in the discussion at the end of this section.

**** The superscripts on $f_1(t)$ and $g_1(t)$ indicate the transition.
The vector and axial form factors $f_1(t)$ and $g_1(t)$ appearing above are referred to in the literature as:

$\begin{align*}
f_1(t) &= \text{vector form factor}, \\
f_2(t) &= \text{weak magnetism}, \\
f_3(t) &= \text{induced scalar} \\
\quad \text{(second class vector form factor).} \\
g_1(t) &= \text{axial vector form factor}, \\
g_2(t) &= \text{pseudotensor form factor} \\
\quad \text{(second class axial form factor),} \\
g_3(t) &= \text{induced pseudoscalar form factor.}
\end{align*}$

In nuclear $\beta$-decay, apart from a few exceptions, $t$ is limited to small values, and one is essentially studying the $t = 0$ limit of the transition matrix elements where the form factors may be replaced by their values at $t = 0$. Furthermore, not all of the form factors are expected to make observable contributions. Let us see this for neutron $\beta$-decay, where

$$\frac{|q_\rho|}{2M} = \frac{|p_\rho - n_\rho|}{2M} \ll 1, \quad \rho = 0, 1, 2, 3.$$ 

Thus the contribution of the form factors $f_2$, $f_3$, $g_2$, and $g_3$ (as compared to that of $f_1$ and $g_1$) is expected to be negligible. Furthermore, $t = 0$, namely:

$$(0.51 \text{ MeV})^2 = m_e^2 \leq t \leq (M_n - M_p)^2 = (1.3 \text{ MeV})^2.$$ 

Thus

$$\langle p | V_\lambda - A_\lambda | n \rangle \approx \frac{1}{\sqrt{4p_\rho n_\rho}} u \gamma^\rho \left( f_1(0) - g_1(0)\gamma_5 \right) u_n.$$ 

Experiments give

$$f_1(0) G_\beta \approx 0.98 G \quad (6.3)$$

*) This argument holds unless one (or more) of the form factors $f_2$, $f_3$, $g_2$, and $g_3$ is anomalously large near $t = 0$. We shall see below that $f_2$ and $f_3$ are well described by CVC and are not anomalously large. The information of $g_2$ and $g_3$ (from $\Sigma^+ \rightarrow \Lambda e^+\nu$ and muon capture, respectively) is also consistent with the absence of anomalous effects.
and
\[
g^{\text{pn}}_1(0)/f^{\text{pn}}_1(0) = (1.25 \pm 0.01). \tag{6.4}
\]

6.2 Conserved isovector (isotriplet) current hypothesis (CIVC\textsuperscript{c})

Relation (6.3) above is quite remarkable because it indicates that the effective β-decay vector coupling constant \( G^\beta_\beta f^{\text{pn}}_1(0) \) is essentially the same \(^*) as the corresponding quantity for muon decay \( G \). For point-like nucleons, we would have expected \( G^\beta_\beta = G, f^{\text{pn}}_1 = 1, g^{\text{pn}}_1 = 1 \) if the transitions \( n \rightarrow p, \mu \rightarrow \nu_\mu, \) and \( e \rightarrow \nu_e \) were the same (a very simple form of lepton-hadron universality). Let us, for the moment, in Eq. (6.3) approximate 0.98 with unity and assume that the basic muon decay and β-decay coupling constants are equal, \( G^\beta_\beta = G \). Then \( f^{\text{pn}}_1(0) = 1 \). One may ask, why should \( f^{\text{pn}}_1(0) = 1 \), as expected for point-like \( n \rightarrow p \) transition? Why is \( f^{\text{pn}}_1(0) \) not changed (renormalized) radically because of strong interactions of \( n \) and \( p \)?

The clue to the answer comes from electromagnetic interactions. We could easily envisage that the electric charges of a point-like (bare) proton and an electron should be equal in magnitude because of universality of the electromagnetic interactions. Experiment tells us that the measured charges of electrons and protons are equal in magnitude to high accuracy, in spite of the fact that the proton is 2000 times heavier than the electron and interacts strongly. Thus the charge is not changed (renormalized) by strong interactions. Why? The answer is because the electromagnetic current is conserved (see below).

6.2.1 Matrix elements of the electromagnetic current

Let us go through the details. For electrons (and positrons) we can write the electromagnetic current in terms of the basic field \( \psi_e(x) \); namely, \( J^\text{electron}_\lambda = \bar{\psi}_e(x) \gamma_\lambda \psi_e(x) \). Similarly, for point-like protons, we have \( J^\text{proton}_\lambda = \bar{\psi}_p(x) \gamma_\lambda \psi_p(x) \).

\(^*) In Cabibbo theory (see Section 9) \( G^\beta_\beta f^{\text{pn}}_1(0) \) gets multiplied with \( \cos \theta_c \), where \( \theta_c \) is the Cabibbo angle. Thus one predicts \( G^\beta_\beta f^{\text{pn}}_1(0) < G \), in better agreement with data.
The matrix element of the electromagnetic current between two electrons is given by (we neglect radiative corrections)

\[ \langle e(p') | J_{\lambda}^{\text{em}}(0) | e(p) \rangle = \frac{1}{\sqrt{4E_0E'_0}} \bar{u}(p') \gamma_{\lambda} u(p), \]  

(6.5)

where \( p \) (\( p' \)) denotes the four-momentum, \( E_0 = p_0 \).

We do not know how to write \( J_{\lambda}^{\text{proton}} \), in terms of \( \psi_p \) and \( \bar{\psi}_p \), for physical protons. However, from general considerations (see the discussion and example 1 at the end of this section) we know that the matrix element of the electromagnetic current between two protons must have the form

\[ \langle p(p') | J_{\lambda}^{\text{em}}(0) | p(p) \rangle = \frac{1}{\sqrt{4E_0E'_0}} \bar{u}(p') \left[ \gamma_{\lambda} F_1^P(t) - i \frac{q_{\lambda\nu} q^\nu}{2M} F_2^P(t) \right] u(p). \]

(6.6)

Again \( p \) (\( p' \)) denote the four-momenta, this time for the particles involved in Eq. (6.6), namely protons. \( m \) is the proton mass, \( q^\nu = (p' - p)^\nu \), \( t = q^2 \), and we have suppressed particle and spin labels on the Dirac spinors; \( F_1^P \) and \( F_2^P \) are the electromagnetic form factors. For point-like (bare) protons, by the universality assumption, we have

\[ \begin{bmatrix} F_1^P(t) \\ F_2^P(t) \end{bmatrix} = 1, \quad \begin{bmatrix} F_1^P(t) \\ F_2^P(t) \end{bmatrix} = 0 \]  

(6.7)

[analogous to the form (6.5)].

The normalization of the electromagnetic form factors is \( F_1^P(0) = 1 \) (see below) and \( F_2^P(0) = \mu_p \), where \( \mu_p \) is the anomalous magnetic moment of the proton.

The electric charge operator \( Q \) is defined by

\[ Q = \int d^3x \ J_0^{\text{em}}(x, t). \]

(6.8)

*) The coupling constant (electric charge) is introduced in the Lagrangian. For electrons we write \( \mathcal{L}_{\text{electrons}} = -e \bar{\psi}_e(x) \gamma_{\lambda} \psi_e(x) A^\lambda(x) \), where \( A^\lambda(x) \) is the photon field. For protons we write \( \mathcal{L}_{\text{protons}} = e J_1^e(\text{protons}) A^\lambda(x). \) Thus the electron and the proton charges are defined in the units \(-e\) and \(+e\), respectively. By this convention, the electron and proton charges are equal.
Note that since the electromagnetic current is conserved, $Q$ is a constant of motion $\left[\frac{\partial J^\text{em}_\lambda}{\partial x_\lambda} = 0 \rightarrow Q(t) = Q(0)\right]$. The charge of the electron is given by the matrix element of $Q$ between states of two electrons with equal spins and four-momenta

$$\langle e(p) | Q | e(p) \rangle = \int d^3x \langle e(p) | J^\text{em}_0(0) | e(p) \rangle = \frac{1}{\sqrt{4E_0^2}} \bar{u}(p) \gamma_0 u(p) = 1,$$

(6.9)

where we have put $V = 1$ and used the formulae in Appendix A.

The charge of the proton (in units of $+e$) is similarly

$$\langle p(p) | Q | p(p) \rangle = \int d^3x \langle p(p) | J^\text{em}_0(0) | p(p) \rangle = \frac{F^p_1(0)}{\sqrt{4E_0^2}} \bar{u}(p) \gamma_0 u(p) = F^p_1(0).$$

(6.10)

We now assume that strong interactions could be switched off slowly. Then, as $t \rightarrow \infty$, the proton would become point-like (bare), and by the universality assumption $F^p_1 \rightarrow [F^p_1]_{\text{point-like}} = 1$. Thus $\langle p(p) | Q(t) | p(p) \rangle \bigg|_{t \rightarrow \infty} = 1$. Since $Q$ is independent of time, we conclude [by comparing with Eq. (6.10)] $F^p_1(0) = 1$. This result [compare Eqs. (6.9 and 6.10)] implies the equality of the experimental charges of the proton and the electron (in units of $+e$ and $-e$, respectively.

**CONCLUSIONS**

The assumption of the equality of bare charges, leads to the equality of physical charges, provided the electromagnetic current is conserved.

Analogously, one would expect the equality of the $\beta$- and $\mu$-decay vector coupling constants* [i.e. $f^{\text{pm}}_1(0) = 1$], if $(G_\beta)_{\text{point-like}} = G$ and the weak vector current in question is conserved, $\partial W/\partial \xi = 0$. This is the conserved vector current hypothesis (CVC).

*) Here the proton-neutron mass difference is neglected.
6.2.2 Isospin structure of vector current

One may go further by trying to understand if there is any reason for the conservation of the vector current. The clue to the answer comes from the structure of the currents in the isospin space.

In the isospin space, the proton and the neutron are the two states of the nucleon corresponding to \( I_3 = \pm \frac{1}{2} \), respectively. We write

\[
N(q) = \begin{pmatrix} p(p) \\ n(p) \end{pmatrix},
\]

where \( p \) is the four momentum.

The transition turning a neutron with four-momentum \( n \) into a proton with four-momentum \( p \) involves

\[
\tilde{p}(p) n(n) = \begin{pmatrix} \tilde{p}(p), \bar{n}(p) \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} p(n) \\ n(n) \end{pmatrix}
\]

\[
= \tilde{N}(p) \frac{\tau_1 + i\tau_2}{2} N(n),
\]

\[
N(n) = \begin{pmatrix} p(n) \\ n(n) \end{pmatrix},
\]

where \( \tau_i \) are the Pauli matrices.

Thus the matrix element of the vector current, given by formula (6.1), may be written as

\[
\langle p|V_\lambda|n\rangle = \frac{1}{\sqrt{4p_0n_0}} \tilde{N}(p) \left[ \gamma_\lambda f_1^n(t) - \frac{i\sigma_\lambda^\gamma}{2M} f_2^n(t) + \frac{q_\lambda}{2M} f_3^n(t) \right] \frac{\tau_1 + i\tau_2}{2} N(n).
\]

(6.11)

A similar expression can also be written for the axial transition, Eq. (6.2).

6.2.3 Assumption of conserved isodoublet vector current

The vector current above increases the third component of the isospin by one unit: \( I_3(p) - I_3(n) = 1 \). Therefore the simplest assumption one can make is that it should be an \( I = 1 \) object. By this assumption there
exists a vector in the isospin space \( \vec{V}_\lambda = (V^1_\lambda, V^2_\lambda, V^3_\lambda) \), and the vector current appearing in the \( n \rightarrow p \) transition is simply

\[
v^{np}_{\lambda} = v^1_\lambda + iv^2_\lambda \quad \text{(isospin current).} \tag{6.12}
\]

Generally, the matrix element of \( V^k_\lambda \) between two nucleon states can be written in the form

\[
\langle N(p) | V^k_\lambda | N(n) \rangle = \frac{\bar{N}(p)}{\sqrt{4p_0n_0}} \left[ \gamma_\lambda f_1(t) - \frac{i\sigma_\lambda v^\mu}{2M} f_2(t) + \frac{q_\lambda}{2M} f_3(t) \right] \frac{\tau_k}{2} N(n),
\tag{6.13}
\]

where \( f_1(t) = f^p_1(t) \). Furthermore, by hermiticity the vector current occurring in transitions \( p \rightarrow n \), e.g. \( \nu_\mu \rightarrow p \rightarrow \mu^- + n \), is given by

\[
V^{np}_{\lambda} = V^1_\lambda - iv^2_\lambda, \tag{6.14}
\]

\[
f^{np}_{\lambda} = f^p_{\lambda} = f_1(t). \tag{6.15}
\]

Having seen that \( V^1_\lambda \) and \( V^2_\lambda \) occur in weak interactions, we ask what is \( V^3_\lambda \)? It is a conserved* isovector neutral current which connects a proton to itself and a neutron to itself. The only such current we know of is the electromagnetic current. Therefore we shall assume that \( V^3_\lambda \) is the isovector part of the electromagnetic current. By this economical assumption [conserved isovector (isotriplet) current hypothesis], weak vector current is obtained by an isospin rotation from the isovector electromagnetic current and therefore it must be conserved if isospin is good.

*) Because we want \( \partial V^k_\lambda / \partial x_\lambda = 0, k = 1, 2, 3. \)
SUMMARY

1) The vector currents in the transitions $n \rightarrow p$ and $p \rightarrow n$ are the $1 + i2$ (isospin-raising) and $1 - i2$ (isospin-lowering) members of a vector in the isospin space $\vec{V}^\lambda = (V^1_\lambda, V^2_\lambda, V^3_\lambda)$.

2) The third member of this vector is the (experimentally well studied) isovector electromagnetic current, $V^3_\lambda = j^\text{em, isovector}_\lambda$.

3) The conservation of the vector current $V^1_\lambda \pm iV^2_\lambda$ follows from the conservation of the electromagnetic current and isospin invariance.

4) The vector form factors are determined from the isovector electromagnetic form factors (see Section 6.3 and Discussion).

GENERALIZATION

Although the discussions above were given for the particular transitions $n \neq p$, by economy, one assumes that the same currents, namely $V^1_\lambda \pm iV^2_\lambda$, operate in all strangeness-conserving transitions. Examples of such transitions are

$$\pi^\pm \rightarrow \pi^0 e^\pm \nu, \quad \Sigma^\pm \rightarrow \Lambda e^\pm \nu, \quad \nu^\pm \mu \rightarrow \mu^- \nu^+,$$

where $A$ and $B$ have equal strangeness quantum number. Note that the currents $V^1_\lambda \pm iV^2_\lambda$ have the quantum numbers of $\pi^\pm$ (excluding parity) in the sense that they carry $I = 1$, $I_3 = \pm 1$, and $S = 0$, where $S$ = strangeness.

6.3 Some consequences and tests of CIVC

The electromagnetic form factors of the neutron and the proton are quite well known. By CIVC, the weak form factors $f_1^{NP}(t)$ and $f_1^{pn}(t)$ are related to the electromagnetic form factors of the nucleons. In order to find the exact relationships, we must simply project out the matrix element of the isovector part of the electromagnetic current (see Discussion). We find

$$f_1^{pn}(t) = f_1^{NP}(t) = f_1(t) = F_1^P(t) - F_1^n(t)$$

(6.16)
\[ f_2^{p} = f_2^{np}(t) = f_2(t) = F_2^p(t) - F_2^n(t) \]  \hspace{1cm} (6.17)

\[ f_3^{p} = f_3^{np}(t) = f_3(t) = 0 \]  \hspace{1cm} (6.18)

where the electromagnetic form factors of the neutron are defined analogously as done in formula (6.6) for the proton. The normalization of the electromagnetic form factors of the neutron is

\[ F_1^{n}(0) = 0, \quad F_2^{n}(0) = \mu_n. \]  \hspace{1cm} (6.19)

Relation (6.19) together with the normalization relation for the proton electromagnetic form factors \([F_1^p(0) = 1, F_2^p(0) = \mu_p]\) may be substituted in formulae (6.16) and (6.17). We find

\[ f_1(0) = 1 \]

which was the relation leading to the CIVC hypothesis, and

\[ f_2(0) = \mu_p - \mu_n. \]  \hspace{1cm} (6.20)

It is not possible to test the prediction (6.20) in the neutron \(\beta\)-decay because the momentum transfer is too small. However, the existence of \(f_2(0)\) with predicted sign and magnitude has been established\(^5\) in the processes \(B^{12} \rightarrow C^{12} + e^- + \bar{\nu}_e\) and \(N^{12} \rightarrow C^{12} + e^+ + \nu_e\) where the energy releases are very large (13 and 16 MeV, respectively) in the nuclear \(\beta\)-decay scale.

CIVC hypothesis has been further tested\(^6\) in the decay \(\pi^+ \rightarrow \pi^0 e^+ \nu_e\), where again agreement is found with theory.

6.4 What about the axial vector current?

In the previous paragraphs we discussed the isospin structure of the vector current for \(\Delta S = 0\) transitions as well as its conservation. What about the axial current? Again, by economy, one assumes that the axial currents in \(\Delta S = 0\) transitions are the \(1 \pm i2\) members of a vector in the isospin space \(\vec{A}_\lambda = (A_\lambda^1, A_\lambda^2, A_\lambda^3)\). Thus these currents have again the quantum numbers of \(\pi^\pm\) in that they carry \(I = 1, I_3 = \pm1\) and \(S = 0\).
However, there is an essential difference between the vector and axial currents for $\Delta S = 0$ transitions, i.e. the axial vector is not conserved $\partial A_\lambda / \partial x_\lambda \neq 0$. Evidence against the conservation of the axial vector current comes from the charged pion decays:

$$\pi^\pm \rightarrow \mu^\pm \nu$$

(with 100% branching ratios) which are forbidden if

$$\frac{\partial A_\lambda}{\partial x_\lambda} = 0$$

(see Exercise 3, Section 7).

Conclusions of this section:

i) The hadronic currents responsible for strangeness-conserving transitions are $V$-$A$ currents, where $V$ and $A$ are isospin-raising or isospin-lowering currents, namely:

$$h^{S=0}_\lambda = v_\lambda^1 + iv_\lambda^2 - (A_\lambda^1 + iA_\lambda^2),$$

$$h^{S=0\dagger}_\lambda = v_\lambda^1 - iv_\lambda^2 - (A_\lambda^1 - iA_\lambda^2).$$

ii) $V^\lambda \equiv j^\text{em, isovector}_\lambda$ and thus $\partial V^k_\lambda / \partial x_\lambda = 0$, $k = 1, 2, 3$, whereas the axial vector current cannot be conserved $\partial A_\lambda^{1\pm i2} / \partial x_\lambda \neq 0$.

We shall see later on that the hadronic strangeness-conserving current is slightly modified in the Cabibbo theory, i.e.

$$h^{S=0}_\lambda = \cos \theta_C (v^{S=0}_\lambda - A^{S=0}_\lambda), \quad v^{S=0}_\lambda = v_\lambda^1 + iv_\lambda^2, \text{ etc.},$$

where $\theta_C$ is the Cabibbo angle ($\sin \theta_C \sim 0.26$, i.e. $\cos \theta_C \approx 1$).

* * *
Question: How do we write down the matrix elements of \( \langle p | V_\lambda | n \rangle \) and/or \( \langle p | A_\lambda | n \rangle \) on general grounds?

Answer: Consider \( \langle p | V_\lambda | n \rangle \). This quantity must transform as a four-vector in momentum space. Furthermore, it must contain a \( \bar{u}_p \) for the emitted proton and a \( u_n \) for the absorbed neutron. These Dirac spinors carry the relevant information on four-momenta, spin, etc. of the particles. Therefore, we write

\[
\langle p | V_\lambda | n \rangle \sim \bar{u}_p \{ \ldots \} u_n,
\]

where \( \{ \ldots \} \) carries the (Minkowski) index \( \lambda \). The only quantities carrying such an index at our disposal are \( \gamma_\lambda, p_\lambda, \) and \( n_\lambda \), where \( p_\lambda (n_\lambda) \) is the four-momentum of the proton (neutron). Thus we write

\[
\langle p | V_\lambda | n \rangle \sim \bar{u}_p \left\{ \gamma_\lambda A + \gamma_\lambda B + C \gamma_\lambda D + E \gamma_\lambda + F n_\lambda \right\} u_n.
\]

Note that \( A, B, C, \) and \( D \) are scalars under Lorentz transformations, however, they could include products of \( \gamma_\mu \) and \( \gamma_n \), i.e. they do not necessarily commute with \( \gamma_\lambda \). We may use the Dirac equation, and its conjugate, which in our case are \( \bar{u}_p \gamma_p = m_\lambda \bar{u}_p \), and \( \gamma_n u_n = m_\lambda u_n \) and the anticommutation relations \( \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu \nu} \) for \( \gamma \)-matrices to get rid of all factors \( \gamma_\mu \) and \( \gamma_n \), e.g.

\[
\bar{u}_p \gamma_n \gamma_\lambda u_n = \bar{u}_p \left[ -\gamma_\lambda \gamma_n + 2n_\lambda \right] u_n = -m_\lambda \bar{u}_p \gamma_\lambda u_n + 2n_\lambda \bar{u}_p u_n
\]

(Convince yourself by taking a few examples). The terms in \( \{ \ldots \} \) can thus be reduced to three terms; namely, \( \gamma_\lambda, p_\lambda, \) and \( n_\lambda \) each multiplied by a form factor (function of the momentum transfer between \( p \) and \( n \)) without any \( \gamma \)-matrices. Thus we find

\[
\langle p | V_\lambda | n \rangle \sim \bar{u}_p \left\{ K_1(t) \gamma_\lambda + K_2(t)p_\lambda + K_3(t)n_\lambda \right\} u_n.
\]
Note that, since the nucleons are on the mass shell, there is only one variable

\[ t = (n - p)^2 , \]

and \( K_1(t) \) are form factors. We have to do a little more work to show the equivalence of this result to formula (6.1); namely,

\[
\begin{align*}
\bar{u}_p \gamma^\nu q_p u_n &= \frac{i}{2} \bar{u}_p \left[ \gamma_\lambda \gamma^{(p-n)} - \gamma^{(p-n)} \gamma_\lambda \right] u_n \\
&= \frac{i}{2} \bar{u}_p \left[ \gamma_\lambda \gamma^p - m_n \gamma_\lambda - m_p \gamma_\lambda + \gamma^n \gamma_\lambda \right] u_n \\
&= \frac{i}{2} \bar{u}_p \left[ -\gamma^p \gamma_\lambda + 2 \gamma_\lambda - \left( m_n + m_p \right) \gamma_\lambda - \gamma^n \gamma_\lambda + 2n_\lambda \right] u_n \\
&= i \bar{u}_p \left[ (p+n)_\lambda - \left( m_n + m_p \right) \gamma_\lambda \right] u_n .
\end{align*}
\]

The second term here can now be added to the \( K_1(t) \gamma_\lambda \) term and the form of Eq. (6.1) results.

The arguments leading to the form (6.2) for \( \langle p | A_\lambda | n \rangle \) are analogous to the above.

**Question:** How do we find the relations (6.16) to (6.18)?

**Answer:** The matrix element of the electromagnetic current between two nucleons is of the form (we drop uninteresting normalization factors)

\[
\langle N(p') | J_{\lambda}^{\text{em}} | N(p) \rangle = \left[ \bar{u}_p(p'), \bar{u}_n(p') \right] \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} u_p(p) \\ u_n(p) \end{pmatrix} .
\]

Any diagonal matrix \( \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \) can be written as a linear combination of the identity and \( \tau_3 \) matrices

\[
\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \frac{a + b}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{a - b}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{a + b}{2} I + \frac{a - b}{2} \tau_3 .
\]

Comparing with Eq. (6.13) we see that the coefficient of \( \tau_3/2 \) gives the isovector form factors. In our case
\[ a = F_1^p(t)\gamma_\lambda - i \frac{\sigma_{\lambda\nu} q^\nu}{2M} F_2^p(t) \]
\[ b = F_1^n(t)\gamma_\lambda - i \frac{\sigma_{\lambda\nu} q^\nu}{2M} F_2^n(t) . \]

Thus
\[ f_1(t) = F_1^p(t) - F_1^n(t) , \]
\[ f_2(t) = F_2^p(t) - F_2^n(t) , \]
\[ f_3(t) = 0 . \]

* * *

**EXERCISES**

1) We have shown that (see the remarks at the end of this Section) the matrix element of a vector operator between two spin-\(\frac{1}{2}\) particles generally depends on three form factors. Thus the most general form of the matrix element of the electromagnetic current between two protons is
\[
\langle p(p')|J^\text{em}_\lambda(0)|p(p)\rangle = \frac{1}{\sqrt{4p_0p'_0}} \bar{u}(p') \left[ \gamma_\lambda F_1^p(t) - i \frac{\sigma_{\lambda\nu} q^\nu}{2M} F_2^p(t) + \frac{q_\lambda}{2M} F_3^p(t) \right] u(p) .
\]

Show that the conservation of the electromagnetic current \(\partial J^\text{em}_\lambda/\partial x_\lambda = 0\) implies \(F_3^p(t) = 0\).

* * *
REFERENCES
(Section 6)


3) See, for example, the compilation by A. Kropf and H. Paul, Z. Phys. 267, 129 (1974).


6) P. Depommier et al., Nuclear Phys. B4, 189 (1968), and references cited therein.
7. DECAYS $\pi^\pm \rightarrow \ell^\pm + \nu$ and $K^+ \rightarrow \ell^+ + \nu$

One very important experimental fact about the decays of charged pions $\pi^\pm$ and kaons $K^\pm$ is that they decay about $10^4$ times more often to the final state $\mu\nu$ as compared to $e\nu$ although the phase space favours the electronic modes. According to the compilation of the Particle Data Group

$$R^\pi = \frac{\Gamma(\pi \rightarrow ev)}{\Gamma(\pi \rightarrow \mu\nu)} = (1.24 \pm 0.03) \times 10^{-4},$$

(7.1)

and

$$R^K = \frac{\Gamma(K \rightarrow ev)}{\Gamma(K \rightarrow \mu\nu)} = (2.17 \pm 0.32) \times 10^{-5}.$$

(7.2)

As we shall show in this section, these experimental facts are very well explained by the conventional current-current theory.

Putting together what has been learned from $\mu$-decay and (nuclear and nucleon) $\beta$-decay, the conventional theory postulates that the weak interaction Lagrangian is given by

$$\mathcal{L} = \frac{G}{\sqrt{2}} (\ell_\lambda + h_\lambda)(\ell^\lambda + h^{\lambda\dagger}).$$

(7.3)

Here $\ell_\lambda$ is the leptonic current (see Eq. 5.3), and $h_\lambda$ is the hadronic current consisting of a vector part and an axial vector part. This Lagrangian, by construction, describes $\mu$-decay and $\beta$-decay. Here we shall consider the decays $\pi^\pm \rightarrow \ell^\pm \nu$ and $K^+ \rightarrow \ell^+ \nu$ where the crossed terms $\ell_\lambda h^{\lambda\dagger}$ and $\ell^{\lambda\dagger} h_\lambda$ giving rise to semi-leptonic processes*) may be studied and the current-current nature of $\mathcal{L}$ may be tested. Note also that in pion decays the strangeness is conserved, while in the kaon decays the strangeness changes by $\pm 1$.

*) In the conventional theory, a weak semi-leptonic process is a process in which a charged lepton together with its neutrino (or antineutrino) and one or more hadrons take part, e.g.

$$\nu + H \rightarrow \ell + H' , \quad \pi^\pm \rightarrow \ell^\pm + \nu , \quad \Lambda \rightarrow p + \ell + \nu ,$$

where $H$ and $H'$ are hadrons.

Weak processes of the type

$$\nu + H \rightarrow \nu + H'' , \quad \ell + H \rightarrow \ell + H'' ,$$

where again $H$ and $H''$ are hadrons, are referred to as semi-leptonic processes mediated by neutral currents. Note that these are forbidden by the conventional theory. We shall discuss them further in Part II.
7.1 Decay matrix elements

The matrix element for the decay

\[ M^-(q) \rightarrow \ell^-(k) + \bar{\nu}(k') , \quad \ell^- = (e^- \text{ or } \mu^-) , \quad \bar{\nu} = (\bar{\nu}_e \text{ or } \bar{\nu}_\mu) , \]

where \( M^- \) is either \( \pi^- \) or \( K^- \), and \( q, k, \) and \( k' \) refer to four-momenta, is given by

\[
\mathcal{M}(M^- \rightarrow \ell^- + \bar{\nu}) = \frac{G}{\sqrt{2}} \langle 0 | h_\lambda | M^-(q) \rangle \frac{u_\ell \gamma^\lambda (1 - a_\lambda \gamma_5)v_\nu}{\sqrt{k_0 k'_0}} .
\] (7.4)

Here we have introduced a constant, denoted by \( a_\lambda \) in front of \( \gamma_5 \), in order to keep track of how the results depend on the V-A structure of the leptonic current. For V-A coupling \( a_\lambda = 1 \) and by \( \mu-e \) universality

\[ a_e = a_\mu . \]

By our hypothesis \( h_\lambda \) consists of vector (\( V_\lambda \)) and axial vector (\( A_\lambda \)) parts. However, since the pions and kaons are pseudoscalar particles, the matrix element \( \langle 0 | V_\lambda | M^+(q) \rangle \) vanishes (Exercise 1) and we are left with \( \langle 0 | A_\lambda | M^-(q) \rangle \). This matrix element must be proportional to \( q_\lambda \) because that is the only four-momentum at our disposal (the pions and kaons have spin 0 and therefore there is no \( \gamma_\lambda \) term). We define

\[
\langle 0 | A_\lambda | M(q) \rangle = \frac{f_M q_\lambda}{\sqrt{2} q_0} ,
\] (7.5)

where \( f_M \) is called the meson (pion or kaon) decay constant. Note that there is no form factor in Eq. (7.5) because the meson is on its mass shell and therefore the only invariant one may construct, namely \( q^2 \), is in fact a constant, \( q^2 = m_M^2 \), where \( m_M \) is the mass of pion or kaon.

The essential point here is that the hadronic matrix element, formula (7.5), is proportional to the four-momentum of the decaying particle \( q_\lambda \). The \( q_\lambda \) multiplied into the leptonic current in formula (7.4) gives the quantity
\[ q_{\lambda} \bar{u}_{\lambda}(k) \gamma^{\lambda}(1 - a_{\lambda} \gamma_{5})v_{\nu}(k') = \bar{u}_{\lambda}(k)\gamma_{k}(1 - a_{\lambda} \gamma_{5})v_{\nu}(k') = \]
\[ = \bar{u}_{\lambda}(k)(\gamma_{k} + \gamma_{k'})^{(1 - a_{\lambda} \gamma_{5})}v_{\nu}(k') = \]
\[ = \bar{u}_{\lambda}(k)\gamma_{k}(1 - a_{\lambda} \gamma_{5})v_{\nu}(k') + \]
\[ + \bar{u}_{\lambda}(k)(1 + a_{\lambda} \gamma_{5})\gamma_{k'}v_{\nu}(k') = \]
\[ = m_{\lambda} \bar{u}(k)(1 - a_{\lambda} \gamma_{5})v_{\nu}(k') . \]  

(7.6)

In formula (7.6) we have used conservation of energy and momentum, \( q = k + k' \),
the Dirac equation, \( \bar{u}(k)\gamma_{k} = m_{\lambda} \bar{u}(k) \), \( \gamma_{k'}v_{\nu}(k') = -m_{\nu}v_{\nu}(k') \),
and \( m_{\lambda} = 0 \).

The proportionality to \( m_{\lambda} \) (i.e. the axial nature of the transition
\( M \to \) vacuum) is indeed the reason for the suppression of the electronic
decay modes. Using formulae (7.4-7.6), after doing spin summations
and putting in the phase-space factor \( T \) (see Appendices A and B) we find

\[ \Gamma(M \to \nu) = \left[ \frac{G_{M}^{2}M_{M}^{2}}{16\pi} (1 + a_{\lambda}^{2}) \right] m_{\lambda} \left( 1 - \frac{m_{\lambda}^{2}}{m_{\lambda}^{2}} \right)^{2} . \]

(7.7)

Here \( \Gamma \) denotes the decay rate, \( f_{M} \) is the decay constant, and \( m \) with appro-
priate subscript refers to the mass.

Assuming \( \mu-e \) universality, in the ratio \( R^{M} \), the factor \( \frac{f_{M}}{f_{\nu}} \) in relation
drops out. We find

\[ R^{M} = \frac{\Gamma(M \to \nu)}{\Gamma(M \to \mu\nu)} = \left( \frac{m_{e}}{m_{\mu}} \right)^{2} \left[ \frac{m_{M}^{2} - m_{\nu}^{2}}{m_{M}^{2} - m_{\mu}^{2}} \right]^{2} . \]

(7.8)

Note that the suppression of the electron mode is due to the factor
\( (m_{e}/m_{\mu})^{2} \sim 2 \times 10^{-5} \). The second factor in the r.h.s. of Eq. (7.8) is
about 5 for pion decay and is near unity in kaon decay.

Numerically we have

\[ R_{\text{theory}} = 1.28 \times 10^{-4} \]
\[ = (1.23) \times 10^{-4} , \]

(7.9)
where the number in parenthesis is the theoretical prediction including radiative correction (due to diagrams below)

\[
\begin{align*}
\pi^+ & \rightarrow \gamma^* \rightarrow \gamma^* \rightarrow \ell^+ \nu \\
\pi^- & \rightarrow \gamma^* \rightarrow \gamma^* \rightarrow \ell^- \nu \\
\pi^0 & \rightarrow \gamma^* \rightarrow \gamma^* \rightarrow \ell^0 \nu
\end{align*}
\]

Thus the experimental result (7.1) is in excellent agreement with the theoretical prediction.

For kaons

\[ R^K_{\text{theory}} = 2.58 \times 10^{-5}, \]

again in accord with experiment, Eq. (7.2).

The agreement between theory and experiment is indeed very good. Below, we discuss some implications of the results given above.

i) Does it test the V-A structure of leptonic current?

The results do not depend on the V-A structure of the leptonic current, i.e. the factor \(1 + a^2\) drops out in the ratio \(R^M\) if \(\mu-e\) universality is assumed. However, if one assumes that the leptonic current should be a mixture of V and A, the agreement between theory and experiment favours the hypothesis of universality although it does not prove it; for example, taking \(a_\mu = -a_\mu\), which violates universality, would give the same ratios \(R^\pi\) and \(R^K\).

ii) Absence of pseudoscalar interaction.

As discussed previously, the most general weak interaction Lagrangian could in principle include scalar (S), pseudoscalar (P), vector (V), axial vector (A), and tensor (T) couplings. Since pions and kaons are pseudoscalar particles, the S, V, and T interactions do not contribute to \(M \rightarrow \ell\nu\) decays. However in addition to the axial vector interaction discussed above, the pseudoscalar interaction could contribute. In that case the matrix element for M due to P would be
8. **EMPIRICAL FACTS ABOUT STRANGENESS-CHANGING PROCESSES**

In this section we shall mainly be concerned with semi-leptonic processes, because they are theoretically much simpler than non-leptonic processes. Semi-leptonic processes may be divided into two classes, namely the strangeness-conserving and the strangeness-changing reactions. Examples of observed semi-leptonic strangeness-conserving and strangeness-changing decays are

\[ \Delta S = 0 \quad \Delta S = 1 \]

\[
\begin{align*}
\pi^+ & \rightarrow \pi^+ + \nu \\
K^+ & \rightarrow \pi^+ + \nu \\
n & \rightarrow p + e^- + \bar{\nu} \\
\Lambda & \rightarrow p + e^- + \bar{\nu}, \\
\Sigma^+ & \rightarrow \Lambda^0 + \pi^+ + \nu, \text{ etc.} \\
\Sigma^+ & \rightarrow n + e^+ + \nu, \text{ etc.}
\end{align*}
\]

Before the Cabibbo theory a number of empirical rules were extracted from study of semi-leptonic and non-leptonic decays, namely:

a) suppression of the \( |\Delta S| = 1 \) decay as compared to \( \Delta S = 0 \);

b) absence of \( |\Delta S| \geq 2 \) decays (semi-leptonic and non-leptonic);

c) the \( \Delta S = \Delta Q \) rule (semi-leptonic)

Below we shall discuss these rules and give the present experimental evidence favouring them.

8.1 **Suppression of strangeness-changing decays**

In the previous section, we found that the strength of strangeness-changing decays \( K^+ \rightarrow \pi^+ + \nu \) (measured by the decay constant which does not involve kinematical factors such as phase-space, etc.) is considerably smaller than the strength of the strangeness-conserving decays \( \pi^+ \rightarrow \pi^+ + \nu \), namely:

\[
\left| \frac{f_K}{f_{\pi}} \right| \approx 0.27.
\]

*) A process where no leptons are involved is called non-leptonic, e.g. \( \Lambda \rightarrow p\pi^- \), \( K^+ \rightarrow \pi^+\pi^+\pi^- \), etc. These processes are assumed to be due to the terms \( h^+ h^+ \) of the conventional theory [Eq. (7.3)].

**) In this section we restrict ourselves to semi-leptonic processes mediated via charged currents, i.e. the processes arising from the terms \( h^+ h^+ \) and \( h^+ h^+ \) of the conventional theory [see Eq. (7.3)].
Similar order of magnitude estimates can be made for other semi-leptonic processes, and one finds that the suppression of $|\Delta S| = 1$ semi-leptonic processes compared to $\Delta S = 0$ ones hold generally.

8.2 Absence of $|\Delta S| > 1$ decays (semi-leptonic or non-leptonic)

All decay processes satisfy $|\Delta S| \leq 1$, where $\Delta S$ is the difference of the strangeness between initial and final states. Accurate tests of this rule in decay processes is difficult, because only in $\Xi$ and $\Omega$ decays the strangeness may change by more than one unit.

The best present limit comes from the study of $\Xi^0$ decays,

$$\Xi^0 \to p\pi^- \quad \Delta S = S_i - S_f = -2$$
$$\Xi^0 \to \Lambda\pi^0 \quad \Delta S = -1$$

where one finds $^*)$ \[ \frac{\Gamma(\Xi^0 \to p\pi^-)}{\Gamma(\Xi^0 \to \Lambda\pi^0)} < 3.5 \times 10^{-5} \text{ , } 90\% \text{ C.L.} \]

8.3 $\Delta S = \Delta Q$ rule$^1$)

The $\Delta S = \Delta Q$ rule applies ONLY to semi-leptonic processes

$$H_i \to H_f + \ell + \nu ,$$

(8.1)

$$\ell + \nu = e^-\bar{\nu}_e \text{ or } (e^+\nu_e) \text{ or with } \mu^-\bar{\nu}_\mu , \ldots$$

where $H_i$ is the decaying hadron and $H_f$ denotes one or more hadrons. We define

$$\Delta S = S_i - S_f , \quad \Delta Q = Q_i - Q_f ,$$

where $S_i, Q_i$ are the strangeness and charge of the initial hadron, etc.

$^*)$ Note that the mode $\Xi^0 \to p\pi^-$ is favoured by phase space. The experimental results (unless indicated) are taken from the compilation by the Particle Data Group.
In the decays (8.1) the leptons together carry a +1 or −1 unit of charge. Therefore ΔQ = ±1. Experimentally one finds that decays with ΔS = −ΔQ are very much suppressed as compared to the ones with ΔS = ΔQ.

In baryon decays, the only present evidence for the ΔS = ΔQ rule comes from the comparison of the modes Ξ⁺ → nK⁺ν (ΔS = −ΔQ) and Ξ⁻ → nK⁻ν (ΔS = ΔQ), where

\[
\frac{\Gamma(\Xi^+ → nK^+\nu)}{\Gamma(\Xi^- → nK^-\nu)} < 0.035 .
\]

In the future, the rule may also be tested in the decay Ξ⁰ → Σ⁻K⁺ν (ΔS = −ΔQ) and Ξ⁰ → Σ⁺K⁻ν (ΔS = ΔQ) and perhaps even in the Ω⁻ decay.

Further evidence for the ΔS = ΔQ rule comes from the comparison of the decays K⁺ → π⁺π⁺e⁻νₑ (ΔS = −ΔQ), K⁺ → π⁺π⁻e⁺νₑ (ΔS = ΔQ), K⁰ → π⁺e⁻νₑ (ΔS = −ΔQ) and K⁰ → π⁻e⁺νₑ (ΔS = ΔQ). One finds

\[
\frac{\Gamma(K^+ → π^+π^+e^-ν_e)}{\Gamma(K^+ → π^+π^-e^+ν_e)} < 0.01 ,
\]

and

\[
\frac{A(K^0 → π^-e^+ν_e)}{A(K^0 → π^+e^-ν_e)} = 0.04 ± 0.03 - i(0.06 ± 0.05) , \quad \text{(Ref. 2)}
\]

where A stands for the amplitude.

In the next Section, we shall show that the rules (a), (b) and (c) follow naturally from the Cabibbo theory.

* * *

REFERENCES

(Section 8)


9. CABIBBO THEORY

Cabibbo theory is a natural extension of the isotriplet vector and axial-vector current hypothesis in the light of the discovery of the invariance of strong interactions under SU(3). We remind the reader that by the isotriplet vector and axial-vector current hypothesis (which was based on the invariance of strong interactions under isospin) the strangeness-conserving vector and axial-vector currents are the \( 1 \pm i2 \) (isospin-raising and -lowering) members \(^a\) of vectors in the isospin space \( V_{\lambda}^{S=0} \) and \( A_{\lambda}^{S=0} \); \( S \) denotes the strangeness.

9.1 Allowed and forbidden transitions

The matrix elements of the vector and axial-vector currents are directly measured in semi-leptonic processes such as \( \nu + H \rightarrow \ell + H' \), \( H + H' \rightarrow \ell + \bar{\nu} \), where \( H \) and \( H' \) are hadrons and \( \ell \) denotes as usual the lepton associated with \( \nu \). For simplicity, we restrict ourselves to the cases where \( H \) and \( H' \) are single hadrons, baryons, or mesons.

The low-lying baryons and mesons are members of octets of SU(3) as shown in Fig. 9.1.

---

\(^a\) For example, by \( 1 \pm i2 \) members of \( V_{\lambda} \) we mean \( V_{\lambda}^1 \pm iV_{\lambda}^2 \), where we have suppressed the strangeness quantum number.
where $I_3$ denotes the third component of the isospin and $Y$ is the hypercharge; $Y = S + B$, $S =$ strangeness, and $B =$ baryon number.

By the isotriplet hypothesis, the allowed semi-leptonic strangeness-conserving transitions (in which a single hadron is changed into another) are transitions by one unit ($\Delta I_3 = \pm 1$) along the horizontal axis ($\Delta Y = \Delta S = 0$) in Fig. 4.1, e.g. \(^*\) $K^0 \rightarrow K^+$, $K^+ \rightarrow K^0$, $\pi^+ \rightarrow \pi^0$, $\Sigma^0, \Lambda^0 \rightarrow \Sigma^\pm$, $\Xi^- \rightarrow \Xi^0$, etc. Only some of these transitions are observed in decay processes, e.g.

\[ \pi^+ \rightarrow \pi^0 + e^+ + \nu, \]
\[ \Sigma^+ \rightarrow \Lambda + e^+ + \nu, \]
\[ n \rightarrow p + e^- + \bar{\nu}, \]

while many others are forbidden by energy-momentum conservation.

Allowed transitions for hypercharge changing (or strangeness changing semi-leptonic decays obey the rules $\Delta S = \Delta Q$ and $|\Delta S| \leq 1$ (see Section 8). For example, $K^{\pm} \rightarrow \pi^0$, $K^0 \rightarrow \pi^-$, $\Sigma^- \rightarrow n$, $\Lambda, \Xi^0 \rightarrow p, \Xi^-$ are allowed, while $\Sigma^+ \rightarrow n$, $\Xi^0 \rightarrow \Sigma^-$, $K^0 \rightarrow \pi^+$ are forbidden. We note that in all the allowed transitions $\Delta Y = \Delta Q = \pm 1$ and $\Delta I_3 = \pm \frac{1}{2}$. The simplest assumption one may make is that the strangeness-changing currents carry $I = \frac{1}{2}$. Thus the strangeness-changing vector and axial-vector currents carry the quantum numbers of $K^\pm$ (again disregarding parity), i.e. $S = Y = \pm 1$, $I = \frac{1}{2}$, and $I_3 = \pm \frac{1}{2}$.

9.2 The currents in terms of quark fields

The allowed transitions are seen most transparently in the triplet representation of SU(3) shown in Fig. 9.2, where $p$, $n$, and $\lambda$ are the three quarks

\(^*\) Only the hadrons involved in the transition are indicated; e.g. $K^0 \rightarrow K^+$ could indicate $K^0 \rightarrow K^+ + e^- + \bar{\nu}_e$, $\nu_e + K^0 \rightarrow K^+ + e^-$, etc.
with the \((I_3, Y)\) quantum number assignments

\[
p = (\frac{1}{2}, \frac{1}{3}), \quad n = (-\frac{1}{2}, \frac{1}{3}), \quad \lambda = (0, -\frac{2}{3})
\]

The allowed transitions (indicated by arrows in Fig. 9.2) are \(n \neq p\) (strangeness-conserving) and \(\lambda \neq p\) (strangeness-changing)*. In terms of the quark fields and SU(3) matrices \(\lambda_j (j = 1, \ldots, 8)\), the vector and axial-vector currents may be written as follows. The strangeness-conserving isospin-raising vector current turns an \(n\)-quark into a \(p\)-quark and is therefore given by

\[
\bar{\psi}_p (x) \gamma_\rho \gamma_5 \psi_n (x) = (\bar{p}, \bar{n}, \bar{\lambda}) \gamma_\rho \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p \\ n \\ \lambda \end{pmatrix} = 
\]

\[
= \bar{q} \gamma_\rho \frac{\lambda_1 + i \lambda_2}{2} q \equiv v_\rho^{1+i2},
\]

where we have used the notation \(p = \psi_p (x)\), etc.; \(q \equiv \begin{pmatrix} p \\ n \end{pmatrix}\). Analogously, the strangeness-conserving isospin-raising axial current is

* By superimposing Fig. 9.2 on Fig. 9.1 one finds the allowed transitions for the octets. This is so because only one quark (or antiquark) inside the hadron is taking part in the transition.
\[ A^{1+12}_\rho = \bar{q} \gamma_\rho \gamma_5 \frac{\lambda_1 + i\lambda_2}{2} q . \] (9.2)

The strangeness-raising vector current which turns a \( \lambda \)-quark into a \( p \)-quark may be written in the form

\[
\bar{\psi}_p(x)\gamma_\rho \psi_\lambda(x) = (\bar{p}, \bar{n}, \bar{\lambda}) \gamma_\rho \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p \\ n \\ \lambda \end{pmatrix} = \bar{q} \gamma_\rho \gamma_\mu + \frac{i\lambda_5}{2} q \equiv \psi^{\mu+5}_\rho
\]

(9.3)

Again the axial counterpart of this current is \( A^{4+5}_\rho \). Similarly the isospin-lowering and strangeness-lowering currents are

\[ \psi^{1-i2}_\rho = \bar{q} \gamma_\rho \frac{\lambda_1 - i\lambda_2}{2} q , \quad A^{1-i2}_\rho = \bar{q} \gamma_\rho \gamma_5 \frac{\lambda_1 - i\lambda_2}{2} q , \quad (9.4) \]

and

\[ \psi^{4-5}_\rho = \bar{q} \gamma_\rho \frac{\lambda_4 - i\lambda_5}{2} q , \quad A^{4-5}_\rho = \bar{q} \gamma_\rho \gamma_5 \frac{\lambda_4 - i\lambda_5}{2} q . \quad (9.5) \]

### 9.3 Cabibbo currents

In the Cabibbo theory\(^2\) the currents (vector as well as axial-vector) are assumed to be members of octets of SU(3). The octet of vector currents consists of eight operators\(^\ast\) \( V_j^\lambda (j = 1, \ldots, 8) \), where \( V^\pm_\lambda \equiv V^1_\lambda \pm iV^2_\lambda \) carry the quantum numbers \( y = S = 0, I = 1, \) and \( I_3 = \pm 1 \), and may be thought of as the \( \pi^\pm \) members of the octet of vector currents (again, we disregard parity). Similarly \( V^{4+5}_\lambda, V^{6+7}_\lambda, V^8_\lambda, \) and \( V^8_\lambda \) correspond respectively to the \( K^\pm, K^0(K^\mp) \), \( \pi^0 \), and \( \eta^0 \) members of the octet of vector currents.

\( \ast \) The octet property has to do with the transformation properties of the operator under rotations in SU(3)-space. The interested reader can find the details in the books cited in Ref. 1 at the end of this Section (see also Section 9.6 below).
Furthermore, eight axial octet operators $A^j_\lambda$ \(j = 1, \ldots, 8\) are assumed to exist. Again $\pi^\pm_\lambda$ are the $\pi^\pm$ members of the octet of axial currents, etc. We shall, therefore, introduce the notations*)

$$\pi^\pm_\lambda = V^1_\lambda \pm iV^2_\lambda - (A^1_\lambda \pm iA^2_\lambda), \quad (9.6)$$

and

$$K^\pm_\lambda = V^4_\lambda \pm iV^5_\lambda - (A^4_\lambda \pm iA^5_\lambda). \quad (9.7)$$

Note that these currents are explicitly of the $V$-$A$ form.

9.4 Cabibbo Lagrangian

Following the discussion above, the simplest assumption one may make for the hadronic current $h_\lambda$ is that it should be a linear combination of the $K^+_\lambda$ term (giving $\Delta S = \Delta Q = 1, \Delta I = \Delta I_3 = \frac{1}{2}$) and the $\pi^+_\lambda$ term ($\Delta S = 0, \Delta I = \Delta I_3 = 1$),

$$h^+_\lambda = a\pi^+_\lambda + bK^+_\lambda, \quad h^+_\lambda = a\pi^-_\lambda + bK^-_\lambda, \quad (9.8)$$

where $a$ and $b$ are real constants **).

We expect the constant $b$ to be smaller than $a$ since $|\Delta S| = 1$ transitions are suppressed as compared to $\Delta S = 0$.

The full Lagrangian now reads

$$\mathcal{L} = \frac{G}{\sqrt{2}} (\pi^+_\lambda + a\pi^+_\lambda + bK^+_\lambda)(\pi^+_\lambda + a\pi^-_\lambda + bK^-_\lambda).$$

This Lagrangian explains all the rules discussed in the previous section. The rule $\Delta S = \Delta Q$ follows by construction since, whenever the current $K^+_\lambda$ acts, $\Delta S = \Delta Q = 1$, and for $K^-_\lambda \Delta S = \Delta Q = -1$. There is no $|\Delta S| > 1$ transition allowed by this Lagrangian to order $G$.

*) Note that $\pi^\pm_\lambda$ and $K^\pm_\lambda$ are not appropriately normalized to be components of (irreducible) tensor operators.

**) That is, time reversal invariance is assumed to be valid. The Cabibbo theory does not account for the small violation of T-invariance observed in the $K^0 - \bar{K}^0$ system.
The full Cabibbo theory goes further. It assumes\(^2\) that

i) the hadronic current \(h_\lambda\) transforms according to the octet representation of \(\text{SU}(3)\);

ii) the vector part of \(h_\lambda\) is in the same octet as the electromagnetic current;

iii) The hadronic current has unit "length", i.e.

\[ a^2 + b^2 = 1 \quad , \quad a = \cos \theta_C \quad , \quad b = \sin \theta_C \quad , \]

where \(\theta_C\) is the Cabibbo angle\(^*\). Thus the Cabibbo Lagrangian is given by

\[
\mathcal{L} = \frac{G}{\sqrt{2}} \left\{ \lambda_\lambda^* + \cos \theta_C \pi^+ \lambda \pm \sin \theta_C K^\pm \lambda \right\} \times \\
\times \left\{ \lambda_\lambda^+ + \cos \theta_C \pi^- \lambda \pm \sin \theta_C K^\pm \lambda \right\} 
\]

(9.9)

where \(\pi^\pm \lambda\) and \(K^\pm \lambda\) are given by relations (9.6) and (9.7).

Below we examine the assumptions of the Cabibbo theory and their consequences in more detail.

9.5 Octet electromagnetic current and conservation of vector currents

First we examine the octet structure of the electromagnetic current. The electric charge \(Q\) is related to the \(\text{SU}(3)\) generators \(I_3\) and \(Y\) by the Gell-Mann-Nakano-Nishijima relation

\[
Q = I_3 + \frac{1}{2} Y .
\]

(9.10)

\(^*\) The numerical value of the Cabibbo angle is obtained empirically from meson and baryon decays as we shall discuss later. The dynamical origin of the Cabibbo angle, in spite of being the subject of several theoretical speculations, is still obscure.
In SU(3), $I_3$ and $Y$ can be expressed, in term of the zeroth component of octet vector currents, as

$$I_3 = \int d^3x \, V^0_\lambda(x,t) , \quad Y = \frac{2}{\sqrt{3}} \int d^3x \, V^\lambda(x,t) . \quad (9.11)$$

The electric charge is the space integral of the charge-density (zeroth component of the current)

$$Q = \int d^3x \, J^{em}_0(x,t) . \quad (9.12)$$

Thus, from (9.10) to (9.12) follows

$$\int d^3x \, J^{em}_0(x,t) = \int d^3x \, V^0_\lambda(x,t) + \frac{1}{\sqrt{3}} \int d^3x \, V^\lambda(x,t) .$$

Thus a **plausible assumption** for the electromagnetic current is

$$J^{em}_\lambda(x) = V^3_\lambda(x) + \frac{1}{\sqrt{3}} V^\lambda(x) . \quad (9.13)$$

Therefore the isovector part of the electromagnetic current is assumed to be the isospin current $V^3_\lambda(x)$ and the isoscalar electromagnetic current is (up to a numerical factor) the (octet) hypercharge current $V^\lambda_\lambda = 2/\sqrt{3} \, V^\lambda_\lambda$.

Note that $V^3_\lambda$ and $V^\lambda$ have respectively the quantum numbers of $\pi^0$ and $\eta^0$ (i.e. $I = 1$, $I_3 = 0$, $S = 0$, and $I = I_3 = S = 0$). Therefore we may write symbolically

$$J^{em}_\lambda(x) = \pi^0_\lambda(x) + \frac{1}{\sqrt{3}} \eta^0_\lambda(x) .$$

Assumption (ii) of the Cabibbo theory is thus a generalized conserved vector current hypothesis as follows. The vector currents are octet operators $V^j_\lambda (j = 1, \ldots, 8)$. By the conservation of the electromagnetic current, two members of this octet, namely, $V^3_\lambda$ (the isovector electromagnetic current) and $V^\lambda_\lambda$ (the isoscalar part), are conserved. If SU(3) is exact, then **all** the members should be conserved:

$$\frac{\partial V^j_\lambda}{\partial x^\lambda} = 0 (j = 1, \ldots, 8) .$$
Note that from the eight members of the octet of vector currents, six enter in interactions; namely, $K^\pm$, $\pi^\pm$ members in weak interactions and $\pi^0$ and $\eta^0$ in electromagnetic interactions. The remaining two are strangeness-changing neutral currents (with the quantum numbers of $K^0$ and $\bar{K}^0$) and seem to be absent (see Part II).

In the axial octet $A_\lambda^j (j = 1, \ldots, 8)$ the four charged members (namely, $\pi^\pm$, $K^\pm$) have established themselves in weak interactions. The strangeness-changing neutral members seem to be absent and the role (if any) played by the strangeness-conserving neutral members $A_\lambda^3$ and $A_\lambda^6$ is not known.

The assumption (iii) of the Cabibbo theory is known as universality. It may be derived by requiring the hadronic current to satisfy the same commutation relations as those of the leptonic current.

9.6 Matrix elements of $V_\lambda$ and $A_\lambda$ between octet states

By Cabibbo theory the structure of the hadronic current is known. Experiments measure the matrix elements of currents $\langle H_1 | V_\lambda | H_2 \rangle$, $\langle H_1 | A_\lambda | H_2 \rangle$, where $H_1$ and $H_2$ are hadrons, in semi-leptonic processes and the matrix element $\langle H_1 | h^\lambda h^\lambda_\ast | H_2 \rangle$ in non-leptonic reactions. Evidently, the matrix elements that are the easiest to analyse theoretically are the one-particle matrix element of $V_\lambda$ and $A_\lambda$ (i.e. when $H_1$ and $H_2$ are single hadrons). Further simplifications are expected to arise if we further restrict ourselves to decay processes, where the four-momentum transfer between the hadrons is small.

We consider first the semi-leptonic decays of baryons:

$$B_1 (Y, I, I_3) \to B_1 (Y_f, I_f, I_{3_f}) + \text{leptons,}$$  \hspace{1cm} (9.14)

where $Y$, $I$, and $I_3$ denote the hypercharge, isospin, and the third component of the isospin. Therefore, we need to calculate the matrix elements of the octet operators ($V_\lambda$ or $A_\lambda$) between two octet states. By the Wigner-Eckart theorem applied to the SU(3) scheme, the matrix element of an octet operator between two octet states may be expressed in terms of two reduced matrix elements (which contain the dynamics) and ordinary Clebsch-Gordan as

*) We do not consider here the cases where $H_1$ (or $H_2$) is the vacuum state (relevant, for example, to $\pi \to \nu \bar{\nu}$ and $K \to \nu \bar{\nu}$ decays).
well as SU(3) isoscalar (Clebsch-Gordan) coefficients which are tabulated in the Particle Data Group tables. There is, however, a slight complication, namely that the Wigner-Eckart theorem is valid only for irreducible tensor operators. In our case, the irreducible vector current operators \( V^1 \) are given by

\[
V^1_{\lambda}^{\nu, \pm 1} = \begin{cases} 
\frac{1}{\sqrt{2}} (V^1_{\lambda} + i V^2_{\lambda}) & \text{for } \nu = 1, 0 \\
\frac{1}{\sqrt{2}} (V^1_{\lambda} - i V^2_{\lambda}) & \text{for } \nu = 0, 0
\end{cases}, \quad V^0_{\lambda}^{0, 0} = V^3_{\lambda}.
\]

The Wigner-Eckart theorem applied to \( V \)'s reads

\[
\langle B_{1}(Y_f, I_f, I_3_f) | V^{Y, I, I_3}_{\lambda} | B_{1}(Y_i, I_i, I_3_i) \rangle = 
\]

\[
= \langle I_i, I_3_i, I, I_3 | I_f, I_3_f \rangle \sum_{\gamma = s, a} (Y_f, I_f, Y_i, I_i)_{\gamma} \times 
\]

\[
x (8 \parallel V_{\lambda} \parallel 8)_{\gamma}.
\]

(9.15)

The first factor in the r.h.s. of Eq. (9.15) is an ordinary Clebsch-Gordan coefficient, while the second one is the relevant SU(3) isoscalar factor for \( 8 \otimes 8 \rightarrow 8' \), where 8 denotes the octet. The index \( \gamma = s, a \) characterizes the symmetric (D-type) or antisymmetric (F-type) nature of the coupling. Finally \( (8 \parallel V_{\lambda} \parallel 8)_{\gamma} \) is the \( \gamma \)-type reduced matrix element. In our case, 8 denotes the baryon octet. Note that the isoscalar factors and the reduced matrix elements are usually different for \( \gamma = s \) and \( \gamma = a \).

The expression for the matrix element of the axial currents is obtained from Eq. (9.15) by changing \( V_{\lambda} \) to \( A_{\lambda} \).

9.6.1 Simplifications

Thus the amplitudes for all the decays (9.14) may be expressed in terms of four reduced matrix elements \( (8 \parallel V_{\lambda} \parallel 8)_{s, a} \) and \( (8 \parallel A_{\lambda} \parallel 8)_{s, a} \). On general grounds (see Discussion at the end of Section 6), we may write the reduced matrix elements in the form
\[ (8|V_\lambda|8)_{s,a} = \frac{\bar{u}(p')}{\sqrt{4p_0p'_0}} \left[ \gamma_\lambda f_{1\lambda}^D F(t) - i \frac{\sigma_{\lambda\nu}q^\nu}{2M} f_{2\lambda}^D F(t) + \frac{q_\lambda}{2M} f_{3\lambda}^D F(t) \right] u(p) , \]

\[ (8|A_\lambda|8)_{s,a} = \frac{\bar{u}(p')}{\sqrt{4p_0p'_0}} \left[ \gamma_\lambda g_{1\lambda}^D F(t) - i \frac{\sigma_{\lambda\nu}q^\nu}{2M} g_{2\lambda}^D F(t) + \frac{q_\lambda}{2M} g_{3\lambda}^D F(t) \right] \gamma_5 u(p) . \]

(9.16)

In (9.16) we have assumed that SU(3) symmetry is exact\(^*)\). The symbol \(8\) denotes the baryon octet with a mass \(M\); \(p\) and \(p'\) are the four-momenta of the initial and final baryons. The twelve quantities \(f_{1\lambda}^D(t)\) and \(g_{1\lambda}^D(t)\) are form factors, where \(t = (p' - p)^2\). The superscripts \(D\) and \(F\) denote respectively the form factors associated with the \(s-\) and \(a\)-type matrix elements.

Evidently, twelve form factors are too many to be determined from experiment. Fortunately, by the assumption (ii) of Cabibbo theory, the vector form factors are all related to the measured electromagnetic form factors and are therefore completely determined. The arguments are analogous to those given in Section 6, where the vector form factors were determined in terms of the isovector electromagnetic form factors (see further Discussion at the end of this Section). Note also that the form factors \(f_{3\lambda}(t)\) and \(f_{3\lambda}^F(t)\) must vanish because the vector current is conserved.

Having determined the vector form factors, we consider the axial form factors \(g_{1\lambda}^D(t)\). Fortunately, in decay processes, the \(g_{2\lambda}^D(t)\) and \(g_{3\lambda}^D(t)\) are expected to make small contributions, because they are multiplied with \(q^2/2M\), where \(|q^2/2M| \ll 1\). Thus we may neglect these form factors in the first approximation. A further simplification occurs because the momentum transfer between the baryons is small, whereby the approximations

\(^*)\) That is, the baryons within the octet have the same mass. The Cabibbo theory is only valid in the limit of exact SU(3) symmetry (where the baryons could not decay!). There is as yet no reliable method for calculating the SU(3) symmetry-breaking effects (see also Section 9.7).
\[ F_1(t) \approx g_1(0) , \quad \text{and} \quad D_1(t) \approx g_1(0) \quad (9.17) \]

should be satisfactory.

Thus, all the baryon decays in relation (9.14) are described with the help of three parameters, namely, \( F_1(0) \), \( D_1(0) \), and the Cabibbo angle \( \theta_C \).

### 9.7 Comparison of the Cabibbo theory with data

The semi-leptonic baryon decays \( B \to B' \ell \nu \), because of richness in variety, provide the best testing ground of the Cabibbo hypothesis. As discussed above, all these decays are described by only three parameters \( D, F \) and \( \theta_C \).

Experimentally, the measurement of the partial decay rate for \( B \to B' \ell \nu \) determines the magnitude of the corresponding \( g_1/f_1 \) (which depends only on \( D, F \) and \( \theta_C \)). Furthermore, the sign of this quantity is known, from using polarized baryons, for three decays, namely \( n \to p e^- \nu , \Lambda \to p e^- \nu \) and \( \Sigma^- \to n e^- \nu \).

The outcome of the latest Cabibbo fit \(^4\) is that the set

\[
\sin \theta_C = 0.230 \pm 0.003 ,
\]

\[ D/(D + F) = 0.658 \pm 0.007 , \]

\[ (\chi^2/DF = 8.4/8) \]

gives a very good description of the data.

The Cabibbo theory is supposed to be valid in the limit of exact SU(3)-symmetry. In the real world, the symmetry is broken and in some cases quite badly (e.g. the \( \pi \) and \( \eta \) are in the same multiplet, however \( m_\eta \approx 4 m_\pi \)). Unfortunately, it is not known how the symmetry breaking should be taken into account. In baryon decays, in a model calculation \(^5\) the SU(3)-breaking effects do not seem to matter much.

Finally, the Cabibbo angle obtained from pion and kaon decays is consistent with the above value from baryon decays.
\[ \langle p | V^0_{\lambda} | n \rangle = \langle \frac{1}{2}, -\frac{1}{2}, 1, 1 | \frac{1}{2}, \frac{1}{2} \rangle \sum_{\gamma} (N, \pi | N) \gamma (8 \| V_{\lambda} \| 8) \gamma , \]
\[ -\sqrt{2} \sigma_{3} \]
\[ \langle p | V^1_{\lambda} | \Lambda \rangle = \langle 0, 0, \frac{1}{2}, \frac{1}{2} | \frac{1}{2}, \frac{1}{2} \rangle \sum_{\gamma} (\Lambda, K | N) \gamma (8 \| V_{\lambda} \| 8) \gamma . \]
\[ 1 \]

From tables of the Particle Data Group, we have
\[ (N, \pi | N) = \frac{3\sqrt{5}}{10} , \]
\[ (N, \pi | N) = \frac{1}{2} , \]
\[ (\Lambda, K | N) = \frac{-\sqrt{5}}{10} , \]
\[ (\Lambda, K | N) = \frac{1}{2} . \]

Thus
\[ \langle p | h_{\lambda} | n \rangle = \frac{1}{\sqrt{3}} \cos \theta_{C} \left[ (8 \| V_{\lambda} \| 8)_{a} + \frac{3}{\sqrt{5}} (8 \| V_{\lambda} \| 8)_{s} \right] - \]
\[ \left[ (8 \| A_{\lambda} \| 8)_{a} + \frac{3}{\sqrt{5}} (8 \| A_{\lambda} \| 8)_{s} \right] \]  
\[ \text{(10)} \]

\[ \langle p | h_{\lambda} | \Lambda \rangle = -\frac{1}{\sqrt{2}} \sin \theta_{C} \left[ (8 \| V_{\lambda} \| 8)_{a} + \frac{1}{\sqrt{5}} (8 \| V_{\lambda} \| 8)_{s} \right] - \]
\[ \left[ (8 \| A_{\lambda} \| 8)_{a} + \frac{1}{\sqrt{5}} (8 \| A_{\lambda} \| 8)_{s} \right] \]  
\[ \text{(11)} \]

Using Eq. (9.16) and relations (5) to (7) above we find, in the low-momentum transfer limit (where the contribution of \( i = 2 \) and \( 3 \) form factors is neglected)
\[ \langle p | h_{\lambda} | n \rangle = \frac{1}{\sqrt{3}} \cos \theta_{C} \left[ \gamma_{\lambda} \left( f_{1}^{F}(t) + \frac{3}{\sqrt{5}} f_{1}^{D}(t) \right) \right] - \]
\[ \gamma_{\lambda} \gamma_{s} \left[ g_{1}^{F}(t) + \frac{3}{\sqrt{5}} g_{1}^{D}(t) \right] \]  
\[ \text{(12)} \]
\[ \langle p | h_\lambda | \Lambda \rangle = -\frac{1}{\sqrt{2}} \sin \theta_C \sum_{\sigma} \gamma_\lambda \left[ f_1^F(t) + \frac{1}{\sqrt{5}} f_1^D(t) \right] - \gamma_\lambda \gamma_5 \left[ g_1^F(t) + \frac{1}{\sqrt{5}} g_1^D(t) \right] u, \]

(13)

where we have suppressed the momenta, spins, normalization factors, and particle indices. Using the approximations \( f_1^D(t) \approx f_1^D(0) \) and \( g_1^D(t) \approx g_1^D(0) \), from relations (5), (6), (12), and (13), we find

\[ \langle p | h_\lambda | n \rangle = \cos \theta_C \sum_{\sigma} \gamma_\lambda \left[ \gamma_\lambda - \gamma_\lambda \gamma_5 (F + D) \right] u, \]

(14)

and

\[ \langle p | h_\lambda | \Lambda \rangle = -\frac{\sqrt{3}}{2} \sin \theta_C \sum_{\sigma} \gamma_\lambda \left[ \gamma_\lambda - \gamma_\lambda \gamma_5 (F + \frac{1}{\sqrt{5}} D) \right] u, \]

(15)

where

\[ F = \frac{1}{\sqrt{3}} g_1^F(0) \quad \text{and} \quad D = \frac{\sqrt{3}}{\sqrt{5}} g_1^D(0). \]

All other semi-leptonic (three-body) baryon decays can be determined analogously. Thus, all these processes are expressible in terms of the three parameters \( F, D, \) and \( \theta_C \).

**REFERENCES**

(Section 9)


3) M. Gell-Mann, Physics 1, 63 (1964).


10. CONCLUSIONS OF PART I AND REMARKS

In the previous Sections (2–9), we have studied the conventional current-current theory of the weak interactions based on the effective interaction Lagrangian

$$\mathcal{L} = \frac{G}{\sqrt{2}} \left[ \bar{\ell}_\lambda \gamma^{\mu}_{\lambda} \lambda^{\mu} + \bar{h}_\lambda \gamma^{\mu}_{\lambda} \lambda^{\mu} \right].$$  (10.1)

Here

$$\ell_\lambda = \bar{\psi}_e \gamma_\lambda (1 - \gamma_5) \psi_e + \bar{\psi}_\mu \gamma_\lambda (1 - \gamma_5) \psi_\mu,$$

$$h_\lambda = \cos \theta_c \left[ V^{1}_\lambda + i V^{2}_\lambda - (A^{1}_\lambda + i A^{2}_\lambda) \right] +$$

$$+ \sin \theta_c \left[ V^{4}_\lambda + i V^{5}_\lambda - (A^{4}_\lambda + i A^{5}_\lambda) \right],$$

where the notation is described in Sections 4, 5, and 9.

The simple form (10.1) is capable of describing very well the muon decay (Section 4) and the semi-leptonic decays of mesons and baryons mediated by charged currents*) (see Section 7 and 9).

In fact the predictions of the Lagrangian (10.1) for all semi-leptonic processes mediated by charged currents are in agreement with data. A very special class of such processes is the high-energy interactions of neutrinos†): no deviation from the form (10.1) has been established up to incoming neutrino energies of about 150 GeV.

We have not discussed the non-leptonic weak interactions, which are assumed to be due to the term $h_\lambda \lambda^{\mu}$ in the Lagrangian (10.1). The operator $h_\lambda \lambda^{\mu}$ is quite complicated, because it is the product of two octet currents. Therefore, the form (10.1) has little predictive power**) as far as non-leptonic processes are concerned.

The experimentally observed phenomena not described by the conventional theory [Eq. (10.1)] are the following:

*) i.e. where a neturino (or antineutrino) and a charged lepton are emitted.

**) i.e. the rule $\Delta I = \frac{1}{2}$ for non-leptonic hyperon decays ($\Lambda \rightarrow N\pi$, $\Sigma^{\pm} \rightarrow N\pi$, and $\Xi^{0} \rightarrow \Lambda\pi$) does not follow from the Lagrangian (10.1).
i) The small CP violation\(^2\) observed in the \(K^0 - \overline{K}^0\) system. All experiments on CP violation are in agreement with the superweak theory\(^3\), where an extremely small CP-violating term is added to the form (10.1).

ii) "Neutral current" processes. These are discussed in Section 13 and in the lectures given by D. Perkins at this School. Here we would like to mention that the observed semi-leptonic neutral current processes are believed to be \(\nu(\overline{\nu}) + N \rightarrow \nu(\overline{\nu}) + \) hadrons, where \(N\) is a nucleon. However, since in the final state only the hadrons may be observed, other possibilities such as production and subsequent decay of a short-lived heavy lepton cannot be excluded.

There is also no contradiction between the existence of strangeness-conserving neutral currents in neutrino interactions and low-energy data. Decays mediated by strangeness-conserving neutral currents are either very hard to see (such as \(\pi^0 \rightarrow \nu(\overline{\nu})\)) or are completely masked by the electromagnetic interactions.

The point we would like to emphasize here is that there is evidence against the existence of strangeness-changing neutral currents at low energies. For example, the ratio \(\Gamma(K^+ \rightarrow \pi^+ + \nu + \overline{\nu}) / \Gamma(K^+ \rightarrow \pi^0 e^+ \nu)\) is less than \(6 \times 10^{-7}\), implying that the effective coupling constant for the strangeness-changing neutral current is less than \(^*\) \(G(\alpha/\pi) \sin \theta_C\). Accepting the evidence for the existence of neutral currents in neutrino interactions, a major question to be answered is, Why are the strangeness-changing neutral currents absent (at low energies)? We shall return to this question in Section 13.

Also the purely leptonic neutral current events (believed to be \(\overline{\nu}_\mu e \rightarrow \overline{\nu}_\mu e\)) observed in the Gargamelle bubble chamber at CERN are not explained by the form (10.1).

Finally, we shall repeat again that the traditional Lagrangian (10.1), in spite of being phenomenologically correct, is theoretically a disaster. It violates unitarity and gives higher-order weak interaction corrections

\(^*\) This is not the best limit, but it has the advantage of being clean (free of electromagnetic background). The tightest upper limit\(^*\) \((\nu G \alpha^2)\) comes from the analysis of the decay \(K_L \rightarrow \mu^+ \mu^-\).
which are formally infinitely large. These infinities may not be removed by renormalization techniques (which work so well in quantum electrodynamics). For these reasons the theoreticians are excited about the so-called gauge theories \(^5\) which are constructed to agree with all the known experimental data and do not suffer from the above-mentioned difficulties. However, they require the existence of as yet undiscovered particles (heavy vector bosons, heavy leptons, etc.).

Several topics, which are very important for a better understanding of the traditional weak interactions, are not discussed in these lectures. Among these we would like to mention:

i) symmetries (C, P, and T);
ii) pole-dominance of the axial vector current;
iii) current algebra.

The ambitious reader is referred to the literature \(^6\).

* * *

REFERENCES

(Section 10)

1) For an extensive review of neutrino interactions before the discovery of neutral currents, see C.H. Llewellyn Smith, Physics Reports 3 C, (1972). Recent results are discussed in D. Perkins, Lectures given at this School. See also P. Landshoff, Lectures given at this School.

2) For the most up to date review of the status of CP violation, see K. Kleinknecht Proc. 17th Int. Conf. on High-Energy Physics, London 1974.


4) See, for example, the compilation by K. Kleinknecht, loc. cit.

11. **INTERMEDIATE VECTOR BOSONS**

The electromagnetic interactions of the charged leptons ($e^\pm$ and $\mu^\pm$) are extremely well described by the Lagrangian of quantum electrodynamics

$$\mathcal{L}^{\text{em}} = e J^{\text{em}}_\lambda(x) A^\lambda(x)$$  \hspace{1cm} (11.1)

where $e$ is the electric charge, $A^\lambda(x)$ is the photon field, and $J^{\text{em}}_\lambda(x)$ is the leptonic electromagnetic current given by

$$J^{\text{em}}_\lambda(x) = \bar{\psi}_e(x) \gamma_\lambda \psi_e(x) + \bar{\psi}_\mu(x) \gamma_\lambda \psi_\mu(x).$$  \hspace{1cm} (11.2)

Note that the ($\mu$-$e$) universality (in electromagnetic interactions) is guaranteed by the form (11.2). Pictorially, $\mathcal{L}^{\text{em}}$ can be represented

$$\mathcal{L}^{\text{em}} = e \left( \begin{array}{c} e \varepsilon e \varepsilon \mu \varepsilon \mu \\ e \gamma_e \gamma \mu \end{array} \right) + \left( \begin{array}{c} \mu \varepsilon \mu \varepsilon e \varepsilon e \\ \mu \gamma_e \gamma e \end{array} \right),$$  \hspace{1cm} (11.3)

i.e. the electromagnetic interactions are mediated by a neutral vector (spin 1) boson (photon).

11.1 **Conventional theory of weak interactions rephrased in terms of $W^\pm$ bosons**

One may ask whether weak interactions are also mediated by one or more vector bosons (vector bosons because the currents are vector and axial-vector). To answer this question, let us consider the leptonic Lagrangian (see Section 5)

$$\mathcal{L}^{\text{lept}} = \frac{G}{\sqrt{2}} \left( \begin{array}{c} \nu \mu \\ \mu e \end{array} \right) \left( \begin{array}{c} \mu \\ \nu_e \end{array} \right) + \left( \begin{array}{c} \nu e \nu \mu \\ e \mu \nu_e \end{array} \right).$$

The simplest possible way of incorporating intermediate bosons is to couple the V-A currents to a charged vector boson field.

Pictorially*)

$$\mathcal{L}^{\text{lept}} = g \left( \begin{array}{c} \nu \mu \nu \mu \mu \mu \\ \mu e \mu e e \mu e \\ e \nu e \nu e \nu e \end{array} \right),$$

*) Note that the last two diagrams (terms) are the Hermitian conjugate of the first two. They must be added in order to make the Lagrangian (Hamiltonian) a Hermitian (real) operator.
i.e.

$$\mathcal{L}^{\text{lept}} = g \left[ \bar{\psi}_\mu \gamma_\lambda (1-\gamma_5) \psi_\mu + \bar{\psi}_e \gamma_\lambda (1-\gamma_5) \psi_e \right] W^{\lambda^+}_\mu + \text{h.c.} \quad (11.4)$$

Here $g$ is a dimensionless constant; $W^{\lambda^+}_\mu$ is the field of the charged vector boson (creates a $W^-$ and absorbs a $W^+$). With this modification all allowed leptonic processes -- such as $\mu$ decay, $\nu_{\mu} e$ scattering -- occur in the second order in $g$ via exchange of a virtual $W$ boson. For example, the muon decay is described by the diagram

and its matrix element is given by

$$M(\mu^- \to \nu_\mu e^+ \bar{\nu}_e) = g \bar{u}_{\nu_\mu} \gamma_\lambda (1-\gamma_5) u_\mu \bar{u}_{e} \gamma_\sigma (1-\gamma_5) v_e \frac{g_{\lambda\sigma} - Q^2 Q_{\lambda\sigma}/M_W^2}{M_W^2 - Q^2},$$

(11.5)

$$Q_\lambda = (q_\mu - q_{\nu_\mu})_\lambda = (q_e + q_{\nu_e})_\lambda. \quad (11.6)$$

In Eq. (11.5) the last factor in the matrix element is the $W$ propagator. $M_W$ is the mass of the vector boson, and for simplicity we have left out the normalization factors.

If the vector boson is heavy ($M_W^2 >> m_{\mu}^2$), the matrix element [Eq. (11.5)] is equivalent to that of the conventional theory as follows. The term proportional to $Q^2$ is negligible [namely, by Dirac equation this term is proportional to $m_{\mu} m_e / M_W^2$, cf. Eq. (7.6)]. Furthermore, the $Q^2$ in the denominator is negligible ($m_e^2 \leq Q^2 \leq m_\mu^2$) as compared to $M_W^2$. Thus the muon decay mediated via a virtual $W$, is equivalent to the point-like decay provided that

$$\frac{g^2}{M_W^2} = \frac{G}{\sqrt{2}}. \quad (11.7)$$
However, if $M_W$ were small, the agreement between theory and experiment would be spoiled because of the $W$-propagator. The lower limit on $M_W$ from $\mu$ decay is, however, quite low ($M_W \geq 2$ GeV).

Similarly, the entire conventional theory discussed in Part I may be rephrased in terms of the $W^\pm$ bosons. All one has to do is to couple the currents to the $W$ field. Thus the Cabibbo Lagrangian, written in the vector boson theory, is given by

$$\mathcal{L} = g \left\{ (\ell_\lambda^\dagger + h_\lambda^\dagger) W^{\lambda^\dagger} + (\ell_\lambda + h_\lambda) W^\lambda \right\}, \quad (11.8)$$

where $\ell_\lambda$ and $h_\lambda$ are the leptonic and hadronic currents discussed in Part I (namely, $h_\lambda = \cos \theta_C [\nu^1_\lambda + i\nu^2_\lambda - (A^1_\lambda + iA^2_\lambda)] + \sin \theta_C [\nu^3_\lambda + i\nu^4_\lambda - (A^3_\lambda + iA^4_\lambda)]$, or in terms of the quark fields $h_\lambda = \bar{q}_\gamma_\lambda (1 - \gamma_5) \left[ \cos \theta_C n + \sin \theta_C \lambda \right]$, where $n$ stands for $\psi_n$, etc.).

We emphasize again (just as shown above for the $\mu$ decay) that the phenomenology of low-energy processes, such as decays, in the vector boson theory is the same as in the conventional theory provided $W$ is heavier than about 2 GeV. However, the vector boson theory possesses a certain esthetic beauty which is lacking in the conventional theory (namely $(e, \nu_e)$, $(\mu, \nu_\mu)$, and hadrons communicate via exchanging $W^\pm$ bosons just as in quantum electrodynamics muons and electrons interact via exchanging virtual photons).

11.2 Decay modes of $W^\pm$

The vector bosons $W^\pm$ decays [according to the Lagrangian (11.8)] into known leptons or hadrons via

$$\begin{align*}
W^- &\rightarrow e^- + \bar{\nu}_e \\
&\rightarrow \mu^- + \bar{\nu}_\mu \\
&\rightarrow \text{hadrons},
\end{align*} \quad (11.9)$$

$$\begin{align*}
W^+ &\rightarrow e^+ + \nu_e \\
&\rightarrow \mu^+ + \nu_\mu \\
&\rightarrow \text{hadrons},
\end{align*} \quad (11.10)$$

(and the charge-conjugate processes for the decay of $W^\mp$). If there are heavy leptons (lighter than the $W$) other leptonic modes may also be possible (see Section 14).
The transition rate for the processes (11.9) is given by

\[ \Gamma(W \to \ell + \nu) = \frac{g^2 M_W}{6\pi} \left( 1 - \frac{m_\ell^2}{M_W^2} \right) \left( 1 + \frac{m_\ell^2}{2M_W^2} \right), \quad (11.11) \]

where we have used the formula (11.4). In the limit \( m_\ell / M_W \ll 1 \), using relation (11.7) we find

\[ \Gamma(W \to \mu + \nu) + \Gamma(W \to e + \nu) = \frac{g^2 M_W}{3\pi} = \frac{GM_W^3}{3\pi \sqrt{2}}, \quad (11.12) \]

\[ GM_W^3 = GM_P^2 \cdot M_p \cdot \left( \frac{M_W}{M_p} \right)^3 \approx 10^{-5} \left( 1.4 \times 10^24 \right) \left( \frac{M_W}{0.94 \text{ GeV}} \right)^3 \text{ sec}^{-1}, \]

where we have expressed \( M_P \) in units of \( \text{sec}^{-1} \), namely \( M_p = c/\lambda_p \); \( \lambda_p \) is the Compton wavelength of the proton. Thus \( \Gamma(W \to \text{leptons}) \approx 1.3 \times 10^{18} (M_W/\text{GeV})^3 \text{ sec}^{-1} \), and the \( W \)-lifetime \( \tau \) is expected to be very short:

\[ \tau = \frac{1}{\Gamma(W \to \text{all})} < \frac{1}{\Gamma(W \to \text{leptons})} \approx \frac{8 \times 10^{-19}}{(M_W/\text{GeV})^3} \text{ sec}. \]

The transition rate for \( W \to \text{hadrons} \) may be estimated using the present experimental information\(^2\) on \( e^+e^- \to \text{hadrons} \) (as explained in the Discussion at the end of this Section). The relevant diagrams are

\[ \begin{array}{c}
\:\\ e^+\\
\gamma\\
e^-\\
\end{array} \quad \begin{array}{c}
hadrons\\\end{array} \quad \begin{array}{c}
W\\\end{array} \quad \begin{array}{c}
hadrons\\\end{array} \]

We estimate (see Discussion)

\[ \Gamma(W \to \text{hadrons}) = \frac{GM_W^3}{4\pi \sqrt{2}} R(M_W^2), \quad (11.13) \]
where
\[
R(s) = \frac{\Gamma(e^+ e^- \to \text{hadrons})}{\Gamma(e^+ e^- \to \mu^+ \mu^-)}
\]

at the centre-of-mass energy \(\sqrt{s}\). Thus the leptonic and hadronic branching ratios are given by
\[
\begin{align*}
\text{BR}(W \to \text{leptons}) &= 4/(4 + 3R) \\
\text{BR}(W \to \text{hadrons}) &= 3R/(4 + 3R),
\end{align*}
\]

where \(R\) is to be evaluated at \(s = M_W^2\). Note that the branching ratio to leptons may be very small if \(R(s)\) keeps on rising with \(s\) and if \(W\) is heavy. For example, if \(R(M_W^2) = 10\), the branching ratio to leptons would be about 12%.

11.3 Production of \(W^+\) and the present experimental situation

\(W\) bosons may be produced in a variety of processes. Here we shall simply list a number of possible mechanisms. The interested reader will find the answer to most of his questions in the articles listed in Ref. 3 at the end of this Section. The most obvious \(W^+\) production mechanisms are the following:

i) In neutrino interactions or in high-energy muon (or electron) interactions \(W^+\) may be produced in the Coulomb field of the nucleus, e.g.

\[
\nu + \text{nucleus} \rightarrow \mu^- + W^+ + \text{anything}
\]
ii) In electromagnetic processes $W$ may be pair-produced

\[ e^+ e^- \rightarrow W^+ W^- , \]

\[ \gamma + \text{nucleus} \rightarrow W^+ + W^- + \text{anything} \]

iii) In hadronic collisions, e.g.

\[ p + p \rightarrow W + \text{anything} . \]

The present experimental limit on the $W$ mass from the above processes is\(^4\) \( M_W > 4.4 \text{ GeV} \) (assuming the leptonic branching ratio is 50\%) obtained in a direct search for the $W$ in neutrino interactions.

Note that this lower limit is still quite low since the "gauge theories" in general require much heavier $W$ bosons. For example, in the Weinberg model (see Section 12) \( M_W > 37 \text{ GeV} \). If the $W$ is so heavy that its direct production is impossible with the present machines, one may look for it indirectly by studying the effects of the $W$ propagator. For instance, in \( \nu + N \rightarrow \mu^- + \text{anything} \)

the square of the propagator \( (q^2 - M_W^2)^{-2} \), \( q^2 = (q_\nu - q_\mu)^2 < 0 \) enters in the expression for the differential cross-section, and the cross-sections become substantially smaller than expected from scaling hypothesis (see also the lectures by D. Perkins given at this School).

The present limit on the $W$ mass (assuming scaling) is\(^5\) \( M_W \geq 10 \text{ GeV} \). In the future one hopes to be able to extend this limit\(^6\) to about 50 \text{ GeV} (or discover the $W$ boson).
11.4 Difficulties of the W-boson theory

The vector-boson version of the conventional theory of weak interactions, in spite of resembling quantum electrodynamics (which is a renormalizable theory, i.e. the higher-order corrections to the measurable quantities are predicted to be finite), is not a renormalizable theory. The reason for this great difference between the two theories is that the W boson is massive, while the photon is massless. A massive vector boson has three possible spin orientations, while the massless one has only two. This extra degree of freedom (longitudinal vector boson) makes the theory non-renormalizable (see the next Section).

* * *

DISCUSSION

**Question:** How does one estimate the branching ratio for W → leptons?

**Answer:** \( \Gamma(W \rightarrow \text{hadrons}) \) may be related to \( \Gamma(e^+e^- \rightarrow \text{hadrons}) \) as follows:

![Diagram showing the reaction e^+e^- → hadrons](image)

The transition rate for \( e^+e^- \rightarrow \text{hadrons} \), where we sum over all possible hadronic states, is given by

\[
\Gamma(e^+e^- \rightarrow \text{hadrons}) = \frac{e^6}{4EE's^2} \sum_f (2\pi)^4 \delta(Q - p_f) \sum_{\text{spin average}} |\bar{\nu}(q')\gamma^\lambda u(q)|^2 \times \langle f|J_\lambda^{\text{em}}|0\rangle^2 ,
\]

where \( s = Q^2 \), \( Q = q + q' \), \( q_0 = E \), \( q'_0 = E' \).

Here \( 4EE's^2 \) comes from our normalization and the square of the photon propagator; \( f \) denotes any hadronic final state with four-momentum \( p_f \). Since we want to compare the above two processes, we shall be interested in the value of the transition rates at \( s = Q^2 = M_W^2 \).
Similarly

$$\Gamma(W \to \text{hadrons}) = \frac{g^2}{2M_W} \sum_f (2\pi)^4 \delta(p_f) \sum_{\text{spin average}} \times
$$

$$\times |\varepsilon^\lambda(f|h_\lambda|0)|^2 .$$

(2)

Here $h_\lambda$ is the Cabibbo current and $\varepsilon^\lambda$ is the polarization vector of the $W$. Since the Cabibbo angle is small, we shall approximate $\theta_C \approx 0$, whereby

$$\langle f|h_\lambda|0\rangle = \langle f|V_{\lambda}^1 + iV_{\lambda}^2 - (A_{\lambda}^1 + iA_{\lambda}^2)|0\rangle ,$$

(3)

relevant to $W^+$ decay. Furthermore, by Eq. (9.13)

$$\langle f|J_{\lambda}^{\text{em}}|0\rangle = \langle f|V_{\lambda}^3 + \frac{1}{\sqrt{3}} V_{\lambda}^8|0\rangle .$$

(4)

Now we do the spin-averaging (using the formulae in Appendix A):

$$\sum_{\text{spin average}} |\bar{v}(q')\gamma^\lambda u(q)\langle f|J_{\lambda}^{\text{em}}|0\rangle|^2 =$$

$$= \frac{1}{4} \sum_{\text{spin}} \bar{v}(q')\gamma^\lambda u(q)\bar{v}(q)\gamma^\sigma v(q')\langle 0|J_{\sigma}^{\text{em}}|f\rangle \langle f|J_{\lambda}^{\text{em}}|0\rangle$$

$$= \frac{1}{4} \text{Tr} \left[\gamma_\lambda \gamma_\sigma \gamma_\gamma\right] \langle 0|J_{\sigma}^{\text{em}}|f\rangle \langle f|J_{\lambda}^{\text{em}}|0\rangle$$

(5)

where we have neglected the electron mass. Substituting relation (5) into Eq. (1), we find
\[ \Gamma(e^+ e^- \rightarrow \text{hadrons}) = \frac{2e^4}{16EE's^2} \text{Tr}[\ldots] T_{\sigma\lambda} , \]  
(6)

where \( \text{Tr}[\ldots] \) stands for the trace in Eq. (5) and

\[ T_{\sigma\lambda} = \sum_f \frac{(2\pi)^4}{2} \delta(Q - P_F) \langle 0 | J_{\sigma}^{\text{em}} | f \rangle \langle f | J_{\lambda}^{\text{em}} | 0 \rangle . \]  
(7)

\( T_{\sigma\lambda} \) is a tensor; it depends on only one four-momentum, namely \( Q \); thus it must be of the form \( Q_\sigma Q_\lambda F_1(s) + g_{\sigma\lambda} F_2(s) \), where \( s = Q^2 \) and \( F_i(s) \) are as yet arbitrary functions. However, the electromagnetic current is a conserved current, i.e. \( (P_F)^\sigma \langle 0 | J_{\sigma}^{\text{em}} | f \rangle = 0 \), etc. Since \( P_F^\sigma = Q^\sigma \), we have \( Q_\sigma T_{\sigma\lambda} = Q_\lambda T_{\sigma\lambda} = 0 \). Therefore, our tensor must be of the form

\[ T_{\sigma\lambda} = (Q_\sigma Q_\lambda - Q^2 g_{\sigma\lambda}) W_1(s) , \]  
(8)

where \( W_1(s) = -F_1(s) = F_2(s)/s \). Multiplying Eq. (8) with \( g_{\sigma\lambda} \) and summing over \( \sigma \) and \( \lambda \) we find

\[ T^\sigma = (Q^2 - 4Q^2) W_1(s) = -3s W_1(s) , \]

i.e.

\[ W_1(s) = -\frac{1}{3s} T_{\sigma\sigma} . \]  
(9)

Putting Eq. (8) into Eq. (6), we may calculate \( \Gamma(e^+ e^- \rightarrow \text{hadrons}) \). Note that

\[ Q_\sigma \text{Tr}[\gamma q' \gamma^\lambda q q^0] = \text{Tr}[\gamma q' \gamma^\lambda q (q q + q q)] = \text{Tr}[\gamma q' (q q)^2] + \text{Tr}[\gamma q' q (q q)] = 0 , \]

because \( (q q)^2 = q^2 = m_e^2 = 0 \) in our approximation. Furthermore

\[ g_{\sigma\lambda} \text{Tr}[\gamma q' \gamma^\lambda q q^\sigma] = \text{Tr}[\gamma q' \gamma \gamma q q^\lambda] = -8qq' = -4s . \]  
\[ -2\gamma q \]
Thus
\[ \Gamma(e^+ e^- \rightarrow \text{hadrons}) = \frac{e^4}{2E E'} W_1(s) = \frac{(4\pi \alpha)^2}{s} W_1(s) = \frac{16\pi^2 \alpha^2 W_1}{s} \] (10)

where \( \alpha \) is the fine-structure constant and we have used \( s = (q + q')^2 = 2m E' \), where \( E' \) is the energy of the positron in the lab. Using the fact that \( \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) = 4\pi \alpha^2/3s \), and the velocity of the incoming positron is unity, we have

\[ R(s) = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} = 12\pi W_1(s) . \] (11)

The expression (9) for \( W_1 \) leads to

\[ W_1(s) = \frac{-1}{3s} \sum_f (...) \langle 0 \mid V^3 + \frac{1}{\sqrt{3}} V^8 \mid f \rangle \langle f \mid V^3 + \frac{1}{\sqrt{3}} V^8 \mid 0 \rangle = \]

\[ = \frac{-1}{3s} \sum_f (...) \left[ \langle 0 \mid V^3 \mid f \rangle \langle f \mid V^3 \mid 0 \rangle + \frac{1}{3} \langle 0 \mid V^8 \mid f \rangle \langle f \mid V^8 \mid 0 \rangle \right] \approx \]

\[ \approx \frac{-1}{3s} \times \frac{4}{3} \sum_f (...) \langle 0 \mid V^3 \mid f \rangle \langle f \mid V^3 \mid 0 \rangle , \] (12)

where the dots stand for \( (2\pi)^6 \delta(Q - P_f)/2; \) the indices \( \sigma \) have been suppressed and we have assumed that the contributions from \( V^3 V^3 \) and \( V^8 V^8 \) are equal \([SU(3)]\)-symmetry]. Note that these are no interference terms because if \( f \) has isospin zero (one) it can only enter in the matrix elements of \( V^8(V^3) \).

We now consider the process \( W \rightarrow \text{hadrons} \). From Eqs. (2) and (3), we find

\[ \Gamma(W \rightarrow \text{hadrons}) = \frac{8^2}{M_w} \epsilon^\lambda \epsilon^\sigma \ T'^\prime_{\sigma \lambda} , \] (13)

where

*) We neglect the muon mass.
\[
T'_{\sigma \lambda} = \sum_f \frac{(2\pi)^4}{2} \delta(m - p_f) \langle 0 | h_{\sigma} | f \rangle \langle f | h_{\lambda} | 0 \rangle .
\]

(14)

Again, we may write

\[
T'_{\sigma \lambda} = (Q \lambda Q - g_{\lambda \sigma} Q^2) W'_1(s) + Q \sigma Q W_2(s) .
\]

(15)

We shall now assume that the hadronic current is conserved, whereby \( W_2 \) vanishes and

\[
W'_1(s) = \frac{-T'_{\sigma \lambda}}{3s}
\]

\[
= \frac{-1}{3s} \sum_{f} (\ldots) 0 | V^1 - iV^2 - (A^1 - iA^2) | f \rangle \langle f | V^1 + iV^2 - (A^1 + iA^2) | 0 \rangle
\]

(16)

\[
\approx \frac{-4}{3s} \sum_{f} (\ldots) 0 | V^1 | f \rangle \langle f | V^1 | 0 \rangle = 3W_1(s) .
\]

In deriving relation (16), we have used the same notations and approximations as in Eq. (12). Moreover, we have assumed that the strength of the axial-vector and vector currents are equal. From relations (13), (15), and (16) we find (note that \( \epsilon Q = 0 \), \( s = Q^2 = M_W^2 \))

\[
\Gamma(W \rightarrow \text{hadrons}) = \frac{-g^2 M_W^2}{3M_W^2} (\epsilon^\lambda \epsilon_{\lambda}) 3W_1(s) ,
\]

(17)

where the factor \( \frac{1}{3} \) come from spin-averaging. Now we use \( g^2/M_W^2 = G/\sqrt{2} \) and

\[
\epsilon^\lambda \epsilon_{\lambda} = -(\epsilon^\lambda Q_{\lambda} / M_W^2) = -3 .
\]

Thus

\[
\Gamma(W \rightarrow \text{hadrons}) = \frac{3GM_W^3 W_1(s)}{\sqrt{2}} ,
\]

i.e. using Eq. (11)

\[
\Gamma(W \rightarrow \text{hadrons}) = \frac{GM_W^3}{4\pi\sqrt{2}} R(M_W^2) .
\]

(18)
12.1 Neutral currents and heavy leptons

The bad high-energy behaviour of the diagram (12.1) may be remedied in two ways:

i) By introducing a neutral vector boson Z:

![Diagram of Z and W^+ and W^-](image)

**Fig. 12.2**

and arranging the couplings such that the diagram (12.2) cancels the bad high-energy behaviour of the diagram (12.1). The existence of Z would introduce neutral vertices (neutral currents) into the theory.

ii) The cancellation mechanism also works if a heavy*) lepton L is introduced in the "crossed channel", namely

![Diagram of L^+ and W^- and W^+](image)

**Fig. 12.3**

Here (by lepton number conservation) L^+ carries the same leptonic number as ν. Thus we need four heavy leptons L^+ = M^+ or E^+, where (ν_e, E^+, e^-) carry n_e = 1 and their antiparticles n_e = -1, and

*) Heavy, because it has not yet been discovered.
similarly for muon-type heavy leptons. Note that if we introduce
the heavy lepton in the direct channel [i.e. we add a diagram simi-
lar to (12.1) where $\ell^-$ is replaced by $L^-$] no cancellation occurs,
but the new diagram is as badly divergent as the original one.

Although the two possibilities above are the simplest for getting rid
of the worst divergences (quadratic), one may construct more complicated
schemes, where both neutral vector bosons and heavy leptons occur.

The electromagnetism comes in when one considers processes such as
$e^+e^- + W^+W^-$ via the diagrams

![Diagrams](image)

(a)  (b)  (c)

Fig. 12.4

The diagram (a) diverges; adding the diagrams (b) and (c) (or one with
a heavy neutrino) and choosing the couplings properly gives the needed
cancellation mechanism. It has been shown\(^2\) that the couplings obtained
in this fashion are those of a Yang-Mills gauge theory\(^3\). Note that
couplings occurring in weak interactions [relevant to the diagrams (b)
and (c)] are thus related to the coupling constant in the diagram (a)
which is the electric charge (unification of weak and electromagnetic
interactions).

Although the worst divergences (quadratic) may be eliminated by in-
troducing neutral vector boson(s) or heavy leptons or both, there remains
some logarithmically divergent terms [namely, $\sigma \sim \ln (s)$, etc.]. These
divergent terms are removed by introducing a number of scalar bosons
(Higgs bosons) into the theory. The Higgs bosons are introduced via the
Higgs' mechanism\(^3\) which gives masses to the particles without spoiling
the renormalizability of the theory.
12.2 An example of gauge theories: 
the Weinberg-Salam model\(^4,5\)

This is the most popular gauge model, as yet not excluded by data. It was also the first model to be constructed\(^4\). The model requires three vector bosons \(W^+, W^-, Z\) and one neutral scalar Higgs boson \(\phi^0\). There is only one additional parameter in the model, namely the Weinberg angle \(\theta_W\). The leptonic vertices occurring in this model are pictorially

\[
\begin{align*}
\ell & \rightarrow W^+ \rightarrow \nu \\
\nu & \rightarrow Z \rightarrow \ell
\end{align*}
\]

The specific coupling constants are given in Ref. 4 and in the lecture notes by D. Perkins. The hadronic sector of the model is more complicated\(^6\), because one must incorporate the well-established Cabibbo structure and suppress the unwanted strangeness-changing neutral currents (see also Sections 13 and 15).

* * *

REFERENCES

(Section 12)

1) For a review of the unified theories of weak and electromagnetic interactions, see

2) C.H. Llewellyn Smith, Phys. Letters 46 B, 233 (1973) and references quoted therein.

3) See the review articles cited in Ref. 1.


5) A. Salam, in Elementary Particle Theory (ed. N. Svartholm, Almqvist and Wiksell, Stockholm, 1968).

13. **NEUTRAL CURRENTS**

Neutral current processes in neutrino interactions are covered in some detail in the lectures given by Perkins at this School. Therefore, our discussion here will be brief, especially since there is as yet no indication of neutral currents outside the domain of neutrino interactions.

If a neutral boson such as a Z exists, it would give rise to neutral current processes. However, a short-lived heavy lepton may fake such processes (see Section 14).

Neutral current processes may be divided into three classes; namely, leptonic, semi-leptonic, and non-leptonic processes.

13.1 **Leptonic neutral currents**

i) In neutrino interactions, the cleanest experimentally accessible neutral current processes are $\nu_\mu (\bar{\nu}_\mu) + e \rightarrow \nu_\mu (\bar{\nu}_\mu) + e$. These reactions are forbidden by the conventional theory, but could take place via the diagram

![Diagram](image)

**Fig. 13.1**

Note that these processes cannot be faked by the production and subsequent decay of a heavy lepton, because the available energy is not enough for producing a heavy object; namely, the variable $s \approx 2m_e E_\nu$ is only 0.1 (GeV)$^2$ for $E_\nu = 100$ GeV. As mentioned previously, two candidates for the process $\bar{\nu}_\mu e + \nu_\mu e$ have been observed\(^1\) in the Gargamelle bubble chamber at CERN. In the context of the Weinberg model, the limit on the Weinberg angle is $0.1 < \sin^2 \theta_W < 0.5$. For the diagonal processes (allowed by the conventional theory) only an upper limit is available for $\bar{\nu}_e e$ scattering (see Section 5.3).
ii) In traditional electromagnetic interactions, the neutral currents would cause deviations\(^2\) from the predictions of quantum electrodynamics, e.g. in

\[ e^+e^- \rightarrow (\text{virtual } \gamma, Z, \phi^0) \rightarrow \mu^+\mu^- . \]

Furthermore, because of Z or \( \phi^0 \) exchange, parity would no longer be conserved in such processes. Owing to the propagator of the exchanged particle, \( 1/(s-m^2) \), the Z exchange should become experimentally observable at high energies. The \( \phi^0 \) exchange is, however, expected to be very small even at high energies.

It is interesting to note that neutral currents also modify the traditional atoms, e.g. the muonium \( (\mu^+e^-) \) bound by electromagnetic forces now becomes

\[
\begin{align*}
&\gamma \quad \mu \quad e \\
&\mu \quad e
\end{align*}
\quad +
\begin{align*}
&Z \quad \mu \\
&\mu \quad e
\end{align*}
\]

Because Z is heavy the second diagram gives a point-interaction. One can calculate the corrections to the energy levels [which are very small i.e. less than \( 10^{-6} \text{ eV} \)] and to the hyperfine splitting of the ground state. If the neutral current coupling is too strong, the agreement between the theory and experiment will no longer hold. However, the present limit is not very good, and with the Weinberg model we find that neutral-current coupling cannot be stronger than 400 times the weak interaction constant.

13.2 Semi-leptonic neutral current

There is now evidence from CERN\(^1\), FNAL\(^3\) and ANL\(^4\) for inelastic neutrino processes where no charged lepton emerges in the final state, namely:

\[
\begin{align*}
\nu_\mu + (p,n) &\rightarrow (\nu_\mu) + \text{hadrons}, \\
\bar{\nu}_\mu + (p,n) &\rightarrow (\bar{\nu}_\mu) + \text{hadrons},
\end{align*}
\]

and where the produced neutrinos cannot be observed.
The responsible diagram could be

where the strangeness of the hadrons is not known. From the decay processes, we know that the strangeness-changing neutral currents (if they exist at all) are highly suppressed at low energies (see Sections 10 and 15).

Assuming that the neutral currents seen in neutrino interactions are strangeness-conserving, one may ask, Why have such processes not been seen in ordinary decay processes? and why are the strangeness-changing neutral currents absent?

The answer to the first question is left to the reader, who may try to write down a number of strangeness-conserving neutral current reactions and convince himself that they all are very hard to detect or are masked by electromagnetic effects.

Nobody knows the reason for the suppression (absence?) of the strangeness-changing neutral currents. The phenomenon could be "explained" if one introduces a fourth quark $p'$, called charm, which carries a new quantum number and which conspires to suppress the strangeness-changing neutral current (see Section 15). Other possibilities have also been put forward, but so far none of these stands on firm ground.

Semi-leptonic neutral currents effect lepton-nucleon or nucleus scattering and, related to it, the ordinary and muonic atoms. The upper limit on the strangeness conserving neutral-current coupling constant from the hyperfine splitting in hydrogen (which is one of the most accurate measurements in physics) is about 60 G. This limit could be greatly improved if the two-photon and size corrections were better known.
13.3 Non-leptonic neutral currents

Here, as always when one is dealing with non-leptonic interactions, the situation becomes complicated. It is also difficult to isolate the effects due to the neutral currents from those due to the charged currents. For instance, parity-violation effects in non-leptonic reactions and in nuclear transitions have been very well established. However, the theoretical predictions based on the conventional theory are usually model-dependent, and there is no over-all agreement between theory and experiment. The existence of neutral currents could drastically alter a number of the previous predictions. However, since the nature of the neutral currents is not yet known, no definite statement may be made. We must wait for future experiments.

Finally, it is generally believed that the neutral currents may play a substantial role in astrophysics in connection with the cooling of the stars and the production of supernovae.

* * *

REFERENCES

(Section 13)

1) See, for example, A. Rousset, Neutral currents, Invited talk at the 4th Int. Conf. on Neutrino Physics and Astrophysics, Downington, Pennsylvania 1974.

2) See, Ref. 3, Section 3.


14. HEAVY LEPTONS

In this Section we shall consider the decay modes and the production mechanism for the "gauge" heavy leptons $M^+$ and $E^\pm$ introduced in Section 12. However, one should keep in mind that, if there are any heavy leptons, they might be of a different kind. In fact, in the past, many heavy leptons have been invented, some in order to resolve the then existing discrepancies between theory and experiment.

Here we do not consider the heavy neutrinos $M^0$, $E^0$, etc., since in a large class of models they may not be produced by neutrinos\(^1\). Our discussion here will be brief; more detailed analysis of the production and decay of the "gauge" heavy leptons may be found in the literature\(^1-4\).

14.1 Decay modes

Remembering that $M^\pm$ have the same muonic number as $\mu^\pm$, the possible decay modes are

\[ M^+ \to \nu_\mu + \mu^+ + \nu_\mu, \quad (14.1) \]

\[ M^+ \to \nu_\mu + e^+ + \nu_e, \quad (14.2) \]

\[ M^+ \to \nu_\mu + \text{hadrons}, \quad (14.3) \]

Similar reactions may be written for $E^+$ and the heavy anti-leptons. $M^+$ may also decay into other heavy leptons.

The rate for the process (14.2) in the limit $M^\pm_w \gg m$, where $m$ is the heavy lepton mass, is obtained from the rate for $\mu$ decay by replacing $m_\mu$ with the heavy lepton mass $m$, namely $\Gamma(M \to \nu e\nu) = G^2m^5/(192\pi^3)$, provided the coupling constant for $M^+\nu_\mu W^+$ vertex is also $g$. The rate for the processes (14.1) turns out to be twice as large\(^1\) (because of the identical $\nu$'s). Thus\(^*)

\[^*)\text{The life-time of the such a heavy lepton must be shorter than} \]
\[ \frac{1}{3}(m_\mu/m)^5 T_\mu \approx 10^{-11}(1/m)^5, \text{ m = heavy lepton mass in GeV.} \]
\[ \Gamma(M^+ \rightarrow \text{leptons}) = \frac{G^2}{64\pi^3} m^5. \]

The rate for the hadronic mode (14.3) may be estimated, using the same method as that used for \( W \rightarrow \text{hadrons} \) (see Discussion, Section 11). Again, the larger the ratio \( R(s) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-) \), in the interval \( 0 \leq s \leq m^2 \), the smaller the leptonic branching ratio. For example, extrapolating the present data on \( R(s) \), for a heavy lepton mass of 10 GeV, the leptonic branching ratio \(^3\) is about 20%. 

14.2 Production mechanism

Heavy leptons may be pair-produced in colliding \( e^+e^- \) beams, e.g.

\[ e^+e^- \rightarrow M^+M^- \rightarrow \nu_\mu + \bar{\nu}_\mu + \text{hadrons} \]

\[ \rightarrow \nu_\mu + e^+ + \nu_e + \bar{\nu}_\mu + \mu^- + \bar{\nu}_\mu, \text{ etc.} \]

where the second chain would fake lepton non-conservation, which has not been seen in the present machines. In any case, with the available energies only leptons with mass less than a few GeV may be produced. At the present time the best place to look for the heavy leptons \( M^\pm \) is in the neutrino interactions \(^*\). For \( M^+ \), we would have

\[ \nu_\mu + (p,n) \rightarrow M^+ + \text{hadrons} \]

\[ \rightarrow \mu^+\nu_\mu\bar{\nu}_\mu \text{ or } e^+\nu_e\nu_\mu \text{ or } \nu_\mu \text{ hadrons}. \]

Note that the first two decay modes fake lepton number non-conservation, while the third one would look like a neutral current process. These effects are in themselves very dramatic. However, one must also consider the experimental conditions, e.g. a \( \mu^+ \) could also be produced by the \( \bar{\nu}_\mu \) impurity in the \( \nu_\mu \) beam, etc. We shall not go into a more careful analysis of these processes but refer the interested reader to the literature \(^1-4\))

\(^*\) The flux of \( \nu_e \) and \( \bar{\nu}_e \) is much smaller than those of \( \nu_\mu \) and \( \bar{\nu}_\mu \) (namely, the \( \pi^0 \)'s and \( K^0 \)'s prefer to decay into \( \mu^- \)'s rather than \( e^- \)'s). Therefore \( E^\pm \) is experimentally less accessible.
14.3 Experimental situation

The heavy lepton \( M^+ \) has been looked for in \( \nu_\mu \) interactions in the Gargamelle bubble chamber\(^5\) (where positrons were searched for) and in a counter experiment\(^6\) at FNAL (where one was sensitive to the \( \mu^+ \) decay mode). In Gargamelle, no positrons were seen and the limit \( m > 2.4 \) GeV was obtained. In the experiment at FNAL, eight events with a secondary \( \mu^+ \) were found. These events were found to be consistent with the expected background due to the \( \bar{\nu}_\mu \) contamination in the \( \nu_\mu \) beam. The limit obtained was \( m \geq 8.4 \) GeV. Note that the mass limits depend on the assumptions made on the \( \bar{\nu}_\mu \) \( W \) coupling constant and the leptonic branching ratio.

* * *

REFERENCES

(Section 14)

2) A. Soni, preprint CO-2271-23 (to be published).
15. **CHARMED PARTICLES**

The notion charm was introduced\(^1\) (already) in 1964. The original motivation for its invention was that since there are four leptons, there should also be four quarks, the fourth one being a charmed quark. More recently, charm is used\(^2\) to suppress the unwanted strangeness-changing neutral currents. The argument is as follows.

In addition to the three quarks \(p, n, \) and \(\lambda\), one introduces an additional quark \(p'\) which has the same quantum numbers as a \(p\)-quark, except for the fact that it is an isosinglet \((I = 0)\) and carries the charm quantum number \(C = 1\). The ordinary quarks have \(C = 0\) and charm is supposed to be conserved in strong interactions. With four quarks, the symmetry scheme for hadrons will no longer be \(SU(3)\) but instead \(SU(4)\), with dramatic consequences for the spectrum of hadrons: One would expect many new bosons and baryons. For example, corresponding to the ordinary proton (made up of \(pnp\)) then will a charmed proton (made up of \(p'pn\)), etc.

15.1 **The need for charm**

The Cabibbo current, in terms of the quark fields, is 

\[
h_{\lambda} = \bar{\nu}_{\lambda} Y_{\lambda} (1 - \gamma_5)(n \cos \theta + \lambda \sin \theta),
\]

where \(\theta\) denotes the Cabibbo angle. In the \(W\)-boson version of the conventional theory, this current gives an effective strangeness-changing neutral current (e.g. it gives \(K_L^0 - K_S^0\)) and also \(|\Delta S| = 2\) transitions contributing to the \(K_L - K_S\) mass difference) that are estimated to be much too large. The relevant diagrams are

![Diagram](image)

**Fig. 15.1**
Note that $\bar{\nu}_\lambda$ in the initial state may be thought of as $\bar{K}^0$ and $\bar{\nu}_\lambda$ as $K^0$. The calculation of these diagrams is made with the help of an arbitrary cut-off parameter $\Lambda$, and the diagrams (a) and (b) are of the order $G(\Lambda^2)$. In order to suppress the effects to the level of experiment, one must take a very small value of the cut-off (a few GeV), which is quite unsatisfactory.

Suppose the charmed quark $p'$ exists and (by assumption) enters into the Cabibbo current in the form $^2$\textsuperscript{2)*)}

$$h_{\lambda, \text{charmed}} = \bar{p}' \gamma_\lambda (1 - \gamma_5)(-m \sin \theta + \lambda \cos \theta).$$

Then, to every diagram with a $p$ exchange, we must also add one with a $p'$ exchange, namely:

The leading (most divergent) term in these diagrams is independent of the masses. This term has the coefficient $\cos \theta \sin \theta$ from $p$ exchange and $-\sin \theta \cos \theta$ from $p'$ exchange and thus vanishes. The next leading term depends on the mass of the quarks. If the mass difference between the $p$ and $p'$ quarks is too large, the unwanted strangeness-changing neutral current and $|\Delta s| = 2$ transitions become unbearably large compared to experiment. Thus one concludes that the low-lying charmed hadrons should not be heavier than a few GeV. Strangeness-changing neutral currents and $|\Delta s| = 2$ transitions show up also in some of the gauge models (such as the Weinberg-Salam model) and are "cured" by charm.

\textsuperscript{2)*) The form (15.1) follows uniquely from only a few assumptions (see Ref. 2).
15.2 Decay of charmed particles

The charmed particle decays weakly. To see the decay modes, we add to the Cabibbo current the charmed piece [Eq. (15.1)]. Then (from the product of the leptonic current with the charmed hadronic current), the charmed quark $p'$ can undergo $\beta$-decay:

\[
p' \rightarrow n\bar{\nu},
\]

\[
p' \rightarrow \lambda\bar{\nu},
\]

where $\bar{e}^+ = \mu^+$ or $\epsilon^+$. Because of the structure (15.1), the rate for the strangeness-conserving process (15.2) is proportional to $\sin^2 \theta$ and for the strangeness changing to $\cos^2 \theta$. This is the major signature of a charmed particle. It decays preferably into strange particles. The lifetime of the charmed baryon is expected to be at least $(\cos \theta / \sin \theta)^2$, i.e. $\sim 20$ times shorter than the lifetime of hyperons. The lifetime could be further reduced by probably the fifth power of the ratio of the masses of $\Lambda$ and the charmed baryon. This gives a lifetime shorter than $10^{-12}$ sec, if the charmed particle is about twice as heavy as the $\Lambda$. Further reduction in the lifetime is expected owing to the non-leptonic decay modes of $p'$.

The charmed bosons can be made out of a quark-antiquark pair, where one of the constituents is charmed, e.g. $(\bar{n}p')$, $(\bar{\Lambda}p')$, etc. Their decay modes depend on their masses, namely they could decay into other charmed particles or leptons.

15.3 Production of charmed particles

Charmed particles may be produced in pairs via strong and electromagnetic interactions:

\[
pp \rightarrow C^+ C' + \text{anything}
\]

\[
e^+ e^- \rightarrow C^+ C^-, \text{etc}.
\]
The charmed particles would then decay (rapidly) non-leptonically or semi-leptonically. The semi-leptonic decay has a clear signature, namely the charged lepton is usually accompanied by strange hadrons. Furthermore, in some fraction of the events one should find $e^+\mu^-$, $e^-\nu^+$ pairs, where, for example, $e^+$ is a decay product of a $C^+$ and $\mu^-$ that of $C^-$. In neutrino interactions, the charmed quark is produced via the elementary neutrino-quark process

$$\nu + n \rightarrow p' + \pi^-,$$  \hspace{1cm} (15.4)

which is, unfortunately, inhibited by the factor $\sin^2 \theta$. If the process (15.4) is followed by the decay (15.3), the over-all process would look, for instance, like

$$\nu_{\mu} + (p,n) \rightarrow \mu^- \nu^+ + \text{hadrons (} s \neq 0 \text{)}$$

$$\rightarrow \mu^- e^+ + \text{hadrons (} s \neq 0 \text{)},$$

where $s$ denotes the strangeness. Near threshold, most of the available energy goes into producing the charmed particle. Therefore, we would expect $\mu^-$ to be slow while $\mu^+$ (or $e^+$) could be quite energetic.

Charmed particle physics is a rapidly growing field\(^3\), partly because of the recent observation\(^4\) of electrons and muons with high transfer momenta in $p(p,n) + \pi^+ + \text{anything}$ and of the two $\mu^+ \mu^-$ events\(^5\) observed in $\nu$ interactions. The mechanism for these processes is not understood and could be due to some new phenomenon (charm?).

Charm-hunting has just started; hopefully, we shall know in the near future whether such particles exist or not.

\* \* \*
REFERENCES
(Section 15)


3) For possible consequences of the existence of charmed particles see, for example:
   A. de Rujula and S.L. Glashow, Phys. Letters 46 B, 381 (1973);
   M.A. Bég and A. Zee, Phys. Rev. D8, 2334 (1973);
   C.H. Albright, Nuclear Phys. B75, 539 (1974);
   48 B, 435 (1974);
   M.K. Gaillard, B.W. Lee and J.L. Rosner, Preprint, Fermilab-Pub-74/34-
   THY (1974).

   F.W. Busser et al., to be published.

5) B. Aubert et al., Experimental observation of $\mu^+ \mu^-$ pairs produced
   by very high energy neutrinos, submitted to the 17th International

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grateful to Mrs M. Guarisco for her efficient typing of the manuscript.
In this appendix, the notations and some formulae used in the text are given.

A.1 **UNITS**: We use the units $\hbar = c = 1$. For example $1/m$ (where $m$ denotes some mass) is $\hbar mc$ in cm and $\hbar mc^2$ in sec.

A.2 **METRIC**: We follow the Feynman metric à la Bjorken and Drell\(^1\). However, for convenience, we also give the "translation" into the Pauli metric à la Källén\(^2\).

<table>
<thead>
<tr>
<th>Feynman metric</th>
<th>Pauli metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = 0, 1, 2, 3$</td>
<td>$\mu = 1, 2, 3, 4$</td>
</tr>
<tr>
<td>$a^\mu = (a^0, a^1, a^2, a^3) = (\vec{a}, a_0)$</td>
<td>$a^\mu = (a_1, a_2, a_3, a_4) = (\vec{a}, i\alpha_0)$</td>
</tr>
<tr>
<td>$x^\mu = (t, \vec{x})$, $x^0 = t$</td>
<td>$x^\mu = (\vec{x}, it)$, $x_0 = t$</td>
</tr>
<tr>
<td>$p^\mu = (E, \vec{p})$, $p^0 = E$</td>
<td>$p^\mu = (\vec{p}, iE)$, $p_0 = E$</td>
</tr>
<tr>
<td>$g_{\mu\nu} = 0$, $\mu \neq \nu$, $g_{00} = 1$</td>
<td>$\delta_{\mu\nu} = 1$, $\mu = \nu$, $\mu, \nu = 1, 2, 3, 4$</td>
</tr>
<tr>
<td>$g_{\kappa\lambda} = -\delta_{\kappa\lambda}$, $\kappa, \lambda = 1, 2, 3$</td>
<td>$\delta_{\mu\nu} = 0$, $\mu \neq \nu$</td>
</tr>
<tr>
<td>$a^\mu g_{\mu\nu} a^\nu = (a_0, -\vec{a})$</td>
<td>$a^\mu a_\mu = \vec{a}.\vec{a} - a_0 b_0$</td>
</tr>
<tr>
<td>$x^\mu = (t, -\vec{x})$, $p^\mu = (E, -\vec{p})$</td>
<td>$a = a^\mu b_\mu = \vec{a}.\vec{b} - a_0 b_0$</td>
</tr>
<tr>
<td>$ab = a^\mu b_\mu = a_0 b_0 - \vec{a}.\vec{b}$</td>
<td>$a^\mu b_\mu = \vec{a}.\vec{b} - a_0 b_0$</td>
</tr>
<tr>
<td>$p^2 = p^\mu p_\mu = E^2 - \vec{p}^2 = m^2$</td>
<td>$p^2 = p^\mu p_\mu = \vec{p}^2 - E^2 = -m^2$</td>
</tr>
</tbody>
</table>
where \( a \) and \( b \) are arbitrary four-vectors, and \( p \) is the four-momentum for a free particle of mass \( m \).

### A.3 DIRAC EQUATION AND \( \gamma \)-MATRICES

#### Feynman metric

\[
(i \gamma^\mu \frac{\partial}{\partial x^\mu} - m) \psi(x) = 0
\]

\[
\{ \gamma^\mu, \gamma^\nu \} = 2 \delta^{\mu\nu}
\]

\[
\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ -\sigma_\mu & 0 \end{pmatrix} = -\sigma_\mu
\]

\[
\gamma_5 = +i \gamma^0 \gamma^1 \gamma^2 \gamma^3
\]

\[
\{ \gamma_5, \gamma_\mu \} = 0
\]

\[
\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]

\[
\gamma^0 = \gamma^0, \quad \gamma^\mu \gamma^\nu = -\delta^\mu\nu
\]

\[
(\gamma_5)^2 = (\gamma^0)^2 = 1, \quad (\gamma^\mu)^2 = -1
\]

#### Pauli metric

\[
(i \gamma^\mu \frac{\partial}{\partial x^\mu} + m) \psi(x) = 0
\]

\[
\{ \gamma^\mu, \gamma^\nu \} = 2 \delta^{\mu\nu}
\]

\[
\gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_\mu = \begin{pmatrix} 0 & -i \sigma_\mu \\ i \sigma_\mu & 0 \end{pmatrix}
\]

\[
\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4
\]

\[
\{ \gamma_5, \gamma_\mu \} = 0
\]

\[
\gamma_5 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
\]

\[
\gamma^\mu = \gamma_\mu
\]

\[
(\gamma_5)^2 = (\gamma^\mu)^2 = 1
\]

Here \( k = 1, 2, 3 \) and \( \sigma_\mu \) are the Pauli matrices

\[
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

The decomposition of the Dirac spinor field is given by (Feynman metric)
\[ \Psi(x) = \sum_{\mathbf{p}, r} \frac{1}{\sqrt{2V_p}} \left\{ e^{i\mathbf{p} \cdot \mathbf{x}} a^{(r)}(\mathbf{p}) \psi^{(r)}(\mathbf{p}) + e^{-i\mathbf{p} \cdot \mathbf{x}} b^{(r)}(\mathbf{p}) \bar{\psi}^{(r)}(\mathbf{p}) \right\}, \]

\[ \bar{\Psi}(x) = \sum_{\mathbf{p}, r} \frac{1}{\sqrt{2V_p}} \left\{ e^{i\mathbf{p} \cdot \mathbf{x}} a^{(r)}(\mathbf{p}) \bar{\psi}^{(r)}(\mathbf{p}) + e^{-i\mathbf{p} \cdot \mathbf{x}} b^{(r)}(\mathbf{p}) \psi^{(r)}(\mathbf{p}) \right\}, \]

\[ \bar{\Psi}(x) = \Psi^+(x) \gamma^0, \quad r = \pm 1 \]

where $V$ is the volume of quantization which, for simplicity, is taken to be unity in the calculations. The quantities $a^{(r)}(\mathbf{p}) \left[ b^{(r)}(\mathbf{p}) \right]$ is the annihilation operator for a particle (antiparticle) with momentum $\mathbf{p}$ and the spin state $r$; $r = \pm 1$ denote, respectively, spin along and opposite to the direction of quantization. For the Pauli metric we need only change the sign of $\mathbf{p} \cdot \mathbf{x}$ in $e^{i\mathbf{p} \cdot \mathbf{x}}$. Furthermore, in this metric one defines $\bar{\psi}(x) = \psi^+(x) \gamma_4$, however $\gamma_4 = \gamma^0$.

The quantities $u^{(r)}(\mathbf{p})$, etc. are the Dirac spinors in the momentum space. For both metrics we have

\[ u^{(r)}(\mathbf{p}) = \sqrt{E + m} \begin{pmatrix} \xi^{(r)}(\mathbf{p}) \\ \sigma \cdot \mathbf{p} \xi^{(r)}(\mathbf{p}) \end{pmatrix}, \quad v^{(r)}(\mathbf{p}) = \pm \sqrt{E + m} \begin{pmatrix} \xi^{(r)}(\mathbf{p}) \\ \sigma \cdot \mathbf{p} \xi^{(r)}(\mathbf{p}) \end{pmatrix} \]

\[ \bar{u}^{(r)}(\mathbf{p}) = u^{(r)}(\mathbf{p}) \gamma_0 = \sqrt{E + m} \begin{pmatrix} \xi^{(r)} \xi^{(r)} \\ -\xi^{(r)} \sigma \cdot \mathbf{p} \end{pmatrix}, \]

\[ \bar{v}^{(r)}(\mathbf{p}) = v^{(r)}(\mathbf{p}) \gamma_0 = \pm \sqrt{E + m} \begin{pmatrix} \xi^{(r)} \xi^{(r)} \\ -\xi^{(r)} \sigma \cdot \mathbf{p} \end{pmatrix}, \]

where the upper sign is for $r = 1$ and the lower sign for $r = 2$. Furthermore $\xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\xi^{(r)}$ is the transposed spin vector.
A.4 DIRAC EQUATION IN THE MOMENTUM SPACE; SPIN SUMMATIONS

Feynman metric  
\[ \gamma_p = \gamma^\mu P_\mu = \begin{pmatrix} E & -\bar{\sigma} \cdot \vec{p} \\ \bar{\sigma} \cdot \vec{p} & -E \end{pmatrix} \]

Pauli metric  
\[ \gamma_p = \gamma^\mu P_\mu = i \begin{pmatrix} E & -\bar{\sigma} \cdot \vec{p} \\ \bar{\sigma} \cdot \vec{p} & -E \end{pmatrix} \]

\[ (\gamma_p - m) \zeta^{(r)}(p) = 0 \]
\[ \bar{u}^{(r)}(p) (\gamma_p - m) = 0 \]
\[ (\gamma_p + m) \zeta^{(r)}(p) = 0 \]
\[ \bar{v}^{(r)}(p) (\gamma_p + m) = 0 \]

\[ \sum_{r=1,2} \zeta^{(r)}(p) \bar{u}^{(r)}(p) = (\gamma_p + m) \zeta^{(r)}(p) \]
\[ \sum_{r=1,2} \zeta^{(r)}(p) \bar{v}^{(r)}(p) = (\gamma_p - m) \zeta^{(r)}(p) \]

Four kinds of spin summations appear often in calculations. One such sum is \[ \sum_{x, x' = 1, 2} |\zeta^{(r)}(p') M \zeta^{(r)}(p)|^2 \], where \( M \) is a \( 4 \times 4 \) matrix, and the other three are obtained by replacing \( u \) or \( \bar{u} \) by \( v \) or \( \bar{v} \). In the Feynman metric

\[ \left( u^{(r)}(p) M \zeta^{(r)}(p) \right)^* = \left( \zeta^{(r)}(p) M^* \zeta^{(r)}(p) \right) \]
\[ = \zeta^{(r)}(p)^* \bar{u}^{(r)}(p) \left( \bar{u}^{(r)}(p) M \right)^* \zeta^{(r)}(p) = \zeta^{(r)}(p) \bar{u}^{(r)}(p) M^* \zeta^{(r)}(p) \zeta^{(r)}(p)^* \zeta^{(r)}(p) \]
where *, +, and \( \gamma \) denote complex conjugation, Hermitian conjugation and transposition, respectively. Note that, the above calculation is also valid in the Pauli metric. However, there the \( \gamma_0 \) is called \( \gamma_\alpha \). Thus we have

\[
\sum_{r, r'} \left| \bar{U}^{(r')} \gamma \bar{U}^{(r)} \right|^2 = \sum_{r, r'} \bar{U}^{(r')} \gamma \bar{U}^{(r)} (\gamma_0 M^+ \gamma_\alpha) \alpha \beta \bar{U}^{(r')} \gamma \bar{U}^{(r)} = (\gamma p + m) (\gamma_0 M^+ \gamma_\alpha) (\gamma p' + m) M_{\alpha \beta} \rho \rho' \rho_\alpha
\]

\[
= \text{Tr} \left[ (\gamma p + m) (\gamma_0 M^+ \gamma_\alpha) (\gamma p' + m) M \right]
\]

\[
= \text{Tr} \left[ (\gamma p' + m) M (\gamma p + m) \gamma_0 M^+ \gamma_\alpha \right]
\]

in the Feynman metric and we have used the relation \( \text{Tr}[AB] = \text{Tr}[BA] \).

Clearly in the Pauli metric, we must replace \( \gamma p \) by \(-i\gamma p\) and change the \( \gamma_0 \) to \( \gamma_\alpha \). Thus we have

\textbf{Feynman metric}

\[
\sum_{r, r'} \left| \bar{U}^{(r')} \gamma \bar{U}^{(r)} \right|^2 = \text{Tr} \left[ (\gamma p' + m) M (\gamma p + m) \gamma_0 M^+ \gamma_\alpha \right],
\]

\[
\sum_{r, r'} \left| \bar{U}^{(r')} \gamma \bar{U}^{(r)} \right|^2 = \text{Tr} \left[ (\gamma p' + m) M (\gamma p - m) \gamma_0 M^+ \gamma_\alpha \right],
\]

\[
\sum_{r, r'} \left| \bar{U}^{(r')} \gamma \bar{U}^{(r)} \right|^2 = \text{Tr} \left[ (\gamma p' - m) M (\gamma p + m) \gamma_0 M^+ \gamma_\alpha \right],
\]

\[
\sum_{r, r'} \left| \bar{U}^{(r')} \gamma \bar{U}^{(r)} \right|^2 = \text{Tr} \left[ (\gamma p' - m) M (\gamma p - m) \gamma_0 M^+ \gamma_\alpha \right].
\]
In the Pauli metric, in these relations we call $\gamma_p$, $\gamma_p'$, and $\gamma_0$ respectively $-i\gamma_p$, $-i\gamma_p'$, and $\gamma_4$.

A.5 THE SCALAR, VECTOR, TENSOR, AXIAL-VECTOR, AND PSEUDOSCALAR INTERACTIONS

There are 16 linearly independent $4 \times 4$ matrices. We may choose these to be (Feynman metric)

$$1, \gamma^0, \sigma^{\mu\nu}, \gamma^0\gamma_5, \text{ and } \gamma_5, \ \ \rho, \mu, \nu = 0, 1, 2, 3$$

where 1 denotes the unit matrix and $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$. Note that because of $\sigma^{\mu\nu} = -\sigma^{\nu\mu}$ there are only six independent $(4 \times 4)$ matrices $\sigma^{\mu\nu}$, namely $\sigma^{12}, \sigma^{23}, \sigma^{31}, \sigma^{01}, \sigma^{02},$ and $\sigma^{03}$. Under Lorentz transformations, the quantities $\bar{\psi}'(x)\Gamma\psi(x)$, where $\psi$ and $\psi'$ denote the field operators for two (not necessarily identical) particles and $\Gamma$ is one of the above $4 \times 4$ matrices, transform as

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>1</th>
<th>$\gamma^0$</th>
<th>$\sigma^{\mu\nu}$</th>
<th>$\gamma^0\gamma_5$</th>
<th>$\gamma_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\psi}(x)\Gamma\psi(x)$</td>
<td>scalar</td>
<td>vector</td>
<td>tensor</td>
<td>axial-vector</td>
<td>pseudoscalar</td>
</tr>
</tbody>
</table>

In the notes these five classes have been denoted by $0_i$, $i = 1, \ldots, 5$. In the Pauli metric, we need only remember that the indices occur as subscripts and go from 1 to 4.

A.6 TRACES OF PRODUCTS OF $\gamma$-MATRICES

In both metrics

$$\text{Tr} \left[ \gamma_{\nu_1} \gamma_{\nu_2} \ldots \gamma_{\nu_n} \right] = 0 \text{ if } n \text{ is odd},$$

$$\text{Tr} \left[ \gamma_5 \right] = \text{Tr} \left[ \gamma_5 \gamma_\alpha \gamma_\beta \right] = 0,$$

$$\text{Tr} \left[ \gamma_{\nu_1} \gamma_{\nu_2} \ldots \gamma_{\nu_n} \right] = \text{Tr} \left[ \gamma_{\nu_1} \ldots \gamma_{\nu_n} \gamma_{\nu_2} \gamma_{\nu_3} \right].$$
Further useful relations are

**Feynman metric**

\[
\text{Tr} [ \gamma_{\alpha} \gamma_{\beta} ] = 4 \delta_{\alpha\beta}, \\
\text{Tr} [ \gamma_{\alpha} \gamma_{\beta} \gamma_{\sigma} \gamma_{\rho} ] = 4 \left[ \delta_{\alpha\beta} \delta_{\sigma\rho} - \delta_{\alpha\sigma} \delta_{\beta\rho} + \delta_{\alpha\rho} \delta_{\beta\sigma} \right], \\
\text{Tr} [ \gamma_{5} \gamma_{\alpha} \gamma_{\beta} \gamma_{\sigma} \gamma_{\rho} ] = 4 \, \epsilon_{\alpha\beta\rho\sigma},
\]

\( \epsilon_{\alpha\beta\rho\sigma} \) is totally antisymmetric

\( \epsilon_{0123} = 1 \)

\( \epsilon_{\alpha\beta\rho\sigma} \epsilon^{\alpha\beta\rho\sigma} = -24 \)

\( \epsilon_{\alpha\beta\rho\sigma} \epsilon^{\alpha\beta\rho'\sigma} = -6 \, g_{\sigma}^{\rho'} \)

\( \epsilon_{\alpha\beta\rho\sigma} \epsilon^{\alpha\beta\rho'\sigma'} = -2 \left( g_{\rho'}^{\rho'} g_{\sigma}^{\sigma'} - g_{\rho}^{\rho'} g_{\sigma'}^{\sigma} \right) \)

**Pauli metric**

\[
\text{Tr} [ \gamma_{\alpha} \gamma_{\beta} ] = 4 \delta_{\alpha\beta}, \\
\text{Tr} [ \gamma_{\alpha} \gamma_{\beta} \gamma_{\sigma} \gamma_{\rho} ] = 4 \left[ \delta_{\alpha\beta} \delta_{\rho\sigma} - \delta_{\alpha\rho} \delta_{\beta\sigma} + \delta_{\alpha\sigma} \delta_{\beta\rho} \right], \\
\text{Tr} [ \gamma_{5} \gamma_{\alpha} \gamma_{\beta} \gamma_{\sigma} \gamma_{\rho} ] = 4 \, \epsilon_{\alpha\beta\rho\sigma},
\]

\( \epsilon_{\alpha\beta\rho\sigma} \) is totally antisymmetric

\( \epsilon_{1234} = 1 \)

\( \epsilon_{\alpha\beta\rho\sigma} \epsilon^{\alpha\beta\rho\sigma} = 24 \)

\( \epsilon_{\alpha\beta\rho\sigma} \epsilon^{\alpha\beta\rho'\sigma} = 6 \, \delta_{\sigma}^{\rho'} \)

\( \epsilon_{\alpha\beta\rho\sigma} \epsilon^{\alpha\beta\rho'\sigma'} = 2 \left( \delta_{\rho'}^{\rho} \delta_{\sigma}^{\sigma'} - \delta_{\rho}^{\rho'} \delta_{\sigma'}^{\sigma} \right) \)
A.7 PHASE-SPACE

The number of one-particle states with the particle momentum \( \vec{p} \) in the interval \( \vec{p} + d\vec{p} \) is given by

\[
dn = \frac{V}{(2\pi)^3} \, d^3p.
\]

The two-particle phase space, where the particle masses are \( m \) and \( m' \) respectively, is given by

\[
L_2(m,m',s) = \int \frac{d^3p}{2p_0} \frac{d^3p'}{2p'_0} \delta(p+p'-Q) = \int d^3p \, d^3p' \, \delta(p^2-m^2) \cdot \delta(p'^2-m'^2) \cdot \delta((p+p'-Q)^2 - s) = \frac{\pi}{s} \sqrt{\lambda(s,m^2,m'^2)},
\]

\( s = Q^2, \quad \lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz, \)

where we have used the Feynman metric. In the Pauli metric, we must let \( p^2 \to -p^2, \quad p'^2 \to -p'^2 \) and define \( s = -Q^2 \).

As a special case of the above formula we have

\[
L_2(0,0,s) = \frac{\pi}{2}, \quad L_2(m,m,s) = \frac{\pi}{2} \sqrt{1 - \frac{4m^2}{s}}.
\]

* * *

REFERENCES

(Appendix A)


In this section, we give a type calculation. We calculate the cross-sections for $\nu_e-e$ and $\nu^e-e$ scattering, using the formulae given in Appendix A. Consider

$$\nu_e(q,r) + e(p,s) \to \nu_e(q',r') + e(p',s'),$$  \hspace{1cm} (1)$$

$$\nu^e(q,r) + e(p,s) \to \nu^e(q',r') + e(p',s'),$$  \hspace{1cm} (2)$$

where $q$ and $r$ refer to the four-momentum and the spin state of the incoming $\nu_e$ (or $\nu^e$), etc.

The S-matrix elements, to the order $G$, are obtained from

$$\langle f|S|i \rangle = i \int d^4x \langle f|L(x)|i \rangle,$$  \hspace{1cm} (3)$$

where $i$ and $f$ refer to the initial and final states and we have used that the Lagrangian density is the negative of the Hamiltonian density. Substituting the leptonic Lagrangian (Eq. 5.3) and using the decomposition formulae for the Dirac spinors given in (A.3), we find

$$\langle \nu_e(q',r'), e(p',s'); S | \nu_e(q,r), e(p,s) \rangle = \frac{i (2\pi)^4 \delta(q' + p' - p - q)}{\sqrt{16 q_0 p_0 q' \cdot p' \cdot r^4}} \times \frac{G c}{\sqrt{2}} \bar{\nu}(q) \gamma(1 - \bar{e} \gamma_5) \nu(p) \bar{\nu}(p') \gamma(1 - \bar{e} \gamma_5) \nu(q').$$  \hspace{1cm} (4)$$

The matrix element $\langle \nu^e e|S|\nu^e e \rangle$ is obtained from (4) by replacing

$$\bar{\nu}(q) \to \nu(r), \quad \bar{\nu}(q') \to \nu(r').$$
The transition rate (per unit time) for process (1) is given by

$$
\Gamma' = \frac{\mathcal{V}(2\pi)^4 \delta(q' + p' - q - p)}{16 q \cdot P \cdot q' \cdot P' \cdot \mathcal{V}^4} \frac{G^2}{2} \left| \bar{u}^{(q')} \ldots \bar{u}^{(q)} \right|^2. \tag{5}
$$

In order to obtain the cross-section we must divide by the flux for the incoming particle (i.e., divide by $v_in/V$, where $v_in$ is the velocity of the incoming particle in the lab. frame) and integrate over the available phase space. Thus

$$
\sigma' = \frac{\mathcal{V}}{v_in} \int \left( \frac{\mathcal{V}}{(2\pi)^3} \right)^2 d^3p' d^3q' \Gamma'. \tag{6}
$$

After slight simplification

$$
\sigma' = \frac{G^2}{8 q_0 \cdot P_0 \cdot v_in \cdot (2\pi)^2} \int \frac{d^3p' d^3q'}{2 P_0' \cdot 2 q_0'} \delta(p' + q' - p - q) \left| \bar{u}^{(q')} \ldots \bar{u}^{(q)} \right|^2. \tag{7}
$$

We introduce the variables

$$
S = (p + q)^2 = (p' + q')^2,
$$

$$
X = (p' - p)^2 = (q' - q)^2,
$$

$$
\Xi = (p' - q)^2 = (q' - p)^2,
$$

and consider the processes (1) and (2) at large $s$, where the electron mass can be neglected. Then
\[ s = 2p'q = 2p'q', \quad t = -2p'p = -2q'q, \]
\[ u = -2p'q = -2q'p, \quad s + t + u = 0. \] \hspace{1cm} (8)

The expression (7) is for the case when all particles are in definite spin states. To obtain the unpolarized cross-section, we must average over the spin of the target electron and sum over the final spin states. Note that, since the neutrino is completely polarized, we are not allowed to average over its spin! Thus we replace

\[ \sum_{s', \bar{s}'} \left| \bar{u}^{(s')} \left( q' \right) \cdots \bar{u}^{(s')} \left( q \right) \right|^2 \to \frac{1}{2} \sum_{s, \bar{s}} \left| \bar{u}^{(s)} \left( q' \right) \cdots \bar{u}^{(s)} \left( q \right) \right|^2. \]

From the formulae in Section (A.4) we have

\[ A'_{\sigma} = \frac{1}{2} \sum_{s, \bar{s}} \left| \bar{u}^{(s)} \left( q' \right) \cdots \bar{u}^{(s)} \left( q \right) \right|^2 = \frac{1}{2} \sum_{s, \bar{s}} \left| \bar{u}^{(s)} \left( q' \right) \cdots \bar{u}^{(s)} \left( q \right) \right|^2 \]
\[ \times \bar{u}^{(s)} \left( q \right) O_{\sigma}' \bar{u}^{(s)} \left( q' \right) \sum_{s, \bar{s}} \left| \bar{u}^{(s)} \left( q' \right) \cdots \bar{u}^{(s)} \left( q \right) \right|^2 O_{\sigma}' \bar{u}^{(s)} \left( q \right) \] \hspace{1cm} (9)

where

\[ O_{\lambda} = \gamma_{\lambda} \left( 1 - \gamma_{\bar{s}} \right), \quad O_{\sigma}' = \gamma_{\sigma} \left[ \gamma_{\sigma} \left( 1 - \gamma_{\bar{s}} \right) \right]^+ \gamma_{\sigma}. \] \hspace{1cm} (10)

The primed matrices may be simplified

\[ O_{\sigma}' = \gamma_{\sigma} \left[ \gamma_{\sigma} \left( 1 - \gamma_{\bar{s}} \right) \right]^+ \gamma_{\sigma} = \gamma_{\sigma} \left( 1 - \gamma_{\bar{s}} \right) \gamma_{\sigma}^+ \gamma_{\sigma} \]
\[ = \left( 1 + \gamma_{\bar{s}} \right) \gamma_{\sigma} \gamma_{\sigma}^+ \gamma_{\sigma} = \left( 1 + \gamma_{\bar{s}} \right) \gamma_{\sigma} = \gamma_{\sigma} \left( 1 - \gamma_{\bar{s}} \right) = O_{\sigma}. \]
where we have used $\gamma_k^+ = -\gamma_k$, $\gamma_0^+ = \gamma_0$, and the anticommutation relations for $\gamma$-matrices. Now, the spin summations in expression (9) lead to

$$A_\nu = \frac{1}{2} \text{Tr} [ \gamma_q' \gamma_\lambda (1-\gamma_5) \gamma_p \gamma_\sigma (1-\gamma_5)] \text{Tr} [ \gamma_p' \gamma^\lambda \gamma^\sigma (1-\gamma_5) ]$$

$$\times (1-\gamma_5) \gamma_q \gamma^\sigma (1-\gamma_5) = 2 \text{Tr} [ \gamma_q' \gamma_\lambda \gamma_p \gamma_\sigma (1-\gamma_5) ]$$

$$\times \text{Tr} [ \gamma_p' \gamma^\lambda \gamma^\sigma (1-\gamma_5) ],$$

where we have used

$$(1-\gamma_5) \gamma_p \gamma_\sigma (1-\gamma_5) = \delta_p (1+\gamma_5) \delta_\sigma (1-\gamma_5) =$$

$$= \delta_p \delta_\sigma (1-\gamma_5)^2 = 2 \gamma_p \delta_\sigma (1-\gamma_5),$$

and a similar relation for the second factor.

For the antineutrino scattering, by the replacement rules, we need only interchange $q$ and $q'$ in relation (11). We now do the traces using the formulae in (A.6)

$$\text{Tr} [ \gamma_q' \gamma_\lambda \gamma_p \gamma_\sigma (1-\gamma_5) ] = q'^\beta p^\beta$$

$$\times \text{Tr} [ \gamma_\alpha \gamma_\lambda \gamma_\beta \gamma_\sigma (1-\gamma_5) ] =$$

$$= q'^\beta p^\beta [ g_{\alpha\lambda} g_{\beta\sigma} - g_{\alpha\beta} g_{\lambda\sigma} + g_{\alpha\sigma} g_{\beta\lambda} -$$

$$- i \epsilon_{\alpha\lambda\beta\sigma} ] = 4 [ q'_\alpha p_\sigma + q'_\sigma p_\alpha - g_{\lambda\sigma} p^\gamma +$$

$$- i \epsilon_{\alpha\lambda\beta\sigma} q'^\gamma p^\beta ],$$
and similarly for the second trace. We multiply the two traces and use

\[ g_{\lambda \sigma} q^{\lambda \sigma} = 4, \quad \varepsilon_{\alpha \lambda \beta \sigma} \varepsilon_{\alpha' \lambda' \beta' \sigma'} = -2(g^{\alpha \alpha'} g^{\beta \beta'} - g^{\alpha \alpha' \beta \beta'} g^{\beta \beta' \alpha \alpha'}), \]

\[ (q_{\lambda} P_{\sigma} + q^{\sigma} P_{\lambda} - g_{\lambda \sigma} q q') (q^{\lambda} q^{\sigma} + q^{\sigma} q^{\lambda} - g^{\lambda \sigma} P q') = 2 \left[ (q q')(q P) + (q q')(P P') \right], \]

\[ \varepsilon_{\alpha \lambda \beta \sigma} q^{\mu \nu} P_{\rho} \varepsilon_{\alpha' \lambda' \beta' \sigma'} P_{\rho'}, q_{\beta} = -2 \left[ (q q')(q q) - (q q')(q q') \right]. \]

Thus

\[ A_\nu = 128 \langle q' q' \rangle \langle q P \rangle = 32 \text{ s}^2 \quad (13) \]

where, we have used relations (8). Interchanging \( q \) and \( q' \), we have for antineutrino scattering

\[ A_{\bar{\nu}} = 128 \langle q P' \rangle \langle q' q \rangle = 32 \text{ u}^2 \quad (14) \]

Since we have neglected the electron mass, \( v_{in} = 1 \) [in the expression (9)]. Furthermore we have, in the laboratory system

\[ s = 2P q = 2P \nu_0 q = 2m E \quad (15) \]

where \( E \) is the incoming neutrino (antineutrino) energy and \( m \) is the electron mass. Thus, from relations (7), (9), (13) and (14) we deduce
\[ \sigma' = \frac{G^2}{4s(2\pi)^2} \left(32s^2\right) \int \frac{d^3p'}{2p'} \frac{d^3q'}{2q'} \delta(p' + q' - p - q) \]
\[ = \frac{G^2s}{\pi}, \]
\[ \sigma = \frac{G^2}{4s(2\pi)^2} 32 \int \frac{d^3p'}{2p'} \frac{d^3q'}{2q'} \delta(p' + q' - p - q) u^2. \]  

It remains to compute the integral in (17). Let us denote this integral by I. Then, from the Section A.7 and the definition of \( u \)

\[ I = \int d^4p' d^4q' \delta(p' + q' - p - q) \delta(p'^2) \delta(q'^2) (2q'p')^2 = \]
\[ = \int d^4q' \delta((p + q - q')^2) \delta(q'^2) (2q'p')^2. \]

We go to the centre-of-mass frame, where

\[ p + q = 0, \quad (p + q)^2 = (P + q_0)^2 = s \]
\[ p = q = p_0 = q_0 = \frac{\sqrt{s}}{2}, \quad (p + q - q')^2 = s - 2\sqrt{s}q_0', \]
\[ q'p = \frac{s}{2} (1 + c \cos \theta). \]
Note that we have suppressed all the superscripts indicating the frame. In this frame, \( I \) (which is frame independent) is given by

\[
I = \int \frac{d^3 q'}{2q'_0} \delta(q_0 - \sqrt{s} q'_0) \left[ \frac{s}{2} (1 + \cos \theta) \right]^2 =
\]

\[
= \frac{q_{0}^{'} \cdot (5/2)^{2}}{(2 q_{0}')(2 \sqrt{3})} \int d \omega (1 + \cos \theta)^{2}
\]

\[
= \frac{\pi s^{2}}{6}
\]

Thus

\[
\sigma_{S} = \frac{G^{2} s}{3 \pi}.
\]

Note that \( \sigma_{S} = 1/3 \sigma_{y} \) at the same value of \( s \).

Numerically

\[
\sigma_{S} \approx 1.7 \ E_{\nu} \ 10^{-41} \ cm^{2},
\]

where \( E_{\nu} \) is in GeV.
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APPENDIX B 113
DEEP INELASTIC PROCESSES

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I. INTRODUCTION

The famous deep inelastic electron scattering experiments at SLAC\(^1\) have given us a tantalising glimpse of the inner structure of the proton and neutron, and have led to the suggestion that they might consist of more elementary constituents, which are called partons. This conclusion is reinforced by the results of experiments in which beams of neutrinos or antineutrinos are scattered on nuclear targets\(^2\). Taken together, the data also point to the strong possibility that the partons are quarks, though this interpretation does raise some problems because free quarks have not yet been seen in any experiment.

In a sense, the electron scattering experiments can be said to probe the structure of the nucleon rather directly. This is because the way in which the electron interacts with another particle is well understood from quantum electrodynamics\(^2\): it emits a virtual photon which is then absorbed by the other particle, Figure 1.1. Because the emission
of the virtual photon by the electron is well understood, it is really
the absorption mechanism that is being studied in the experiments, and
it is what we learn about this that provides the basis for building
models of the structure of the nucleon. The same can be said about the
neutrino scattering experiments. Here it is the weak interaction that
is involved, instead of the electromagnetic (Figure 1.2). From $\beta$-decay
we think that we know how this interacts with the neutrino, though
obviously we cannot be at all certain that what we have learnt from such
a very low energy reaction as $\beta$-decay will still be valid at high
energy. One remarkable result of the neutrino scattering experiments is
that it does seem to be, to surprisingly good accuracy, and once we
have established this we can regard the experiments as giving us infor-
mation about how the weak interaction probes the inner structure of the
nucleon target. What we learn about the nucleon structure from the
neutrino scattering experiments is complementary to the information from
electron scattering; in fact from an analysis of the electron scattering
results it was possible to make good predictions$^3$) about the neutrino
scattering.

Having learnt about the inner structure of the proton, it is im-
portant to try and test it in further experiments, and I shall discuss
some of these. In particular, there is a lot of interest at present in
the production of hadrons emerging with large transverse momentum from
nucleon-nucleon collisions. There are reasons to suppose that this depends on the inner structure of the nucleons but, because here only strong interactions are involved, things are theoretically much less certain. In spite of this, we are beginning to build up what we think may be quite a good picture of what is going on, though very much more experimental data will be needed.

2. **DEEP INELASTIC ELECTRON SCATTERING**

The unknown part of Figure 1.1 is the lower amplitude. To calculate the cross-section we must square its modulus and, if we do not look at any of the final-state hadrons, we must sum over all possible final states of hadrons. By the optical theorem, this gives the imaginary part $W^{\mu\nu}$ of the elastic Compton scattering amplitude evaluated at zero momentum transfer (Figure 2.1). Suppose that the target is not

\[ \sum \left| \frac{q}{p} \right|^2 = 2 \text{Im} \left| \frac{q}{p} \right| \]

Fig. 2.1

The optical theorem

polarised, so that we must average over its two possible spin states.

The labels $\mu$ and $\nu$ are the Lorentz indices of the two photons in the Compton amplitude. $W^{\mu\nu}$ is a tensor, and must be constructed out of the basic tensors

\[ p^\mu q^\nu, \quad p^\mu q^{\prime \nu}, \quad p^{\prime \mu} q^\nu, \quad p^{\prime \mu} q^{\prime \nu}, \quad g^{\mu \nu}, \quad \epsilon^{\mu \nu \alpha \beta} p_\alpha q_\beta \]

But the electromagnetic current is conserved:

\[ q_\mu W^{\mu\nu} = 0 \]

and $W^{\mu\nu}$ has positive parity, so that the most general form can be
written

\[ W_{\mu\nu} = - (q_\mu q_\nu - \frac{q^2 q^2}{q^2}) W_1 + (p^\mu - \frac{p \cdot q}{q^2} q^\mu) (p^\nu - \frac{p \cdot q}{q^2} q^\nu) W_2 \]

(2.2)

where \( W_1 \) and \( W_2 \) are functions of the two independent Lorentz invariants \( \nu = p \cdot q \) and \( q^2 \). To determine \( \nu \) and \( q^2 \) one measures the energy of the electron before and after scattering, and its scattering angle. In the laboratory frame, where the target nucleon is at rest,

\[ \nu = M(E - E') \quad , \quad \nu_{\text{max}} = ME \]

\[ q^2 = -4EE' \sin^2 \frac{1}{2} \theta \quad \leq 0 \]

(2.3)

with \( M \) the nucleon mass.

The differential cross-section is, neglecting the electron mass,

\[ \frac{d^2 \sigma}{dq^2 d\nu} = \frac{4\pi \alpha^2}{q^4} \frac{E'}{E} \left[ W_2 \cos^2 \frac{1}{2} \theta + \frac{2W_1}{M^2} \sin^2 \frac{1}{2} \theta \right] \]

(2.4a)

Actually nowadays, with plans for electron-proton colliding beam machines, it is probably more sensible to write this in invariant form, rather than referring to a particular frame:

\[ \frac{d^2 \sigma}{dq^2 d\nu} = \frac{4\pi \alpha^2}{q^4} \left[ W_2 (1 - \frac{\nu}{\nu_{\text{max}}}) + (M^2 W_2 - 2W_1) \frac{q^2}{4\nu_{\text{max}}^2} \right] \]

(2.4b)

In spite of appearances, this does not become singular when \( q^2 \to 0 \).

This is because, from its definition as the imaginary part of the Compton amplitude, \( W_{\mu\nu} \) remains finite as \( q^2 \to 0 \), so from (2.7), as \( q^2 \to 0 \).
\[ W_2 \rightarrow 0 \]
\[ W_1 + \frac{\nu}{q^2} W_2 \rightarrow 0 \]  

(2.5)

Virtual photons may be polarised either longitudinally or transversely, but real photons must be polarised transversely. The second condition in (2.6) removes the contribution from longitudinal photons as \( q^2 \rightarrow 0 \).

According to (2.3), a given pair of values of \( \nu \) and \( q^2 \) can be achieved at various values of \( \Theta \), by taking different \( E \) and \( E' \). So since \( W_1 \) and \( W_2 \) depend only on \( \nu \) and \( q^2 \), from measurements of (2.4) one can extract \( W_1 \) and \( W_2 \) separately. For deep inelastic events, that is large \( \nu \) and large \( q^2 \), the data seem to give

\[ W_1 \rightarrow F_1(\omega), \quad \nu W_2 \rightarrow F_2(\omega) \]  

(2.6)

\[ \omega = \frac{2\nu}{1q^2} \]

and

\[ F_1(\omega) = \frac{1}{2} \omega F_2(\omega) \]  

(2.7)

The first of these results, that asymptotically \( W_1 \) and \( \nu W_2 \) do not vary with \( q^2 \) at fixed \( \omega \), is known as Bjorken scaling (Figure 2.2). The second result, (2.7), implies that at large \( q^2 \) the scattering is dominated by the exchange of transversely polarized virtual photons (compare it with the second condition in (2.6), which must be satisfied by real photons).

The square of the invariant mass of the final-state system of hadrons in Figure 1.1 is

\[ W^2 = (p + q)^2 = M^2 + 2\nu (1 - \frac{1}{\omega}) \]  

(2.8)
Fig. 2.2 Verification of Bjorken scaling (E.M. Riordan, Ph.D. Thesis)
By baryon conservation, the final state must contain at least one nucleon. Thus \( W^2 > M^2 \), and

\[
\omega \geq 1
\]

with \( \omega = 1 \) corresponding to \( W^2 = M^2 \), that is elastic scattering. One often writes

\[
x = \frac{1}{\omega}
\]

(2.9)

Experimental results for the dimensionless function \( F_2 \), for proton and neutron targets, are shown in Figure 2.3. Remember that the data for \( F_2^p \) are actually extracted from a deuterium target, and that there are some uncertainties about correcting for this at large \( q^2 \).

3. **THE PARTON MODEL**

If we put (2.6) into (2.4b) we have, at large \( \nu \),

\[
\gamma_{\text{max}} \frac{d^2 \sigma}{dw dy} = \frac{2 \pi \alpha^2}{y^2} \left[ F_2(\omega) (1-y) + F_1(\omega) \frac{y^2}{\omega} \right]
\]

\[
y = \frac{\nu}{\gamma_{\text{max}}}
\]

(3.1)

No fixed dimensional parameter appears on the right-hand side; it seems that deep inelastic electron scattering is independent of any basic scale of length. This is interpreted as meaning that whatever structure within the nucleon is responsible for scattering must be pointlike in character, and leads to the idea that the nucleon is composed of pointlike constituents, or partons.

The parton model can be formulated in different ways:

a) the "naive" parton model

b) using the language of quantum field theory

c) in terms of the algebra of currents on the light cone

Provided that they are formulated with suitable care, all three approaches give equivalent results, though the light-cone methods seem to be rather less powerful and so I shall not discuss them. In the "naive" approach, the nucleon is thought of as consisting of partons in a concrete way,
Fig. 2.3
very much like we think of a nucleus as composed of nucleons. However, there is an important difference: a nucleus is rather lightly bound, which means that the nucleons are very nearly real particles. But the partons are very tightly bound inside the nucleon and so are highly virtual. Virtual particles change their character when one passes from one Lorentz frame to another, and so the choice of Lorentz frame is important, and it turns out that one moving with infinite momentum is the most suitable. The ultimate theory must, of course, be Lorentz invariant, and for this reason I myself prefer the other approach to the parton model: formulating it in the language of field theory enables us to give a covariant treatment from the start, with no preferred Lorentz frame. It also has the advantage that one can more easily make contact with various familiar ideas of strong-interaction theory, such as Regge poles.

The basic idea, then, is that a parton is struck by the virtual photon and ejected (Figure 3.1). It might subsequently decay into other

![Diagram of deep inelastic scattering in the parton model.](image)

Fig. 3.1
Deep inelastic scattering in the parton model.

particles. (This picture may need to be refined if one is going to suppose that the partons are quarks which cannot completely escape. That is, it may be necessary to allow the parton to interact with the remaining components of the nucleon after it has been struck by the photon, though in most simple models one can show that this does not happen when $\nu$ and $q^2$ are large$^7$). Taking the squared modulus of Figure 3.1,
and summing over possible final states, as in the left-hand side of the equation in Figure 2.1, we find that we have to calculate the imaginary part of Figure 3.2. Here $T$ is the strong-interaction amplitude that describes the emission of the virtual parton $k$ from the nucleon and its subsequent recapture.

In Figure 3.2 we have to integrate over all possible values of $k$. Because the parton $k$ is to thought of as a constituent of the nucleon $p$, we expect that in the rest frame of $p$, $k$ will be essentially confined to finite values. Suppose we choose to expand $k$ as a linear combination of $p$, $q$ and a vector transverse to each of them:

$$k = x p + y q + k_T$$

$$k_T \cdot p = k_T \cdot q = 0$$

(3.2)

In the rest frame of $p$, the components of $q$ are of order $\nu$, so to keep $k$ finite we must make $y$ of order $\nu^{-1}$:

$$y = \frac{\bar{y}}{\nu}$$

(3.3)

The basic assumption (which may be wrong in the real world) is that $x$, $\bar{y}$ and $k_T$ remain bounded as $\nu \to \infty$. Since we need the imaginary part of Figure 3.2, we must put the parton $(q + k)$ on its mass shell - this is the final-state parton in Figure 3.1. Thus
\[ \mu^2 = (q + \kappa)^2 = 2\gamma \left( \frac{x}{\omega} + 1 \right) \left( x - \frac{\omega}{x} + 1 \right) + x^2 M^2 + k_T^2 \]

Hence for large \( \gamma \), we need the \( \delta \)-function

\[ \delta \left[ 2\gamma \left( x - \frac{1}{\omega} \right) - \mu^2 \right] \]  

so that

\[ x \sim \frac{1}{\omega} \]  

(3.5)

Thus the variable \( x \) of (2.9) is actually the fraction of the momentum \( p \) of its parent nucleon carried by the parton that is struck by the virtual photon, as it is defined in (3.2).

We next have to ask how the virtual photons in Figure 3.2 couple to the parton, and this depends on the spin of the parton. If the parton has spin 0, and so is described by a scalar field \( \phi \), the most simple form for the electromagnetic current is

\[ J^\mu = i Q \left[ \phi^+ \partial^\mu \phi - \partial^\mu \phi^+ \phi \right] \]  

(3.6)

where \( Q \) is the charge of the parton. Then from the two vertices we get the factor

\[ Q^2 (2k^\mu + q^\mu)(2k^\nu + q^\nu) = Q^2 \left[ 4x^2 p^\mu p^\nu + 2x(2y+1)(p^\mu q^\nu + q^\mu p^\nu) + (2y+1)^2 q^\mu q^\nu + 4k_T^\mu k_T^\nu \right] \]

Since there is no term here proportional to \( g^{\mu\nu} \), we have no structure function \( W_1 \); from (2.2) \( W_1 \) is the coefficient of \( g^{\mu\nu} \) in \( W^{\lambda\nu} \). (In fact, this is not quite right; when the \( k \) integration has been done, the \( k_T^\mu k_T^\nu \) term gives a part involving \( g^{\mu\nu} \), but
this also gets a factor $\nu^{-1}$, so that it gives a contribution to $W_i$ that vanishes in the limit $\nu \to \infty$. Hence scalar partons are no good, they do not give the transverse-photon result (2.7).

The next thing to try is a parton of spin $\frac{1}{2}$, with

$$J^\mu = Q \bar{\psi}(\gamma^\mu)\psi$$

(3.7)

The amplitude $T$ in Figure 3.2 now has to be a $4 \times 4$ Dirac matrix. We include in $T$ the propagators of the two partons $a$ and $\bar{a}$ and expand it: the most general parity-conserving form is

$$T = T_0 + T_1 \gamma^\mu p + T_2 \gamma^\nu \kappa + T_3 \sigma^{\alpha\beta} p_\alpha \kappa_\beta$$

(3.8)

Then we have to perform the trace, to sum over the polarisations of the parton *)

$$\text{Tr} \left\{ T \gamma^\mu \left[ \gamma \cdot (\kappa + q) + \mu \right] \gamma^\nu \right\}$$

$$\sim 4 \gamma \left[ p^\mu (q^\nu + \kappa^\nu) + (q^\mu + q^\nu) p^\nu - \nu \delta^{\mu\nu} \right]$$

+ lower order terms

$$T = T_1 + x T_2$$

(3.9)

Now we do have a $\delta^{\mu\nu}$ term, and in fact the transverse-photon result (2.7) is satisfied, since the ratio of the coefficients of $\delta^{\mu\nu}$ and $p^\mu p^\nu$ is just $\frac{1}{2} \frac{\nu}{x} = \frac{1}{2} \omega_{\nu}$. The function $\gamma$ is a function of the Lorentz scalars

$$s' = (p - \kappa)^2 - 2(1 - x) \vec{\kappa} + (1 - x)^2 M^2 + \vec{s}_T^2$$

(3.10)

Because $s_T$ is by definition orthogonal to both $p$ and $q$, it is actually spacelike:

*) The factor $\gamma \cdot (k + q) + \mu$ is the numerator of the propagator for the parton line that joins the two photon vertices.
\[ \kappa_T^2 = -\kappa^2 \]  

(3.11)

To complete the calculation, we change the integration from \( \int d^+k \) to one over \( k, s' \) and \( \kappa \):

\[ \int d^+k \rightarrow \frac{1}{2(1-\alpha)} \int d\alpha \ ds' \ d^2 \kappa \]  

(3.12)

(Being orthogonal to both \( p \) and \( q \), \( \kappa \) is effectively two-dimensional).

The final result is

\[ F_2(\omega) = \frac{2Q^2}{(2\pi)^3} \frac{1}{\omega-1} \int ds' d^2 \kappa \ \text{Im} \ \Im^\prime(s', \kappa^2) \]  

(3.13)

\[ \kappa^2 = -\frac{s' + \omega \kappa^2}{\omega-1} + \frac{M^2}{\omega} \]

Hence we have Bjorken scaling so long as the integral in (3.13) converges. In fact if one takes various field-theory models for \( \Im^\prime \) in most cases one finds that it does not, and so if scaling is really a fact this tells us something altogether non-trivial about basic dynamics.

I started off by saying that the partons are pointlike. I have used this assumption in not putting in form factors at the two vertices. Perhaps the assumption is wrong, and we should include\(^8\) factors something like \( (1 - \frac{\kappa^2}{M_0^2})^{-n} \) associated with their form factors. (Here \( M_0 \) is some mass characteristic of the form factor, possibly associated with a heavy vector meson). Alternatively, one could imagine that the partons are themselves composed of partons, and so on, so that perhaps there are successive bands of energy such that in each band there is scaling, but \( F_1(\omega) \) and \( F_2(\omega) \) change from band to band, so revealing successive stages in the hierarchical structure.

Observe that \( \sqrt{s'} \) is the invariant mass of the lower group of final-state particles in Figure 3.1, and so \( s' \) is certainly positive.
Thus from (3.13) $q^2$ is negative (spacelike) throughout most, or even all, of the integration. This means that the partons are certainly virtual, and one must take care in pressing too far the analogy between the nucleon parton model and the nucleus.

Now some words about the elastic form factor of the nucleon. For elastic scattering, the quark that has been struck by the virtual photon has to recombine to re-form the proton, Figure 3.3. At large $q^2$ it is moving very fast after it has been struck and so it is hard for it to recombine; this is why the elastic form factor decreases fairly rapidly at large $q^2$, and elastic scattering contributes a very small part of the total electron scattering cross-section. At $q^2 = 0$ the amplitude $T$ in Figure 3.3 is exactly the same as the one in Figure 3.2. One can calculate Figure 3.3 by methods similar to those that I have described, and finds that at $q^2 = 0$ it is related closely to Figure 3.2. This is because integrating Figure 3.2 with respect to $\omega$ just removes the $\delta$-function (3.4), that is it suppresses the parton line joining the two vertices in Figure 3.2. In fact

$$\int_1^\infty \frac{d\omega}{\omega} \langle \text{Figure 3.2} \rangle = Q \langle \text{Figure 3.3 evaluated at } q^2 = 0 \rangle \quad (3.14)$$

I will come back to this sum rule when I discuss the quark model. It is closely related to a sum rule derived in the naive parton model \(^4\), where what is left inside the nucleon after the parton $Q$ has been emitted is supposed to consist of a further set of partons: there one finds

$$\int_1^\infty \frac{d\omega}{\omega} F_2(\omega) = \text{expected value of the sum of the squares} \quad (3.15)$$

of all the partons in the nucleon.
In the naive parton model one derives a further sum rule
\[ \int_0^\infty \frac{d\omega}{\omega} F_2(\omega) = \text{average value of the square of the charge on the partons} \quad (3.16) \]

I am a bit suspicious of this second sum rule, because it does not seem possible to derive it in the covariant formalism. Perhaps, in a highly relativistic situation, one should beware of letting one's picture of the nucleon as being composed of a number of constituents become too concrete.

Now for the final piece of abstract formalism in these lectures. Let us ask what happens in Figure 3.2 when \( \omega \) becomes large. From the second equation in (3.13),
\[ \mathcal{T} \propto \omega |s'| \quad (3.17) \]

Now \( \mathcal{T} \) is an elastic strong-interaction amplitude, and so when its energy variable \( s' \) becomes large we expect it to be dominated by Regge-pole exchange
\[ \mathcal{T} \sim \sum \beta^{(i)}(s') s' \alpha_0^{(i)} \quad (3.18) \]

where \( \alpha_0^{(i)} \) are the intercepts of the various Regge trajectories that can be exchanged (Figure 3.4). Thus from (3.17), \( \mathcal{T} \) behaves like a sum

\[ \text{Regge exchange in } \mathcal{T} \text{ at large } s'. \]

of terms \( \omega^{\alpha_0^{(i)}} \), and taking account of the factor \( (\omega-1)^{-1} \) in (3.13), \( F_2^{ep} \) behaves like a sum of terms \( \omega^{\alpha_0^{(i)}-1} \). In the case of \( F_2^{eN} \) or \( F_2^{en} \), one can exchange the pomeron, \( \alpha_0 = 1 \).
and so $F_2 \rightarrow \text{constant as } \omega \rightarrow \infty$. Since the pomeron contributes equally to $F_{2ep}$ and $F_{2en}$ (it carries zero isospin), the leading behaviour of the difference $F_{2ep} - F_{2en}$ as $\omega \rightarrow \infty$ is dominated by tensor meson exchange, $\alpha_0 = \frac{1}{2}$. Thus $F_{2ep} - F_{2en} \sim \omega^{-\frac{1}{2}}$. These behaviours seem to be consistent with the data in Figure 2.3, though it would be very useful to have much more accurate experimental information about the $\omega \rightarrow 0$ behaviour. Notice that a consequence of the constant asymptotic behaviour of $F_{2ep}$ or $F_{2en}$ is that, according to (3.15), the total number of partons inside the nucleon in the naive parton picture is infinite!

4. THE QUARK MODEL

The fact that we want the partons to have spin $\frac{1}{2}$ suggests that the most economical assumption is that they are the quarks which are so successfully used in hadron spectroscopy. Notice, though, that the hadron spectroscopist thinks of the nucleon as being composed of just three quarks, while we have already seen, from the fact that the sum rule (3.15) diverges, that for us the picture cannot be so simple. We think of the nucleon as containing a "sea" of an infinite number of quark-antiquark pairs, as well as the three "valence" quarks. Perhaps the two pictures are related by a mathematical transformation, or perhaps the spectroscopists' simple picture is too simple. I am going to take just ordinary Gell-Mann-Zweig quarks. Maybe one does need something more fancy, such as three triplets differing only in their "colour", but for many purposes this will not make much difference.

Consider electroproduction on a proton target. The virtual photon can hit any one of the three types of quark, or any of the three types of antiquark, this is we have six terms like Figure 3.2. If I write in explicitly the factors $Q^2$ arising from the quark charges at the two photon vertices, then in an obvious notation

$$F_{2ep} = \left(\frac{2}{3}\right)^2 (F^b + F^d) + \left(-\frac{1}{3}\right)^2 (F^n + F^\alpha + F^\lambda + F^\Sigma)$$

(4.1)

Each function $F^q, F^\Sigma$ is given by an integral like (3.13), and
involves the amplitude $\gamma$ for the emission by the target proton of the quark $q$ or antiquark $\bar{q}$. Thus $F^p(\omega)$, for example, involves the amplitude $\gamma^p$ for the emission of a $p$-quark by the proton. By charge symmetry, this amplitude is equal to that for the emission of an $\bar{n}$-quark by a neutron, and similarly for the other amplitudes:

$$\gamma^p = \gamma^n, \quad \gamma^\bar{p} = \gamma^\bar{n}, \quad \gamma^\lambda = \gamma^{\bar{\lambda}}$$

(4.2)

So we may write $F_2^{eN}$ in terms of the same six functions as appear in (4.1):

$$F_2^{eN} = \left(\frac{2}{3}\right)^2 (F^n + F^\bar{n}) + (-\frac{1}{3})^2 (F^p + F^\bar{p} + F^\lambda + F^{\bar{\lambda}})$$

(4.3)

Consider now the sum rules (3.14). Write

$$\int_1^\infty \frac{d\omega}{\omega} F^2(\omega) = H^2$$

(4.4)

$$\int_1^\infty \frac{d\omega}{\omega} F^\bar{2}(\omega) = \bar{H}^2$$

Then according to (3.14), the Dirac charge form factor of the proton at $q^2 = 0$ is

$$1 = \frac{2}{3}(H^p - H^{\bar{p}}) - \frac{1}{3}(H^n - H^{\bar{n}} + H^\lambda - H^{\bar{\lambda}})$$

(4.5)

Similarly for the neutron

$$0 = \frac{2}{3}(H^n - H^{\bar{n}}) - \frac{1}{3}(H^p - H^{\bar{p}} + H^\lambda - H^{\bar{\lambda}})$$

(4.6)

We can also consider the form factor of the baryon current instead of the electromagnetic current, and so make sure that the proton has the correct baryon number. Because each quark has baryon number $\frac{1}{3}$, and
each antiquark $\frac{1}{3}$, 

$$1 = \frac{1}{3} (H^p - H^\bar{p} + H^n - H^\bar{n} + H^\lambda - H^\bar{\lambda})$$  \hspace{1cm} (4.7)

From (4.5), (4.6) and (4.7)

$$\int_1^\infty \frac{dw}{w} (F^p - F^\bar{p}) = 2$$  \hspace{1cm} (4.8)

$$\int_1^\infty \frac{dw}{w} (F^n - F^\bar{n}) = 1$$

$$\int_1^\infty \frac{dw}{w} (F^\lambda - F^\bar{\lambda}) = 0$$

Now write each $F^\ell$, $F^\bar{\ell}$ as a sum of contributions from valence quarks and from the sea of quark-antiquark pairs:

$$F^\ell (\omega) = V^\ell (\omega) + S^\ell (\omega) \quad (V^\lambda = 0)$$

$$F^\bar{\ell} (\omega) = S^\bar{\ell} (\omega)$$  \hspace{1cm} (4.9)

It seems reasonable to suppose that the sea has zero isospin and is even under charge conjugation:

$$S^p = S^n = S^\bar{p} = S^\bar{n}, \quad = S(\omega) \quad \text{say}$$

$$S^\lambda = S^\bar{\lambda}$$  \hspace{1cm} (4.10)

so that the proton, neutron, antiproton and antineutron all have the same sea, differing only because of their valence quarks. Then from (4.8)

$$\int_1^\infty \frac{dw}{w} V^p (\omega) = 2$$  \hspace{1cm} (4.11)

$$\int_1^\infty \frac{dw}{w} V^n (\omega) = 1$$

Also,

$$F^\ell_{e^p} - F^\ell_{e^n} = \frac{1}{3} (V^p - V^n)$$  \hspace{1cm} (4.12)
so that

$$\int \frac{d\omega}{\omega} \left( F_2^{eP} - F_2^{eN} \right) = \frac{1}{3}$$  \(4.13\)

This sum rule seems to be compatible with the data in Figure 2.3, though as it is slow to converge and so sensitive to what happens at small \(x\), one cannot yet be sure. In the upper part of Figure 4.1 I have plotted the function

$$\frac{5}{16} (1-x)^2 \sqrt{x}$$  \(4.14\)

which is the simplest function that fits the data reasonably well, satisfies (4.13) and also has the correct Regge behaviour \(x^2\) as \(x \to 0\).

To extract further information, one has to make some assumptions about the relative shapes of the two functions \(V^P\) and \(V^N\). For want of any theoretical ideas about this, the simplest thing to assume is that they have the same shape. Because of (4.11), \(V^P\) must then be twice \(V^N\):

$$V^P(\omega) = 2V^N(\omega), \quad = 2V(\omega) \quad \text{say.}$$  \(4.15\)

Then \(F_2^{eP} - F_2^{eN} = \frac{2}{3}V\), that is \(V\) is three times the curve in the upper part of Figure 4.1. But now

$$F_2^{eP} = V + \text{(contribution from the sea)}$$  \(4.16\)

I have plotted \(V\) in the lower part of Figure 4.1, so that the difference between this curve and the data is the contribution from the sea.

Obviously this is not quite right, because the contribution from the sea surely cannot be negative. It seems that (4.15) must break down
at least near $\omega = 1$, and if one takes the data literally it must be that $V^q(\omega)$ is actually very much less than $V^p(\omega)$ near $\omega = 1$, so that here the $p$ quarks are doing nearly all the scattering. The crude fits in Figure 4.1 will suffice for my purposes; more sophisticated analyses of the data are to be found in the literature. Notice that, when the necessary adjustment has been made so as to bring the curve down to the data near $\omega = 1$, the sea contribution will be very small or zero there; it does not begin to become important until $\omega \geq 2.5$. That is, the sea contribution comes from partons with fairly small fractional longitudinal momentum $\kappa$; these are sometimes called "wee" partons. The sea contribution $\rightarrow$ constant at large $\omega$, corresponding to pomeron exchange. Thus the pomeron is somehow closely connected with wee partons: in $p\bar{p}$ collisions pomeron exchange is thought to arise from some sort of intermingling of the wee partons of the two protons.

5. **NEUTRINO PROCESSES**

Having constructed this picture of the proton from the electroproduction data, we want to test it in other experiments. In particular, it seems to be strikingly successful in making quantitative predictions about deep inelastic neutrino and antineutrino scattering.

Our ideas about weak interactions come from particle decays, which are all very low energy reactions. It is far from obvious that these ideas will still be right at very high energy, but let us suppose that they are. Then Figure 1.2 is closely similar to Figure 1.1 and we must now calculate Figure 2.1 with the lines $q_2$ associated with the weak-interaction current instead of the electromagnetic current. An important question is whether the weak-interaction is a point-like Fermi interaction, or whether it is mediated by an intermediate vector boson; for the moment let us suppose the former is the case.

Because the weak interaction current is not conserved, and because it is a mixture of vector and axial parts, the expansion of $W^\mu$ now involves all the basic tensors (2.1):
\[ W^{\mp \mu \nu} = -g^{\mu \nu} W_1^{\mp} + B^\nu B^\mu W_2^{\mp} + \\
+ \frac{1}{2} i \epsilon^{\mu \nu \xi \rho} q_\xi p_\rho W_3^{\mp} + \ldots \]

(5.1)

The \( \mp \) signs refer respectively to the reactions

\[ \nu + p \rightarrow (\mu^- \text{ or } e^-) + \text{hadrons} \]

\[ \bar{\nu} + p \rightarrow (\mu^+ \text{ or } e^+) + \text{hadrons} \]

(5.2)

The terms that are not written in explicitly in (5.1) are all proportional to \( q^\mu \) or \( q^\nu \). Since \( W^{\mu \nu} \) must be multiplied by lepton-current factors, using the Dirac equation for the lepton spinors these give terms proportional to the muon or electron mass and so should be negligible. The structure functions \( W_1^{\mp} \) and \( W_2^{\mp} \) are each the sum of two parts, vector-vector and axial-axial, while \( W_3^{\mp} \) is the interference term, with one of the two currents in the right-hand term of Figure 2.1 vector, and the other axial. Instead of (2.4), \( G \) is the weak-interaction coupling constant,

\[ \frac{d^2 \sigma}{d^2 q d \omega} = \frac{G^2}{4 \pi} \frac{E'}{E} \left[ W_2^{\mp} \cos \frac{1}{2} \theta + \frac{2W_1^{\mp}}{M^2} \sin \frac{1}{2} \theta \mp \frac{E + E'}{M^3} W_3^{\mp} \sin \frac{1}{2} \theta \right] \]

(5.3)

The different signs for the \( W_3^{\mp} \) contributions come about because the vector-axial interference term in \( W^{\mp \mu \nu} \) couples to the vector-axial interference term in the contribution from the lepton current. This involves a \( \chi_s \), which effectively changes sign when one goes from lepton to antilepton.

The two processes in (5.2) differ in the sign of the charge carried by the weak-interaction current. By charge symmetry, the amplitude for the \( \mp \) charged current scattering on a proton is equal to that for the \( \mp \) charged current scattering on a neutron, so that changing
to a neutron target has the effect of interchanging the labels $\frac{1}{2}$ on $W_1, W_2, W_3$ in (5.3), though of course it has no effect on the sign in front of the $W_3$ term.

In the large $\nu$ and large $q^2$ limit, the parton model again makes Figure 3.2 the dominant contribution to $W^\pm \nu \nu$, with the currents now the weak currents. One finds the Bjorken scaling laws

$$W_1^\pm (\nu, q^2) \to F_1^\pm (\omega), \quad \nu W_2^\pm \to F_2^\pm (\omega),$$
$$\nu W_3^\pm \to F_3^\pm (\omega)$$

(5.4)

The total cross section is

$$\sigma^\pm = \frac{G^2 M E}{\pi} \int_1^\infty d\omega \left[ \frac{F_2^\pm}{2\omega^2} + \frac{F_1^\pm}{3\omega^3} + \frac{F_3^\pm}{3\omega^3} \right]$$

(5.5)

so one has a total cross section that rises linearly with the neutrino beam energy $E$. The data from CERN and from NAL are shown in Figure 5.1, and seem to agree very well with this. However, one knows from unitarity that the linear rise cannot continue indefinitely with increasing energy. If the weak-interaction is mediated by an intermediate vector boson $W$, the square of its propagator must be included in (5.3); because of the way that $G$ is defined this means an additional factor

$$\left( \frac{M_w^2}{M_w^2 - q^2} \right)^2$$

When the energy $E$ rises to a value such that $|q^2|$ becomes comparable with $M_w^2$, this will cause the total cross-section to flatten off; Figure 5.1b shows the effect that a $W$-boson of mass 10 or 20 GeV would have. Whether or not there exists a $W$-boson, at high enough energy higher-order weak-interaction effects will begin to become
important, that is the simple picture of Figure 1.2 with just a single weak current will break down. It is obviously of great interest to study these effects, and this is one of the things one would hope to do with an $e^P$ colliding-beam facility. There one would look at the reactions inverse to (5.2):

$$
e^- + p \rightarrow \nu + \text{hadrons}$$

$$e^+ + p \rightarrow \bar{\nu} + \text{hadrons}$$

(5.6)

If the partons have spin $\frac{1}{2}$, Figure 3.2 gives

$$F_1^+ = \frac{1}{2} \omega F_2^+$$

$$F_3^+ = -2 F_1^+$$

for partons

$$= +2 F_1^+$$

for antipartons.

(5.7)

In the quark-parton model we can relate these functions to the structure functions measured in electroproduction. Instead of the electromagnetic current, which is written in terms of the quark fields as

$$\frac{2}{3} F G^{1/2} - \frac{1}{3} \bar{\lambda} G^{1/2} - \frac{2}{3} \bar{\lambda} G^{1/2}$$

(5.8)

we have the weak current

$$F G^{1/2} (1 - G^{1/2}) [\lambda \cos \Theta_c + \lambda \sin \Theta_c]$$

(5.9)

where $\Theta_c$ is the Cabibbo angle. The Cabibbo angle as measured from particle decays is quite small, $\Theta_c \approx 15^0$. It will be of interest to check whether this is still correct at high energies, so that it is comparatively rare that the final-state system of hadrons in (5.2) has non-zero total strangeness. If it is, it is a good approximation to put $\Theta_c = 0$. 
Then, in terms of the functions $F^2$, $F^\bar{2}$ that I introduced in (4.1),

$$
\begin{align*}
F_2^{\nu P} &= F_2^{\nu N} = F_2^{-} = 2(F^n + F^\bar{n}) \\
F_3^{\nu P} &= F_3^{\nu N} = F_3^{-} = -2\omega (F^n - F^\bar{n}) \\
F_2^{\bar{\nu} P} &= F_2^{\bar{\nu} N} = F_2^{+} = 2(F^p + F^\bar{n}) \\
F_3^{\bar{\nu} P} &= F_3^{\bar{\nu} N} = F_3^{+} = -2\omega (F^n - F^\bar{n})
\end{align*}
$$

(5.10)

For example, in the first term of $F_2^{\nu P}$, the positively charged current strikes an $n$ quark that has been emitted by the proton, and changes it into a $p$ quark. This changes back into an $n$ when it emits the right-hand current in Figure 3.2. The factor of 2 arises because there are two equal terms, one from $\gamma$ coupling at each vertex and one from $\gamma^5$. As I said earlier, the cross terms, with $\gamma$ at one vertex and $\gamma^5$ at the other, give rise to $F_3$.

There are three things one can check even without going on to build up the detailed valence quark/sea picture of the proton; these are consequences just of the assumption that the partons are quarks:

(a) Because $F^\lambda$ and $F^\bar{\lambda}$ appear in (4.1) and (4.3) only in the combination $F^\lambda + F^\bar{\lambda}$, there are five independent functions on the right of (4.1), (4.3) and (5.10). Hence there is a single linear relation

$$
6\omega (F_2^{eP} - F_2^{eN}) = F_3^{\nu P} - F_3^{\nu N}
$$

(5.11)

This is not yet tested, because the structure functions $F_3$ are not yet known well enough.

(b) Because each of the contributions $F^2$, $F^\bar{2}$ is surely positive, there are some inequalities, for example
\[ F_2^{eP} + F_2^{eN} \geq \frac{5}{18} \left( F_2^{\gamma P} + F_2^{\gamma N} \right) \]  

(5.12)

(c) Using the integral relations (4.8), one finds the sum rules

\[ \int_1^\infty \frac{dw}{w} \left( F_2^{\gamma N} - F_2^{\gamma P} \right) = 2 \]  

(5.13)

\[ \int_1^\infty \frac{dw}{w^2} \left( F_3^{\gamma N} + F_3^{\gamma P} \right) = -6 \]  

(5.14)

The first of these, the Adler sum rule, can actually be derived without quark assumptions, but the second, the Gross-Llewellyn Smith sum rule, relies on the quark model.

Consider now the particular picture of the nucleon that I built up in section 4. Suppose we take the sea as \( SU_3 \) singlet, so that in addition to (4.10),

\[ S^\lambda = \bar{S}^\lambda = S(\omega) \]  

(5.15)

Then from (4.9) and (4.15)

\[ F_2^{eP} - F_2^{eN} = \frac{1}{3} V(\omega) \]

\[ F_2^{eP} = V(\omega) + \frac{4}{3} S(\omega) \]  

(5.16)

and from (5.10)
\[ F^\mu_p = F^\mu_N = 2V + 4S \]
\[ F^\mu_p = F^\mu_N = -\omega V \]
\[ F^\mu_p = F^\mu_N = 4V + 4S \]
\[ F^\mu_p = F^\mu_N = -2\omega V \]

\[(5.17)\]

My rough fit to \( \frac{1}{3} V(\omega) \) was the curve drawn in the upper part of Figure 4.1, and the corresponding \( V(\omega) \) is the curve drawn in the lower part of Figure 4.1; the difference between this and the data is \( \frac{4}{3} S(\omega) \).

Hence I know completely the neutrino structure functions (5.17).

Consider the total cross section (5.5). Because of the \( \int \frac{d\omega}{\omega^2} \) that operates on \( F_2 \), and because the sea contribution \( S(\omega) \) is very small for \( \omega \leq 2.5 \), the sea contributes very little to the total cross section. If its contribution were zero, then on a target with equal numbers of protons and neutrons one would have

\[ \frac{\sigma^p}{\sigma^n} = \frac{1}{3} \]

\[(5.18)\]

Putting in the contribution from the sea increases this ratio; it becomes about \( \frac{2}{5} \). Compare this with the data in Figure 5.2.

For the total cross-section one gets

\[ \sigma^\nu = 0.47 \ \frac{G^2 ME}{\pi} \ \text{per nucleon} \]

\[(5.19a)\]

compared with the experimental value

\[ (0.45 \pm 0.09) \ \frac{G^2 ME}{\pi} \ \text{per nucleon} \]

\[(5.19b)\]
What is known about the various structure functions also seems to agree well with theoretical prediction. For example, in Figure 5.3 is plotted data for

\[ q(x) = F^b + F^n \]
\[ \bar{q}(x) = F^\bar{b} + F^\bar{n} \]

together with the theoretical curves

\[ q(x) = 3V + 2S \]
\[ \bar{q}(x) = 2S \]

as extracted from Figure 4.1.

6. **THE FINAL STATE**

It goes without saying that, both in electroproduction and in neutrino scattering, one can learn more about what is going on if one, or preferably more than one of the final-state hadrons can be detected. However, a word of warning: the discussion so far has been about theoretical expectations at asymptotic values of \( \nu \) and \( q^2 \). We are very lucky that the values of \( \nu \) and \( q^2 \) that we are able to achieve with present accelerators are apparently large enough to be regarded as in asymptopia. However, it is a familiar feature of strong interactions that total cross-sections appear to come close to their asymptotic forms at lower energies than single-particle distributions, and single-particle distributions at lower energies than two-particle distributions, and so on. Similarly, one cannot expect that the electroproduction coincidence experiments that have been done so far give more than a hint of what the final state will look like at higher energies.

Because the strong interaction that produces the final-state hadrons is between the virtual photon (or weak current) and the nucleon target, it is relative to these that one discusses momentum distributions. So far as the transverse momentum of the emerging hadrons is concerned, it is widely expected that, as in ordinary strong interactions, this remains small as the energy increases. (However, there have been some suggestions
that, on the contrary, the average transverse momentum does increase with $|q^t|$). To discuss the longitudinal momentum, it is convenient to work in terms of the rapidity, defined in terms of a particle's energy $E'$ and longitudinal momentum $p'$ by

$$
\gamma = \frac{1}{2} \log \frac{E' + p'}{E' - p'}
$$

(6.1)

This variable, which is borrowed from strong interaction theory, has the convenient property that a longitudinal boost to a different Lorentz frame just changes it by a constant depending only on the boost. The overall range of variation of $\gamma$, from its minimum to its maximum, is $\log \nu$. Because of our expectation that the transverse component of momentum of the final-state hadrons will be small, it does not make much difference whether we take for $p'$ the longitudinal momentum or the total momentum of the particle.

Consider the parton model of Figure 3.1, allowing the emerging parton to decay into a system of hadrons, Figure 6.1. The way in which the

![Fig. 6.1](image)

Deep inelastic scattering, with a fragmenting parton.

parton fragments should, of course, be exactly the same as in the scaling limit of $e^+e^-$ annihilation, if our picture of what will happen in annihilation at sufficiently high energy is correct; see Professor Gatto's lectures. The simplest assumption would be that the number of hadrons in each of the two bunches in Figure 6.1 does not change with increasing energy, so that there are two jets of particles moving in opposite directions. Very few particles would be moving slowly in the centre-of-mass frame, so that the centre of the rapidity plot is empty and the two jets are well separated in rapidity, Figure 6.2a.
This is the picture at moderate values of $\omega$. As we saw in (3.17), when $\omega$ becomes large the squared invariant mass $s'$ of the lower bunch of hadrons in Figure 6.1 grows linearly with $\omega$, and so in the lower part of the diagram we have a purely strong interaction that at large $\omega$ occurs at high energy. We expect this to produce the same sort of final state as in, say, $\bar{p}p$ collisions at the ISR. In the $\bar{p}p$ collisions the rapidity plot has essentially three regions (Figure 6.2c); at either end there are the fragments of the two protons, while in the middle there is the "pionisation" plateau. Dynamically, the $\bar{p}p$ reaction is thought to proceed mainly through the exchange of Regge poles (pomerons), Figure 6.3a.

In $e^p$ collisions at large $\omega$, then, we expect the left-hand jet in Figure 6.2a to widen and become like Figure 6.2c; this is shown in Figure 6.2b. In the $\bar{p}p$ collisions, the width of the rapidity distribution is $\log s$, so here it is $\log s' \propto \log \omega$. The extreme left of the distribution corresponds to proton fragmentation, just as in the $\bar{p}p$ collisions, and there is a pionisation plateau, just as in the $\bar{p}p$ collisions. The number of particles, and their nature, in each of these two regions should be precisely similar in the two processes; for example in the pionisation region we learn from the $\bar{p}p$ collisions that there are about ten times as many $\pi$ as $K$. Unlike in Figure 6.2c, the right-hand edge of the pionisation plateau is different from the left-hand edge in Figure 6.2b; it corresponds to hadron fragments resulting from the emission of the parton, and is called the "hole" fragmentation region.

The multiplicities in the two bunches of particles in Figure 6.1 actually need not remain finite as $|q^2|$ rises, even at finite $\omega$. For example, the upper virtual parton might give a multiplicity that rises logarithmically with its squared four-momentum $\sigma$, or even as fast as $\sqrt{\sigma}$. There are some constraints on the way in which this is allowed to happen, if one is still to have momentum-conservation and scaling, and it would in fact correspond to the gap in the rapidity plot in Figure 6.2a or 6.2b being filled in. A popular conjecture is that the gap is indeed filled in by another plateau, so giving a multiplicity that rises logarithmically with $|q^2|$. If one guesses that the height
Fig. 6.2

Rapidity plots: (a) for electroproduction at moderate $\omega$; (b) for electroproduction at large $\omega$; (c) for proton-proton collisions. Is the gap in the plots (a) and (b) filled in somehow?
Fig. 6.3
The various regions in the rapidity plots of Figure 6.2 populated by pomeron exchange: (a) in $\bar{P}P$ collisions; (b) in $eP$ collisions at large $\omega$. 
of this plateau is the same as the height $C_A$ of the pionisation plateau in Figure 6.2b, then at large $\omega$ and large $|q^2|$ the multiplicity would rise as

$$c_A \left[ \log |q^2| + \log \omega \right] \propto c_A \log \omega$$

(6.2)

which is to be compared with the rise $c_A \log s$ in $\bar{p}p$ collisions. However, it is not at all obvious that the second plateau has its origins in pomeron exchange, so that there is no reason why its height should be $C_A$, and so the coefficients of $\log |q^2|$ and $\log \omega$ might well be different. Also there is no strong reason to suppose that the composition of the second plateau is the same as that of the first, for example the $\pi/K$ ratio need not be 10. In fact there is quite a strong possibility that the gap is not filled in by a plateau at all, but in some other way. In this case the multiplicity need not rise logarithmically; any behaviour less than $\sqrt{|q^2|}$ is theoretically feasible.

The investigation of the gap is of great importance to our picture of what is happening in electroproduction and neutrino scattering, for the following reason. If the partons are quarks, and quarks are not seen as free particles, there must be some force that binds the quarks to the nucleon so strongly that they cannot escape. One might hope that, although this force is so strongly attractive at large distance that it prevents escape, its short-distance character is such as to have little effect on our discussion of the parton model. The mechanism by which this might occur and its consequences are far from being understood, and are an active area of theoretical study\textsuperscript{12}. However, it presumably requires some sort of interaction between the two bunches of particles in Figure 6.1, and so is likely to result in some filling in of the gap in the rapidity plot.

Of course a detailed study of the parton decay fragments will give interesting information about the nature of the parton. Pantin, and several authors subsequently\textsuperscript{13}, pointed out that for electroproduction the quark model would lead to a $\pi^+ / \pi^-$ ratio greater than one from the
parton fragmentation, perhaps even as great as 8. This is because the proton is more likely to emit a $\bar{p}$ quark than an $n$ quark, the electromagnetic current couples more strongly to a $\bar{p}$ quark than an $n$ quark, and a $\bar{p}$ quark, being positively charged, is more likely to emit a $\pi^+$ than a $\pi^-$. 

7. **MUON-PAIR PRODUCTION**

Having deduced from electron and neutrino scattering that the nucleon apparently has a particular quark structure, one obviously tries to test this in other reactions.

One important way of testing the quark-parton idea, and obtaining further information about the structure of the nucleon, is to do deep inelastic electroproduction experiments with a polarised electron beam on a polarised nucleon target\(^{14}\). Then there are two new structure functions, in addition to $W_1$ and $W_2$. They are extracted from the difference between electroproduction with total helicities $\frac{1}{2}$ and $\frac{3}{2}$ in the initial state. Scaling laws have been predicted for these structure functions, and the quark model involves them in certain sum rules of fundamental interest; for example one of them relates the structure functions to the ratio $g_\pi/g_\nu$ measured in beta decay of the neutron.

Here I shall choose to talk about a reaction which can be discussed in terms of the structure functions $W_1$ and $W_2$ that we already know about. This is the production of "massive" muon (or electron) pairs in $\bar{p}p$ collisions:

$$\bar{p} + p \rightarrow \mu^+\mu^- + \text{hadrons}$$  \hspace{1cm} (7.1)

To lowest order in the electromagnetic interaction, this reaction proceeds via the production of a virtual photon:

$$\bar{p} + p \rightarrow \gamma + \text{hadrons} \rightarrow \mu^+\mu^-$$

In saying that the muon pair is massive, we require that the square of the four-momentum $q^2$ of the virtual photon be large. Notice that in
the experiment it is important to detect both muons in coincidence, to
be sure that one is not just seeing meson decay. Since the
muons tend to go off to opposite sides of the beam at the ISR, this
requires two detectors.

Drell and Yan\cite{15}) showed that in the parton model a dominant con-
tribution to the process (7.1) at large $q^2$ and large centre-of-mass
energy $\sqrt{s}$ arises from Figure 7.1. Here one nucleon emits a quark and

![Diagram of the Drell-Yan mechanism](image)

Fig. 7.1

The Drell-Yan mechanism for heavy muon-pair production.

the other an antiquark, which annihilate each other to produce the vir-
tual photon. The two bubbles in Figure 7.1 are each exactly similar to
the bubble in the electroproduction diagram, Figure 3.1, so that it
turns out that when one calculates the contribution to Figure 7.1 (for
example by covariant methods similar to those that I described in sec-
tion 3) one finds that the answer can be expressed in terms of the func-
tions $F_2(\omega)$, $F_{\overline{2}}(\omega)$ that I introduced in section 4: at large $s$
and large $q^2$

$$
\frac{d\sigma}{d\sqrt{q^2}} \sim \frac{16\pi\alpha^2}{3(q^2)^{3/2}} \sum_{\text{quarks}} Q_q^2 \int_1^\infty d\omega_1 d\omega_2 \delta(\omega_1\omega_2 - \frac{s}{q^2}) F_2(\omega_1) F_{\overline{2}}(\omega_2)
$$

(7.2)

Here $Q_q$ is the charge carried by the quark $q$. 
We can calculate (7.2) by inserting for \( F_1^u, \overline{F}_2^u \) their expressions in terms of the functions \( V \) and \( S \). The only data so far is for a proton beam scattering on a uranium target; this is compared with the calculated curve in Figure 7.2. Notice that the experiment had a cut-off \( q_{\min} \) on the momentum of the muon pair, and this has been taken into account in the calculation. A consequence of it is that one cannot take advantage of the simple scaling property of (7.2):

\[
\frac{d\sigma}{d\sqrt{q^2}} \sim \frac{1}{(q^2)^{3/2}} \times \left( \text{dimensionless function} \right) \quad (7.3)
\]

to predict from the data in Figure 7.2 what will be found at higher energies. Rather, one must calculate (7.2) afresh. It will be very interesting to test the scaling when higher-energy experiments are done without a substantial cut-off, or with a cut-off that itself scales.
One would assume that the discrepancy between the data and the curve for $\sqrt{s} \approx 2.5$ GeV is because these values of $\sqrt{q^2}$ are too low for the asymptotic theory to be valid. In electroproduction it seems that the asymptotic scaling law becomes valid for $|t| \gg 1$, but this is not immediately relevant here because now $q$ is timelike instead of spacelike. For example, in muon-pair production resonances in the virtual-photon channel are important for low enough $q^2$. Bearing in mind what has been found in $e^+e^-$ annihilation, where $q$ is also timelike, one might wonder whether any of the values of $q^2$ in Figure 7.2 can be considered to be in asymptopia. Because we do not know what is causing the slow approach to asymptopia in $e^+e^-$ annihilation, this question cannot of course be answered. But I would point out one important difference between the parton models for $e^+e^-$ annihilation and muon-pair production: in annihilation the two partons that one assumes are produced by the virtual photon in the scaling limit both have timelike momenta, while the two partons in Figure 7.1 each have spacelike momenta.

The sharp fall-off of the calculated curve in Figure 7.2 can be traced back to the fact that the antiquark functions $F_T(\omega)$ in (7.2) are just $S(\omega)$, since there are no valence antiquarks. It is the property of $S(\omega)$ that it is essentially zero for $\omega \ll 2.5$ that causes the sharp fall-off in the calculated curve. If one were to do the experiment with a pion beam, the fall-off would be much slower, because in this case the antiquark can be a valence parton of the pion. 17) This observation could be of importance in the analysis of the data in Figure 7.2: because uranium is such a heavy nucleus, one would expect the incident proton beam to generate a substantial number of pions by collisions within the nucleus. Most of these pions will be of comparatively low energy, but since for a pion beam the calculated curve falls so much more slowly with $q^2$ may-be there are enough of high energy for it to be important to consider the effect of these secondary pions colliding with a nucleon and producing a muon pair. Of course it will be of great interest to have the muon-pair production experiment repeated with a pion beam, so as to learn directly about the structure function of the pion.

I mentioned earlier that if instead of a single triplet of ordinary quarks one takes three triplets, it makes little difference for many purposes (This is the "coloured" quark model6): each of the three familiar
quarks now is found in three indistinguishable versions, red, white and blue). This is true in the analysis of electroproduction and neutrino scattering: each equation (4.1), (4.3), (5.10) giving the structure functions in terms of $F^l$, $F^\bar{l}$ now has three times as many terms, but this is compensated by each $F^l$, $F^\bar{l}$ being only $\frac{1}{3}$ as big. However, (7.2) is quadratic in $F^l$, $F^\bar{l}$, so that although again there are three times as many terms, now the calculated curve would be only a $\frac{1}{3}$ as big.

The muon-pair production process involves the production of a virtual photon. If the intermediate vector boson of weak interactions exists, then because of the quark-model connection (5.8), (5.9) between weak, and electromagnetic currents one would expect it also to be produced by the mechanism of Figure 7.1. If it can only be produced as a virtual particle the cross-section for its production and decay is proportional to the square $G^2$ of the weak interaction constant, but if it is light enough to be produced as a real particle the cross-section is proportional to $G$ and so is quite large. It has been calculated\textsuperscript{18} that at the ISR the cross-section should be about $5 \times 10^{-33}$ cm$^2$, provided that the mass of the vector boson is rather less than the energy of each beam.

Finally, an important theoretical matter. I said that the Drell-Yan term of Figure 7.1 gives a dominant contribution to the muon-pair production process. In fact there is another contribution that gives a result with the same scaling property (7.3), at least up to a logarithmic factor. This contribution arises from a mechanism like Figure 7.1, but with a final-state interaction between the two bunches of hadrons. There are arguments that, although it scales in the same way as the Drell-Yan term, it is numerically rather smaller, but the arguments are a little weak and it is quite possible that they are not in fact correct.

The possible importance of this additional contribution makes the theoretical status of muon-pair production rather different from that of electroproduction or neutrino scattering. For the latter processes, well-defined mathematical assumptions lead to the result that Figure 3.1 by itself dominates in the asymptotic limit; although these basic assumptions may be wrong, they do lead in a clear mathematical way to this
A SCALING OF THE INCLUSIVE $\pi^0$ CROSS SECTION
AT LARGE TRANSVERSE MOMENTA

\[ p_{1}^{824}, E \frac{d^3\sigma}{dp^3} \]

\[ 10^{-26} \]

\[ 10^{-27} \]

\[ 10^{-28} \]

\[ 0.04 \quad 0.06 \quad 0.08 \quad 0.10 \quad 0.12 \quad 0.14 \quad 0.16 \quad 0.18 \quad 0.20 \]

\[ p_{1} / \sqrt{s} \]

SYMBOL

- ● 23.5
- △ 30.6
- □ 44.8
- ▼ 52.7
- ○ 62.4

Fig. 8.2
everybody more or less agrees what is meant by the parton model, but for the hadronic large-$p_T$ processes there is no such thing as the parton model. Everybody has a different idea about what is going on, and models are based as much on simplicity as on anything else.

For inclusive leptonic processes, at asymptotic values of the variables one expects to find scaling laws, in the sense that the inclusive cross-sections become independent of any fixed dimensional parameter. This is a feature of the formula (2.5) for deep inelastic lepton scattering, and (7.2) for muon-pair production. Similarly, it was originally hoped that one would find such a scaling law for the inclusive process (8.1) at large $p_T$. This would require $n=2$ in (8.4). An explicit model for this was proposed$^{21}$ (Figure 8.5): each proton emits a virtual parton, the two partons scatter at wide angle, and then both
decay into systems of hadrons, one of which is the detected pion. The mechanisms by which the parton is emitted from the nucleon, and by which it ultimately decays, are exactly the same as in electroproduction, Figure 6.1. The only new element in Figure 8.3 is the wide-angle parton-parton scattering in the centre. To obtain the dimensionless scaling result $n = 2$ for the overall process, one must assume that this elastic parton-parton scattering scales in a dimensionless fashion at large $t$, that is it behaves like (8.6) with $m = 2$. A particular mechanism by which this might occur is where the large $-t$ parton-parton scattering is dominated by the exchange (Figure 8.6) of a single elementary particle ("gluon"), whose spin can be either 0 or 1.

However, because experiment does not give $n = 2$, but rather a scaling law (8.4) with the function $f$ carrying dimension, it seems that the quark-quark scattering at large $t$ does not have the dimensionless character $m = 2$. Further evidence for this comes from elastic scattering, for which a dimensionless quark-quark scattering
would result in \( m = 8 \), instead of the experimental value of 10 or 12. The value \( m = 8 \) would arise from the diagram of Figure 8.7, where the proton is pictured as being composed essentially of three quarks, each of which scatters by gluon exchange through the same large angle on one of the quarks of the other proton, so that after the scatterings the quarks are moving more or less in the same direction and so can easily recombine to make a proton. The result \( m = 8 \) follows after a non-trivial calculation.

Most of the other models for large \(-T\) inclusive processes again have the basic structure of Figure 8.3, but with some or all of the four quarks involved replaced by ordinary hadrons. (If either of the particles C or D is an ordinary hadron, it need not subsequently decay). That the models nearly all have this structure is not always immediately obvious from their description in the literature, because the papers are written with various different terminologies - words such as "hadronic bremsstrahlung" or "Multiperipheral model" usually conceal a basic structure like Figure 8.3. In each case it is necessary to make an assumption about the large \(-\mathcal{t}\) elastic or quasi-elastic scattering in the centre of the diagram. One can either assume some explicit mechanism for this, or another possibility is the attractive 'counting law\(^{23}\)'. This says that the wide-angle scattering \( A + B \rightarrow C + D \) has the behaviour \((8.6)\), with

\[
m = m_A + m_B + m_C + m_D - 2
\]  
\((8.8)\)
where \( m_H \) is the number of quarks + antiquarks required to construct hadron \( H \) in the naive quark model. For \( \bar{P}P \) elastic scattering (8.8) gives \( m = 10 \), and for \( \pi P \) or \( K P \), \( m = 8 \), which seems good. It had been thought that (8.8) would follow from fairly general principles, for example in a world where quark-quark forces are mediated by gluon exchange, but the value \( m = 8 \) obtained for \( \bar{P}P \) scattering from Figure 8.7 unfortunately destroys this hope. Maybe the counting law is true, but for special rather than general reasons.

Another model for the inclusive process, which does not have the structure of Figure 8.3, is that of Figure 8.8. Here a quark emitted by

![Fig. 8.8](image)

one proton scatters directly at wide angle on the other, and then fragments. The fact that in this model the second proton inevitably recoils with large transverse momentum could explain the surprisingly large number of large \(-p_T\) protons found in the experiments; at NAL the \( P/\pi \) ratio even approaches 1.

I think that the present situation is that it is hard to tell which, if any, of the various models is closest to the truth. For a more detailed discussion of their differing properties, and lists of references, the reader should consult the appropriate Conference reports\(^{19}\).

9. **Connection Between Inclusive and Exclusive Processes**

It is perhaps not surprising that exclusive processes are very much more model-dependent than inclusive processes. For example, to determine the large \(-q^2\) behaviour of the elastic form factor of the proton, Figure 3.3, we have to make assumptions about how the quark recombines
to reform the proton after it has been scattered by the virtual photon. For the inclusive process, on the other hand, the quark is assumed just to escape, or even if it does not we do not care what happens to it. In this section I discuss a possible relationship between exclusive and inclusive processes which, although it is essentially a dynamical principle rather than in any way fundamental, may well be valid for a wide class of different models.

This is the "correspondence principle". It was first stated in general form by Bjorken and Kogut\textsuperscript{24}, though it had previously been applied to the special case of electron-proton scattering by Bloom and Gilman\textsuperscript{25}. Consider an inclusive reaction

\[ a + b \rightarrow c + X \]  

(9.1)

where X is an undetected system of hadrons (Figure 9.1a). Define

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig.png}
\caption{Inclusive and exclusive processes thought to be connected by the correspondence principle.}
\end{figure}

\[ t = (p_c - p_a)^2 \]  and the missing mass \( m \), where \( m^2 \) is the square of the total four-momentum of the system X. For most high-energy events \( m \) will be large, but if one looks at the small- \( m \) events one expects to find a few prominent resonances. At such a resonance \( R \), one effectively has an exclusive reaction

\[ a + b \rightarrow c + R \]  

(9.2)
The correspondence principle assumes that the dynamics is so smooth that if one extrapolates the large-\(m\) inclusive differential cross-section down into the resonance region, one obtains quite a good description of the exclusive processes. Because it is a dynamical principle rather than a fundamental one, the principle is not a very precise one, but it reads something like

\[
\int_{\text{Resonance region}} d\Omega^2 \frac{d^2\sigma(a\bar{b} \rightarrow cR)}{dt \, d\Omega^2} \approx \sum_{\text{Resonances}} \frac{d\sigma(a\bar{b} \rightarrow cR)}{dt}
\]  

(9.3)

Perhaps one can even be bolder, and say that a relation like (9.3) holds for individual resonances, that is replace the sum on the right by a single term, and integrate only over the region of the corresponding resonance.

In particular, consider electroproduction on a nucleon target. Here particles \(a\) and \(c\) are the initial and final electrons, \(t = q^2\) and \(m\) is just the variable \(W\) of (2.8):

\[
m^2 = M^2 + q^2 + 2\nu = M^2 + t(1 - \omega)
\]  

(9.4)

Hence at large \(t\) the resonance region corresponds to \(\omega\) near to 1. With the scaling laws (2.6) and the transverse-photon result (2.7), one can write (2.4b) at large \(\nu, t\) and \(m \gg M\) as

\[
s \frac{d^2\sigma}{dt \, d\Omega^2} = \frac{4\pi a^2}{t^2} \frac{s^2 + u^2}{s(s+u)} F_2(\omega)
\]  

(9.5)

where

\[
s = 2\nu_{\text{max}} = (p_a + p_e)^2, \quad u = (p_e - p_c)^2
\]

\[
1 - \omega = \frac{m^2}{t}
\]  

(9.6)

Suppose that near \(\omega = 1\)
REFERENCES


4) R.P. Feynman, Photon-hadron interactions (Benjamin).


7) A model in which such a final-state interaction does occur is that of G. Preparata, Phys. Rev. D7, 2973 (1973).


9) This is called the Melosh transformation; see J. Weyers, CERN-TH 1743.


14) See, for example, the review by A.J.G. Hey, CERN-TH 1841 or by F.E. Close, Daresbury preprint DNPL/P154 and CERN-TH 1843.


27) D.M. Scott, preprint DAMTP 73/37.
as a $\mu^+$, also right-handed, if we are to conserve angular momentum. The CMS angular distribution is not isotopic, but $(1-\cos\theta)^2$. Put simply, the initial state has $J = 1$, but only one of the $|\mathcal{E} J + 1| = 3$ components is allowed in the final state. Thus if all partons are particles of spin $1/2$, we expect $R = \frac{\sigma(\overline{v})}{\sigma(v)} = \frac{1}{3}$. For any other spin assignment 0, 1, 3/2 --- one can easily see that the ratio $R > \frac{1}{3}$. For spin 0, or for spin $\gg \frac{1}{2}$, $R = 1$. Furthermore, if the nucleon contains both particle and antiparticle constituents, clearly $R \gg \frac{1}{3}$, and $R = 1$ if left and right-handed constituents occur equally.

The data are shown in Fig. 2. The NAL counter experiment gives a value up to $E = 75$ GeV of $R = 0.34 \pm 0.08$, while the CERN Gargamelle experiment ($E = 1-10$ GeV) gives $R = 0.38 \pm 0.02$. Thus the observed ratio $R \sim \frac{1}{3}$ or perhaps a little more. This tells us therefore three things:-

(i) partons have spin $\frac{1}{2}$

(ii) there are few antipartons

(iii) the coupling is $V-A$

To be exact, the CERN result $R = 0.38 \pm 0.02$ suggests that $>87\%$ of partons have spin $1/2$, and $<7\%$ are antipartons. The SLAC-MIT data on electron-nucleon scattering measure the ratio of magnetic to electric scattering (i.e. the gyromagnetic ratio) and indicate $\sim 94\%$ of partons have spin $1/2$. (The electron experiments tell us nothing about antipartons, since they measure the (charge)$^2$, which is the same for particle and antiparticle). Again, we note that the ratio $\sim \frac{1}{3}$ holds over a large energy range (2-75 GeV). At the low energy end, one should worry if the parton is really relativistic. If it is not, then the formula becomes

$$R = \frac{1}{3} \frac{(1 + 3z + 3z^2)}{(1 + z)^2}$$

where $z = m/2E$, $m$ being the parton mass. For $m = 0.25$ GeV, $R = 0.37$ at $E = 2$ GeV, so our relativistic approximation is satisfactory.

Our data so far seems to be consistent with the nucleon being built from pointlike objects (quarks) of spin $1/2$, with few antiparticle constituents. We can get no further without a more detailed analysis; which I shall sketch briefly. From the neutrino data and the electron
electron data taken together we can in principle find out, for example (i) the quark charges (ii) the number of quarks (iii) the fractional nucleon momentum carried by the quarks. Inelastic lepton cross-sections are usually expressed in terms of so-called structure functions (the analogue of form-factors for elastic scattering) and it is necessary at this point to explain how they are defined. According to the Bjorken scaling hypothesis, we get the following formulae:-

Electron Scattering:-

\[ \frac{d^2\sigma^e}{dx dy} = \frac{4\pi\alpha^2}{q^4} \left[ F_2^e(x)(1-y) + 2xF_1^e(x)\frac{y^2}{2} \right] \]  \hspace{1cm} (2)

Neutrino Scattering:

\[ \frac{d^2\sigma^\nu,\bar{\nu}}{dx dy} = \frac{G^2ME}{\pi} \left[ F_2^\nu(x)(1-y) + 2xF_1^\nu(x)\frac{y^2}{2} \pm xF_3^\nu(x)\left(y-\frac{y^2}{2}\right) \right] \]

In these formulae, \( x = q^2/2Mv \) and \( y = v/E \), where \( E \) is the energy of the incident lepton, \( q^2 \) is the 4-momentum transfer squared, and \( v \) is the energy transfer from lepton to hadron, measured in the rest-frame of the target nucleon. The assumption of single vector particle exchange means that there are, for neutrino and antineutrino scattering, 3 independent polarization states for the exchanged particle \( (2J+1 = 3) \) and thus 3 structure functions \( F_1, F_2, F_3 \). In electron scattering, the \( F_3 \) term is absent because of parity conservation. The above formulae represent averages over lepton and nucleon polarizations (if we consider particular polarization states, there are twice as many functions for neutrinos, i.e. 6, and 4 for electrons). For spin \( \frac{1}{2} \) constituents, one expects \( 2xF_1 = F_2 \), which as discussed above, seems to be the case in both the electron and neutrino data.

3. Quark Charge Sumrules

If the constituents are pointlike - and from now on, we think of them as quarks or antiquarks - then the electron experiments simply measure the Rutherford scattering from the quarks. The cross-section, or rather the part in the forward direction, \( x \rightarrow 0 \), determined by \( F_2 \) only, therefore measures the product of the number of quarks times the (charge)\(^2\) of each. From the neutrino scattering, we get a measure of
the number of quarks, and thus between the two results, we should be able to find the charges themselves.

The electron experiments measure cross-sections, as described above, in terms of the structure function $F_2^\text{EN}(x)$, where the scaling variable $x = q^2/2M_\gamma$ is the fractional nucleon momentum carried by the quark. If $u(x)$ stands for the probability of finding, in a proton, an isospin "up" quark with momentum $x$, $d(x)$ that for an isospin "down" quark, $s(x)$ that for a strange quark, and $\bar{u}, \bar{d}$ and $\bar{s}$ stand for the corresponding antiquarks, then for a proton target

$$F_2^\text{EP}(x) = x\left(\frac{2}{3}\right)^2 [u(x) + \bar{u}(x)] + x\left(\frac{1}{3}\right)^2 [d(x) + \bar{d}(x)]$$

$$+ x\left(\frac{1}{3}\right)^2 [s(x) + \bar{s}(x)]$$  \hfill (3)

since in the usual Gell-Mann/Zweig quark model, we have for the quark charges:-

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Charge</th>
<th>$I_3$</th>
<th>$S$</th>
<th>Symbol</th>
<th>Charge</th>
<th>$I_3$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;p quark&quot;</td>
<td>$u$</td>
<td>$+\frac{2}{3}$</td>
<td>$+\frac{1}{2}$</td>
<td>0</td>
<td>$\bar{u}$</td>
<td>$-\frac{2}{3}$</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>&quot;n quark&quot;</td>
<td>$d$</td>
<td>$-\frac{1}{3}$</td>
<td>$-\frac{1}{2}$</td>
<td>0</td>
<td>$\bar{d}$</td>
<td>$+\frac{1}{3}$</td>
<td>$+\frac{1}{2}$</td>
</tr>
<tr>
<td>&quot;\lambda quark&quot;</td>
<td>$s$</td>
<td>$-\frac{1}{3}$</td>
<td>0</td>
<td>$-1$</td>
<td>$\bar{s}$</td>
<td>$+\frac{1}{3}$</td>
<td>0</td>
</tr>
</tbody>
</table>

For a neutron target, we simply interchange $u$ and $d$ quarks in the formula. So, for a neutron-proton average (i.e. anucleon), we get

$$F_2^\text{EN}(x) = \frac{1}{2} \left\{ \frac{4}{9}[u(x) + \bar{u}(x) + d(x) + \bar{d}(x)] + \frac{1}{9}[d(x) + \bar{d}(x) + u(x) + \bar{u}(x)]
+ \frac{2}{9}[s(x) + \bar{s}(x)] \right\}$$  \hfill (4)

Since we already know from the neutrino data that antiquarks make little contribution, one could at this point consider a nucleon built from 3 quarks only, half 'u' and half 'd'. If they account for all the nucleon momentum, then we should have

$$\int_0^1 x \left[ u(x) + d(x) \right] \frac{1}{x} \, dx = 1$$
Hence
\[
\frac{18}{5} \int_{0}^{1} F_2 e^N(x) \, dx = 1
\]  \hspace{1cm} (5)

Experimentally, the SLAC-MIT data on electron-proton and electron-neutron scattering in the scaling region give
\[
\frac{18}{5} \int_{0}^{1} F_2 e^N(x) \, dx = 0.51 \pm 0.08
\]  \hspace{1cm} (6)

One way of interpreting this is to say that the active quarks only account for half of the momentum of the nucleon - a point we return to later.

Now consider inelastic neutrino scattering. We deal with the weak analogue of $F_2 e^N(x)$, called $F_2 e^\nu_N(x)$. A neutrino transforms to a $\bar{\nu}$, thus it has to scatter off an isospin "down" quark, denoted $d$, or off $\bar{d}$ so that the transformations are:

\[
v + d \rightarrow \nu^- + u; \hspace{1cm} v + \bar{u} \rightarrow \mu^- + \bar{d}
\]

Charge, $Q/e$:
\[
0 \hspace{1cm} -\frac{1}{2} \hspace{1cm} -1 \hspace{1cm} +\frac{1}{2} \hspace{1cm} 0 \hspace{1cm} -\frac{1}{2} \hspace{1cm} -1 \hspace{1cm} +\frac{1}{2}
\]

Similarly, antineutrinos transform to $\bar{\nu}^+$ and scatter from $u$ or $\bar{d}$ quarks. Thus one finds, in analogy with (2):

\[
F_2 e^\nu_p(x) = 2x \left[ d(x) + U(x) \right]
\]  \hspace{1cm} (7)

The factor 2 in (7) arises because one naturally assumes, for pointlike quarks, equal contributions from vector and axial-vector coupling. We have neglected strange quarks in (6), because their coupling is suppressed by a factor $\tan^2 \theta_{\text{Cabibbo}} \approx 0.05$. For a neutron-proton average,

\[
F_2 e^\nu_N(x) = x[d(x) + u(x) + \bar{u}(x) + \bar{d}(x)]
\]  \hspace{1cm} (8)

Comparing (4) and (8), we obtain the prediction

\[
\int F_2 e^\nu_N(x) \, dx / \int F_2 e^N(x) \, dx \leq \frac{18}{5}
\]  \hspace{1cm} (9)

where the inequality becomes an equality if we neglect the effects of strange quarks and antiquarks in electron scattering.

Experimentally, $\int F_2 e^\nu_N(x) \, dx = 0.51 \pm 0.03$ from the CERN-Gargamelle experiment, that is, from the slope of Fig. 1, so that the
observed ratio is

\[ \frac{\int F_2^{\nu N}(x) dx}{\int F_2^{eN}(x) dx} = 3.6 \pm 0.3 \] (10)

The agreement between (9) and (10) is a confirmation of the "conventional" quark charges. However, there are other, quite different, arguments which arrive at a similar ratio. For example, it is known that the vector part of the strangeness-conserving weak interactions (which involve the isospin raising and lowering operators \( I^+ \) and \( I^- \)) and the isovector part of the electromagnetic interactions are connected by \( \Delta I = 1 \) rule - they are different components of an isospin 1 current, carrying electric or weak "charge" as the case may be, just as \( \pi^+ \), \( \pi^- \) and \( \pi^0 \) are different components of an isovector pion. Then from the Clebsch-Gordan coefficients we get

\[ F_2^{\nu}(\text{Vector}) = 2 F_2^{e}(\text{Isovector}) \]

Also, if we assume that axial vector and vector contributions to the weak scattering are equal, we expect

\[ F_2^{\nu}(A_{1V}) = 4 F_2^{e}(\text{Isovector}) \]

From photoproduction data, isoscalar contributions in electromagnetic processes are typically at the 10% level, so one might suppose that \( F_2^{e}(\text{Isovector}) \sim 0.9 F_2^{e}(\text{Isovector} + \text{Isoscalar}) \). Hence \( F_2^{\nu} \sim 3.6 F_2^{e} \), exactly the same as the quark model prediction.

The confrontation with the constituent models becomes more impressive when one considers differential cross-sections, that is \( F_2^{eN}(x) \) or \( F_2^{\nu N}(x) \) in unintegrated form. Fig. 3 shows the CERN data in the scaling region, compared with the SLAC scaling curve. The evidence is rather compelling that electrons and neutrinos are seeing the same substructure inside the nucleon, with absolute rates standing exactly in the ratio predicted by the quark charge assignments.

4. **Quark Counting. The Gross-Llewellyn Smith Sumrule**

For spin \( \frac{1}{2} \) constituents, there are two independent structure functions describing neutrino scattering on nucleons, called \( F_2(x) \) and \( xF_3(x) \), as in (2) above. While \( F_2 \) contains both \( A \) and \( V \) parts in
quadrature, \( xF_3 \) is the V-A interference term, and changes sign under interchange of neutrino and antineutrino. We adopt the positive sign for neutrinos. The antineutrino/neutrino ratio shows that \( xF_3 \) is nearly maximal, and indeed the ratio one finds by integrating (2) over \( y \) is:

\[
R = \frac{\sigma(\bar{\nu})}{\sigma(\nu)} = \frac{2 - B}{2 + B} \quad \text{where} \quad B = \int xF_3dx \quad \frac{1}{\int xF_2dx} = 0.85
\]  

(11)

The quantity \( xF_3 = \pm F_2 \) for collisions of neutrinos on particles (antiparticles) respectively, so that from (8),

\[
xF_3^{\nu}(x) = x[d(x) + u(x) - \bar{u}(x) - \bar{d}(x)]
\]

Hence

\[
\int_0^1 xF_3^{\nu}(x) \frac{dx}{x} = \frac{1}{\int_0^1 xF_3^{\nu}(x) dx}
\]

measures the difference in the number of quarks and antiquarks (while \( \int_0^1 xF_2^{\nu}(x) dx/x \) measures the sum). For the Gell-Mann/Zweig quark model, we therefore expect

\[
\int_0^1 xF_3^{\nu}(x) dx = 3 \quad \text{(Gross/Llewellyn-Smith sumrule)}
\]

(13)

The sumrule can be investigated experimentally by forming the neutrino-antineutrino cross-section difference (see equation (2)):

\[
F_3(x) = \frac{3\pi}{2G^2ME} \left[ \frac{1}{x} \frac{d\sigma(\nu)}{dx} - \frac{1}{x} \frac{d\sigma(\bar{\nu})}{dx} \right]
\]

(14)

There are difficulties if one is to remain always in the scaling region of \( q^2 > 1 \text{ GeV}^2 \). On the basis of the distribution function \( u(x), d(x) \) etc. obtained by empirical fits to the electron-scattering data, it turns out that, near \( x = 0 \), \( F_3(x) \propto x^{-1/2}(1-x^2)^3 \) so that contributions to the integral \( \int_0^1 F_3(x) dx \) near \( x = 0 \) are very important. In fact, these fits suggest that 20% of the integral arises from values of \( x < 0.01 \). Since \( x = q^2/2Mv \), this implies that a neutrino energy of several hundred GeV is required to get within 20% of the full integral. Furthermore, the absolute cross-sections must be measured accurately.

No test of (13) has yet been made for events in the scaling region. A test has been made in the CERN experiments, where it is observed (Fig. 4) that if no scaling cuts are made, then even at low energies (\( E \sim 2 \text{ GeV} \)), the values of \( F_2^{\nu}(x') \) are in good agreement with
3.6 \frac{F_2}{eN(x')}{x'} from the SLAC experiments. Here \( x' = \frac{q^2}{(2M\nu + M^2) \nu} \) is a modified scaling parameter introduced by Bloom and Gilman. Empirically one finds that, if no kinematic constraints \( q^2 > q_{\text{min}}^2, \nu > \nu_{\text{min}} \) are applied to the data, then one observes some average type of "precocious" scaling behaviour in \( x' \) when averaged over all values of \( q^2 \) and \( \nu \) allowed by conservation of energy and momentum. The result of the analysis is

\[
\frac{1}{\int_{0}^{x'} F_2(x')dx'} = 3.1 \pm 0.4
\]

(15)

There is no observed dependence of the integral on neutrino energy, so the hope is that this value also applies in the true scaling region. At the very least, (15) is an interesting result.

5. **Gluon Contributions**

So far, the simple quark model looks quite good, barring one mysterious result. From (8), we see that \( \int F_2 \omega dx \) measures the total momentum of the (non-strange) quarks and antiquarks, and should be unity if nucleon is composed entirely of such objects. However, for both electron and neutrino scattering, one obtains

\[
\frac{(\text{Total Quark + Antiquark Momentum})}{(\text{Nucleon Momentum})} = \frac{1}{2} (0.51 \pm 0.08 \text{ for e})
\]

(0.51 \pm 0.03 \text{ for } \nu)

The extra, unaccounted momentum is ascribed to neutral "gluons" which provide the quark binding forces. What is mysterious is that these contribute exactly half of the nucleon mass. So far, no one has been able to explain this particular magic number.

6. **General Comments on the Quark Model of Inelastic Lepton Scattering**

I conclude with a few general comments about the quark model. In the context of inelastic lepton scattering, it is an extremely useful form of shorthand, enabling one to compute cross-sections for all types of leptons on nucleons, over a wide range of dynamical variables. This does not imply at all that one has to take the model literally as meaning that a nucleon is actually built from three quarks; indeed there are good reasons for believing that, in a sense, such an idea is not meaningful.
Let us look at some of problems. First, the parton model seeks to describe the deep-inelastic scattering of leptons by nucleons as incoherent elastic scattering of the lepton by one quasi-free constituent. The interaction of the constituents among themselves must be neglected. So the model should only work at high $q^2$, where the impulse approximation can hold. How large has $q^2$ to be? Generally one expects that one should have $q^2 \gg M^2$. That means that, by the uncertainty principle, one is isolating a very small region ($\sim 0.1$ fermi for $q^2 = M^2$) inside the nucleon, where there is zero probability that one will find 2 constituents together, that is one hits either 0 or 1 parton. However, one is embarrassed to find that scaling, that is the independence of $F_2(x)$ on $q^2$, holds to a precision of order 1% or so, right down to $q^2 \sim 1$ GeV. Even worse, if we modify our $x$ to $x'$, we get some average type of scaling, with the cross-sections determined by the single dimensionless parameter $x'$, down to ridiculously low values of $E$ and average $q^2$. The asymptotic sum rules work beautifully in this non-asymptotic region of energy. So, precocious scaling is a fact of life which was certainly not expected and is not easy to explain in our parton picture.

Setting these troubles aside, there are deeper problems, or solutions - it is hard to know which! Quarks are not observed as free particles, and in a way this is just as well, for they have properties which are unacceptable for free particles. I am of course referring to the spin-statistics relation, which has been thoroughly tested for free particles; photons, electrons, protons, neutrons, muons. Quarks do not obey this relation, for they have spin $1/2$ and yet 3 p-quarks with spins parallel, in an S-state, form the $\Delta^{++}$-resonance. This is perfectly admissible if they can never get out, for example if they were held in a simple harmonic oscillator or other potential which increased fast enough with the quark separation. Their "reality" or otherwise then depends on whether the observer is inside or outside the potential well. In the same way, an observer inside a 'black hole' (as the existing universe may well be) can observe photons; he can define them as free particles. An observer outside the black hole cannot receive those photons, although he might in principle deduce their existence as a
contribution to the energy/momentum of the black hole, and hence the gravitational field outside it, which he can detect.

Let me now try to summarize. The quark model of baryons is to say the least, a very useful mnemonic way of describing an enormous mass of inelastic lepton-nucleon scattering data. It has drawbacks in that as far as leptons are concerned, it appears to work only (but rather too well) for states of baryon number 1 (i.e. nucleons) and for spacelike momentum transfers (i.e. scattering). Even then it throws up some unexplained numbers, like the 50% gluon contribution. It fails dismally for timelike processes involving leptons and hadrons of $B = 0$, that is $e^+e^- \rightarrow$ hadrons, where the predicted cross-sections, via $e^+e^- \rightarrow q\bar{q}$ are significantly less than what are observed in the recent colliding beam experiments at CEA and SPEAR.

LECTURE II

Neutral Weak Currents

The outstanding development in weak interactions in the last year has been the observations of neutral weak currents, that is, interactions in which a neutrino is scattered, elastically or inelastically, without change of charge. The effects have been observed in four independent experiments at three accelerators, and their existence is firmly established, with couplings comparable with those of the charge-changing weak currents.

It is useful to discuss briefly the theoretical importance of neutral currents. One of the outstanding problems in physics is to understand the inter-relation between the apparently independent fundamental interactions; The strong, weak, electromagnetic and gravitational couplings, not to mention the possible superweak and superstrong interactions. The discovery of neutral currents, levels observed, is very strongly suggestive, for the first time, of a basic unification of two of these interactions, the weak and electromagnetic. This is a profound step forward.

It had long been recognised that neutral currents and/or new heavy leptons might be the key to renormalizability of the weak interactions. Let us recall that the old Fermi recipe for $\beta$-decay, although
successful at low energies, became badly divergent at high energy. For example,

\[
\sigma \sim G^2 mE
\]

point, s-wave scattering of neutrinos of energy \(E\) by electrons of mass \(m\) had a cross-section

\[
\sigma(v_e + e^- + e^- + v_e) = \frac{4G^2}{\pi}, \quad p^2 = \frac{2G^2 m E}{\pi}
\]

where \(p\) is the CMS momentum. From ordinary wave theory, the maximum elastic scattering cross-section for CMS wavelength \(\lambda\) is

\[
\sigma_{\text{max}} = \frac{\pi \lambda^2}{2} \sum (2k + 1)
\]

\[
= \frac{\pi}{2p^2} \quad \text{for s-waves}
\]

Thus, when

\[ p \geq G^{-1/2} = 300 \text{ GeV} \]

the Fermi cross-section exceeds the wave theory limit, which is impossible. This difficulty could be avoided by introducing the charged intermediate vector boson \(W^\pm\) to "spread" the weak interaction, thus giving a constant high energy cross-section:

\[
\sigma = \frac{G^2}{\pi} \int_0^{2mE} \frac{dq^2}{(1+q^2)^2} \approx \frac{G^2}{\pi} M_w^2 \quad \text{at large } E
\]

Unfortunately, there are other amplitudes such as that in (a) below, which are found to be divergent even in first order. A possible
cure is to introduce extra amplitudes which are arranged to exactly cancel the divergent terms. These could be in the t-channel in the form of a new heavy lepton \( \ell^* \), as in (b), or in the s-channel via a neutral boson \( Z^0 \), that is a neutral current, in the context of the Salam-Ward-Weinberg theory.

The possible renormalizability of weak interactions, that is the finiteness of amplitudes to all orders in the coupling constant and at all energies, is sought in analogy with the prototype renormalizable interaction, namely quantum electrodynamics. Here, renormalizability is connected with gauge invariance and the zero photon mass, and the new theory enlarges the gauge symmetry to include weak processes, by introducing further charged and neutral vector bosons, in addition to the photon. In the infinite energy limit, all particle masses, both of leptons and bosons, can be neglected, and the coupling of the members of the boson family to the leptons is essentially specified by a single coupling constant, \( \lambda \) (the fine structure constant). In the Salam-Ward-Weinberg model, the bosons consist of an "isospin" triplet \( W^+, W^-, W^0 \) and an "isosinglet" \( B_0 \).

In the real world, such zero-mass, charged fields cannot exist since all charged particles have mass. The mass is supposed to be acquired as a result of some symmetry-breaking mechanism, which however leaves the couplings unaltered. Thus the intrinsic couplings of weak and electromagnetic interactions are supposed to be identical (and the actual ones would be, at sufficiently high energy). At normal energies however, the effective couplings are very different, since the weak interaction is mediated by massive bosons and is consequently of short range, while the electromagnetic interaction is of infinite range (zero photon mass). It is easy to compute the approximate boson
mass required to produce the observed effective coupling ratios:

![Diagrams showing charged boson interactions](image)

From (a) we see that the Fermi coupling $G$ is given by

$$G = \frac{\alpha^2}{q^2 + 0 \left(q^2 + M_W^2\right)},$$

so that $M_W = g/\sqrt{G}$. If (a) weak and (b) electromagnetic, interactions have the same coupling $g = e$, then $M_W = e/\sqrt{G} \sim 30$ GeV. So the charged boson mass in this model is huge, simply expressing the fact that weak interactions are extremely short range, of order $10^{-2}$ fermi. In detail, the neutral bosons $W^\pm$ and $B_\omega$ mix to form

$$Z^0 = W^\pm \cos \theta_W + B^\pm \sin \theta_W,$$

$$\gamma = B^\pm \cos \theta_W - W^\pm \sin \theta_W,$$

where $\theta_W$, the Weinberg mixing angle, is the only free parameter of the theory. As a result of the gauge symmetry-breaking mechanism, which we do not discuss here, three of the four bosons acquire mass. $W^\pm$ mediate the charged weak currents; $Z^0$ and the massless $\gamma$ mediate the neutral weak and electromagnetic interactions respectively. The masses are given by

$$M_{W^\pm}^2 = \sqrt{2} \frac{e^2}{(8G \sin^2 \theta_W)}; \quad M_W = \frac{37}{\sin \theta_W} \text{ GeV},$$

$$M_{Z^0}^2 = \sqrt{2} \frac{e^2}{(8G \cos^2 \theta_W \sin^2 \theta_W)}; \quad M_{Z^0} = \frac{37}{(\sin \theta_W \cos \theta_W)} \text{ GeV} \quad (16)$$

and

$$M_\gamma = 0 \quad \geq 74 \text{ GeV}$$

Finally, it should be mentioned that two isodoublets of scalar bosons have to be introduced in the model (to provide the symmetry-breaking mechanism). There is no prediction on their masses. A
necessary consequence of the model is that neutral and charged weak bosons have comparable couplings (in the limit $\theta_w = 0$, the amplitude ratio (neutral/charged) = $1/2$, which is simply a Clebsch-Gordan coefficient). It is most important to understand that this model is one of leptons and mediating bosons; hadrons do not enter specifically, and one has to make further hypotheses to include them.

We now discuss the experimental situation. We expect the experiments of course to tell us much more than simply whether or not neutral currents exist. For example they should determine the Weinberg angle, or, equivalently, the relative amount of V and A coupling (and any other) associated with neutral currents. In addition to the space transformation properties, one should get indications, when dealing with hadrons, on the isospin properties, whether the inclusive neutral current interactions obey scaling, and so on.

(i) Leptonic Neutral Currents - the CERN Gargamelle Experiment

The CERN experiment uses a so-called focussed wideband beam, obtained by directing 26 GeV protons into a beryllium target, thus producing charged pions and kaons which decay to neutrinos in a 60 m long decay tunnel. Hadrons and muons are filtered out by a 22 m steel shield. The beam of about $10^9$-$10^{10}$ neutrinos/sec traverses a large bubble chamber (Gargamelle, 1.8 m diameter x 5 m long) filled with heavy liquid (CF$_3$Br, density 1.5). The useful mass of liquid is about 10 tons. The pions and kaons of one sign of charge from the target can be partly focussed by specially-shaped, pulsed conductors, and one can select a beam of $\nu_\mu$ (positives focussed) or $\bar{\nu}_\mu$ (negatives focussed). The neutrino spectrum is continuous, with a peak at 2 GeV and falling off rapidly at higher energy - see Figs. 5 and 6. In addition to $\nu_\mu$, a small component ($\sim 1/2\%$) of $\nu_\mu$, from $\mu$ and Ke3 decay, is present.

The lepton couplings in the Weinberg theory are given in the accompanying table. The differential cross-section for projecting an electron with energy $E$ is

$$\frac{d\sigma}{dE} = \frac{G^2 m}{2\pi} \left[ (g_V + g_A)^2 + (g_V - g_A)^2 (1 - E/E_0)^2 \right]$$

(17)
<table>
<thead>
<tr>
<th>Elastic Scattering</th>
<th>Weinberg Theory</th>
<th>V-A Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e + e^- \rightarrow \frac{1}{2} + 2\sin^2 \theta \nu_e + e^- \rightarrow \frac{1}{2} + 2\sin^2 \theta$</td>
<td>$g_V$</td>
<td>$g_A$</td>
</tr>
<tr>
<td>$\bar{\nu}_e + e^- \rightarrow \frac{1}{2} + 2\sin^2 \theta \bar{\nu}_e + e^- \rightarrow \frac{1}{2} + 2\sin^2 \theta$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_\mu + e^- \rightarrow -\frac{1}{2} + 2\sin^2 \theta \nu_\mu + e^- \rightarrow -\frac{1}{2} + 2\sin^2 \theta$</td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td>$\bar{\nu}<em>\mu + e^- \rightarrow -\frac{1}{2} + 2\sin^2 \theta \bar{\nu}</em>\mu + e^- \rightarrow -\frac{1}{2} + 2\sin^2 \theta$</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

$\theta$ = Weinberg Angle

for a neutrino energy $E_\nu >> m$, the electron mass. The total cross-sections obtained by integrating this expression are shown in Fig. 7.

While electron neutrinos can scatter via both charge-changing and neutral currents, muon neutrino-electron scattering can take place only via a neutral current:

\[
\begin{align*}
\nu_e &\rightarrow e^- + W^+ \\
\bar{\nu}_e &\rightarrow e^+ + W^-
\end{align*}
\]

\[
\begin{align*}
\nu_\mu &\rightarrow e^- + Z^0 \\
\bar{\nu}_\mu &\rightarrow e^+ + Z^0
\end{align*}
\]

\[
\begin{align*}
\nu_e &\rightarrow e^- + e^-
\end{align*}
\]

\[
\begin{align*}
\nu_\mu &\rightarrow e^- + e^-
\end{align*}
\]

Note that not only the total cross-section but also the recoil energy spectrum depends quite critically on $\theta_\nu$. We shall be concerned mostly with $\bar{\nu}_\mu$ interactions, for which the spectrum is fairly flat for $\sin^2 \theta_\nu \approx \frac{1}{2}$, but is sharply peaked to low values, and of the form $(1-E/E_\nu)^2$, for small $\theta_\nu$. The minimum cross-section for $\bar{\nu}_\nu$ on electrons occurs for $\sin^2 \theta_\nu = 0.125$, and has the value $G^2 m^2 E_o/8\pi = 1.06 \times 10^{-42} E_{\text{cm}^2/\text{electron/GeV}}$. For a 10-ton detector, $10^9 \nu_e/m^2/\text{pulse}$, this corresponds to one interaction in every million pulses. This dismally low rate is compensated by the fact that the interactions have very
clear signatures, since the maximum angle of emission of the electron is small:

$$\theta_e = \frac{2m}{\sqrt{2mE}} \left( 1 - \frac{1}{E_0} \right) < \frac{2m}{\sqrt{2mE}}$$

For $E > 200$ MeV, $\theta_e < 4^\circ$.

The Gargamelle experiment is still in progress. The runs started in 1971, and are being continued (through 1973/4) with the CERN booster ($\approx 5 \times 10^{12}$ ppp). From 0.3 million antineutrino pictures without, and 0.5 million with, the booster, the very preliminary results, for single $e^-$ or $e^+$ events, or events of undetermined sign, with $\theta_e < 5^\circ$, looks roughly as follows:

\[\begin{array}{c}
\bar{\nu}_e^- & e^+ \quad \bar{\nu}_e^+ \\
\end{array}\]

The radiation length in the liquid freon employed is 0.11 m; this means that an $e^+$ or $e^-$ may undergo bremsstrahlung with pair conversion after only a few cms., and it is then not possible to measure the sign of charge from curvature. In events of the type $\nu_e + n \rightarrow e^- + $ hadrons, it is known that such confusion arises in 30% of the events.

The main source of $e^+$ events is the elastic reaction produced by

\[\bar{\nu}_e^+ + p \rightarrow n + e^+\]  

(18)

where the $e^+$ has $\theta < 5^\circ$, and there is correspondingly small $q^2$ of order 0.01 or less. Analysis of numerous events of the type $\nu_\mu + n \rightarrow p + \mu^+$ allows one to evaluate the probability of such low $q^2$ processes which are of course suppressed by the Pauli principle in complex nuclei. Since the $e^+$ carries off essentially the full energy of the incident $\bar{\nu}_e$, and the cross-section at low $q^2$ is independent of energy, we expect the positron spectrum to follow that of the antineutrinos, with a pronounced peak at 2 GeV.
Background single electron events can arise from the reaction

$$\nu_e + n \rightarrow e^- (p)$$  (19)

where, on account of the low $q^2$, the proton is not observed. As compared with (18), the event rate should be suppressed by a factor 5-10 (depending on energy), since in the antineutrino runs, the neutrino parents ($\pi^+$ and $K^+$) are defocussed. The energy spectrum of $e^-$ and $e^+$ from the elastic interactions on nucleons will be similar.

The important features of the signal and background processes is that electrons from neutral current interactions will be of predominantly low energy (less than 1 GeV usually), while $e^+$ and $e^-$ from the background will be of high energy (>2 GeV usually). This feature is illustrated in Fig. 8, and seems to be borne out by the data. The table below gives an approximate calculation of expected numbers of signal and background events.

<table>
<thead>
<tr>
<th>Film</th>
<th>Flux/m²</th>
<th>Weinberg $\nu$, max/ min</th>
<th>$\nu, e^-$ ($p$)</th>
<th>$\nu, e^+$, (n)</th>
<th>Observed (θ&lt; 5°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>$2.10^{15}$</td>
<td>6.0/0.6</td>
<td>0.3 ± 0.2</td>
<td>0</td>
<td>$e^-$</td>
</tr>
<tr>
<td>$\bar{\nu}$</td>
<td>$3-4.10^{15}$</td>
<td>20/1</td>
<td>0.12 ± 0.04 ($&gt;1$ GeV)</td>
<td>5+4</td>
<td>$e^-$ (&lt;1 GeV)</td>
</tr>
</tbody>
</table>

The 90% confidence levels from the experiment to date are

$$0.6 > \sin^2 \theta_w > 0.1$$
(ii) **Inclusive Neutral Current Interactions on Nucleons. The CERN and NAL experiments.**

These experiments have observed examples of the process \( \nu_\mu, \bar{\nu}_\mu + \text{nucleon} \rightarrow \nu_\mu, \bar{\nu}_\mu + \text{hadrons}. \) The CERN experiment was carried out in the heavy liquid chamber Gargamelle (Hasert et al 1973) with a wideband beam produced by 26 GeV protons. The NAL experiment employed a large liquid scintillator calorimeter, followed by a muon spectrometer, and used both wideband and narrowband neutrino beams, produced by 200-400 GeV protons (Benvenuti et al 1974).

There is not space to go into these experiments in detail, but the principals of the experiments were as follows. In the CERN experiment, events were recorded which contained **identified hadrons only** (so-called "NC" events), with no other event in the same picture. The main problem was to establish that these were not due to neutrons produced by neutrino interactions in the shielding or other material surrounding the bubble chamber liquid. This was done by recording associated star ("AS") events, in which a neutrino interaction (with muon secondary), together with the interaction of a secondary neutron, was observed:

- **Charged current (CC) event**
- **Neutral current (NC) event**
- **Associated star (AS) event.**

From a detailed analysis of the event distributions, and Monte Carlo calculations of neutron production and propagation through the shielding, it was concluded that only 10% of the NC events could be ascribed to neutrons, and the remainder must be ascribed to some new process; neutral currents are the obvious candidates. For hadron energies >1 GeV, the observed ration \( R_\nu = \frac{\text{NC events}}{\text{CC events}} \) = 0.26 ± 0.03 for neutrinos and \( R_{\bar{\nu}} = 0.46 ± 0.09 \) for antineutrinos.
The NAL counter experiment observed "muonless" events as those which gave a certain minimum hadron energy release in the calorimeter, but no signal in the spark chambers of the downstream spectrometer. From these events must be subtracted those with muons which missed the spectrometer. The muon detection efficiency was measured and the conclusion, after some 6 months of hesitation on the part of the groups involved, was that a genuine neutral current signal remained. In order to improve the muon detection efficiency, the experiment was modified to require that muons should only penetrate a 30 cms thick iron plate at the rear of the calorimeter; one then has to correct genuine NC events for the hadron punch-through probability, that is, the probability that a hadron would give a track in a counter behind the iron plate. This can be measured using events with an identified muon in the spectrometer. The experiments used a wideband, unseparated or partly separated beam, and the early results gave for the ratio \( R = \frac{\text{NC events}}{\text{CC events}} = 0.27 \pm 0.09; \) the later runs gave \( R = 0.20 \pm 0.05. \) In a still later version, using a narrow-band, dichromatic beam, separate values for neutrino and antineutrino runs have been given: \( R_\nu = 0.13 \pm 0.06, \ R^-_\nu = 0.34 \pm 0.12. \)

The results from both CERN and NAL experiments are given in the following table:-

<table>
<thead>
<tr>
<th>Laboratory</th>
<th>Min Hadron energy</th>
<th>Mean neutrino energy</th>
<th>( R_\nu )</th>
<th>( R^-_\nu )</th>
<th>Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN</td>
<td>&gt; 1 GeV</td>
<td>~3 GeV</td>
<td>0.26±0.03</td>
<td>0.46±0.09</td>
<td>Wideband separated</td>
</tr>
<tr>
<td>NAL</td>
<td>&gt; 6 GeV</td>
<td>~50 GeV</td>
<td>0.13±0.06</td>
<td>0.34±0.12</td>
<td>Narrow Band separated</td>
</tr>
</tbody>
</table>

We note that the two sets of experimental results are not very consistent; this may be attributable to different values of the ratio (hadron energy)/(mean neutrino energy), to the different energy range, or, more likely, systematic errors or biases arising from the experimental criteria.
The results may be interpreted in the Weinberg theory, if we are prepared to make additional assumptions regarding the coupling of hadrons to neutral currents. These interpretations are very model-dependent.

As an example, Fig. 9 shows $R_V$ plotted against $R_V$. The curve shown indicates the variation expected for equal $V$ and $A$ coupling with maximum $V$-$A$ interference (as is the case for charged currents). This curve represents a lower limit to the ratios $R_V$ and $R_V$, for pure isovector coupling and zero isoscalar contribution. While the CERN result sits on the limiting curve, the NAL result is not consistent with it. Because of these discrepancies, it is hard to draw conclusions. There is however some other circumstantial evidence to suggest that isoscalar contributions to the neutral coupling are unimportant. For example, the ratio of neutral to charged pion secondaries observed in the Gargamelle experiment is $0.9 \pm 0.1$ in both charged and neutral current events, as expected if they are the $I_+^1$ and $I_3^1$ components of an isospin 1 current.

If the neutral current inclusive cross-sections for neutrino and antineutrino are denoted $\sigma_0$ and $\bar{\sigma}_0$, then the result $\sigma_0 = \bar{\sigma}_0$ could indicate that the coupling is pure $V$ or pure $A$, with no $V/A$ interference term. The results observed are

$$\frac{\sigma_0}{\bar{\sigma}_0} (\text{CERN}) = 0.46 \pm 0.12; \quad \frac{\sigma_0}{\bar{\sigma}_0} (\text{NAL}) = 0.9 \pm 0.5$$

Thus, pure $A$ or pure $V$ coupling does not appear very likely, but it must be borne in mind that the selection of events introduces possible biases and one has to exercise caution.

In summary, the study of inclusive hadron processes has demonstrated the existence of semi-leptonic neutral currents. It is too early to draw definite conclusions regarding their transformation properties, but it seems likely that they are predominantly isovector with both $V$ and $A$ spatial components. A value of the Weinberg angle $\sin^2 \theta_W = 0.3-0.5$ is consistent with the rates observed.
(iii) **Exclusive Hadronic Neutral Currents - Single Pion Production**

It has long been recognized that more precise information regarding the nature of semi-leptonic neutral currents could be obtained, with fewer theoretical assumptions, by studying exclusive final states with well-defined properties, such as isospin.

The cleanest experiment of this type has been carried out in the ANL 12' λ2/λ2 chamber, over the past 2-3 years. Because of the kinematic constraints and high precision of the hydrogen bubble chamber technique, it is possible to evaluate backgrounds in a rather direct way; for example, the neutron background flux can be measured by observing events of the type np → ppπ⁻ (1C fit). There is not space here to do justice to a very clever analysis, which is unfortunately and rather naturally limited by poor statistics. After making all background corrections, the single pion production channels (either a \( \pi^+ \) or a \( \pi^0 \) giving, on occasion, a single \( \gamma \rightarrow e^+e^- \)) lead to the results

\[
R_+ = \frac{\nu p \rightarrow \nu p\pi^+}{\nu p \rightarrow \mu^- p\pi^+} = 0.17 \pm 0.08 \quad \text{and} \quad 0.06 \leq R_+ \leq 0.17
\]

\[
R_0 = \frac{\nu p \rightarrow \nu p\pi^0}{\nu p \rightarrow \mu^- p\pi^+} = 0.48 \pm 0.24 \quad \text{and} \quad 0.06 \leq R_0 \leq 0.22
\]

There is therefore a clear neutral current signal (13 events against a background of \( \sim 2.4 \)), the rates being consistent with various models, but favouring \( \sin^2 \theta_W < 0.5 \).

The interesting result \( R_0/R_+ = 2.9 \pm 2.0 \) can be compared with the expectation of 2 for a final state of \( I = \frac{3}{2} \) (therefore \( \Delta I = 1 \)), and \( \frac{1}{2} \) for a final state of \( I = \frac{1}{2} \) (isoscalar current). Again, isovector neutral currents are favoured.

(iv) **Conclusions**

Four different experiments find significant evidence for neutral weak currents, in both leptonic and semi-leptonic processes. We can regard the effect as well established.

This result must be set against the very stringent limits on the complete absence of hadronic neutral currents in \( \Delta S = 1 \) decay processes, for example \( K^+ \rightarrow \pi^+ \nu\bar{\nu} \) has a branching ratio \( <10^{-6} \). How
these results can be reconciled on the basis of a hadron model is not clear. One suggestion has been to cancel the $\Delta S = 1$ amplitude by introducing extra quantum numbers. The Glashow, Iliopoulos, Maiani SU4 scheme has a quarter of quarks; the usual $p, n, \lambda$ plus a "charmed" quark $p'$, with the correct Cabibbo coupling to cancel the $\Delta S = 1$ neutral current amplitude. The $p'$ has charge $+2/3$, $S = -1$, $I = 0$ and the extra "charm" quantum number ($C = 1$). If the new quark is heavier than $p, n, \lambda$, one can ascribe the success of SU3 to the fact that observed baryon resonances are below the "charm" threshold. The mass difference cannot be more than a few GeV however, otherwise the forbidden $\Delta S = 1$ neutral current transitions would proceed in second order at an unacceptably high rate.

The fact that the charged-current processes show no evidence for charmed particle production may be due to the fact that charmed particles contribute only a small part of the cross-section. Thus, in the GIM model, either transitions are of the $\Delta S = 1$ type $n \to p'$, and therefore suppressed by the Cabibbo factor $\sin^2 \theta_C \sim 0.05$; or they are $\Delta S = 0$, such as $\lambda \to p'$, where the target quark $\lambda$ can arise only from the quark-antiquark sea, again making $\sim 5\%$ contribution to the cross-section. Thus, more refined tests are required. The question of charmed particles is at present an open one.

Finally, one might speculate (perhaps idly) about the space properties of neutral lepton currents. Maximum economy of hypothesis could have been obtained by having pure $V$ electromagnetic interactions, pure $A$ neutral weak currents, and charged weak currents with equal $V$ and $A$ contributions, with maximum interference (i.e. maximum parity violation). The first and last options were taken up by nature, but the second, apparently, was not. It would require $\sin^2 \theta_W = 0.25^+$ which is on the verge of exclusion by the experimental data.

See the table on leptonic couplings in the first part of this lecture.
REFERENCES

Total Cross-Sections

CERN: Eichten et al., PL 46B, 281 (1973)
NAL: Benvenuti et al., PRL 30, 1084 (1973)

Neutral Currents

Leptonic; CERN, Hasert et al., PL 46B, 121 (1973)
Inclusive: CERN, Hasert et al., PL 46B, 138 (1973)
   NAL, Benvenuti et al., PRL 32, 800 (1974)
Exclusive (1π); Barish S. et al., Private Communication (1974)

Other References

Gross D. and Llewellyn-Smith C., NP B14, 337 (1969)
Pais A and Treiman S.B., PR D6, 2700 (1972)
Paschos E.A. and Wolfenstein L., PR D7, 91 (1973)
Weinberg S., PRL 19, 1264 (1967)
Salam A and Ward J.C., PL 13, 168 (1964)
NEUTRINO TOTAL CROSS-SECTIONS

- CERN wideband (1973)
- NAL wideband (1973)
- NAL narrowband (1974)

Fig. 1
ANTINEUTRINO/NEUTRINO CROSS-SECTION RATIO

\[ R = \frac{\sigma(\bar{\nu})}{\sigma(\nu)} \]

- CERN wideband
- NAL 1A narrowband

\[ \begin{align*}
0.38 \pm 0.02 & \quad \text{spin 1/2 only} \\
0.34 \pm 0.08 & \quad \text{no antiquarks} \\
R = \frac{1}{3} & \quad V-A \text{ coupling}
\end{align*} \]

Fig. 2
STRUCTURE FUNCTIONS FOR EVENTS IN THE SCALING REGION
$q^2 > 1 \text{ GeV}^2$
$W^2 > 4 \text{ GeV}^2$

- $3.6 F_2^e(x) - \text{SLAC}$
- Modified by Fermi motion & measurement errors

Curve computed from empirical fit to electron data.

Fig. 3
- All events, $E = 1-11$ GeV
- Elastic events
- $3.6 F_{2}^{sN}(x)$, SLAC
- --- With Fermi smearing and measurement errors

Fig. 4
FLUXES IN NEUTRINO RUNS

(Positive Particles Focussed)

\[ \text{FLUX / m}^2 \cdot \text{GeV/ proton averaged over 0.6M radius} \]

\[ \nu_\mu \]

\[ \nu_e \]

\[ \bar{\nu}_e \]

\[ E_\nu \text{ GeV} \]

Fig. 5
FLUXES IN ANTINEUTRINO RUNS

(Negative Particles Focussed)

Fig. 6
NEUTRINO-ELECTRON SCATTERING
CROSS-SECTIONS IN WEINBERG
MODEL

$$\sigma_0 = \frac{G^2 m}{2\pi} = 4.24 \text{ cm}^2/\text{GeV} \times 10^{42}$$

$$\nu_e + e^- \rightarrow e^- + \nu_e$$

$$\bar{\nu}_e + e^- \rightarrow e^- + \bar{\nu}_e$$

$$\nu_\mu + e^- \rightarrow e^- + \nu_\mu$$

$$\bar{\nu}_\mu + e^- \rightarrow e^- + \bar{\nu}_\mu$$

$$\sigma_{(V-A)} \text{ for } \nu_e + e^-$$

$$\sigma_{(V-A)} \text{ for } \bar{\nu}_e + e^-$$

$$\sigma_{0/4} = 1.06 \times 10^{42}$$

Fig. 7
ELECTRON RECOIL SPECTRA IN $\bar{\nu}$ FILM, AVERAGED OVER $\bar{\nu}_\mu$ or $\nu_e$ SPECTRUM

Area B = 0.082
Area A

$\sigma(\bar{\nu}_\mu e \rightarrow e^{-} \bar{\nu}_\mu)$

$\sigma(\nu_e n \rightarrow p e^{-}(<5^\circ))$

$G^2 m / 4 \pi$

$e^2 / g^2 = 1$

$e^2 / g^2 = 1 / 2$

$e^2 / g^2 = 1 / 4$

$e^2 / g^2 = 0$

$0 \text{p} + 1 \text{p events}$

$x \sum \phi_{\nu_e} / \sum \phi_{\nu_\mu}$

$0 \text{ proton events}$

Fig. 8
NEUTRAL/CHARGED CURRENT INCLUSIVE CROSS-SECTIONS

○ CERN
× NAL

Paschos-Wolfenstein
Pais-Treiman
lower limit (no
isoscalar) assuming
scaling and $|V|=|A|$

Fig. 9
1974 CERN SCHOOL OF PHYSICS, WINDERMERE, ENGLAND

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