A SPLIT FIELD MAGNET GEOMETRY FIT PROGRAM: NICOLE

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ABSTRACT

A program is described which will perform the geometrical fit of $pp \rightarrow (n\pi^\pm)p$ data collected by the Split Field Magnet Detector. Some resolution results are presented, and program extensions for higher multiplicity experiments are indicated.
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1. **PRELIMINARY REMARKS**

The reason for the preparation of the initial version of the geometry fit program NIOLE was to have a means to study the measurement of isobar production at ISR energies in the Split Field Magnet Detector (SFMD) \(^1\). The program in fact rapidly became very useful in dialogues with the SFM construction group on such subjects as the number of planned chambers, wire spacing, holes and frames, the amount of matter to be allowed in the tube wall and the chamber planes, chamber spacing, etc.

For these reasons it seemed indispensable to have a complete fit program available, so that the upper limit of the resolution capability of the detector could be attained, random processes such as chamber resolution and multiple scattering could be reasonably well weighted, and so that full use could be made of the constraints provided by the equations of motion. In this way systematic errors can be excluded and good background separation achieved.

The program came to serve also as a help in the development of others, such as in the establishment of the necessary accuracy of the field representation, the level of permitted ambiguity from a pattern recognition program, or as a means of measuring the loss of information caused by such procedures as parametrization. Special attention was paid to the necessary accuracy of the tracking method, the most heavily used section of the program. Here a novel method was used, which due to the field inhomogeneity brings no significant improvement on the Runge-Kutta method.

Since the program will be used on a production basis, the computation speed is of the utmost importance, as the proportional wire chambers used in the SFMD allow very high data rates and in many experiments the bottle-neck in the data flow lies probably in the reconstruction stage. It has appeared desirable to fit the reconstructed tracks singly and only then to perform a vertex fit of those remaining, as this permits a fast test of the pattern recognition.

The program, originally written for two charged particles, is now available for high multiplicity events. We should not, however, fail to mention that the choice of variables is optimized for the forward detector, as is the precision of the field model, and, in particular, the approximations made for the treatment of the multiple scattering in the proportional chambers are not suitable for all experiments. In order to permit the program to be "tuned" for other experiments, these points will be expanded upon below.

2. **THE FIT PROCEDURES**

2.1 **Calculation of the individual particle trajectories**

2.1.1 **Representation of a particle trajectory in a form suitable for fitting**

In the SFM the particle trajectories are measured by multiwire proportional chambers; a coordinate \(c_i\) on the \(i^{th}\) plane is determined by a wire signal on that wire plane. If the actual intersection coordinate on the plane is \(g_i\), the measurement error is given by \(g_i - c_i\).

The trajectory of a charged particle in an inhomogeneous magnetic field is determined by a non-linear second-order differential equation. If the initial conditions are given on a
reference surface, then the trajectory is defined by five independent parameters $p_1, \ldots, p_5$, where

$$g_i = f_i(p_1, \ldots, p_5) = f_i(P).$$

Conversely, in the general case, five coordinates define a trajectory and thus the determination of the trajectory parameters becomes a fit problem, which may be solved by a least squares fit procedure.

If

$$Q^2 = D^T W D$$

then

$$\chi^2 = \min Q^2$$

where, if $d_i$ is an element of $D$,

$$d_i = f_i(P) - c_i$$

and where, if $\omega_{ij}$ is an element of $W$,

$$\omega_{ij} = \delta_{ij}/\sigma_i^2,$$

where

$$\delta_{ij} =
\begin{cases} 
1 & \text{for } i = j \\
0 & \text{for } i \neq j
\end{cases}$$

and

$$\sigma_i^2 = \frac{(g_i - c_i)^2}{1/2}$$

$$= \frac{1}{12} \text{ (wire spacing on } i^{th} \text{ chamber)}^2,$$

(from the variance of a flat distribution).

The dependence of $g_1$ on the five independent parameters $P$ can be expressed by a closed-form solution for certain special field forms, of which the SPM is not an example. Here $g_1$ is taken to be a linear function of $P$ in the neighbourhood of the approximate value $\hat{P}$:

$$g_1 = f_1(P) = \hat{f}_1 + \sum_{j=1}^{5} \hat{a}_{ij} [P_j - \hat{P}_j]$$

where

$$\hat{f}_1 = f_1(\hat{P})$$

and

$$\hat{a}_{ij} = \left. \frac{\partial f_1}{\partial P_j} \right|_{\hat{P}}$$

hence

$$G = E(P) = \hat{E} + \hat{A}[P - \hat{P}].$$
The actual intersection point is, of course, unknown. For the calculation of \( \mathbf{F} \) and \( \mathbf{A} \) we calculate the intersection points of six different tracks with the relevant wire planes:

- **track 0:** \( \hat{\mathbf{p}}_1, \ldots, \hat{\mathbf{p}}_5 \rightarrow \hat{\mathbf{t}}_i \)
- **track 1:** \( \hat{\mathbf{p}}_1 + \epsilon_1, \hat{\mathbf{p}}_2, \ldots, \hat{\mathbf{p}}_5 \rightarrow \hat{\mathbf{t}}_i + \eta_{ij} \)

etc. (Fig. 1), giving

\[
\hat{a}_{ij} = \frac{\eta_{ij}}{\epsilon_j}.
\]

Attention must be paid to the avoidance of even small discontinuities in the tracking of these six tracks.

Before the least squares fit is performed, the multiple scattering terms must be introduced.

### 2.1.2 Choice of the parameters

The choice of the parameters involves two considerations:

1. The \( f_i \) should be well approximated by a linear dependence on these parameters in an interval of several standard deviations of the measurement error around the true impact coordinates \( g_i \). Then the fitted values of \( \mathbf{F} \) are distributed symmetrically around the true values of \( \mathbf{F} \), and a Gaussian error on \( g_i \) leads to a Gaussian error on \( \mathbf{F} \), which is important for the vertex and kinematical fits. In fact, the dependence is not linear, and the fit must be performed by a series of iterations whose convergence is at least accelerated by the approximate linear dependence, which is especially useful for poorly initialized tracks.

2. The multiple scattering on the tube should lend itself to simple treatment.

Since the chamber wires run parallel to the \( x \) or \( z \) axes (Fig. 2), the choice of a plane normal to the \( y \) axis as reference plane suggests itself, and similarly, the choice of \( x \) and \( z \) on that plane as representative parameters \( p_1 \) and \( p_2 \). The choice of the \( y \) coordinate of the reference plane at the exit point of the particle from the tube enables...
the simple introduction of the multiple scattering correction in the forward cone along the $y$ axis. The choice of

$$p_3 = \frac{dx}{ds} \quad (s = \text{path length})$$
$$p_u = \frac{dz}{ds}$$
$$p_s = \frac{1}{\text{momentum}}$$

leads to a good linear representation of the functions $f_i$ in the neighbourhood of $g_i$.

The advantage of this choice of parameters for the treatment of the beam tube multiple scattering will become clear later.

2.1.3 Minimization of the sum of squares

The minimization of the generalized least squares sum is performed by an iterative procedure, which is terminated when $Q^2$ is close enough to its minimum value. The fitted parameters of one iteration serve as the initial parameters of the succeeding one. The construction of the derivative matrix $\Lambda$ and the vector of residuals $\Delta$ has been explained in Section 2.1.1. The weight matrix of the measured coordinates $\mathcal{W}$ is simply the inverse of the error matrix defined by

$$e_{ij} = \frac{[f_i(P_{true}) - c_i] [f_j(P_{true}) - c_j]}{\sigma_i \sigma_j}$$

$$= \delta_{ij} \sigma_i \sigma_j + \text{a multiple scattering term.}$$
We distinguish the actual intersection coordinate \( x_i \), and that of a non-scattered particle \( f_i (\mathbf{p}) \), such as is calculated in the single fit procedure (SINFIT). For simplicity the differences \( [f_i (\mathbf{p}_{\text{true}}) - g_i] \) (multiple scattering term) and \( [g_i - c_i] \) (chamber resolution) are assumed to follow Gaussian distributions, and there should be no correlation between \( g_i - c_i \) and \( g_j - c_j \) for \( i \neq j \), or between \( g_i - c_i \) and \( f_j (\mathbf{p}_{\text{true}}) - g_j \) for any \( i, j \).

If the trajectory intersects \( n \) wire planes, \( \mathbf{e} \) is an \( n \times n \) matrix.

The search for

\[
\chi^2 = \min \left\{ \mathbf{D}^T \mathbf{W} \mathbf{D} \right\}
\]

yields

\[
\mathbf{P} = \mathbf{P}_0 - (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W} \mathbf{D}
\]

(\( \mathbf{P} \) are the parameters as defined in Section 2.1.1).

The matrix of variances of the fitted parameters is

\[
\mathbf{E} = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1}.
\]

Since the procedure becomes lengthy due to the necessity of tracking the variations through the inhomogeneous field, \( \mathbf{A} \) and \( \mathbf{W} \) are normally completely calculated in the first iteration only, although the column of \( \mathbf{A} \) in \( \partial / \partial (1/p) \) (p = momentum) may have to be recalculated in the second iteration too. Moreover in the forward detector it is possible to set terms of the form \( \partial / \partial x \) and \( \partial / \partial z \) to zero or one, according to the plane considered.

At the end of the single fit procedure one has available the best estimate for \( \mathbf{P} \) [i.e. \( \hat{x}, \hat{z}, \hat{x}', \hat{z}', (1/p) \)] for each charged particle, the variables being given at the y coordinate of the track intersection with the beam tube, at which point one has erected a virtual scattering plane parallel to the wire chambers.

### 2.1.4 Multiple scattering in the chambers

As the multiple scattering in a series of chambers can involve an error of the same order as the chamber resolution it is essential that it be considered. Formally, one modifies the error matrix used by SINFIT. To do that, it is necessary to be able to express the deviation of the trajectory on a given plane caused by a preceding plane. Then one determines the global deviation due to the succession of scatters. Finally, this allows one to calculate the covariance matrix of these deviations, which is added to the error matrix as set up for the chamber resolution only. The two matrices are additive, as there is no correlation between the errors due to the chamber resolution and those due to the chamber multiple-scattering (see Section 2.1.3).

The calculation of the deviations caused by the multiple scattering of a particle may be simplified by making certain approximations, which are valid for the experiment considered here, in which the tracks have a small curvature in the vertical projection and remain near the horizontal median plane, whilst possibly having a large curvature in that projection. The approximations are specified in Appendix A (see also Figs. 3 and 4).
Fig. 3  Space representation of the multiple scattering on plane i

Fig. 4  xy projection of Fig. 1
To calculate the deviation caused on a plane \( k \) by the multiple scattering on a plane \( i \) we first calculate \( \Delta x_{ik} \), the horizontal deviation (xy plane), and \( \Delta z_{ik} \), the vertical deviation (yz plane), the former being a function of the horizontal scattering angle \( \theta_{i}^{H} \), the latter of the vertical scattering angle \( \theta_{i}^{V} \). This calculation is given in Appendix A. One obtains functions of the form

\[
\Delta x_{ik} = f_{ik}(a_i, a_k) \theta_{i}^{H}
\]

where \( a_i, a_k \) are angles determining the track between the two planes, as shown in that appendix.

The over-all deviations on the \( k^{th} \) wire plane, due to the scattering on the preceding planes, are thus:

\[
\Delta x_k = \sum_{i<k} \Delta x_{ik} = \sum_{i<k} f_{ik}(a_i, a_k) \theta_{i}^{H},
\]

and

\[
\Delta z_k = \sum_{i<k} \Delta z_{ik} = \sum_{i<k} g_{ik}(a_i, a_k) \theta_{i}^{V},
\]

where \( \theta_{i}^{H} \) and \( \theta_{i}^{V} \) are non-correlated normally-distributed random variables, whose variances are given by Molière's equation:

\[
\sigma_{\text{proj}}^2 = L \left( \frac{0.015}{p} \right)^2
\]

where

\( p \) = momentum of the particle in GeV/c,

\( L \) = thickness of matter traversed in radiation lengths,

\( \theta \) is in radians and small, with \( \theta \sim 1 \) assumed,

whence

\[
\langle \Delta x_k \rangle = \langle \Delta z_k \rangle = 0
\]

\[
\langle \Delta x_k \Delta z_k \rangle = 0 \quad \text{as} \quad \theta_{i}^{H} \text{ and } \theta_{i}^{V} \text{ are not correlated}
\]

\[
\langle \Delta x_k \Delta x_j \rangle = \sum_{i<j<k} f_{ik}(a_i, a_k) f_{ij}(a_i, a_j) \langle (\theta_{i}^{H})^2 \rangle \quad \text{as} \quad \langle \theta_{i}^{H} \theta_{j}^{H} \rangle = 0,
\]

and similarly for \( z \), with

\[
\langle (\theta_{i}^{H})^2 \rangle = \langle (\theta_{i}^{V})^2 \rangle = L_i \left( \frac{0.015}{p} \right)^2.
\]

The complete expressions are given in Appendix A.
2.2 Determination of the vertex and treatment of the multiple scattering on the tube

Knowing \([\xi, \eta, \xi', \eta', (1/p)]\) at the \(\gamma\)-intersection of each track with the beam tube, the problem is to fit a common vertex and at the same time to fit the momenta using the common vertex as a constraint, taking into account the fact that the particles have been deflected by multiple scattering whilst passing through the tube wall. The variables fitted by SINFIT are thus used as virtual measurements by the vertex fit procedure TOFFIT. If there were no multiple scattering the matrix of variances of the virtual measured quantities would be simply the error matrix of the parameters fitted by SINFIT, calculated in Section 2.1.3. The multiple scattering is dealt with by adding variances and covariances to this error matrix. The random uncertainties caused by the multiple scattering affect only \(x'\) and \(z'\), and are not correlated with the errors of SINFIT; thus only four elements of the matrix \(E\), i.e. \(\sigma_{x'^2}, \sigma_{z'^2}, \sigma_{x'z'}, \sigma_{z'x'}\), have to be modified by additive terms.

The calculation made in Appendix B yields the following terms

\[
\sigma_{x'}^2 = e_{xx} = (1 - x'^2) \langle \sigma_{\text{proj}}^2 \rangle \\
\sigma_{z'}^2 = e_{zz} = (1 - z'^2) \langle \sigma_{\text{proj}}^2 \rangle \\
\sigma_{x'z'} = e_{xz} = e_{zx} = -z'x' \langle \sigma_{\text{proj}}^2 \rangle
\]

where \(\langle \sigma_{\text{proj}}^2 \rangle\) is the variance of the projected scattering angle, being a function of the momentum and of the amount of matter traversed.

Thus the error matrix is completely determined. It is composed of the individual non-correlated \(5 \times 5\) error matrices \(E\) of the tracks, which permits the decomposition of the matrix operations for the fit procedure. Using an identical procedure to that in SINFIT, varying the vertex parameters \(x, y, z\) and the track parameters \(x', z', (1/p)\) at the vertex for each track in turn, we have

\[
\Delta E = \left[ \sum_N \left( \Delta \mathbf{M} \mathbf{W} \Delta \right) \right]^{-1} \sum_N \left( \Delta \mathbf{M} \mathbf{W} \Delta \right), \quad (N = \text{number of tracks})
\]

where, for each track, \(\mathbf{W}\) is now the inverse matrix of \(E\).

The initial vertex is calculated by tracking back the particles to the neighbourhood of the origin and taking the centre point of the line of shortest distance of approach.

It is possible to measure the \(z\) profile of the beam by the method of Van der Meer, which permits the addition of an (optional) constraint to the vertex fit.

We note here that it would also be possible to treat the multiple scattering in the beam tube wall by using a plane at \(y = 0\) as a reference plane, and then using the same procedure on the tube wall as is used for the chambers. This would have the disadvantage of imposing the same approximations as are used for the chamber scattering, with a consequent reduction in accuracy since the tube scattering is the most important and can have a large "moment", but could make a possible parametrization of the tracking simpler.
3. **TRACKING THROUGH THE FIELD**

3.1 **Method**

The tracking method chosen for NICOLE is a special application of the solution of the equation of motion of a charged particle in a magnetic field, in which the field representation is transformed into a system with its origin at the starting point of each integration step. The tracking routines are very heavily used, and every effort has been made to optimize the implementation of the method.

3.2 **Field representation, storage and access**

The field has been analysed, for the sake of simplicity of use, by dividing one quadrant of the magnetic volume into four regions: the main magnet, the large compensator magnet, and two fringing field regions at the side of the main magnet (which are not used by the program version described here), and by further subdividing each region into equal boxes, 1552 in all. Within each such box the coefficients of polynomial expressions for the field components have been determined. These polynomials, which satisfy Maxwell's equations, are of order one, two or three, the analysis program choosing the lowest order compatible with the maximum residual in the box considered being less than an amount which ensures that, in the fit procedures, errors in the tracking due to errors in the determination of the field will be less important than the errors due to the resolution of the wire chambers (Appendix C).

The analysis of the four magnetic regions yields a total of about 27,000 coefficients which are stored in the Large Core Memory (LCM) of the CDC 7600 at execution time of the program. They are accessed via the routine FIELD, which provides the field components, and their derivatives if required, at a given point. On its first call, FIELD reads the coefficients from a permanent disc file and stores them into LCM, and at each call calculates the box in which the given point lies, transfers the relevant coefficients into Small Core Memory (SCM) and calls FASTER which performs the evaluation of the appropriate polynomials. FIELD performs such symmetry operations as are necessary.

Appendix D contains a chart which relates all the first-, second-, and third-order derivatives of the field to those which are independent, these relationships being determined by the application of Maxwell's equations, \( \text{div} \, \mathbf{B} = 0 \) and \( \text{curl} \, \mathbf{B} = 0 \). The symmetry operations are also listed.

3.3 **Tracking routines**

The tracking is performed by two routines, **STEP** and **TRACE**. **STEP** is given the position of the point from which tracking is to be begun, together with the three direction cosines, the charge and the momentum of the particle to be tracked. It then proceeds to track to a given plane, which in general is normal to one of the three principal axes of the system. It determines where the initial point lies within a box and calls **TRACE** to perform the integration to the nearest box wall. From here on the particle is tracked through successive boxes until the desired end plane is reached. **STEP** controls the integration onto a wall or end plane by making an initial linear estimation of the required path length and varying it so as to arrive within a small tolerance on either side of the wall. Here the
integration coefficients in TRACE are re-used, which makes an iterative approach to a surface very efficient.

Account is taken of undershoots, overshoots, change of direction, field free regions, and of the different coordinate system of the large compensator magnets. As an option, the maximum possible step size in a box may be redefined, so that in the limiting case only the error due to the field representation remains. Also, when tracking through a series of planes, several of which may be in one box, the same initial conditions may be used to track to each plane and to the exit wall.

The form of the equations which are written out explicitly in TRACE is shown in Appendix E. The order of tracking is a function of the momentum and of the highest power of the field polynomials of the box under consideration.

3.4 Timing considerations

The evaluation of the field components and their derivatives at a point is a potentially time-consuming calculation, and an attempt has been made to reduce to a minimum the number of calls to FIELD. Since the trajectories tracked for the variations in SINFIT and TOTFIT closely follow the unvaried tracks, and since the unvaried tracks in the second and subsequent iterations follow fairly closely those in the first iteration, the field data calculated for these basic tracks are stored and re-used for all their modifications. In general, a modified track will pass close to a basic track on each entry wall of the boxes traversed, and a Taylor series in two variables can be used to calculate new field data from the basic stored data on that wall. The logic of this procedure, which has to take account of deviations from the basic track which are too large, multiple FIELD calls in a single box, and the special case of the box containing the vertex for TOTFIT, is contained in SWATH. The series evaluations are performed by TAYLOR.

3.5 Adjustment of tolerances

STEP and SWATH both contain tolerances whose optimum values have to be established. The principal ones are STOL in STEP, which is the tolerance in the path length on either side of a wall within which a linear interpolation is made, and LAUXEM which defines the order of the tracking integrations in TRACE (it is added to the highest power of the field polynomials in the box to give the tracking order). Both these quantities have been determined by varying them and observing the effect of the variations on the tracking accuracy, and then fixing them at values (which are functions of the momentum) such that the tracking errors are small compared with the errors due to the chamber resolutions.

SWATH contains tolerances which determine which, if any, of the derivatives are to be recalculated in TAYLOR, and to which precision. These tolerances have been established empirically by observing the effect of varying them on the fit procedure in SINFIT.

Tables 1a, b, and c show how these tolerances guide the operation of TAYLOR (' denotes d/ds in all these tables).
Table 1
Operation of TAYLOR

a)  

<table>
<thead>
<tr>
<th>Highest field power</th>
<th>Quantities available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B B'</td>
</tr>
<tr>
<td>2</td>
<td>B B' B''</td>
</tr>
<tr>
<td>3</td>
<td>B B' B'' B'''</td>
</tr>
</tbody>
</table>

b)  

<table>
<thead>
<tr>
<th>Distance between tracks</th>
<th>Quantities recalculated for the variations</th>
<th>Quantities copied</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; TOL 1</td>
<td>$\vec{B} = \vec{B}(B, B')$</td>
<td>$B', B'', B'''$</td>
</tr>
<tr>
<td>&gt; TOL 1 and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt; TOL 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vec{B} = \vec{B}(B, B', B'')$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vec{B}' = \vec{B}'(B', B'')$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vec{B}'' = \vec{B}''(B'', B''')$</td>
<td></td>
<td>$B'''$</td>
</tr>
</tbody>
</table>

*) $\vec{B}$, etc. are the new field components calculated from the old values, for use in tracking the modified tracks.

c)  

<table>
<thead>
<tr>
<th>Tracking order</th>
<th>Quantities required</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$B$</td>
</tr>
<tr>
<td>3</td>
<td>$B, B'$</td>
</tr>
<tr>
<td>4</td>
<td>$B, B', B''$</td>
</tr>
<tr>
<td>5</td>
<td>$B, B', B'' B'''$</td>
</tr>
</tbody>
</table>

4. ORGANIZATION OF THE PROGRAM

If we ignore housekeeping and input/output routines, the program can be considered to consist of four sections:

i) the field utility FIELD-(FASTER);
ii) the tracking utility STEP-(TRACE-SNATH-TAYLOR);
iii) the single track-fitting procedure SINFIT-(TUBE-MOMNTM-COVAR);
iv) the vertex fitting procedure COMPAT-(TOTFIT-GRAPH-LOOP-VERTEX).

A block diagram of the program is shown in Appendix F.

The tracking utility is used extensively by the two fit procedures, and in its turn it uses the field utility. These utilities are also called from MOMNTM, the routine controlling the momentum initialization.

The single fit procedure enables a fit to be performed separately on each track proposed by a track-finding program, thus allowing the introduction of the beam-tube multiple scattering correction in a simple manner, as well as having the natural advantage of removing bad tracks (i.e. wrongly associated coordinates) before they reach the vertex fit procedure, thereby greatly simplifying the task of its track selection logic. TUBE finds the intersection of the track with the beam tube, and COVAR calculates the error matrix for SINFIT (Section 2.1).

The vertex fit procedure is controlled by COMPAT, which first requires GRAPH and LOOP to provide the largest set of mutually compatible tracks remaining after SINFIT (compatible tracks have no common wires). This process is trivial for the simple two charged particle events considered here. The resulting set of tracks is used by VERTEX to calculate an initial vertex which is then fitted by TOTFIT. COMPAT has the ability to remove or replace any tracks from the proposed vertex which cause TOTFIT difficulties (i.e. causing the fit to diverge), provided that the bad track can be identified and that at least two tracks remain.

The final output consists of the fitted parameters of the accepted tracks, the corresponding error matrix, and codes defining which of the input tracks have been accepted and which rejected.

The program, whose longest overlay occupies 24 K words in SOM on the CDC 7600, conforms largely to ANSI programming standards.

5. EXTENSIONS OF THE PROGRAM

The program as described here was originally conceived for the CERN-Hamburg-Orsay-Vienna (CHOV) pp → Np experiment[6]. A considerable effort has been expended to extend the program to handle data from the other proposed experiments to be performed at the SPS, some of which have very high multiplicities. Features which are or will be provided to cater for these experiments include:

i) the acceptance of up to 60 tracks by the single fit procedure;
ii) the development (for complicated cases) of the selection logic for tracks to be fitted to a vertex;
iii) the acceptance of up to 30 tracks by the vertex fit procedure, which increases the program size to 31 K;

iv) the incorporation of the logic for the central detector;

v) the adjustment of the tracking utility to accommodate low-energy particles;

vi) the incorporation of whole or parts of the program into iterative and/or interactive track-finding programs.

These points and the use of the program are more fully discussed elsewhere\textsuperscript{7}). Low-energy multiple scattering corrections and energy losses must also be dealt with.

In addition, a test will be performed to see whether there could be any advantage to be gained in replacing the tracking method described here by a standard Runge-Kutta tracking procedure, or better still by a parametrization method\textsuperscript{8}).

6. RESULTS

The following figures show some results of NICOLE, obtained by processing simulated CHOW data generated by a Monte Carlo program, and tracked at high precision by the program TRACOM\textsuperscript{9}). For these tests two types of data were used:

a) perfect data (no multiple scattering and zero wire spacing);

b) data with multiple scattering added and handled on the beam tube and chambers, and with the wire spacing considered.

The plots, in which \( x, y, z \) refer to the vertex, are:

i) Fig. 5 \( x_{\text{fitted}} - x_{\text{true}} \) (cm)

ii) Fig. 6 \( y_{\text{fitted}} - y_{\text{true}} \) (cm)

iii) Fig. 7 \( z_{\text{fitted}} - z_{\text{true}} \) (cm)

iv) Fig. 8 \( (p_{\text{fitted}} - p_{\text{true}})/p_{\text{true}} \) \( \% \) proton

v) Fig. 9 \( (p_{\text{fitted}} - p_{\text{true}})/p_{\text{true}} \) \( \% \) pion

vi) Fig. 10 \( x_{\text{fitted}} - x_{\text{true}} \) (cm)

vii) Fig. 11 \( y_{\text{fitted}} - y_{\text{true}} \) (cm)

viii) Fig. 12 \( z_{\text{fitted}} - z_{\text{true}} \) (cm)

ix) Fig. 13 \( (p_{\text{fitted}} - p_{\text{true}})/p_{\text{true}} \) \( \% \) proton

x) Fig. 14 \( (p_{\text{fitted}} - p_{\text{true}})/p_{\text{true}} \) \( \% \) pion

xi) Fig. 15 \( \chi^2_{\text{TOTALFIT}} \)

We note in Fig. 11 the relatively poor \( y_{\text{vertex}} \) resolution due to the near collinearity of the two tracks.

Data of type b require 100 msec processing time per event on the CDC 7600, and the time increases roughly linearly with the number of input tracks.
Acknowledgements

We wish to express our gratitude to Drs. G.R. Macleod, M. Vivargent, K. Winter and P. Zanella for their continuing support of our work.

We are indebted to Miss. F. Ranjard for the help she has given us in the testing of the program, and to Dr. D. Drijard for a number of valuable suggestions.

* * *

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8) J.J. Aubert and C. Broll, Track parametrization in the inhomogeneous field of the Split Field Magnet, Institut de Physique Nucléaire, Orsay, France, preprint IPN-HE-75-01.
9) F. Ranjard, A description of the programs TRACK, TRACMS and REREAD (unpublished).
Fig. 5 \( x_{\text{fitted}} - x_{\text{true}} \) (cm)
Fig. 7  \( z_{\text{fitted}} - z_{\text{true}} \) (cm)
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**Fig. 10**  
$x_{\text{fitted}} - x_{\text{true}}$ (cm)
Fig. 11 $y_{\text{fitted}} - y_{\text{true}}$ (cm)
Fig. 13 \( \frac{(p_{\text{fitted}} - p_{\text{true}})}{p_{\text{true}}} \times 100 \% \) proton
Fig. 14 \( \frac{(p_{\text{fitted}} - p_{\text{true}})}{p_{\text{true}}} \) pion
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**Fig. 15** $\chi^2_{TOTFIT}$
APPENDIX A

MULTIPLE SCATTERING IN THE CHAMBERS

THE APPROXIMATIONS

In Fig. 3:

01 is the trajectory without scattering with its tangent OTₐ at 0,
02 is the trajectory with scattering with its tangent OT₂ at 0,
θ₁ is the scattering angle on the plane i.

a) We assume that 01 (and also 02, since θ₁ is small) is near to the horizontal plane OXY.
   We can, therefore, take the projected angle to be the angle \( \theta^H_1 \) between the tangents at
   OA and OB at 0.

b) We assume that projections of 01 and 02 on the vertical planes are straight lines.
   Thus, 01 and 02 project onto the plane OT₂ like OC and OD, and the second scattering
   angle \( \theta^V_1 \) is the angle between OC and OD.

CALCULATION OF THE HORIZONTAL DEFLECTION \( \Delta X_{ik} \)

In Fig. 4:

\[
\begin{align*}
    r &= |\overrightarrow{OA}| \\
    \Delta X_{ik} &= AB \\
    a &= |\overrightarrow{AM}| \\
    s &= |\overrightarrow{AN}|
\end{align*}
\]

and the various angles are defined in the figure; \( \alpha_i \) is the horizontal projection of the
angle of incidence of the trajectory on the \( i \) th plane, \( \beta_{ik} \) the angle of \( OY \) with the line \( OA \).
As \( \theta^H_1 \) is small, we assume that \( AN \) is perpendicular to \( OA \) and that \( AM \) is a normal common to
the two trajectories, with and without scattering.

Thus

\[
\Delta X_{ik} = \frac{a}{\cos(\pi - \alpha_i)} = -s \cos(\alpha_k - \beta_{ik}) \frac{\cos(\alpha_k - \beta_{ik})}{\cos \alpha_k} \theta^H_1
\]

\[
= -\frac{y_k - y_1}{\cos \beta_{ik}} \frac{\cos(\alpha_k - \beta_{ik})}{\cos \alpha_k} (y_1 - y_k)(1 + \tan \beta_{ik} \tan \alpha_k) \theta^H_1,
\]

where \( y \) is the plane coordinate.

CALCULATION OF THE VERTICAL DEFLECTION \( \Delta Z_{ik} \)

In Fig. 3, since OTₐ lies nearly in the horizontal plane, we can write directly

\[
\Delta Z_{ik} = CD = OE \times \theta^V_i = \frac{y_k - y_1}{\cos \alpha_i} \theta^V_1.
\]
ERROR MATRIX

We can now substitute into the equations mentioned in the text (Section 2.1.4). We have seen that:

\[ \langle (a_i^y)^2 \rangle = \langle (a_i^y)^2 \rangle = L_i \left( \frac{0.015}{p} \right)^2 . \]

If \( d \) is the thickness in radiation lengths of a plane of a chamber and the trajectory is assumed to be close to the horizontal plane, the incident angle is given approximately by \( \alpha_i \) defined above. Thus \( L_i = d/|\cos \alpha_i| \) for the \( i \)th plane. Putting \( c = (0.015/p)^2 d \), we obtain

\[
\langle \Delta x_k \Delta x_j \rangle = c \sum_{i \neq j} (y_k - y_i) (y_j - y_i) \frac{(1 + \tan \beta_{ik} \tan \alpha_i) (1 + \tan \beta_{ij} \tan \alpha_j)}{|\cos \alpha_i|} \\
\langle \Delta z_k \Delta z_j \rangle = c \sum_{i \neq j} \frac{(y_k - y_i) (y_j - y_i)}{|\cos \alpha_i|^3} \\
\langle \Delta x_k \Delta z_j \rangle = 0 .
\]
MULTIPLE SCATTERING IN THE BEAM TUBE WALL

Let $\mathbf{r}_o = (x', y', z')$ be the vector representing the direction of the particle at its intersection with the beam tube before the multiple scattering, and $\mathbf{r}_1 = (x' + dx', y' + dy', z' + dz')$ be the vector after scattering.

Let us construct an orthogonal system about $\mathbf{r}_o$, which is normalized:

$\mathbf{h}_1 = (x', y', z')$ (coincident with the initial vector)

$\mathbf{h}_2 = (y'/\sqrt{x'^2 + y'^2}, -x'/\sqrt{x'^2 + y'^2}, 0)$

$\mathbf{h}_3 = (z'x'/\sqrt{x'^2 + y'^2}, z'y'/\sqrt{x'^2 + y'^2}, -\sqrt{x'^2 + y'^2})$.

Thus, after a small angle scatter, we have

$\mathbf{r}_1 = \mathbf{r}_o + d\mathbf{r} = \mathbf{h}_1 + \mathbf{h}_2 = \mathbf{h}_1 + \theta_1 \mathbf{h}_2 + \theta_2 \mathbf{h}_3$,

where $\theta_1$ and $\theta_2$ are non-correlated random variables. The vector $d\mathbf{r}$ therefore decomposes into:

$dx' = \frac{y'}{\sqrt{x'^2 + y'^2}} \theta_1 + \frac{z'x'}{\sqrt{x'^2 + y'^2}} \theta_2$

$dy' = \frac{-x'}{\sqrt{x'^2 + y'^2}} \theta_1 + \frac{z'y'}{\sqrt{x'^2 + y'^2}} \theta_2$

$dz' = -\sqrt{x'^2 + y'^2} \theta_2$,

showing $dx'$, $dy'$, and $dz'$ to be functions of random variables,

$\langle \theta_1^2 \rangle = \langle \theta_2^2 \rangle = \langle \theta^2 \rangle = \left( \frac{0.015}{p} \right)^2 L$,

where

$L = \text{thickness of traversed material in radiation lengths}$

$p = \text{momentum in GeV/c}$.

By propagation of variances we obtain:

$\left( \sigma_{x'}^2 \right)_{ms} = \left( \frac{\langle dx' \rangle}{\theta_1} \right)^2 + \left( \frac{\langle dx' \rangle}{\theta_2} \right)^2 = \frac{y'^2 + z'^2 x'^2}{x'^2 + y'^2} \langle \theta^2 \rangle = (1 - x'^2) \langle \theta^2 \rangle$

$\left( \sigma_{y'}^2 \right)_{ms} = \left( \frac{\langle dy' \rangle}{\theta_1} \right)^2 + \left( \frac{\langle dy' \rangle}{\theta_2} \right)^2 = \langle \theta^2 \rangle = (1 - z'^2) \langle \theta^2 \rangle$

and

$\left( \sigma_{x'y'} \right)_{ms} = \left( \frac{\langle dx' \rangle}{\theta_1} \right) \left( \frac{\langle dz' \rangle}{\theta_2} \right) + \left( \frac{\langle dx' \rangle}{\theta_2} \right) \left( \frac{\langle dz' \rangle}{\theta_1} \right) = -z'x' \langle \theta^2 \rangle$ (ms standing for multiple scattering).
SCHEMATIC COMPARISON OF THE ERROR CONTRIBUTIONS DUE TO FIELD RESIDUAL ERRORS AND CHAMBER RESOLUTION ON THE MOMENTUM AND VERTEX RESOLUTIONS FOR SINGLE TRACKS

(We note that the final vertex and therefore the momentum resolutions are strongly dependent on the event multiplicity.)
### APPENDIX D

#### DERIVATIVE DEPENDENCE CHART$^1$)

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$^1$) See notes of explanation on p. 31.
EXPLANATION OF THE DERIVATIVE Dependence CHART (ON PREVIOUS PAGE)

Each derivative is numbered with the number of that derivative to which it is identical. The first appearance of a derivative in the chart is marked by an asterisk.

For clarity, the dependent third-order derivatives are written out in full only in those boxes where there is a new independent second derivative.

Examples:

i) $\partial^2 Bz/\partial x\partial y$ is numbered 5, and is therefore equal to $\partial^2 Bx/\partial y\partial z$;

ii) $\partial^3 Bz/\partial y^2\partial z$ is numbered -(4 + 8), and is therefore equal to $-[(\partial^3 Bz/\partial x^3 y^2) + (\partial^3 Bx/\partial y^3)]$.

SYMMETRY OPERATIONS

The field, having been measured in one quadrant only, needs to have symmetry operations performed upon it when components in another quadrant are requested. The following quantities are multiplied by -1 for the specified reflection (numbering as on the chart):

| Reflection in y: | Field components | 1 | 3 |
| | 1st derivatives | 1 | 3 | 4 |
| | 2nd derivatives | 1 | 3 | 4 | 7 |
| | 3rd derivatives | 1 | 3 | 4 | 7 | 8 |

| Reflection in z: | Field components | 1 | 2 |
| | 1st derivatives | 1 | 2 | 4 |
| | 2nd derivatives | 1 | 2 | 4 | 6 |
| | 3rd derivatives | 1 | 2 | 4 | 8 | 8 |
THE DERIVATIVES OF THE EQUATIONS OF MOTION, AS WRITTEN OUT EXPLICITLY IN TRACE

\[ x^\prime\prime\prime/c = x^\prime\prime B_k - x^\prime B_j + \sum_a \left\{ x^\prime_a x_a \frac{\partial B_k}{\partial x_a} - x^\prime_k x^\prime_a \frac{\partial B_j}{\partial x_a} \right\} \]

\[ x^\prime\prime\prime/c = x^\prime\prime B_k - x^\prime B_j + \sum_a \left\{ 2 \left( x^\prime\prime_a x^\prime_a \frac{\partial B_k}{\partial x_a} - x^\prime_k x^\prime_a - x^\prime_k x^\prime_a \frac{\partial B_j}{\partial x_a} \right) + x^\prime_j x^\prime_a \frac{\partial B_k}{\partial x_a} - x^\prime_k x^\prime_a \frac{\partial B_j}{\partial x_a} \right\} \]

\[ + \sum_{\alpha \beta} \left\{ x^\prime\prime_a x^\prime_a \frac{\partial^2 B_k}{\partial x_\alpha \partial x_\beta} - x^\prime_k x^\prime_a x^\prime_\alpha \frac{\partial^2 B_j}{\partial x_\beta \partial x_\gamma} \right\} \]

\[ x^\prime\prime\prime/c = x^\prime\prime\prime B_k - x^\prime\prime\prime B_j \]

\[ + \sum_a \left\{ \left( x^\prime\prime\prime_a x^\prime_a \frac{\partial B_k}{\partial x_a} - x^\prime\prime\prime_k x^\prime_a \frac{\partial B_j}{\partial x_a} \right) + x^\prime\prime\prime_j x^\prime_a \frac{\partial B_k}{\partial x_a} - x^\prime\prime\prime_k x^\prime_a \frac{\partial B_j}{\partial x_a} \right\} \]

\[ + 3 \sum_{\alpha \beta} \left\{ x^\prime\prime\prime_a x^\prime_a x^\prime_\alpha \frac{\partial^2 B_k}{\partial x_\beta \partial x_\gamma} + x^\prime\prime\prime_j x^\prime_a x^\prime_\alpha \frac{\partial^2 B_j}{\partial x_\beta \partial x_\gamma} \right\} \]

\[ + \sum_{\alpha \beta \gamma} \left\{ x^\prime\prime\prime_a x^\prime_a x^\prime_\alpha x^\prime_\gamma \frac{\partial^3 B_j}{\partial x_\beta \partial x_\gamma \partial x_\delta} - x^\prime\prime\prime_k x^\prime_a x^\prime_\alpha x^\prime_\gamma \frac{\partial^3 B_j}{\partial x_\beta \partial x_\gamma \partial x_\delta} \right\} , \]

where

' denotes \( d/ds \)

\( i, j, k \) denote the three coordinates cyclically

\( \alpha, \beta, \gamma \) are summation indices running from 1 to 3,

whence, for example:

\[ \Delta x_1 = x_1 s + x_1\prime s^2 \frac{2!}{2!} + x_1\prime\prime s^3 \frac{3!}{3!} + x_1\prime\prime\prime s^4 \frac{4!}{4!} + x_1\prime\prime\prime\prime s^5 \frac{5!}{5!} \]

and

\[ \Delta x'_1 = x'_1 s + x'_1\prime s^2 \frac{2!}{2!} + x'_1\prime\prime s^3 \frac{3!}{3!} + x'_1\prime\prime\prime s^4 \frac{4!}{4!} . \]
BLOCK DIAGRAM OF THE PROGRAM

PRINCIPAL PROGRAM SECTIONS

Main program
Setting-up and termination
Event processing control
Data input and conversion
Single track fitting
Track selection and vertex fit
Data output

FIELD UTILITY
FIELD Control and symmetry
FASTER Polynomial evaluation

TRACKING UTILITY
STEP/TRACE Path integration
SWATH Field data storage
TAYLOR Series evaluation