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STRUCTURE OF NEW MINIMAL SUPERGRAVITY

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ABSTRACT. We derive the gravitational multiplets of new minimal Poincaré supergravity and obtain a tensor calculus which allows us to compute general matter couplings in arbitrary R-symmetric supergravity theories. In particular, we derive Chern-Simons multiplets and super topological invariants. The relevance of these results for the description of the effective interactions of the massless excitations of four-dimensional heterotic superstrings with $N = 1$ space-time supersymmetry is stressed.

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1. Introduction.

General matter couplings in $N = 1$ four-dimensional (Poincaré) supergravity have been extensively investigated over the last decade. These couplings can be obtained either in superspace¹ or in component form using $N = 1$ tensor calculus.² However, since different $N = 1$ superspace geometry exists, there is actually more than one tensor calculus. The different formulations of Poincaré supergravity get unified in the framework of conformal supergravity.² In fact in the latter formulation different super-Poincaré geometries just reflect different choices of superconformal compensators. The simplest examples are the so-called old and new minimal formulations^{3,4} of $N = 1$ Poincaré supergravity each of which contains six bosonic auxiliary degree of freedom. In the superconformal approach,² they correspond to a chiral and a real linear multiplet compensator respectively.

In the Poincaré framework, the old minimal supergravity has the basic gravitational multiplet given by³

$$e_{a\mu}, \quad \psi_{\mu}, \quad A_{\mu}, \quad u$$

while in the new minimal formulation it is

$$e_{a\mu}, \quad \psi_{\mu}, \quad A_{\mu}, \quad B_{\mu\nu}$$

Here $e_{a\mu}$ and ψ_{μ} denote the physical vierbien and gravitino fields, u is a complex scalar, $B_{\mu\nu}$ is an antisymmetric tensor with gauge invariance

$$\delta B_{\mu\nu} = \partial_{\mu} \xi_{\nu} - \partial_{\nu} \xi_{\mu}$$

and A_{μ} is a vector auxiliary field which, in the new minimal formalism, has an

associated R gauge invariance

$$\delta A_\mu = \partial_\mu \phi.$$

In theories with no higher derivative interactions, it was shown that the two formulations become equivalent,⁵ at least at the classical level since Legendre transform can be performed which change the linear compensator into a chiral compensator and vice versa. The new minimal theories turn out to be a subset of the old minimal theories, namely those which obey a restriction due to the R-symmetry, gauge by the auxiliary A_μ connection. This implies a restriction on the Kähler potential and on the superpotential for these theories.

The situation changes when higher derivative interactions are added to the lowest order supergravity Lagrangian because the dependence on the auxiliary fields is no longer algebraic. In particular it was shown that general theories with R^2 terms are physically inequivalent in the two-minimal formulations of $N = 1$ supergravity.⁶

The problem of having the correct geometrical formulation of $N = 1$ supergravity geometry becomes therefore an obvious task in the framework of superstring theories⁷ which give rise to effective Lagrangians for the massless states containing arbitrary high powers of the gravitational and Yang-Mills curvatures.

A recent study^{8,9} of 4D $N = 1$ heterotic superstrings¹⁰ has revealed that the natural superspace geometry for these theories correspond to the new minimal formulation of $N = 1$ supergravity. This can be understood by explicit construction of the gravitational supervertices and by studying the superspace Bianchi identities for the vertex supermultiplets.⁸ Moreover, there is a deep connection between the $U(1)$ gauge symmetry of the A_μ connection and the $U(1)$

Kac-Moody symmetry of the $N = 2$ superconformal algebra of the underlying two-dimensional superconformal theory.¹¹ This recent investigation is the key motivation for a renewed interest in new minimal supergravity and the explanation of its consequences in the dynamics of the light degree of freedom of four-dimensional superstrings.

The present paper is devoted to a derivation of the basic geometrical quantities of new minimal Poincaré supergravity and general matter couplings. In particular, we derive Chern-Simons multiplets¹² which play an important role in the four-dimensional version of the Green-Schwarz mechanism¹³ occurring in superstring theories. A short version of some of the results contained in this paper were reported in a previous note.¹⁴

This paper is organized as follows: in Section 2, we begin by deriving new minimal supergravity at the linearised level, reviewing the results of Ref. 6. We then generalize at the nonlinear level giving the algebra supercovariant derivatives and their commutation relations. Finally, we discuss the various supercovariant curvatures, and derive the transformation rules and Bianchi identities. In Section 3, we discuss the multiplets of new minimal supergravity¹⁴. We first consider general multiplets and then, by imposing suitable constraints, we get other multiplets (chiral, linear, etc. ...). Explicit expressions for spinor and vector derivatives are given. In Section 4, we derive the gravitational curvature multiplets and their related Bianchi identities¹⁴. We then discuss super Yang-Mills theories and give the chiral and Lorentz connection gauge multiplets. The commutation relations of spinor and vector derivatives are obtained. In the last section we give the F and D type density formulae and then discuss general matter couplings. The equivalence with standard (old minimal) supergravity is explicitly shown. Finally, all topological invariants and Chern-Simons

multiplets are derived.

2. New Minimal Supergravity Algebra.

2.1. Linearized New Minimal Supergravity.

In global supersymmetry, gauge theories are formulated by introducing a real superfield along with superspace gauge transformations. The extra fields are then eliminated by choosing the Wess-Zumino gauge, where we are left with the physical gauge multiplet (V_μ, λ, D) and the usual space-time gauge transformations. We now generalize this procedure to describe gravity.⁶ The obvious starting point is to introduce a real superfield with a vector index,

$$\begin{aligned} \phi_\mu = & C_\mu + \theta \chi_\mu + \bar{\theta} \bar{\chi}_\mu + \theta^2 F_\mu + \bar{\theta}^2 \bar{F}_\mu + \bar{\theta} \sigma_\nu \theta (h_{\nu\mu} + B_{\nu\mu}) \\ & + \frac{1}{2} \bar{\theta}^2 \theta^2 \left(A_\mu + \frac{1}{2} \partial^2 C_\mu \right) + i \bar{\theta}^2 \theta \left(\psi_\mu + \frac{1}{2} \not{\partial} \bar{\chi}_\mu \right) - i \theta^2 \bar{\theta} \left(\bar{\psi}_\mu - \frac{1}{2} \not{\partial} \chi_\mu \right) \end{aligned} \quad (1.1)$$

where $g_{\mu\nu} = \eta_{\mu\nu} + 2h_{\mu\nu}$ is the graviton and ψ_μ the gravitino.

Then let us define Poincaré supergravity transformations as

$$\delta \phi_\mu = S_\mu + \bar{S}_\mu + \partial_\mu \phi \quad (1.2)$$

with ϕ real and S_μ chiral (i.e. $\bar{\nabla}_{\dot{\alpha}} S_\mu = 0$). There follow from the superconformal transformations, $\delta \phi_\mu = \bar{\nabla}_\sigma \lambda - \nabla_\sigma \bar{\lambda}$, by restricting λ to satisfy $\bar{\nabla}^2 \lambda + \nabla^2 \bar{\lambda} = 0$.⁶ In any case, the above Poincaré transformation allow us to choose a gauge where $C = \chi = F = 0$. The remaining transformations are then

$$\begin{aligned} \delta h_{\mu\nu} &= \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \\ \delta \psi_\mu &= \partial_\mu \epsilon \\ \delta A_\mu &= \partial_\mu \phi \end{aligned}$$

$$\delta B_{\mu\nu} = \partial_\mu \xi'_\nu - \partial_\nu \xi'_\mu . \quad (1.3)$$

The first two are the desired Poincaré supergravity transformations and the remaining two are needed to give us a multiplet with equal (12+12) off-shell degrees of freedom. The multiplet $(e^a_\mu, \psi_\mu, A_\mu, B_{\mu\nu})$ is precisely the new minimal graviton multiplet with a gauge vector and antisymmetric tensor as auxiliary fields.^{2,4}

The supersymmetry transformations in the above gauge are derived as usual by adding compensating gauge transformations. They are

$$\begin{aligned} \delta \psi_\mu &= -D_\mu^+ \epsilon \\ \delta(h_{\mu\nu} + B_{\mu\nu}) &= \frac{i}{2} \bar{\epsilon} \gamma_\mu \psi_\nu \\ \delta A_\mu^+ &= \bar{\epsilon} \gamma_5 \not{\partial} \psi_\mu \end{aligned} \quad (1.4)$$

where

$$\begin{aligned} \omega_{\mu\lambda\nu}^+ &= \omega_{\mu\lambda\nu} + H_{\mu\lambda\nu} \\ \omega_{\mu\lambda\nu} &= -(\partial_\mu h_{\lambda\nu} - \partial_\nu h_{\lambda\mu}) \\ H_{\mu\nu\lambda} &= \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu} . \end{aligned} \quad (1.5)$$

We identify the vector in the new minimal formalism with $A_\mu = A_\mu^+ + H_\mu$ ($H_\mu = -\frac{1}{3!} \epsilon_{\mu\nu\rho\lambda} H_{\nu\rho\lambda}$) instead of A_μ^+ . Also note that the vector $A_\mu^- = A_\mu^+ - 2H_\mu$ has a covariant (i.e. depends on $\psi_{\mu\nu} = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu$) transformation

law,

$$\delta A_{\mu}^{-} = -\frac{1}{4} \bar{\epsilon} \gamma_{\mu} \gamma^{\sigma} S_{\rho\lambda} \psi_{\rho\lambda} \quad (1.6)$$

Similarly, the connection $\omega_{\mu\lambda\nu}^{-} = \omega_{\mu\lambda\nu} - H_{\mu\lambda\nu}$ transforms as

$$\delta \omega_{\mu\lambda\nu}^{-} = -\frac{1}{2} \bar{\epsilon} \gamma_{\lambda} \psi_{\mu\nu} \quad (1.7)$$

where we have neglected a Lorentz transformation. The transformations (1.6) and (1.7) are similar to those for gauge vectors, i.e. $\delta V_{\mu} = -\frac{1}{2} \bar{\epsilon} \gamma_{\mu} \lambda$. In Section 4.6 we will indeed find chiral and Lorentz connection multiplets with A_{μ}^{-} and $\omega_{\mu\lambda\nu}^{-}$ as their gauge vectors.

The chiral field strength multiplet invariant under the Poincaré supergravity transformation is given by

$$\begin{aligned} T_{\mu\nu} &= \frac{1}{4} \bar{\nabla}^2 \nabla (\partial_{\mu} \phi_{\nu} - \partial_{\nu} \phi_{\mu}) \\ &= \psi_{\mu\nu} - i \left(\frac{1}{2} \sigma_{\rho\lambda} \theta R_{\rho\lambda\mu\nu}^{+} + F_{\mu\nu}^{+} \right) \theta + i \bar{\theta} \bar{\psi}_{\mu\nu} \theta^2 + \dots \end{aligned} \quad (1.8)$$

We call this the Riemann multiplet. Another invariant multiplet is the real linear Einstein multiplet,

$$\begin{aligned} E_{\mu} &= -\frac{1}{4} \epsilon_{\mu\nu\rho\lambda} \bar{\nabla}_{\sigma} \nabla \partial_{\rho} \phi_{\lambda} \\ &= H_{\mu} - i \bar{\theta} \gamma_5 r_{\mu} - \frac{1}{4} \bar{\theta} \gamma_5 \gamma_{\nu} \theta (E_{\mu\nu}^{+} - {}^* F_{\mu\nu}^{+}) + \dots \end{aligned} \quad (1.9)$$

where r_{μ} is the Rarita-Schwinger operator and $E_{\mu\nu}$ the Einstein tensor.

The relation between these multiplets and the other gravitational multiplets, i.e. Weyl and scalar curvature, will be given in detail in Sections 4.1 and 4.2.

2.2. Graviton Multiplet and Local Algebra.

We now consider new minimal supergravity at the nonlinear level.^{2,4} It is very convenient to use the super-covariantized field strength of the antisymmetric tensor,

$$H = dB + \frac{i}{8} \bar{\psi} \gamma \psi \quad (2.1)$$

to define three sets of chiral and Lorentz connections. Specifically,

$$\begin{aligned} \omega_{abc}^{\pm} &= \omega_{abc} \pm H_{abc} \\ A_{\mu}^{+} &= A_{\mu} - H_{\mu} \\ A_{\mu}^{-} &= A_{\mu} - 3H_{\mu} \end{aligned} \quad (2.2)$$

where $\omega_{abc} = \omega_{abc}(e, \psi)$ is the usual supercovariantized Lorentz connection with torsion

$$T^a = -\frac{i}{4} \bar{\psi} \gamma^a \psi \quad (2.3)$$

and

$$H_a = -\frac{1}{3!} \epsilon_{abcd} H_{bcd} \quad \text{or} \quad H_a e^a = *H. \quad (2.4)$$

The torsion for the connections $\omega_{abc}^{\pm}$ is given by $T^{\pm a} = \pm H_{bc}^a e^b e^c - \frac{i}{4} \bar{\psi} \gamma^a \psi$.

The three covariant derivatives are then defined as

$$D = d + \delta_L(\omega_{ab}) + \delta_A(A) \quad (2.5a)$$

and

$$D^\pm = d + \delta_L(\omega_{ab}^\pm) + \delta_A(A^\pm) \quad (2.5b)$$

where

$$\delta_A(\phi) \Phi = i n \phi \Phi$$

$$\delta_L(\Lambda) \Phi = \frac{i}{2} S_{ab} \Lambda_{ab} \Phi . \quad (2.6)$$

For the gravitino $S_{ab} = \frac{\sigma_{ab}}{2}$ and $n = -\frac{\gamma_5}{2}$.

These derivatives are related by

$$D_a^- = D_a - i {}^*S_{ab} H_b - 3i n H_a$$

$$D_a^+ = D_a + i {}^*S_{ab} H_b - i n H_a . \quad (2.7)$$

In particular, for the gravitino

$$\begin{aligned} D_a^+ \epsilon &= (D_a^- + i \gamma_5 H_a) \epsilon \\ &= \left(D_a - \frac{1}{2} i \gamma_5 \gamma_a H \right) \epsilon . \end{aligned} \quad (2.8)$$

The nonlinear supergravity transformation of the graviton multiplet

$(e^a_\mu, \psi_\mu, A_\mu, B_{\mu\nu})$ are taken to be^{2,4}

$$\begin{aligned}\delta e^a &= \frac{i}{2} \bar{\epsilon} \gamma^a \psi \\ \delta \psi &= -D^+ \epsilon \\ \delta B &= \frac{i}{4} \bar{\epsilon} \gamma \psi \\ \delta A^- &= \frac{i}{4} \bar{\epsilon} \gamma \gamma_5 \sigma_{ab} \psi_{ab}\end{aligned}\tag{2.9}$$

where $\psi_{\mu\nu}$ is the gravitino curvature with plus connections, i.e. $\psi_{\mu\nu} = D_\mu^+ \psi_\nu - D_\nu^+ \psi_\mu$. Note that the above transformation are simple covariantizations of those at the linear level (2.1.4 and 2.1.6). The transformation of the Lorentz connection can then be calculated in the standard manner, it is

(2.1.7)

$$\delta \omega^-_{ab} = -\frac{i}{2} \bar{\epsilon} \gamma \psi_{ab} .\tag{2.10}$$

Also the variation of H_a is given by

$$\delta H_a = -\frac{i}{2} \bar{\epsilon} \gamma_5 r_a .\tag{2.11}$$

The transformations (2.9) form an algebra along with general coordinate, Lorentz, chiral and B-gauge transformations. The variation of the gauge fields under these transformations is given by

$$\begin{aligned}\delta_L(\Lambda) \omega_{ab} &= -D\Lambda_{ab} \\ \delta_A(\phi) A &= -d\phi \\ \delta_B(\xi') B &= -d\xi'\end{aligned}\tag{2.12}$$

where in the term $\hat{D}_a^-(\delta_Q \Phi)$ the derivative does not act on ϵ .

Due to the presence of general coordinate transformations in the algebra, covariant fields can only have tangent space indices. When using differential forms, it is very convenient to formally define

$$\hat{D}e^a = 0 \quad (3.5)$$

or explicitly, $\hat{D}e^a = De^a + \frac{1}{2} \delta_Q(\psi) e^a$. Then

$$\hat{D}^\pm e^a = \pm H_{bc}^a e^b e^c. \quad (3.6)$$

Also we define a new supersymmetry variation for forms by

$$\delta_Q' V = \frac{1}{p!} \delta_Q V_{a_1 \dots a_p} e^{a_1} \dots e^{a_p}. \quad (3.7)$$

The analog of (3.2) and (3.4) in differential forms is

$$\hat{D}\hat{D} = \delta_L(\hat{R}) + \delta_A(\hat{F}) + \delta_Q(D^+\psi)$$

$$\hat{D}^\pm \hat{D}^\pm = \delta_L(\hat{R}^\pm) + \delta_A(\hat{F}^\pm) + \delta_Q(D^+\psi)$$

$$\hat{D}^- V = D^- V + \delta_Q'(\psi) V + \frac{i}{4} \bar{\psi} \gamma^a \psi \frac{1}{(p-1)!} V_{aa_2 \dots a_p} e^{a_2} \dots e^{a_p} \quad (3.8)$$

and

$$\delta_Q(\hat{D}^- V) = \hat{D}^-(\delta_Q V) + \delta_L \left(-\frac{i}{2} \bar{\epsilon} \gamma \psi_{ab} \right) V + \delta_A \left(\frac{i}{2} \bar{\epsilon} \gamma \gamma_5 \gamma \cdot r \right) V +$$

All fields except $B_{\mu\nu}$ are inert under the B gauge transformation. The verification of the algebra is quite trivial using differential forms and identifying $\delta_{\text{GCT}}(\xi) = \mathcal{L}_\xi = d i_\xi + i_\xi d$. It is also convenient to let ϵ transform as a spinor under Lorentz, as $\delta_A \epsilon = -\frac{i\gamma_5}{2} \phi \epsilon$ under chiral (i.e. in two-component notation, ϵ has $n = 1/2$ and $\bar{\epsilon}$ has $n = -1/2$), and all parameters are assumed to transform covariantly under general coordinate transformations. The non-vanishing commutators are then given by

$$[\delta_L(\Lambda), \delta_L(\Lambda')] = -\delta_L([\Lambda, \Lambda'])$$

$$[\delta_{\text{GCT}}(\xi), \delta_{\text{GCT}}(\xi')] = \delta_{\text{GCT}}(\xi \cdot \partial \xi' - \xi' \cdot \partial \xi) \quad (2.13)$$

and

$$\begin{aligned} [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] &= \delta_{\text{GCT}}(\xi^\lambda) + \delta_L(\xi^\lambda \omega_{a\lambda b}^-) + \delta_A(\xi^\lambda A_\lambda^-) + \delta_Q(\xi^\lambda \psi_\lambda) \\ &\quad + \delta_B\left(\xi^\lambda \left(B_{\lambda\mu} + \frac{1}{2} g_{\lambda\mu}\right)\right) \end{aligned} \quad (2.14)$$

with $\xi^\lambda = \frac{1}{2} \bar{\epsilon}_1 \gamma^\lambda \epsilon_2$.^{2,4} On covariant fields (i.e. those transforming without derivatives of any parameter in the above algebra) inert under the B gauge transformations, the $\{Q, Q\}$ anticommutator defines a supercovariant derivative,²

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] \Phi = \xi \cdot \hat{D}^- \Phi \quad (2.15)$$

where

$$\hat{D}^- = D^- + \delta_Q(\psi) . \quad (2.16)$$

Similarly, $\hat{D}^+ = D^+ + \delta_Q(\psi)$, $\hat{D} = D + \delta_Q(\psi)$ and these derivatives are related as in (2.7).

Finally, we give for reference the supergravity transformation of the other connections

$$\begin{aligned} \delta A^+ &= i\bar{\epsilon}\mathbb{M}\psi - i\bar{\epsilon}\gamma_5 \left(r + \frac{1}{2} \gamma\gamma \cdot r \right) \\ \delta\omega_{ab}^+ &= -\frac{1}{2} \bar{\epsilon}\gamma_5 \{ \sigma_{ab}, \mathbb{M} \} \psi - \frac{1}{4} \bar{\epsilon}(\sigma_{ab}, \gamma_c) \psi_{cd} e^d \\ \delta A &= \frac{3}{2} i\bar{\epsilon}\mathbb{M}\psi - \frac{3}{2} i\bar{\epsilon}\gamma_5 \left(r + \frac{1}{3} \gamma\gamma \cdot r \right) \\ \delta\omega_{ab} &= -\frac{1}{4} \bar{\epsilon}\gamma_5 \{ \sigma_{ab}, \mathbb{M} \} \psi - \frac{1}{4} \bar{\epsilon}(\gamma_a \psi_{cb} + \gamma_c \psi_{ab} + \gamma_b \psi_{ac}) e^c . \end{aligned} \quad (2.17)$$

2.3 Gravitational Curvatures.

The chiral and Lorentz connections defined in the last subsection can be used to define three sets of supercovariant curvatures. Using (2.9), (2.10), and (2.17), these are given by

$$\begin{aligned} \hat{F}^- &= F^- + \frac{i}{2} \bar{\psi}\gamma\gamma_5\gamma \cdot r \\ \hat{F}^+ &= F^+ + \frac{i}{2} \bar{\psi}\mathbb{M}\psi - i\bar{\psi}\gamma_5 \left(r + \frac{1}{2} \gamma\gamma \cdot r \right) \\ \hat{F} &= F + \frac{3i}{4} \bar{\psi}\mathbb{M}\psi - \frac{3i}{2} \bar{\psi}\gamma_5 \left(r + \frac{1}{3} \gamma\gamma \cdot r \right) \\ \hat{R}_{ab}^- &= R_{ab}^- - \frac{i}{2} \bar{\psi}\gamma\psi_{ab} \end{aligned}$$

$$\begin{aligned}\hat{R}_{ab}^+ &= R_{ab}^+ - \frac{1}{4} \bar{\psi} \gamma_5 \{M, \sigma_{ab}\} \psi - \frac{1}{4} \bar{\psi} (\sigma_{ab}, \gamma_c) \psi_{cd} e^d \\ \hat{R}_{ab} &= R_{ab} - \frac{1}{8} \bar{\psi} \gamma_5 \{M, \sigma_{ab}\} \psi - \frac{1}{4} \bar{\psi} (\gamma_a \psi_{cb} + \gamma_c \psi_{ab} + \gamma_b \psi_{ac}) e^c.\end{aligned}\quad (3.1)$$

For the gravitino there is only one supercovariant curvature and hence we omit the plus sign and hat on ψ_{ab} .

These curvatures can be used to explicitly give the commutator of supercovariant derivatives as²

$$\begin{aligned}(\hat{D}_a, \hat{D}_b) &= \delta_L(\hat{R}_{ab}) + \delta_A(\hat{F}_{ab}) + \delta_Q(\psi_{ab}) \\ (\hat{D}_a^\pm, \hat{D}_b^\pm) &= \delta_L(\hat{R}_{ab}^\pm) + \delta_A(\hat{F}_{ab}^\pm) + \delta_Q(\psi_{ab}) \mp 2H_{abc} \hat{D}_c^\pm.\end{aligned}\quad (3.2)$$

In particular, for a field of spin S_{ab} , chiral weight n and transforming under supersymmetry as $\delta C = -\frac{1}{2} \bar{\epsilon} \chi$,

$$(\hat{D}_a^-, \hat{D}_b^-) C = \left[\frac{1}{2} S_{cd} \hat{R}_{cdab}^- + i n \hat{F}_{ab}^- \right] C - \frac{1}{2} \bar{\psi}_{ab} \chi - 2\epsilon_{abcd} H_c \hat{D}_d^- C. \quad (3.3)$$

Another very useful identity is the supersymmetry variation of covariant derivatives,²

$$\begin{aligned}\delta_Q(\epsilon)(\hat{D}_a^- \Phi) &= \hat{D}_a^-(\delta_Q(\epsilon)\Phi) + \delta_L \left[-\frac{1}{2} \bar{\epsilon} \gamma_a \psi_{cd} \right] \Phi + \delta_A \left[\frac{1}{2} \bar{\epsilon} \gamma_a \gamma_5 \gamma \cdot r \right] \Phi \\ &\quad + \delta_Q \left[-i \gamma_5 M \gamma_a \epsilon \right] \Phi\end{aligned}\quad (3.4)$$

$$+ \delta'_Q (-i\gamma_5 H \gamma \epsilon) V + \frac{i}{2} \bar{\epsilon} \gamma^a D^+ \psi \frac{1}{(p-1)!} V_{aa_2 \dots a_p} e^{a_2} \dots e^{a_p} . \quad (3.9)$$

The different curvatures are then related as

$$\hat{F}^+ = \hat{F} - \hat{D}^* H$$

$$\hat{F}^- = \hat{F} - 3\hat{D}^* H$$

$$\hat{R}^\pm_{ab} = \hat{R}_{ab} \pm \hat{D} H_{adb} e^d + H_{adc} H_{ceb} e^d e^e \quad (3.10)$$

or in components

$$\hat{F}^+_{ab} = \hat{F}_{ab} - (\hat{D}_a H_b - \hat{D}_b H_a)$$

$$\hat{F}^-_{ab} = \hat{F}_{ab} - 3(\hat{D}_a H_b - \hat{D}_b H_a)$$

$$\hat{R}^\pm_{abcd} = \hat{R}_{abcd} \pm (\hat{D}_c H_{adb} - \hat{D}_d H_{acb}) + (H_{ace} H_{edb} - H_{ade} H_{ecb})$$

$$\hat{R}^\pm_{ab} = \hat{R}_{ab} \pm \epsilon_{abcd} \hat{D}_c H_d + 2g_{ab} H_d^2 - 2H_a H_b$$

$$\hat{R}^\pm = \hat{R} + 6H_a^2 . \quad (3.11)$$

Here R_{ab} is the Ricci tensor and R the scalar curvature.

The Bianchi identities for the curvatures are trivial to derive using the above formalism. The identity for the three-form H is

$$\begin{aligned}\hat{D}H &= 0 \quad \text{or} \\ \hat{D}_a H_a &= 0 .\end{aligned}\tag{3.12}$$

This follows as $\hat{D}H = dH + \text{explicit } \psi \text{ terms} = d(dB) + \text{explicit } \psi \text{ terms} = 0$. As H is supercovariant and so is $\hat{D}H$, the explicit ψ terms are precisely those needed for supercovariantization and hence can be ignored. Also note that for

$$H_a, \hat{D}_a^\pm H_b = \hat{D}_a H_b \quad (\text{see (2.7)}). \quad \text{Actually for any uncharged vector } \hat{D}_a^\pm V_a = \hat{D}_a V_a.$$

Similarly for the gravitino curvature

$$\hat{D}^+(D^+\psi) = 0$$

or

$$\begin{aligned}\epsilon_{abcd} \hat{D}_b^+ \psi_{cd} + 4\psi_{ab} H_b &= 0 \\ \hat{D}_a^- r_a &= 0 .\end{aligned}\tag{3.13}$$

The Lorentz curvature satisfies

$$\begin{aligned}\hat{R}_{abcd} + \hat{R}_{acdb} + \hat{R}_{adbc} &= 0 \\ \hat{R}_{abcd}^+ &= \hat{R}_{cdab}^- \\ \epsilon^{abcd} \hat{R}_{abcd}^\pm &= 0 \\ \hat{R}_{abcd}^\pm - \hat{R}_{cdab}^\pm &= \pm 2(\hat{D}_a H_{bcd} - \hat{D}_b H_{acd}) .\end{aligned}\tag{3.14}$$

The Lorentz and chiral curvatures also satisfy the following differential Bianchi identities,

$$\begin{aligned}
 \hat{D}^+ \hat{F}^- &= -\frac{i}{2} \overline{D^+ \psi} \gamma_5 \gamma \cdot r \\
 \hat{D}^+ \hat{F}^+ &= -i \overline{D^+ \psi} \gamma_5 \left(r + \frac{1}{2} \gamma \gamma \cdot r \right) \\
 \hat{D} \hat{F} &= -\frac{3i}{2} \overline{D^+ \psi} \gamma_5 \left(r + \frac{1}{3} \gamma \gamma \cdot r \right) \\
 \hat{D}^+ \hat{R}_{ab}^- &= -\frac{i}{2} \overline{D^+ \psi} \gamma \psi_{ab} \\
 \hat{D}^+ \hat{R}_{ab}^+ &= -\frac{1}{4} \overline{D^+ \psi} (\sigma_{ab}, \gamma_c) \psi_{cd} e^d \\
 \hat{D} \hat{R}_{ab} &= -\frac{i}{4} \overline{D^+ \psi} (\gamma_a \psi_{cb} + \gamma_c \psi_{ab} + \gamma_b \psi_{ac}) e^c .
 \end{aligned} \tag{3.15}$$

For the derivation of the Einstein multiplet in Section 4.1, we need the following identities, which are derivable from (3.15),

$$\begin{aligned}
 \hat{D}_b^+ \hat{F}_{ab}^+ &= 2 \hat{F}_{ab}^+ H_b + \overline{\psi}_{ab} r_b \\
 \hat{D}_b^+ \hat{E}_{ab}^+ &= -4 H_a \hat{D} H_a + \frac{i}{2} \overline{\psi}_{ac} \gamma_d \psi_{cd} .
 \end{aligned} \tag{3.16}$$

The supersymmetry transformations of the curvatures are quite straightforward to derive. For example,

$$\delta' (D^+ \psi) \sim -D^{+2} \epsilon \sim - \left(\frac{i}{4} \sigma_{ab} R_{ab}^+ - \frac{i \gamma_5}{2} F^+ \right) \epsilon \quad \text{or}$$

$$\delta'(D^+\psi) = - \left(\frac{1}{4} \sigma_{ab} \hat{R}_{ab}^+ - \frac{i\gamma_5}{2} \hat{F}^+ \right) \epsilon .$$

The explicit ψ terms and $D^+\epsilon$ can be ignored in these derivations. The reason being that, as $D^+\psi$ is supercovariant and the algebra closes on $D^+\psi$, its variation has to be a supercovariant quantity. As another example, consider

$$\delta'\hat{F}^- \sim d\delta A^- \quad \text{or} \quad \delta'\hat{F}^- = \frac{1}{2} \bar{\epsilon} \hat{D}^+ (\gamma \gamma_5 \gamma \cdot r) \quad \text{or} \quad \delta'\hat{F}^- = \frac{1}{2} \bar{\epsilon} \gamma \hat{D}^- \gamma_5 \gamma \cdot r .$$

In the last equation we have used the supercovariantization of the identity $D\gamma = \mathcal{I}$, namely $\hat{D}\gamma = 0$ and $\hat{D}^\pm \gamma = \mp i\gamma_5 \gamma \mathcal{H} \gamma$. Summarizing,

$$\delta\psi_{ab} = - \left(\frac{1}{4} \sigma_{cd} \hat{R}_{cdab}^+ - \frac{i\gamma_5}{2} \hat{F}_{ab}^+ \right) \epsilon$$

$$\delta\hat{F}_{ab}^+ = \frac{1}{2} \bar{\epsilon} \gamma_5 \hat{\mathcal{D}}^- \psi_{ab}$$

$$\delta\hat{R}_{abcd}^+ = - \frac{1}{4} \bar{\epsilon} (\sigma_{ab}, \hat{\mathcal{D}}^-) \psi_{cd} . \quad (3.17)$$

Finally, let us give the transformation of the Rarita Schwinger operator,

$$\delta r_a = - \frac{1}{4} \gamma_b (\hat{E}_{ab}^+ - {}^*\hat{F}_{ab}^+) \epsilon + \frac{1}{2} \gamma_5 \hat{\mathcal{D}} H_a \epsilon$$

with

$$\hat{E}_{ab}^\pm - {}^*\hat{F}_{ab}^\pm = \hat{E}_{ab} - {}^*\hat{F}_{ab} - g_{ab} H_d^2 - 2H_a H_b . \quad (3.18)$$

3. New Minimal Supergravity Multiplets.

3.1. General Multiplet.

A general multiplet of new minimal supergravity,^{2,4,15}

$$V = (C, \chi, H, K, V_a, \lambda, D) \quad (1.1)$$

is specified by the spin and chiral weight of its first component, i.e.

$$\delta_L C = \frac{1}{2} \Lambda_{ab} S_{ab} C \text{ and } \delta_A C = i\phi n C. \text{ We also define } F = (H+iK)/2 \text{ and } \bar{F} = (H-iK)/2.$$

The supersymmetry transformation rules of this multiplet are given by

$$\begin{aligned} \delta C &= -\frac{1}{2} \bar{\epsilon} \chi \\ \delta \lambda &= \frac{1}{2} \{ i \hat{D}^- C + \gamma_5 V + H - i \gamma_5 K \} \epsilon (-)^F \\ \delta(H \pm iK) &= -i \bar{\epsilon} \frac{1 \pm \gamma_5}{2} \{ \gamma_5 \lambda + \hat{D}^- \chi - 2i \gamma_5 M \chi - i \xi C \} \\ \delta V_a &= -\frac{1}{2} \bar{\epsilon} \{ \gamma_a \lambda + \gamma_5 \hat{D}_a^- \chi - i \gamma_a M \chi \} \\ \delta \lambda &= - \left\{ \frac{1}{4} \sigma_{ab} \hat{G}_{ab} + \frac{i \gamma_5}{2} D \right\} \epsilon (-)^F - \frac{1}{2} \xi (\bar{\epsilon} \gamma_5 \chi) \\ \delta D &= -\frac{1}{2} \bar{\epsilon} \gamma_5 \{ \hat{D}^- \lambda - \gamma_a \xi V_a + i \Delta \chi \} . \end{aligned} \quad (1.2)$$

Here

$$\begin{aligned} \xi &= \frac{1}{2} \psi_{ab} S_{ab} - i \gamma_5 \gamma \cdot r n \\ \Delta &= -\frac{1}{2} \hat{F}_{ab}^+ S_{ab} - \frac{1}{2} \hat{R}^- n \end{aligned} \quad (1.3)$$

and

$$\begin{aligned}\hat{G} &= \hat{D}^- V + \frac{1}{2} \overline{D^+ \psi} \gamma_5 \chi \\ &= D^- \left(V + \frac{1}{2} \bar{\psi} \gamma_5 \chi \right) - \frac{1}{2} \bar{\psi} \gamma_5 \chi\end{aligned}$$

or

$$\hat{G}_{ab} = \hat{D}_a^- V_b - \hat{D}_b^- V_a - 2H_{abd} V_d + \frac{1}{2} \bar{\psi}_{ab} \gamma_5 \chi. \quad (1.4)$$

The factor $(-)^F$ accounts for the Fermi or Bose statistics of the first component. These factors disappear if ϵ and ψ are placed to the left of the other fields. Note that ζ and Δ only involve the spin and chiral generators of the first component (e.g. the spin matrix for χ is $S_{ab} + \frac{\sigma_{ab}}{2}$). In two component notation $\psi_\mu, \epsilon, \theta$ have chiral weight 1/2 and $\bar{\psi}_\mu, \bar{\epsilon}, \bar{\theta}$ have -1/2. The chiral weights of the other components are determined using this (e.g. χ has chiral weight $n-1/2$ and $\bar{\chi}$ has $n+1/2$).

Let us briefly explain the derivation of the above transformation rules. The first equation in (1.2) is the definition of χ . The second is the general form for $\delta\chi$ so that the algebra (2.15) closes on C . The third equation is a definition of λ and the fourth is determined by closing the algebra on χ . The fifth equation is the general form of $\delta\lambda$ to close the algebra on H and K . A term like $D'\epsilon$ in $\delta\lambda$ is ruled out by requiring the algebra close on V_a . Finally the last equation is determined by closing the algebra on λ . The identities (2.3.3) and (2.3.4) are very useful in these derivations.

As ϵ is assigned to transform as a spinor under Lorentz transformations, the Lorentz algebra is manifest. Similarly, the chiral algebra is easily

checked by comparing the chiral weights of the left- and right-hand sides of (1.2).

The appearance of ξ and Δ in the above transformation rules is analogous to the contribution of λ and D terms of gauge multiplets in the Wess-Zumino gauge. Indeed ξ (and Δ) will be identified with the λ (and D) component of the chiral and Lorentz connection gauge multiplet in Section 4.5.

Let us now give some other transformation rules, which are needed in deriving the constrained multiplets,

$$\begin{aligned}
 \delta \xi &= - \left\{ \frac{1}{4} \sigma^{ab} \left(\frac{1}{2} S_{cd} \hat{R}_{cdab}^- + i n \hat{F}_{ab}^- \right) + \frac{1}{2} \gamma_5 \Delta \right\} \epsilon \\
 \delta' \hat{G} &= \frac{1}{2} \bar{\epsilon} \gamma (\hat{D}^- \lambda - \xi V) + \frac{1}{2} \left(\frac{1}{2} S_{cd} \hat{R}_{cd}^- + n \hat{F}^- \right) \bar{\epsilon} \gamma_5 \chi \\
 \delta (i \hat{D}_a^- C \pm V_a) &= -i \bar{\epsilon} \frac{1 \pm \gamma_5}{2} \hat{D}_a^- \chi + \frac{1}{2} \frac{1 \pm \gamma_5}{2} \gamma_a \left\{ \gamma_5 \lambda - 2i \gamma_5 M \chi - i \xi C \right\} \\
 &\quad - \frac{1}{2} \bar{\epsilon} \frac{1 \mp \gamma_5}{2} \gamma_a \left(\gamma_5 \lambda + i \xi C \right) \\
 \delta \{ \gamma_5 \lambda + \hat{D}^- \chi - 2i \gamma_5 M \chi - i \xi C \} &= \frac{1}{2} \hat{D}^- (H - i \gamma_5 K) \epsilon \\
 &\quad - \frac{1}{2} \epsilon \left\{ D + \hat{D}^- C - 2H \cdot V - \frac{1}{4} \bar{\psi}_{ab} \sigma_{ab} \chi - \bar{\xi} \chi \right\} \\
 &\quad + \frac{1}{2} \gamma_5 \epsilon \left\{ \hat{D}^- \cdot V + 2H \cdot \hat{D}^- C + \frac{1}{4} \bar{\psi}_{ab} \sigma_{ab} \gamma_5 \chi - i \bar{\xi} \gamma_5 \chi - \Delta C \right\} . \quad (1.5)
 \end{aligned}$$

The multiplication rule of these multiplets, i.e. the components of an arbitrary function $F(V)$ are,¹⁵

$$C_F = F(C)$$

$$\chi_F = \chi^j_{F_j}$$

$$H_F = H^j_{F_j} + \frac{1}{4} \bar{\chi}^j \chi^k_{F_{kj}}$$

$$K_F = K^j_{F_j} + \frac{1}{4} \bar{\chi}^j i \gamma_5 \chi^k_{F_{kj}}$$

$$V_{aF} = V^j_{a F_j} + \frac{1}{4} \bar{\chi}^j \gamma_5 \gamma_a \chi^k_{F_{kj}}$$

$$\begin{aligned} \lambda_F = & \lambda^j_{F_j} + \frac{i\gamma_5}{2} \{ i \hat{D}^- C^j + \gamma_5 \mathcal{Y}^j - H^j - i\gamma_5 K^j \} \chi^k_{F_{kj}} \\ & - \frac{1}{8} \{ \gamma_5 \chi^j (\bar{\chi}^k \chi^\ell) - \chi^j (\bar{\chi}^k \gamma_5 \chi^\ell) \} F_{\ell kj} \end{aligned}$$

$$\begin{aligned} D_F = & D^j_{F_j} + \frac{1}{2} \{ -V^j \cdot V^k - \hat{D}^- C^j \cdot \hat{D}^- C^k + H^j H^k + K^j K^k \} F_{kj} \\ & + i \bar{\chi}^j \left\{ \gamma_5 \lambda^k + \frac{1}{2} \hat{D}^- \chi^k - \frac{3i\gamma_5}{2} \mathcal{H} \chi^k - i \xi^k C^k \right\} F_{kj} \\ & + \frac{1}{4} \bar{\chi}^j \{ H^k + i\gamma_5 K^k + \gamma_5 \mathcal{Y}^k \} \chi^\ell_{F_{\ell kj}} \\ & - \frac{1}{4} \bar{\chi}^j \frac{1+\gamma_5}{2} \chi^k \chi^\ell \frac{1-\gamma_5}{2} \chi^m_{F_{m\ell kj}} . \end{aligned} \quad (1.6)$$

These multiplication rules are encoded in the superfield representation

$$\begin{aligned} V = & C - \bar{\theta} \chi - \frac{1}{2} \bar{\theta} \{ H - i\gamma_5 K + \gamma_5 \mathcal{Y} \} \theta + i \bar{\theta} \theta \bar{\theta} \left\{ \gamma_5 \lambda + \frac{1}{2} \hat{D}^- \chi - \frac{3i\gamma_5}{2} \mathcal{H} \chi - i \xi C \right\} \\ & + \frac{1}{4} (\bar{\theta} \theta)^2 \left(D + \frac{1}{2} \hat{D}^- C \right) . \end{aligned} \quad (1.7)$$

3.2. Complex Linear Multiplet.

We now impose constraints on the general multiplet of the last section to get the various constrained multiplets. The complex linear multiplet, $\mathcal{L}$, is defined by requiring $\bar{F} = 0$ or $\bar{V}^2 \mathcal{L} = 0$. (The spinor derivatives will be defined in Section 3.6.)^{2,15} The independent components are C , χ , F , V_a and λ_R . The D and λ_L components are given by

$$\lambda = \hat{D}^- \bar{\chi} + 2iH\bar{\chi} - i\xi C$$

$$D = -\hat{D}^- C + 2H \cdot V + i\hat{D} \cdot (V + 2CH) - i\Delta C - \frac{1}{2} \bar{\psi}_{ab} \bar{\sigma}_{ab} \bar{\chi} - 2\bar{\xi} \bar{\chi} . \quad (2.1)$$

3.3. Real Linear Multiplet.

The real linear multiplet, L , is defined by $F = \bar{F} = 0$ or $V^2 L = \bar{V}^2 L = 0$.^{2,15} It must have zero chiral weight. The independent components are C , χ , V_a , with the constraint on the vector,

$$\hat{D}^- \cdot (V + 2CH) + \frac{1}{2} \hat{F}_{ab}^+ S_{ab} C + \frac{1}{2} \bar{\psi}_{ab} \gamma_5 \left(S_{ab} + \frac{\sigma_{ab}}{2} \right) \chi = 0 . \quad (3.1)$$

The transformation rules of the real linear multiplet are

$$\delta C = -\frac{1}{2} \bar{\epsilon} \chi$$

$$(-)^F \delta \chi = \frac{1}{2} (i\hat{D}^- C + \gamma_5 V) \epsilon$$

$$\delta V_a = -\frac{1}{2} \bar{\epsilon} (\gamma_5 \sigma_{ab} \hat{D}_b^- - \gamma_a H) \chi - \frac{1}{4} \bar{\epsilon} \gamma_5 \gamma_a \psi_{cd} S_{cd} C . \quad (3.2)$$

The λ and D components of this multiplet are given by

$$\begin{aligned}\lambda &= -\gamma_5 \hat{D}^- \chi + 2iM\chi - \frac{i}{2} \gamma_5 \psi_{ab} S_{ab} C \\ D &= -\hat{D}^- C + 2H \cdot V + \frac{i}{2} \bar{\psi}_{ab} \left(S_{ab} + \frac{\sigma_{ab}}{2} \right) \chi\end{aligned}\quad (3.3)$$

and the embedding in the general multiplet is

$$V(L) = (C, \chi, 0, 0, V_a, \lambda, D) . \quad (3.4)$$

For $S_{ab} = 0$ the constraint (3.1) can be explicitly solved in terms of an antisymmetric tensor, $b_{\mu\nu}^{2,4}$,

$$*V = db - 2CdB + \frac{1}{4} \bar{\psi} \gamma \gamma \chi$$

and

$$\delta b = \frac{1}{4} \bar{\epsilon} \gamma \gamma \chi + \frac{1}{4} C \bar{\epsilon} \gamma \psi . \quad (3.5)$$

3.4. Chiral Multiplets.

Chiral multiplets, Σ , are defined by the constraint $\chi_R = 0$ or $\bar{V}_\alpha \cdot \Sigma = 0$.^{2,4,15} The transformation rules of the chiral multiplet are

$$\Sigma = (z, \chi, h)$$

$$\delta z = \frac{1}{2} \epsilon \chi$$

$$(-)^F \delta \chi = i \hat{D}^- z \bar{\epsilon} + h \epsilon$$

$$\delta h = \frac{1}{2} \bar{\epsilon} (i \hat{D}^- + 2H) \chi + \bar{\epsilon} \bar{\xi} z . \quad (4.1)$$

The embedding in a general multiplet is given by

$$V(\Sigma) = (z, \chi_L, h, -ih, -i \hat{D}_a^- z, -i \xi z, -i \Delta z) . \quad (4.2)$$

The multiplication rule is^{4,15}

$$f(\Sigma) = \left(f(z), \chi^j f_j, h^j f_j - \frac{1}{4} \chi^j \chi^k f_{kj} \right) \quad (4.3)$$

which implies the chiral superfield representation

$$\Sigma = z + \theta \chi + \theta^2 h . \quad (4.4)$$

From an arbitrary multiplet V we can form a chiral multiplet with weight $n+1$ by the chiral projection,⁴

$$\Pi(V) = - \frac{1}{4} \bar{V}^2 V . \quad (4.5)$$

The components of $\Pi(V)$ are given by

$$z_\pi = \bar{F}$$

$$\chi_\pi = i(\hat{D}^- \bar{\chi} + 2iH\bar{\chi} - \lambda - i\xi C)$$

$$h_{\pi} = \frac{1}{2} \left\{ D - i(\hat{D} - 2iH) \cdot (V + i\hat{D}C) + i\Delta C + \frac{1}{2} \bar{\psi}_{ab} \bar{\sigma}_{ab} \bar{\chi} + 2\bar{\xi} \bar{\chi} \right\}. \quad (4.6)$$

Similarly, the kinetic multiplet, $T(\Sigma) = -\frac{1}{4} \bar{V}^2 \Sigma$, is⁴

$$z_T = \bar{h}$$

$$\chi_T = i\hat{D} \bar{\chi} - 2H\bar{\chi} + 2\xi \bar{z}$$

$$h_T = \hat{D} \bar{z} + i\Delta \bar{z} + \frac{1}{4} \bar{\psi}_{ab} \bar{\sigma}_{ab} \bar{\chi} + \bar{\xi} \bar{\chi}. \quad (4.7)$$

The chiral weight of $T(\Sigma)$ is $1-n$ (Σ has weight n and $\bar{\Sigma}$ has $-n$).

3.5. Gauge Multiplet.

From a real scalar multiplet we can form the gauge multiplet

(V_{μ}', λ, D) ,^{2,4} where

$$V' = V + \frac{1}{2} \bar{\psi} \gamma_5 \chi. \quad (5.1)$$

These fields transform among themselves, up to a gauge transformation,

$$\delta V_{\mu}' = -\frac{1}{2} \bar{\epsilon} \gamma_{\mu} \lambda - \partial_{\mu} \left(\frac{1}{2} \bar{\epsilon} \gamma_5 \chi \right)$$

$$\delta \lambda = - \left(\frac{1}{4} \sigma_{ab} \hat{G}_{ab} + \frac{i\gamma_5}{2} D \right) \epsilon$$

$$\delta D = -\frac{1}{2} \bar{\epsilon} \gamma_5 \hat{D} \bar{\lambda} \quad (5.2)$$

with $\hat{G} = dV' - \frac{1}{2} \bar{\psi} \gamma \lambda$. The complete analysis of Abelian and non-Abelian gauge

theories will be given in Sections 4.3, 4.4, and 4.5.

3.6. Spinor and Vector Derivatives.

We now discuss the derivative operations on these multiplets. The spinor derivative is defined by¹⁵

$$C(VV) = \chi \quad (6.1)$$

i.e. the first component of the multiplet VV is χ (or $V \sim -\frac{\partial}{\partial \theta} + \dots$). This corresponds to the usual superspace derivative. We will give the explicit component expansion of $\bar{V}_\alpha V$ (two-component notation). The derivative $V_\alpha V$ is then determined by complex conjugation, i.e. $V_\alpha V = (-)^F (\bar{V}_\alpha V^*)^*$.

The multiplet $\bar{V}_\alpha V$ turns out to be complex linear, just like in global supersymmetry. Its components are given by

$$C(\bar{V}V) = \bar{\chi}$$

$$\chi(\bar{V}V) = i\hat{D}^- C + Y$$

$$\bar{\chi}(\bar{V}V) = 2\bar{F}$$

$$F(\bar{V}V) = i(\bar{\lambda} + i\xi C)$$

$$V_a(\bar{V}V) = -\bar{\sigma}_{ab} \hat{D}_b^- \bar{\chi} + i\sigma_a(\lambda - 2iH\bar{\chi} + i\xi C)$$

$$\bar{\lambda}(\bar{V}V) = iD + (\hat{D} - i2H) \cdot (V + i\hat{D}^- C) - \Delta C - \frac{i}{4} \bar{\psi}_{ab} \bar{\sigma}_{ab} \bar{\chi} - \frac{3}{2} \bar{\xi} \bar{\chi} +$$

$$+ \frac{1}{4} \bar{\sigma}_{ab} \left\{ \bar{\psi}_{ab} \bar{\chi} - \frac{1}{2} \bar{\xi} \bar{\sigma}_{ab} \bar{\chi} \right\} . \quad (6.2)$$

Here the spinor index of the derivative is always on the left (e.g. $\chi_\alpha (\bar{\nabla}_\alpha V) = (i \hat{D} C + \mathcal{Y})_{\dot{\alpha}\alpha}$).

The λ_L and D components of $\bar{\nabla}_\alpha V$ are given by 3.2.1. In using this formula, one should remember that $\bar{\nabla}_\alpha V$ has chiral weight $n+1/2$ and spin $S_{ab} + \frac{\sigma_{ab}}{2}$.

From the third equation in (6.2), we can see that

$$\{\bar{\nabla}_\alpha, \bar{\nabla}_\beta\} = 0 \quad (6.3)$$

exactly like in global supersymmetry. The full set of commutation relations will be given in Section 4.6.

The vector derivative is defined by¹⁵

$$C(\nabla_a V) = \hat{D}_a C . \quad (6.4)$$

The χ component of this multiplet is

$$\chi(\nabla_a V) = \hat{D}_a \chi - i \gamma_5 \gamma_a M \chi + i \gamma_a \xi C . \quad (6.5)$$

The second equation in (6.2) implies that this derivatives can also be defined as

$$\nabla_a = \frac{1}{4i} (\nabla_\sigma \bar{\nabla}_a + \bar{\nabla}_\sigma \nabla_a)$$

or

$$\{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\} = -2i\sigma_{\alpha\dot{\alpha}}^a \nabla_a \quad . \quad (6.6)$$

Similarly, we can define the other vector derivatives ∇_a, ∇_a^+ by giving their first components. Due to 2.2.7, these derivatives are related as

$$\begin{aligned} \nabla_a^- &= \nabla_a - i^* S_{ab} E_b - 3inE_a \\ \nabla_a^+ &= \nabla_a + i^* S_{ab} E_b - inE_a \end{aligned} \quad (6.7)$$

where E_a is the Einstein multiplet with first component H_a . The vector derivative ∇_a corresponds to the usual superspace derivative.

4. Curvature Multiplets.

4.1. Gravitational Curvature Multiplets.

In new minimal supergravity, there are two fundamental gravitational curvature multiplets, the Einstein multiplet, E_a , and the Riemann multiplet, T_{ab}^α . The irreducible pieces of the Riemann multiplet are the scalar curvature multiplet, T , and the Weyl multiplet, W_{ab}^α . In Section 2.1, we derived the Einstein and Riemann multiplets at the linearized level. Here we take the first component of these expressions, i.e. H_a and ψ_{ab} respectively, to derive the corresponding multiplets at the nonlinear level.

The Einstein multiplet is a real linear multiplet (i.e. $\bar{\nabla}^2 E_a = 0$, $E_a = E_a^*$) with components (see (2.2.11), (2.3.18);

$$E_a = \left(H_a, i\gamma_5 r_a, \frac{1}{2} (\hat{E}_{ab}^+ - {}^* \hat{F}_{ab}^+) \right). \quad (1.1)$$

The embedding in a general multiplet is given by (3.3.4). It can be shown to satisfy the linear multiplet constraint (3.3.1) by using the Bianchi identities (2.3.16).

The Riemann multiplet is chiral ($\bar{\nabla}_\alpha T_{ab}^\alpha = 0$) with components (see (1.3.17)^{1,6}

$$T_{ab} = \psi_{ab} - \left(\frac{1}{2} \sigma_{cd} \hat{R}_{cdab}^+ + i \hat{F}_{ab}^+ \right) \theta + i \hat{D}^- \bar{\psi}_{ab} \theta^2. \quad (1.2)$$

The scalar curvature multiplet is given by

$$\begin{aligned} T &= \frac{1}{2} \sigma_{ab} T_{ab} \\ &= \sigma \cdot \bar{r} - \frac{1}{2} (\hat{R}^- + \sigma_{ab} \hat{F}_{ab}^-) \theta - i \hat{D}^- \sigma \cdot r \theta^2. \end{aligned} \quad (1.3)$$

Similarly, the Weyl multiplet is

$$\begin{aligned}
 W_{ab} &= \frac{1}{24} (3\sigma_{cd}\sigma_{ab} + \sigma_{ab}\sigma_{cd}) T_{cd} \\
 &= \frac{1}{2} \left(T_{ab} + i^* T_{ab} - \frac{1}{3} \sigma_{ab} T \right) \\
 &= \frac{1}{24} (3\sigma_{cd}\sigma_{ab} + \sigma_{ab}\sigma_{cd}) \psi_{cd} - \frac{1}{2} \sigma_{cd} \hat{\theta} \hat{W}_{cdab} \\
 &\quad - \frac{1}{3} \left\{ \hat{F}_{ab} + i^* \hat{F}_{ab} - \frac{1}{2} \sigma_{ac} \hat{F}_{cb} + \frac{1}{2} \sigma_{bc} \hat{F}_{ca} \right\} \theta \\
 &\quad + \{ \hat{D}_a^- \phi_b - \hat{D}_b^- \phi_a + i^* (\hat{D}_a^- \phi_b - \hat{D}_b^- \phi_a) \} \theta^2 .
 \end{aligned} \tag{1.4}$$

where $\phi_a = r_a + \frac{1}{3} \gamma_a \gamma \cdot r$ and $\hat{W}_{abcd}$ is the Weyl tensor .

The matrix multiplying T_{ab} in the above expression is the projection operator for the σ_a traces, i.e. $\sigma_a W_{ab} = 0$. Equivalently, in two component spinor notation,

$$W_{\alpha\beta\gamma} = \frac{1}{4} \sigma_{\alpha\beta}^{ab} W_{ab\gamma} \tag{1.5}$$

is completely symmetric in α, β and γ .

The Riemann multiplet can be reconstructed in terms of the three irreducible multiplets E_a , W_{ab} and T as

$$T_{ab} = W_{ab} - \frac{1}{3} \sigma_{ab} T - i\epsilon_{abcd} \sigma_c \bar{\nabla} E_d . \tag{1.6}$$

We can also construct other gravitational multiplets by applying the

spinorial derivatives to the Riemann multiplet. For example,

$$\begin{aligned}
 F_{ab}^+ &= -\frac{i}{4} \bar{\nabla}_Y T_{ab} \\
 &= (\hat{F}_{ab}^+, \dots) \\
 F_{ab}^- &= \frac{i}{4} \bar{\nabla}_\sigma T_{ab} \\
 &= (\hat{F}_{ab}^-, \dots) \\
 R_{cdab}^- &= R_{abcd}^+ \\
 &= \frac{i}{4} \bar{\nabla}_\sigma T_{cd} \\
 &= (\hat{R}_{cdab}^-, \dots) .
 \end{aligned} \tag{1.7}$$

These multiplets are related as in Section (2.3), with the replacements $\hat{D}_a \rightarrow \nabla_a$, $\hat{F}_{ab} \rightarrow F_{ab}$, $\hat{R}_{abcd} \rightarrow R_{abcd}$, $H_a \rightarrow E_a$, $\psi_{ab} \rightarrow T_{ab}$, $\delta_Q(\psi_{ab}) \rightarrow \frac{1}{2} \bar{T}_{ab} \nabla$. For example,

$$F_{ab}^- = F_{ab}^+ - 2(\nabla_a E_b - \nabla_b E_a) . \tag{1.8}$$

4.2. Bianchi Identities.

The Bianchi identities for the gravitational curvature multiplets are derived by considering the first components and using the Bianchi identities for the curvatures given in Section (2.3). For example, $C(\nabla_a E_a) = \hat{D}_a H_a = 0$ (see 2.3.12) implies that $\nabla_a E_a = 0$. We list here

some of the Bianchi identities^{1,6}

$$\nabla_a E_a = 0$$

$$T = -i\sigma_a \bar{\nabla} E_a, \quad \bar{\nabla} E_a = \frac{i}{2} \sigma_b^* T_{ba}$$

$$\nabla T + \bar{\nabla} \bar{T} = 0$$

$$\nabla T_{ab} = -\bar{\nabla} \bar{T}_{ab} = 2i(\nabla_a E_b - \nabla_b E_a) + \frac{i}{2} \epsilon_{abcd} (\bar{\nabla}_c V - \nabla_c \bar{V}) E_d$$

$$\nabla W_{ab} = 2i(\nabla_a E_b - \nabla_b E_a) - 2^*(\nabla_a E_b - \nabla_b E_a) + \frac{1}{3} \nabla_{ab} T. \quad (2.1)$$

4.3. Supersymmetric Yang-Mills.

Gauge theories are formulated exactly as in the global case. We introduce a real superfield V , which transforms under gauge transformations as^{1,2,4,16}

$$e^{-2iV} \rightarrow e^{\bar{\Lambda}} e^{-2iV} e^{-\Lambda} \quad (3.1)$$

where $\Lambda = i\Lambda_A T_A$ and $\bar{\Lambda} = i\bar{\Lambda}_A T_A$ are chiral and antichiral respectively. The chiral field strength multiplet is defined by

$$W = -\frac{i}{8} \bar{\nabla}^2 (e^{2iV} \nabla e^{-2iV}) \quad (3.2)$$

and it transforms as $W \rightarrow e^{\Lambda} W e^{-\Lambda}$. In the Wess-Zumino gauge, $C = \chi = F = 0$, the remaining gauge transformations are, $\Lambda_A = \bar{\Lambda}_A = \phi_A$,

$$\delta_G V_\mu = -D_\mu \phi$$

$$\delta_G \lambda = \phi \lambda$$

$$\delta_G D = \phi D \quad (3.3)$$

and now the derivative includes the Yang-Mills connection, i.e. $D_\mu = \partial_\mu + V_\mu$.
The supersymmetry transformations in the Wess-Zumino gauge are given by¹⁶

$$\delta_Q^{WZ}(\epsilon) = \delta_Q(\epsilon) + \delta_G \left(i\theta \gamma \bar{\epsilon} + \theta^2 \bar{\epsilon} \left(\bar{\lambda} - \frac{1}{2} \sigma_a \gamma \bar{\psi}_a \right) \right) \quad (3.4)$$

and

$$\delta V_\mu = -\frac{1}{2} \bar{\epsilon} \gamma_\mu \lambda$$

$$\delta \lambda = - \left(\frac{1}{4} \sigma_{ab} \hat{G}_{ab} + \frac{i\gamma_5}{2} D \right) \epsilon$$

$$\delta D = -\frac{1}{2} \bar{\epsilon} \gamma_5 \hat{\not{D}} \lambda$$

with

$$\hat{G} = dV + V^2 - \frac{1}{2} \bar{\psi} \gamma \lambda \quad (3.5)$$

The component expansion of the field strength multiplet is

$$W = \lambda + i\theta \left(\frac{1}{2} \sigma_{ab} \hat{G}_{ab} + D \right) + i\hat{\not{D}} \bar{\lambda} \theta^2 \quad (3.6)$$

and its square is given by

$$W^2 = \lambda^2 + 2i\theta \left(\frac{1}{2} \sigma_{ab} \hat{G}_{ab} + D \right) \lambda + \theta^2 \left\{ 2\lambda i \hat{D} \bar{\lambda} + \frac{1}{2} \hat{G}_{ab} \hat{G}_{ab} + \frac{1}{2} \hat{G}_{ab} {}^* \hat{G}_{ab} - \frac{1}{2} D^2 \right\}. \quad (3.7)$$

Under Yang-Mills transformations, chiral multiplets transform as $\Sigma \rightarrow e^{\Lambda} \Sigma$. The minimal coupling to Yang-Mills is then given by replacing $\bar{\Sigma}$ with $\bar{\Sigma} e^{-2iV}$, which transforms as $(\bar{\Sigma} e^{-2iV}) \rightarrow (\bar{\Sigma} e^{-2iV}) e^{-\Lambda}$. In the Wess-Zumino gauge, the supersymmetry transformations of the chiral multiplet are given by¹⁶

$$\begin{aligned} \delta z &= \frac{1}{2} \epsilon \chi \\ \delta \chi &= i \hat{D} \bar{z} \epsilon + h \epsilon \\ \delta h &= \frac{1}{2} \bar{\epsilon} (i \hat{D} + 2H) \chi + i \bar{\epsilon} \bar{\xi} z + i \bar{\epsilon} \bar{\lambda} z \end{aligned} \quad (3.8)$$

where the derivatives now include the Yang-Mills connection.

4.4. Supergravity and Super-Yang-Mills Algebra.

In the last subsection we described Yang-Mills theories in the standard manner. In this section we give an alternate method which includes the Yang-Mills algebra directly in the supergravity algebra.² We define a gauge multiplet (V_μ, λ, D) by the transformation rules 4.3.5 and 4.3.3. We now extend the supergravity algebra 2.2.9 by including these. The commutator of two supersymmetry transformations is then given by 2.2.14 with the addition of $\delta_G(\xi \cdot V)$ on the right-hand side. On covariant fields, this again defines a

supercovariant derivative

$$\hat{D}^- = d + \delta_L(\omega_{ab}^-) + \delta_A(A^-) + \delta_G(V) + \delta_Q(\psi) . \quad (4.1)$$

The commutator of two supercovariant derivatives is now given by 2.3.3 with the addition of $\delta_G(\hat{G}_{ab})$. Similarly 2.3.4 is modified by adding $\delta_G\left(-\frac{i}{2}\bar{\epsilon}\gamma_a\lambda\right)$. Actually all the results of Section 2 are valid if we make the replacements

$$\begin{aligned} \xi &\rightarrow \frac{i}{2}\psi_{ab}S_{ab} - i\gamma_5\gamma\cdot r n + i\lambda\cdot T \\ \Delta &\rightarrow -\frac{i}{2}\hat{F}_{ab}^+S_{ab} - \frac{i}{2}\hat{R}^-_n + iD\cdot T \end{aligned} \quad (4.2)$$

and $\hat{D}_a$ now includes the Yang-Mills connection.

The Yang-Mills field strength multiplet cannot be defined as 4.3.2 (as we only have covariant spinor derivatives). However, we can define it to be a chiral multiplet with first component λ . The component expansion is, of course, the same as 4.3.6. These field strength multiplets satisfy the identity

$$-\bar{\nabla}W = \nabla W + \bar{\nabla}\bar{W} = 0 . \quad (4.3)$$

From the field strength multiplet we can form other multiplets with first components given by $\hat{G}_{ab}$ and D ,

$$G_{ab} = \frac{i}{4}\bar{\nabla}\sigma_{ab}W$$

$$= (\hat{G}_{ab}, \dots)$$

$$D = \frac{i}{4} \bar{\psi} \gamma_5 \psi$$

$$= (D, \dots) . \quad (4.4)$$

4.5. Chiral and Lorentz Connection Multiplets.

There are two important gauge multiplets in the supergravity algebra itself. The Poincaré or chiral gauge multiplet is given by (see 2.2.9, 2.3.17, 3.3.5)⁴

$$V_p = \left(A_\mu^-, -\gamma_5 \gamma_\mu r, -\frac{1}{2} \hat{R}^- \right) . \quad (5.1)$$

The field strength of this multiplet is the scalar curvature multiplet (3.1.3), i.e.

$$T = W(V_p) . \quad (5.2)$$

The Lorentz connection gauge multiplet is given by (2.2.10, 2.3.17, 3.3.5)

$$\Omega_{ab} = (\omega_{ab}^-, \psi_{ab}, -\hat{F}_{ab}^+) . \quad (5.3)$$

The field strength of this multiplet is the Riemann multiplet (3.1.2),

$$T_{ab} = W(\Omega_{ab}) . \quad (5.4)$$

4.6. Commutation Relations of Spinor and Vector Derivatives.

Let us now give the commutation relations of the various derivatives,^{1,15}

$$\{\nabla, \bar{\nabla}\} = 2i\bar{\nabla}$$

$$(\nabla_a^-, \nabla) = \gamma_a \left\{ \frac{1}{2} T_{bc} S_{bc} + T_n + W \cdot T \right\} - i\gamma_a \gamma_5 E V$$

$$(\nabla_a^-, \nabla_b^-) = \frac{i}{2} S_{cd} R_{cdab}^- + i n F_{ab}^- + i T \cdot G_{ab} - 2E_{abc} \nabla_c^- + \frac{1}{2} \bar{T}_{ab} \nabla \quad (6.1)$$

with $E_{abc} = -\epsilon_{abcd} E_d$. The first equation is exactly analogous to the global case and defines ∇_a^- . The second can be viewed as a geometric definition of the field strength multiplets. The identity $\nabla W + \bar{\nabla} \bar{W} = 0$ follows as $\{\bar{\nabla}, (\gamma_a \nabla, \nabla_a^-)\} \sim (\nabla_a^-, \nabla_a^-) = 0$. The third equation (and similar equations for ∇_a^+ and the superspace derivative ∇_a) are derived from 2.3.2 and the substitutions given at the end of Section 4.1. As this equation shows, the Riemann and Einstein multiplets can be interpreted as superspace torsion components.

5. Lagrangians.

5.1. F Type Density Formula.

Chiral multiplets with chiral weight $n = 1$ can be used to form invariant actions by the F density formula^{2,4}

$$[\Sigma]_F = e \left\{ h + \frac{i}{2} \chi \sigma \cdot \bar{\psi} + \frac{i}{2} z \bar{\psi}_a \bar{\sigma}_{ab} \bar{\psi}_b \right\}. \quad (1.1)$$

In superfield notation this can be written as¹

$$[\Sigma]_F = \int d^2\theta \mathcal{Z} \Sigma$$

with

$$\mathcal{Z} = e \left\{ 1 - i\theta \sigma \cdot \bar{\psi} + \frac{i}{2} \theta^2 \bar{\psi}_a \bar{\sigma}_{ab} \bar{\psi}_b \right\}. \quad (1.2)$$

The restriction $n = 1$ follows as $d\theta$ has $n = -1/2$ (θ has $n = 1/2$).

The Lagrangian for Yang-Mills theories is then given by (see 4.3.7)

$$\begin{aligned} e^{-1} [W^2]_F &= \frac{1}{2} G_{ab} G_{ab} - D^2 - \bar{\lambda} i \not{D} \lambda - \frac{1}{2} \left(G_{ab} - \frac{i}{2} \bar{\psi}_a \gamma_b \lambda \right) \bar{\psi}_c \sigma_{ab} \gamma_c \lambda \\ &+ \frac{i}{2} G_{ab} {}^* G_{ab} + \frac{i}{2} e^{-1} \partial_\mu (e \bar{\lambda} \gamma_5 \gamma^\mu \lambda) \end{aligned} \quad (1.3)$$

as the field strength multiplet, W , has $n = 1/2$.

5.2. D Type Formula.

From a multiplet with chiral weight $n = 0$ we can form an invariant action as^{2,4}

$$\begin{aligned}
 [V]_D &= 2 \int d^4\theta \, EV \\
 &= e \left\{ D - \frac{1}{2} \bar{\psi} \cdot \gamma \gamma_5 \lambda + \left(V_\mu + \frac{i}{2} \bar{\psi} \gamma_\mu \gamma_5 \chi \right) \epsilon_{\mu\nu\rho\lambda} \partial_\nu B_{\rho\lambda} \right\} \\
 &\quad + \text{surface terms}
 \end{aligned} \tag{2.1}$$

where

$$E = e \left\{ 1 + \frac{i}{2} \bar{\theta} \gamma \cdot \psi + \frac{i}{16} \bar{\theta} \theta \bar{\psi}_a \sigma_{ab} \psi_b - \frac{i}{16} \bar{\theta} \gamma_5 \theta \bar{\psi}_a \gamma_5 \sigma_{ab} \psi_b + \bar{\theta} \gamma_5 \gamma_a \theta X_a + \bar{\theta} \theta \bar{\theta} Y \right\}. \tag{2.2}$$

Note that E has no $\theta^2 \bar{\theta}^2$ component, i.e. $[1]_D = 0$.^{1,4} The F and D type formulae are related as²

$$[V]_D = 2[\Pi(V)]_F + \text{S.T.} \tag{2.3}$$

To verify the above equation we need the expression for the supercovariant Laplacian²

$$\hat{\square} C = \frac{1}{e} \partial_\mu (e \hat{D}^\mu C) - \frac{1}{2} \bar{\psi} \cdot \hat{D} \chi - \frac{i}{2} \bar{\psi} \cdot \gamma \psi \cdot \hat{D} C \tag{2.4}$$

which is valid for $n = S_{ab} = 0$.

From (2.3), it is easy to see that all the spinor and vector derivatives can be integrated by parts in the D density formula. Also for a chiral multiplet Σ and linear multiplet L we have $[\Sigma]_D = [L]_D = [\Sigma L]_D = 0$, up to surface terms. The last equation implies that linear multiplets can be duality transformed to chiral multiplets.

Finally, the action for Poincaré supergravity is given by the Fayet-Iliopoulos term of the chiral gauge multiplet,⁴

$$\begin{aligned} -e^{-1} [V_p]_D &= \frac{1}{2} R^- + \bar{\psi} \cdot r - A_{\mu}^- \epsilon_{\mu\nu\rho\lambda} \partial_{\nu} B_{\rho\lambda} \\ &= \left(\frac{1}{2} R + \bar{\psi} \cdot r \right) \Big|_{A=H=0} - 2A \cdot H - 3H_a^2 \end{aligned} \quad (2.5)$$

or in differential forms

$$-[V_p]_D(dx) = \frac{1}{2} R_{ab}^{-*} e^a e^b + \frac{1}{2} \bar{\psi} \gamma_S \gamma D^+ \psi + 2A^- dB. \quad (2.6)$$

5.3 Genral Matter Couplings

The genral supergravity Lagrangian containing at most two derivatives is^{5,16}

$$\mathcal{L} = \frac{1}{2} [F(\Sigma, \bar{\Sigma}, L)]_D + 2 \operatorname{Re}[f(\Sigma)]_F - \frac{1}{2} \operatorname{Re}[f_{AB}(\Sigma) W^A W^B]_F - [V_p]_D \quad (3.1)$$

where the superpotential f has $n = 1$,

$$(nz)_j f_j = f \quad (3.2)$$

and f_{AB} , F have $n = 0$,

$$(nz)_j f_{ABj} = 0$$

$$(nz)_j F_j = (nz)_j^- F_j^- \quad (3.3)$$

The bosonic components of this Lagrangian are given by

$$\begin{aligned}
 e^{-1} \mathcal{L}_{\text{BOS}} = & \frac{1}{2} e^{2\sigma} (R + 6H_\mu^2) + 2 \left\{ A_\mu^- + \frac{i}{2} F_j D_\mu^- z_j - \frac{i}{2} F_j^- D_\mu^- \bar{z}_j \right\} H_\mu \\
 & + \frac{1}{4} F_{LL'} \partial C^L \cdot \partial C^{L'} - \frac{1}{4} F_{LL'} V^L \cdot V^{L'} + \left\{ \frac{i}{2} F_{Lj} D_\mu^- z_j - \frac{i}{2} F_{Lj}^- D_\mu^- \bar{z}_j \right\} V_\mu^L \\
 & - F_{ij}^- D^- z_i \cdot D^- \bar{z}_j + F_{ij}^- h_i \bar{h}_j + h_j f_j + \bar{h}_j \bar{f}_j \\
 & - \frac{1}{4} \text{Re } f_{AB} F_{ab}^A F_{ab}^B + \frac{1}{4} \text{Im } f_{AB} F_{ab}^A * F_{ab}^B \\
 & + \frac{1}{2} \text{Re } f_{AB} D^A D^B + (T_A z)_j F_j D_A
 \end{aligned} \tag{3.4}$$

with

$$V_\mu^L = -\frac{1}{2} \epsilon_{\mu\nu\rho\lambda} \partial_\nu b_{\rho\lambda}^L - 2C^L H_\mu \tag{3.5}$$

$$e^{2\sigma} = 1 - (nz)_j F_j. \tag{3.6}$$

Let us restrict ourselves to the case with only chiral multiplets, as linear multiplets can always be duality transformed to chiral multiplets.^{2,5} The field H_μ can be made unconstrained by adding the Lagrange multiplier, $2H_\mu \partial_\mu \phi$. Then ϕ can be eliminated by performing a chiral gauge transformation (i.e. fixing the gauge for chiral transformations). As a result the vectors A_μ and H_μ can be eliminated algebraically.^{4,5}

The scalar potential, after elimination of the auxiliary fields is

$$V = e^{-4\sigma} f_i^{-1} F_{ij}^- \bar{f}_j^- + \frac{1}{2} e^{-4\sigma} \text{Re } f_{AB}^{-1} F_i (T_A z)_i F_j (T_B z)_j. \tag{3.7}$$

However, F_{ij}^{-} is no longer the kinetic matrix for the scalars and hence need not be positive definite.⁵ Actually the positivity of the scalar kinetic terms requires that

$$F_{ij}^{-1} + 3e^{-2\sigma}(nz)_i (nz)_j^{-} \quad (3.8)$$

be a positive definite matrix. These results will be derived in detail in Section 5.4. However, the validity of (3.8) can be seen directly by recalling that the negative contribution to the scalar potential in $N = 1$ supergravity¹⁶ is $3m_{3/2}^2$, and here the gravitino mass is given by $m_{3/2}^2 = e^{-6\sigma}|f|^2$.

5.4. Equivalence to Standard Supergravity.

Matter couplings in $N = 1$ supergravity theories are specified by two functions, the Kähler potential K and the superpotential f .¹⁶ In the presence of Yang-Mills theories, we also have another function f_{AB} . The bosonic Lagrangian in terms of these is

$$e'^{-1}\mathcal{L}_{BOS} = \frac{1}{2} R' - K'_{i,j} Dz'_i \cdot Dz'_j - \frac{1}{4} \text{Re } f'_{AB} F_{ab}^A F_{ab}^B + \frac{1}{4} \text{Im } f'_{AB} F_{ab}^A * F_{ab}^B - V \quad (4.1)$$

and the scalar potential is

$$\begin{aligned} V &= e^{G'} [G'_{\bar{k}}, G'^{-1}_{k',k}, G'_{k'} - 3] + \frac{1}{2} \text{Re } f'^{-1}_{AB} D_A D_B = \\ &= e^{K'} [(\bar{f}'_{\bar{k}'} + K'_{\bar{k}'} \bar{f}') K'^{-1}_{k',k}, (f'_{k'} + K'_{k'} f') - 3|f'|^2] \\ &+ \frac{1}{2} \text{Re } f'^{-1}_{AB} K'_{i'} (T_A z')_{i'} K'_{j'} (T_B z')_{j'} \end{aligned} \quad (4.2)$$

where $G'(z', \bar{z}') = K'(z', \bar{z}') + \ln|f'(z')|^2$. The negative contribution to the scalar potential is proportional to the gravitino mass squared, $m_{3/2}^2 = e^{G'}$.

We now explain how to cast the Lagrangian of Section 5.3 in standard supergravity form, i.e. the explicit relation between K and F . In the old minimal supergravity formalism, the relation between the D term $\left(\text{i.e. } \frac{1}{2} [\phi]_D \right)$ and the Kähler potential is $K = -3 \ln \left(-\frac{1}{3} \phi \right)$.¹⁶ As ϕ is an arbitrary function, so is K . However, in new minimal supergravity, F is subject to the constraint 5.3.3. The Kähler potential can be shown to satisfy a similar constraint. Hence, with new minimal supergravity, we can only write Lagrangians with the Kähler potential and superpotential subject to the constraint 5.3.2 and 5.3.3, or in other words we can only couple R invariant Lagrangians.^{5,16}

To put the Einstein term in canonical form, we have to perform the following Weyl scaling on the Lagrangian 5.3.4,

$$e'^a_{\mu} = e^{\sigma} e^a_{\mu} . \quad (4.3)$$

As is well known from superconformal tensor calculus,² the Weyl and chiral weights of chiral multiplets are related as $w = 3n$. Therefore, we need to perform a Weyl rescaling on z as well. However, as σ itself depends on z , it is not clear what the transformation should be. We make the ansatz

$$z'_i = e^{-3n_i(\sigma + (1/6)K)} z_i \quad (4.4)$$

where K is for now an arbitrary function of z (it will turn out to be the Kähler potential). The transformation (4.4) is nonholomorphic, nevertheless we can define a new supersymmetry, $\delta'_Q = \delta_Q + \delta_A$, such that $\delta'_Q z' = \frac{1}{2} \epsilon' \chi'$.

After eliminating the auxiliary fields we find that the $\partial z' \partial z'$ terms

cancel if we choose

$$K = F + 3e^{2\sigma} - 6\sigma - 3 . \quad (4.5)$$

In fact the Lagrangian then reduces to the standard supergravity form (4.1), (4.2) with the function K in (4.5) as the Kähler potential. Note that (4.5) gives K as a function of z . We then have to solve the algebraic equation (4.4) to determine z as a function of z' and substitute it in K to get the Kähler potential in terms of its natural variable z' (e.g. the kinetic term for the scalar is $K'_{i,j} \partial z'_i \cdot \partial \bar{z}'_{\bar{j}}$ with

$$K'_{i,j} = \frac{\partial}{\partial z'_i} \frac{\partial}{\partial \bar{z}'_{\bar{j}}} K'(z', \bar{z}') \quad \text{and} \quad K'(z', \bar{z}') = K(z, \bar{z}) .$$

We give a few identities which are needed to explicitly verify the above results

$$e^{2\sigma} = 1 - (nz)_j F_j = \frac{1}{1 + (nz')_j K'_{j'}} \quad (4.6)$$

and

$$e^{2\sigma} + 3(nz)_i F_{ij} (\bar{n}\bar{z})_{\bar{j}} = \frac{1}{e^{-2\sigma} - 3e^{2\sigma} (nz')_i K'_{i,j} (\bar{n}\bar{z}')_{\bar{j}}}$$

$$F_i = e^{2\sigma} e^{-3n_i(\sigma + (1/6)K)} K'_{i'}$$

$$F_{ij}^{-1} + 3e^{-2\sigma} (nz)_i (\bar{n}\bar{z})_{\bar{j}} = e^{-2\sigma} e^{3n_i(\sigma + (1/6)K)} \left\{ \delta_{ik} + (nz')_i K'_{k'} \right\}$$

$$\begin{aligned}
 & \times K'^{-1}_{k'k'} \left\{ \delta_{\bar{k}}^{\bar{j}} + K'_{\bar{k}'} (n\bar{z}')_{\bar{j}} \right\} e^{3n_j(\sigma+(1/6)K)} \\
 f(z) &= e^{3(\sigma+(1/6)K)} f'(z') \\
 f_i(z) &= e^{3(\sigma+(1/6)K)} e^{-3n_i(\sigma+(1/6)K)} f'_i(z') . \tag{4.7}
 \end{aligned}$$

Let us explicitly state how to go from F to K (or vice versa). We first calculate σ by (4.6). Then K (or F) is determined by (4.5). Finally, one solves for the relation between z and z' by (4.4) and substitutes it in K (or F). The equation (4.4) can always be solved. To see this, let⁵

$$\begin{aligned}
 z'_i &= (z'_0)^{3n_i} \tilde{z}_i \\
 z_i &= (z_0)^{3n_i} \tilde{z}_i . \tag{4.8}
 \end{aligned}$$

Then Eq. (4.4) reduces to a single algebraic equation between $(\bar{z}'_0, z'_0)$ and $(\bar{z}_0, z_0)$ which can always be solved.

We remark, in passing, that the Kähler transformations $K'(z', \bar{z}') \rightarrow K'(z', \bar{z}') + g'(z') + \bar{g}'(\bar{z}')$ and $f'(z') \rightarrow e^{-g'(z')} f'(z')$ correspond to $F(z, \bar{z}) \rightarrow F(z, \bar{z}) + g(z) + \bar{g}(\bar{z})$ and $f(z) \rightarrow f(z)$. Here $g(z)$ has chiral weight zero. The latter transformation is an invariance, as in the new minimal formalism $[g(\Sigma)]_D = 0$, and the corresponding field strength is $W_F = -\frac{1}{4} \bar{\nabla}^2 \nabla F$.

There are two examples where the Weyl scaling on the scalars are not needed. The first is when all fields have zero chiral weight. Then the Kähler potential $K = F$. However, in this case we cannot have a superpotential. The other example is $F = \Sigma_0 \Sigma_1 \tilde{F}(\Sigma)$ with Σ_0, Σ_1 having chiral

weights $n_0 = 1/3$, $n_1 = 0$ respectively. In this case, the Kähler potential is given by $K = -3 \ln \left(1 - \frac{1}{3} F \right)$.

We mentioned before that new minimal supergravity can only be used to write R invariant Lagrangians. However, one can explicitly break the chiral symmetry to write arbitrary Lagrangians. Let us consider the Lagrangian $\mathcal{L} = -\frac{1}{2} [\Sigma_0 \bar{\Sigma}_0]_D$, where Σ_0 has $n = 1/3$. Upon eliminating the auxiliary fields and assuming $\langle z_0 \rangle \neq 0$ (as the Weyl rescaling factors are proportional to $\bar{z}_0 z_0$), we get the Poincaré supergravity Lagrangian. Similarly, the

$$\mathcal{L} = \frac{1}{2} [\Sigma_0 \bar{\Sigma}_0 \phi(\Sigma, \bar{\Sigma})]_D + 2 \operatorname{Re}[\Sigma_0^3 f(\Sigma)]_F - \frac{1}{2} \operatorname{Re}[f_{AB}(\Sigma) W^A W^B]_F$$

where Σ have $n = 0$; upon elimination of all auxiliary fields and Weyl scalings, gives the general $N = 1$ supergravity Lagrangians.

Finally, let us give the effective Lagrangian for the superstring¹⁷ in the new minimal formalism. As the superpotential is cubic in C , we assign C to have chiral weight $1/3$ and S, T are assigned chiral weight zero. The Kähler potential is

$$K' = -3 \ln(T' + \bar{T}' - \bar{C}' C')(S' + \bar{S}')^{1/3} . \quad (4.9)$$

We then use (4.6) to determine σ , and substitute it in (4.4) to get the relation between the primed and unprimed fields,

$$C = \frac{C'}{\sqrt{(T' + \bar{T}')(S' + \bar{S}')^{1/3}}} \quad (4.10)$$

$$T = T' , \quad S = S' .$$

The function F is given by (4.5),

$$F = -3 \ln(T+\bar{T})(S+\bar{S})^{1/3} + 3\bar{C}C(S+\bar{S})^{1/3} . \quad (4.11)$$

The effective superstring Lagrangian¹⁷ can then be written as

$$\begin{aligned} \mathcal{L}_S = & \frac{1}{2} [F]_D + 2 \operatorname{Re}[d_{ijk} C^i C^j C^k]_F - [V_p]_D \\ & - \frac{1}{2} \operatorname{Re}[SW^2]_F + \frac{\alpha}{2} \operatorname{Re}[S \, iT_{ab} {}^*T_{ab}]_F . \end{aligned} \quad (4.12)$$

The last two terms, needed for the anomaly cancellation,¹³ will be discussed in the next subsection.

5.5. Topological Invariants and Chern-Simons Multiplets

The space-time topological invariants (i.e. Chern, Pontrjagin and Euler classes) can all be supersymmetrized using the chiral field strength multiplets. The imaginary part of the F density for the field strength multiplet squared is a total derivative (see 5.1.3),

$$\operatorname{Im}[W^2]_F (dx) = -G^2 + \frac{1}{2} d^*(\bar{\lambda}\gamma_5\lambda) \quad (5.1)$$

with

$$\begin{aligned} G^2 &= d \left\{ v \left(G - \frac{1}{3} v^2 \right) \right\} \\ &= d\Omega . \end{aligned} \quad (5.2)$$

The Yang-Mills field strength multiplet gives the Chern class. The Riemann multiplet (i.e. the field strength of the Lorentz connection multiplet) gives the Hirzebruch signature.

The formula (5.1) can be generalized by replacing W^2 by $f_{AB} W^A W^B$, with f_{AB} a real, constant, gauge covariant matrix. Using this generalization for the Riemann multiplet, we get the Gauss-Bonnet combination as $\text{Im}[T_{ab}^* T_{ab}]_F$.^{6,18} Actually, both the real and imaginary parts of $[T_{ab}^* T_{ab}]_F$ are total derivatives.

This can be seen using the Bianchi identities (4.1.6 implies that $[(T_{ab} - i^* T_{ab})^2]_F \sim [E_a^2]_D + \text{S.T.}$) or directly in components. Incidentally, this implies that the Weyl multiplet also satisfies $\text{Im}[W_{ab}^2] = \text{S.T.}$ (see 4.1.4). The components of the Gauss-Bonnet multiplet are

$$\begin{aligned} T_{ab}^* T_{ab} = & \psi_{ab}^* \psi_{ab} + \theta \{ i \sigma_{cd} \hat{R}_{cdab}^+ - 2i \hat{F}_{ab}^+ \}^* \psi_{ab} \\ & + \theta^2 \left\{ 2\psi_{ab} i \hat{D}^+ \psi_{ab}^* - \hat{F}_{ab}^+ \hat{F}_{ab}^+ + \frac{1}{2} \hat{R}_{abcd}^+ \hat{R}_{abcd}^+ + \frac{1}{2} \hat{R}_{abcd}^+ \hat{R}_{abcd}^{+*} \right\}. \end{aligned} \quad (5.3)$$

It can be expressed in terms of the irreducible multiplets as

$$i T_{ab}^* T_{ab} = W_{ab} W_{ab} - \frac{4}{3} T^2 - 2V^2 (E_a E_a) .$$

The F density is given by

$$\begin{aligned} [T_{ab}^* T_{ab}]_F (dx) = & -R_{ab}^+ R_{ab}^+ - i R_{ab}^+ \hat{R}_{ab}^+ + 2F^{+2} + \\ & - d \left\{ \overline{8D^+ \psi} \frac{1-\gamma_5}{2} \left(r + \frac{1}{2} \gamma \gamma \cdot r + \mathcal{H} \bar{\psi} \right) \right\} \\ = & d \left\{ \omega_{ab}^+ \left(R^+ - \frac{1}{3} \omega^{+2} \right)_{ba} + 2A^+ F^+ \right\} + \end{aligned}$$

$$\begin{aligned}
& - \text{id} \left\{ \frac{1}{2} \epsilon_{abcd} \omega_{ab}^+ \left(R^+ - \frac{1}{3} \omega^{+2} \right)_{cd} \right\} \\
& - d \left\{ \overline{4D^+ \psi} \left(r + \frac{1}{2} \gamma \gamma \cdot r - \gamma_5 \mathcal{H} \psi \right) \right\} \\
& - \text{id} \left\{ \overline{4D^+ \psi} i \gamma_5 \left(r + \frac{1}{2} \gamma \gamma \cdot r - \gamma_5 \mathcal{H} \psi \right) \right\} .
\end{aligned} \tag{5.4}$$

The space-time Chern-Simons forms, (5.2), can also be extended to real Chern-Simons multiplets by¹²

$$W^2 = 2\pi(\Omega) . \tag{5.5}$$

If we let $W^2 = (z, \chi, h)$, then Ω is

$$\begin{aligned}
\Omega = & \left(C^\Omega, \chi^\Omega, \text{Re } z, -\text{Im } z, v_a^\Omega, -\frac{1}{2} \gamma_5 \chi - \gamma_5 (\hat{D}^- - 2i\gamma_5 \mathcal{H}) \chi^\Omega, \right. \\
& \left. \text{Re } h - \hat{D} C^\Omega + 2\mathcal{H} \cdot V^\Omega + \frac{1}{4} \bar{\psi}_{ab} \sigma_{ab} \chi^\Omega \right)
\end{aligned} \tag{5.6}$$

with the constraint on the vector

$$\hat{D} \cdot (V^\Omega + 2C^\Omega \mathcal{H}) + \frac{1}{4} \bar{\psi}_{ab} \gamma_5 \sigma_{ab} \chi^\Omega = -\text{Im } h . \tag{5.7}$$

As Ω is determined up to a linear multiplet, we can consider $\Omega - L_0$ and choose a gauge where $C^\Omega = C^{L_0}$, $\chi^\Omega = \chi^{L_0}$.¹² Then

$$\Omega - L_0 = (0, 0, \text{Re } z, -\text{Im } z, v_a, -\frac{1}{2} \gamma_5 \chi + \frac{1}{2} \gamma_a \gamma_5 (\gamma_5 \not{x} + \text{Re } z + i\gamma_5 \text{Im } z) \psi_a ,$$

$$\text{Re } h + 2H \cdot v + \frac{1}{4} \bar{\psi}_a (\gamma_5 \not{v} + \text{Re } z + i\gamma_5 \text{Im } z) \psi_a \quad (5.8)$$

and the constraint is explicitly solved as

$$d^*v = - \text{Im}[W^2]_F (dx)$$

$$v_a = v_a^\Omega - v_a^{L_0} . \quad (5.9)$$

Here we have used the relation $d^*v = \partial_\mu (ev^\mu) (dx) = (D_a v^a - T_{ab}^a v^b) e(dx)$.

These Chern-Simons multiplets are needed for the consistent coupling of linear multiplets.¹² The Green-Schwarz mechanism to cancel anomalies requires that linear multiplets should appear in the combination $L + \Omega$.^{12,13} In terms of the duality transformed theories ($L \rightarrow S$), this implies the coupling $\text{Re}[SW^2]_F$. For example, consider the term $\text{Re}[S i T_{ab}^* T_{ab}]$ in the superstring Lagrangian (5.4.12). In components, this implies the coupling of $\partial_\mu \text{Im } S$ with the Lorentz Chern-Simons form (see (5.4)). If we now perform a duality transformation in components, this term modifies the field strength of the antisymmetric tensor by the addition of the Lorentz Chern-Simons form. Also, note that this term contains the coupling of $\text{Re } S$ with the Gauss-Bonnet integrand (see (5.4)).

The superstring lagrangian in terms of a linear multiplet is obtained by replacing $S + \bar{S}$ by U and adding the lagrange multiplier $1/4[U L]_D$. Eliminating U we then get

$$\begin{aligned} \mathcal{L}_S = & 1/2[-3 \ln(T + \bar{T}) - 3 \ln g(x) + 3x g(x) + \ln \hat{L} - V_P]_D \\ & + 2\text{Re} [d_{ijk} C_i C_j C_k]_F \end{aligned} \quad (5.10)$$

with $x = 2/3 \, C\bar{C} \, e^{-1/3 \ln \hat{L}}$

$$g(x) = (1 + (1+x^3)^{1/2})^{1/3} - (-1 + (1+x^3)^{1/2})^{1/3} \quad (5.11)$$

and $\hat{L} = L - \Omega + \alpha \Omega'$

$$W^2 = 2 \, \Pi(\Omega)$$

$$iT_{ab}^* T_{ab} = 2 \, \Pi(\Omega'). \quad (5.12)$$

6. Conclusions.

In the present paper, we have derived the tensor calculus of new minimal Poincaré supergravity and constructed general matter couplings. This investigation was motivated by a renewed interest of new minimal supergravity as the natural framework to describe the dynamics of 4D heterotic superstrings. The results derived here provide the necessary tools to construct any higher order coupling in the gravitational and Yang-Mills curvatures.

Appendix. Conventions and Useful Formulae.

We use a Minkowski metric, $g_{ab} = (-+++)$, $\epsilon_{0123} = +1$. The Dirac matrix conventions are $\{\gamma_a, \gamma_b\} = -2g_{ab}$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma_{ab} = \frac{i}{2}(\gamma_a\gamma_b - \gamma_b\gamma_a)$, ${}^*\sigma_{ab} = \frac{1}{2}\epsilon_{abcd}\sigma^{cd} = i\gamma_5\sigma_{ab}$, $\bar{\psi} = \psi C$. Left- and right-handed components are defined by

$$\psi = \frac{1-\gamma_5}{2} \psi_L + \frac{1+\gamma_5}{2} \psi_R, \quad \bar{\psi} = \bar{\psi}_L \frac{1-\gamma_5}{2} + \bar{\psi}_R \frac{1+\gamma_5}{2}.$$

In a Majorana representation $C = \gamma^0$ and the Majorana condition is $\psi = \psi^*$.

The two-component spinor formalism is derived from the following chiral representation of the Dirac matrices,

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \gamma_a = \begin{pmatrix} 0 & \bar{\sigma}_a \\ \sigma_a & 0 \end{pmatrix}, \quad \sigma_a = (1, \sigma_k), \quad \sigma_{ab} = \begin{pmatrix} \sigma_{ab} & 0 \\ 0 & \bar{\sigma}_{ab} \end{pmatrix},$$

$$\psi = \begin{pmatrix} \psi \\ \bar{\psi} \end{pmatrix} \quad \text{and} \quad \bar{\psi} = - \begin{pmatrix} \bar{\psi} \\ \psi \end{pmatrix}.$$

Some useful Dirac algebra identities are

$$\gamma_a \gamma_b \gamma_c = g_{ac} \gamma_b - g_{ab} \gamma_c - g_{cb} \gamma_a - \epsilon_{abcd} i \gamma_5 \gamma_d$$

$$\sigma_{ab} \sigma_{cd} = g_{ac} g_{bd} - g_{ad} g_{bc} - \epsilon_{abcd} i \gamma_5 + i(g_{ac} \sigma_{bd} - g_{ad} \sigma_{bc} + g_{bd} \sigma_{ac} - g_{bc} \sigma_{ad})$$

and the Fierz identity for anticommuting spinors,

$$\begin{aligned} \psi \cdot \bar{\psi}' = & -\frac{1}{4} \left\{ (\bar{\psi}' \psi) + \gamma_5 (\bar{\psi}' \gamma_5 \psi) - \gamma_a (\bar{\psi}' \gamma_a \psi) + \gamma_5 \gamma_a (\bar{\psi}' \gamma_5 \gamma_a \psi) \right. \\ & \left. + \frac{1}{2} \sigma_{ab} (\bar{\psi}' \sigma_{ab} \psi) \right\} \end{aligned}$$

or in two components,

$$\psi \cdot \psi' = -\frac{1}{2} (\psi \psi') + \frac{1}{8} \sigma_{ab} (\psi \sigma_{ab} \psi')$$

$$\psi \cdot \bar{\psi}' = -\frac{1}{2} \sigma_a (\psi \sigma_a \bar{\psi}') .$$

The gravitational curvature and torsion are given by

$$R^a_b = d\omega^a_b + \omega^a_c \omega^c_b = \frac{1}{2} R^a_{b\mu\nu} dx^\mu dx^\nu ,$$

$$T^a = de^a + \omega^a_b e^b = \frac{1}{2} T^a_{\mu\nu} dx^\mu dx^\nu$$

with

$$\omega_{ab} = -\omega_{ba} = \omega_{a\lambda b} dx^\lambda .$$

The Ricci, scalar curvature, Einstein and Weyl tensors are

$$R_{ab} = R^d_{adb} , \quad R = R^d_d , \quad E_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = {}^*R^{*d}_{bda} ,$$

$$W_{abcd} = \frac{1}{2} (R_{abcd} - {}^*R^*_{cdab}) - \frac{1}{12} (g_{ac}g_{bd} - g_{ad}g_{bc}) R .$$

The connection is explicitly solved as $\omega_{abc} = \omega_{abc}^{(e)} + \frac{1}{2} (T_{abc} + T_{bac} + T_{cab})$.

Under a variation of the vierbein and torsion, the change in the connection is given by

$$\delta\omega_{apb} = \frac{1}{2} (x_{apb} + x_{\mu ab} + x_{ba\mu})$$

with

$$x^a_{\mu\nu} = -(D_\mu \delta e^a_\nu - D_\nu \delta e^a_\mu) + \delta T^a_{\mu\nu}.$$

Note that D_μ only includes connections for the tangent space indices whereas ∇_μ includes connection for both the tangent and curved space indices.

The gravitino curvature is given by $\psi_{\mu\nu} = D_\mu^+ \psi_\nu - D_\nu^+ \psi_\mu$ and $\psi_{ab} = e^\mu_a e^\nu_b \psi_{\mu\nu}$. The Rarita-Schwinger operator is $r_a = \frac{1}{2} \gamma_5 \gamma_b^* \psi_{ba}$. Some useful identities are

$$\phi_a = r_a + \frac{1}{3} \gamma_a \gamma \cdot r, \quad \frac{1}{2} \gamma_b \psi_{ba} = -\frac{1}{2} \gamma_b \gamma_a r_b = r_a + \frac{1}{2} \gamma_a \gamma \cdot r,$$

$$\gamma_d \sigma_{ab} r_d = \psi_{ab} + i \gamma_5^* \psi_{ab}, \quad \sigma_{ab} \phi_b = i r_a, \quad \gamma \cdot r = \frac{1}{2} \sigma_{ab} \psi_{ab} = -3 \gamma \cdot \phi.$$

Finally, to convert our formulae to the Euclidean conventions of Kugo and Uehara or van Nieuwenhuizen,² the following replacements have to be made:

$$\epsilon_{abcd} \rightarrow i \epsilon_{abcd}, \quad \gamma_a \rightarrow -i \gamma_a, \quad \gamma_5 \rightarrow -\gamma_5, \quad \sigma_{ab} \rightarrow -2i \sigma_{ab}, \quad C \rightarrow -C,$$

$$e^a_\mu \rightarrow e^a_\mu, \quad \psi_\mu \rightarrow -\psi_\mu, \quad \epsilon \rightarrow \epsilon, \quad \omega_{\mu\nu b} \rightarrow \omega_{\mu\nu b}, \quad S_{ab} \rightarrow -i \Sigma_{ab},$$

$$A_\mu \rightarrow -\frac{3}{2} A_\mu, \quad n \rightarrow \frac{1}{3} n, \quad B_{\mu\nu} \rightarrow -i a^0_{\mu\nu}, \quad H_\mu \rightarrow -\frac{1}{2} B^0_\mu.$$

For a vector multiplet

$$C \rightarrow C, \quad \chi \rightarrow i \gamma_5 \mathcal{X}, \quad H \rightarrow -\mathcal{H}, \quad K \rightarrow -\mathcal{K}, \quad V_a \rightarrow -B_a, \quad \lambda \rightarrow \Lambda, \quad D \rightarrow D.$$

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