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HETEROTIC SUPERSTRINGS IN AUXILIARY FIELD BACKGROUND *)
AND EXTENDED SPACE-TIME SUPERSYMMETRY

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ABSTRACT

We introduce the superstring vertices of the auxiliary connections of extended supergravity and elucidate the relation between the internal symmetry of the space-time superconformal algebra and the Kac-Moody symmetry of the 2d superconformal algebra. The microscopic computation of effective Lagrangians is outlined: it turns out that the low-energy effective Lagrangians for 4D heterotic superstrings are of a standard supergravity form.

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It is well known that at low energy the dynamics of any superstring model preserving at least one space-time supersymmetry is effectively described by some supergravity theory [1]. In particular, this is true for the recently constructed heterotic superstrings in four dimensions [2].

Then a natural question arises: are the resulting supergravity Lagrangians of the standard known form [3]? In more technical terms, we ask whether the relevant supergravities are obtainable from the usual rules of the tensor calculus or not [4]. This question is not a rhetorical one, since, in the extended case, exceptional supergravities are known to exist [5] and a complete classification is missing. Moreover, the computation of the low-energy effective Lagrangians (see Ref. 6) is based on the assumption that they can be constructed from the standard tensor calculus for all $N \geq 1$.

That this assumption is correct, i.e. that no "exotic" supergravities can emerge as low-energy limits of 4D heterotic superstrings, can be proven directly from stringy microscopic arguments, i.e. by studying the superconformal structure of the underlying 2d σ -model [7], describing the motion of the string on an arbitrary background for the light fields.

The argument goes as follows. The tensor calculus for extended supergravity contains--besides the physical propagating fields--a set of dummy fields [4] (auxiliary fields, superconformal connections, compensators, etc.) whose symmetry properties identify the tensor calculus as well as help to determine the structure of the possible Lagrangians. In particular, they fix the geometry of the scalars' σ -model manifold. Now, it turns out that (in the covariant formulation) the structure of the underlying 2d superconformal algebra is such that one can introduce explicit vertex operators for these dummy fields; they differ from the usual vertices for physical states only because they are not BRS-invariant, the 2d version of the fact that they

correspond to non-propagating space-time degrees of freedom.

For the $N = 1$ case, these auxiliary vertices were constructed in Refs. 8 and 9 for the matter multiplet, and for the supergravity multiplet in Ref. 10.

The purpose of the present letter is two-fold. First, we shall give the basics of the auxiliary field vertices for the extended case and also explain their relationship with the extended tensor calculus. Then we shall give some hints on our general strategy to compute the full **nonlinear** effective Lagrangian from 2d superconformal considerations [the extended auxiliary vertices are also useful to simplify the computation for the $N = 1$ case]. We shall present some illustrative examples below.

The problem of constructing the light fields' effective Lagrangian (at the full **nonlinear** level) is then reduced to a quite elementary exercise in 2d superconformal theory.

Our basic tool in constructing the auxiliary vertices will be the remarkable theorem [11-9] relating the number of space-time supersymmetries to the extension of the underlying 2d superconformal algebra. We state this theorem in a slightly different way to emphasize its connections with the Haag-Lopuszanski Sohnius theorem [12].

Theorem. Let G be the internal part of the 4D superconformal group [12] ($U(N)$ for $N \neq 4$, $SU(4)$ for $N = 4$). If a 4D heterotic string has N unbroken supersymmetries, at the world sheet level, we have (at least) a left-moving G Kac-Moody algebra, corresponding to a symmetry which does not commute with 2d SUSY (i.e. the KM currents are not last components of SKM multiplets).

"Macroscopic Proof." In 4D SUGRA G is gauged by auxiliary-field connections. Since G is a "space-time symmetry," its generators give a KM algebra at the 2d

level. Because the corresponding connections are not physically propagating fields, the corresponding vertices should not be BRS invariant, that is, the corresponding KM currents are not the last components of a SKM algebra.

Remark. The fact that in the $N = 2$ case the group G is not simple is equivalent to say that the space-time scalars' manifold is a product manifold of the form $(\text{Kähler}) \otimes (\text{Quaternionic Kähler})$.

This theorem allows us to construct vertices for the auxiliary fields with the structure predicted by the tensor calculus. However, in specific models, it may happen that we have actually **more** auxiliary vertices than that! This strange phenomenon can be understood from supergravity considerations alone. Consider, e.g., the set of all $N = 2$ SUGRA's. A special subset is given by those obtainable by consistent truncation of $N = 4$ SUGRA's. For this subset we have actually two possible tensor calculi: the $N = 2$ one and the consistently truncated $N = 4$ one. This last calculus has fewer auxiliary fields in the matter sector (since less couplings are possible than in a general $(N = 2)$ theory) but has more of them in the gravity sector (since the scalars' and vectors' kinetic terms have more geometrical structure than in a generic $N = 2$ SUGRA). This explains why--in **specific** models--we find more auxiliary-field vertices than predicted by either 4D or 2d superconformal invariance: indeed, we find the vertices corresponding to **both** the possible tensor calculi. This observation enormously simplifies the microscopic computation of the effective Lagrangian in many complicated situations.

N = 4 Auxiliary Vertices for the Supergravity Sector.

In $N = 4$ supergravity the auxiliary fields [13] of the gravitational (more precisely, the Weyl) multiplet are known, but not those of the matter multiplet. The situation at the string level is more promising since one can get a **partial** set of auxiliary fields in the matter case too. However, since these last are not needed for the applications, we shall limit ourselves to the supergravity multiplet.

The auxiliary fields which are relevant for us are:^{*} a_μ , γ_μ^{ij} , and $T_{\mu\nu}^{ij}$ ($i, j = 1, \dots, 4$) with $\gamma_\mu^{ij} \in \text{Lie } \text{SU}(4)$ and $T_{\mu\nu}^{ij} = -T_{\nu\mu}^{ij} = -T_{\mu\nu}^{ji}$. Not all the auxiliary components of the Weyl multiplet have a vertex operator in the NSR formalism; this should be no surprise to SUGRA experts. Indeed (neglecting higher derivative terms), the other gravitational auxiliary fields D^{ij} and χ_k^{ij} are Lagrange multipliers enforcing geometrical constraints on the kinetic terms which, in the string case, are automatically satisfied by the very structure of the 2d σ -model (i.e. by 2d duality). The corresponding vertices would then have all correlation function vanishing. The last auxiliary field E_{ij} simply vanishes on **all** bosonic background (probably it has a vertex, but this is not relevant for our purposes).

The auxiliary field vertices are defined by their SUSY transformation properties. We can see SUSY transformations in two different ways. From the passive (or dual) point of view, SUSY acts on the vertices themselves. As explained in Ref. 10, in this approach one identifies the auxiliary vertices by showing that certain Bianchi identities hold. Instead, the active SUSY transformations act directly on the background fields. The two approaches are

^{*}Actually, a_μ is not an auxiliary field, but a dummy connection for the local $U(1)$ symmetry, which is defined in terms of the other fields by a constraint. However, it is convenient to treat a_μ on the same footing as the $\text{SU}(4)$ connection γ_μ^{ij} , to make more clear the relation with the $N = 1, 2$ cases.

equivalent, but the last one has the advantage of allowing a simpler identification of the fields, so that one can go a step further in abstraction, working in the superconformal rather than Poincaré calculus. In the active case, the procedure is as follows (for illustrative purposes, we work at the linear level). The 2d σ -model Lagrangian reads as (to the first order in the background fields)

$$\mathcal{L} = \mathcal{L}_0 + V^A(z, \bar{z}) \phi_A[x^\mu(z, \bar{z})] , \quad (1)$$

where $\mathcal{L}_0$ is the 2d Lagrangian describing the vacuum configuration (which is a free theory in the $N = 4$ case), $V^A(z, \bar{z})$ are the various vertices and $\phi_A(x^\mu)$ the corresponding background fields. To keep the formulae as simple as we can, we assume a purely bosonic background. Then, in this bosonic background, the SUSY transformation of the fermionic fields $\delta_\epsilon \psi_A$ is defined by

$$Q_{(-1/2)}^\alpha(z) \epsilon_\alpha V_{(0)}^A(w, \bar{w}) \phi_A \sim (z-w)^{-1} V_{(-1/2)}^A(w, \bar{w}) \delta_\epsilon \psi_A + \dots \quad (2)$$

where $Q_{(-1/2)}^\alpha(z)$ is the $(-1/2)$ -picture SUSY generator, and $V_{(-1/2)}^A$ are the fermionic vertices. The main idea is that now the sum $\sum V_{(0)}^A \phi_A$ in Eqs. (1) and (2) also contains the auxiliary field vertices. So we have also an auxiliary field background; of course, the requirement of BRS-invariance of Eq. (1) implies that **on-shell** the auxiliary-background is not independent: it is an algebraic function of the physical fields' background. Thus the situation is just the same as in field theory. A complete discussion of the **microscopic** origin of the auxiliary fields' equations of motion will be published elsewhere; however, below we give some simple applications.

In the $N = 4$ superconformal tensor calculus, the transformation of the gravitino reads

$$\begin{aligned} \delta\psi_\mu^i = & \left(\partial_\mu - \frac{1}{2} \omega_{\mu ab} \sigma^{ab} - \frac{1}{2} \gamma_S^a \partial_\mu \right) \epsilon^i - \gamma_\mu^i \epsilon^j \\ & - \frac{1}{2} \sigma^{ab} T_{ab}^{ij} \gamma_\mu \epsilon_j - \gamma_\mu \eta^i + \text{three-fermion terms} . \end{aligned} \quad (3)$$

Here η^i is the S-supersymmetry parameter [4]; it is a function of ϵ as well as of the fields. Since the η -transformation is rather tricky, in this letter we shall limit ourselves to the identification of the other terms in Eq. (3).

Now, for the auxiliary field background, we write

$$\begin{aligned} V_{(0)}^A \phi_A \Big|_{\text{AUX}} = & \frac{1}{2} \hat{H}^\sigma(x) \epsilon_{\mu\nu\rho\sigma} \psi^\nu \bar{\psi}^\rho \partial x^\mu - 2\gamma_\mu^i(x) J_j^j(z) \bar{\partial} x^\mu \\ & - T_{\mu\nu}^{ij}(x) Z_{ij}(z) \psi^{[\mu} \bar{\partial} x^{\nu]} \end{aligned} \quad (4)$$

where $J_j^i(z)$ are the right-moving $SU(4)$ Kac-Moody currents (not commuting with T_F) whose existence is guaranteed by the unbroken space-time $N = 4$ SUSY. The Abelian SKM currents $Z_{ij}(z) = -Z_{ji}(z)$ are defined by the graviphoton vertices in the (-1) picture

$$e^{-\phi} Z_{ij}(z) \bar{\partial} x^\mu(\bar{z}) e^{ik \cdot x} . \quad (5)$$

Using the $N = 4$ OPE's

$$[Z_{ij} \psi^\mu](z) Q_\alpha^k(w) \sim \frac{1}{2} (z-w)^{-1} (\gamma^\mu)_\alpha^{\hat{\beta}} \delta_{[j}^k Q_{i]\hat{\beta}}(w) + \dots$$

$$J_j^i(z) Q_\alpha^k(w) \sim -\frac{1}{2} (z-w)^{-1} \delta_j^k Q_\alpha^i(w) + \dots \quad (6)$$

it is easy to see that the **stringy** expression for $\delta_\epsilon \psi_\mu^i$, as computed from Eqs. (2) and (4), just reproduces the field theory result, Eq. (3) (up to a η -transformation), with the identification $a_\mu \equiv \hat{H}_\mu$. In particular, the covariant derivative term in Eq. (3) is also reproduced (from the "physical" part of $V_{(0)}^A \phi_A$).

This unambiguously identifies the $N = 4$ auxiliary field vertices as the coefficients of $\hat{H}_\mu$, $\gamma_{\mu j}^i$ and $T_{\mu\nu}^{ij}$ in Eq. (4). The fact that a_μ is not an independent auxiliary field in $N = 4$ SUGRA has an important consequence to be discussed in the applications below.

$N = 2$ Auxiliary Vertices: Gauge Sector.

In $N = 2$ superconformal tensor calculus [16] the SUSY transformation of the gaugino Ω_i^A ($i = 1, 2$; A adjoint rep. index) reads

$$\delta \Omega_i^A = \not{D} X^A \epsilon_i + \frac{1}{2} \sigma^{ab} F_{ab}^A \epsilon_{ij} \epsilon^j - g f_{BC}^A X^{B-C} \epsilon_{ij} \epsilon^j + \frac{1}{2} Y_{ij}^A \epsilon^j + X^A \eta_i, \quad (7)$$

and X^A are the scalar superpartners of the gauge vectors. The auxiliary field $Y_{ij}^A = Y_{ji}^A$ satisfies the reality condition

$$Y^{Aij} \equiv (Y_{ij}^A)^* = \epsilon^{ik} \epsilon^{jl} Y_{kl}^A \quad (8)$$

which means that Y_{ij}^A is a triplet under the $SU(2)$ rotating the two gravitini.

From the general theorem of Ref. 9, we know that $N = 2$ space-time SUSY requires $N = 4$ superconformal invariance at the world sheet level. In particular, we should have a $SU(2) \otimes U(1)$ right-moving Kac-Moody algebra not

commuting with 2d SUSY. Let $J^i_j(z)$ ($i, j = 1, 2$) and $J(z)$ be the corresponding holomorphic currents. They satisfy OPE's similar to Eq. (6).

The auxiliary background is

$$V^A_{(0)}\phi_A|_{AUX} = Y^A_{ij}\epsilon^{ik}J^j_k(z) T_A(\bar{z}) + iC^A(x) J(z) T_A(\bar{z}) \quad (9)$$

where $T_A(\bar{z})$ are the Kac-Moody currents such that the gauge vectors' vertices read

$$e^{-\phi}\psi_\mu T_A(\bar{z}) e^{ik\cdot x} \quad (10)$$

Inserting Eq. (9) in Eq. (2) we get Eq. (7) (modulo η -transformations) provided we identify $iC^A(x)$ with $gf^A_{IJ}X^I\bar{X}^J$. [The reader can easily check that the same identification is obtained if the Lagrangian (1) is required to be BRS-invariant: Again, all other terms in Eq. (7) come from the transformation of the "physical" terms in $V^A\phi_A$.

Then Eq. (9) identifies the vertex of Y^A_{ij} to be

$$\epsilon^{(ik}J^{j)}_k(z) T_A(\bar{z}) e^{ik\cdot x} \quad (11)$$

N = 2 Auxiliary Vertices: Gravitational Sector.

In the $N = 2$ superconformal tensor calculus Eq. (3) still holds, where now $i, j = 1, 2$. The relevant auxiliary background is

$$\begin{aligned} V^A_{(0)}\phi_A|_{AUX} = & \frac{1}{2} \hat{H}^\sigma \epsilon_{\mu\nu\rho\sigma} \psi^\nu \psi^\rho \bar{\partial}x^\mu - 2V_\mu^i{}_j(x) J^j_i(z) \bar{\partial}x^\mu \\ & - \frac{1}{2} B_\mu^{ij}(z) \bar{\partial}x^\mu - T_{[\mu\nu]}^{ij} Z_{ij} \psi^{[\mu} \bar{\partial}x^{\nu]} \quad (12) \end{aligned}$$

(Z_{ij} complex; it comes from the (2,0) sector of the underlying 2d theory [9].)

Again, we see that $\delta_\epsilon \psi_\mu^i$, as computed by Eqs. (2) and (12) (plus the "physical" background) agrees--modulo η -transformations--with the standard tensor calculus result, provided one identifies the auxiliary U(1) connection A_μ with $\hat{H}_\mu + B_\mu$. The fact that the auxiliary field A_μ gets contribution from two different 2d operators, means that the manifold for the vector multiplet scalars is in fact a product manifold. [This is a first example of the special geometrical structures implied by the stringy realization of the SUGRA tensor calculus].

Computations of Low-Energy Lagrangians.

We would like to show how the above results can be used to compute the low energy effective Lagrangians. Consider the simplest case, i.e. $N = 1$. Then we have a vertex for the auxiliary field A_μ which can be used to couple an auxiliary background to the 2d σ -model. The requirement of BRS-invariance of the resulting 2d theory gives the auxiliary field equation of motion (as well as those of the physical fields). If K is the manifold on which the 4D scalars live, a scalar background can be seen as a map

$$S: (4D \text{ space-time}) \rightarrow K .$$

Since the one-form A is not an independent field, its equation of motion should have the general form (locally)

$$A = S^* \alpha + \text{higher-derivative corrections} ,$$

where α is a one-form on K . Then, more or less by the definition of A , K is a

Kähler-Hodge manifold, whose Kähler form can be written (locally) as $\omega = d\alpha$. In particular, the local R-invariance is realized as the pullback to space-time of the Kähler invariance. Thus, the equation for A_μ obtained by requiring BRS-invariance of the underlying 2d theory, gives directly the Kähler metric of the scalars' σ -model. For $N > 1$, the situation is similar; the vector auxiliary fields are a $U(1) \otimes SU(N)$ connection, which (up to higher correction) is the pullback of a "Yang-Mills" connection α on K . Then the Kähler form is replaced by the non-Abelian field strength $\Omega = d\alpha + \alpha \wedge \alpha$. From this Ω one can determine the σ -model metric by a canonical procedure, just as in the Kähler case. This general geometric structure on K is due to the fact that the auxiliary fields have the structure of composite gauge fields, the corresponding gauge-invariance being the local holonomy on K , acting on "flat" indices.

As an illustrative example of the method we compute the Kähler potential of the simplest case, i.e. the Z_3 -invariant truncation of the torus compactification [17] (neglecting the twist sector). The vertex of A_μ in the $N = 1$ case, is just the same as in the $N = 2$ case, Eq. (12). From Eq. (12) we see that we have two terms contributing to the A_μ background; i.e. $\hat{H}_\mu$ and B_μ . From the 2d point of view, they are independent, so we get two separate equations of motion for them. By the above discussion, this means that the Kähler manifold is actually a product manifold. To the lowest order in α' , the BRS-invariance condition on the auxiliary-field background is just the requirement that their background value equals that function of the physical background which makes the 2d Lagrangian (1,0) supersymmetric. In the well known (1,0) invariant σ -model, there is a term

$$\frac{1}{2} \hat{H}_{\mu\nu\rho}(B) \psi^\mu \psi^\nu \bar{\partial} x^\rho, \quad (13)$$

where the 3-form $\hat{H}_{\mu\nu\rho}$ is the (improved) curl of the 2-form $B_{\mu\nu}$ (the axion). Then the equation of motion for $\hat{H}_\mu$ reads

$$\hat{H}^\mu = \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} \hat{H}_{\nu\rho\sigma}(B) + \text{higher order corrections} . \quad (14)$$

If we perform a duality transformation from $B_{\mu\nu}$ to a pseudoscalar b , and also switch on the dilaton (which is the scalar partner of $B_{\mu\nu}$), we get from this sector the Poincaré metric on the $SU(1,1)/U(1)$ manifold. The Kähler manifold has thus a $SU(1,1)/U(1)$ factor. Since this coset space comes from the space-time part of the 2d σ -model, it is **universal**, i.e. $SU(1,1)/U(1)$ is a factor of the scalars' σ -model for all 4D heterotic strings, independently of N . Indeed, the equations for $\hat{H}^\mu$ in the $N = 2$ case, and in the $N = 4$ case (the BRS-invariance of Eqs. (12) and (4)) are equal to Eq. (14). In particular, the $N = 4$ "auxiliary" field a_μ turns out to be the pullback of the standard $U(1)$ connection on $SU(1,1)/U(1)$, which is exactly the solution to the constraint defining it in terms of the physical fields in the $N = 4$ tensor calculus.

The same strategy yields the equation for B_μ . In this case the superconformal $U(1)$ current reads

$$J(z) = \frac{i}{2} \sum_{i=1}^3 \bar{\chi}^i \chi^i , \quad (15)$$

so the B_μ equation of motion is obtained by comparing the background expression

$$\frac{1}{4} B_\mu(x) \bar{\chi}^i \chi^i \partial x^\mu \quad (16)$$

with the general Lagrangian for the (1,0) σ -models. Then, B_μ is identified, up to normalization, with the U(1) part of the connection for the complex internal manifold. As it is well known, this connection has a torsion equal to the internal part of the 3-form $H = dB$. Here one encounters a slight complication in the computation, since this torsion is modified by σ -model loop effects. The only relevant modifications come from the 2d σ -model anomalies [18]. Their effect is two-fold. First of all, in order to cancel them, we add the internal gauge Chern-Simons form to the torsion; and moreover, to put the anomaly in a suitable form we have to add local counterterms;* physically these counterterms are irrelevant since they amount to field redefinition of the background. But for our purposes it is crucial to select the counterterms in such a way that they correspond to holomorphic redefinitions of the chiral fields (this requires us to go a step further in the specification of the σ -model for the auxiliary background, since we need to work beyond the linear level in the background fields). This can be done easily with a bit of care using the explicit form for the 2d σ -model anomaly given by Sen [19]. In an alternative approach [20] where the heterotic fermions are replaced by bosons moving on the gauge group manifold the "anomaly" contribution with the **right** counterterms is already present at the classical level.

Thus, after the Z_3 invariant truncation, we are led to the following identification for the chiral multiplet T

$$T_{\alpha\beta}^- = g_{\alpha\beta}^- + b_{\alpha\beta}^- + \frac{\alpha'}{4} A_\alpha^\mu A_\beta^\mu. \quad (17)$$

*They can be seen as a specification of the regularization.

Computing the $U(1)$ part of the internal connection, one then finds the potential for the second factor in the scalars' Kähler manifold

$$K = -\ln \det(T + \bar{T} - 2\bar{C}C) \quad (18)$$

where C_α^a is the Z_3 invariant part of the "internal" gauge fields A_α^μ .

The above is a very simple example of the method. But it illustrates well two rather general features: (i) the fact that we can introduce more independent auxiliary backgrounds than expected from SUSY consideration alone* (thus getting more equations) gives severe constraints on the geometrical structure of the scalars' σ -model. This observation allows us to compute the effective kinetic terms for complicated models (where no conclusive answer can be obtained by SUGRA arguments alone) by back-of-envelope calculations. (In the interesting models, we have much more independent auxiliary field backgrounds). (ii) The tricky part of the procedure is the correct identification of fields and multiplets, at the nonlinear level. Since the auxiliary fields have a geometrical meaning in 2d, the nonlinear identification is possible, but some care is needed, especially in the presence of anomalies or chiral bosons. This problem is in some sense natural, since also in Kaluza-Klein the main difficulty is to find the field-redefinition putting the Lagrangian in canonical form. In this case, the problem is solved by the existence of "hidden" symmetries; the nice field parametrization is that one which makes the symmetry as manifest as possible. The same idea can be used to identify the convenient field variables in the string case too. In many

*Actually, in this case, the two auxiliary fields $\hat{H}_\mu$ and B_μ are predicted by SUSY considerations, since we have an auxiliary 2-form (see Ref. 10).

interesting models, we have "hidden" symmetries [21] realized as 2d generalized duality rotations. For instance, in the above example, the results of Ref. 21 implies the existence of an $SU(3, 3(n+1))$ (n = number of C_α^a) hidden symmetries [17], broken by gauge interactions. This is enough to obtain Eq. (17) up to a multiplicative factor.

The above ideas, allows us to compute the effective Lagrangians directly for the extended case. Consider, for instance, the $N = 4$ case; the most convenient choice of variables is the one related to the hidden symmetry. Unfortunately, working with these variables, the $SO(6, \dim G)$ partial symmetry is essentially manifest, so we have that the scalars' σ -model metric is the round metric on the coset

$$SO(6, \dim G)/SO(6) \otimes SO(\dim G) , \quad (19)$$

So we do not need to solve the auxiliary equation of motion. [This is just a manifestation of what we claimed in the introduction: the superstring predicts **standard** low-energy supergravities. In fact all standard $N = 4$ supergravities are based on the σ -model in Eq. (18).]

However, it is an interesting exercise to obtain this metric from the equation of motion for v_μ^i . Of course, the above ones are rather simple examples where no computation is actually required. Less trivial situations require some more technical tricks.

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