SYMPOSIUM ON NUCLEON-ANTINUCLEON ANNIHILATIONS

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PROCEEDINGS
edited by
L. Montanet

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ERRATUM FOR

SYMPOSIUM ON NUCLEON-ANTINUCLEON ANNHIILATIONS

Proceedings edited by L. Montanet

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INTRODUCTION

With the exception of the 1969 Lund Conference, the organizers of large international meetings do not, as a rule, set aside a special period of their programme for the discussion of nucleon-antinucleon interactions. Consequently, many of our colleagues in this field were of the opinion that a topical symposium, mainly oriented towards specific properties of such interactions, i.e. the annihilation channels, would be of particular interest. It was therefore with this object in mind that a small symposium took place at Chexbres on March 27th to 29th, 1972.

Among current topics of special interest discussed during the meeting were:

- the attempts made since 1961 - and largely continuing to this day - to use annihilation to study the properties of meson resonances;
- the more recent use of formation experiments;
- the advent of Veneziano amplitudes and their application to several annihilation channels;
- the apparent lack of agreement between certain experimental results on annihilations at rest and their implication as regards the nature of the initial state.

The number of participants in the symposium was limited to 60 to ensure that the atmosphere remained informal. This is, in fact, one of the reasons which has prompted us both to bring out the Proceedings as rapidly as possible and to do all in our power to make them reasonably complete for the benefit of those physicists who were unable to be present. As a consequence, we must beg the reader's indulgence for any errors, typographical or otherwise, that may have crept into the text. We have also tried to reproduce to the best of our ability the comments and discussions that followed the talks and there again we tender our apologies for any important omissions.

The Organizing Committee expresses its appreciation to the rapporteurs who had the difficult task of preparing their talks and compiling their notes. It wishes to thank the secretaries of the sessions for their work in taking the minutes; also the two secretaries, Miss Gill Elvin and Mrs. Monique Burkhalter for their active help and devotion in organizing the symposium and Mrs. Edith Feldmann for typing the final version of the Proceedings. The Organizing Committee gives its thanks to the Swiss Cantonal Authorities as well as to the CERN Direction for their financial support.

The Organizing Committee
PRESENT STATUS ON THE "S-MESON" REGION

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When a signal was first observed in the vicinity of 1930 MeV/c², the words "new boson" were immediately put forward and the object projected onto the A_2 Regge trajectory. As sound as this hypothesis has appeared, we shall try to avoid confusing analogous structures observed afterwards, if they display clear discrepancies.

In the frame of this brief description of the present situation, I shall restrict to the main aspects of the experimental results, because deeper considerations of the problems involved (quality and validity of the methods) would lead us much too far. (They are tackled in discussions which followed the speech).

First, to fix ideas let us, perhaps arbitrarily, limit the S region by the mass interval 1920-1975 MeV/c². Results have been presented from several kinds of experiments.

1. MISSING MASS EXPERIMENTS.

The first observation was made by Chikovani et al. at CERN (1968)\(^{(1)}\); looking at the X⁻ missing mass of the reaction π⁻ p → p X⁻ (at 12 GeV/c), they obtained a peak at M = 1929 ± 14 MeV/c² (only in the 3 charged prongs + possible neutrals), corresponding to a cut : 0.22 < |t| < 0.43 (GeV/c)². The width was compatible with zero, as the mass resolution was 22 MeV/c²; they gave an upper limit of \( \Gamma \leq 35 \) MeV/c². The significance was 4.6 standard deviations.

But soon after, Focacci et al. (ref. 2), varying somewhat the cut (0.22 < |t| < 0.38) reached a 5.5 standard-deviation significance. The rate signal/background was 1/7. These results were guaranteed against instrumental biases by a repetition of the experiment in 3 runs corresponding to different technical conditions.

So, the evidence seems conclusive, but due to the large errors quoted on cross sections and to the poor energy resolution, a somewhat broader structure would not be incompatible with the data. Actually, Cotteron et al. (Paris, ref. 3), reanalysing these data and adding some others at 11.5 GeV/c, obtained a shifted enhancement at 1945 MeV/c², \( \Gamma \leq 20 \) MeV/c² (4.2 standard deviations), with the cut 0.212 < |t| < 0.412 (GeV/c)². Now, they found also evidence in the events with one charged particle (in addition to those with three).

2. BUBBLE CHAMBER PRODUCTION.

In 1968, Boesebeck et al. (using photographs of the 81 cm Sacly B.C. and of the 2 m CERN B.C., ref. 4) pointed out a small shoulder at \( M = 1900 \pm 40 \), \( \Gamma = 216 \pm 105 \) MeV/c², in the \( \pi^+ \pi^0 \) mass distribution of the reaction \( \pi^- p \rightarrow p \pi^+ \pi^0 \) (at 8 GeV/c).

The evidence was poor but, in spite of a large width, apparently not incompatible with the observation of Chikovani et al.
However, in the same reaction at 13.1 GeV/c (82° HBC, at the Stanford Linear Accelerator Center), Kramer et al. (1970, ref. 5) noticed a rather significant \((\pi^+ \pi^0)\) peak at \(M = 1975 \pm 12\), with \(\Gamma < 52\) MeV/c. Moreover, from comparison with results of \(\pi^+ p \rightarrow \pi^+ \pi^\pm n\) (on the basis of isospin invariance), they deduced that the object could only be isodiplet; so \(J^G = I^G = 1^+, P = -1 = (-1)^I\). Let us remark that the mass was too high to agree with the MMSP result; moreover, a very strong discrepancy was noticed for the \((1 \text{ charged} / 3 \text{ charged})\) rate (which was nearly zero for Chikovani et al).

At the same mass, in 1970 also, a structure was pointed out in the \((\rho^- \pi^+ \pi^-)\) distribution of the reaction \(\pi^- p \rightarrow \rho^- \pi^+ \pi^- p (\rho^- + \pi^- \pi^0)\) at 11.2 GeV/c, by Caso et al. (from photographs of the 1.5 m British National B.C. and the 2 m CERN B.C., ref. 6); after exclusion of \(\eta, \omega\) and \(\Delta^\pm\) events, they find:

\[
M = 1973 \pm 15, \quad \Gamma = 80 \text{ MeV/c}^2 \quad (3 \text{ standard deviations}).
\]

3. FORMATION EXPERIMENTS.

\(a)\) Let us first point out the observation of Benvenuti et al. (1971, ref. 7). The cross-section of the reaction \(\bar{p} p \rightarrow K_L^0 K_L^0\), below 800 MeV/c, shows an enhancement at \(M = 1968\) MeV/c with \(\Gamma = 35\) MeV/c. The angular distribution, fitted to an expansion in Legendre polynomials, suggests \(J^{PC} = 1^{--}\); this indication, as the authors said, is in no way conclusive, but in good agreement with previous results of Lørsåd et al. (ref. 8).

\(b)\) \(\bar{p} p\) backdward elastic scattering.

The \(\bar{p} p\) elastic scattering is mainly diffractive, thus resulting in a strong forward peak. So, the backward hemisphere is particularly favorable to a search for a resonant amplitude, although the diffractive part \textit{a priori} may not be negligible at all; this is an acute and delicate problem which has not yet really been solved.

Two studies have been carried out in nearly identical conditions; therefore I find it useful to present them in parallel so as to compare the methods used (30° BNL B.C. and 80 cm Saclay B.C., respectively; momentum intervals: 250 - 740 MeV/c, and 350 - 675 MeV/c soon to be enlarged; at the University of Wisconsin and at Collège de France, Paris).

The first problem which occurs is the very selection of events. In both experiments (ref. 9, 10), most very backward events in fact had been lost after a first scan; this was due to the quite peculiar topologies the scatters display: very short secondary anti-protons, often indistinguishable, yielding an aspect of odd-pronged events (resembling annihilations). A complementary search specially devoted to them thus appeared necessary.

There is no doubt that the correctness of either result is mainly dependent on the quality of the scannings.

Now, the low energy \(\bar{p} p\) data display a wide energy spectrum (dispersion at chamber entrance and strong energy loss along the tracks), which makes the cross section normalization an uneasy task. The Wisconsin group determined a pathlength function from direct measurement of a beam sample (without distinguishing between interacting and not inter-
acting tracks). The CDF group adopted a more complicated technique: they selected the interacting tracks, and from a weighting procedure calculated the interaction probabilities as an energy function. Indeed I know very little about the real procedure used by the former group, all that I will say about the latter method is that it yields very precise results in thin energy bins (and weakly depends on possible biases on the $Q_{\text{tot}}$ values used as input).

Now, both experiments show a peak in the extreme backward direction between $\sim 400$-$600$ MeV/c, which afterwards appears to vanish. As its width is nearly constant and as it does not move forward in the angular distribution, the resonant interpretation is natural and even seems preferable (x). Thus, I think the authors were quite justified to follow fixed $\theta^K$ bin cross sections so as to get an idea of the energy variation. In that manner, a strong energy structure appears, possibly split into two bumps centered at the very masses found in the missing mass experiment. Wisconsin group gives:

\[
\begin{align*}
\text{for one Breit - Wigner:} & \quad M = 1940 \pm 8 \quad \Gamma = 49 \pm 9 \quad \text{MeV/c}^2; \\
\text{for two incoherent B.W.:} & \quad M_1 = 1925 \pm 2.3 \quad \Gamma_1 = 7.6 \pm 3.5 \quad \text{MeV/c}^2; \\
& \quad M_2 = 1947 \pm 16 \quad \Gamma_2 = 52 \pm 10 \quad \text{MeV/c}^2.
\end{align*}
\]

The CDF group finds:

\[
\begin{align*}
\text{for one Breit - Wigner:} & \quad M = 1939 \quad \Gamma = 63 \quad \text{MeV/c}^2 \\
(\text{the errors are meaningless as the shape differs widely from that of a B.W.},) & \\
\text{for two incoherent B.W. (plus a third at 1970 MeV/c^2)xx:} & \quad M_1 = 1929 \pm 4 \quad \Gamma_1 = 36 \pm 3 \quad \text{MeV/c}^2; \\
& \quad M_2 = 1954 \pm 3 \quad \Gamma_2 = 12 \pm 1.5 \quad \text{MeV/c}^2.
\end{align*}
\]

(Measurement errors on effective masses are $\sim 2 \quad \text{MeV/c}^2$, one tenth those of the MM results, ref. 1 - 2).

We first observe that the very backward results of both experiments are rather similar (we use the 1970 Klev data of the Wisconsin group, as they have strongly modified their results since a first publication in 1968 : ref. 11). But in the forward part of the distributions, clear discrepancies appear; however, there is a perfect agreement between the CDF data and those of Conforto et al., studying the complete elastic distributions (but with a much weaker precision in the backward region, even about $\cos \theta = 0$ ; ref. 12).

(x) Bizzari (Roma) has achieved a rather good description of the CDF data in their main features, from a crude boundary - condition model (needing no parameter to be adjusted). But, as it requires very high angular momenta ($\ell \sim 10$), this model cannot be regarded as a diffractive picture at such low c.m. energies.

Besides, if a resonant term occurs in the elastic channel, it must also give a contribution in the inelastic ones (simply related by the elasticity, if the value of J is unique). In that manner, Bizzari et al. looked in the total $p$-$p$ and $p$-$n$ annihilation cross sections; as they observe no big enhancement (simply a shoulder at $\sim 1955$ MeV/c$^2$), they infer that the "S-meson" may not be strongly coupled to the elastic channel. To reach that result, they had to estimate the maximum height of a small differential effect and to assume $\Gamma < 40 \quad \text{MeV/c}^2$; due to statistical and measurement errors, this seems hard, and the test is much less significant for $\Gamma = 50 - 60 \quad \text{MeV/c}^2$ (as observed in elastics).

(xx) The introduction of this third B.W. curve may explain most of the discrepancy between the given values of $\Gamma_2$. 
The Wisconsin group described their backward angular distributions by Legendre polynomial expansions. (This procedure poses problems on simple mathematical grounds, as the fits might yield anything in the forward region: in spite of an allusion, they do not indicate clearly how they have simultaneously tested the corresponding experimental values). They need as far as an $A_6$ term, and observe structures for the $A_2/A_0$ parameters.

The interpretation in orbital momenta is far from being straightforward: all that may be said a priori is that an $L \geq 3$ contribution would therefore be necessary. Moreover, the variation of the $180^\circ$ point thus extrapolated reproduces the peak at $\sim 1925$ MeV/c$^2$ (but the second peak at $\sim 1947$ MeV/c$^2$ vanishes)\(^{(*)}\).

To fit the entire angular distributions to Legendre polynomial expansions, the CDF group did not need to go further than an $A_4$ term. Moreover, their $A_4/A_0$ variations are quite smooth and do not display any structure. So, they simply get $L \geq 2$ (which is not very interesting, as unitarity alone implies $J \geq 2$, according to the high inelastic $p\bar{p}$ cross section). Besides, a partial-wave analysis is impossible because of the complication of the $p\bar{p}$ formalism; that is the reason also why there exists apparently no diffractive model working out the spins correctly (which seems absolutely necessary for the study of fine structures). So the CDF people were led merely to assume a fixed $J^P$ resonant term over a negligible diffractive contribution (this may be justified somewhat by the qualitative arguments previously presented; a study of a unitary limit on the diffractive amplitude is now being performed, but not yet finished). Now, they adapted the analytic form allowed by unitarity, to the region $\cos \Theta \leq -0.6$, for the momentum band $375 - 575$ MeV/c. All unnatural parity hypotheses seem to be excluded (even if C mixtures are considered), except $J^{PC} = 2^{--}$. So is it for $J^P = 1^-$ and $6^+$ among the natural parity ones (where necessarily $C=\bar{P}$); $2^+$, $3^-$ and $5^-$ are still possible\(^{(xx)}\) and an excellent agreement is obtained for $4^+$.

The $4^+$ value would be in agreement with what is expected from the $A_2$ Regge trajectory (but the $3_{1/2}$ state predicted by the quark model does not agree alone with the data), since $I = 1$ if the effect is identified with the M.M. one (and then $\tilde{G} = \bar{s}_+^3$). When the $4^+$ best-fit amplitude is extracted, one finds an equivalent cross section of 5 mb, yielding an elasticity of 1/3 in the $\Lambda_N$ channel (1/6 for $\Lambda\bar{p}$).

CONCLUSION.

There probably exist several objects in the $S$ region. First, convergent indications have been presented about 1970 MeV/c$^2$ (let us notice that the CDF backward data also present a small "accident" at this mass, whereas Wisconsin's display a shoulder). Missing mass studies and elastic scattering both give structures at about 1925 - 1930 and 1945 - 1955 MeV/c$^2$ (which may be a double structure: let us think that Cotteron partly used the Chikovani data to display his 1945 peak).

This agrees with some theoretical ideas suggesting multipole resonances on the $A_2$ trajectory (ref. 13) or quasi-nuclear structures at about $M = 2 \mathrm{MeV}$ (ref. 14).

\(^{(*)}\) The validity of these results has been discussed by Bizerri during the session (see discussion).

\(^{(xx)}\) The fits have been slightly modified since the session, so that the $3^-$ and $5^-$ cases appear less probable.
REFERENCES

(1) G. Chikovani et al. (CERN), P.L., 22, 233 (June 1966).
(2) M.N. Focacci et al (CERN), P.R.L. 17, 890 (October 1966).
(3) Cotteron et al. (Laboratoire de Physique Générale, Paris), quoted in Maglic's report on meson resonances, Lund Conference (June-July 1968).
(6) C. Caso et al. (Genova - Hamburg - Milano - Saclay), L.N.C. 3, 707 (June 1970).
(7) A. Benvenuti et al. (Univ. of Wisconsin, Madison), P.R.L. 27, 283 (August 1971).
(8) B. Lärrstad et al. (Collège de France, Paris, and CERN), quoted in Montanet's report on antiproton reactions, Lund Conference (June-July 1969).
(9) D. Cline et al. (Univ. of Wisconsin, Madison), paper presented at the Kiev Conference (August-September 1970). See also: J. English, thesis presented to the University of Wisconsin (1969).
(10) Ch. d'Andlau et al. (Collège de France, Paris), to be published. Preliminary results presented at the Amsterdam Conference (June-July 1971) and at the Lausanne Symposium are superseded by the present ones.
(11) D. Cline et al. (University of Wisconsin, Madison), P.R.L. 21, 1268 (October 1968).
(12) B. Conforto et al. (CERN - Rome - Trieste), N.C. 54 A, 441 (March 1968).
(13) Chen et al. (Yale Univ., New Haven, Connecticut, and Univ. of Glasgow, Glasgow), P.R.L. 25, 482 (August 1970).
(14) G.D. Dalkarov et al. (Institute for Theoretical and Experimental Physics, Moscow), N.F., B 21, 88 (August 1970).
FIGURE CAPTIONS

Missing mass results

1) \( \pi^- p \rightarrow p X^- \) (12 GeV/c) :
   angular (and mass) distributions of recoil protons in a narrow band of momentum
   \( (0.50 < p_T < 0.63) \); \( \Theta_3 \) is the laboratory angle. The same data in triple bins are
   shown at the side of each distribution (extracted from ref. 1).

2) The same in all decay modes; the "roof" above the distribution represents the
   100% efficiency region of the spectrometer (ref. 1).

3) Cumulated data for the same reaction at 12 GeV/c and 11.5 GeV/c (ref. 3).

Production experiments

4) \( \pi^+ p \rightarrow p \pi^+ \pi^- \) at 13.1 GeV/c :
   \((\pi^+ \pi^-)\) effective-mass spectrum (ref. 5).

5) \( \pi^- p \rightarrow p^- \pi^+ \pi^- p \) \((\pi^- \pi^+ \pi^-)\) at 11.2 GeV/c :
   \((\pi^- \pi^+ \pi^-)\) mass distribution, after exclusion of \( \eta, \omega \) and \( \Delta^{++} \) events (ref. 6).

Formation experiments

6) \( \bar{p} p \rightarrow K^0_S K^0_L \) below 800 MeV/c :
   cross section variation (ref. 7).

   \( \bar{p} p \) backward elastic scattering : Wisconsin results (ref. 9).

7) Angular distributions in 50 MeV/c momentum bins (lab. system).

8) Energy variation of the \(-0.9 > \cos \theta^X \bar{p} p > -1\) bin and of the 180° extrapolated point.

9) Variation of the \((a_2/e)\) parameter ratios from Legendre polynomial expansions of
   the backward angular distributions.

   \( \bar{p} p \) backward elastic scattering : CDF (Paris) results (ref. 10).

10) Angular distributions in 50 MeV/c momentum bins (lab. system).

11) Angular distributions in 20 MeV/c momentum bins (lab. system).

12) Energy variation of the \(-0.9 > \cos \theta^X \bar{p} p > -1\) bin.

13) Energy variation of the \(-0.8 > \cos \theta^X \bar{p} p > -1\) bin.

14) Spin-parity tests on the backward peak \(-0.8 > \cos \theta^X \bar{p} p > -1\) in the whole
    375-575 MeV/c lab. momentum interval.

    For natural parity, \( C = P \).

    For unnatural parity, \( C \) may be \( P \) or \(-P \); only fixed-C curves are presented
    because fits do not get much better for \( C \) mixtures (which are incoherent).
Fig. 1
Fig. 2
Runs: $\pi^- p$ 12 GeV $\{1+3$ charged $\}$ $\pi^- p$ 11.5 GeV $\{1+3$ charged $\}$

$0.212 < t < 0.412$

$1945\text{ MeV}$ $2.002\text{ MeV}$

4.2 s.d. 5.6 s.d

$\downarrow$ 300 $\downarrow$ 500 $\downarrow$

number of events in peaks

1.8 1.9 2.0 2.1 2.2 Mass GeV

Fig. 3
Fig. 5
\textbf{Fig. 6}
Fig. 7
\( \bar{p}p \) ELASTIC SCATTERING

\[ \cos \theta_{\bar{p}p} = 0.9 \text{ to } -1.0 \]

\[ \begin{align*}
\text{mb/sr} \\
\text{MeV/c}
\end{align*} \]

\[ \begin{align*}
\text{2 peak BW } X^2 = 20.9 \\
\text{1 peak BW } X^2 = 30.8
\end{align*} \]

\[ \begin{align*}
\text{MeV/c}
\end{align*} \]

\( \bar{p}p \) ELASTIC SCATTERING EXTRAPOLATED TO 180°

\[ \Gamma \approx 17.6 \text{ MeV} \]
\[ M \approx 1926 \text{ MeV} \]

Fig. 8
$\sigma_n/\sigma_0$ FOR $\bar{p}p$ ELASTIC

$$d\sigma/d\Omega = \sum_n a_n P_n$$

$1.0 \leq \cos \theta_{\bar{p}p} \leq +0.2$

Fig. 9
DIFFERENTIAL CROSS-SECTIONS

Fig. 10
Fig. 11
\( \cos \theta \ \bar{P} = -0.9 \) to \(-1.0\)

C.M. ENERGY (MeV)

\( \sigma \) mb

- Experimental points
  - 3 curves \( \chi^2 = 11 \), conf. level = 96%
  - 2 curves \( \chi^2 = 19.5 \), conf. level = 72%
  - 1 curve \( \chi^2 = 24.9 \), conf. level = 63%

LAB. BEAM MOMENTUM

Fig. 12
$\cos \theta \cdot \vec{P} = -0.8 \text{ to } -1.0$

C.M. ENERGY (MeV)

BREIT WIGNER CURVES ADJUSTMENT

- experimental points
- 3 curves
  \$\chi^2 = 15.4 \text{, conf. level } = 80\%$
- 2 curves
  \$\chi^2 = 26.4 \text{, conf. level } = 33\%$
- 1 curve
  \$\chi^2 = 27.1 \text{, conf. level } = 46\%$

Fig. 13
SPIN - PARITY HYPOTHESES

J =

1 2 3 4 5 6

P = +1

\( \lambda_{\pi}^n \) of fr. (c level)

- 68.9/15 (0.00%)
- 21.6/13 (6.10%)
- 314.0/15 (0.00%)
- 8.6/13 (80.32%)
- 315.0/15 (0.00%)
- 32.6/13 (0.20%)

- 140.7/15 (0.00%)

P = -1

\( \lambda_{\pi}^n \) of fr. (c level)

- 68.9/13 (0.00%)
- 129.3/15 (0.00%)
- 29.5/13 (0.56%)
- 338.0/15 (0.00%)
- 25.6/13 (1.93%)
- 345.0/15 (0.00%)

- 26.4/15 (3.40%)

- 242.7/15 (0.00%)

- 148.5/15 (0.00%)

BACKWARD PEAK (cos \( \theta^* \)) in the momentum interval 375 - 575 MeV

differential cross section in arbitrary scale

For natural parity: \( C \equiv P \)
For unnatural parity: \( C \) may be + or -

--- for \( C = P \) (singlet state)
--- for \( C = P \) (triplet state)

Fig. 14
DISCUSSION AND COMMENTS

Mr. Bizzarri: First I want to give an information. The extrapolation to 180° of the measured pp elastic cross-section made by the Wisconsin Group has been mentioned by Mr. Laloum. This extrapolation contains errors, as can be immediately suspected if the extrapolated cross sections are plotted on the same figure with the measured ones. We have re-done in Rome the Legendre polynomial fit of the Wisconsin data and I am informed by Prof. Cline that also in Wisconsin they have re-done the fit. The new results do not show any structure in the energy dependence of the coefficients a_n of the Legendre polynomials, which vary monotonically with energy. Furthermore, the cross section extrapolated to 180° is now not significantly different from the average cross section measured in the interval -1 ≤ cos θ ≤ -0.9.

I want also to make a brief remark on the effect of diffraction scattering on the backward cross section. The reason why people try to find resonances in the backward scattering is because the diffraction scattering is supposed to be small. This is not easy to estimate: An expression like

\[ e^{-\frac{R^2}{2} |t|} \]

is good only forward, while an expression like \[ J_1^2(R/\sqrt{|t|})/|t| \] is valid only for kR ≫ 1. For the case under consideration we have to estimate backward scattering for energies such that 1/k ≲ R ≲ 1 fm.

To this end, we have taken the old boundary condition model of Feshback and Weisskopf which was applied to the pp interaction by Koba and Takeda 14 years ago. We assume that the antiproton-proton annihilation region is a totally absorbing sphere radius R_0. At the boundary of this sphere there are only incoming and not outgoing waves and the wave function inside is therefore

\[ \psi(R) \sim \frac{1}{2} \left( e^{-iKR} \right) \]

Dr. F. Marzano in Rome has computed the backward angular distribution with this very rough model assuming R_0 = 1 fm and K = k = \( \hat{p} \) wave number in the pp c.m.s. The result (dotted curves) is compared with the experimental points from College de France (fig. 1).

Since absorption is not the only thing, we add the real potential due to one pion exchange, without taking in account heavier exchanges which will be of shorter range. The scattering amplitude for this potential is computed in the Born approximation for the highest waves only (L > 2), because the lowest ones are absorbed any way. The results are the full line curves, which agree very well with the observations, except at the highest energies.

This is not a fit: We used the value 1 fm for the interaction radius as deduced from the annihilation cross section measurements at these energies. We have taken the wave number inside the sphere equal to the wave number outside. This ought to be correct at high enough energies.
This diffractive cross section is not stable in t. It is also very high and comparable with the measured cross sections. Therefore, the hope that in the backward direction the diffractive background would be negligible is not so founded and to study formation via backward scattering one must work very seriously on what the diffraction is doing. We can not escape this conclusion even if the model itself might not be very realistic.

One might further speculate that in the double structure observed, the first one is diffraction and the second one is an s-channel effect. But to do this one has to believe that the model gives a good representation of the diffractive background, which is not really proven.

Mr. French: How good is the agreement between Wisconsin and Collège de France?

Mr. Laloum: In backward direction, they are qualitatively in good agreement. There are however some discrepancies on the fine structures like the widths of the bumps.

Mr. Astier to Mr. Bizzarri: How did you obtain your backward diffractive contribution? Did you normalize the angular distribution to the forward peak?

Mr. Bizzarri: There is no arbitrary normalization. The model gives also a rough agreement for the forward region.

Mr. French to Mr. Bizzarri: Can you generate narrow peaks in the region ~1.0 to ~0.9 of the form which is reasonably well established?

Mr. Bizzarri: I can show you the 180° cross section versus mass generated by the model (fig. 2). There is a peak at 1930 MeV with a width of about 40 MeV, in reasonable agreement with the data. The data seems to show a second peak or to move this peak slightly forward.

Mr. Kalogeropoulos: The π⁺π⁻ angular distribution which is very striking at low energy became almost isotropic above 400 ~ 500 MeV/c, whereas the cross sections drops in a normal exponential way. The KK-system seems to become crazy somewhere above 500 MeV/c, the π⁺ system is crazy below 400 MeV/c, and the elastic scattering may have structures. We are faced with very contradictory phenomena changing with energy, which may depend of final states.

Mr. Bizzarri: I am convinced that there is something in the s-channel. But I am not sure that π⁺π⁻ backward scattering is a good place to look for it. Certainly the KK and π⁺π⁻ final states show funny things going on. But we are not in a position now to put a mass and a width on these effects.

Mr. Butterworth: Did anybody looked for π⁺π⁻ in production experiments?

Mr. Donald: We have examined the data from our U-film for final states N̅N selected events with no N̅̅̅̅ and examined the NN mass distribution. It is quite smooth.
Mr. Bettini: What is the variation with energy of the position in \( t \) of the dip in the backward \( \bar{p}p \) differential cross section?

Mr. Laloum: It varies strongly; for example, between about 400 MeV/c to \( \sim 600 \) MeV/c the position in \( t \) varies nearly by a factor 2.

Mr. Bizzarri: As I already stated one does not expect diffraction scattering at these energies to be stable in \( t \). At low energy the important parameter is rather \((R+\delta)t\), and the slope of the forward peak goes as \((R+\delta)^2/4\). This is the reason for the antishrinkage observed at low energy by Mr. B. Conforto et al, (Nuovo Cimento 54A (1968) 441) some years ago.

Mr. Butterworth: A more sophisticated fit to the forward peak, like the Bryan and Phillips one, gives also large bumps in the backward direction, which may not be quantitatively correct.
$\cos \theta^{p\bar{p}} = -0.9 \text{ to } -1.0$

**Fig. 2**
REPORT ON $\bar{p}p \to K^- K^-$ AT LOW MOMENTUM

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(presented by Mr. Benvenuti)

In this note we report some preliminary results for the reaction:

$$\bar{p}p \to K^+ K^- \quad (1)$$

with incident $\bar{p}$ momentum between 300 and 800 MeV/c. These data are part of a bubble chamber study of $\bar{p}p$ interactions at low momentum currently in progress at the University of Wisconsin. Several papers have already appeared on the subject\(^1\) and we refer to them for the description of the bubble chamber exposure and other experimental details.

The cross-section distribution for reaction (1) is presented in Fig. 1 together with the cross-sections for the following reactions\(^2\):

$$\bar{p}p \to K^0_S K^0_L \quad (2)$$

$$\bar{p}p \to K^0_S K^0_S + K^0_L K^0_L \quad (3)$$

$$\bar{p}n \to K^- K^0 \quad (4)$$

Our data for reaction (1) are based on a sample of 106 events and have not been corrected for scanning efficiency. The good agreement between our data and the available measurements in the same momentum range corroborate our estimate that such correction should be small.

The data presented in Fig. 1 show some interesting features:

a) Around 300 MeV/c the cross-section for $K^0_S K^0_L$ is equal to that for $K^+ K^-$ within errors, while the cross-section for $K^0_S K^0_S + K^0_L K^0_L$ is very small and consistent with being zero.

b) Above 1000 MeV/c the cross-section for $K^0_S K^0_S + K^0_L K^0_L$ rises slowly to the value of the cross-section for $K^0_S K^0_L$.

c) Between 300 and 800 MeV/c something remarkable occurs:

the cross-section for $K^0_S K^0_L$ shows a very sharp structure which has been interpreted elsewhere\(^1d\) as evidence for a new vector meson with mass 1970 MeV and width 35 MeV\(^3\). The cross-section for $K^+ K^-$ presents a peak around 500 MeV/c just where the $K^0_S K^0_L$ cross-section has a sharp dip.
Each partial wave amplitude for the reactions (1), (2), (3) is of the form:

\[ A(\overline{p}p \rightarrow K^+K^-) = A_1^+ + A_0^+ + A_1^- + A_0^- \]

\[ A(\overline{p}p \rightarrow K_S^0K_L^0) = A_1^- + A_0^- \]

\[ A(\overline{p}p \rightarrow K_S^0K_S^0 + K_L^0K_L^0) = A_1^+ - A_0^+ \]

where the subscripts stand for the isospin and the superscripts for the C parity. In the \( \overline{p}p \) momentum range under consideration (300 - 800 MeV/c) the cross-section for reaction (3) is negligible. Therefore, either all the \( C = +1 \) amplitudes vanish or the \( I = 1, C = +1 \) amplitudes cancel the \( I = 0, C = +1 \) amplitudes in each partial wave. The presence of \( C = -1 \) amplitudes would be revealed in reaction (1) by a sizable forward-backward asymmetry. Our data shown in Fig. 2 do not support any significant asymmetry, thus the \( C = +1 \) amplitudes are apparently negligible below 800 MeV/c.

Studies of \( \overline{p}p \) annihilation at rest show that the rates for reaction (1) and (2) are approximately equal. Furthermore, as noted above, the cross-sections for these reactions are equal at 300 MeV/c. Hence, either one of the \( C = -1 \) I-spin amplitudes is negligible or they are \( 90^\circ \) out of phase in each partial wave. At rest the \( \overline{p}p \) annihilation rate into \( K\bar{K} \) is \( 1.8 \pm .2 \times 10^{-3} \) while the rate for \( \overline{p}n \) annihilation into \( K^+K^- \) is \( 5.2 \pm .8 \times 10^{-3} \). This would indicate that at rest these annihilations occur from a pure \( I = 1 \) state since for this case we would expect a value of 2 for the ratio \( \overline{p}n \rightarrow K^+K^-/\overline{p}p \rightarrow K\bar{K} \). It is therefore plausible to assume that reactions (1) and (2) below 300 MeV/c are dominated by the isospin 1 amplitude.

With these assumptions the structures observed in the cross-section distribution for reaction (1) and (2) can be interpreted as due to a resonance with isospin zero interfering with a background with isospin one. In support of this interpretation the momentum dependence of the ratios:

\[ R_1 = (\sigma(\overline{p}p \rightarrow K^+K^-) + \sigma(\overline{p}p \rightarrow K^0K^0))/\sigma_A \]

\[ R_2 = (\sigma(\overline{p}p \rightarrow K^+K^-) - \sigma(\overline{p}p \rightarrow K^0K^0))/\sigma_A \]

is given in Fig. 3 where \( \sigma_A \) is the annihilation cross-section. The strength of the interference is illustrated by the sharp structure in the distribution of \( R_2 \) while \( R_1 \) has a reasonably smooth behavior.
The data for reaction (1) have been fitted to a series of Legendre polynomials. The ratio $a_1/a_0$ is consistent with zero, Fig. 4, while $a_2/a_0$ shows a peak around 700 MeV/c implying the presence of at least a $3D_1$ wave and/or interference with higher partial waves. Given the statistical limitations of the data we cannot determine the spin of this resonance.

This situation may be resolved by a systematic study of reaction (1) (3) to be undertaken by us in Fall 1972 using the Argonne National Laboratory 12' hydrogen bubble chamber.

REFERENCES


3. The vector meson interpretation of the $K^-K^+$ data presented in Ref. 1a must be re-examined in view of the present results.

TOTAL CROSS SECTION FOR $\bar{p}p \rightarrow K^+ K^-$ BELOW 2 GeV/c

- LÖRSTED et al
- BENVENUTI et al
- BARLOW et al
- SEGAL
- NICHOLSON et al
- U.W. PRELIMINARY
- BIZZARRI et al
- SEGAL
- SEGAL

$(K^+ K^- + K^0 K^-)$

ABOVE $P_L \sim 1200$ MeV/c

Fig. 1 Total cross-section for $\bar{p}p + K\bar{K}$ below 2 GeV/c.
Fig. 2 Distributions of \((F - B)/(F + B)\) and \((P - E)/(P + E)\) ratios for \(\bar{p}p \rightarrow K^+K^-\) data from this work and from Ref. 2e.
Fig. 3 Distributions of $(\sigma(\bar{p}p + K^+K^-) + \sigma(\bar{p}p + K^0\bar{K}^0))/\sigma_{\text{ANN}}$ and of $(\sigma(\bar{p}p + K^+K^-) - \sigma(\bar{p}p + K^0\bar{K}^0))/\sigma_{\text{ANN}}$. 
Fig. 4 Legendre coefficients for $\bar{p}p \rightarrow K^+K^-$ data from this experiment and from Ref. 2b and 2e.
FORMATION EXPERIMENTS: T REGION

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1. Evidence for a boson resonance of mass about 2200 MeV was first reported by the CERN M.M.S. group\(^1\), that measured the missing mass distribution in the reaction \(\pi^+ p \rightarrow p + MM^-\) at small \(t\), and found a five standard deviation peak, the \(T\)-meson, at a mass of \(2195 \pm 15\) MeV with a width \(\Gamma \approx 13\) MeV. Since then, many formation and production experiments have been performed, in the attempt to verify the existence of the \(T\)-resonance and possibly to measure its quantum numbers.

Although I am not going to discuss in detail production experiments, it is probably worthwhile to present a table showing a listing of the peaks seen in production experiments in the mass range 2000 to 2300 MeV.

| Table 1 |

<table>
<thead>
<tr>
<th>Authors</th>
<th>Reactions</th>
<th>(M) (MeV)</th>
<th>(\Gamma) (MeV)</th>
<th>Decay modes observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chikovani et al.</td>
<td>(\pi^- p \rightarrow p MM^-) at 12.0 GeV/c (small (t))</td>
<td>2195 ± 15</td>
<td>(\leq 13)</td>
<td>3 charg. (+ poss. neutr.) (\approx 94%)</td>
</tr>
<tr>
<td>Anderson et al.</td>
<td>(\pi^- p \rightarrow p MM^-) at 16.0 GeV/c (small (u))</td>
<td>2086 ± 38</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Alles-Borelli et al.</td>
<td>(\bar{p}p \rightarrow 2\pi^- 2\pi^+\pi^0) at 5.7 GeV/c</td>
<td>2207 ± 13</td>
<td>62 ± 52</td>
<td>(\pi^+\pi^-\pi^0)</td>
</tr>
<tr>
<td>Clayton et al.</td>
<td>(\bar{p}p \rightarrow 3\pi^- 3\pi^+\pi^0) at 2.5 GeV/c</td>
<td>2190 ± 10</td>
<td>130</td>
<td>(\Lambda_2\omega)</td>
</tr>
<tr>
<td>Caso et al.</td>
<td>(\pi^- p \rightarrow p \pi^-\pi^+\pi^-) at 11.2 GeV/c</td>
<td>2207 ± 22</td>
<td>130</td>
<td>(p\pi^+\pi^-)</td>
</tr>
<tr>
<td>Kramer et al.</td>
<td>(p p \rightarrow p p\pi^+\pi^0) at 13.1 GeV/c</td>
<td>2157 ± 10</td>
<td>68 ± 22</td>
<td>(\pi^0)</td>
</tr>
<tr>
<td>Dagan et al.</td>
<td>(\bar{p}p \rightarrow 3\pi^- 3\pi^+\pi^0) at 6.94 GeV/c</td>
<td>2140 ± 20</td>
<td>&lt; 30</td>
<td>(f^0\pi)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2210 ± 40</td>
<td>100 ± 80</td>
<td>(g^0\pi)</td>
</tr>
</tbody>
</table>

The first entry in the table refers to the CERN missing mass experiment just mentioned. Later on, Anderson et al.\(^2\) studied the production of high mass bosons in the same reaction at small \(u\) and observed enhancements at masses consistent with the \(R\) and \(U\) masses, but not with the \(S\) and \(T\) masses. Actually, the masses and widths corresponding to the peaks closest
to the $T$-meson mass are $(2086 \pm 38, 150)$ MeV and $(2260 \pm 18, \leq 25)$ MeV.

The other five entries refer to bubble chamber experiments. The peak seen by Alles-Borelli et al. $^3$ in the negative $G$-parity state $\pi^+\pi^-\pi^0$ is about two standard deviations above the over-all phase space and five standard deviations above the adjacent regions. Mass and width are consistent with the values found by Chikovani. Clayton et al. $^4$ saw a structure at 2190 MeV in the $(6\pi)^\pm$ effective mass distribution, where a $\pi^+\pi^-\pi^0$ combination in the $6\pi$ state was required to be in the $\omega$ region and the remaining $(3\pi)^\pm$ were required to have an effective mass in the $A_2$ region. Another positive $G$-parity structure was observed by Caso et al. $^5$, who reported a four-standard deviation peak at about the $T$ mass in the $2\pi\pi\pi^0$ mass distribution with at least one $\pi^-\pi^0$ combination in the $\rho$ region. Events with a $\pi^+\pi^-\pi^0$ combination in the $\eta$ or $\omega$ region, as well as events with a $\pi^+p$ combination in the $\Delta^{++}(1236)$ region, were excluded from the plot.

Next in the list, we have an $J^P = 1^+$ peak reported by Kramer et al. $^6$ at a smaller mass. Besides the mass shift with respect to the CERN M.W.S. experiment, also the $1$ charged/3 charged ratios disagree. Assuming they are observing a new boson resonance ($T'$-meson), the authors put an upper limit $J^P \leq 5^-$ to its spin. Finally, Ogan et al. $^7$ reported some indications of structures in the $f^0\pi^\pm$ and $g^0\pi^\pm$ channels, where $g^0$ really stands for a peak at 1650 MeV.

The $\bar{p}N$ total cross section measurements by Abrams et al. $^8$ also give evidence for an $l = 1$ structure at 2190 $\pm$ 10 MeV. Additional structures were observed at 2350 and 2375 MeV, respectively in the $l = 1$ and $l = 0$ cross sections.

This experiment has a high statistical accuracy and great care was exercised to avoid energy-dependent errors larger than $\pm$ 0.1%. The data points in the $T$-region were taken at momentum settings corresponding to mass intervals of 18 MeV, the mass resolution of the apparatus being 15 MeV full width at half-height.

The height of the enhancement at 2190 MeV is about 5.5 mb: if interpreted as a boson resonance it would have approximately: $\Gamma = 85$ MeV, $\frac{1}{2}(J + \frac{1}{2}) = 0.36$, where $x$ is the elasticity of the resonance (the statistical uncertainty being smaller than $\pm$ 15% for the height and $\pm$ 30% for the width, with that particular choice of the background).

As pointed out by the authors, an alternative explanation of the bump could be a sharp rise of the $N\Delta(1236)$ ($\bar{N}\Delta$) cross section, whose threshold is near the energy of the observed structure. To investigate this possibi-
lity, single $\pi$ production in hydrogen$^{9,10,11}$ and deuterium$^{12,13}$ was studied by several groups. The reactions involved, in hydrogen and deuterium, are:

\[
\begin{align*}
\bar{p}p &\rightarrow \bar{p}p \pi^0 \quad a) \\
\bar{p}n &\rightarrow \bar{p}n \pi^+ \quad b) \\
p\bar{n} &\rightarrow np \pi^- \quad c) \\
n\bar{n} &\rightarrow nn \pi^0 \quad d)
\end{align*}
\]  

(1)

\[
\begin{align*}
\bar{p}n &\rightarrow \bar{p}p \pi^- \quad a) \\
\bar{n}n &\rightarrow \bar{n}n \pi^- \quad b) \\
\bar{p}n &\rightarrow \bar{p}n \pi^0 \quad c)
\end{align*}
\]  

(2)

Reactions 1d), 2b) and 2c) cannot be measured in a bubble chamber. The analysable final states, however, are in agreement with the assumption that the one pion production is dominated by a $N\Delta + \bar{N}\Delta$ intermediate state, which allows to correct for the unseen reactions. In this case, only the $I = 1$ amplitude contributes to the one pion production so that

\[
\sigma(\bar{p}n \rightarrow \bar{N}\pi) = 2\sigma(\bar{p}p \rightarrow N\pi)
\]

where the factor 2 takes into account the isospin normalization of the initial state. Fig. 1 shows the $\bar{p}n \rightarrow \bar{N}\pi$ cross section and the $\bar{p}p \rightarrow N\pi$ cross section multiplied by 2 as functions of total energy. (In comparing the two sets of data, one should consider the fact that the deuterium data of ref. 13 are not corrected for the screening effect inside the deuterium nucleus). The one pion production cross section rises with nearly constant slope after threshold. This excludes that the bump observed byAbrams et al. in the $p\bar{N}$ cross section at 1.3 GeV/c is mainly due to the isobar threshold effect.

3. If it is assumed that the bump is due to the formation of a boson resonance lying, as conjectured, on the normal Regge trajectory $\rho$, $\Lambda_2$, with constant slope 1 GeV$^{-2}$, it would have spin $J = 5$. From the known value of $(J + \frac{1}{2})x$, we can expect an effect of about 200 $\mu$b in the $\bar{p}p$ elastic cross section. This is very difficult to see in the total elastic cross section$^{9,14-17}$(Fig. 2), which is dominated by the large forward diffraction peak.

Cline$^{18}$ has suggested that backward scattering should be more sensitive to the presence of direct channel resonances.

A CalTech-BNL-Rochester collaboration$^{19}$ has performed a counter experiment to measure the $\bar{p}p$ elastic scattering for $\cos \theta \leq -0.98$
and -1.0 and incident momenta between 0.70 and 2.16 GeV/c, at approximately 100 MeV/c intervals. The momentum dependence of the cross section exhibits a sharp dip at 0.9 GeV/c and a broad maximum at about 1.4 GeV/c (Fig. 3). No evidence for the narrow peaks observed in the CERN M,M,S experiment was found, but the energy resolution might not be sufficient to resolve them. On the other hand, the authors were able to obtain, on the basis of three resonances corresponding to the Abrams peaks, a 180° dσ/dΩ which approximately reproduces their data. Alternatively, a diffraction model too gives an over-all view in agreement with data.

Bubble chamber data are also available, both for p̅p⁹,¹⁰,¹¹,²⁰,²¹ and p̅n backward scattering²²). Fig. 4 presents a compilation by Vallet et al.²¹) of the p̅p average differential cross sections in the interval -1.0 < cos θ C,M, < -0.8. Fig. 5 presents preliminary results on the p̅n backward scattering in the interval -0.95 < cos θ C,M, < -0.8. The momentum dependence of the deuterium data looks similar to that observed by Yoh et al. in their p̅p experiment in the same energy interval, and an interpretation in terms of an optical model is being tried.

In conclusion, most backward scattering data may be interpreted in terms of optical models, although it is probably not possible to exclude resonant effects.

The charge-exchange reaction p̅p→p̅n has been also studied. Brimman et al.²³) measured the cross section for this reaction at various incident momenta between 1 and 3 GeV/c. Although the over-all normalization is somewhat uncertain, the detection of a structure over a small momentum interval should not be impaired. No significant enhancement in the T-region was observed, but the presence of a resonance of spin as high as 5 or more is compatible with the data.

Recently, a Stony Brook-Wisconsin collaboration²⁴) has measured total and differential cross sections for the same reaction. The total cross section presents some structures which resemble the Abrams peaks (Fig. 6). The results of this experiment are however still preliminary and the authors do not claim at present that these structures are real effects. If confirmed, these results would favor a resonant interpretation of the Abrams peaks.

4. If the Abrams peak at 2190 MeV corresponds to a resonance, it must have small elasticity (and correspondingly high spin), and should show up in some inelastic channel. The p̅p topological cross sections¹⁰,¹⁶,²⁵)(Fig. 7) do not give clear indications about where to look for it. Various annihila
tion channels have been or are being studied in the attempt to find specific channels where an effect is present over a relatively small background.

4.1 A counter-wire chamber experiment to study the reactions \( \bar{p}p \rightarrow \pi^- \pi^+ \) and \( \bar{p}p \rightarrow K^- K^+ \) between 0.7 and 2.4 GeV/c has been performed by a CalTech-Rochester-B.N.L. collaboration\(^{26}\). By combining these data with the ones obtained from a previous CalTech-B.N.L. collaboration\(^{27}\), it was possible to obtain folded differential cross sections \( \frac{d\sigma}{d\Omega} (\Theta_{C,M_N}) + \frac{d\sigma}{d\Omega} (\pi - \Theta_{C,M_N}) \) at 13 different momenta. Figs. 8 and 9 show respectively the folded differential cross sections at some momenta and the coefficients \( a_l \) of their expansion in terms of (even) Legendre polynomials.

The two pion angular distributions seem to be dominated by one set of states below 1 GeV/c and by another set above 1.7 GeV/c with interference in between. Therefore they have been fitted with two direct channel resonances and constant background. The results of the fit, shown in Fig. 9, are obtained with a \( J=3 \) resonance with \( M = 2.12 \) GeV, \( \Gamma = 0.249 \) GeV and a \( J=5 \) resonance with \( M = 2.29 \) GeV, \( \Gamma = 0.165 \) GeV.

Both resonances, decaying into two pions, must have \( l = 1, P = -1, G = +1 \). The position of the enhancements found by Abrams et al. are about 70 MeV away from these values and the widths are quite different. As for the narrow \( T \) and \( U \) resonances found by the M.M. spectrometer group, it is possible that the energy resolution was not good enough to reveal their presence.

The two pion (and two kaon) annihilation has been also studied in bubble chamber by T.C. Bacon et al.\(^{28}\) in the more restricted C.M. energy range 2150 to 2240 MeV (Figs. 10 and 11). No structure in the \( T \)-region is observed.

In the region of overlap between the counter experiment mentioned above and this experiment, the \( a_4/a_0 \) and \( a_6/a_0 \) ratios are in reasonable agreement, but \( a_2/a_0 \) is somewhat smaller in the latter. This experiment yields also the odd Legendre coefficients. It is interesting to note that the forward and backward hemisphere have very different distributions and the low order odd coefficients are non-zero. Unfortunately, the statistical significance of this data does not allow a detailed direct channel analysis, but more high statistics counter data are forthcoming\(^{29}\).

4.2 In the same bubble chamber experiment mentioned above, the channel \( \pi^+ \pi^- \pi^0 \) has been studied. In particular, the amount of production of \( \rho^0, \rho^+, \rho^- \) has been determined. With the possible exception of a relatively small \( \rho^0 \) cross section at 1.36 GeV/c, the resonance fractions and cross
sections are approximately constant (Table II) and do not show evidence of direct channel effects. A compilation of the 3-pion cross section is given in Fig. 12\textsuperscript{26,28}.

**Table II**

3-pion cross sections in microbarns

<table>
<thead>
<tr>
<th>( p(\text{GeV/c}) )</th>
<th>1.23</th>
<th>1.30</th>
<th>1.36</th>
<th>1.43</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{p}p \rightarrow \pi^+ \pi^- \pi^0 )</td>
<td>2019 ± 75</td>
<td>1894 ± 78</td>
<td>1719 ± 72</td>
<td>1742 ± 71</td>
<td>1843 ± 46</td>
</tr>
<tr>
<td>( \rho^+ \pi^- )</td>
<td>313 ± 84</td>
<td>308 ± 85</td>
<td>151 ± 82</td>
<td>215 ± 78</td>
<td>260 ± 39</td>
</tr>
<tr>
<td>( \rho^0 \pi^- )</td>
<td>255 ± 61</td>
<td>304 ± 71</td>
<td>216 ± 64</td>
<td>251 ± 61</td>
<td>264 ± 32</td>
</tr>
<tr>
<td>( f^0 \pi^- )</td>
<td>356 ± 71</td>
<td>300 ± 78</td>
<td>290 ± 76</td>
<td>295 ± 73</td>
<td>313 ± 38</td>
</tr>
</tbody>
</table>

The process \( \bar{p}p \rightarrow \pi^+ \rho^- \) has been studied by Yoh et al.\textsuperscript{30} at several incident momenta between 1 and 2 GeV/c in the angular range of \( \cos \Theta_{CM} (\vec{p} \pi^+) \) between 0.96 and 1.0. Forward emitted positive particles were momentum analyzed by a magnet-wire spark chamber spectrometer. A Cherenkov counter and a time-of-flight system allowed the identification of these particles. From the missing mass distributions, the amount of \( \pi^+ \rho^- \) production was evaluated at the various momenta. Fig. 13 shows the differential cross sections at an average value of \( \cos \Theta_{CM} (\vec{p} \pi^+) \) of 0.99. The broad peak could suggest the existence of a \( G = -1 \) resonance near 2.25 GeV with a width of about 200 MeV. The peak covers both the T and U regions and might have been caused by more than one resonance.

4.3 No s-channel effect in the final state \( \bar{p}p \rightarrow 2 \pi^- 2 \pi^+ \) has been reported. Cooper et al.\textsuperscript{10} fitted the 4\( \pi \) annihilation to an incoherent superposition of phase space and sums and products of Breit-Wigner cross sections for \( \rho^0, f^0 \) and \( \rho^0 f^0 \). About 1/3 of the cross section was attributed to \( \rho^0 f^0 \) production, but it stays approximately constant with momentum.

4.4 On the contrary a bump of about 0.8 mb at about the T mass was reported by Kalbfleisch et al.\textsuperscript{16,31} in the channel \( \bar{p}p \rightarrow 2 \pi^- 2 \pi^+ \rho^0 \). Among the substates of the five-pions, \( \rho^0 \) was found in particular to show an enhancement and to be accompanied by a second \( \rho^0 \) at 1.33 GeV/c, but not at the other momenta (1.11 and 1.52 GeV/c). The significance of these data and a comparison between them (Fig. 14) and the analogous data from a A.N.L. group\textsuperscript{32} (Fig. 15) were discussed extensively by Kalbfleisch in his report.
on the T-region at the 1970 Philadelphia Conference\textsuperscript{31}). His conclusion was that the $\bar{p}p$ system is forming a meson state $\pi(2190)$ of width between 20 and 80 MeV, which decays into $\rho^0\rho^0\pi^0$.

The $\rho^0\rho^0\pi^0$ cross section at the peak was estimated to be about 0.4 mb. No signal was seen in the $\rho^0\rho^0\pi^0$ channels but the $\pi^+\pi^-\pi^0\pi^0$ cross section was reported to give probable evidence for a $\rho^+\rho^-\pi^0\pi^0$ decay of the $\pi(2190)$. The same cross section was measured by Bacon et al.\textsuperscript{28} at 1.23, 1.30, 1.36 and 1.43 GeV/c. Fig. 16 shows a comparison between the two sets of data. According to Bacon et al., the difference between them might be due to a different treatment of events with identifiable $K$-mesons. They concluded that in the $\pi^+\pi^-\pi^0\pi^0$ cross section there is no significant evidence of structure in the T-region.

4.5 Before turning to the $\bar{p}n$ annihilation, let me recall the results for the reactions $\bar{p}p \rightarrow k\bar{k}\pi$ and $\bar{p}p \rightarrow \bar{p}k\pi$\textsuperscript{33}(Fig. 17). If the excess of events at 1.3 GeV/c is attributed to resonance formation, a fit through the $k,\bar{k},\pi$ points gives $M = 2176 \pm 5$ MeV, $\Gamma = 20\pm16$ MeV. This state would correspond to $I^G = 0^-$ or $1^+$: both assignments are incompatible with a $\rho^0\rho^0\pi^0\pi^0$ state.

4.6 The data which will be discussed in this section are preliminary results on $\bar{p}d$ interactions at seven momenta between 1.0 and 1.6 GeV/c, from the Lawrence Berkeley Laboratory-Padova-Pisa-Torino collaboration. The results relative to one-pion production and $\bar{p}n$ backward elastic scattering have been presented in sects. 2 and 3 respectively.

Fig. 18 gives some topological cross sections. The odd prong topologies have been fitted to a background of the form $a + b/p$ plus two Breit-Wigner functions which represent the Abrams et al. bumps in $l = 1$. The smearing effect of the Fermi momentum of the neutron and of the beam momentum spread have been taken into account. The $\eta\pi$ (2190) contribution to the topological cross sections results to be $5.1 \pm 2.4$ mb for the 1-prong events, $2.3 \pm 1.0$ mb for the 3-prongs and $3.0 \pm 1.4$ for the 5-prongs (all 2-standard deviation effects). If we look at the physical channels (Fig. 19) we get: $0.2 \pm 0.09$ mb for the $3\pi$ state and $0.5 \pm 0.2$ mb for the $3\pi$ state, whereas the $4\pi$ and $6\pi$ states do not show appreciable effects. Although these effects are probably not meaningful by themselves, they may provide some insight into the nature of the statistically much more meaningful peak found by Abrams et al. at 2190 MeV.

The $4\pi$, $5\pi$ and $6\pi$ final states have been studied in some detail to
determine the amount of production of the various $2\pi$ and $3\pi$ resonances.

Figs. 20 and 21 show some distributions relative to the $5\pi$ final state, relevant for the $\rho\rho\pi$ problem discussed in sect. 44. Since a background subtraction, as made by Kalbfleisch et al., is difficult to make objectively, a maximum likelihood fit to the data of combinations of amplitudes representing the various intermediate states has been performed. The results are shown in Table 3. Each amplitude has been symmetrized incoherently with respect to the exchange of like pions, but a coherent symmetrization does not alter the energy dependence of the $\rho\rho\pi$ percentage. It is impossible to draw from these data alone definite conclusions about the existence of the $\rho\rho\pi$ effect.

**Table 3**

$p\pi \rightarrow 3\pi^+ 2\pi^-$. Estimated channel percentage. With the particular parametrization used, the errors are about 0.05.

<table>
<thead>
<tr>
<th>$E_{c.m.}(\text{MeV})$</th>
<th>2130-2150</th>
<th>2150-2165</th>
<th>2165-2180</th>
<th>2185-2210</th>
<th>2210-2240</th>
<th>2240-2290</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho 3\pi$</td>
<td>0.25</td>
<td>0.21</td>
<td>0.19</td>
<td>0.11</td>
<td>0.26</td>
<td>0.00</td>
</tr>
<tr>
<td>$f 3\pi$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>$\xi 3\pi$</td>
<td>0.01</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>$\rho\rho\pi$</td>
<td>0.04</td>
<td>0.02</td>
<td>0.05</td>
<td>0.17</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>$\xi\xi\pi$</td>
<td>0.22</td>
<td>0.08</td>
<td>0.00</td>
<td>0.00</td>
<td>0.09</td>
<td>0.15</td>
</tr>
<tr>
<td>$\xi\rho\pi$</td>
<td>0.05</td>
<td>0.24</td>
<td>0.45</td>
<td>0.20</td>
<td>0.19</td>
<td>0.06</td>
</tr>
<tr>
<td>$f\rho\pi$</td>
<td>0.01</td>
<td>0.00</td>
<td>0.03</td>
<td>0.00</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>$f\xi\pi$</td>
<td>0.23</td>
<td>0.20</td>
<td>0.18</td>
<td>0.27</td>
<td>0.19</td>
<td>0.09</td>
</tr>
<tr>
<td>$A^+_2 2\pi$</td>
<td>0.00</td>
<td>0.02</td>
<td>0.03</td>
<td>0.10</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>$A^-_2 2\pi$</td>
<td>0.05</td>
<td>0.00</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.26</td>
</tr>
<tr>
<td>$A^-_2 \xi$</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>$A^-_2 \rho$</td>
<td>0.14</td>
<td>0.08</td>
<td>0.00</td>
<td>0.15</td>
<td>0.08</td>
<td>0.00</td>
</tr>
</tbody>
</table>

As for the $4\pi$ and $6\pi$ channels, the various percentages are roughly constant.

5. In conclusion, the situation in the T-region, as it results by the various formation experiments, is not yet clarified, but some progress has been made. Backward elastic scattering has not given so far conclusive results. The new data on the $p\pi \rightarrow p\pi$ reaction are more promising, but we ha
ve to wait for the final results. The momentum dependence of the one-pion production cross section leads to exclude that the Abrams et al. enhancement at 2190 MeV is mainly due to a $\Delta$-threshold phenomenon. It is a difficult task to interpret without much ambiguity the various annihilation channels. Annihilation in deuterium, however, indicates that $G = -1$ states are probably important in this region. Finally, there are indications, both from production and formation experiments, for more than one resonance in the T-region, but they need confirmation.

In preparing this talk, I have benefited by interesting conversations with Drs. A. Astbury, A. Benvenuti, A. Bettini, G. Borreani, M. Cresti, M.L. Good, B. Guassiat, A. Werbrouck and by the excellent reviews in which this subject has been discussed, mainly those by L. Montanet$^{34}$ and M. Derrick$^{35}$. I wish also to thank Drs. E. Flaminio and I. Mannelli for useful comments on this paper.

REFERENCES


12) J. Berryhill, U. Camerini and D.B. Cline, see ref. 10.


27) D. Fong, thesis, California Institute of Technology.


29) A. Astbury and P. Kalmus, private communication.


32) W.A. Cooper, S. Egli, L.G. Hyman and W. Manner, Paper submitted to the Boulder Conference, Colorado, 1969; quoted in ref. 35.


Fig. 1 Cross sections for \( \bar{p}n \rightarrow \bar{p}p\pi^- \), \( \bar{p}n \rightarrow N\bar{N}\pi^- \) (closed symbols) and \( \bar{p}p \rightarrow N\bar{N}\pi^- \) (open symbols). The \( \bar{p}p \) cross sections have been multiplied by two.
Fig. 2 $\bar{p}p$ elastic cross section.
Fig. 3 $\bar{p}p$ elastic differential cross section for $\cos \theta_{c.m.}$ between -1.0 and -0.98. Open circles: data from Yoh et al.; closed circles: data from Cline et al.. The curve represents the predictions of a diffraction model.
Fig. 4 A compilation of bubble chamber data on $\bar{p}p$ elastic scattering for $\cos \theta_{c.m.}$ between -1.0 and -0.8.

$\blacktriangle$ ref. 9, $\blacksquare$ ref. 10, $\square$ ref. 11, $\bigcirc$ ref. 20, $\bullet$ ref. 21
$\bar{p}n$ ELASTIC CROSS SECTION

$-0.95 < \cos \theta_{C.M.} < -0.8$

---

Fig. 5 $\bar{p}n$ elastic differential cross section for $\cos \theta_{C.M.}$ between -0.95 and -0.8.
Fig. 6 Total cross section for the reaction $\bar{p}p \rightarrow \bar{n}n$, from the Stony Brook–Wisconsin collaboration.
Fig. 7 $\bar{p}p$ topological cross sections.
Fig. 8 Folded differential cross sections for $\bar{p}p \rightarrow \pi^- \pi^+$ and $\bar{p}p \rightarrow k^- k^+$ from ref. 26. The curves represent Legendre polynomial fits.
Fig. 9 Legendre coefficients from fits to the folded differential cross sections of fig. 8. The curves represent the resonance fit reported in the text.
Fig. 10 Cross section for $\bar{p}p \rightarrow \pi^-\pi^+$
Fig. 11 Cross section for $\bar{p}p \rightarrow K^-K^+$
$\bar{p}p \rightarrow \pi^+ \pi^- \pi^0$

**Fig. 12** Cross section for $\bar{p}p \rightarrow \pi^+ \pi^0$
Fig. 13 Differential cross section for $\bar{p}p \rightarrow \pi^+ \rho^-$ at $\cos \theta_{\text{C.M.}}(\rho^+) = 0.99$ from ref. 30.
Fig. 14  $\pi^+\pi^-$ effective mass distribution for the reaction $\bar{p}p \rightarrow \rho^0 \pi^+ \pi^- \pi^0$ (Kalbfleisch et al. data). In the upper graphs the $\rho^0$ is in the $\rho$ band; in the lower graphs the $'\rho^0'$ is in mass regions adjacent to the $\rho$ band.
Fig. 15  \( \pi^+\pi^- \) effective mass distribution for the reaction \( \bar{p}p \rightarrow \rho^0 \pi^+\pi^- \pi^0 \) (Cooper et al. data). In the upper graphs the \( \rho^0 \) in the \( \rho \) band; in the lower graphs the '\( \rho' \) is in mass regions adjacent to the \( \rho \) band.
Fig. 16 Cross section for $\bar{p}p \rightarrow \pi^+ \pi^- + \text{neutrals.}$
Fig. 17 Cross sections for $\bar{p}p \rightarrow k^- k^+ \omega$ and $k^- k^+ \omega$. 
Fig. 18 Cross sections for the one, three and five prong topologies (with and without spectator proton). $\sigma(\bar{p}n)$ is the total $\bar{p}n$ cross section from Abrams et al.
Fig. 19 Cross sections for $\bar{p}n \rightarrow 2\pi^-\pi^+$ and $3\pi^- 2\pi^+$ and their sum.
\[ \bar{p} n \rightarrow 3 \pi^- 2 \pi^+ \quad M (\pi^+ \pi^-) \text{ in MeV/c}^2 \]

Fig. 20 \( \pi^+ \pi^- \) effective mass distributions from \( \bar{p} n \rightarrow 3 \pi^- 2 \pi^+ \).
\( \bar{p}n \rightarrow \rho^0 \pi^+ \pi^- \pi^- \quad M(\pi^+\pi^-) \) in MeVs^2

Fig. 21 \( \pi^+ \pi^- \) effective mass distributions from \( \bar{p}n \rightarrow \rho^0 2\pi^- \pi^+ \). The \( \rho^0 \) is in the mass region 700-820 MeV.
DIRECT S-CHANNEL EFFECTS IN THE FINAL STATE $K^0_L K^0_L$ IN THE REGION OF THE $T$-MESON

LIVERPOOL - LPHE (PARIS) COLLABORATION

This collaboration presented to the Lund Conference a preliminary report which gave evidence in favour of the existence of a direct channel effect in the reaction

$$\bar{p}p \rightarrow K^0_L K^0_L$$

at centre of mass energies spanning the $T$ meson. That report was based on half the statistics of the experiment, and on a preliminary method of background subtraction.

This article is a report on the analysis of the full statistics of the experiments. The conclusion of this analysis is that we no longer require the hypothesis of a direct channel contribution to explain the data. The photographs used came from several exposures at incident momenta of 1.1, 1.15, 1.24, 1.36, 1.31, 1.44 GeV/c, all taken in the CERN 200 cm. chamber, with a magnetic field of 9.5 KG, and an older exposure at 1.18 GeV/c in the Saclay 81 cm. chamber with a magnetic field of 20 KG. The total sample contained 18 900 events with at least one visible $V^0$ and 1416 events in the channel

$$\bar{p}p \rightarrow K^0_L K^0_L \pi^+ \pi^-$$

The various exposures have missing mass squared distributions which show that there is no significant contamination by other topologies. There are overlaps in the centre of mass energy ranges of the separate exposures. Accordingly, for the purpose of determining cross-sections, the events were divided into groups by the centre of mass energy in steps of 0.02 GeV/c$^2$, irrespective of the exposure from which they came. The centre of mass energy of each event was determined by the fitted value of the antiproton momentum at the annihilation point. Cuts were made on the direction of the antiproton to eliminate events which deviate from the general direction of the beam; these events may have undetected scatterings on the antiproton with a consequent error in the centre of mass energy. After these cuts 1278 events were left. Table 1 contains the number of events in each energy range after these cuts.

<table>
<thead>
<tr>
<th>Energy Range (GeV)</th>
<th>2.10</th>
<th>2.12</th>
<th>2.14</th>
<th>2.16</th>
<th>2.18</th>
<th>2.20</th>
<th>2.22</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0_L K^0_L \pi^+ \pi^-$</td>
<td>263</td>
<td>330</td>
<td>195</td>
<td>106</td>
<td>153</td>
<td>128</td>
<td>103</td>
</tr>
</tbody>
</table>

Table 1
The path length in each energy range was determined by studying a representative sample of tracks and normalising accordingly. The cross-sections so determined are thus independent of any other experiment. We have also determined cross-sections by normalising our events to the total cross-section measurements of Abrams et al.\textsuperscript{1).} to check that there were no significant systematic errors in the path length determination.

Since we wish to establish whether or not the cross-sections require a Breit-Wigner component in the energy variation, it is important to calculate the errors accurately, including all systematic effects. Therefore, we detail how both the cross-sections and their errors were calculated.

Table 2 contains the cross-sections for the reaction

\[(\bar{p}p \to K^0_L K^0_L \pi^+ \pi^-)\]

as directly determined.

<table>
<thead>
<tr>
<th>Mean Energy (GeV)</th>
<th>2.11</th>
<th>2.13</th>
<th>2.15</th>
<th>2.17</th>
<th>2.19</th>
<th>2.21</th>
<th>2.23</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma(K^0_L K^0_L \pi^+ \pi^-)_{\mu b})</td>
<td>135±8.5</td>
<td>150±8.5</td>
<td>115±8.5</td>
<td>153±11.5</td>
<td>155±12</td>
<td>170±15</td>
<td>163±16</td>
</tr>
</tbody>
</table>

However, to calculate the cross-sections for the process

\[(\bar{p}p \to K^0_L K^0_L \omega)\]

these results are not used directly. A straight line is fitted to the data of table 2 and the fitted line used to find the best values of the cross-section at each energy. To calculate the uncertainties on these values, the cross-sections of table 2 were used to generate by Monte Carlo methods 200 sets of cross-sections with values and spreads corresponding to the values and errors in Table 2. Each set was fitted by a straight line and the cross-sections were read off at the appropriate energies. The quoted errors are the standard deviations of the 200 values at each energy. The results are contained in Table 3.

<table>
<thead>
<tr>
<th>Mean Energy (GeV)</th>
<th>2.11</th>
<th>2.13</th>
<th>2.15</th>
<th>2.17</th>
<th>2.19</th>
<th>2.21</th>
<th>2.22</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma(K^0_L K^0_L \pi^+ \pi^-)_{\mu b})</td>
<td>133±6</td>
<td>138±4.5</td>
<td>142±4</td>
<td>147±4.6</td>
<td>152±6</td>
<td>157±8</td>
<td>162±10</td>
</tr>
</tbody>
</table>
Many resonances are produced in this final state. Although, the channel $K_1^0 K_1^0 \omega^0$ represents some 30% - 40% of all events there is appreciable production of $K^*, \rho, S^*$, and possible production of other resonances. Since an adequate description of the background under the $\omega^0$ is essential for this study, a maximum likelihood fit was carried out at each energy range, allowing for both single and associated resonance production. Owing to the low statistics at each energy range, substantial fluctuation in the percentages of the background channels occur. It is not pretended that the energy dependence of each channel has been determined. The fits are used solely to provide the best estimate of the background, and hence of the proportion of $K_1^0 K_1^0 \omega^0$. Table 4 contains the fitted fractions of the channel $K_1^0 K_1^0 \omega^0$ and the corresponding cross-section.

<table>
<thead>
<tr>
<th>Mean Energy (GeV)</th>
<th>2.11</th>
<th>2.13</th>
<th>2.15</th>
<th>2.17</th>
<th>2.19</th>
<th>2.21</th>
<th>2.23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitted fraction(%)</td>
<td>40±4</td>
<td>42±4</td>
<td>42±6</td>
<td>38±6</td>
<td>30±6</td>
<td>24±5</td>
<td>30±5</td>
</tr>
<tr>
<td>$\sigma(K_1^0 K_1^0 \omega)$</td>
<td>53.6</td>
<td>58.6</td>
<td>60.8±4</td>
<td>56±9</td>
<td>46±9</td>
<td>38±8</td>
<td>49±10</td>
</tr>
</tbody>
</table>

The data of Table 4 are consistent with the energy variation being a straight line over this energy region ($\chi^2 = 3$ for 5 degrees of freedom). On this basis we conclude that no direct channel effect is necessary to explain the data. Fig. 1 shows our data together with results from experiments at neighbouring energies.

References
2) Duboc et al. Submitted to Nuclear Physics.
DISCUSSION AND COMMENTS

Mr. Benvenuti: Concerning the differences in the total cross-section for $p\bar{p} \to \bar{n}n$ as measured by CERN and the Stony Brook and Wisconsin Groups:

1) We believe that our data will not change as far shape and magnitude are concerned.

2) We are now investigating the data in more detail, to reduce the point to point discrepancies which should enable us to make some more definite statements on the Abrams et al. bumps.

Mr. Montanet: About new data on $p\bar{p}$ charge exchange, I would like to indicate the new results obtained at Tokyo, for $p\bar{p}$ annihilations at 700 MeV/c:

"$p\bar{p}$ elastic and charge exchange scattering at 230 MeV"


Cross-section for $p\bar{p} \to \bar{n}n$ in found to be $9.1 \pm 0.6$ mb. The differential cross-sections are well explained by the phenomenological theory of Bryan and Phillips.

Mr. Donald: In the case of $p\bar{p} \to 5\pi$ we must realise that the annihilation process is very complex, $\omega^0, \rho^+, \rho^0, \pi^0$ resonances are produced singly and in association and one must look carefully at the analysis before drawing conclusions regarding the existence of the $T$ mesons. The data from the Liverpool - LPMHE (Paris) finds quite substantial amounts of $\rho^+ \pi^-$, but the amounts of $\rho^0 \pi^0$ varies strongly with the details of the analysis - introduction of final states such as $A_2^{\pi\pi}$ or $A_2^{\rho}$ reduces it by a factor of 2. So it is hard to know what is the correct amount of such a channel.

As far as $\rho^0 \pi^0$ is concerned, we found small amounts of this ($\sim 0.3$ mb) with no strong structure across the T-region.

We now have completed the study of the state $k^0, k^0, \omega^0$ with full statistics and we now probably do not require a direct channel effect.

Going on to the 4$\pi$ final state, if we select $\rho^0 \rho^0$ events by a straight forward mass cut and examine the angular distribution of the $\rho^0$ in the overall C.M.S. with respect to the beam direction, there is a (strong) structure which changes rapidly with energy. We are not able to reach a conclusion yet since the structures do not have an obvious simple explanation.

Mr. Allison: 1. At 2.3 GeV/c $p\bar{p} \to \omega\pi\pi$ we see a strong asymmetry in $\pi\pi$ angular distribution at large $\pi\pi$ mass suggesting strong contribution from double peripheral dynamics. If this is true, it ruins any attempt to analyse multiple resonance production.

2. In the T-region, if there are resonances in one $G$ state and not in another, one expects phase or coherence or cross-section ratio of $\rho\pi\pi/\omega\pi\pi$ to change with mass. It is not observed to do so. One can put some limits on this.
Mr. Bettini: I want to make a comment on resonance hunting. The situation seems to be easier to handle with \( \bar{p}n \) than with \( \bar{p}p \). A track sensitive target device would be the way to study the many \( \pi^0 \) systems. What is the situation in this respect at Nimrod?

Mr. Butterworth: The situation at Nimrod is not very good. The efficiency for detecting more than one \( \pi^0 \) is very small.

Mr. Astbury: There is an experiment at CERN on \( \bar{p}p \to \pi^+ \pi^- \) which will give full angular distribution in T and U region. If polarisation is also measured, it will be possible to do phase-shift analysis because there are only two invariant amplitudes in this channel.

Mr. Lillestøl: I have heard that Hyams has some data on the reaction \( \pi^+ n \to (\bar{p}p) p \). Using the method of extrapolation to the pion pole corresponding to the diagram

![Diagram]

the reaction \( \pi^+ \to \bar{p}p \) may be studied for a large range of \( \bar{p}p \)-effective mass using only one incoming \( \pi^+ \) momentum.
FORMATION EXPERIMENTS IN THE U-REGION.

A. Astbury,
Rutherford Laboratory.

INTRODUCTION - MISSING MASS EXPERIMENTS.

By way of introduction it is interesting to consider the evidence for the existence of the U-meson which comes from missing mass experiments.

The first evidence for a state U came from the CERN missing mass\(^1\) experiment. The reaction used was \(\pi^- + p \rightarrow p + \chi^-\) at 12 GeV/c. The object \(\chi^-\) has I-spin 1 or 2, and its mass is observed by the Jacobian peak method. The recoil proton is identified by range and time of flight, and its angle measured accurately by sonic spark chambers. Fig. 1 shows the evidence for the U in the angular distribution of the protons, and the missing mass plot. A statistical significance of 6.3 standard deviations is obtained by comparing \(\chi^2\) for a straight line fitted through the data, with, and without the peak channels. The width \(\Gamma_u\) is smaller than the mass resolution of the spectrometer in the U-region (62±3 MeV). The mass and width quoted for the state are 2.382±.024 GeV and \(0<\Gamma_u<.030\) GeV.

When this peak, along with others (R-S-T) from the same experiment were plotted in order, against their masses squared, one obtained a straight line extrapolation of the \(\rho-A_2\) trajectory, the U-peak having J=6. This fact gave the state a strong theoretical acceptability since it appeared to be narrow, had a high J value, and was on a linear trajectory. However the J value is purely conjecture, the surprising fact is that this very important experiment, with its many new states, was not repeated until recently.

The Northeastern Stonybrook\(^2\) missing mass experiment has produced preliminary results on the mass region above the R. The production process, incident momentum and technique are all very similar to those of the CERN experiment. Fig. 2 shows the data from the two experiments in the R-S-T mass region. The Northeastern Stonybrook group have no evidence for the high mass, narrow states.

There is also data from the CERN-IHEP Boson Spectrometer\(^3\) group using the same Jacobian peak mode, with incident pion momenta of 25 and 40 GeV/c at Serpukhov. They have surveyed the mass region up to 4.5 GeV, and for masses \(>2\) GeV place a limit of \(\Delta\sigma/\Delta t < 3\mu b/GeV^2\) for the production of any narrow resonances.
The experiment of Anderson et al. completes the picture of missing mass experiments. The channel was \( \pi^-\to p \) at 16 GeV/c looking at backward production. They have evidence for enhancements at the \( R \) and \( U \) but not the \( S \) and \( T \). The authors state that a smooth curve fits the high mass region with a \( \chi^2 \) probability of 5%.

The missing mass experiments are summarised in table 1.

**CONCLUSIONS ON THE U-FROM MISSING MASS**

1) An enhancement has been seen by two experiments\(^1,4\) although neither with overwhelming significance. The high statistics experiment of Northeastern Stonybrook\(^2\) has no evidence for the \( U \) nor does the CERN-BHEP\(^3\) group, although the latter could be explained by a rapid fall off in production cross section as the incident momentum is increased.

2) It is by no means clear that a narrow state at 2.38 GeV is established.

**FORMATION EXPERIMENTS IN THE U-REGION**

In the absence of a well established state we will take the \( U \)-region as 2.29-2.5 GeV, which corresponds to a momentum range \( \gamma/1.6-2.2 \) GeV/c for formation in antiproton reactions. The formation experiments will be considered in order of complexity of final states, namely total cross sections, elastic cross sections, two body channels, and many body channels.

**TOTAL CROSS-SECTIONS**

The experiment of Abrams et al.\(^5\) measured \( \bar{p}-p \) and \( \bar{p}-d \) total cross sections in the momentum range 1.00-3.3 GeV/c. They were able to extract the pure I-spin cross sections using the relations that \( \sigma_t(p-p) = \frac{1}{2}(\sigma_0 + \sigma_1) \) and \( \sigma_t(p-d) = \frac{1}{2}(\sigma_0 + 3\sigma_1) - 6\sigma \), where \( \sigma \) is the screening correction for the deuteron. Fig.3 shows the resonant cross sections of a few mb, which could be measured above smooth total cross sections, \( \sim 100 \text{mb} \) in \( \bar{p}-p \), and \( \sim 170 \text{mb} \) in \( \bar{p}-d \).

In order get resonance parameters a simple Breit-Wigner shape was assumed,

\[
\sigma_t(E) = \frac{4\pi\Gamma^2}{(2S_a+1)(2S_b+1)\left((E-E_0)^2/\Gamma^2+1\right)}
\]

The values of the parameters are given in table 2.

The state at 2350 could be the object seen in the CERN missing mass experiment but the agreement on width is poor. The elasticity \( x \), gives the branching ratio for the decay into the initial state of \( \bar{p}p \), i.e. \( x = \Gamma \bar{p}p/\Gamma \). For both possible mass peaks in the \( U \)-region \( x \) is \( \sim 10 \%), assuming \( J=5 \) or 6.
CONCLUSIONS OF TOTAL CROSS SECTION MEASUREMENTS

The total cross section measurement provides some well established bumps but the interpretation is not clear. They could be resonances, but as pointed out often, the mass value, for example, in the U (2350) corresponds suggestively to \( M_{\text{nucleon}} + M_{N^*(1400)} \) indicating a possible threshold effect. The question is open in the U-region, but there have been unsuccessful attempts\(^6\) to explain the T (2190) enhancement as the onset of significant single pion production associated with \( N^*(1236) \). The momentum dependence of the pion production cross section does not permit a complete explanation.

ELASTIC CHANNEL

Fig. 4 shows the existing data on the total elastic cross section through the U mass region. Within the errors there is no obvious structure. The situation could be improved by a single experiment covering the region, since even with the significant systematic uncertainties of extracting the total elastic cross section, the relative errors may not destroy any small structure. However if we take the resonance parameters determined from the experiment of Abrams et al\(^5\) a purely resonant contribution to the total elastic cross section of \( \sim 200 \mu b \) is estimated from a direct channel state. This assumes \( J=5 \) or 6 which gives an elasticity \( x^2 \), the contribution enters as \( x^2 \). The effect could be greater if there is some enhancement from interference due to the presence of a background with the same quantum numbers at the resonance.

The possibility of a phase shift analysis to extract resonance effects is remote, since in \( \bar{p}-p \) scattering we have two spin 1 particles, requiring 5 amplitudes, each complex, and from I-spin 1 or 0, we have 20 parameters.

The elastic scattering is dominated by the diffraction peak, so that in order to see contributions based on a \( \sim 200 \mu b \) enhancement in the total elastic cross section there is some attraction in looking in the backward hemisphere. In particular close to 180° it may be hoped that opposite parity background terms may cancel with opposite signs, and we may be left, in an ideal situation, with a differential cross section consisting of a resonance squared term. This has lead to a good deal of speculation as to the meaning of the "ups" and "downs" of the backward elastic cross section when plotted as a function of momentum.

Fig. 5 shows the backward elastic cross section of Yoh et al\(^7\). There have been suggestions that the effects seen are due to towers of resonant states\(^8\) in the S-T and U-regions. The authors can generate enough cross section at 180° from the total cross section states of Abrams\(^5\) et al. provided they make certain assumptions about them. However, they prefer to describe the data in terms of a diffraction model, deep structure at 0.9 GeV/c (t=7 GeV\(^2\) for 180°) corresponding to a second diffraction minimum.
The existence of this second minimum was not well established in a single experiment, in this momentum range, at the time of the interpretation of Yoh\textsuperscript{7)} et al. Fig.6 shows the preliminary data at 1.3 GeV/c from a QMC, Liverpool, DNPL-RHEL Collaboration\textsuperscript{9}). Two minima are clearly visible at \(t=0.37\) and \(-1.0\) GeV\(^2\).

Odorico\textsuperscript{10)} has pointed out that if one is prepared to overlook the existence of five independent amplitudes, and to consider \(\bar{p}-p\) elastic scattering in terms of one general amplitude, it is possible to obtain a systematic picture of the behaviour of the dips in the \(\bar{p}-p\) elastic channel in terms of intersecting resonances in the \(s-t-u\) plot. Fig.7 shows the \(s-t-u\) plot for \(\bar{p}-p\) and \(p-p\) elastic scattering; the \(u\)-channel being defined as \(p-p\). Data on the two established dips is shown. Odorico suggests that the dips trace out hyperbolae DIP I crosses the intersection of the two pion poles. DIP II (the second minima suggested by by Yoh et al) crosses the intersection of the \((\text{pp})\) systems with the pion poles, and a third dip, DIP III as yet not established should cross the intersection of the \(f^0A_2\) systems with the pions and the intersection of the two \((\text{pp})\) systems.

Pinsky\textsuperscript{11)} has proposed a new set of kinematic variables

\[
u \text{ and } \cos^2 \phi = \frac{s t}{(5 A^2)(t-4m^2)}
\]

with values of \(\cos \phi = -0.45\) and \(-0.66\) one obtains curves similar to those proposed by Odorico, except for the behaviour outside the physical region. It is not clear, as can be seen in fig.7, how the backward elastic data of Cline\textsuperscript{12)} et al. and the backward point of Yoh et al. fit into the scheme of two well behaved dips.

**CONCLUSIONS OF THE ELASTIC CHANNEL**

1) Because of the complexity, a phase shift solution is highly unlikely. The near backward differential section could show resonance squared terms in the absence of interfering background, which is probably an unrealistic model.

2) A clear systematic mapping of diffraction minima is needed before there can be any meaningful speculations about the momentum dependent structure at 180\(^0\) in terms of direct channel states. There could exist at least three minima, their \(t\) values being dependent on momentum, particularly at low momenta.

**THE CHARGE EXCHANGE CHANNEL, \(\bar{p}p \rightarrow \bar{\text{n}}n\)**

There are two experiments in this channel in which data has been collected with sufficient precision to see the formation of high mass states.
The first of these Bricman et al. took data from 1-3 GeV/c with a statistical accuracy of \( \sim 1 \% \). The momentum bite of the incident beam was equivalent to \( \pm 7 \) MeV in mass, and data points taken at approximately 7 MeV steps. The apparatus consisted of a hydrogen target surrounded by a box of veto counters, for charged particles, and a lead scintillator sandwich outside this, (\( \sim 3 \) radiation lengths) for \( \gamma \)-rays. The beam enters and leaves through a hole, and downstream there is another veto system. The principle of the measurement is that an antiproton enters the target and no detected particle leaves. The result requires corrections for the reaction leading to \( K^0_2 \ K^0_2 \), which is negligible; \( .06 \) mb out of 4-12 mb. A much more serious correction is the amount of the \( \bar{\eta}n \) final state which registers in the veto shield. This can be measured in principle, and turns out to be \( \sim 50 \% \). A knowledge of this factor is important for a good absolute measurement, but an error would probably not obscure any structure in the cross section.

Fig. 8 shows the data for the total cross section, the dotted lines show the scale uncertainty. The upper graph shows the deviations from the smooth curve, divided by \( 4 \pi \alpha^2 \). A fit was made to a smooth background plus resonances with masses and widths fixed by the data of Abrams et al. Table 3 shows the results. The column \( \Delta \text{ch-ex. expected} \), assumes \( J = 6 \) for states in the \( U \) region. Their best fit shows a bigger resonance contribution than expected at mass 2380 MeV, but the data is less precise in this region and allows the fit considerable freedom.

The second experiment is a recent Stonybrook-Wisconsin collaboration. They have measured the total charge exchange cross section and the differential cross sections in the near-forward and backward regions, \( |t| < 0.1 \), and \( |u| < 0.1 \) GeV\(^2\). The principle of the experiment is the same as that of Bricman et al. The hydrogen target is subdivided into three sections to localise the interaction, and surrounded by veto shields for charged particles and \( \gamma \)-rays. The new feature being the inclusion of steel-scintillator sandwiches specifically aimed at anti-neutron detection for the differential cross section measurements. The experiment has the significant correction for the detection of the \( \bar{\eta}n \) system in the veto shield. Data was collected at 40 momenta between 1 and 3.17 GeV/c. The statistical errors are \( \sim 1 \% \) and an overall scale uncertainty of \( 15 \% \) is claimed.

Figure 9 shows the data along with a representative curve for the results of Bricman et al. There is an obvious scale discrepancy. Also shown is the \( \bar{p}-p \) total cross section data of Abrams et al. scaled by 1/10. The total charge exchange cross section of Stonybrook-Wisconsin shows structure very similar to that in the \( \bar{p}-p \) total cross section.
The same collaboration \(^{15}\) has preliminary data on the backward cross section. The scope for systematic error is considerably increased. The detection efficiency for antineutrons varies from 60 % to 30 % through the momentum range. It can be measured by detecting the forward neutron for which the efficiency is \(\sim 10 \%\). Around 90\(^{\circ}\) in the lab., only \(\sim 50 \%\) of the slow antineutrons emerge from the target, and time of flight is used to improve their separation from \(\gamma\) rays. The backward differential cross section is shown in fig. 10. The authors point out a "shelf" at the \(U\) mass.

CONCLUSIONS FROM THE CHARGE EXCHANGE EXPERIMENTS

1) The difficulty of the experiments is considerable, hence a large scale discrepancy between the measurements.

2) The Stonybrook Wisconsin data shows confirmatory evidence for the structures seen in the \(\bar{p}p\)-total cross section measurement.

3) In view of the interpretational problems in the backward elastic cross section, one has considerable reservations about taking the structure in the backward differential cross section as any evidence of a state in the \(U\) region. It will probably be necessary to obtain a clear picture of structures in the crossed channels \(\bar{p}n \rightarrow \bar{p}n\) and \(np \rightarrow np\) before drawing conclusions.

THE CHANNEL \(\bar{p}p \rightarrow \pi^{+}\pi^{-}, K^{+}K^{-}\)

These channels are particularly interesting because they are described by two independent amplitudes and offer, in principle, the possibility of a phase shift analysis if \(d\sigma/d\Omega\) and polarization effects are measured.

Only the initial triplet state of \(\bar{p}p\) enters. This can be seen by considering parity conservation, \(P_{(pp)} = (-1)^{I+1} = P_{(\pi\pi)} = (-1)^{J}\). Also the \((\pi^{+}\pi^{-})\) final state has a \(G\)-parity constraint such that if \(J\) is odd \(I = 1\), and for \(J\) even \(I = 0\), \((G_{(\pi\pi)} = +1 = -1^{J+I})\)

There are several relevant experiments in the \(U\)-region.


b) Preliminary data is emerging from a Bubble Chamber collaboration of Liverpool, Glasgow, Saclay, Lausanne-Neuchatel \(^{17}\). The exposure spans the region 1.56-2.04 GeV/c.
c) Queen Mary College, Liverpool University, Daresbury, Rutherford 9) collaboration have preliminary results from a counter and spark chamber experiment.

d) University of Michigan Bubble Chamber group (Chapman18) et al) 1.6-2.2 GeV/c.

e) The CERN-Munich 19) group have data from the process $\bar{p}p + \pi^-$ at high energy, where single pion exchange gives them inverse formation.

f) A Yale20) counter and spark chamber experiment has data on polarization effects in $\bar{p}p + \pi^+\pi^-.$

The above experiments will be used to try to give a current picture of the state of knowledge of the $\pi^+\pi^-$ channel. The total cross section for this channel shows no marked structure.

The data of Nicholson 16) et al. came from two experiments; one near forward and backward directions, where the sign of the outgoing pion was known, and the other around 90° in the centre of mass, which was a by product of a search for $\bar{p}p \to e^+e^-,$ and had no magnet to give sign information. They produced folded angular distributions, which gave marked structure in the even Legendre coefficients used in fitting. At 2.0 GeV/c (mass 2.43 GeV) $A_{10}$ was significant, which could mean the presence of angular momentum states up to $J = 5.$

Nicholson16) et al. proposed a very simple model to fit their data. It is comprised of constant complex backgrounds in $J = 1,2$ and 3 partial waves, and has resonances $J = 3$ $M = 2.132$ GeV $\Gamma = .320$ GeV $I = 1$

and $J = 5$ $M = 2.287$ GeV $\Gamma = .159$ GeV $I = 1.$

The interference between the $J = 3$ and $J = 5$ states reproduced the observed enhancement in the $A_8$ coefficient.

There are several experiments which confirm the strong presence of $A_8$ in the U-region. The energy averaged data from 1.6-2.2 GeV/c of Chapman18) et al. shows structure requiring $A_8.$

Figure 11 shows very early data from a Liverpool-Glasgow-Saclay-Lausanne-Neuchatel 17) collaboration in the region 1.56-1.75 GeV/c. Their fit gives $A_8/A_0 = 2.14 \pm .5.$ The full experiment should produce $\sim 200\pi\pi$ events at each of 6 momenta between 1.56 and 2.04 GeV/c.

Figure 12 shows preliminary data from the QMC-Liverpool-DNPL-RHEL 9) collaboration. The fit gives $A_8/A_0 \sim .9 \pm .16.$ This experiment when completed should produce $\sim 2000\pi\pi$ events per angular distribution at $\sim 20$ momenta in the range 0.7-2.4 GeV/c.
The even Legendre expansion coefficients of Nicholson\textsuperscript{16}) et al. for $\bar{p}p \rightarrow \pi^+\pi^-$ are shown in figure 13 along with the curves produced by their model. The very preliminary data from the QMC-Liverpool-DNPL-RHEL\textsuperscript{9}) experiment is also plotted at the few momenta which have been partially analysed. The two measurements agree fairly well on the event coefficients and the model reproduces the general shape if not the detailed structure. The early indications are that the model of Nicholson et al.\textsuperscript{16}) does not fit the odd coefficients of ref. 9 very well although definite conclusions must await a full analysis. The general trends of the even coefficients do not appear to accommodate a narrow resonance of $\sim 30$ MeV in the U region in this process. This is confirmed quantitatively by the widths of the proposed states of ref. 16.

A Yale\textsuperscript{20}) group has a measurement of the asymmetry for the production of $\pi^-\pi^-$ off polarised protons at an incident antiproton momentum of 1.64 GeV/c. They have 350 events, and their result is shown in figure 14 along with the curve representing the prediction of the model of Nicholson\textsuperscript{16}) et al. The simple model clearly does not fit the data.

A further interesting source of information on $\bar{p}p \rightarrow \pi^+\pi^-$ comes from the CERN-Munich\textsuperscript{19}) group. They have studied the process $\pi^-p \rightarrow n\bar{p}p$ at 19.0 GeV/c. If one pion exchange dominates, the reaction may be represented by the following diagram.

An extrapolation to the pion pole means that they can measure the inverse formation reaction $\pi^-\pi^- \rightarrow \bar{p}p$. They have $\sim 10,000$ events covering an effective mass for the $\bar{p}p$ system from 2 nucleon masses to $\sim 3$ GeV. A comparison of the angular distribution at an effective mass of 2.53 GeV with the data of Allison\textsuperscript{21}) et al. tends to confirm the model of single pion exchange dominating the process. An experiment like this could produce data in the interesting region of effective mass, namely from 2 nucleon masses to $\sim 2$ GeV, which for physical experiments, using antiprotons, means momenta below 700 MeV/c, where the low flux limitations for spark chambers are severe, and it is unlikely that bubble chambers will ever produce $\sim 2000$ events per angular distribution because of the massive measuring effort required.
CONCLUSIONS ON \( \bar{p}p \rightarrow \pi\pi \)

The model of Nicholson\(^{16}\) et al, which required a state of \(J=5\) in the \(U\) region, is too simple to explain the new polarisation data. One should soon have data capable of making statements about the existence of resonances, but polarisation measurements will be required, since the channel may be complicated by the presence of several states.

MANY BODY FINAL STATES

The many body final states in formation is a field specially suited to bubble chamber study. The acceptance calculations for a conventional spark chamber experiment make the measurements prohibitive, although they may be feasible in spectrometers like Omega.

THE CHANNEL \( \bar{p}p \rightarrow \bar{K}K\pi\pi \)

The University of Michigan bubble chamber group has data on this channel at six momenta in the range 1.6–2.2 GeV/c (Chapman\(^{22}\) et al.)

The channels studies are the following

\[
\begin{align*}
\bar{p}p &\rightarrow K^0_L K^0_L \pi^+ \pi^- \pi^0 \quad (207 \text{ events}) \\
\bar{p}p &\rightarrow K^0_L K^+ \pi^+ \pi^- \quad (347 \text{ events}) \\
\bar{p}p &\rightarrow K^0_L K^0_2 \pi^+ \pi^- \pi^0 \quad (518 \text{ events}) \\
\end{align*}
\]

The reactions (1)(2) and (4) are overconstrained in the chamber, but reaction (3) contains two unseen particles and therefore cannot be distinguished from \( K^0_L K^0_2 \pi^+ \pi^- \pi^0 \quad m > 1 \), however it is included because it provides an upper limit for \( K^0_L K^0_2 \omega \).

A very careful analysis gives a clean sample of reaction (4) free from multi-pion background. The authors quote possibly 31 events ambiguous with all pions. Therefore they believe that they have a sample of \( K^+ K^- \pi^+ \pi^- \pi^0 \) essentially free from background, and when weighted, without bias with respect to any particular region of phase space.

The cross sections are shown in figure 15 along with other data at higher and lower momenta. In the channel \( K^+ K^- \pi^+ \pi^- \pi^0 \) at 2347 MeV there appears to be a three standard deviation enhancement above the mean of the other five points. There is evidence for strong \( \omega \) production in \( \pi^+ \pi^- \pi^0 \) and by making mass cuts the following channels, \( K^+ K^- \omega \) (with \( \omega \mathregular{\sim} 23 \) \( \uparrow \) non \( \omega \) background inside the mass cut),
and $K_1^0 K_1^0 \omega$ (≈10 % non $\omega$) can be measured. The cross sections are shown also in figure 15. The data does not confirm a earlier enhancement in $K_1^0 K_1^0 \omega$ suggested by the same group at 2368 MeV. They derive a 3-4 standard deviation enhancement of $146 \pm 49 \text{mb}$ at 2347 MeV in the $K^+ K^- \omega$ channel.

If the 2347 MeV enhancement is due to a direct channel state, then it should be seen, depending on $C$ quantum number in $K^0 K^0 \omega$ channels. For $C$ odd, an enhancement is expected in $K_1^0 K_1^0 \omega$, or for $C$ even, $K_1^0 K_2^0 \omega$ should show a peak. Neither channel confirms the $K^+ K^- \omega$ state.

The group see $K^*(890)$ in reactions (1)(2), and (4), but do not confirm a previous enhancement of Oh et al.\textsuperscript{23} at 2.35 GeV in $K^* K^{\pi \pi}$

CONCLUSIONS ON $KK^{\pi \pi}$

(1) It is hard to interpret the 3-4 standard deviation effect in $K^+ K^- \omega$ at 2347 MeV as a direct channel effect since it fails to appear in $K^0 K^0 \omega$.

(2) The previous states in $K_1^0 K_1^0 \omega$ at 2370 MeV and $K^* K^{\pi \pi}$ 2360 MeV are not confirmed.

We conclude the review of experiments in the $U$ region by considering a measurement which is producing preliminary data in the many body channels, but in a novel way, in which no attempt is made at identification of final particles, but an effort is made to determine the multiplicity, and how peripheral the production process was.

THE RUTGERS ANNIHILATION SPECTROMETER \textsuperscript{24}

The principle behind the apparatus is to enhance effects from resonance formation. If a high mass state is formed, it could be narrow and have a high $J$ value with a preferred decay mode by pion cascade through other states. Evidence for such a state formed in $\bar{p}-p$ could appear as a non peripheral process, with a high final multiplicity, as distinct from elastic scattering or $N^*$ production e.g. at laboratory angles $>20^\circ$ only $\approx 1 \%$ of the elastic cross section remains, whereas $\approx 50 \%$ of the six prong annihilation remains.

The apparatus consists of a 60 cm hydrogen target with scintillation counters downstream which subtend $2^\circ$, $5^\circ$, $10^\circ$ and $20^\circ$. These counters measure the degree of "anti-peripheralism" in four bins $> 20^\circ$, $10^\circ-20^\circ$, $5^\circ-10^\circ$, and $2^\circ-5^\circ$. The target is surrounded by 32 scintillators which determine the multiplicity in eight groups $0,1,2\ldots8$. Data is collected in 7 momentum
bins for a given incident central momentum and by taking in .5 % steps in
momentum data is obtained in overlapping regions. At 2.0 GeV/c the mass
resolution is ±5 MeV and ~10^6 events are collected in a 5 MeV mass bite.

The data in the U region is shown in figure 16. There is a clear
enhancement around the U mass. The data can be subdivided into 32 classes;
8 levels of multiplicity for each of four levels of "anti peripheralism". The
enhancement is most pronounced in the 2.5°-5° data and with multiplicity
equal to 0 or 1. The results are therefore a complete reversal of what might
have been expected, the enhancement is produced peripherally, has low
multiplicity and is broad. The mass value is ~2360 MeV with a width ~100 MeV.

The same experiment has preliminary data through the T and U regions.
The data is shown in figure 17. The resonant cross sections extracted from
this data are shown in figure 18 along with the resonant cross sections from
Abrams et al. The similarity to the bumps in the total cross sections is
remarkable.

The experiment has no statistical limitations, so the precision lies
in understanding systematic effects. The results raise interpretational
problems because of the nature of the enhancements. The authors speculate
that the enhancement in the U region could be due to the onset of N(400)
production.

A SUMMARY OF THE U-REGION; SPECULATIONS AND RESULTS

We can ask the question whether we have any idea of what states we expect
to see, or rather what we would like to see, in this mass region. If the
p-A_2-g trajectory is extrapolated it should have a state on it at 2380 MeV
with J=6; also we might reasonably expect a set of parallel trajectories
opening up the possibility of states from J=0 up to J=6 - a tower of meson states.

If we allow ourselves the freedom of taking indications from theoretical
models we can get an idea of the expected width. Chan and Tsou have
calculated the decay patterns and widths for high spin states within the
framework of a Veneziano model. They suggest that \( \Gamma \propto 1/\sqrt{J} \) which gives
\( \Gamma \propto \text{mass} \propto \text{constant}. \) In order to fix the scale they insert \( \Gamma^0 = 100 \text{ MeV} \)
for \( J=1 \) which predicts \( \Gamma = 24 \text{ MeV} \) for \( J=9 \) or approximately, \( \Gamma \propto 30 \text{ MeV} \) for \( J=6 \)
one sees how neatly the proposed states of the CERN missing mass experiment
fit the scheme. It is comparatively easy to obtain a mass resolution of
±5 MeV in \( \bar{p}-p \) formation experiments so states with \( \Gamma \sim 30 \text{ MeV} \) should be
readily observable.
Chan and Tsou (25) also compute the widths of daughter states. The first daughter could have a width \( \approx \)5 times the leading state i.e. \( \approx \)150 MeV. Subsequent daughters get wider and consequently could be hard to observe. Thus the theoretical model suggests that we might expect a state of \( J=6 \) with \( \Gamma \approx 30 \) MeV plus a state \( J=5 \) with \( \Gamma \approx 150 \) MeV; plus of course other daughter states.

In \( \bar{p}-p \) formation, if one takes a simple impact parameter model and a radius of 1 fm, we can only excite angular momentum states up to \( J=4 \). However if we extract an effective radius from the position of the first diffraction minimum in the elastic scattering, we may reach \( J=5 \). Therefore on this very simple minded approach, if the \( U \) exists with \( J=6 \), we could not form it in \( \bar{p}-p \), however we may be able to reach a state with \( J=5 \), and the model (25) suggests it might have \( \Gamma \approx 150 \) MeV.

We stop speculating and examine the accumulated evidence presented in figure 19. The original CERN missing (1) mass evidence is not confirmed in the Northeastern Stonybrook (2) experiment hence the question mark. The \( \pi\pi \) state of Nicholson (16) et al is an oversimplication of that channel. The \( K^+K^-\pi^-\pi^+ \) state of Oh (23) et al is not confirmed, and the 3-4 standard deviation effect of Chapman (22) et al in \( K^+K^-\omega \) which does not appear in \( K^0\bar{K}^0\omega \) must await confirmation. There remain three observations of very similar effects. The total cross section data, the charge exchange data and the RAS results all have an enhancement at \( \approx 2380 \) MeV with a width \( \approx 100 \) MeV. The \( \bar{p}-p \), \( \bar{p}-d \) data of ref. 5 is able to give two \( I\pi\pi \) states \( I=0 \) and \( I=1 \) with roughly similar parameters.

We can ask the question as to whether we are seeing a threshold effect or a resonance - perhaps a daughter state?

**Threshold?**

The data of RAS shows the state to be peripherally produced and of low multiplicity they suggest \( N^*(1400) \) production. The data to check this speculation already exists on bubble chamber film. One is not too hopeful for this solution, since single pion production could not fully account for the \( T \) region enhancement (6) in the total cross section.

**Resonance?**

In order to establish a resonance state we will either have to examine the behaviour of the Real and Imaginary parts of an amplitude in an Argand diagram, or isolate a particular set of quantum numbers and study variations as a function of \( s \). What are the future possibilities?
In the elastic channel we are presently far removed from a phase shift analysis. We may be fortunate in the backward hemisphere if we can observe the effects of resonance squared terms in the angular distribution, however there remains the question of background amplitudes.

The elastic channel is potentially very interesting since we have the problem of an illegal duality diagram²⁷).

\[ \begin{array}{c}
\text{\includegraphics[width=0.3\textwidth]{diagram}} \\
\end{array} \]

The u-channel with $B=2$ cannot contain particles, and we cannot construct $\bar{q}q$, or $qqq$ states in both $s$ and $t$ channels in the above diagram.

The situation corresponds to the 'null solution' with real amplitudes and no particles present. However experimentally we know resonances participate in $\bar{NN}$ scattering. One solution to the $\bar{NN}$ problem suggested by Rosner²⁸) is to have exotic states in the multiplets (10), (10) and (27) which couple only to $\bar{NN}$. It is a pity that interesting predictions are made for this channel, which presents such significant analysis difficulties.

The $\pi\pi$ channel shows perhaps the best hope of providing evidence for resonances in formation. We have two amplitudes, and differential cross section and polarization measurements, could lead to a phase shift solution. We should not however push the similarity with the $\pi-p$ system too far, since we do not have several of the simplifying features of the $\pi-p$ channel - not the least of which, is the existence of known states to help fix the phase at some points. Also the $\pi\pi$ channel will have considerable angular momentum barriers which mean one is looking mainly for low $J$, and perhaps very wide resonances.

The bubble chambers can continue to study the multibody final states e.g. $\rho\pi$, $\rho\rho$, $A_2\rho$, $KK\omega$ etc. but the need for significant increases in the number of events is ever apparent. Should a clear resonance emerge then there are many other channels which one would be encouraged to explore e.g.

\[ \begin{array}{l}
\bar{p}p \rightarrow \pi^0\eta^0, \quad \pi^0\eta \quad \text{- already a proposed counter experiment} \\
\bar{p}p \rightarrow K^0\eta^0_1k^0_2 \quad \text{- a feasible counter experiment with angular momentum constraints built in.} \\
\bar{p}n \rightarrow K^-k^0 \quad \text{- would help unravel I spin in the KK channel.} \\
\end{array} \]

Plus of course the massive BC efforts in many body channels.
A final personal conclusion is that we do not have evidence for a narrow state in the U region from formation experiments. We do have several pieces of evidence for a broad enhancement. The question of whether or not it is a resonance is open and intriguing.

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* * *

REFERENCES


2) The results of the Northeastern Stonybrook experiment were reported at the International Conference on High Energy Collisions (4th Stonybrook) Oxford 1972. Figure 2 of their preliminary data is taken from a talk by D. Garellick at the Summer Workshop on Meson Spectroscopy held at ANL, July 1971.


9) Preliminary unpublished data from QMC-Liverpool-DNPL-RHEL
   E. Eisenhandler, W.R. Gibson, C. Ho\'vat, P.I.P. Kalmus,
   L.L. Lee Chi Kwong, T.W. Pritchard, E. Usher, D.T. Williams,
   M. Harrison, W.H. Range, M.A.R. Kemp, A.D. Rush, J.N. Woulfs,


12) D. Cline, J. English, D.D. Reeder (University of Wisconsin report 1971,
    to be published).

13) C. Bricman, M. Ferro-Luzzi, J.M. Perreau, J,K. Walker, G. Bizard,
    Y. Declais, J. Duchon, J. Séguinot, G. Valladas, Phys. Lett. 29B

14) D. Cutts, R. Pittman, Y.Y. Lee, M.L. Good, J. Storer, D.D. Reeder -
    preprint - October 1972.

15) Stonybrook - Wisconsin collaboration, private communication of preliminary data.

16) H. Nicholson, B.C. Barish, J. Pine, A.V. Tollestrup, J.K. Yoh, C. Delorme,
    F. Lobkovicz, A.C. Melissinos, Y. Nagashima, A.S. Carroll, R.H. Phillips,

17) Preliminary data from U-region exposure 1.56-2.04 GeV/c, Liverpool,
    Glasgow, Saclay, Lausanne, Neuchâtel: Collaboration.

    Lett. 21 (1968) 1718.

19) G. Grayer, B. Hyams, C. Jones, P. Schlein, P. Weilhammer, W. Blum, H. Dietl,
    W. Koch, E. Lorenz, G. Lütjens, W. Männner, J. Meissburger, W. Ochs,

20) R.D. Ehrlich, A. Etkin, P. Gledis, V.W. Hughes, K. Kondo, D.C. Lu,

21) T. Fields, W.A. Cooper, D.S. Rhines, W.W.M. Allison, Oxford University
    report October 1971.

22) J.W. Chapman, R. Green, J. Lys, C.T. Murphy, H.M. Ring, J.C. Vander Velde,

23) B.Y. Oh, D.L. Parker, P.S. Eastman, G.A. Smith, R.J. Sprafka, Z. Ma,

    F. Sannes, D. Van Harlingen, G. Cuijanovich, M. Martin, Rutgers Report
    HEP-72-104.


    Lett. 17 (1966) 890.


FIGURES

2) Data from the CERN missing mass experiment and Preliminary data from the Northeastern Stonybrook experiment.
3) Resonant cross sections from total cross sections of Abrams et al.
4) Data on the total elastic cross section in the U-region.
5) Backward elastic cross section data of Yoh et al. (-0.98 < cosθ* < -1.0)
6) Preliminary data on the differential cross section for elastic scattering at 1.5 GeV/c from the Queen Mary College-Liverpool-DNPL-RHEL collaboration.
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11) Preliminary data for the angular distribution of pp → π⁺π⁻ 1.57-1.75 GeV/c.
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13) Preliminary data from QMC-Liverpool-DNPL-RHEL pp → π⁺π⁻ at 1.99 GeV/c.
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15) The angular distribution of the asymmetry of π⁺π⁻ produced from a polarised target. -Yale-
16) Cross sections for the reactions pp → KKππ 1.6-2.2 GeV/c, Chapman et al.
17) Preliminary data in the U-region from the Rutgers Annihilation Spectrometer (RAS)
18) Preliminary data in the Tand U-region from RAS.
19) Resonant cross sections in T and U from RAS and p-p total cross section measurements.
20) Summary of data in U-region.

* * *

TABLES

Table 1 : Summary of Missing Mass experiments
Table 2 : Resonant parameters from total cross section data of Abrams et al.
Table 3 : Fils of Bricman et al. for resonance contributions in pp → n̄n
Table 1

Summary of Missing Mass experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Incident Momentum GeV/c</th>
<th>t-Range GeV²</th>
<th>Mass Resolution MeV</th>
<th>dσ/dt</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN MISSING MASS</td>
<td>12</td>
<td>.28&lt;</td>
<td>t</td>
<td>&lt;.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(252 events in U peak)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(signal to noise 1:6)</td>
</tr>
<tr>
<td>NORTHEASTERN STONYBROOK</td>
<td>13.5</td>
<td>.2 &lt;</td>
<td>t</td>
<td>&lt; .3</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>.2 &lt;</td>
<td>t</td>
<td>&lt; .3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Γ_U=0) 4±2µb/GeV²</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Γ_U=30) 6±4µb/GeV²</td>
</tr>
<tr>
<td>CERN IHEP BOSON SPEC.</td>
<td>25</td>
<td>.17&lt;</td>
<td>t</td>
<td>&lt;.35</td>
</tr>
</tbody>
</table>

Table 2

Resonant parameters from total cross section data of Abrams et al.

<table>
<thead>
<tr>
<th>MASS MeV</th>
<th>Γ MeV</th>
<th>P GeV/c</th>
<th>I</th>
<th>H (mb)</th>
<th>4πx² (mb)</th>
<th>(J+1)x</th>
<th>x(J=6)</th>
<th>x(J=5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2190±10</td>
<td>85</td>
<td>1.32</td>
<td>1</td>
<td>5.5</td>
<td>15.4</td>
<td>.36</td>
<td>-</td>
<td>.13</td>
</tr>
<tr>
<td>2350±10</td>
<td>140</td>
<td>1.77</td>
<td>1</td>
<td>3.2</td>
<td>9.8</td>
<td>.32</td>
<td>.10</td>
<td>.12</td>
</tr>
<tr>
<td>2375±10</td>
<td>190</td>
<td>1.84</td>
<td>0</td>
<td>2.5</td>
<td>9.2</td>
<td>.27</td>
<td>.08</td>
<td>.10</td>
</tr>
</tbody>
</table>
Table 3

Fits of Brice\textit{man et al.} for resonance contributions in $\bar{p}p + \bar{n}n$

<table>
<thead>
<tr>
<th>MASS MeV</th>
<th>$\Gamma$ MeV</th>
<th>$\Delta TOT$</th>
<th>$\Delta$CH-EX EXPECTED</th>
<th>$\Delta$CH-EX FITTED</th>
<th>ASSUME J</th>
</tr>
</thead>
<tbody>
<tr>
<td>2190</td>
<td>85</td>
<td>0.4</td>
<td>0.013</td>
<td>0.004</td>
<td>5</td>
</tr>
<tr>
<td>2345</td>
<td>140</td>
<td>0.3</td>
<td>0.007</td>
<td>0.001</td>
<td>6</td>
</tr>
<tr>
<td>2380</td>
<td>140</td>
<td>0.2</td>
<td>0.004</td>
<td>0.014</td>
<td>6</td>
</tr>
</tbody>
</table>
END OF 100% EFFICIENCY
FIRST OBSERVATION IN ANGULAR DISTRIBUTION

A.

100% EFFICIENCY
COMBINED DATA
MAY 1965
NOVEMBER 1965
ALL DECAY MODES

$\mu = 2.382.$

B.

Fig. 1
CERN MMS.

\[ \frac{d\sigma}{d\Delta M} \left[ \frac{\mu b}{(GeV)^2/10MeV} \right] \]

- 7,12 GeV/c
- 12 GeV/c

MASS (GeV)

PRELIMINARY DATA
NORTHEASTERN/STONY BROOK

MASS \( \times \) GeV

DETECTION EFFICIENCY

Fig. 2
Fig. 3
Fig. 5
Fig. 6

\[ \frac{d\sigma}{d\Omega} \quad \text{mb/sr} \quad \text{vs} \quad \cos \theta^* \]
\[
\cos \phi = -0.659
\]

\[
\cos \phi = -0.45
\]

- CLINE ET AL
- YOH ET AL
- BACON ETAL
- QMCL'POOL-DNPL-RHEL
- CERN-HOLLAND
- KATZ ET AL
- CERN-PARIS-ORSAY-STOCKHOLM
- ODORICO (CURVES NOT FITTED)
- PINSKY

---

\[ (f, A_2) \]

\[ (\omega p) \]

s=0

π

pp - pp

\[ t=0 \]

\[ \pi \]

Fig. 7
Fig. 9
$\bar{p}p$ CHARGE EXCHANGE SCATTERING

BACKWARD RATE (PRELIMINARY DATA)

$-0.95 > \cos \theta_{cm} \geq -1.0$

Fig. 10
\( \bar{p} p \rightarrow \pi^+ \pi^- \) \( 1.56 - 1.75 \text{ GeV/c} \)

PRELIMINARY DATA

No. of EVENTS

\( \cos \theta_{cm} \)

Fig. 11
\( \bar{p}p \rightarrow \pi^+\pi^- \) 1.99 GeVc
PRELIMINARY ANALYSIS OF
\(-950\) EVENTS
\( \bar{p}p \rightarrow \pi^+\pi^- \) EVEN LEGENDRE COEFFICIENTS

- Nicholson et al.
- Preliminary analysis of QMC - Liverpool-DNPL-RHEL collaboration.

Fig. 13
YALE POLARISATION
DATA 1.64 GeVc \bar{p} p \to \pi^+\pi^-
CURVE CAL. TECH. ROCH. BNL MODEL (NICHOLSON) ET AL

Fig. 14
\begin{center}

\begin{figure}[h]
\includegraphics[width=\textwidth]{figure15}
\caption{Fig. 15}
\end{figure}
\end{center}
CROSS SECTION > 2 1/2°
ALL MULTIPLICITY
(PRELIMINARY DATA)

Fig. 16
\( \theta \geq 2.5^\circ \)

\( M = 0.7 \)

6 MeV BINNING

(PRELIMINARY)
R.A.S.
(PRELIMINARY RESULTS)

\[ \sigma_R \text{ (mb)} \]

\[ \begin{array}{c|c|c|c|c|c}
2.1 & 2.19 & 2.28 & 2.37 & 2.46 \\
0 & 0.8 & 1.6 & 2.4 & 2.4 \\
\end{array} \]

MASS (GeV)

ABRAMS ET AL
\( \bar{p} - p \) \( \sigma_T \) BUMPS.

\[ \begin{array}{c|c|c|c|c|c|c}
2.1 & 2.2 & 2.3 & 2.4 & 2.5 \\
0 & 1 & 2 & 3 & 0 \\
\end{array} \]

MASS GeV

RESONANT CROSS SECTION mb

Fig. 18
CERN MM.\textsuperscript{1)} "ORIGINAL U"

BROAD $\sigma_T$ $\bar{p}p$ BUMP.\textsuperscript{5)}
$\sigma(pp), \sigma(\bar{p}d)$ ALLOWS I SPIN DETERMINATION

$\sigma_T (\bar{p}p - n\bar{n})$\textsuperscript{14)}

$\bar{p}p - \pi^+ \pi^-$\textsuperscript{16)} EVEN LGND COEFF.

$K^+ K^- \pi^+$\textsuperscript{23)}
$(\bar{p}p, \bar{p}d)$

$K^+ K^-$\textsuperscript{22)}

RAS.\textsuperscript{24)} PERIPHERAL, LOW MULT.

Fig. 19
Mr. Armenteros: What did you comment on this particular quark diagram representing elastic scattering in connection with $\bar{p}p \rightarrow \pi^+\pi^-$?

Mr. Astbury: The problem in fact does not arise in $\pi^+\pi^-$ because I think we can draw a perfectly valid quark diagram. This is not the case for the elastic scattering. Here the u-channel does not have particles because $B = 2$, and one cannot cut the diagram in the t and s channels and have non-exotic particles as duality requires. One solution is to call this a null solution with no particles in any channel and real amplitudes. But this statement is physically not correct, because one knows that resonances are exchanged in the elastic channel. So to get around this, it has been suggested that there exists a set of new resonant states which belong to 10, 10 and 27 and couple only to $\bar{p}p$.

Mr. Armenteros: In this counter experiments the acceptance problem is a serious one, because the data are strongly corrected.

Mr. Astbury: In principle the problem is simple: you generate events with a Monte-Carlo program, you know the limiting apertures of the apparatus from the measured events and surveying, you can impose these apertures on the generated events and build up a geometrical efficiency curve in the centre of mass. Then you take the events found in the actual experiment and you correct them with the geometrical curve to get the differential cross section. To test the method you have only to do the same thing for an experimental well known distribution.

Mr. Jeannet: Presents some preliminary results on $K\bar{K}\omega$ coming from a $\bar{p}p$ formation experiment between 1.5 and 2.05 GeV/c, done in collaboration with Paris-Liverpool-Lausanne and Glasgow. (fig. 1).
MODELS FOR NUCLEON-ANTINUCLEON ANNIHILATIONS

J. Vandermeulen
Université de Liège - P.M. 72/3

1. INTRODUCTION

This is a partial review. It concentrates on a few general features of phenomenological character and describes models of statistical type and related, which attempt at systematizing the properties like the structure of the multiplicity distribution, the magnitude and energy variation of the multipion cross-sections.

Nucleon-antinucleon annihilations provide multiple production (mainly of pions) already at low kinetic energies. Since the reaction implies the conversion of the nucleon masses, a rather large total energy is available for the products, but it is remarkable that a substantial (∼ 40%) fraction is used up for the pion (and kaon) masses. Table 1 shows the difference in the average multiplicity of produced pions between production in pp collisions and p̅p annihilations (Q is the available energy, equal to \( \sqrt{s} \) for annihilation, to \( \sqrt{s} - 2M_p \) for pp collisions).

<table>
<thead>
<tr>
<th>lab. momentum (GeV/c)</th>
<th>Q(GeV)</th>
<th>(&lt;n_p&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp</td>
<td>19</td>
<td>4.25</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>5.10</td>
</tr>
<tr>
<td>p̅p</td>
<td>0</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>3.85</td>
</tr>
</tbody>
</table>

Nucleon-antinucleon annihilation thus appears as a special type of multiple production at low and intermediate energies. The connections with other aspects of hadron collisions have not yet been much explored. For example, there might exist a relation between the annihilation and the process of pionization in high energy collisions, if such a mechanism distinct from the excitation of the incident particles exists.

The annihilation cross-section is large at low kinetic energies and is still of the same magnitude as the non-annihilation inelastic channels near 6 GeV/c. If the difference \( \sigma_{\text{tot}}(p\bar{p}) - \sigma_{\text{tot}}(pp) \) at high energy is attributed to annihilation, \( \sigma_{\text{ann}} \) is still equal to 5mb at 50 GeV (lab. energy). The empirical formula

\[
\sigma_{\text{ann}} = \frac{45}{\sqrt{s} - 1.4} \text{ (millibarn)}
\]
reasonably fits the annihilation cross-section (including channels with kaon production) from 1.2 GeV/c ($\sqrt{s} = 2.15$ GeV$^2$) to 50 GeV ($\sqrt{s} = 10$ GeV$^2$). Formula (1) should not be extrapolated down towards threshold. The annihilation cross-section in the very low energy region grows very large; it is not known precisely close to threshold. The onset of partial waves is rapid; the application of the unitarity limits shows that the minimum number of partial waves is already 2 at 50 MeV lab. kinetic energy and 3 beyond 100 MeV; the analysis of the angular distribution of elastic scattering requires up to at least D-waves between 60 and 100 MeV and at least F-waves from 100 MeV cm$^{-1}$.

The relative magnitude of the absorption in the various partial waves is not known. Evidence that S-capture for annihilation strongly dominates in annihilation at rest, in similarity to other hadronic atoms, has recently been challenged by an evidence for a substantial $2^0/n^0/n^+$ ratio$^2$.

The comparatively large value of the average pion multiplicity - and the narrowness of the multiplicity distribution - in annihilation reflects the fact that the final particles use up much more of the available phase space than the particles produced in high energy collisions do. In that sense, annihilations could be regarded as "central" collisions as opposed to peripheral ones. Let

$$<p_t^{(O)}> \lesssim 0.35 \text{ GeV/c} \quad (2)$$

be the "universal" average transverse momentum. In an isotropic distribution (as results from uniform phase space population)

$$<p_t> = \frac{n}{4} <p> \quad (3)$$

Therefore, a necessary condition for the realization of a peripheral event (i.e., one where extra energy is drained into the longitudinal motion) is:

$$<e> \lesssim <p> \gtrsim \frac{4}{n} <p_t^{(O)}> \lesssim 0.45 \text{ GeV} = e^0 \quad (4)$$

A critical multiplicity of produced pions:

$$N^{(O)} = \frac{\sqrt{s^2}}{c^0} \lesssim 2.2 \sqrt{s} \quad (5)$$

defines a kind of threshold for peripherality. Comparing the values of $N^{(O)}$ with those of $<\lambda>$ in Table 1, it is seen that even in the higher region of the studied energy domain the kinematical limitations are important for an appreciable fraction of the multiplicity distribution.
2. THE MULTIPLICITY DISTRIBUTION OF PIONS

2.1 The multiplicity of charged pions is determined more easily than the multiplicity including the neutral ones. The dependence of $<n_c>$ as a function of $\sqrt{s}$ can be fitted by a formula

$$<n_c> = a + b \sqrt{s}$$ (6)

in reasonable approximation from 0 to 7 GeV/c ($s = 4$ to 15) but the data suggest a flattening at higher energy.

The shape of the multiplicity distribution (including neutral and charged pions) obtained for annihilation at rest$^3$ is shown in Fig. 1, where the predictions of the statistical model of uncorrelated pion production are shown for comparison. A Poisson shape with $<N> = 5$ is also shown. The qualitative agreement (the fit is very poor, see Appendix I) between experiment and the statistical model suggests that the narrowness of the multiplicity distribution derives from the kinematical restrictions. This feature still holds at higher energies, but we believe that comparisons with Poisson shapes or variants should then be applied to the whole inelastic channels, including pion production without annihilation.

2.2 Branching ratios between charge configurations

The cross-sections for annihilation into pions are given in terms of "topologies":

$$p + \bar{p} \rightarrow k(\pi^+ + \pi^-)$$

(7a)

$$\rightarrow k(\pi^+ + \pi^-) + \pi^0$$

(7b)

$$\rightarrow k(\pi^+ + \pi^-) + \Lambda\bar{\Lambda}$$

(7c)

where $k = 1, 2$, etc., and where $\Lambda\bar{\Lambda}$ means a system of at least two neutral pions. The bubble chamber technique presently does not allow the specification of the $\Lambda\bar{\Lambda}$ content in individual events. The contribution of channels of type c) is large and increases with energy: $\sim 60\%$ of all annihilations at rest, $\sim 85\%$ at 5.7 GeV/c. We call "leading" channel the channel of type a) for multiplicity 2k, of type b) for multiplicity (2k + 1).

The structure of the pion collection in charge and number is governed by: a) the multiplicity distribution: what percentage of the annihilations gives $2\pi$, $3\pi$, $4\pi$, etc.; b) the branching ratios between the various charge configurations for each multiplicity, for example: in the $3\pi$ states, how much do $2\pi + 2\pi^0$, $\pi^+ + 3\pi^0$, and $5\pi^0$ contribute. Topologies of the type c) of course contain the contribution of several multiplicities; as long as the neutral pions will not be identified nor their momenta measured, the study of the properties of annihilation will be hindered.

Already welcomed is the determination of the average number of neutral pions associated to a given number 2k of charged pions. The failure of isospin statistical branching ratios in this matter is to be recalled; more details are found in Appendix 2.
2.3 How to estimate the average number of neutral pions associated to a given number of charged pions?

The average number \( \langle \lambda_k \rangle \) of missing \( \pi^0 \)'s in (MM) for a given \( k \) depends of course on \( k \) and \( \sqrt{s} \). One tends to believe that \( \langle \lambda_k \rangle \) should decrease when \( k \) increases, as it is the case for example in statistical models. This is not necessarily the rule and it is amusing to mention a simple model case (not realized in annihilation!): if pions were produced through \( I = 0 \) pairs exclusively, with a Poisson law for the pair multiplicity \( \lambda \), then

\[
\langle \lambda_k \rangle = \sum_{i=k}^{\infty} 2(i-k)e^{-\lambda}\frac{\lambda^i}{i!} \binom{i-k}{k} \binom{i}{k} / \ldots
\]

\[
/ \sum_{i=k}^{\infty} e^{-\lambda}\frac{\lambda^i}{i!} \binom{i-k}{k} \binom{i}{k} = \frac{2}{3} \lambda = \frac{1}{3} \langle \lambda \rangle
\]

would be independent of \( k \).

The simplest method to find out the average number of missing \( \pi^0 \)'s assumes that - in all pion multiplicities - the total energy is shared equally between the pions irrespectively of charge, an assumption which can be checked in the leading channels and seems to be a very good approximation, at least for the higher multiplicities. Some (weak) deviation should come from the association of charge with larger longitudinal momentum.

A slightly different method which is in principle a finer analysis consists in fitting the missing mass distribution (mass spectrum of the MM object) as a combination of contributions with \( k = 2, 3, \ldots \) of variable weights. A model has thus to be chosen for the properties in multiplicities \( (2k + 2), (2k + 3), \ldots \). The simplest model is "phase space", namely the assumption that the invariant mass of the group of \( \pi^0 \)'s corresponds to a constant matrix element in every multiplicity. The approach is especially worthwhile when it extrapolates the information gained in the detailed study of "leading" channels, thus replacing the constant matrix element by a definite structure with resonant factors, etc. This is what has been done for annihilation at rest.

The information obtained by these methods must in principle be used when comparison is made between experiment and the models which predict globally the properties of the multiplicity distribution. However, the models, some of which will be reviewed now, have not yet reached the stage of such moderately ambitious tests.

3. FERMI_STATISTICAL MODELS

3.1 The relative probability for observing \( N(n_+^* n_0^* n_-) \) particles of mass \( m \), members of an isospin multiplet, is written:

\[
P(S; I; n_+^* n_0^* n_-) = \frac{1}{n_+^* n_0^* n_-} W(I, n_+^* n_0^* n_-) R_N(S; N, m) \frac{K-2N}{(2N)^N}
\]
where $W$ is a spin-isospin weight, $R_N$ is the invariant momentum space integral and $K$ is a parameter with the dimensions of a mass. The correspondence:

$$m^{-1}_\pi \kappa^{-2} = \Omega = \lambda \Omega_0$$  \hspace{1cm} (10)

where $\Omega_0 = \frac{4}{3} \pi m^{-3}_\pi$ is the "normal" interaction volume is often made. It is known that the traditional model of uncorrelated pion production requires $\lambda \approx 5\text{--}10$ to fit the average pion multiplicity$^4,5)$. The large value of $\lambda$ is often supposed to indicate that resonances should be incorporated into the model.

3.2 A treatment$^6)$ by Lamb has included resonance production by extending the usual statistical treatment of isospin invariance to SU(3) invariance and by introducing several multiplets. The following set was retained: $0^-$, $1^-$ and $2^+$ mesons; any selection is of course arbitrary.

There are successes in this approach: a) fits of topologies are fairly good; b) symmetry violation is interpreted by a ratio $\Omega_K/\Omega_\pi$ independent of energy. But the energy dependence of the interaction volume is strong and even leads to a volume equal to zero at finite energy. Even then, the fast decrease of the $2\pi$ channel when energy increases is not accounted for (see Fig. 2); the disagreement between the $2\pi$ contribution and the average multiplicity is a recurrent shortcoming in variants of the statistical hypothesis.

3.3 Note that Lamb has formulated the model to compute not simply rates but also the magnitude and energy variation of the cross-sections. Quite generally we can write the cross-section for a particular channel in the form:

$$\sigma_N(\sqrt{s}; n_+, n_0, n_-) = \frac{1}{p_c\sqrt{s}} \frac{1}{n_+! n_0! n_-!} R_N(\sqrt{s}; N, m) \frac{\langle |T_N|^2 \rangle}{(2\pi)^{3N}}$$ \hspace{1cm} (11)

where it is again assumed for simplicity that one definite isospin multiplet is treated. $\langle |T_N|^2 \rangle$ is the average matrix element squared, including the spin-isospin dependence. Mairhead and Poppleton$^7)$ have used the experimental values of the multipion cross-sections to study the behaviour of the dynamics of pion production. We shall come to this later.

4. RESTRAINED MULTIPERIPHERALISM

The model initiated by Chan et al.$^8)$ is devised to interpolate between high energy peripheral scattering, described by the multi-Regge behaviour, and low energy scattering dominated by phase space behaviour. The application to $pp$ annihilations has been made by Chen$^9)$. The cross-section for the production of $N(n_+ n_0 n_-)$ pions in a definite ordering (multiperipheral graph) is written:

$$\sigma_N(\sqrt{s}) = \frac{1}{p_c\sqrt{s}} \frac{1}{n_+! n_0! n_-!} \int |A|^2 dR_N$$
where

\[ |A| = b^{N-1} \prod_{i=1}^{N-1} \left( 1 + \frac{h}{1 + \frac{s_i}{a}} \right) \left( 1 + \frac{s_i}{b} \right)^a \left( 1 + \frac{b}{b} \right)^{b t_i} \]  

(13)

the \( s_i \) and \( t_i \) are the sub-energies and momentum transfer squared; \( a \) and \( b \) are the parameters of the linearized nucleon trajectory, taken as -0.38 and 0.88 from high energy fits. \( h \) is taken equal to 0.077, the value found by Chan et al. \(^8\) for other reactions and the two energy scale factors \( a \) and \( b \) are chosen 0.1 and 3.0 respectively to bring the best agreement to experimental data.

A rather good fit is obtained for the pion transverse momentum, longitudinal momentum, and c.m. angular distributions. The results are not very sensitive to the choice of parameters, but it is essential to set \( b \) much larger than \( a \), to make the angular distributions sufficiently isotropic. This underlines the qualitative difference between annihilation and high energy production mentioned in the opening.

The cross-sections are normalized individually. A fair fit to the energy variation of the cross-sections in the lab. momentum range 1.6 - 7 GeV/c is then obtained (an extrapolation for 4\(\pi\) and 5\(\pi\) at 12 GeV/c also fits but a result \(^9\) for 9\(\pi\) at 7 GeV/c : (0.65 ± 0.05)mb is in disagreement with the prediction : 0.37 mb). It must be noticed that for the 2\(\pi\) channel the sharp decrease of the cross-section is not understood and that the observed angular distribution is much more isotropic than predicted.

When the relative magnitudes of the cross-sections for the various multiplicities are considered, the model is in difficulty. In conclusion, the model offers a parametrization of the amplitude which rather well reproduces properties like the variation of the average transverse momentum with multiplicity and the magnitude of the forward-backward asymmetry; this is obtained by deviating from a parameter value obtained in high energy fits. The relevance of the model for the calculation of the magnitude of the cross-sections is not obvious.

5. **QUARK MODELS**

5.1 The quark rearrangement model \(^{11}\) goes beyond SU(3) invariance by the dynamical assumption that the number of quarks and antiquarks is conserved (at least in first approximation). Therefore, nucleon-antinucleon annihilation would yield three and only three mesons. The particular scheme of rearrangement (no spin exchange, which implies uncoupling of spin and orbital momenta, and no isospin and strangeness exchange) implies the production of pseudoscalar and vector non strange mesons, with definite rates. A detailed comparison with experiment has shown that there are severe discrepancies between the model and the experiment \(^{12}\). The predominance of three-meson production, even in a more general framework than rearrangement does not correspond to the data. If the 2\(\pi\) mode is indeed low, other two-body modes (\(\rho\pi, \rho\rho, \rho\omega, f\pi\), etc.) exist and they make up globally a non
negligible fraction of the total cross-section, at least at low energy. Less conventional
two-body modes become perhaps operative at higher energy. The average multiplicity predic-
ted by the quark rearrangement model, correct at threshold, increases too slowly with
energy; two possible corrections exist:

a) the production of heavier mesons;
b) the emission of "bremsstrahlung" pions before the actual reorganization of the
quark matter.

5.2 A possible recasting of the quark rearrangement or recombination idea was conceivable
by referring to diagrams with quark lines. A diagram for three-meson annihilation would
then be:

The creation of an extra meson would be represented by the intrusion of a new quark line:

and these contributions would be treated in a kind in a kind of perturbation approach.
Two-meson annihilation would correspond to a diagram:

and both the smallness and fast decrease of these processes would be associated to the
large change in momentum for the returning quark line.

The approach fails for several reasons, the main one being the following: the compara-

tion of the $2K$ and $2\pi$ modes

requires $\sigma(K\bar{K}) \ll \sigma(\pi\pi)$ and a faster decrease of the former when $s$ increases. But over
the whole energy range where measurement have been made the two cross-sections are in a
practically constant ratio $\sim 1/3$. The consideration of the quark content does not seem
a fruitfull way to examine annihilation processes.
6. A TWO-BODY MODEL

The three-body filter does not work. We switch to another filter and present a model\(^{14}\) based on the assumption that quasi two-body states dominate the annihilation. As an essential ingredient, the two-body amplitude implements the principle of dominance of nearly threshold in the s-channel.

We have pointed out that the \(2\pi\) and \(2K\) cross-sections show a very rapid decrease when \(s\) increases; this behaviour seems to be shared by the other identified two-body modes \((\rho\pi, \rho f, \omega\pi, \ldots)\). It is then tempting to assume that it is a fundamental feature and to investigate whether a dominance of two-body processes with parallel energy dependence would account for the properties of the multipion cross-sections.

The cross-section of a particular channel

\[
p + \bar{p} \rightarrow A + B
\]

where \(A\) and \(B\) are any meson (resonance) is written:

\[
\sigma(p\bar{p} \rightarrow AB) = \frac{1}{p_c^2} \frac{\alpha(AB)}{\Delta=0} \int_{t_0}^{t_1} |R_{AB}(s,t,\Delta\lambda)|^2 dt\tag{14}
\]

where \(J = J_A + J_B\) (spins) and \(\alpha(AB)\) is the spin-isospin weight; \(\Delta\lambda\) is the helicity-flip; \(t_0\) and \(t_1\) of course depend on \(s\) and on the masses.

What is the structure of the amplitude? \(R_{AB}\) is written as the product of two factors:

\[
R_{AB}(s,t,\Delta\lambda) = R^{(s)}_{AB}(s) \cdot R^{(t)}_{AB}(s,t,\Delta\lambda)\tag{15}
\]

where \(R^{(t)}\) is the conventional dual amplitude and \(R^{(s)}\), assumed to depend only on the direct-channel parameters, is a correction which is expected to be important because of the failure of the dual approach to describe annihilation processes. The specific forms retained for \(R^{(s)}\) and \(R^{(t)}\) are:

\[
R^{(t)}(s,t,\Delta\lambda) = \left(\frac{t_1 - t}{t_0}\right)^{\Delta\lambda} \exp \left[A(t_1 - t)\right]\tag{16}
\]

\[
t_0 = 1 \ (\text{GeV}/c)^2
\]

\[
R^{(s)}(s) = C \left[\frac{s - s_{AB}}{s}\right]^{1/2} \exp \left[-a\left(s - s_{AB}\right)^{1/2}\right]
\]

\[
s_{AB} = (m_A + m_B)^2\tag{17}
\]
The assumption that the normalization constant \( C \) is the same for all channels is of course a crude approach. An elaborate fit procedure has not been attempted; to fix the parameters, \( a \) and \( C/A \) have been determined by fitting the magnitude and energy variation of the \( 2\pi \) cross-section (Fig. 2), and the parameter \( A \) by fitting the ratio between the \( 2\pi \) and the total annihilation cross-section at a fixed \( s \) \( (s = 4.5 \, \text{(GeV)}^2) \); the results do not critically depend on \( A \). The retained values are \( a = 1 \), \( A = 2 \).

The essential content of the model is a rapid increase of each two-body cross-section at the channel threshold followed by an exponential decrease (remember that a number of channels have their threshold below \( 2M_p \)).

The correct order of magnitude is obtained for the \( 3\pi \), \( 4\pi \), and \( 5\pi \) cross-sections (see Fig. 3). For higher multiplicities, a severe underestimation occurs but this is to be expected as a consequence of the incomplete knowledge of the meson spectrum, especially beyond 1700 MeV. (Note that the branching ratio for \( g \to 4\pi/2\pi \), here taken to be \( 3/1 \), has a serious impact on the results; cf. the results of Matthews et al. (Nucl. Phys. B33 (1971) 1) on the \( g \) meson).

In conclusion, quasi-particles seem to play a very important role and it seems plausible that most final states are made of them. The principle of nearby threshold dominance seems to be required by the data and its implementation leads, for the magnitude and energy variation, to a qualitative agreement between the \( 2\pi \) channel and the higher multiplicities, a result which was not reached in other models.

7. THE EXTRACTION OF THE AVERAGE MATRIX ELEMENT SQUARED

7.1 The cross-section for the production of \( N \) pions is written:

\[
\sigma_N(\sqrt{s}; n_+, n_0, n_-) = \frac{1}{n_+! n_0! n_-!} \frac{1}{p_c \sqrt{s}} \frac{1}{(2\pi)^3 N} \int |T_N|^2 dR_N
\]

\[
= \frac{1}{n_+! n_0! n_-!} \frac{1}{p_c \sqrt{s}} \frac{1}{(2\pi)^3 N} \langle |T_N|^2 \rangle \delta_N. \tag{18}
\]

Muirhead and Poppleton\(^7\) have investigated the properties of \( \langle |T_N|^2 \rangle \) by calculating

\[
\frac{p_c \sqrt{s}}{R_n} \tag{19}
\]

[The \( (n_+!n_0!n_-!) \) factor does not appear in their formulation]. The gross behaviour can be expressed by the equality of (19) with

\[
A_N s^{-5}. \tag{20}
\]
The s-dependence is thus the same for all N; besides, \( A_N \) is of the form \( a^N \), where \( a \) is a constant; more precisely, the value of \( a \) is different for \( N \) odd and for \( N \) even. The existence of (20) may suggest the relevance of uncorrelated pion production, possible final state interactions being then dependent on the basic processes of pion creation, but in our opinion, such a conclusion is not at all compulsory.

The agreement between the form (20) and the data is crude. Fluctuations can certainly be tolerated in such an approach, but there furthermore appears a systematic trend depending on multiplicity, as we shall soon show.

Hansen et al.\(^{15} \) have used a different version to analyse many-body cross-sections from various hadron collisions by considering the following quantities:

\[
\sigma_A = \sigma_N \frac{p_{\text{lab}}^{N-2}}{R_N^N} \quad .
\]

Interesting systematics arises; it is found that

\[
\sigma_A \propto p_{\text{lab}}^{-n_A}
\]

where \( n_A \) (not the multiplicity !) depends on the type of reaction, more precisely on the dominant exchange in a multi-Regge graph but not on the multiplicity. However, the reactions of annihilation are found anomalous in that framework: when \( n_A \% 4 \) is obtained for nucleon exchange in high energy reactions, it is equal to \( \% 3 \) for small annihilation multiplicities and \( \% 2 \) for higher multiplicities (the anomaly persists with \( s \) instead of \( p_{\text{lab}} \) as variable).

7.2 In view of these contradictions, we have made again the calculations along the same lines\(^{7} \), and computed

\[
<|T|^2> = n_{+}n_{0}^{'}n_{-}^{'} \frac{p_{\text{lab}} \sqrt{s}}{R_N^N} \sigma_N \quad .
\]

The presence of the factor \( (n_{+}n_{0}^{'}n_{-}^{'}) \) brings about a more uniform behaviour for even and odd multiplicities (although we do not claim that it is the adequate factor for the branching ratios within multiplicities). Fig. 4-6 show the comparison of the data with two laws

\[
<|T|^2> \propto s^{-4} 8^N
\]

and

\[
<|T|^2> \propto s^{-5} 8^N \quad .
\]

It is seen that a rather regular variation of the exponent in the s-dependence variation from \( \% 4 \) to \( \% 6 \) is observed when the multiplicity increases, an effect already partially noticed\(^{10} \).
It is obvious that a $s^{-c}$ dependence with fixed $c$ cannot be tolerated at high energy, since this would entail\textsuperscript{15}:

$$
\sigma_N \sim \frac{1}{s} s^{-c} s^{N-2} = s^{N-c-3}
$$

(26)

as $R_N$ is proportional to $s^{N-2}$ asymptotically. The cross-sections for $N > c+3$ would grow without limit!

In terms of a statistical model, the observed behaviour implies a very weak dependence on energy of the parameter governing the ratio between the pion multiplicities but a drastic dependence of an extra factor governing the reaction probability. The heuristic importance of the approach seems limited. Quite a range of models are certainly apt to reproduce the observed gross features.

In connection with phase space effects, the consideration of the $K\bar{K}n\pi$ channels might be instructive and their comparison with the purely pionic channels would shed light on the way the kinematical limitations operate. For example, the fact that the ratio of $\sigma(K\bar{K})$ to $\sigma(2\pi)$ seems practically independent if $s$ rises the question whether this becomes true at sufficiently high energy for $\sigma(K\bar{K}n\pi)/\sigma((n-2)\pi)$.

7.3 Many aspects have not been touched in this review, in the first place the Veneziano approach. Let us just mention very briefly two points. It is possible that $s$-channel resonances below threshold play a role. The low multiplicity cross-sections could therefore be governed by the tails of such resonances whereas higher multiplicities would originate in other mechanisms. Another way to look at the problem is to ask whether to different channels could correspond different interaction ranges: would this explain the $2\pi^0 - K\bar{K}^0$ contradiction in annihilation at rest?

A more differential approach than those reported here is probably necessary to gain insight into the annihilation mechanism. A feature such as the rapid change with energy of the angular distribution of $p\bar{p} \rightarrow \pi^+\pi^-$ is certainly a very significant information, whose understanding will reach beyond the particular considered channel.

The interrelation between the hadrons is dramatically illustrated by the fact at the annihilation of the nucleon with its twin particle, the antinucleon, -a reaction which reorganizes at least two GeV of matter-energy- is a marvellous factory of mesons. We can pick up unknown mesons there, find out their mass, spin, etc., but the elucidation of the reaction mechanism remains a huge task.
STATISTICAL MODEL AND ANNihilation AT REST

Fig. 1 presents the percentage distribution of the pion multiplicity in annihilation at rest compared with:

A) the Poisson shape (normalized to 1 from $N = 2$ to $\infty$);
B) the predictions of the statistical model of pion production, without resonances, adjusted to $<N> = 5$ (Exp. $5.0 \pm 0.15$);
C) the same adjusted to $<N_c> = 3.05$ (Exp. $: 3.05 \pm 0.10$).

The fit of the statistical model is very poor, but the trend suggests that the narrowness of the multiplicity distribution is related to phase space restrictions; annihilations, at least at low energy, should therefore not be tackled with asymptotic models. When the table of topologies is examined, it is seen that a systematic discrepancy exists between the statistical model and the data at the level of branching ratios. Especially, the properties of the 5*-states, as they are presented $\text{3)} (2\pi^+2\pi^-\pi^0/5* = 0.43 \pm 0.03)$, deviate very much from the statistical predictions. The introduction of selection rules in the statistical framework $\text{4)}$ is no cure (see Table Al.1). Neither is there a straightforward explanation in terms of resonances in the statistical framework; the occurrence of complicated interferences is probably the source of the difficulty. Pais$^1$ scheme$\text{18)}$ defines classes with definite permutation symmetry (in general several classes correspond to one value of the isospin), the branching ratios varying greatly from class to class). It might perhaps be feasible to find out the dominant classes and then check the compatibility between $\bar{p}p$ and $\bar{p}n$ results.

**TABLE Al.1**

Frequencies (%) of the topologies in annihilation at rest. Several sets of weights for the four 5-sectors, compatible with $R_D/R_I = 1.33$ in deuterium, have been tried for the calculation "with selection rules"; they always give comparable results.

<table>
<thead>
<tr>
<th>Topology</th>
<th>Experiment</th>
<th>Without selection rules</th>
<th>With selection rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>zero-prong</td>
<td>~ 3.</td>
<td>2.2</td>
<td>1.1</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>0.375 ± 0.03</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^0$</td>
<td>6.9 ± 0.35</td>
<td>4.1</td>
<td>4.8</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi\pi$</td>
<td>35.8 ± 0.8</td>
<td>32.9</td>
<td>32.4</td>
</tr>
<tr>
<td>$2\pi^+2\pi^-$</td>
<td>6.9 ± 0.6</td>
<td>10.0</td>
<td>8.5</td>
</tr>
<tr>
<td>$2\pi^+2\pi^-\pi^0$</td>
<td>19.6 ± 0.7</td>
<td>26.0</td>
<td>30.4</td>
</tr>
<tr>
<td>$2\pi^+2\pi^-\pi\pi$</td>
<td>20.8 ± 0.7</td>
<td>17.9</td>
<td>16.4</td>
</tr>
<tr>
<td>$3\pi^+3\pi^-$</td>
<td>2.1 ± 0.25</td>
<td>4.3</td>
<td>3.8</td>
</tr>
<tr>
<td>$3\pi^+3\pi^-\pi^0$</td>
<td>1.85 ± 0.15</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>$3\pi^+3\pi^-\pi\pi$</td>
<td>0.3 ± 0.1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>
BRANCHING RATIOS

The use of statistical branching ratios, which assumes that all allowed isospin states for the final particles have the same weight\(^\text{16,17}\), is known to fail when applied to annihilation (Table A2.1 shows the result for three energies).

**TABLE A2.1**

Cross-sections for the MM-channels. A) observed; B) deduced from the magnitude of the leading channels by means of the statistical branching ratios.

<table>
<thead>
<tr>
<th></th>
<th>1.6 GeV/c</th>
<th></th>
<th>5.7 GeV/c</th>
<th></th>
<th>7 GeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>(2π^+ 2π^-) MM</td>
<td>12.0 ± 1.5</td>
<td>6.6 ± 0.5</td>
<td>8.3 ± 1.4</td>
<td>3.2 ± 0.15</td>
<td>10.5 ± 1.5</td>
</tr>
<tr>
<td>(3π^+ 3π^-) MM</td>
<td>1.05 ± 0.25</td>
<td>0.2 ± 0.05</td>
<td>4.8 ± 0.15</td>
<td>1.95 ± 0.15</td>
<td>3.9 ± 0.5</td>
</tr>
<tr>
<td>(4π^+ 4π^-) MM</td>
<td>0.96 ± 0.07</td>
<td>0.42 ± 0.08</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The question is whether other assumptions for the branching ratios could avoid the discrepancy. We present in Table A2.11 the branching ratios for multiplicities 4 to 10 in several assumptions. The first four columns refer to uncorrelated pion production with I-spin conservation in the statistical framework. Note that the method of averaging between the two I-states is not important for the conclusion that the leading channels are severely overestimated.

Columns 5-11 present the branching ratios obtained in several conditions of resonant production, calculated in the following way. The resonances are treated as stable particles in the statistical isospin distribution and then allowed to decay to build up the multipion states. It is seen that the general trend of \(\rho\) and \(\omega\) mesons (but not of the tensors) is to deplete the leading channels; this is in the direction which is needed to bring closer model and experiment.

Other situations in uncorrelated pion production also lead to too large percentages for the leading channels. This is shown in the next columns. Column 12 corresponds to the simple statistical factor \((n_+ n_0 n_\lambda)\)^\(-1\); column 13 to multiperipheral production with incoherent graphs of nucleon exchange; column 14 the same with \(\Delta\) exchange.

Isospin invariance does not at all necessarily imply that

\[ <m(\pi^+) = 2 <m(\pi^0) = \frac{2}{3} N \]  

**\text{(A2.1)}**
is obeyed in each multiplicity $N$ (nor, of course, that $\langle n_c \rangle = \frac{2}{3} \langle N \rangle$ holds for the distribution). In the statistical framework for uncorrelated pion production (A2.1) would be generally true only if all isospins up to $N$ were available; the restriction to definite isospins destroy the relation, except for the case $I = 0$ where all directions in isospace are on equal footing.

However, in a variety of schemes, (A2.1) is almost satisfied, the better in general the larger $N$ is. It can be verified by means of Table A2.II that the result for $\langle m(\pi^\pm) \rangle / N$ falls within a few percent of the value $\frac{2}{3}$ (except a few deviations at small $N$), in spite of large variations in the branching ratios between the charge configurations.

Finally, let us point out that $\bar{p}n$ annihilation being a pure $I$-state allows a more precise comparison between model and results. The presently available data are, to my knowledge, still scarce, but the future will certainly bring interesting information. As a general remark, let us urge the model builders to treat in parallel $\bar{p}p$ et $\bar{p}n$. 

REFERENCES

1) Cf. D. Cline, in Symposium on Nucleon-Antinucleon Interactions, Argonne (1968) ANL/HEP 6812
3) CERN-Colle ge de France results, presented by C. Ghesquiere at the Aix-en-Provence Conference on Elementary Particles, Sept. 1970
8) Chan-Hong-Mo et al., Nuovo Cimento 57A (1968) 93
9) Fong-Ching Chen, Nuovo Cimento 62A (1969) 113
13) This investigation was suggested by Prof. L. Van Hove in a private communication
15) J.D. Hansen et al., Nuclear Physics B25 (1971) 605
18) A. Pais, Annals of Physics 9 (1960) 548

FIGURE CAPTIONS

Fig. 1 Pion multiplicity distribution in p̅p annihilation at rest. Curve A: Poisson form with <N> = 5. Curve B: statistical model with <N> = 5. Curve C: statistical model with <N> = 3.06.

Fig. 2 π⁺π⁻ cross-section (in microbarn) vs s. Both coordinates in logarithmic scale. Full curve: model of Ref. 14; Dashed curve: Lamb's result (Ref. 6).

Fig. 3 π⁺π⁻⁰ cross-section. Curve: Ref. 14.

Fig. 4 Averaged matrix element squared divided by s⁴ (arbitrary units) vs s (logarithmic scales), for the 2π, 3π, 4π and 5π cross-sections.

Fig. 5 The same as in Fig. 8 for 6π and 7π. Also shown is the s⁻⁵ dependence.

Fig. 6 The same as in Fig. 9 for 8π, 9π, 10π and 11π.
Fig. 2
Fig. 3
Fig. 6
1. TOTAL CROSS SECTIONS FOR ANNHIILATION

In looking at the gross features of the nucleon - antinucleon annihilation process one is immediately struck by the lack of systematic data. Even quantities like total cross sections are badly known. Most groups do not systematically examine all the possible channels and sum them to give a total cross section, and so in assessing total cross sections one is forced to make indirect approaches. The total annihilation cross section $\sigma_a$ is given by

$$\sigma_a = \sigma_T - (\sigma_e + \sigma_i)$$

where the subscripts $T$, $e$ and $i$ refer to the total, elastic and inelastic channels respectively. The total and elastic cross sections are well known from counter experiments, the charge exchange elastic cross section is poorly measured but, fortunately, it is small. This leaves the inelastic cross section which is reasonably well recorded from threshold up to $\sim 2$ GeV/c. But above this figure the total inelastic cross sections become increasingly vague as more and more channels open up. The net position can be summarised in figure 1 where the solid lines indicate data which is fairly well established and the dashed lines the less certain data. The few direct attempts at measurement of annihilation cross sections above 1 GeV/c are included for comparison.

When one turns to the antiproton data in deuterium the situation becomes even more vague. The only reasonably well tabulated data is for the $\bar{p}n$ total cross section. Even here the data tend to rely on single experiments, in the sense that one group has tackled one momentum band whilst another group has examined a different momentum band - fortunately the data seems to fit reasonably well together. The only other data which show reasonable consistency is the inelastic cross section between threshold and $\sim 2$ GeV/c, and so one must make a guess at the elastic cross section. Since it is a shadow of the total cross section I have assumed that the ratio of elastic to total cross sections is the same for $\bar{p}p$ and $\bar{p}n$ interactions - the net result is shown in figure 2. Here I have made a very rough Glauber correction to the Melbourne data (1) on $\sigma_T$ below 1 GeV/c. The cross represents the sole experimental point for $\sigma_a$ (2).
2. ISOSPIN DEPENDENCE OF $\sigma_a$

What can one learn from these curves? Apart from the fact that the total annihilation cross section is falling with increasing laboratory momentum, which means that experiments at higher momentum become increasingly difficult as more and more high multiplicity channels open up, the diagrams show that the total annihilation cross sections are nearly independent of isospin above 1 GeV/c.

In figure 3 the ratios of the cross sections for $\bar{p}p$ and $\bar{p}n$ interactions are displayed. The trend of the data fits reasonably to the ratio $1.33 \pm 0.05$ quoted by Bizzari (3) for interactions at rest. If we write

$$\sigma_a(\bar{p}p) = \frac{1}{3}(\sigma_0 + \sigma_1)$$

$$\sigma_a(\bar{p}n) = \sigma_1$$

where the subscripts 0 and 1 refer to the isospin channels, then the ratio of the cross sections in the two channels may be obtained. The result is also displayed in figure 3 and shows that whilst the isospin zero channel is dominant near zero incident momentum, this dominance has virtually vanished by $\sim 2$ GeV/c.

3. G DEPENDENCE

Thus the isospin of the $a$ channel does not appear to play any prominent role in the annihilation process beyond the region of low incident momentum. Another internal quantum number which could influence the $a$ channel is the $G$ parity, since if we concentrate on annihilation to pions only ($\sim 90\%$ of the annihilation process) in principle one has a good determination of the $G$ parity from

$$G = (-1)^n$$

where $n$ is the number of pions in the final state. In practice the problem is not so simple since we lack information on the multi-pi zero ($m^0$; $m > 2$) cross sections. In principle we can correct for these $m^0$ events if we can replace some reliance on a theoretical model. The first which comes to mind is the isospin statistical model. Using the tables of Cerulus (4) and the published data on 4C and 1C events, a typical distribution of channel cross sections shown by the solid line in figure 4 (data from the 1.6 GeV/c Berkeley experiment (5)) can be converted into $m^0$ cross sections (dashed line). One then finds

$$\sigma_{\text{even}} = 10.5 \text{ mb}$$
\[ \sigma_{\text{odd}} = 22.6 \text{ mb} \]

and could argue that annihilation into negative G parity states predominate at 1.6 GeV/c. Unfortunately 10.5 plus 22.6 yields 33 mb, whilst the measured total annihilation cross section is 50 mb (the errors are of the order of a few mb). Kaon channels account for a few mb of this discrepancy, but this explanation alone is insufficient. This fact was first noted by Fridman and his collaborators (6). They performed similar analyses on other experiments (table 1) and found that the numbers of multi \( \pi^0 \) events were consistently underestimated.

### Table 1

<table>
<thead>
<tr>
<th>Incident momentum (GeV/c)</th>
<th>( \sigma_a ) (mb)</th>
<th>Measured</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.61</td>
<td>51 ± 3</td>
<td>33 ± 5</td>
<td></td>
</tr>
<tr>
<td>3.28</td>
<td>31 ± 3</td>
<td>24 ± 3</td>
<td></td>
</tr>
<tr>
<td>5.7</td>
<td>22 ± 2</td>
<td>16 ± 4</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>25 ± 5</td>
<td>13 ± 5</td>
<td></td>
</tr>
</tbody>
</table>

This underestimation of the number of multi \( \pi^0 \) states also shows in the individual pion channels. An estimation of the cross sections for forming \( m^0 \) systems can be made by fitting phase space curves to the missing mass spectra. The method assumes the absence of resonances and isotropic angular distributions – both conditions are fairly well satisfied for high multiplicities and the modest initial momenta at which these analyses have so far been applied. The results are displayed in figure 5. In this figure the ratios of the cross sections of the \( m^0 \) systems to the 4\( \pi \) and 1\( \pi \) cross sections are plotted as a function of laboratory momentum. The bands indicate the ratios expected from pure \( I = 0 \) or 1 states from the isospin statistical model. It is apparent that

1. there is reasonable consistency between different experiments,
2. the ratios do not change violently with laboratory momentum,
3. the \( m^0 \) states occur with greater frequency than the isospin statistical model would predict (confirming the conclusion of the Strasbourg group),
4. the ratio of \( m^0 \) states appears to be higher for \( 6 \) and \( 8\pi \) systems (positive G
parity) than for 7 and 9π (negative G parity), suggesting that the apparent predominance of the negative G parity states could disappear if better information was available concerning the π^0 systems.

The influence of the π^0 states can also be seen if we compare πn interactions with p̅p. Here the equivalent (in ease of recognition) of the 4π state in p̅p → 2π^- 2π^+ is π^- π^+ π^0 etc. In figure 6 the ratios of cross sections for 4π and 1C events for p̅p and π^- n are displayed. Bands are also given for the ratios expected from the isospin statistical weights. Although we are sadly lacking in data at higher moments the experimental ratios do not appear to be violently dependent upon momentum. Two other points emerge from this display:

1. the strong influence of the π^0 on the magnitude of the cross section - the cross section is always much bigger if a π^0 is present than if all the mesons are charged,

2. the isospin statistical weights compare unfavourably with the experimental data.

Insufficient data exists to comment on the possible effects of G parity on the π^- n system.

4. ENERGY DEPENDENCE

Returning to the p̅p system, it would appear that the process of antiproton annihilation is probably not strongly coupled to any initial isospin or G parity state.* The absence of prominent resonances in the formation experiments have also shown that the initial nucleon - antinucleon system is not strongly coupled to any specific angular momentum or parity states. The question might therefore be asked whether the behaviour of the annihilation process is dependent on any initial condition. I should now like to show that it is very sensitive on the energy of the initial state. In figure 7 the total cross sections for annihilation into π^- π^+, 3π^- 3π^+ and 4π^- 4π^+ 4π^0 channels are presented as a function of laboratory momentum. The broken lines are eyeball fits. It can be seen that the 2π cross section fall rapidly with increasing momentum whilst the 9π rises rapidly. A steady change between these two extremes are seen at intermediate multiplicities - the 6π cross section is included as an example. Only one experimental error is inserted on all the data points - typically these are quoted as in the 5 to 10

* This conclusion contradicts the hypothesis of Rubinstein (7), who suggested that the nucleon - antinucleon system was strongly coupled to the pion trajectory, i.e. I = 1, G = -1.
per cent region, but the fluctuations in cross sections between different experiments suggest that the errors are consistently underestimated.

Now the cross section for the production of \( n \) pions is given by

\[
\sigma_n = \frac{1}{\text{Flux}} \sum_i \sum_f |T_{fi}|^2 \text{ phase space (}i \rightarrow n\text{)}
\]

where

\[
\text{Flux} = p_L = \text{incident laboratory momentum.}
\]

The product of the phase space and flux terms for \( 2\pi \) and \( 9\pi \) are shown as solid lines on figure 7. It is apparent that the cross sections fall more rapidly with increasing momentum than these two terms alone would warrant. Furthermore the \( 2\pi \) cross section shows relatively little dependence on \( p_L \) when \( s \) is nearly constant, but then it dives rapidly with increasing \( s \). In 1969 Poppleton and the present author (8) decided to look at the behaviour of the data available at that time in a systematic way. We defined a mean square matrix element

\[
|T_{fi}|^2 = \frac{\sigma_n \times \text{Flux}}{\text{phase space}}
\]

and found that the data were consistent with

\[
|T_{fi}|^2 = a \ b_{e,o}^n \ s^{-5} \quad (1)
\]

where \( a \) and \( b_{e,o} \) were constants (the value of \( b \) depended on whether \( n \) was even (e) or odd (o)). This empirical relation fitted all the measured cross sections, available at that time, reasonably well as can be seen in the display in figure 8. However, it cannot be true in general since

\[
\frac{\text{phase space}}{\text{Flux}} \quad \rightarrow \quad s^n \rightarrow 3 \quad \text{as} \quad s \rightarrow \infty
\]

hence the relation (1) implies

\[
\sigma_n \rightarrow s^{-5} \quad \text{for} \quad s \rightarrow \infty
\]

which is embarrassing for \( n > 3 \).

Subsequent workers (9) have shown that in fact the \( s^{-5} \) relation is only very approximate and that the power increases with \( n \). We have reanalysed the data with many more points available and results are displayed in figure 9. The resulting powers of \( s \) are given in table 2.
Table 2

| \( n \) \(, \) \( \beta \) in \( s^{-\beta} \) |
|---|---|
| \( \pi^+ \pi^- \) | 3.9 ± 0.2 |
| \( \pi^+ \pi^- \pi^0 \) | 2.9 ± 0.2 |
| \( 2\pi^+ 2\pi^- \) | 3.7 ± 0.1 |
| \( 2\pi^+ 2\pi^- \pi^0 \) | 4.4 ± 0.1 |
| \( 3\pi^+ 3\pi^- \) | 4.8 ± 0.1 |
| \( 3\pi^+ 3\pi^- \pi^0 \) | 5.1 ± 0.1 |
| \( 4\pi^+ 4\pi^- \) | 5.5 ± 0.2 |
| \( 4\pi^+ 4\pi^- \pi^0 \) | 5.5 ± 0.2 |

By now sufficient data has also become available to see that \( |\overline{T}_{fi}|^2 \) does not follow a power law even for simple values of \( n \), but rather than hunt for fresh empirical relations some effort must be spent in understanding the underlying physics (if any!). Before embarking on this task it should be remarked that the distortion of phase space due to resonance formation and the alignment of outgoing particles with incoming particles is relatively minor for the high multiplicity processes, so that the strong \( s \) dependence of \( |\overline{T}_{fi}|^2 \) probably has something to tell us.

Let us first consider the two body process. If we examine it in terms of conventional Regge amplitudes (figure 10) we can write in the simplest approximation

\[
|\overline{T}_{fi}|^2 = \sigma_2 \frac{\nu_{\text{max}}}{\text{phase space}} = \frac{1}{p \sqrt{s}} \int p(u) \left( \frac{q}{s_0} \right)^{2(\alpha_0 + \alpha_u)du}.
\]

The numerical evaluation of the integral is slightly complicated by the kinematic limits for \( u \) (figure 10). We have chosen 0.9 for \( \alpha \) and a simple exponential form for \( F(u) \) and found that, virtually independent of the details of \( s_0 \) and \( F(u) \), an \( s^{-4} \) dependence for \( |\overline{T}_{fi}|^2 \) requires \( \alpha_0 \) to be in the region of -1.25 to -1.5. Now the expected intercept for a Reggeized nucleon would be \( \alpha_0 = -4 \), so we appear to have one power of \( s \) too many.

The same problem arises in the multipion states. We have examined the \( s \) dependence of the cross section using a CIA amplitude (10).
\[ T = \prod_{i=1}^{n} \frac{1}{s_i - a} \left( s_i + a \right) \left( s_i + b \right) \alpha^{(a)} \alpha^{(b)} \]

and have obtained a reasonable fit to the experimental cross sections, as shown in figure 11. The parameters used were:

- \[ a = 0.1 \]
- \[ b = 5 \]
- \[ \alpha = 0.88 \]
- \[ r = 0.4 \]
- \[ \alpha^{(a)} = -0.38 \]

We may have gotten a better fit by further adjustment of the parameters, but the game is not worth pursuing too far. The important fact to emerge from the study was that in order to get a reasonable fit at all to the cross sections as a function of \( s \) we had to make a almost zero so that for all practical purposes we were introducing an extra \( s^{-1} \) term in the amplitude as in the \( 2n \) case.

If one ignores explanations in terms of Regge amplitudes then what alternative approaches exist to explain the \( s^{-1/4} \) behaviour? One possibility is the destructive interference of two subthreshold resonances:

\[ T \sim \frac{1}{m_1^2 - s} - \frac{1}{m_2^2 - s} = \frac{m_2^2 - m_1^2}{(m_1^2 - s)(m_2^2 - s)} \]

\[ \longrightarrow \frac{1}{s^2} \quad \text{for} \quad s \gg m_1^2, \]

\[ s \gg m_2^2. \]

The copious production of vector mesons in antiproton annihilation suggest the \( \rho \) and \( \omega \) mesons as possible candidates, however the measured difference in phase for \( \omega - \rho \) interference is \( \sim 0^0 \) (11) and furthermore the angular distributions in \( 2n \) and higher annihilation processes indicate high angular momentum states. Thus we need new candidates for \( m_1 \) and \( m_2 \).

Another possible explanation is that we are witnessing here some connection with the nucleon form factors. The \( t^{-2} \) amplitude observed in elastic electron proton scattering should cross to imply an \( s^{-2} \) amplitude for \( \bar{p}p \rightarrow e^-e^+ \). It is then reasonable to infer that \( \bar{p}p \rightarrow n\pi \) will have an amplitude of \( s^{-2} \) for vector states. Of course we cannot infer from this what will happen for another angular momentum state but the factor of \( s^{-2} \) is intriguing.
5. RESONANCE PRODUCTION

In all this work the effect of the resonances has been neglected. What we know may be summarised as

1. the 

2. the frequency of occurrence of \( \omega \) in 5\( \pi \) and 7\( \pi \) events suggest that the proportion of resonances falls slowly with increasing \( s \) (figure 12).

3. double production of resonances occurs, but only weakly — for example in 5\( \pi \) events we find \( \omega \rho \) and \( \omega f \), the data runs typically as shown in table 3.

<table>
<thead>
<tr>
<th>Laboratory momentum</th>
<th>Fraction of events containing ( \omega )</th>
<th>Fraction of ( \omega ) events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \omega \rho )</td>
</tr>
<tr>
<td>1.2 GeV/c</td>
<td>.29 ± .02</td>
<td>.18 ± .04</td>
</tr>
<tr>
<td>2.3 GeV/c</td>
<td>.16 ± .02</td>
<td>.15 ± .04</td>
</tr>
<tr>
<td>3.6 GeV/c</td>
<td>.09 ± .02</td>
<td>.15 ± .05</td>
</tr>
</tbody>
</table>

4. the mean square matrix elements for the production of two resonances show the same steep \( s \) dependence noted for the final \( n \pi \) systems — graphs for \( \rho \), \( \pi f \), \( \omega \rho \) and \( \omega f \) are given in figure 13 and slopes (assuming a power law in \( s \)) in table 4.

| Reaction | \(|T_{\text{fi}}|^2\) |
|----------|------------------------|
| \( \bar{p}p \rightarrow \pi^+\pi^- \) | -3.9 ± 0.2 |
| \( \pi^0 \rho^0 \) | -5.0 ± 0.7 |
| \( \pi^0 f \) | -5.5 ± 0.7 |
| \( \omega \rho^0 \) | -3.3 ± 0.4 |
| \( \omega f \) | -3.9 ± 0.7 |
Thus to the crude approximation multiperipheral type models appear to average over the resonance region and lead to the same basic a dependence of $|T_{\pi\pi}|^2$.

What is not clear is how big a role massive resonances can play in the high multiplicity processes. Consider for example $\bar{p}p \rightarrow 3\pi^+ 3\pi^-$. If this channel arose from reactions of the type $\bar{p}p \rightarrow \pi^-X^+$, where $X$ is a resonance decaying to 5 pions, we should pick up bumps in the $3\pi$ mass spectrum fairly easily since we should be looking at 3 combinations per event. None appear to be seen, thus ruling out $\bar{p}p \rightarrow \pi^-X, X \rightarrow 5\pi$. On the other hand if we had a reaction $\bar{p}p \rightarrow \pi^-\pi^+X^0$ where $X^0$ decayed to 4 mesons we have to plot 9 combinations per event and the problem of isolating a resonance becomes correspondingly worse. One way of improving one's chances is to try to isolate the leading pion, and hopefully, this approach should get better with higher momentum $\bar{p}$ beams. In figure 14 preliminary data from our experiment at 4.5 GeV/c is plotted for the channel $\bar{p}p \rightarrow 2\pi^- 2\pi^+$. The unselected $\pi^-\pi^+$ mass spectrum contains 4 combinations per event; if we separate off the events containing a $\pi^- (\pi^+)$ lying close to a $\overline{p}$ (p) (the limit $1.0 < \cos \theta_\pi < 0.5$ was used) then this pion is unlikely to form a resonance (section labelled remainder on diagram). We may take two or one combinations per event from the uncorrelated pions and the result can be seen to contain a high proportion of the $\rho$ and $f$ resonances. In addition to giving no evidence for the coupling of more massive 2\pi resonances than $f$ to the $\bar{p}p$ system, this result seems to show that there can be very little double resonance production in the $2\pi$ channel, since we have isolated most of the $2\pi$ resonances and know that at least one of the remaining pions is not a member of a resonance.

We hope to conduct a search for massive resonances in this manner in other channels in our 4.5 GeV/c and 9.0 GeV/c experiments. If they are not found then $\bar{p}p$ interactions will probably have to be described in terms of processes of the multiperipheral type.

6. ACKNOWLEDGEMENTS

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G. Alexander (Tel Aviv)
A. Astbury (Rutherford)

* 20 per cent of the events had two correlated pions; only one combination was then taken from the uncorrelated pions.
S. Bettini (Padua)
R. Bizzarri (Rome)
A. Cooper (Open University)
C. Defoix (Collège de France)
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G. Smith (Michigan State University)

and to my colleagues at the University of Liverpool (E. Donald, D. Everett, F. Gregory, F. Grossmann and P. Mason) who freely gave me their help and time.

7. REFERENCES

Fig. 1  Cross sections for the $\bar{p}p$ interaction. Solid lines indicate cross sections which are reasonably well established. The experimental points represent direct determinations of the annihilation cross section above 1 GeV/c.
Fig. 2: Cross sections for the $\bar{p}n$ interaction.
Fig. 3  Ratios of annihilation cross sections extrapolated to the determination of Bizzari (3) at rest.
Fig. 4  Solid line - cross sections for the production of \( n \) pions at 1.6 GeV/c in the 4C and 1C modes (5).  Dashed line - data corrected for \( m \leq 6 \) states \( (m \geq 2) \) using the isospin statistical coefficients (4).
Fig. 5  Ratios of cross sections for the production of n pions in $m^0$, $4\pi$ and $1\pi$ modes. The horizontal bands represent the results expected for pure isospin 0 and 1 states using isospin statistical coefficients.
Fig. 6  Ratios of cross sections for the production of n pions from \( \bar{p}n \) and \( \bar{p}p \) annihilations.
Fig. 7 Cross sections for the production of $n$ pions as a function of laboratory momentum. The dashed lines represent fits by eye.
Fig. 8  Ratios of the measured cross section to the empirical relation \( a^n s^{-5} \) (phase space)(flux)^{-1} (8).
Fig. 9 \( \sigma(\bar{p}p \rightarrow n\pi) \) (flux) (phase space)\(^{-1} \) versus \( s \).
Fig. 10 Definition of the kinematic terms used in the text together with physical limits on $u$ for $\bar{p}p \rightarrow \pi^+\pi^-$. 
Fig. 11  Ratios of measured cross section to calculated values obtained from a CLA model.
Fig. 12 Numbers of $\omega$ mesons per event for $\bar{p}p \rightarrow 3\pi^+ 3\pi^- \pi^0$ and $2\pi^+ 2\pi^- \pi^0$. 
Fig. 13 $\sigma(\text{flux}) (\text{phase space})^{-1}$ versus $s$ for two meson final states.
Fig. 14. Effects of excluding correlated pions (see text) on the $\pi^+\pi^-$ mass spectra for $pp \rightarrow 2\pi^+2\pi^-$ at 4.5 GeV/c.
Mr. Allison: The $(\rho/\omega)$ relative phase observed in the $2\pi$ spectrum is $90^\circ$, but if you take out the $90^\circ$ associated with the mass mixing of $\rho$ and $\omega$ (which is just associated with the decay process), you get a phase difference for the amplitudes of $0^\circ$.

Mr. De la Vaissière: I agree with what you have said about the channel $\rho^0\pi^+\pi^-$ in the $4\pi$ annihilations (enhancement of $\rho^0$ and $\omega^0$ signal when excluding forward $\pi^+$). I have made the same cuts at 3.0 GeV/c, and observed the same features. One other way to see that, is to consider the angular distributions of the $2\pi^\pm$ external to a possible $\rho^0$ meson. This distribution exhibits an forward-backward asymmetry much higher than normal. It means the important contribution of diagrams of the type

```
\begin{tikzpicture}
\node (p) at (0,0) {$\pi^-$};
\node (pbar) at (0,-1) {$\rho^0$};
\node (pbarp) at (0,-2) {$\pi^+$};
\node (pbarb) at (0,-3) {$\pi^+$};
\node (pbari) at (0,-4) {$\pi^-$};
\draw (pbar) -- (pbarp);
\draw (pbar) -- (pbarb);
\draw (pbar) -- (pbari);
\end{tikzpicture}
```
or

```
\begin{tikzpicture}
\node (p) at (0,0) {$\pi^-$};
\node (pbar) at (0,-1) {$\rho^0$};
\node (pbarp) at (0,-2) {$\pi^+$};
\node (pbarb) at (0,-3) {$\pi^-$};
\node (pbari) at (0,-4) {$\pi^+$};
\draw (pbar) -- (pbarp);
\draw (pbar) -- (pbarb);
\draw (pbar) -- (pbari);
\end{tikzpicture}
```

The same features could be observed in the very similar, from a kinematical point of view, $\omega\pi^+\pi^-$ channel. Moreover, the shape of the angular distributions of the 2 external pions are about identical, which suggests similar mechanism.

Mr. Allison: We have attempted a study of two body meson channels at 2.3 GeV/c and found qualitative agreement with a simple model with a phase space factor. This model is just a factorisable exchange process. Our conclusion is, that at this momentum S-channel quantum numbers are not important. This is in agreement with your general conclusions.
INITIAL ANGULAR MOMENTUM STATE IN $\bar{p}p$ ANNIHILATION AT REST

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1. INTRODUCTION

During the last ten years the antiproton-proton (and antiproton-deuteron) annihilations at rest have been extensively studied on the assumption that, because of the Stark mixing effect as first suggested by Dalley and Snow and Sucker$^1$, annihilation is occurring from atomic orbitals of high principal quantum number $n$ ($n \approx 10 \div 20$) and angular momentum $L = 0$.

This assumption has been basic in the interpretation of the properties of the mesonic clouds emitted in the annihilation process and, vice-versa, the analysis of these mesonic states has never given evidence for the presence of initial states with orbital angular momentum bigger than zero. It must however be emphasized that, mostly because of our ignorance of the properties of the $P$-wave $\bar{N}N$ annihilation, it has not been possible to place a quantitative limit on the $P$-wave contribution to the $\bar{p}p$ annihilation at rest.

In this situation, the recent observation by a Columbia-Syracuse Collaboration$^2$ of the reaction $\bar{p}p \rightarrow \pi^0\pi^0$, which is forbidden for $S$-state annihilation, makes very desirable a rediscussion of the subject.

2. ANNIHILATION RATES

For a bound $\bar{p}p$ atom the $S$ and $P$-wave annihilation rates can be expressed, following Jackson et al.$^3$ as

$$ r_{\bar{p}p} = \frac{\lambda_n}{M'} b_0 |\psi_{nS}(0)|^2 $$

$$ r_{\bar{p}p} = \frac{\lambda_{n+1}}{M'} b_1 |\psi_{nR}(0)|^2 $$

where $M'$ is the reduced mass of the $\bar{p}p$ atom, $\psi_{nS}(0)$ and $\psi_{nR}(0)$ are the values of the atomic wave function and its gradient evaluated in the origin, $b_0$ is the zero energy $S$-wave absorption length and $b_1$ is the corresponding $P$-wave absorption volume. For small $\bar{p}p$ separations $r$ and for $\kappa \ll n$, the atomic wave function can be written$^4$

$$ \psi_{nS}(r,\theta,\phi) = a^{-3/2} n^{-3/2} \frac{(2r/a)^{2\kappa}}{(2\kappa + 1)!} \frac{1}{2\kappa + 1} \psi_{nS}(0,\phi) $$

$a$ being the radius of the fundamental Bohr orbit. For protonium it is $a = 57.5$ fm. We have then

$$ |\psi_{nS}(0)|^2 = 4 \pi (an)^{-3} \frac{1}{4\pi} $$

$$ |\psi_{nR}(0)|^2 = 4 \pi (an)^{-3} \frac{1}{4\pi} \frac{1}{3a^2} $$

and therefore

$$ \frac{\psi_{nR}(0)}{|\psi_{nS}(0)|} = \frac{1}{3a^2} = 10^{-4} \text{ fm}^{-2}. $$
To evaluate the capture rates one has to estimate the values of $b_0$ and $b_1$. An order of magnitude estimate can be obtained by using the fact that a qualitatively rather good description of the $\bar{p}p$ interaction can be obtained representing the annihilation process by a boundary condition model\(^5\). One assumes the $\bar{p}p$ interaction to be effective only for $\bar{p}p$ separations $R < R_0$, where it is so strong as to lead always to annihilation. This means that for $R = R_0$ the $\bar{p}p$ radial wave function must describe an incoming wave: $u(R)R=R_0 \sim e^{-iKR}$ $R_0$ and $K$ (the $\bar{p}$ wave number inside the interaction region) are the only two parameters of this description.

The experimental existing data on the low energy $\bar{p}p$ cross sections require $R_0 \lesssim 1 \text{ fm}^6$, while $K$ can be written $K^2 = K_0^2 + k^2$, $k$ being the wave number of the incident $\bar{p}$ in the c.m.s. and $K_0$ a parameter which cannot be well determined by the existing experimental data but it is presumably not bigger than $1 \text{ fm}^{-1}$. Evaluating the zero energy cross sections in this model one obtains

$$b_0 = \frac{1}{K_0} \quad b_1 = \frac{R_0^2}{K_0} \frac{(K_0 R_0)^2}{1+(K_0 R_0)^2}$$

This suggests $b_0 \gtrsim 1 \text{ fm}$, while $b_1$ can be written $b_1 = R_0^3 \frac{K_0 R_0}{1+(K_0 R_0)^2}$ and it is maximum for $K_0 R_0 = 1$, being then $b_1 \lesssim \frac{1}{2} R_0^2 \sim 0.5 \text{ fm}^2$. Or one might consider the ratio

$$\frac{b_1}{b_0} = \frac{R_0^2}{1+(K_0 R_0)^2}$$

this is maximum for large $K_0$ (small $b_0$) and it is $b_1/b_0 \lesssim R_0^2 \sim 1 \text{ fm}^2$.

More elaborate models have been developed to describe the low energy $\bar{N}N$ interaction.

The model proposed by Bryan and Phillips\(^7\) describes the $\bar{N}N$ annihilation by means of a phenomenological imaginary potential while the real part of the potential is obtained from the Bryan and Scott\(^8\) OBE $\bar{p}p$ potential by changing sign to the terms corresponding to odd G-parity exchange. This model gives a quantitatively good description of the $\bar{p}p$ interaction in the momentum range 300-600 MeV/c. When extrapolated down to zero energy it predicts the absorption lengths shown in Table 1. These now depend on the value of the spin $S$ and the I-spin because of the effect of the real potential (mostly one pion exchange). One can see that the ratio $b_1/b_0$ is smaller than $1 \text{ fm}^2$ except for the state $I = 0 \ S = 1$ where it is slightly bigger, mostly because of the strong attraction due to the OBE potential in the $^3p_0$ state.

In Table 1b) the results obtained by Desai\(^9\) and based on the Ball and Chew model\(^10\) are also shown. Here the ratio $b_1/b_0$ is still smaller, while the differences between different spin and I-spin states are enhanced, these are presumably consequences of the smaller radius assumed for the annihilation interaction.

From the above discussion we can conclude that for protonium the S-wave annihilation rate is about four orders of magnitude bigger than the P-wave rate. To avoid this conclusion one should either assume a much larger annihilation radius $a$: very low energy or introduce a strong dependence of the annihilation potential from the $\bar{p}p$ orbital angular momentum. Both possibilities appear very unsatisfactory.
TABLE 1

\( \bar{\eta}N \) Absorption lengths

a) from Bryan and Phillips\(^7\)

\[
\begin{array}{|c|c|c|}
\hline
 S = 0 & I = 0 & I = 0 \\
 b_0 &= 1.33 & b_0 = 0.60 \\
 b_1 &= 1.00 & b_1 = 0.40 \\
 S = 1 & b_0 = 0.65 & b_0 = 0.87 \\
 b_1 &= 0.97 & b_1 = 0.74 \\
\hline
\end{array}
\]

b) from Besal\(^9\)

\[
\begin{array}{|c|c|c|}
\hline
 S = 0 & I = 0 & I = 1 \\
 b_0 &= 3.57 & b_0 = 0.96 \\
 b_1 &= 0.01 & b_1 = 0 \\
 S = 1 & b_0 = 2.20 & b_0 = 1.72 \\
 b_1 &= 1.65 & b_1 = 0.13 \\
\hline
\end{array}
\]

3. STARK MIXING

The fact that for any given \( n \) the \( S \)-wave capture rate is about four orders of magnitude bigger than the \( P \)-wave capture rate means that, if for protonium the angular momentum is not a good quantum number, the \( S \)-wave annihilation will dominate. This is in fact the situation when the protonium is in the electric field of neighbouring molecules (Stark mixing)\(^1,9\).

During the time between collisions the protonium is not under the influence of a strong electric field and its angular momentum has a definite value. However for \( n \lesssim 5 \), the reciprocal of the time between collisions is larger than the expected \( P \)-wave annihilation rate. From the estimated \( S \)-wave capture rate it can be expected that, before reaching down \( n \lesssim 10 \) the \( \bar{p} \) has already had ample opportunity to undergo \( S \)-wave annihilation during collisions. Although it cannot be excluded that some \( \bar{p} \) might reach down \( n \lesssim 5 \) when \( P \)-wave annihilation will begin to compete, their number is expected to be very small.

4. EXPERIMENTAL DETERMINATION OF THE INITIAL ANGULAR MOMENTUM

In principle from the detailed study of the final states from \( \bar{p}p \) annihilation one could deduce the quantum numbers of the initial state. In practice the properties of the meson clouds emitted in the process are so little known that this determination is practically possible only for a small number of particularly simple final states. Unfortunately these final states are all relatively rare and it is very dangerous to extrapolate to the totality of the annihilations the results obtained for any of them. I shall briefly discuss some of them:

a) \( \bar{p}p \rightarrow \pi^+ \pi^- \). This is a relatively frequent reaction representing \((6.9 \pm 0.4\%)\) of all annihilations at rest\(^11\). 84% of the reaction proceeds from the \( 3S_1 \) initial state and it is dominated by the intermediate state \( \rho \pi \). The distribution \( w(\theta) \) of the \( \rho \) decay
angle with respect to its line of flight is very different according to the initial state:

\[ {}^{1}\text{S}_0 \, \bar{p}p \quad w(\theta) \sim \cos^2 \theta \]
\[ {}^{3}\text{S}_1 \, \bar{p}p \quad w(\theta) \sim \sin^2 \theta \]
\[ {}^{3}\text{P}_1 \text{ or } {}^{1}\text{P}_1 \, \bar{p}p \quad w(\theta) = \text{const.} \]

The experimental data\(^{11}\) are shown in fig. 1. The bump due to the reflection of the two other p's is clearly visible but the angular distribution is very well fitted assuming \(^{3}\text{S}_1\) initial state and in particular goes to zero at the edges of the Dalitz plot. The \(^{1}\text{S}_0\) does not contribute more than 2.4\% of this channel and similarly the P-wave contribution should not exceed 5\%.

\[ \theta^\pm \text{ angular distribution} \]

\[ \begin{align*}
\text{---} & \quad {}^{3}\text{S}_1 \quad W(\theta)\sin^2 \theta \\
\text{---} & \quad {}^{1}\text{S}_0 \quad W(\theta)\cos^2 \theta \\
\text{---} & \quad {}^{3}\text{P}_1 \text{ or } {}^{1}\text{P}_1 \quad W(\theta)\text{const.}
\end{align*} \]

Fig. 1 Decay angular distribution of charged p's produced in the reaction \( \bar{p}p \rightarrow \pi^+\pi^-\rho^0 \) (ref. 11). The curves are the expected distributions for different initial states.
b) \( \bar{p}p \rightarrow KK\pi\). The missing mass spectrum of the reaction \( \bar{p}p \rightarrow K^0 K^0 + \) neutrals is dominated by the three narrow peaks due to \( \pi^0, \omega^0 \), and \( \phi^0 \) production. The CERN-Collège de France Collaboration\(^{12} \) finds on a sample of \( \sim 1.35 \times 10^6 \) annihilations at rest 568 events 0 prong + 2 \( \pi^0 \), when subtracting the contribution of \( \pi^0, \eta, \omega \) production one is left \( \leq 44 \) events \( K^0 L^0 \rightarrow \pi^0 \). This number is very small when compared to other four body annihilations:

\[
\begin{align*}
K^0 L^0 & \rightarrow \pi^0 \rightarrow 1300 \text{ events} \\
K^0 (K^0) & \rightarrow \pi^0 \rightarrow 3060 " \\
K^0 K^\pm & \rightarrow \pi^0 \rightarrow 4080 " \\
K^0 K^0 & \rightarrow \pi^0 \rightarrow 44 " 
\end{align*}
\]

The interesting point is that this reaction is expected to be strongly inhibited by angular momentum barriers if coming from \( S \)-state while no such effects are present from \( P \)-Wave annihilation. If we consider the angular momentum in the \( (K^0, K^0) \) coupling, since Bose statistic requires \( \mu \) and \( \nu \) even, we have for the lowest angular momenta

\[
\begin{align*}
\text{1}_{S_0} & \rightarrow \mu = 2, \nu = 2, L = 1 \\
\text{3}_{P_0} & \rightarrow \mu = 0, \nu = 0, L = 0 
\end{align*}
\]

Similarly in the \( (K^0, \pi^0) \) \( (K^0, \pi^0) \) coupling

\[
\begin{align*}
\text{1}_{S_0} & \rightarrow \lambda = 1, \lambda' = 1, \kappa = 1 (K^0 K^0, \xi = 1) \\
\text{3}_{P_0} & \rightarrow \lambda = 0, \lambda' = 0, \kappa = 0 \\
\text{3}_{P_0} & \rightarrow \lambda = 1, \lambda' = 1, \kappa = 0 (K^0 K^*, \xi = 0 \\
& \rightarrow \lambda = 0, \lambda' = 1, \kappa = 1 (K^0 K^*, \xi = 1) 
\end{align*}
\]

The low frequency of this channel would be therefore very difficult to explain if an important \( P \)-wave contribution were present.

c) Two neutral boson annihilation. A system of two bosons with angular momentum \( \kappa \) has a parity \( P = (-1)^{\kappa} \). A \( NN \) system of orbital angular momentum \( L \) has a parity \( P = (-1)^{L+1} \). Annihilation into two bosons requires therefore a change in orbital angular momentum \( \kappa = L \pm 1 \) and can proceed only from the triplet \( \bar{p}p \) initial state. If the two bosons are identical \( (K^0 K^0, K^0 K^0, \pi^0 \pi^0) \) the Bose statistic requires \( \kappa \) even and therefore \( L \) odd, and the initial state must be \( \text{3}_{P_0} \) or \( \text{3}_{P_2} \). If the two bosons have opposite eigenvalues under \( C \) conjugation \( (K^0 K^0, \kappa \) must be odd and the reaction proceeds from the
Fig. 2
initial state\(^{13}\).

Experimentally only 4 events \(K_{1}\bar{K}_{1}^{0}\) have been observed out of 787 two body reactions with only one visible \(K_{1}\) decay\(^{13}\). Taking into account the probability of observation of the different \(K_{0}^{0}\bar{K}_{0}^{0}\) states, we obtain

\[
\frac{\Gamma(\bar{p}p \rightarrow K_{0}^{0}\bar{K}_{0}^{0})_{L\ odd}}{\Gamma(\bar{p}p \rightarrow K_{0}^{0}\bar{K}_{0}^{0})_{L\ even + L\ odd}} \lesssim 1.5 \times 10^{-2}
\]

(the inequality being due to the fact that it cannot be excluded a small contamination of in flight events).

At variance with this result is the recent observation\(^{2}\) with a spark chamber experiment of the annihilation reaction \(\bar{p}p \rightarrow \pi^{0}\pi^{0}\). The evidence for these events is rather clear and comes primarily from the events with 4\(\gamma\) detected. From these events the rate \(\bar{p}p \rightarrow 2\pi^{0}\) is estimated to be \(4.8 \times 10^{-4}\) and the ratio

\[
\frac{\Gamma(\bar{p}p \rightarrow 2\pi^{0})_{L\ odd}}{\Gamma(\bar{p}p \rightarrow 2\pi^{0})_{L\ odd + L\ even}} = 0.39 \pm 0.08
\]

The results for \(\pi^{0}\pi^{0}\) and \(K_{0}^{0}\bar{K}_{0}^{0}\) are not necessarily in contradiction. The fact is that both these channels are very rare and dynamical effects might suppress or enhance one of them by large factors.

5. INFORMATION FROM ANNIHILATIONS IN FLIGHT

One might attempt to deduce some informations on the properties of the P-wave annihilation from the \(\bar{p}p\) interactions in flight.

To have a rough estimate of the expected contribution of different angular momenta to the annihilation cross section I have recourse again to the boundary condition model with \(R_0 = 1\ \text{fm}\) and \(K_0 = 1\ \text{fm}^{-1}\). The resulting cross-sections are plotted vs the \(\bar{p}\) momentum in Fig. 2a and vs the \(\bar{p}\) residual range in Fig. 2b. One sees immediately that the region of dominant S-wave annihilation is not experimentally accessible with liquid targets and therefore a direct experimental determination of the S-wave scattering length is not possible with standard techniques. Even the region with S and P waves only is not easily accessible. Considering that a good \(\bar{p}\) beam has a spread in residual range \(\sim 5\ \text{cm}\) of liquid H\(_2\) and that one needs some 10 or 20 cm of target to have a large number of interactions, we might

\(\ast\) These numbers are obtained adding the results of three experiments yielding respectively:

\[
\begin{align*}
\text{no \(K_{1}\bar{K}_{1}^{0}\) on } & \text{54 \(K_{0}^{0}\bar{K}_{0}^{0}\) (Armenteros et al., Ref. 14)} \\
1 \(K_{1}\bar{K}_{1}^{0}\) on & \text{239 \(K_{0}^{0}\bar{K}_{0}^{0}\) (Baltay et al., Ref. 15)} \\
3 \(K_{1}\bar{K}_{1}^{0}\) on & \text{494 \(K_{0}^{0}\bar{K}_{0}^{0}\) (CERN-Colloge de France Collaboration, unpublished)}
\end{align*}
\]
conclude that a large statistic experiment will have difficulties in going below 300-350 MeV/c. At this momentum 5% of the total annihilation cross section might be expected to be due to initial D states, thus making S-D and P-D interference quite important.

Even with these difficulties it might be interesting to look to the annihilations at low energy. For the channel K_1^{0}K_1^{0} it has been observed by CERN-Collège de France\textsuperscript{16} and by Wisconsin\textsuperscript{17} that even in flight the ratio K_1^{0}K_1^{0}/K_2^{0}K_2^{0} is very small. The Wisconsin Group finds at various energies below 700 MeV/c 89 events K_1^{0}(K^0) and only 2 events K_2^{0}K_1^{0}. The CERN-Collège de France Collaboration finds at 700 MeV/c 124 K_1^{0}(K^0) and 4 events K_2^{0}K_1^{0}. If we consider, for definitiveness this last result we see that at 700 MeV/c

$$\frac{\bar{p}p \rightarrow (K^{0}K^{0})_{l=\text{even}}}{\bar{p}p \rightarrow (K^{0}K^{0})_{l=\text{even}+\text{odd}}} = 0.11$$

while, for instance from the boundary condition model, one would have expected the odd waves (P and F) to represent about 45% of the annihilating states. This fact might be considered as an indication that the channel K^{0}K^{0} from P-wave \bar{p}p is depressed by a factor \approx 5 as compared from the same channel from S-wave \bar{p}p.

As for the \pi \pi channel there are no data on the cross section for \bar{p}p \rightarrow \pi^{+}\pi^{-} at low energy. The cross section for \bar{p}p \rightarrow \pi^{+}\pi^{-} has been measured at 400 MeV/c\textsuperscript{18} and this channel appears to be about twice as frequent as at rest. Assuming that the P-wave contribution to the total annihilation cross-section at this energy is \approx 60\%, one might consider this as an indication that the 2\pi channel is \approx 3 times more frequent from P-wave than from S-wave annihilation.

If we take seriously these indications from the cross-sections in flight, we might conclude that for some dynamical reasons P-wave \bar{p}p annihilation into K^{0}K^{0} is depressed while annihilation into \pi \pi is enhanced, and the results obtained for these two channels could be reconciled assuming a P-wave contamination at rest of \approx 10\%.

We must however remember that the comparison with in flight annihilations has been possible only with a great number of auxiliary assumptions. In particular we have assumed:

1) that the fraction of annihilations into K^{0}K^{0} or \pi \pi for any angular momentum state does not depend on energy (this, for instance, excludes coupling to narrow resonances);

2) that the contribution of each partial wave to the annihilation cross-section is reasonably estimated by the boundary condition model. In particular any dependence from the \bar{p}p spin and I-spin has been neglected;

3) the contribution of higher waves (D or F) to the channels under discussion has been assumed to be similar to the lower waves of the same parity (S or P).
Informations on the P-Wave $\bar{p}p$ Annihilation from Antiproton-Deuteron Annihilations at Rest

If we consider the annihilation of antiprotons at rest in liquid deuterium the situation is quite different. The arguments in favour of the dominance of annihilation from S-orbitals can be repeated, with small differences, for the $\bar{p}d$ atom. But now the annihilation is on a bound nucleon and because of its Fermi motion the $NN$ system will not be in an eigenstate of L, but higher angular momentum waves will be present\textsuperscript{19}). The wave function of the $\bar{p}d$ atom can be written

$$\psi(\vec{r}, \vec{\rho}) = \psi_{n}(\vec{\rho}) \psi_{N}(\vec{r})$$

$\psi_{n}(\vec{\rho})$ is the atomic wave function describing the antiproton electromagnetically bound to the deuteron, $\vec{\rho}$ being the distance from the $\bar{p}$ to the deuteron center of mass. $\psi_{N}(\vec{r})$ is the nuclear wave function of the deuteron, $\vec{r}$ being the distance between proton and neutron.

To study the interaction of the $\bar{p}$ with one nucleon, for instance the neutron, the appropriate coordinates to use are the $\bar{p}n$ distance $\vec{R}$ and the distance $\vec{X}$ between the spectator $p$ and the $\bar{p}n$, center of mass. These new coordinates are related to $\vec{\rho}$ and $\vec{r}$ by the linear transformation

$$\vec{X} = -\frac{1}{2} \vec{\rho} + \frac{3}{4} \vec{r}$$
$$\vec{R} = \vec{\rho} + \frac{1}{2} \vec{R}$$

The relation between these vectors is illustrated in fig. 3.

![Fig. 3](image)

The wave function $\psi(\vec{\rho}, \vec{r})$ is an eigenfunction of the angular momenta associated with the two variables $\vec{\rho}$ and $\vec{r}$ but it is not eigenfunction of the angular momenta associated with the two other variables $\vec{X}$ and $\vec{R}$. Therefore one can have S and P and higher waves in the $\bar{p}n$ system even when $\psi_{n}(\vec{\rho})$ describes an S orbital.

The contribution of the S and P waves to the annihilation can be evaluated via formulae (1) and (2) but now
\[ |\psi(0)|^2 = \int |\psi(R, x)|_{R=0}^2 \, d^3x \]

\[ |\psi(R)|^2 = \int |\psi(R, x)|_{R=0}^2 \, d^3x \]

the \( \psi \) operation being made with respect to the variable \( R \). For an annihilation radius \( \sim 1 \text{ fm} \) this zero range approximation should be still adequate and the contribution of higher waves negligible. For the atomic wave function the limiting expression (3) for small \( \rho \) can be used, and we obtain the following results

S-orbitals: \( \psi = 2(\alpha n)^{-3/2} \psi_N(\vec{r}) \)

\[ |\psi(0)|^2 = 4(\alpha n)^{-3} \frac{1}{4\pi} \]

\[ \psi_{R=0} = 2(\alpha n)^{-3/2} \frac{1}{\sqrt{4\pi}} \frac{1}{2} \psi_N(\vec{r}) \]

\[ |\psi| = (\alpha n)^{-3} \left< \frac{p^2}{4\pi} \right> \]

\(<p^2>\) being the average of the nucleon momentum squared in the deuteron. From the deuteron wave function\(^{20}\) one can compute \(<p^2> = (150 \text{ MeV/c})^2 = 0.58 \text{ fm}^{-2}\) and

\[ |\psi|^2 / |\psi|^2 = <p^2>/4 = 0.15 \text{ fm}^{-2} \]

(much bigger than the value \( 4 \times 10^{-4} \text{ fm}^{-2} \) obtained in hydrogen). Assuming the absorption lengths estimated by Bryan and Phillips, one is lead to conclude that the P-wave annihilation rate from S-orbitals is 0.4 times the S-wave rate, a very large value.

It is important to note that the P-wave annihilation gives rise to a spectrum of the spectator nucleon which is very much different from the S-wave spectrum. This is because of the factor \( p^2 \) in the \( |\psi|^2 \) which enhances the high energy part of the spectrum. The expected spectra for S and P wave annihilation are shown in fig. 4.
In computing these spectra the fact that the high momentum spectators carry a non negligible energy, thus reducing the energy available for annihilation, has not been taken into account. We do not know the dependence of the annihilation rate from the available energy, but in first approximation we might consider it proportional to the available phase space for the final state. The dotted curve in fig. 4 show the effect of this phase space factor if the final state is assumed to consist of 5 pions. The high momenta are depressed considerably and the rms spectator momentum is reduced to 113 MeV/c, thus decreasing the expected P/S ratio to 0.23. This ratio now depends on the final state considered, for instance for annihilations into $K\bar{K}$ the phase space effect is much smaller and the expected P/S ratio is 1.7 times bigger. We shall consider the 5π phase space as representative of the average annihilation reaction and assume a P/S ratio of ~ 0.25. This is based on the Bryan and Phillips absorption lengths. Both the boundary condition model and the calculations by Desai would give a smaller ratio.

7. OBSERVATION OF THE REACTION $\bar{p}p \rightarrow K^0 L^O K^0$ IN DEUTERIUM

The Rome-Syracuse collaboration has measured on a sample of $\sim 10^6 \bar{p}d$ annihilations at rest all the annihilations with no charged prongs and with one or two associated $V^O$. These events must have a neutron in the final state and can therefore be interpreted as $\bar{p}p$ annihilations. From all the events with one associated $K^0$ the mass of the missing mesons has been computed on the assumption that the neutrons is at rest. The resulting mass spectrum is shown in fig. 5. The peak due to the reaction $\bar{p}p \rightarrow K^0_1 K^0_2 n$ is clearly visible and contains about 230 events, which become ~ 250 if we consider the contribution of the spectator neutrons with momentum high enough to bring the $K^0_1$ momentum out of the peak (i.e. $\geq 200$ MeV/c).

If the $K^0_1 K^0_2 n$ events were to be produced with the same frequency as in $H_2$, we should observe 1.3 events $K^0_1 K^0_1 n$. In fig. 6 the missing mass squared spectrum for events with two visible $K^0$ is shown for the events below the threshold $(m_n + m_{n^0})^2$ for $n^0$ production. A peak of 14 events at the neutron mass is clearly visible.

These have the appearance of being all genuine at rest events. The momentum distribution of the incident $\bar{p}$ is correctly centered around zero and no correlation is observed between the neutron momentum and the $\bar{p}$ direction, as should be the case for in-flight events.

The distribution of the neutron momenta is shown in fig. 7 and compared with the expected distribution for S and P wave annihilation from S-orbitals (P-wave annihilation from P-orbitals being the same as S-wave from S-orbitals). The agreement with the expectation for P-wave annihilation is quite good. The phase space curve for the final state $K^0_1 K^0_1 n$ is also shown. In fact our events could be attributed to three body interactions in which the neutron is not a spectator, but in this case a distribution like the phase space could be expected. The observed distribution is on the contrary very different. To make more clear this point, in fig. 8 the neutron momentum spectrum is compared with the $\Lambda$ spectrum from the final state $\Lambda K\bar{K}$ (which certainly comes from a three body annihilation). The difference between the two reactions is evident.
Fig. 5

Fig. 6
We can therefore attribute the 14 events observed in deuterium to P-wave \( \bar{p}p \) annihilation to obtain the ratio
\[
\frac{\bar{p}p \rightarrow (K^0\bar{K}^0)_{C=+}}{\bar{p}p \rightarrow (K^0\bar{K}^0)_{C=-}} = \frac{14 \times \left(\frac{3}{2}\right)^2}{250 \times \frac{3}{2}} = \frac{63}{375} = 0.17
\]

In hydrogen the similar ratio was instead 0.015.

Assuming for the total of the \( \bar{p}p \) annihilations in deuterium a ratio P/S \( \sim 0.25 \), one can compute the similar ratio in hydrogen
\[
P/S \approx \frac{0.015}{0.17} \times 0.25 \times 1.7 = 3.8 \times 10^{-2}
\]

The factor 1.7 is the ratio of the values of \( \langle p^2 \rangle \) computed weighting the deuterium wave function with the phase space factors appropriate for the \( K\bar{K} \) and \( 5\pi+n \) final states respectively.

8. CONCLUSIONS

It is clear from the above discussion that no quantitative statement on the relative importance of initial P states in \( \bar{p}p \) annihilations at rest can be made.

Annihilations in flight indicate that P-wave annihilation into \( K^0\bar{K}^0 \) is inhibited while annihilation into \( \pi\pi \) is enhanced and might suggest a P-wave contamination \( \sim 10\% \).

The observation of the final state \( K^0\bar{K}^0 \) n from \( \bar{p}d \) annihilations at rest indicates on the contrary that the depression of the \( K^0\bar{K}^0 \) final state is not so important and suggests a P-wave contamination smaller than 4%.

Furthermore the successes obtained in the analysis of various final states on the assumption of S-wave annihilation in my opinion (admittedly subjective) would be hard to reconcile with a P-wave contribution bigger than \( \sim 5\% \).

The final part of this talk is based on unpublished results from a Rome-Syracuse Collaboration. I wish to acknowledge the contribution of Professor U. Dore and Dr. M. Gaspero to the interpretation of these data.

REFERENCES

5) H. Feshbach and V.F. Weisskopf, Phys. Rev. 76 (1949) 1550;  

6) U. Amaldi jr., B. Conforto, G. Fidecaro, H. Steiner, G. Baroni, R. Bizzarri, P. Guidoni,  
V. Rossi, G. Brentti, E. Castelli, M. Ceschia, L. Chersovani and M. Sessa, Nuovo Cimento  
46 (1966) 171.


11) M. Foster, Ph. Gavillet, G. Labrosse, L. Montanet, R.A. Salmeron, P. Villemoes,  

12) B. Conforto, B. Marechal, L. Montanet, M. Tomas, C. d'Andlau, A. Astier, J. Cohen-Ganouna,  
M. Della Negra, M. Baubillier, J. Duboc, F. James, M. Goldberg and D. Spencer, Nucl.  
A. Astier, J. Cohen-Ganouna, M. Della Negra, B. Marechal, L. Montanet, J. Zoll,  


14) R. Armenteros, L. Montanet, D.R.O. Morrison, S. Nilsson, A. Shapira, J. Vandermeulen,  
Ch. D'Andlau, A. Astier, J. Ballam, C. Ghesquière, B.P. Gregory, D. Rahm, P. Rivet  
and F. Solmitz, Proceedings of the International Conference on High Energy Nuclear  
Physics, Geneva 1962, p. 351.

15) C. Baltay, N. Barash, P. Franzini, N. Gelfand, L. Kirsch, G. Lütjens, D. Miller,  
J.C. Severiens, J. Steinberger, T.H. Tan, D. Tycko, D. Zanello, R. Goldberg and  

16) B. Löhrstad, Ch. D'Andlau, A. Astier, J. Cohen-Ganouna, M. Della Negra, M. Aguilar-Benitez,  
J. Barlow, L.D. Jacobs, P. Malechi and L. Montanet, paper presented at the Lund  

17) A. Benvenuti, D. Cline, D.D. Reeder, R. Rutz and V. Scherer, Lett. Nuovo Cimento 2  
(1971) 881.

18) R. Bizzarri, P. Guidoni, F. Marzano, G.C. Moneti, P. Rossi, D. Zanello, E. Castelli and  


FIGURE CAPTIONS

Fig. 1 Decay angular distribution of charged ρ's produced in the reaction  \( \bar{p}p \rightarrow \pi^+ \pi^- \eta^0 \) (ref. 11).  
The curves are the expected distributions for different initial states.

Fig. 2 \( \bar{p}p \) annihilation cross section for different initial angular momentum states as  
predicted from the boundary condition model plotted vs. the \( \bar{p} \) momentum (a) and the  
\( p \) residual range (b).

Fig. 3 Coordinates used to describe the \( \bar{p}d \) system in the \( \bar{p} + d \) coupling (\( \bar{p} \) and \( \bar{p} \)) or in  
the \( (\bar{p} + n) + p \) coupling (\( \bar{R} \) and \( \bar{X} \)).

Fig. 4 Expected spectrum for the spectator nucleon for S and P wave annihilation from \( \bar{p}d \)  
S-orbitals. Full line curves assume the annihilation rate to be independent from  
the available energy, dotted line curves assume it to be proportional to the phase  
space for 5π production.
Fig. 5 Missing mass spectrum from the reaction $\bar{p}d \to k_1^0 + n + MM$, computed in the assumption that the neutron is at rest.

Fig. 6 Missing mass spectrum for the reaction $\bar{p}d \to k_1^0 k_1^0 + MM$, below the threshold for $\pi^0$ production. The events not dashed fit the reaction $\bar{p}d \to k_1^0 k_1^0 n$.

Fig. 7 Expected distributions for the neutron momentum from the reaction $\bar{p}d \to k_1^0 k_1^0 n$:
- P-wave annihilation from P orbitals
- P-wave annihilation from S orbitals
- Three body annihilation (phase space).

The vertical bars below the abscissa axis indicate the measured momenta for the 14 observed events.

Fig. 8 Neutron momentum spectrum from $\bar{p}d \to k_1^0 k_1^0 n$ compared to the $\pi^0$ momentum spectrum from $\Lambda^0 K^\pm$. Curves are phase space predictions.
M. Edwards
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The recent Columbia-Syracuse experiment on \( p\bar{p} \to s^{0+}_{0} \) indicates about 40% odd orbital angular momentum for the \( p\bar{p} \) system at rest. If the high effective-mass shoulder is attributed to annihilations in flight it would appear that this unexpected ratio persists up to a few hundred MeV/c. This has consequences, which can be tested, on the elastic scattering \( p\bar{p} \to p\bar{p} \).

For a simple model, neglecting spin, we have at any energy

\[
\begin{align*}
\sigma_{el}(\theta) &= \frac{\pi}{R^2} \sum_{\ell} (2\ell+1) \frac{1}{2} (1-\eta_k) Y_{\ell m}(\theta) \, R \, (2) \, 2 \\
\sigma_{el} &= \frac{\pi}{R^2} \sum_{\ell} (2\ell+1) |1-\eta_k|^2 \\
\sigma_{ab} &= \frac{\pi}{R^2} \sum_{\ell} (2\ell+1) (1-|\eta_k|^2)
\end{align*}
\]

The elastic differential cross-sections\(^1\), after correction for pure Coulomb effects, was fitted at intervals over the energy range 10-75 MeV (momenta 135-350 MeV/c) by a polynomial in the c.m. scattering angle

\[
\sum_{n=0}^{m} A_n \cos \eta_0 .
\]

At each energy the sum was truncated when a good fit to the data was obtained. Below 20 MeV, \( m = 1 \) sufficed; \( m = 2 \) became necessary at 60 MeV and \( m = 3 \) at the highest energies. The coefficients are shown in the figure: the curves are only to guide the eye. Of course angular distributions can be flat because of the presence of many interfering partial waves but the incidence of \( A_2 \) and \( A_4 \) at about the energy expected from a classical impact parameter model gives some confidence that we are seeing the onset of P and D waves. A study of the equations shows that we cannot have absorption of a particular wave without a corresponding contribution to the elastic scattering; 40% P waves even if highly inelastic appear incompatible with this data in the range 10-30 MeV.

We can use the measured absorption cross-section\(^1\) to estimate the inelasticities of S and P waves.

At 20 MeV

\[
\frac{\sigma_{ab}}{\pi^2} = 1.44 = 1- |\eta_5|^2 + 3|\eta_p|^2
\]

hence

\[
|1-\eta_5|^2 < 0.72 \quad \text{so} \quad 0.15 \leq |\eta_5| \leq 1.0
\]

\[
\frac{\sigma_{ab}}{\pi^2} = 0.72 = 1- |\eta_5|^2 + 3|\eta_p|^2 \quad \text{so} \quad 0.72 \leq |\eta_p| \leq 0.91
\]
Thus we see that although S waves may be (and probably are) strongly absorbed (low $\eta$) the P waves have relatively little absorption.

Another way of looking for P waves is for people who have measured many apparently stopped antiproton annihilations to examine their fitted events for momentum imbalance along the direction of the incident beam track. At 20 MeV this corresponds to 200 MeV/c and should be noticeable for 4-C events. Making the reasonable assumption that the fraction of 4C events is constant over small energy intervals, a more accurate absorption cross-section can be obtained; if greater than $\pi\lambda^2$ P waves must be contributing. The present data $^1$ gives for the ratio

$$\frac{\sigma_{al}}{\pi\lambda^2} = 1.44 \pm .20 \text{ at } 20 \text{ MeV},$$

and is based on only 44 events.

REFERENCE


Mr. Armenteros : Some comments on the $K^0_{12}K^0$ problem at 700 MeV/c : The data seems to indicate that it was mainly D waves, so the connection between the two effects at rest and at 700 MeV/c is certainly not a very straightforward one.

Mr. Bizzari : At 700 MeV/c the ratio $K^0_{11}/K^0_{12}$ gives information to the P+F/S+D. Since F is small compared to P, and S is also small compared to D, this is not a very relevant number. But we can reasonably assume that the cross section $\sigma(K^0_{11})$ is mostly due to P-wave annihilation. To estimate the total P-wave contribution to the annihilation cross section one can use the boundary condition model : $\sigma_{odd} = 0.45 \sigma_{ann}$ (always remembering that the numbers are very uncertain). What is relevant, is :

$$\frac{\sigma(K^0_{11})}{0.45 \sigma_{ann}} = \text{Ratio of } K^0_{11} \text{ on the total P wave annihilation}$$

And the trouble comes from the fact that non only at rest $K^0_{11}$ is a small fraction of the annihilation cross section, but also in flight $K^0_{11}$ is a small fraction of the estimated P wave annihilation cross section.

The $K^0_{12}$ events are not very relevant except to show that the D waves go into $K\bar{K}$ with about the same frequency as S waves.
Mr. Kalogeropoulos: Mr. Bizzarri do not mentioned the existence of the $\pi^0\pi^0$ events at low energy.

We have done an analysis of $4\gamma$ events, all events were passed through the same kinematical procedure, introducing only informations about the beam. We have plotted the angular distributions of the $\pi^0$.

The distributions are isotropic for the region (1), and strongly forward peaked for the region (2), which represents in flight events (Ref.: S. Devons, T. Kozlowski, P. Nemeth) S. Shapiro, N. Horwitz, T. Kalogeropoulos, J. Skally, R. Smith, H. Uto. Phys. Rev. Letters 27 (1971) 1614).

Mr. Bizzarri: I agree with Ted that the Columbia-Syracuse data do contain some informations on the $\pi^0\pi^0$ annihilation cross section at low energy. To decide if the observed tail of in flight events is or is not anomalously high I thing it would require a careful consideration of the expected P-wave contribution to the low energy annihilation.
MODELS FOR ANALYSIS OF ANNIHILATIONS

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I - INTRODUCTION

Mainly aimed at opening a discussion on the models we have tried to use in the analysis of antiproton annihilations, this talk will briefly review the main trends of the analysis in the last years. But, I will raise questions about what looks to me open to criticism, hoping that together we might reach some conclusion about the future.

Looking at what we have searched for in the annihilation processes, I see more or less three periods in the game. The first, extending from 1956 to the early 60 's when statistics were scarce, is oriented towards global processes, concentrating on multiplicities, angular correlations...

Soon after opened the wide field of boson resonances, and the annihilations in \( \eta \) and \( K \) appeared as a very valuable tool to look for bosons in the absence of baryon fields. Most of the effort came in the study of resonant amplitudes: branching ratios for production, quantum numbers and decay properties. This research appeared very fruitful, it being needless to remind you of the many discoveries: \( C, D, F_1, E \)...

The model in favour for reaching this goal was the Final-State-Interaction model.

Trying to refine on the use of Final-State-Interactions, including higher terms and a more complete approach, some attempts have been made to solve the three-body problem, through the use of Faddeev type equations, but difficulties showed up with the problem of introducing the effect of initial states.

Very fortunately, as at the time this model was going into such intricacy as to become useless, there arrived a very attractive and clear way to deal with all crossed channels involved in the annihilation: I mean duality and the Veneziano formulation of duality. Its success in finding a simple explanation to the puzzling problem of \( \bar{p} \to \eta^+ \eta^- \) at rest gave a boost that brought hope to experimentalists.

These models being well known, it would be a tedious job to go through them in detail. I will rather, from an experimentalist 's point of view, go through some definite applications and try to pinpoint what problems we are confronted with when applying the models.

As a remark I would also mention that I shall not talk about the treatment of angular momenta and spins. As the kinematical part of the matrix element will be the subject of the next talk, I shall stick to the dynamics.
II-Model of Final State Interaction (F.S.I.)

In a paper in 1952 (I) Watson formulated his model of F.S.I., which was later developed by many others -Blackenbecker for instance(2)- Essentially, the idea is that if several particles - π and K in our case - are produced within a rather small interaction volume \( \frac{4/3 \pi \, r_s^3}{\hbar} \) at low relative momentum \( q \), if the relation

\[ q \, r_s < \frac{\hbar}{\pi} \]

is fulfilled, they interact strongly between themselves. At the limit we could forget about the initial production mechanism, and assume that the interaction between final particles dominates. The elementary cross-section in then proportional to \( \sigma_q \) between two particles.

\[ d\sigma \sim \sigma_q \, dq \]

If the two-body interaction in a state of angular momentum \( L \) saturates the unitarity, we can make the approximation

\[ d\sigma \sim \frac{\sin^2(\delta_L)}{q^{2L}} \, dq \]

This condition is well fulfilled when a resonance is very strong in the \( L \) channel (\( \rho, K^* \) for instance) and we can make the realistic approximation

\[ \Gamma q \, \delta_L = \frac{\Gamma_L}{E_L - E} \]

and all calculations being done

\[ \sigma = \frac{\pi (2L+1) \, \Gamma_L \, \Gamma_{\rho}}{k^2 \left( E - E_L \right)^3 + \frac{1}{4} \Gamma_L^2} \]

This formula describes the two-body interaction. We now have to sum over all possible pairs of particle and integrate it over the available phase space.

In fact, in most of the analysis of \( NN \) states, all waves have been approximated either by \( \sin \delta_L = c \, \delta_L \) or resonant.

The model has in fact been reduced in its use to the isobaric model, but we could as well use the phase shift expression extracted from \( \pi \, \pi \) or \( K \, \pi \) scattering obtained by some other method (3).

At this point I would make three remarks:

1) First is that we have no proof that the condition \( r_s \, q < \frac{\hbar}{\pi} \)
defined by Watson holds.
We know almost nothing about the interaction range. In the first attempts of the early experiments in the late 50's \((4,5,6,7,8)\) trying to explain the mean multiplicities in terms of a statistical model, in order to reproduce the data they had to play with the only available parameter, the interaction volume, and they currently found of the order of 10 times the elementary volume defined from the pion compton wavelength

\[
V \neq 10 \left[ \frac{4}{3} \pi \left( \frac{\hbar}{m_{\pi}c} \right)^3 \right]
\]

If this were true the condition would be very far from being filled. Many theoreticians have tried to explain an increase of multiplicity avoiding such a large interaction volume. I may quote Gatto (9), Sudarshan (10), even Pomeranchuk (II) Koba and Takeda (I2).

The Ball and Chew (I3) and the Briyan and Phillips (I4) models for low energy \(\bar{p}p\) interaction lead to the conclusion

\[
r \ll 1 \text{ Fermi}
\]

The best answer to these divergent opinions may be to say from an experimental point of view, that since we did apply this model with some success, there must be something realistic in it.

2) The next remark is relative to the parametrization of the phase - shift. Trying to go beyond the usual expression for the phase-shift, R. Bizzari et al (I5) have tried to include in their analysis of the channel

\[
\bar{p}p \rightarrow \omega^+ \pi^+ \pi^- \quad \text{at rest}
\]

a more sophisticated expression for \(\delta_o\), in order to explain a rather peculiar behaviour of the \(\pi^+ \pi^-\) effective mass distribution (Fig. I) and extended it later to the analysis of the channel

\[
\bar{p}p \rightarrow \pi^+ \pi^+ \pi^- \quad (I6)
\]

Although their result is inconclusive between a type of formula with a C.D.D. pole

\[
\frac{q}{m} \cot \theta_o \delta_o = a \frac{(b-m^2)(c-m^2)}{(d-m^2)}
\]
or a phase shift à la Baton (3) (Fig.2) with a strong absorption opening around 960 MeV/c, it shows that we can probably do better than the rough isobar approximation -even with some correction for variable width.

3) The last remark is about the symmetrization of the matrix element. It is an essential feature, though not the only one if we want to reproduce the so called Goldhaber effect (17) between the angular distribution of like and unlike pions.

In 1962, Bouchiat and Flamand (18) have shown how the angular distribution for the $p$ decay could be influenced by the proper symmetrization of the amplitudes.

Nevertheless the effect of symmetrization is much less marked on the effective mass distribution, and the relative amount of resonance is not much affected by the model used.

Take for example the result shown in table 1 (extracted from (19)) showing the percentage of resonant states in $\bar{p}p$

<table>
<thead>
<tr>
<th>$GeV/c$</th>
<th>0.</th>
<th>1.2</th>
<th>2.5</th>
<th>3.28</th>
<th>5.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{p}p \rightarrow 3\pi \rho^+$</td>
<td>0.28±0.04</td>
<td>0.34±0.04</td>
<td>0.51±0.09</td>
<td>0.29±0.07</td>
<td>0.26±0.03</td>
</tr>
<tr>
<td>$3\pi \rho^+$</td>
<td>0.34±0.04</td>
<td>0.41±0.04</td>
<td>0.49±0.09</td>
<td>0.21±0.07</td>
<td>0.45±0.07</td>
</tr>
<tr>
<td>$3\pi \rho^,$</td>
<td>0.06±0.01</td>
<td>0.16±0.05</td>
<td>0.13±0.06</td>
<td>0.09±0.03</td>
<td></td>
</tr>
<tr>
<td>$3\pi \eta$</td>
<td>0.026±0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2\pi \omega$</td>
<td>0.35±0.03</td>
<td>0.29±0.02</td>
<td>0.29±0.03</td>
<td>0.10±0.03</td>
<td>0.09±0.02</td>
</tr>
</tbody>
</table>

The partial percentages seems to have a rather smooth behaviour, although they have been obtained with widely different methods (20).

- At rest : smooth polynomial background + resonances, $\chi^2$ fit over scatter plot.
- 1,2 and 2,5 GeV/c : Incoherent or coherent addition of amplitudes gives comparable amount of resonance. Incoherent gives better agreement with experiment. Maximum likehood fit.
- 3,28 GeV/c : Phase space + Breit Wigner (or gaussian) resonances. $\chi^2$ fit over histograms.
- 5,7 GeV/c : Phase space + Resonant symmetrized amplitudes $\chi^2$ over histograms.

The F.S.I. model such as it has been used is nevertheless a first approximation model for the particles produced and neglects all successive reinteractions. Some attempts have been done to introduce these effects into the analysis.
III- The three-body problem

The three-body channels have proven in $\bar{N}N$ annihilations, as well as in other reactions, one of the most valuable and easy to study. It is why around 1964, following the work of Faddeev (21), some Theorists (22,23) tried to adapt to the high energy three-body problem the method of resolution derived from the non-relativistic one. Lovelace on one side (24), Basdevant and Kreps on another, (25) developed an application to $3\pi$ production.

Roughly they tried to get the realistic three-body scattering amplitude

$$T = T_1(z) + T_2(z) + T_3(z)$$

from the "off the energy shell" scattering amplitude for two particles $t_i(z)$, through the use of the coupled Faddeev integral equations

$$T_i(z) = t_i(z) + t_i(z) G_0(z) \left[ T_j(z) + T_k(z) \right]$$

$$T_j(z) = t_j(z) + t_j(z) G_0(z) \left[ T_i(z) + T_k(z) \right]$$

$$T_k(z) = t_k(z) + t_k(z) G_0(z) \left[ T_i(z) + T_j(z) \right]$$

for more details see for instance (26)

This has the nice features of having the three-body unitarity built in and of taking into account all orders of rescattering in the final state.

\[
\begin{array}{ccc}
\text{graph} & = & \text{graph} \\
\end{array}
\]

The usual F.S.I. used only the first graph.

This model led finally to computational difficulties and also to trouble with the assumption concerning the propagator for three free particles $G_0(z)$.

The essential result was that the effect of these refinements becomes sensitive to experimental test only when the three-body unitarity saturates the amplitude, which is in fact the way a three $\pi$ or a $K\bar{K}\pi$ resonance occurs, and this suggests caution in two cases. First when we use the F.S.I. to study the quantum numbers of some resonance extracted from the data $D \rightarrow K\bar{K}\pi$ or $\eta\pi\pi$, $E^* \rightarrow K\bar{K}\pi$, $A_2 \rightarrow 3\pi$.

Second, if it occurs that in our formation experiments at some energy
the $\bar{p}p \to 3$ body is almost saturated by some decay of a resonant state. The effect is especially sensitive at the crossing of two resonance bands inside the Dalitz plot limits.

This model has been nicely used by Hopkinson and Roberts (27) to extract the phase shift information from the reaction $\bar{p}p \to 3\pi$. Using the known F and D waves as input, they get the $S, I = 0$ phase shift and find a solution indicating an $\epsilon'$ at a mass $\cong M_\rho$ and $\Gamma \approx 480$ MeV, (Fig. 3), which may not be too bad due to the scarceness of the $S$ wave contribution to the data. But they do not succeed in fitting the $\bar{p}n \to \pi^+\pi^-\pi^-$ data with $\rho$ and $f$ as input.

These efforts to solve this three-body problem could have given more results if it had not been for a more promising insight into the annihilation process which came in the form of dual models.

IV - Duality and the Veneziano model (28)

Another aspect of annihilation appears when the momentum increases. Peripheral aspects begin to play a role marked by the forward - backward asymmetry of charge emission.

Several exchange models have been introduced but it seems that the C.L.A. model (29) has given some proof that it may reproduce most of the features in many annihilation channels (30,31).

The amplitude then has the following form

$$|A| = \prod_{i=1}^{n-1} \left( \frac{g_i + C_i}{\delta_i + a_i} \right)^{\alpha_i} \left( \frac{\delta_i + b_i}{l_i} \right)^{\alpha_i'(o)l_i}$$

It has the nice feature that for large intermediate subenergies $\delta_i$, it has a Regge behaviour

$$|A_i|_{\delta_i \to \infty} \sim g_k \left( \frac{\delta_k}{\alpha_k} \right)^{\alpha_k(o)} e^{\Omega_i l_i}$$

where

$$\Omega_i = -\alpha_i'(o) \log \frac{\delta_k}{\alpha_k} + \alpha_k(o) \log \frac{\delta_k}{\alpha_k}$$

and at small $\delta_i$, has a statistical behaviour

$$|A_i|_{\delta_i \to 0} \sim c$$

In Fig. 4 and 5 are shown some nice results obtained by Chen (31) using this model.

We can wonder to what extent the mass spectra can be influenced by this exchange model.

Take for instance the I,7 GeV/c $\bar{p}p$ enhancement observed by Braun et Al. in $\bar{p}p \to 3\pi^+ 3\pi^-\pi^0$ at 5,7 GeV/c (32). Fig. 6.
Could the bump observed at the top of phase space be explained by a multi Regge exchange model which favours small transfers, i.e. low effective masses? Even if a real \( p \bar{p} \) resonant effect exists, its share of the reaction must be very difficult to evaluate and the branching ratio is crucially model dependent.

I must quote along these lines a very interesting development of the C.L.A. model by de la Vaissière (33) in the \( \bar{p} p \rightarrow \pi^+ \pi^+ \pi^- \pi^- \) reaction at 3.6 GeV/c - Where he includes in a simple way, following the ideas of Ranft (35) a certain number of resonances in the multiperipheral chain.

Table II

<table>
<thead>
<tr>
<th>2 body states</th>
<th>3 body states</th>
<th>4 body states</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{p} )</td>
<td>( p )</td>
<td>( \pi^- )</td>
</tr>
<tr>
<td>( \bar{p} \rightarrow \pi^- \pi^+ )</td>
<td>( p \rightarrow \pi^+ \pi^- )</td>
<td>( \pi^+ )</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \pi^- )</td>
<td>( \pi^+ )</td>
<td>( \pi^- )</td>
</tr>
</tbody>
</table>

Table II shows the graphs entering in the analysis. Assuming some simple mass dependence for the coupling constant of different resonances, neglecting the exchange of \( \Delta \) lines, this still rather crude model is able to reproduce quite nicely the mass and angular distribution and looks very promising Fig. 7 and 8.

The Regge exchange could be formulated in some other ways and one of the most attractive is the Veneziano one.

Many applications of this model have now been made to annihilation processes (34, 35, 36, 37, 38, 39, 40, 41), to be compared with the same F.S.I. analysis (35, 42, 43, 44, 45).

Though the Veneziano model may not be, strictly speaking, a model for analysis, and some believe it to be the real dynamics which underlay the strong interaction, some studies were conducted in parallel over the same sample of events and a review of the convergences, as well as divergences, may cast some lights on its utility as a tool for analysis.

In the Table III, IV, V have been sketched the main parameters implied in the analysis and the conclusion reached by the authors, as well as some features of the F.S.I. analysis to be compared, this for the three
reactions
\[ \bar{p} n \rightarrow \pi^+ \pi^- \pi^- \]
\[ \bar{p} p \rightarrow \eta^* \pi^+ \pi^- \]
\[ \bar{p} p \rightarrow \omega^* \pi^+ \pi^- \]

**TABLE III**

\[ ^{1}S_0 \rightarrow (A_2)_{\Delta} \rightarrow \]
\[ (\pi \rho) \eta \]
\[ \bar{p} p \rightarrow \eta \pi^+ \pi^- \]

\[ ^{3}S_2 \rightarrow (A_2)_{\Delta} \rightarrow \]
\[ \eta \]

**Total Amplitude**
\[ L = \left| \sum A_{\pi} \right|^2 + (1 - f) \left| \sum A_{\eta} \right|^2 + \alpha \text{ Background} \]
\[ A_{\pi} = \beta_\pi F_\pi + \beta_1 F_1 + \beta_2 F_2 \]
\[ F_\pi = V (111, \alpha_4, \alpha_4) + V (111, \alpha_4, \alpha_4) + V (111, \alpha_4, \alpha_4) \]
\[ F_1 = V (112, \alpha_4, \alpha_4) + V (112, \alpha_4, \alpha_4) \]
\[ F_2 = V (112, \alpha_4, \alpha_4) \]

A relation between the \( \beta \) exclude an unwanted \( \eta \pi \) resonance at 870 MeV.
\[ A_{\pi} = \gamma_\pi G_\pi + \gamma_1 G_1 + \gamma_2 G_2 \]
\[ G_\pi = V (112, \alpha_4, \alpha_4) - V (112, \alpha_4, \alpha_4) + V (112, \alpha_4, \alpha_4) \]
\[ G_1 = V (223, \alpha_4, \alpha_4) + V (223, \alpha_4, \alpha_4) - V (223, \alpha_4, \alpha_4) \]
\[ G_2 = V (213, \alpha_4, \alpha_4) + V (213, \alpha_4, \alpha_4) \]

**Trajectories**
\[ f = \alpha(t) = 0.39 + 1.06 t + i 0.18 (t - t_0)^{1/2} \]
\[ f = \alpha(t) = 0.39 + 1.06 t + i 0.18 (t - t_0)^{1/2} \]
\[ A_\pi = \alpha(s \pi u) = 0.20 + 1.06 x + i 0.18 (x - x_0)^{1/2} \]

**Fit without satellites (only \( F_\pi \) and \( G_\pi \))**
\[ \chi^2 = 70/35 \text{ cells} \]

**Fit with satellites**
\[ \chi^2 = 40/35 \text{ cells} \]

**Results of Fit**

<table>
<thead>
<tr>
<th>Veneziano</th>
<th>F.S.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>fitted parameters</td>
<td>1.20</td>
</tr>
</tbody>
</table>

| \( ^{1}S_0 \rightarrow A_2 \pi \) = \( 0.60 \) |
| \( ^{3}S_1 \rightarrow A_2 \pi \) = \( 1 \) |
TABLE IV

\[ \begin{align*}
\bar{p}n &\rightarrow \pi^+\pi^-\pi^- \\
\pi^- &\rightarrow (\pi^0) \leftrightarrow \\
\pi^- &\rightarrow \pi^- \\
' S_0 \rightarrow (\pi^0) &\rightarrow \\
\end{align*} \]

Total Amplitude

\[ L = \alpha |A(s,t)|^2 \]

\[ A'_{S_0} = \left[ 0.885 (s,t) - 0.034 \right] \frac{\Gamma(1-\alpha(s)) \cdot \Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)) \cdot \Gamma(2-\alpha(t))} \]

Trajectory

\[ \alpha(z) = 0.483 + 0.885 s + i \cdot 0.28 (s - 4 \mu^2)^{1/2} \]

Hypothesis

\( \rho \) and \( \rho^* \) are decoupled

Fitted parameters

\( \alpha \) (normalisation) and width of \( \epsilon^* \)

---

TABLE V

\[ \begin{align*}
\bar{p}p &\rightarrow \omega\pi^+\pi^- \\
\omega &\rightarrow (\pi^+\pi^-) \leftrightarrow \\
\omega &\rightarrow (\pi^+\pi^-) \\
\end{align*} \]

\[ \begin{align*}
'S_0 \rightarrow (p) &\rightarrow (p',3) \\
\pi &\rightarrow (p',3) \\
3'S_1 \rightarrow (B) &\rightarrow (p',3) \\
\pi &\rightarrow (p',3) \\
\end{align*} \]

\[ A'_{S_0} = \beta_s B_0 + \beta_s B_1 + \beta_s B_2 + \beta_s B_3 \]

\[ B_0 = V(112, \alpha_s, \alpha_s) + V(112, \alpha_s, \alpha_s) + V(112, \alpha_u, \alpha_u) \]

\[ B_1 = V(223, \alpha_s, \alpha_s) + V(223, \alpha_s, \alpha_s) + V(223, \alpha_u, \alpha_u) \]

\[ B_2 = V(224, \alpha_s, \alpha_s) + V(224, \alpha_s, \alpha_s) \]

\[ B_3 = V(224, \alpha_s, \alpha_u) \]

Trajectories

\[ \alpha_p(x) = -0.56 + 1.06 i + 0.28 i (x - x_0)^{1/2} \]

\[ \alpha_{\rho^*}(x) = 0.39 + 1.06 i + 0.38 i (x - x_0)^{1/2} \]

A'_{S_1} has 5 independent spin Amplitudes

\[ A_{S_1} = e_{4\mu} e_{3\nu} \left[ g A_3 - P_1 P_2 A_3 - P_2 P_2 A_3 + P_1 P_2 A_3 + P_2 P_2 A_3 \right] \]

Good fit only with satellites - especially in 'S_0 - 10 parameter fit

Results of Fit

<table>
<thead>
<tr>
<th>Veneziano</th>
<th>F.S.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>'S_0/S_1 \approx 0.09</td>
<td>0.25</td>
</tr>
<tr>
<td>B comes from 'S_1</td>
<td>B comes from 'S_1</td>
</tr>
<tr>
<td>B + P_2</td>
<td>B + P_2</td>
</tr>
<tr>
<td>p' Bump due to ( \epsilon^* )</td>
<td>Good ( \rho ) signal</td>
</tr>
<tr>
<td>Dip at 15 MeV &amp; Adequate ( \rho_0 ) reproduce</td>
<td></td>
</tr>
<tr>
<td>at 1.05 GeV &amp; the dip and bump</td>
<td></td>
</tr>
<tr>
<td>not reproduced &amp; structure</td>
<td></td>
</tr>
</tbody>
</table>
Some assumption have been added to the original Veneziano formulation.

- The fact that $\bar{p}p$ or $\bar{n}n$ at rest annihilate in well defined initial states simplifies the application of the Veneziano formula from a 5 point to a 4 point function and avoids most of the troubles coming from the $N$ and $\bar{N}$ spins.

\[
\begin{array}{c}
\bar{N} \\
N
\end{array} \rightarrow \begin{array}{c}
\bar{N} \\
N
\end{array}
\]

at the expense of dealing with an "Off mass shell" boson. As Lovelace remarks (34) the Regge trajectory exchanged cannot depend on the external mass, but the relative amount of the amplitude may vary and its significance is not obvious.

- The resonances are normally represented by zero-width poles - This unrealistic and non-unitary technique is turned round by introducing an imaginary part to the trajectory. For instance

\[
\alpha_{\text{p}}(s) = 0.39 + 1.06 s + i \ 0.18 (s - \Delta_s)^{1/2}
\]

This is of course just a trick and may appear as just an arbitrary parameter, but on the other side it does not allow an individual adjustment of the width along the same trajectory.

- We see also that all attempts to fit the data, in order to get goodness comparable to the one obtained by F.S.I, have to decouple some trajectory, the in $\bar{p}n \rightarrow 3\pi$; or introduce satellites terms, in $\bar{p}p \rightarrow \eta^\pi\pi$ (table IV) where the part of the satellites is larger than the leading trajectory.

This tendency is emphasized by Altarelli and Rubinstein (35) or Barger (36): in the $\bar{p}n \rightarrow \pi^+\pi^-\pi^-$ fit they introduce a general form of formula

\[
H(s,t) = \sum_{n=1}^{\infty} \sum_{m=0}^{n} C_{nm} \frac{\Gamma(n - \alpha(s)) \Gamma(m - \alpha(t))}{\Gamma(n + m - \alpha(s) - \alpha(t))}
\]

An important question is how far we should go in applying Veneziano. Should we look more for general features, or should we try to reproduce finer details at the expense of adding many parameters and questionable assumptions. In fig. 9 you can see for instance the difference between a fit with only leading trajectories and with satellites.

I have the feeling that the first solution gives a more general insight into physics, especially if we relate crossed reactions. On the
other end it may help also to reproduce some features that were not easy to obtain by the isobaric model, for instance the shape of \( \rho' \) in \( \bar{p} p \rightarrow \pi^+ \pi^- \pi^- \) (Fig. 10).

It is worth mentioning that the results of Veneziano analysis are not always consistent with those obtained by F.S.I. I do not mention the \( \bar{p} n \rightarrow 3 \pi^- \pi^- \) analysis, where Jengo and Remiddi (37) have shown that introducing the \( \rho' \) and \( \varepsilon' \) hypothesis in the isobaric model gives a prediction not too far from Veneziano Fig. II. But consider for instance the \( A_2 \) production in \( \bar{p} p \rightarrow \eta \pi^+ \pi^- \), the F.S.I. model gives

\[
\begin{align*}
4S_0 & \rightarrow A_2 \pi^+ \pi^- \\
3S_1 & \rightarrow A_2 \pi^-
\end{align*}
\]

Whereas Veneziano gives 1 (39)

And in \( \bar{p} p \rightarrow \bar{K} K \pi^- \), F.S.I. gives

\[
\begin{align*}
4S_0 & \rightarrow A_2 \pi^+ \\
3S_1 & \rightarrow A_2 \pi^-
\end{align*}
\]

Whereas Veneziano gives 0 (41).

Another example \( \bar{p} p \rightarrow \omega \pi^+ \pi^- \) in F.S.I. gives \( \omega J \rightarrow 25 \% \) and evidence for \( \rho_v \) at 1250 MeV/c^2, Veneziano gives essentially no \( \omega J \) production, replaced by \( \omega \varepsilon' \) and a \( \rho_s \) at 1250 MeV/c^2. Where is the truth?

V - CONCLUSION

Now at the time to reach some conclusion, I feel rather embarrassed. First I apologize for all the nice and important work I did neglect, but this talk does not intend to be an exhaustive survey, rather it should raise question and discussion.

It is trivial to say that in spite of some progress we are far from having a clear view of what to use as a model for analysis. The same could be said for the whole field of strong interaction.

My feeling is that dual models may be the more promising, as they permit connexion with many crossed channels \( \pi^p, K \rho \), \( \omega \varepsilon' \), Veneziano formalism is a way to tackle the problem, first results are not too discouraging and work could be continued along these lines. Some attempts are now being done to go beyond the four-point function, and it looks worthwhile to extend the duality ideas to annihilations in flight and more complicated final states, with \( n \pi \) and \( K \bar{K} n \pi \). Data are already in our hands.

Acknowledgements

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REFERENCES

1) The Effect of Final State Interactions on reaction Cross-Section K.M. Watson P.R. 88 (1952) II63
2) Construction of unitary scattering amplitudes Blackenbecler P.R. I22 (1961) 983
3) See for instance one of the latest review Meson - Meson scattering J.L. Petersen Physics Reports - 2 C - 3 (1971)
4) Antiproton-Nucleon annihilation process. W.H. Barbas et Al. P.R. I05 (1957) I037
5) Antiproton-Nucleon Annihilation process C. Chamberlain et Al. P.R. II3 (1959) I615
6) Low-Energy Antiproton interactions in hydrogen and Deuterium. N. Horwitz et Al. P.R. II5 (1959) 472
9) R. Gatto N.C. 3 (1956) 468
10) Annihilation of antinucleons G. Sudarshan P.R. I03 (1956) 777
11) Pomeranchuk Doklady Ac. N. USSR 73 (1951) 88
12) Z. Koba and G. Takeda Pr. Th. Ph. I9 (1958) 269
13) Nucleon-antinucleon interaction at intermediate Energies J.S. Ball and G.F. Chew. P.R. I09 (1958) I385
15) Experimental results on the $\pi\pi$ and $\pi K$ systems as observed in the $\bar{p}p$ annihilations at rest : R. Bizzari et Al. N.P. B I4 (1969) I69
16) $\bar{p}p$ annihilations at rest into four pions. J. Dazio et Al. N.P. B I6 (1970) 239
17) Pion-pion correlations in antiproton annihilation events G. Goldhaber et Al. P.R.L. 3 (1959) I8I
18) Final State interaction in proton-Antiproton Annihilations at rest C. Bouchiat and G. Flamand N.C. XXIII (1962) I3
19) The annihilation process $\bar{p}p \rightarrow 2 \pi^+ 2\pi^- \pi^0$ at 2.5 GeV/c Clayton et Al. N.P. B 30 (1971) 605
   - J. Clayton et Al. N.P. - B 30 (1971) 605
   - T. Perbel et Al. P.R. - I43 (1966) I096
   - V. Alles - Borelli et Al. N.C. - 50 (1967) 776
21) L.D. Faddeev. Soviet Physics JEPT 12 (1961) 1014

22) Three-Particle scattering - A Relativistic Theory
V. Allessandri and R. Omnes
Doklady 6 (1961) 384

23) Practical Theory of Three-Particles states
D.Z. Freedman, C. Lovelace and J.M. Namyslowski
N.C. 43 (1966) 258

24) Practical Theory of Three Particles States - Non-relativistic
C. Lovelace
P.R. 139B (1964) 1225

25) Relativistic Three Pion Calculations
J.L. Basdevant and R.E. Kreps
P.R. 141 (1966) 1398

R. Omnes - J.L. Basdevant - Int. report Lab de Ph. Th. et H.E. CERN

27) Pion-pion phase shift information from the reaction \( \bar{p}p \rightarrow 3\pi \)
J.F.L. Hopkinson and R.G. Roberts
N.C. 59 A (1969) 181

28) Construction of a crossing-symmetric, Regge-behaved Amplitude for
linearly Rising Trajectories
G. Veneziano
N.C. 57 A (1968) 190

29) A Reggeized Multipheripheral Model for inelastic processes at High Energy
Chen Hong Mo, J. Losblewicz and W.W. M. Allison
N.C. 57 A (1968) 93

30) The annihilation reaction \( \bar{p}p \rightarrow 2\pi^+2\pi^- \) at high Energy and a
multi-Regge Model.
G. Ranft
N.C. 58 A (1968) 425

31) \( \bar{p}p \) annihilation into pions in a Reggeized Multipheripheral Model
Fong-Ching-Chen
N.C. 62 A (1969) 113

32) Further evidence for a \( 2\pi^+2\pi^- \) \( I=7 \) GeV/c² enhancement observed
in the \( \bar{p}p \rightarrow 3\pi^+3\pi^- \) reaction at 5.7 GeV/c.
H. Braun et Al.
N.C. B 30 (1971) 213

33) Analysis of the annihilation reaction \( \bar{p}p \rightarrow 2\pi^+2\pi^- \) at 3.6 GeV/c
in terms of a multiperipheral model including resonances.
C. de la Vaissière
Submitted to N.C.

34) A novel Application of Regge trajectories
C. Lovelace
P.L. 28 B (1968) 264

35) Dalitz plot analysis including Duality.
G. Altarelli and H. Rubinstein
P.R. 183 (1969) 1469

36) E.L. Berger Paper given at the conference on \( \pi\pi \) and \( \pi K \) scattering.
A.N.L. Argonne I 11 (1969)

37) An analysis of the Reaction \( \bar{p}p \rightarrow 3\pi \) by means of Veneziano.
R. Jengo and E. Rømmedal

38) J.F.C. Hopkinson and R.G. Roberts
39) Analysis of the annihilation process $\bar{p}p \rightarrow \eta \pi^+ \pi^-$ at rest using
Veneziano type amplitudes,
Chung - Rencroft - Montanet
N.P. B 31 (1971) 261

40) Analysis of the annihilation process $\bar{p}p \rightarrow \omega \pi^+ \pi^-$ at rest using
Veneziano type amplitudes,
Chung - Rencroft - Montanet
N.P. B 30 (1971) 525

41) A study of the $KK\pi$ final states for $NN$ annihilations at rest in
The Veneziano language.
C. Ben fatto - Cassandro - Lusignoli - Nicoli -
N.C. 1 A (1971) 255

42) Production of three charged Pions in $\bar{p} + n$ Annihilations at rest.
P. Anninos et Al.
F.R.L. 20 (1968) 402

43) $\bar{p}p$ annihilation at rest into $\eta \pi \pi$
P. Espigat et Al.
N.P. B 36 (1971) 93

44) Production of three pions $\bar{p}p$ annihilation at rest.
A. Foster et Al.
N.P. B 6 (1968) 107

45) The annihilation at rest $\bar{N}N \rightarrow K\bar{K}\pi$
A. Bettini et Al.
N.C. 68 A (1969) 1199
FIGURE CAPTION

Fig. 1: Extracted from (15)
Histogram of the effective mass $m^* m$ in $\bar{p}p \to \omega^* \pi^* \pi^-$
at rest. Bumps are hidden by $\rho^*$ and $f^0$ contribution. The
dip at $\approx 950$ MeV/c^2 is clearly visible.

Fig. 2: Extracted from (15)
Shape of the $\delta^*$ phase shift introduced to best fit the
data.

Fig. 3: Extracted from (32)
$\delta^*$ phase shift from the three-body analysis of
$\bar{p}p \to \pi^* \pi^- \pi^*$ at rest.

Fig. 4: Extracted from (23)
Histograms of transverse momentum distributions, for
various multiplicities at 5.7 GeV/c $pp$ annihilations.
Solid lines are the fit with C.L.A. model.

Fig. 5: Extracted from (23)
Histograms of single pion angular distribution in the
c.m. $\pi^-$ and reflected $\pi^*$ distributions are added. Dashed
lines are from the model.

a) $2 \pi^* \to \pi^- \pi^+$ at 3.28 GeV/c
b) $2 \pi^* \to \pi^- \pi^+$ at 3.28 GeV/c
c) $2 \pi^* \to \pi^- \pi^+$ at 5.7 GeV/c
d) $2 \pi^* \to \pi^- \pi^+$ at 5.7 GeV/c

Fig. 6: Extracted from (24)
Effective mass $2 \pi^* \pi^-$ distribution selected for $2 \pi^* \pi^-$
systems having two $\pi^* \pi^-$ mass combinations in the various
$\pi^* \pi^-$ indicated. Curves on a, b, and c are phase space
predictions. Curve on d is (phase space + B.W.) fit.

Fig. 7: Extracted from (25)
Effective mass distribution of $\pi^* \pi$ from $\bar{p}p \to \pi^* \pi^* \pi^- \pi^-$
at 3.6 GeV/c. Dashed line is the prediction of the simple
C.L.A. model without resonance. Full line best fit with $\rho$
and $f$.

Fig. 8: Extracted from (25)
Outer left: Angular distribution of $\pi^*$ in the production
c.m. in $\bar{p}p \to \pi^* \pi^- \pi^+$ at 3.6 GeV/c. Dashed line is the
prediction of the model.
Three righ: Angular distribution of the $\pi^* \pi^-$ combinations
in the center of mass system with several selection criteria. Full lines are prediction of the model.

Fig. 9: Extracted from (39)
Effective mass distribution for $\eta \pi^*$ and $\pi^* \pi^-$ in the reac-
tion $\bar{p}p \to \eta \pi^* \pi^-$ at rest.
Solid lines are Veneziano best fit. Dotted lines are the
results of Veneziano fit without satellites terms.
Fig. I0 : Extracted from (37)

Comparison between various models and the experimental \( \pi^+ \pi^- \) mass squared distribution in \( \bar{p}p \)

Model I = Basic Veneziano  
Model 2 = Virasoro formulation  
Model 3 = Simple isobaric formula

Fig. II : Extracted from (37)

Comparison between the Veneziano model and the B.W. model with some initial assumptions, for the mass squared distributions in \( \bar{p}n \rightarrow \pi^+ \pi^- \pi^- \) at rest.
Fig. 1
Fig. 2
Fig. 4
Fig. 6
Fig. 11

\[ \bar{p} n \rightarrow \pi^+ \pi^- \pi^- \]

- Lovelace
- Isobaric model

\[ \pi^+ \pi^- \text{ EFFECTIVE MASS SQUARED GeV}^2 \]

\[ \text{NUMBER OF EVENTS/0.5 GeV}^2 \]

150 100 50
ANALYSIS OF THE ANNIHILATION REACTION $\bar{p}p \rightarrow 2\pi^* 2\pi^-$ at 3.6 GeV/c IN TERMS OF A MULTI-PERIPHERAL MODEL INCLUDING RESONANCES

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SUMMARY OF THE COMMUNICATION

The annihilation reaction $\bar{p}p \rightarrow 2\pi^* 2\pi^-$ at 3.6 GeV/c appears to proceed through 9 intermediate channels involving meson resonances $\rho^0$, $\omega^0$, $\phi^0$. The multi Regge model of Chan-Loskiewicz and Allison (CLA), which is found to give a good description of pion distributions, such as c.m.s. angles and momenta, is inadequate for resonance formation. The introduction of resonances in the CLA model made by Plathe and Roberts leads to a rather poor description of the data and give no information on the branching ratios of the intermediate states. So we attempt to introduce the resonances in the CLA model by considering that they can be coupled to the nucleon Regge trajectory in the same way as pions through phenomenological coupling constants. Then, the amplitudes of all the multi-peripheral graphs that can be drawn are added incoherently.

This phenomenological model introduces one extra parameter/resonance relative to the original CLA model. It may be extended to higher multiplicities.

We found that this sum of amplitudes approximates well the physical situation at 3.6 GeV/c, equally well for pions and multi-pions distributions as for resonance formation. A straightforward extrapolation of the model to 1.2 and 5.7 GeV/c leads only to minor discrepancies.
DISCUSSION AND COMMENTS

Mr. Bizzari: According to duality it is illegitimate to add diagrams like

\[ \begin{array}{cc}
\text{and} \\
\includegraphics[width=0.2\textwidth]{Diagram1.png}
\end{array} \]

Now you propose to add diagrams like

\[ \begin{array}{cc}
\text{and} \\
\includegraphics[width=0.2\textwidth]{Diagram2.png}
\end{array} \]

I wonder if this is consistent with duality?

Mr. Butterworth: (to Mr. De La Vassière) How well do you fit the 3-body mass in the 4-body channel?

Mr. De La Vassière: Very Well. (A slide was shown).

Mr. Butterworth: About the analysis of the $\pi^+\pi^- S$-wave in $\bar{p}p \rightarrow \omega \pi$, how do you get rid of the $\rho$?

Mr. Montanet: $\bar{p}p \rightarrow \omega \rho$ comes from $^1S_0$, it does not interfere with $^3S_1$ initial state which is contributing to the final state with a $\pi^+\pi^- S$-wave.

Mr. Armenteros: How do your results compare with those of Baton?

Mr. Butterworth: Are you saying that you can exclude the so-called up-down solution?

Mr. Ghesquière: The last results for the $\pi^+\pi^- S$-wave parametrization, as observed in $\bar{p}p \rightarrow \omega^0 \pi^+$, give the following results for the parametrization of the $\pi^+\pi^- S$-wave:

1) assuming a slowly varying inelasticity, $\delta \gamma$ goes from $90^\circ$ at 840 MeV to $270^\circ$ at $\approx 1000$ MeV, via $180^\circ$ at 940 MeV;

2) or, assuming $\delta \gamma$ to be close to $90^\circ$ in the mass range 800-1000 MeV, the inelasticity parameter has a rapid decrease immediately followed by a rapid increase around 940 MeV.

For more details, see the article "$\omega\pi$ resonances and $\pi\pi S$-wave structures as observed in $pp$ annihilations at rest" - P. Frenkiew, C. Ghesquière, E. Lillesøe, E. Chung, J. Diaz, A. Ferrando and L. Montanet - CERN/D.Ph.II/PHYS 72-9.

So, you see that we find two possible solutions: in terms of the usual terminology, we could say that we favour the down-up and the down-down solutions (with a sharp absorption at 940 MeV in the last case).
Mr. Kalogeropoulos: We observe also a dip in \( \pi^+ \pi^- \) at 970-980 MeV for \( \bar{p} n \rightarrow 4\pi \) at rest (16 000 events).

Mr. Ghesquière: The dip observed in \( \pi^+ \pi^- \) for \( \bar{p} p \rightarrow \omega \pi^+ \pi^- \) is also observed in \( \pi^+ \pi^- \) for \( \bar{p} p \rightarrow \rho \pi^+ \pi^- \). See: J. Diaz, Ph. Gavillet, G. Labrosse, L. Montanet, W. Swanson, P. Vellemees, M. Bloch, P. Frenkel, C. Ghesquière, E. Lillestol, A. Volte - Nuclear Physics - B 16 (1970) 239.

Mr. Moneti: What is the relationship between the parametrization used by Bizzari et al. for the dip in the \( \pi^+ \pi^- \) mass distribution in the \( \omega \pi \pi \) state, and the description of a very similar phenomenon observed by Flatté et al. in the \( \pi \pi \) system?

Mr. Ghesquière: Flatté's solution is quite close to our second solution, with inelasticity opening abruptly at some value. Our dip is a bit lower (950 instead of 990 MeV).

Mr. Fridman: Mr. Ghesquière said that the symmetrization procedure in \( \bar{p} p \) annihilation processes can explain the Goldhaber effect. I do not think that this is true and this for two reasons:

- This model does not take into account the resonance production;
- The symmetrization as used by experimentalists is not always made in a correct way because the constraints due to the initial isospin are not taken into account. See a paper by Pais on the subject.

Mr. Ghesquière: I have been a little rough in saying that B.E. symmetrization explains all of Goldhaber's effect. We have to introduce resonance production.

Mr. Kalogeropoulos: It is clear that Goldhaber's effect is not only a question of B.E. symmetrization. But the application of symmetrization always goes in the right direction (decreases \( \chi^2 \), for instance).

Mr. Donald: Regarding the effect of parametrizing the Goldhaber effect, one can see effects in the mass plots e.g. in \( \pi^+ \pi^- \) masses. It also tends to alter fractions obtained from simple incoherent Breit-Wigner-type fits. For instance, the amount of single \( \rho \) production is reduced and the amount of phase-space increased.

Mr. Kalogeropoulos: (to Mr. Muirhead) Some time ago, you published a comparison between the mass plots of like and unlike pion pairs. Could you account for the difference in terms of symmetrization.

Mr. Muirhead: Our analysis of \( \pi^+ \pi^- \) and \( \pi^- \pi^- \) mass correlations was not satisfying, as the system we examined had too many particles.
It is worth noting that the CLA model satisfactorily reproduces the opening angles for \( \pi^+ \pi^+ \) and \( \pi^+ \pi^- \).

Mr. Allison: There is a very significant effect in mass plots corresponding to the Goldhaber effect. It covers the range 280-380 MeV in mass (data was shown).

Mr. Armenteros: I have seen a paper by the \( K^\ast \) collaboration about high multiplicity \( K^\ast \) interactions. They see correlations which are the opposite of what you expect from B.E. statistics. For more details contact Mr. Goldschmidt-Clermont.
ANGULAR MOMENTUM ANALYSIS, ZEMACH/HELCITY

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1. INTRODUCTION

Apart from a few tentatives using the Veneziano model, most of the data from antiproton-proton annihilations have been analyzed in terms of final state interaction models.

In most of the cases the angular correlations in the final states have been interpreted using angular momentum tensors according to the prescriptions given by C. Zemach\textsuperscript{1}).

For $\bar{p}p$ annihilations at rest this method has been used extensively by several groups and with very good results. This was also the route followed when our group tried to interpret the data from the reaction

$$\bar{p}p(\text{at rest}) \rightarrow \omega \pi^+\pi^-$$

Fitting the $\omega\pi\pi$ Dalitz plot using standard $\chi^2$-methods it turned out that two ($\omega\pi$)-resonances, both around 1250 MeV/c\textsuperscript{2}, had to be introduced simultaneously in order to fit the data\textsuperscript{2}). However, an ambiguity in the choice of the spin-parities of these two "objects" could not be resolved from fits on the Dalitz plot. It also turned out that the ambiguity could be resolved by studying the angular correlations between the $\omega\pi\pi$ plane and the $\omega$-decay plane, giving $l^+$ and $l^-$ for the spin parities of the two ($\omega\pi$)-resonances.

In order to increase our confidence in this interpretation, we decided to check the results using a different approach and analyze the data in terms of helicity amplitudes. At the same time we decided to use maximum likelihood fitting working in the full $5\pi$ final state. Our experience in the use of helicity amplitudes stems mainly from the analysis of this particular final state.

Being limited in "space-time", I will here give a very brief introduction to the Zemach tensors and the helicity amplitudes. For the practical applications of the methods I will concentrate on a few examples.

I will also discuss a method of angular selections which has proved very useful, in particular in the analysis of the reaction

$$\bar{p}p(\text{at rest}) \rightarrow \pi^+\pi^+\pi^-\pi^-$$
2. CONSTRUCTION OF ZEMACH TENSORS

2.1 Non relativistic tensors

Consider a particle X with spin $j$ and magnetic quantum number $m$ decaying into $n$ spinless particles with momenta $\vec{p}_1, \vec{p}_2, \ldots \vec{p}_n$, defined in the center of mass of the decaying particle.

The spin function for the decaying particle is defined by the tensor

$$ (Sm)_{j}^{i_1, i_2, \ldots, i_j} $$

The decay matrix element is now obtained by multiplying the spin function $(Sm)$ by a sufficient number of Cartesian components of the vectors $\vec{p}_a, \vec{p}_b, \ldots \vec{p}_j$, chosen from among $\vec{p}_1, \vec{p}_2, \ldots \vec{p}_n$, $\vec{p}_1 \times \vec{p}_2$, $\vec{p}_k \times \vec{p}_l$, with possible repetitions among the $\vec{p}_i$. A contraction of all the Cartesian indices is then performed.

The decay matrix element is written

$$ \langle \text{final} | D | X \rangle = D(E_1, E_2, \ldots E_n) T_j^{i_1, i_2, \ldots, i_j} (Sm)_{j}^{i_1, i_2, \ldots, i_j} $$

where summation over repeated indices is assumed.

$T_j^{i_1, \ldots, i_j}$ is a Cartesian tensor of rank $j$ in three space. The function $D(E_1, \ldots E_n)$ called a form factor, is a function of the energies only and not of the directions of the particles in the final state.

The tensor $T_j^{i}$ being symmetrical and traceless in each pair of indices has $2j+1$ independent components.

As an example let the particle X be the $\bar{p}p$ system at rest, the final state consisting of $n$ pions. The initial states are the singlet $^1S_0$ and the triplet $^3S_1$ states with spin functions $e^0$ and $e^1$ respectively. ($e^0$ is a scalar and $e^1$ a vector.) The decay matrix element is written

$$ \langle \pi_1, \ldots \pi_n | D | \bar{p}p \rangle = D^S(E_1 \ldots E_n) T_0^0 e^0 + D^T(E_1 \ldots E_n) T_1^1 e^1 $$

$D^S$ and $D^T$ are the singlet and triplet form factors respectively and

$$ e^0 e^0 = e_1^0 e_1^0 = e_2^0 e_2^0 = e_3^0 e_3^0 = 1 $$

The absolute square of the matrix element is now

$$ |\langle \pi_1, \ldots \pi_n | D | \bar{p}p \rangle|^2 = |D^S|^2 |T_0^0|^2 + |D^T|^2 |T_1^1 e_1^1 e_1^1 + 2 \text{Real}(D^S D^T T_0^0 T_j^1 e^0 e_j^1) $$

Integrating over all directions in space (or if the antiproton is non polarized) the term linear in $e^1$ will be equal to zero, and for the bilinear
\[ e_{ij}^1 = \delta_{ij} \]  

(6)

giving

\[ |\langle \pi_1 \ldots \pi_n | D | \bar{p} p \rangle|^2 = |D^S|^2 |T^0|^2 + |D^T|^2 T_1^i T_i^1 \]  

(7)

This is an important result which shows that the matrix element squared for the \( \bar{p} p \)-decay can be considered as an incoherent sum of the four terms \( T^0 T_1^i, T_1^i T_i^1 \).

With a large number of particles in the final state, the cartesian tensor \( T^j \) has been constructed as products of tensors representing two body and quasi two body states. Some typical examples are

\[ \bar{p} p \rightarrow \pi A_2 \]
\[ l_\pi p \]
\[ l_\pi \pi \]

\[ \bar{p} p \rightarrow \pi B \]
\[ l_\omega \omega \]
\[ l_\pi \pi \pi \]

For completeness I give a few examples on the construction of the tensor \( T^j \).

Considering a system of two spinless particles \( a \) and \( b \) with momenta \( \vec{p}_a \) and \( \vec{p}_b \) respectively (\( \vec{p}_a = -\vec{p}_b = \vec{p} \)), a spin one is simply described by

\[ T_1^i = p_i \quad \text{i} = 1,2,3 \]  

(8)

a spin two by

\[ T_2^{ij} = p_i p_j - \frac{1}{3} \delta_{ij} \vec{p} \cdot \vec{p} \quad \text{i,j} = 1,2,3 \]  

(9)

and a spin three by

\[ T_3^{ijk} = p_i p_j p_k - \frac{1}{5} (\vec{p} \cdot \vec{p})(p_i \delta_{jk} + p_j \delta_{ik} + p_k \delta_{ij}) \quad \text{i,j,k} = 1,2,3 \]  

(10)

2.2 Relativistic covariant tensors

The above tensors are written in the rest frame of the particles \( a \) and \( b \). In order to go from this frame to another, the vectors \( \vec{p}_a \) and \( \vec{p}_b \) must be Lorentz transformed.

This may be avoided by writing the tensors \( T^j \) directly in a relati-
vistic covariant form.

Defining $X_\mu$ as the four momentum of the decaying particle using the notation

$$p^\mu = (p^0, \vec{p})$$  \hspace{1cm} (11)

and the metric

$$g_{\mu\nu}: (1,-1,-1,-1)$$  \hspace{1cm} (12)

the relativistic covariant tensors are obtained directly from the non relativistic ones by the simple replacement

$$\vec{p} \rightarrow \vec{p}_\mu \hspace{1cm} \text{where}$$

$$\vec{p}_\mu = p_\mu - X_\mu \frac{(p^\gamma X_\gamma)/X_\gamma)}{(X^\gamma X_\gamma)}$$  \hspace{1cm} (13)

and by the replacement

$$\delta_{ij} \rightarrow g_{\mu\nu} - X_\mu X_\nu/(X^\gamma X_\gamma)$$  \hspace{1cm} (14)

Pseudo vectors are replaced by

$$q^\mu = \varepsilon^{\mu\nu\rho\sigma} p_{a\nu} p_{b\rho} X_\sigma/(X^\gamma X_\gamma)^{\frac{3}{2}}$$  \hspace{1cm} (15)

where $\varepsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor.

When the non relativistic formulation is used, and used correctly, the relativistic one adds nothing except being more compact and in my experience simpler to use. However, sometimes the non relativistic forms are used incorrectly. Let me first mention one example where this is done purposely:

In the study of the reaction $\bar{p}p$(at rest) $\rightarrow \omega \pi^+\pi^-$ the $\omega$ has been treated as a stable particle, the spin vector of the $\omega$ being defined as the normal to the $\omega$-decay plane in the $\omega$-rest frame. This spin vector has not been Lorentz-transformed when constructing the full $\omega\pi\pi$ tensors. In order to do this correctly, it is necessary to work in the full five particle final state. However, it has been checked by Monte Carlo methods that in this particular case an incorrect use of the non relativistic tensors do not lead to observable systematic effects.

On the other hand let me mention one example where an incorrect use would have led to a very bad result:

Consider the reaction $\bar{p}p$(at rest) $\rightarrow \rho \pi^+\pi^-$ and take a typical value of 0.5 GeV/c$^2$ for the $\pi^+\pi^-$ effective mass. Let us assume the $\bar{p}$ being
unpolarized and consider the triplet annihilation only. Without Lorentz transforming from the $\rho$-rest frame to the $\bar{p}p$ rest frame one would conclude that the $\rho$-decay should be flat in $\cos\theta^*$, whereas the correct formula gives an angular distribution close to $1 + 3 \cos^2\theta^*$.

3. ANGULAR SELECTIONS

In order to disentangle the contributions of different amplitudes in a particular reaction, we have made extensive use of a method of angular selections.

As mentioned in section 2.1 a transition amplitude can be written (apart from a complex constant) as $a_i = A_i D_i$ where $A_i$ is the angular part and $D_i$ the dynamical part of the amplitude $i$.

Customarily, dynamical effects - resonances - are observed as peaks in histograms of effective masses. In order to determine the spin of the resonances, different angular distributions are studied for the events in the peaks, the "background" contributions being obtained from equivalent distributions on both sides of the peaks. Essentially, this method consists in making cuts on $D_i$ and plotting the corresponding $A_i$.

Our method is just the contrary. In order to isolate the dynamical effects, we make cuts on $A_i$ and plot the corresponding $D_i$.

It should be clear that such a method would be powerful if the angular parts of the amplitudes contributing to reaction in study were sufficiently different.

As an example consider the reaction $\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^-$. We have observed that the possible amplitudes can be divided in two groups, group 1 representing the singlet amplitudes and group 2 the triplet amplitudes. While the differences between the angular parts of the amplitudes within each of the groups are small, substantial differences can be observed when comparing the amplitudes in group 1 with those in group 2.

In other words, with the method of angular selections it is relatively easy to isolate the singlet from the triplet amplitudes, but more difficult to isolate individual amplitudes within one group.

Attributing the vectors $\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4$ to the $\pi^+, \pi^-, \pi^+,$ $\pi^-$ in the final state, it turns out that all the singlet amplitudes have angular parts being well represented by

$$A = (\vec{p}_1 \times \vec{p}_2) \cdot (\vec{p}_3 - \vec{p}_4)$$

It is therefore possible to enhance the singlet contribution by selection of events with large values of $|A|^2$ and correspondingly the triplet part taking events with small values of $|A|^2$. 
As explained by Ghesquière at this conference, the $\pi\pi$ S-wave may be studied in the two reactions

\begin{align*}
a) \quad & \bar{p}p\text{(triplet)} + \omega^0\pi^+\pi^- \\
& \bar{p}p\text{(triplet)} + \rho^0\pi^+\pi^-
\end{align*}

In order to observe the same structure in the $(\pi^+\pi^-)$ mass spectrum for reaction $b$ as for reaction $a$, one may use the method of angular selections. As an example of the usefulness of this method, I therefore show you in two figures the $\pi^+\pi^-$ effective mass spectrum of reaction $b$, fig. 1 corresponding to the total sample and fig. 2 to a sample where the triplet part is enhanced according to the method explained above.

4. HELICITY AMPLETDUES

4.1 Introduction

I will here present a very brief introduction to the helicity amplitudes, picking out from Ref. 4 only the minimum required to describe an annihilation reaction.

We start with a reaction

\[ a + b \rightarrow c + d \] (1)

Let $\vec{p}_a$, $S_a$, $\lambda_a$, $\eta_a$ be the momentum, spin, helicity, and intrinsic parity, respectively of particle $a$, $\vec{p}_f$ ($\vec{p}_f'$) the C.M. momentum of $a$ ($c$) and $\omega_0$ the C.M. energy. The transition amplitude for reaction (1) may thus be written

\[ <\Omega_0^a c d | T(\omega_0) | 00 \lambda_a \lambda_b> \]

\[ = \text{Const.} \Sigma (2J+1) <\lambda_\lambda | \pi^J(\omega_0) | \lambda_a \lambda_b> \times \]

\[ \times D^J_{\lambda\lambda}'(\phi_0, \theta_0, 0) \] (2)

Here $\vec{p}_i$ has spherical angles $(0,0)$ and $\vec{p}_f$ $\Omega_0 = (\theta_0, \phi_0)$.

$\lambda = \lambda_a - \lambda_b$ and $\lambda' = \lambda_c - \lambda_d$. $D^J_{\lambda\lambda}'(\phi_0, \theta_0, 0)$ is the usual rotation function:

\[ D^J_{m'm}(\alpha, \beta, \gamma) = e^{-i m' \Delta} \frac{\sin^J \gamma}{\sin \gamma} d^{J}_{m'm}(\beta) e^{-i m \gamma} \] (3)

with

\[ d^{J}_{m'm}(\beta) = <j m'| e^{-i \beta J} | j m> \] (4)
Parity conservation gives
\[ <\lambda_{c}\lambda_{d}|T^{J}(w_{o})|\lambda_{a}\lambda_{b}> = \eta_{c}\lambda_{d}|T^{J}(w_{o})|\lambda_{a}\lambda_{b}> \]
(5)

with
\[ \eta = \eta_{a}\eta_{b}\eta_{c}\eta_{d}(-1)^{s_{c}^{+}s_{d}^{-}s_{a}^{-}s_{b}^{+}} \]
(6)

Let us next consider the two body decay
\[ J \rightarrow 1 + 2 \]
(7)

The decay amplitude may be written
\[ A = \langle \lambda_{1}, \lambda_{2}|M|J, 3 \rangle \]
\[ = N_{J}F_{\lambda_{1}\lambda_{2}}^{J}D_{\lambda_{1}^{*}}^{\lambda_{2}}(\phi, \theta, 0) \]
(8)

where \( N_{J} \) is a normalization constant, \((\theta, \phi)\) are the decay angles, \( \lambda = \lambda_{1} - \lambda_{2} \), and the helicity amplitude
\[ F_{\lambda_{1}\lambda_{2}}^{J} = 4\pi \left( \frac{3}{p} \right)^{\frac{1}{2}} \langle JM|\lambda_{1}\lambda_{2}|J, 3 \rangle \]
(9)

In practical use, when fitting a reaction, one usually splits out of \( F \) energy dependent factors like for instance a Breit-Wigner function and leave the rest of \( F_{\lambda_{1}\lambda_{2}}^{J} \) as (energy independent) parameters in the fit.

The number of such parameters is reduced by imposing parity conservation, giving
\[ F_{\lambda_{1}\lambda_{2}}^{J} = \eta_{1}\eta_{2}(-1)^{J-s_{1}^{+}s_{2}^{-}}F_{\lambda_{1}^{-}\lambda_{2}^{+}}^{J} \]
(10)

Under certain conditions, when for instance the above resonance dominates the \( J \)th partial wave in the elastic process
\[ 1 + 2 \rightarrow 1 + 2 \]

time reversal arguments may be applied giving
\[ F_{\lambda_{1}\lambda_{2}}^{J} = \text{real} \]
(11)

To be complete we also need to consider the reaction
\[ a + b \rightarrow c + J \leftrightarrow 1 + 2 \]
(12)
Defining the helicity, effective mass, and decay angles (Jackson frame) for $J$ to be $\Lambda$, $w$ and $\Omega = (\theta, \phi)$, respectively, the amplitude for the reaction may be written

$$M_{fi} = \Sigma_{\Lambda} \langle p_f \lambda_1 \lambda_2 | M | J \Lambda > \langle p_f \lambda_c \Lambda | T(w_o) | p_i \lambda_a \lambda_b \rangle$$

(13)

Summing over all variables except the decay angles $\Omega$ we obtain the differential cross section

$$\frac{d\sigma}{d\Omega} = \int d\omega K(\omega) \Sigma_{\Lambda} |M_{fi}|^2$$

(14)

where all the energy dependent quantities including the square of the Breit-Wigner function has been put into $K(w)$. Defining the spin density matrix of $J$ as

$$\rho_{\lambda\lambda'}^J = \int d\Omega \Sigma_{\lambda\lambda'} \langle p_f \lambda_c \Lambda | T(w_o) | p_i \lambda_a \lambda_b \rangle \times$$

$$\times \langle p_f \lambda_c \Lambda' | T(w_o) | p_i \lambda_a \lambda_b \rangle^*$$

(15)

and assuming (as is usually done) that it is independent of $w$ over the width of the resonance, one obtains

$$\frac{d\sigma}{d\Omega} = N^2 J \Sigma_{\lambda\lambda'} \rho_{\lambda\lambda'}^J \Delta_{\lambda\lambda'}^{\phi, \theta, 0} \Delta_{\lambda\lambda'}^{J} \phi, \theta, 0 \delta_{\lambda_1 \lambda_2}^J$$

(16)

where

$$\delta_{\lambda_1 \lambda_2}^J = \int d\omega K(\omega) |F_{\lambda_1 \lambda_2}^J|^2$$

(17)

Imposing

$$\Sigma_{\lambda} \rho_{\lambda\lambda}^J = 1 \quad \text{and} \quad \Sigma_{\lambda_1 \lambda_2} \delta_{\lambda_1 \lambda_2}^J = 1$$

(18)

we obtain a normalized angular distribution.

With the quantization axis in the production plane parity conservation gives

$$\rho_{\Lambda \Lambda'}^J = (-1)^{\Lambda - \Lambda'} \rho_{-\Lambda - \Lambda'}^J$$

(19)

Finally we want to consider the decay

$$J \to 1 + 2 + 3$$

(20)
Defining the standard orientation of the three particles in the rest frame of $J$ with the $xy$ plane coinciding with the decay plane such that $y \parallel -\vec{p}_3$ and $p_{1x} > 0$, one can write

$$A = \langle a \beta \gamma, E_{1} \lambda_{1} | M | JM \rangle$$

$$= \frac{N_J}{\sqrt{2\pi}} F^J_{\mu}(E_{1} \lambda_{1}) D^{J}_{M\mu}(a, \beta, \gamma)$$

(21)

where

$$F^J_{\mu}(E_{1} \lambda_{1}) = \langle JM u E_{1} \lambda_{1} | M | JM \rangle$$

(22)

$\mu$ is the $z$-component of $\vec{J}$ corresponding to the standard orientation. Parity conservation gives

$$F^J_{\mu}(E_{1} \lambda_{1}) = \eta \eta_2 \eta_3 (-1)^{s_1 + s_2 + s_3 + \mu} F^J_{\mu}(E_{1} - \lambda_{1})$$

(23)

4.2 Helicity amplitudes for the reaction $\bar{p}p(\text{at rest}) + \omega \pi \pi$.

As an example on the use of helicity amplitudes, I will write down the amplitudes used in a fit of the reaction $\bar{p}p(\text{at rest}) + \omega \pi^+ \pi^-$. Since the $\bar{p}p$ system is at rest, we have taken as the initial $z$-direction the direction of the $\omega$ in the lab. system.

Corresponding to the two types of sequential decays

a) $\bar{p}p \rightarrow \pi X$

$b) \bar{p}p \rightarrow \gamma \omega$

we define two series of decay angles shown in fig. 3 and 4, respectively. For the sequential decay (a) the amplitude is written

$$A_M = D_{\lambda}^*(\pi\theta_0) D_{\lambda}^*(\omega \theta_0) f_{\lambda \lambda} D_{\lambda}^*(\phi \theta_1) x$$

$$\times \text{Breit-Wigner} (X) + \text{Bose symmetric term}$$

(24)

where (JM), (SA), ($\sigma, \lambda$) are the spin and helicities for the $\bar{p}p$, the $X$, and the $\omega$, respectively. $\Omega_1 = (\theta_1, \phi_1)$ is the direction of the normal to the $\omega$ decay plane in the system $(x'', y'', z'')$, (see fig. 3). The spin
parity of the $\omega$ is $\sigma^p = 1^-$. Considering the reaction $\bar{p}p$ (triplet) $\rightarrow B\pi$. With spin parities $J^p = 1^-$ for the $\bar{p}p$ system and $S^p = 1^+$ for the $B$, we get from eq. 10

$$F^\Lambda = F^{-\Lambda} \quad \text{and} \quad f^\lambda = f^{-\lambda}$$  \hspace{1cm} (25)

From eq. 18 and 25 we have

$$2|f^1|^2 + |f^0|^2 = 1$$  \hspace{1cm} (26)

$$2|F^1|^2 + |F^0|^2 = 1$$

such that the amplitude may be parametrised in terms of $\alpha$ and $\beta$:

$$\sqrt{2} f^1 = \cos \alpha \quad f^0 = \sin \alpha$$

$$\sqrt{2} F^1 = \cos \beta \quad F^0 = \sin \beta$$  \hspace{1cm} (27)

In the case $X$ is a $1^-$ resonance, one obtains instead of eq. 25

$$F^\Lambda = - F^{-\Lambda} \quad \text{and} \quad f^\lambda = - f^{-\lambda}$$  \hspace{1cm} (28)

For the sequential decay (b) the amplitude is written

$$A_M = D^J_{MA}(000)F^J_{\lambda_1 \lambda_2}D^{S^*}_{\lambda_1 0}(00 \theta_3 0)D^{\sigma^*}_{\lambda_2 0}(\phi_2 \theta_2 0) \times$$

$$\times \text{Breit-Wigner (Y)}$$

$$= F^J_{\lambda_1 \lambda_2} D^{S^*}_{\lambda_1 0}(00 \theta_3 0)D^{\sigma^*}_{\lambda_2 0}(\phi_2 \theta_2 0) \times \text{BW (Y)}$$  \hspace{1cm} (29)

with $\lambda_1 = M + \lambda_2$

Two examples:

i) $\bar{p}p$ (singlet) $\rightarrow \rho^0 \omega^0$

$$J = M = 0 \quad \text{and} \quad \lambda_1 = \lambda_2$$  \hspace{1cm} (30)

$$\rho : \quad S^p = 1^- \quad \text{and} \quad \omega : \quad \omega^p = 1^-$$  \hspace{1cm} (31)

From eq. 10 we obtain

$$F_{11} = - F_{-1} \quad \text{and} \quad F_{00} = 0$$  \hspace{1cm} (32)
ii) \( pp(\text{triplet}) \rightarrow f^0_\omega^0 \)
\[ s^p = 2^+ \]  
(33)
giving
\[ F_{\lambda_1\lambda_2} = F_{\lambda_1\lambda_2} \]  
(34)
From eq. 18 and 34 we can write
\[ F_{12}^2 + F_{01}^2 + F_{10}^2 = \delta_1^2 \]  
(35)
\[ 2 F_{11}^2 + F_{00}^2 = \delta_2^2 \]
such that
\[ F_{12} = \delta_1 \sin \alpha \cos \beta \]  
\[ F_{01} = \delta_1 \sin \alpha \sin \beta \]  
\[ F_{10} = \delta_1 \cos \alpha \]  
\[ F_{11} = \frac{1}{2} \delta_2 \cos \gamma \]  
\[ F_{00} = \delta_2 \sin \gamma \]  
(36)
where the parameters are \( \alpha, \beta, \gamma, \delta_1, \delta_2 \).

The complete transition amplitude is now written
\[ |T|^2 = |\sum_i C_i A_i^\dagger(\text{Singlet})|^2 + \sum_M \sum_j C_j A_j^\dagger M(\text{Triplet})|^2 \]  
(37)
where the \( C \)'s are complex constants.

As mentioned in the introduction, the amplitude has been fitted using maximum likelihood methods. It should here be noted that such a method is both time consuming and requires a large memory computer. Without going into details concerning the computing, I should nevertheless like to mention that a rather simple fit on a full sample of some 8000 events takes between 1 and 2 hours on a CDC 6600, occupying most of the memory of the machine.

The details of the results obtained may be found in Ref. 5.

REFERENCES
2) R. Bizzarri, M. Foster, Ph. Gavillet, G. Labrosse, L. Montanet,


4) S.U. Chung, CERN Lectures; CERN 71-8

5) P. Frenkel, C. Ghesquiere, E. Lillestøl, College de France; S.U. Chung, J. Díaz, A. Ferrando, L. Montanet, CERN; To be published.
Fig. 1

Fig. 2
SUMMARY OF THE ANALYSIS OF $\bar{p}p \rightarrow \omega \pi^{+}\pi^{-}$

(as given by L. Montanet at the last session of the Conference)

Facts: We have 8000 events of stopped $\bar{p}$ leading to $\omega \pi^{+}\pi^{-}$. We have the $\omega \pi^{+}\pi^{-}$ Dalitz-plot and the angular correlations between the normal to the production plane and the normal to the $\omega$ decay plane (these angular correlations are found to be important to discriminate between several solutions).

Assumptions: - Initial state is $^1S_0$ and $^3S_1$ - (no P-waves)
  - Dominance of final state interaction is assumed:
    $$\bar{p} + p = \sum \bar{p} + p$$
  - Consider $\omega\pi$ and $\pi\pi$ final state interactions
    $^1S_0$ is $C = +1$ so $\pi^{+}\pi^{-}$ must be $C = -1$; $\rho$
    $^3S_1$ is $C = -1$ so $\pi^{+}\pi^{-}$ must be $C = +1$; S and D waves
      (no $I = 2$)

Techniques: Two techniques have been tried:
  1) Zemach + $\chi^2$ on Dalitz-plot and projection of the results on the angular distributions.
  2) Helicity formalism + maximum likelihood fit on 5 body.

Results:
  1) Interpretation of the $\omega\pi$ system ($B, \rho'$) and $\pi\pi$ S-wave are independent.
  2) $\omega\rho$: 25%, can be easily "removed".
  3) $\omega\pi$: 10-15% but small interferences with $B, \rho'$... may however spoil the conclusions on high mass $\pi^{+}\pi^{-}$.
  4) One ($\omega\pi$) resonance only gives bad results. Need two ($\omega\pi$) resonances: $1^+$ and $1^-$. 
    $$M (B) = 1228 \pm 5 \quad \Gamma (B) = .26 \pm 10$$
    $$M (\rho') = 1256 \pm 10 \quad \Gamma (\rho') = 129 \pm 20 \text{ MeV}.$$ 
  5) For $\pi^{+}\pi^{-}$ S-wave, see Ghesquière's discussion.
DISCUSSION AND COMMENTS

Mr. Duboc: For decay-processes, there is a method by Berman and Jacob (SLAC 43) based on the helicity formalism. Did you try it?

Mr. Lillestøl: No. The advantage of the helicity formalism is that you need not worry about the angular momentum between particles. It is taken care of automatically.

Mr. Kalogeropoulos: What would be the Dalitz-plot of $\omega^{0}_{p^{0}}$?

Mr. Bizzarri: Just the same, but of course with no $\omega^{0}_{rho^{0}}$.

Mr. Mairhead: I believe that a $\rho^{\prime}$ has been reported from photo-production work. Is it the same as yours?

Mr. Montanet: There are two results from photoproduction. One of them looks like this one (mass $\sim$ 1500 MeV, width $\sim$ 200 MeV rather than 130). The other one has been reported by Salvini from Frascati (colliding beams). They see something at a mass like 1500-1600 MeV, width 200-250 MeV.

Mr. Bizzarri: I want to make a remark on this $\rho^{\prime}$ business. We have:

$$^{1S_{0}} \bar{p}n \rightarrow \rho^{\prime}\pi$$

and

$$^{3S_{1}} \bar{p}p \not\rightarrow \rho^{\prime}\pi$$

if we consider the decay $\rho^{\prime} \rightarrow \pi^{+}\pi^{-}$. Now considering $\rho^{\prime} \rightarrow \omega\pi$, it seem that:

$$^{1S_{0}} \bar{p}p \not\rightarrow \rho^{\prime}\pi$$

and

$$^{3S_{1}} \bar{p}p \rightarrow \rho^{\prime}\pi$$

This is rather puzzling to me.

Mr. Montanet: The second statement may not be correct ($^{3S_{1}} \bar{p}p \not\rightarrow \rho^{\prime}\pi$, $\rho^{\prime} \rightarrow \pi^{+}\pi^{-}$), because we have always trouble understanding the 3-pion annihilation.

Antonio Ferrando repeated the old fit of the Dalitz plot, introducing the $\rho^{\prime}$; we can explain the Dalitz plot with the $\rho^{\prime}$ added coherently. However, the mass and width we find for this $\rho^{\prime}$ are difficult to reconcile with the mass and width we find for $\omega\pi$. $M = 1400$ MeV and $\Gamma \sim 300$ MeV.

Mr. Bettini: I have a question about Zemach analysis. The relativistic treatment is correct, the non-relativistic one is sometimes correct, sometimes not. In Zemach's work, it is stated that the non-relativistic formulation is equivalent to the relativistic one, and that relativity is an unnecessary complication. What is correct?

Mr. Lillestøl: If you use Zemach correctly, they are indeed equivalent.
TWO ELEMENTARY METHODS OF ANALYZING \( \bar{\eta}N \rightarrow 3\pi \)

Jens Bjørneboe

The Niels Bohr Institute, University of Copenhagen, Denmark

I am going to describe briefly two elementary methods, which may be useful to analyze the reactions

\[
\bar{p} \quad p \rightarrow \pi^+ \pi^0 \pi^-
\]

\[
\bar{p} \quad N \rightarrow \pi^+ \pi^- \pi^-
\]

1. ISOSPIN ANALYSIS

The first method is based on some work by Z. Koba, N. Törnqvist, and myself which is already published \(^1,^2\).

The aim of this work was to formulate the consequences of charge independence in such a way that they are directly applicable to the kind of result obtained in a typical bubble chamber experiment.

As you all know, the conventional formulation of charge independence is trivial, but useless in this case, since it applies to an ideal situation in which everything is measured with infinite precision. In reality, however, our knowledge is limited since only cross sections integrated over a finite part of phase space and summed over nucleon spins can be determined with reasonable accuracy.

In terms of amplitudes, charge independence simply means that the amplitude for a particular charge channel can be expressed in terms of a (small) number of isospin amplitudes, \( M_i \):

\[
T_k = \sum_i c_i^k M_i
\]

which can be chosen in a number of ways. The integrated cross sections can then be expressed as

\[
\sigma_k = \sum_{i,j} c_i^k c_j^k m_{ij}
\]

with

\[
m_{ij} = \sum_{\text{spin}} \int d\tau \text{ Re } (M_i^* M_j)
\]
so that by solving these equations for \( m_{ij} \), we can get insight into the isospin structure of the reaction.

However, this system of equations is often underdetermined: e.g. in the case \( \bar{N} N \rightarrow 3\pi \), the first of the channels mentioned above gives six cross sections (taking into account permutations), the second one three, whereas the number of different \( m_{ij} \)'s is ten.

What we found was that the consequences of charge independence were not exhausted by these equations, but that, in addition, a number of inequalities have to be satisfied. To see how this comes about, we note that it follows from the very definition of \( m_{ij} \) that

\[
m_{ij}^2 \leq m_{ii} m_{jj}
\]

which is nothing but the well-known Cauchy-Schwartz inequality. The point is that this inequality should hold not only for all (different) values of the indices \( i \) and \( j \), but also for any choice of isospin amplitudes.

Without going into details, I just mention that all these requirements can be summarized by the inequalities

\[
m_{ii} > 0, \quad \left| \begin{array}{cc} m_{ii} & m_{iz} \\ m_{iz} & m_{zz} \end{array} \right| > 0,
\]

etc.

These conditions, together with the equations above, exhaust the requirements of charge independence for integrated cross sections.

I would like to stress the flexibility of this method: the more detailed the experimental information, the narrower the limits on \( m_{ij} \).

2. GRAND ANGULAR MOMENTUM ANALYSIS

For annihilation at rest it is, however, possible to get a more detailed description for the following reasons:

i) the energy to be shared between the pions is limited,

ii) they come from a region in space with linear dimensions \( \approx \) a couple of fermis, and

iii) only \( J = 0 \) (or 1) are allowed (hopefully),
so that, intuitively, one would expect the number of states accessible to
the three pions to be limited.

To indicate how one could exploit this, I shall first discuss the
well-known example of a two-particle system produced by an interaction of
finite range, \( R \).

The c. m. configuration of such a system is conveniently described
by the relative momentum
\[
\mathbf{P} = \sqrt{\frac{2}{\lambda}} (\mathbf{q}_1 - \mathbf{q}_2)
\]
and the surface \( E = \text{const.} \) (the "mass shell") is then a sphere in the three-
dimensional \( P \)-space.

The wave function of the system can therefore be expanded in terms
of the spherical harmonics \( Y^m_l(\hat{P}) \), and because of the finite range of
the interaction, \( \lambda \) (and thereby the complexity of the angular distribution)
must be limited:
\[
\lambda \leq P \cdot R
\]
For a three-body system, two relative momenta are necessary, e.g.
\[
\mathbf{P}_1 = \sqrt{\frac{4}{\lambda}} (\mathbf{q}_1 + \mathbf{q}_2 - 2\mathbf{q}_3), \quad \mathbf{P}_2 = \sqrt{\frac{4}{\lambda}} (\mathbf{q}_1 - \mathbf{q}_2),
\]
and the mass shell in this case is a sphere in the six-dimensional \( (P_1 P_2) \)-
space; but only for non-relativistic kinematics. For the (relevant) case of
relativistic kinematics, it is a complicated algebraic surface.

Disregarding for a moment this important limitation, one can pro-
ceed as before and introduce spherical harmonics for the hypersphere, la-
belled by an index \( \lambda \), commonly denoted the "grand angular momentum"\(^3\).
This quantum number is the natural generalization of the orbital angular
momentum; in particular, it is restricted by the inequality
\[
\lambda \leq P \cdot R \quad (P = \sqrt{P_1^2 + P_2^2})
\]
and it controls the complexity of the state, this time not only with re-
spect to the over-all angular distribution, but also with respect to the
distribution of energy among the particles, i.e. the complexity of the
Dalitz-plot.
A generalization of this to the relativistic case was proposed two years ago by K.-E. Eriksson\textsuperscript{4,5} by defining a mapping of a non-relativistic mass-shell onto the relativistic ones. The whole formalism of the non-relativistic case could then be extended to the general case, and he was also able to show that the rough inequality for $\lambda$ still holds.

I thought that this kind of description might be useful, and decided to start with the reaction

$$\overline{\nu} \quad N \to \pi^+ \pi^- \pi^-$$

From a preliminary and rather crude study with the Veneziano-amplitude of Gopal et al.\textsuperscript{6}, it turns out that the most pronounced structure of the Dalitz-plot found by Anninos et al.\textsuperscript{7} (the bump and the hole) are produced by the lower waves ($\lambda = 0, 2, 4$), whereas the resonance bands are produced by higher waves.

Using the crude relation between $\lambda$ and the distance, we get the rather reasonable picture that a resonance, in order to decay undisturbed (and therefore be observed as a resonance), must be created in the outer part of the annihilation zone, and that the bump is a genuine three-body effect having no direct connection to resonance formation.

ACKNOWLEDGEMENT

I am indebted to many people, in particular to Ziro Koba and Niels Törnqvist, for discussions and constructive criticism; but none of these should be blamed for mistakes, omissions etc. in the present paper.

REFERENCES

3) F. T. Smith, Phys. Rev. \textbf{120} 1058 (60).
4) K.-E. Eriksson, Physica Scripta \textbf{1} 73 (70).
5) K.-E. Eriksson, Physica Scripta \textbf{2} 1 (70).
Mr. Mairhead: In \( \bar{p}n + \pi^+ \pi^- \) you failed to reproduce the resonance band. Did you also fail to reproduce the hole?

Mr. Bjørneboe: Yes.

Mr. Armenteros: Have you tried to extrapolate your model to annihilations in flight?

Mr. Bjørneboe: No. The number of waves increases rapidly. I am not sure that it is worth doing.

Mr. Astier: Your formalism should be compared to the one of Michel and De Rafael, who took directly relativistic representation of the decays.
THE ANNIHILATION REACTION $p \bar{p} \rightarrow 3 \pi^+ 3 \pi^-$ WITH AN ISOBARIC AMPLITUDE

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ABSTRACT

A good overall description of the experimental data for the reaction $p \bar{p} \rightarrow 3 \pi^+ 3 \pi^-$ is achieved with an isobaric amplitude $A_0$.

1. INTRODUCTION

A successful overall description of annihilation processes has been first obtained by dual amplitudes for reactions at rest, namely for $p \bar{p} \rightarrow n \rightarrow \pi^+ \pi^- \pi^-$ \(^1\), $p \bar{p} \rightarrow \pi^+ \pi^- \pi^0$ \(^2\), and $p \bar{p} \rightarrow 2\pi^+ 2\pi^-$ \(^3\). More recent investigations have shown that a similar agreement with the experimental data can also be achieved using simple isobaric amplitudes \(^4\). The attempt to describe an annihilation reaction in flight with an isobaric amplitude is based on the results of a further study of the process $p \bar{p} \rightarrow 2 \pi^+ 2 \pi^-$ at rest \(^5\). The distributions from the dual $B_5$ computation (Ref.3) can be reproduced almost point by point by a simple isobaric amplitude $A_0^\pi$ with only one resonance pole and irrespective of the use of "kinematical factors". The same holds true for an isobaric $A_0$.

It is therefore suggestive to look for a n-body final state $p \bar{p}$ annihilation in flight dominated by a single resonance, and attempt its description by an $A_0^\pi$ amplitude without caring for the construction of "kinematical factors". Such a candidate is the reaction $p \bar{p} \rightarrow 3 \pi^+ 3 \pi^-$, as the data show a dominant $\rho^0$ peak in $M(\pi^+ \pi^-)$, a moderate asymmetry in the CM angular distributions of the pions and for the other one-dimensional distributions a rather statistical behaviour.

2. THE MODEL

Starting from the graph shown

(\(p \bar{p}\) adjacent, no exotic resonances or exchanges) an ansatz for the amplitude is made:

$$A = c \sum_{\text{perm}} (-1)^{\text{perm}} A_0(1,2,3,4,5,6,7,8),$$

where $c$ means a normalization constant. "Perm" stands for the 36 permutations from the symmetrization in the positive and negative pions, respectively. The evaluation of $A_0$ follows from the required symmetry and factorization properties of the amplitude:
\[ A_8 = \frac{1}{x_{12}} A_7 (12, 3, 4, 5, 6, 7, 8) + \frac{1}{x_{23}} A_7 (\ldots) + \ldots \]

\[ = \frac{1}{x_{12}} A_6 (123, 4, 5, 6, 7, 8) + \ldots \]

\[ = \frac{1}{x_{1234}} A_5 (1234, 5, 6, 7, 8) + \ldots \]

The essential term from this expression is:

\[ A_8 = \left( \frac{1}{x_{1234}} + \frac{1}{x_{3456}} \right) \cdot \frac{1}{x_{12}} \cdot \frac{1}{x_{34}} \cdot \frac{1}{x_{56}} \cdot \frac{1}{x_{78}} \]

\[ + \left( \frac{1}{x_{2345}} + \frac{1}{x_{4567}} \right) \cdot \frac{1}{x_{23}} \cdot \frac{1}{x_{45}} \cdot \frac{1}{x_{67}} \cdot \frac{1}{x_{81}} \]

where \( x_{ij} \) and \( x_{ij} \) mean:

\[ x_{ij} = 1 - \alpha_e (s_{ij}) \]

(\( \scriptscriptstyle \text{P} \) trajectory in case of a resonance in \( s_{ij} \)),

\[ x_{ij} = \frac{1}{2} - \alpha_e (t_{ij}) \]

(\( \scriptscriptstyle \text{N} \) trajectory in case of an exchange with \( t_{ij} \)).

This result corresponds to the diagram:

\[ A_8 = \]

3. RESULTS AND DISCUSSION

With the above amplitude a good overall description of the reaction \( \scriptscriptstyle \text{P} \scriptscriptstyle \text{P} \rightarrow 3\pi^+ 3\pi^- \) at various incoming momenta is obtained. The model reproduces both the asymmetry in the CM production angles of the pions (Fig.1) and the resonance production in \( M(\pi^+ \pi^-) \), as shown in Fig.2.

<table>
<thead>
<tr>
<th>( \scriptscriptstyle \text{P} ) (GeV/c)</th>
<th>exp</th>
<th>( A_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.94 ± 0.06</td>
<td>1.9</td>
</tr>
<tr>
<td>3.6</td>
<td>0.92 ± 0.06</td>
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<tr>
<td>5.7</td>
<td>0.31 ± 0.06</td>
<td>0.34</td>
</tr>
<tr>
<td>7.0</td>
<td>0.25 ± 0.03</td>
<td>0.20</td>
</tr>
</tbody>
</table>

\(+\) \( \alpha_e (s) = 0.48 + 0.9 s + i 0.2 (s - s_0)^{\frac{1}{2}} \)

\( \alpha_e (t) = -0.87 + 0.9 t \)

\(\scriptscriptstyle \text{++} \) The (forbidden) \( e^+ e^- e^0 \) coupling is used only for computational purposes. Obviously the exact numerical value of the meson \( e^0 \) is of minor importance here.
The predicted cross sections, normalized to the 3.6 GeV/c value, are listed in Table 1. Detailed references to the various experimental results are given in Ref. 6.

We want to stress that in constructing our model no attempt was made to parametrize the data but rather to give an acceptable global description. We found that within the framework of the model, interferences play an essential role. Neglecting the Bose-Einstein symmetrization, or adding the parts of the amplitude incoherently, yields a much poorer agreement with the data. This may be regarded as a hint of the importance of interference phenomena in this kind of interactions.

It is a pleasure for me to acknowledge the excellent collaboration with Dr. J. Boguta throughout this work.

REFERENCES

1) C. Lovelace, Phys. Letters 26B, 264 (1968)
4) J. Boguta, Nucl. Physics, B13, 537 (1969)
5) J. Boguta and B. Nellen, \( p\bar{p} \rightarrow 2\pi 2\nu \) in an isobaric \( A_0 \) model
   (in preparation)

---

Fig. 1 The c.m. angular distributions of the pions for the reaction \( pp \rightarrow 3\pi 3\pi^* \) at (a) 5.7 GeV/c and (b) 3.6 GeV/c. The solid curve is from the isobaric model.
Fig. 2 $M(\pi^+\pi^-)$ for the reaction $pp \rightarrow 3\pi^+3\pi^-$ at (a) 5.7 GeV/c and (b) 3.6 GeV/c. The solid curve is from the isobaric model.
Mr. Bettini: I think that the theoretical models, when put out, should be tested against all the published data relevant for the model itself. The Hopkinson and Roberts work for example tests the $\Delta S$ model only against the $\bar{p}p \to 4\pi$ annihilation at rest and not against the published data on $\bar{p}n \to 2(\pi^-\pi^+)\pi^0$ at rest; in this case there are more different mass distributions, whose behaviour must be predicted by the model, and hence the test would be more stringent e.g. (experimentally $\rho^0$, $\rho^+$ and $\rho^-$ are produced with different intensities).

Also the model just presented by Nellen is not tested against these data.

Mr. Lillestøl: Sorry for Nellen's statement about the fair agreement between the model and the experimental data. At least concerning the application of $B_\Delta$ to the data of $\bar{p}p$ at rest into $4\pi$, my personal opinion is that the disagreement is so large that one should rather conclude that it is complete!

Mr. Nellen: My main concern in discussing the dual approach to the reaction $\bar{p}p_{\text{rest}} \to 2\pi^+2\pi^-$ was to show that it can be simplified (drop of kinematical factor). This simplification then motivated our isobaric $A_\Delta$-Ansatz.
AN ANALYSIS OF THE REACTION $\tilde{p}p \rightarrow 2\pi^+ 2\pi^- \pi^0$ AT REST USING A SIMPLIFIED VERSION OF THE
ISOBAR MODEL

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1. INTRODUCTION

1.1 Over the last many years a lot of detailed experimental informations has accumulated on
antinucleon-nucleon ($\tilde{p}p$ and $\tilde{p}n$) reactions, specially in some channels which are abundant-
ly produced. In contrast, theory lags behind and at present there is no theory or model
which can in general be used to explain the experimental data, in the form of various
effective mass and angular distributions.

The dual Veneziano model has the potential of explaining experimental data and has
been successfully used by Lovelace\textsuperscript{1) to explain the reaction $\tilde{p}n \rightarrow 2\pi^+ \pi^-$ and by Hopkinson
and Roberts\textsuperscript{2) to explain the reactions $\tilde{p}p \rightarrow 2\pi^+ 2\pi^-$. Owing to the complexity of the dual $B_n$
($n>5$) functions and the presence of the nucleon (antinucleon) spins, it is not possible to
extend this model to $\tilde{p}p \rightarrow n\pi^+$, $n \geq 5$.

1.2 Recently Boguta\textsuperscript{3}, has tried to understand and extract the useful features of the dual
$B_n$ functions and to incorporate them into a simpler model which can readily be generalized
to higher orders (for higher multiplicity channels). He has called this the Isobar model.
In reactions where both models can be applied, the Isobar model gives almost identical
results as the dual model\textsuperscript{4). Encouraged by this, the Isobar model was extended to the
reaction $\tilde{p}p \rightarrow 3\pi^+ 3\pi^-$ by Nellen\textsuperscript{5) and provided reasonable agreement with the experimental
data.

It must be emphasized that although the agreement with data is not excellent by $\chi^2$
standards, the essential points are:

a) that one can at least speak of a model which gives a reasonable description of
experiment whereas earlier there was no theory or model at all, and

b) this is actually a zero parameter model which gives absolute predictions.

It is in this spirit that we have applied a slightly differing version of the Isobar
model to the reaction $\tilde{p}p \rightarrow 2\pi^+ 2\pi^- \pi^0$ at rest.

2. THE MODEL

2.1 The detailed features of the Isobar model have been described by Boguta\textsuperscript{3,4) and
Nellen\textsuperscript{5}). We describe here briefly the basic features and the later difference in approach
followed by us. The basic amplitude is the 4-point function $A_n$ (1,2,3,4) described in
fig. 1.

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Colaba, Bombay-5, India.
In order to write an amplitude for a graph which has >4 incoming and outgoing lines one uses the property of factorisation whose use for \( A_5 \) \((1,2,3,4,5)\) is illustrated in fig. 2.

2.2 In this subsection we describe how we have proceeded to write down the amplitude for the reaction \( \tilde{p}p \rightarrow 2\pi^+ 2\pi^- \pi^0 \) at rest.

a) **Choice of Permutations**

In the reactions \( \tilde{p}p \rightarrow 2\pi^+ 2\pi^- \) and \( \tilde{p}p \rightarrow 3\pi^+ 3\pi^- \) investigated so far if one assumes that \( p \) and \( \tilde{p} \) lines are adjacent and no two like pion lines should be adjacent, then one is left with only one permutation to deal with fig. 3a. However, in the reaction \( \tilde{p}p \rightarrow 2\pi^+ 2\pi^- \pi^0 \), the \( \pi^0 \) can be permuted with \( \pi^\pm \) and again assuming \( p \) and \( \tilde{p} \) to be adjacent we can have fine permutations, as shown in fig. 3b.

To ease the computational task, we made a selection of permutations (i), (ii), (iii) on the physical basis that these permutations lead to maximum \( \pi^0 \) production, and experimentally, we observe copious production of \( \omega^0 \) in \( \tilde{p}p \rightarrow 2\pi^+ 2\pi^- \pi^0 \) at rest.

b) **Choice of Factorized Diagrams**

Because of the presence of seven lines in each selected permutation and the possibility of factorizing in a very large number of ways (again due to the \( \pi^0 \)), we were forced to make a physical selection of the factorized diagrams which we would use. It is at this point that our approach differs from earlier use of the Isobar model. We neglected those factorizations in which:

i) four mesons were factorized to a single branch (because energy available is small and we thought we could neglect 4-particle resonances),

ii) the proton (or antiproton) was factorized with \( \geq 2 \) mesons (i.e. we took only \( (p\pi) \) or \( (\bar{p}\pi) \)),

iii) the proton and antiproton were factorized with \( \geq 1 \) mesons (i.e. we took only \( (\bar{p}p) \)).

Thus we first drew 16 diagrams for each possible permutation and then wrote down the amplitude for each by tracing its history from the original \( A_7 \) \((1,2,3,4,5,6,7)\) up to the final factorized diagram corresponding to \( A_4 \) \((i,j,k,l)\).

c) **Choice of Trajectories**

For baryon we use the \( N^* \) trajectory below threshold for the baryon pole\(^5\):

\[
a_B = 1 + 0.37 - 0.9 \, S_B
\]
For \((\pi^+ \pi^- \pi^0)\) combination on one branch of the final factorized diagram we use the \(\omega^0\) trajectory:

\[
\alpha_{\omega^0} = 1 - 0.48 - 0.85 S_M - i 0.05 \sqrt{S_M - S_O}
\]

where the imaginary part is adjusted to give proper width.

For other \((\pi^+ \pi^- \pi^0)\) combinations, \((\pi^+ \pi^-)\), \((\pi^+ \pi^0)\) we use the degenerate \(\rho^0 - f^0\) trajectory\(^5\):

\[
\alpha_{\pi^0} = 1 - 0.48 - 0.9 S_M - i 0.2 \sqrt{S_M - S_O}
\]

For \((\pi^\pm \pi^0)\) combinations we use pure \(\rho\) trajectory as used by Jengo and Remiddi\(^6\):

\[
\alpha_{\rho} = 1 - 0.48 - 0.9 S_M - i 0.13 \sqrt{S_M - S_O}
\]

**Comments:** i) by using different trajectories for \((\pi^+ \pi^- \pi^0)\) combinations which are and are not on one branch of the final 4-branch diagram, we have incidentally given a very direct physical meaning to the diagrams. What it means is: if an \(\omega^0\) is actually formed in a diagram, we use the \(\omega^0\) trajectory whereas if a \((\pi^+ \pi^- \pi^0)\) combination occurs incidentally when we factorize the final \(A4\) \((i,j,k,l)\) we describe it by a general degenerate trajectory.

ii) We have used different trajectories for \((\pi^+ \pi^-)\) and \((\pi^\pm \pi^0)\) combinations because we feel that \((\pi^+ \pi^-)\) system is strongly coupled to both \(\rho^0\) and \(f^0\) and should be described by a degenerate trajectory whereas \((\pi^\pm \pi^0)\) system is coupled strongly only to \(\rho^\pm\) and therefore a pure \(\rho\) trajectory may be used.

d) **Addition of Amplitudes**

In each permutation we have 16 amplitudes corresponding to 16 diagrams. We sum up these 16 amplitudes coherently to get three amplitudes, one for each permutation:

\[
A_i = \sum_{k=1}^{16} A_{ik}, \quad i = 1,2,3
\]

We add the absolute squares of these three amplitudes to get the matrix elements:

\[
|A|^2 = |A_1|^2 + |A_2|^2 + |A_3|^2
\]

e) **Symmetrization of Amplitude for like pions**

In order to symmetrize the matrix element, one should, in principle include the effects of diagrams in which the like pions have been permuted. In our case this would lead to a 4-fold increase in computational time. To ease the situation we have simply multiplied the matrix element \(|A|^2\) described in the previous subsection, by a Goldhaber\(^7\) type symmetrizing weightfunction. Thus our final matrix element is:
\[ \text{ME} = |A|^2 \cdot \text{WTFN} \]

where
\[
\text{WTFN} = (1 + e^{\frac{\lambda(M_{\text{eff}}(\pi^+ \pi^-) - 4m^2)}{m}}) \cdot (1 + e^{\frac{\lambda(M_{\text{eff}}(\pi^- \pi^-) - 4m^2)}{m}})
\]

\[ \lambda = 6.15 \]

This matrix element is used as a weight in the FOWL Monte Carlo program.

3. RESULTS

The results are displayed in Figs 4-7. The full line histogram represents the experimental results, and the dashed lines give the prediction of the model. As one can see there is a deficiency of events at \(\approx 1\) GeV in the 3-particle effective mass distributions. Ad hoc we included a 30% contribution of a broad 3-pion trajectory corresponding to an enhancement centered at \(\approx 1\) GeV and with a \(\approx 200\) MeV width. The results of this are shown by the full lines. As can be seen, the agreement with experiment improves considerably in all distributions.

4. CONCLUSIONS

We have described the application of the Isobar model to the reaction \(\bar{p}p \rightarrow 2\pi^+ 2\pi^- K^0\) at rest. We notice it gives modest agreement with experiment, but as emphasized in the beginning, this was precisely our aim in a situation where no model exists. For a zero-parameter model (or one parameter, if we include a new trajectory as described in section 3 above) it gives a good general description of the experimental data.

We are at present continuing this work in trying to understand the effects of the permutations and diagrams we have neglected and also trying to use this model at higher energies.

REFERENCES

1. C. Lovelace, Phys. Letters 28B (1968) 264
4. J. Boguta, Bonn Univ. PI 2-98, July 1971
5. B. Nellen, Bonn Univ. THESIS: The Annihilation Reaction \(\bar{p}p \rightarrow 3^+ 3^-\) at 3.6 GeV/c.
8. F. James, CERN - Program Library (W 505).
FIGURE CAPTIONS

1. Isobaric A4 (1,2,3,4)
2. Isobaric A5 (1,2,3,4,5)
3. Permutations in various final states
4. Distributions of 2-pion effective masses experimental histogram and dashed line - theoretical histogram for zero parameter model (see text) solid line - theoretical histogram for one parameter model, i.e. assuming 50% contribution from another 3-pion trajectory (see text)
5. Distributions of 3-pion effective masses various histograms as in 4
6. Distributions of 4-pion effective masses various histograms as in 4
7. Distributions of various angular correlations various histograms as in 4
Isobaric $A_4(1,2,3,4)$

With one pole in each adjacent channel

$$A_4(1,2,3,4) = \frac{1}{a_{12}} + \frac{1}{a_{23}}$$

$$a_{ik} = \alpha_0 + b S_{ik} + i \theta (S_{ik} - S_{th}) C (S_{ik} - S_{th})^{-\frac{1}{2}}$$

$\alpha_0$, $b$, $c$ are fixed constants corresponding to the appropriate Regge trajectory.

Fig. 1

Factorization Property of $A_n(1,2,...,n)$ Amplitudes

Example $A_5(1,2,3,4,5)$

Formally:

$$A_5(1,2,3,4,5) = \frac{1}{a_{12}} A_4(12,3,4,5) + \frac{1}{a_{23}} A_4(1,23,4,5) + \ldots + \frac{1}{a_{51}} A_4(51,2,3,4)$$

Fig. 2
Possible permutations for

\[ p\bar{p} \rightarrow 2\pi^+ 2\pi^- \]

\[ p\bar{p} \rightarrow 3\pi^+ 3\pi^- \]

Fig. 3a

Possible Permutations for \( p\bar{p} \rightarrow 2\pi^+ 2\pi^- \eta^0 \)

(i) \hspace{1cm} (ii) \hspace{1cm} (iii)

(iv) \hspace{1cm} (v)

Fig. 3b
Fig. 4a

Fig. 4b
Fig. 4c

Fig. 5a
Fig. 5b

Fig. 5c
Fig. 6a

Fig. 6b
Fig. 7
STATUS REPORT ON THE 2.3 GEV/C \( \bar{p}p \) EXPERIMENT

W.W.M. Allison
Nuclear Physics Laboratory, University of Oxford

This experiment is the work of a number of people, D.S. Rhines, T. Fields, Y. Oren, J. Whitmore (Argonne), W.A. Cooper (formerly Argonne now at the Open University), W.W.M. Allison (formerly Argonne now at Oxford).

The data were taken in the ANL 30" chamber and 340,000 events were automatically scanned and measured by POLLY II\(^1\). The \( V^0 \) events were scanned by hand and measured by POLLY later - these data are preliminary.

We have already published our data\(^2\) on \( \rho/\omega \) interference. They suggest that the production amplitudes for the states \( \rho^0 \pi^+\pi^- \) and \( \omega^0 \pi^+\pi^- \) are approximately equal and in phase at this momentum and others where data is available above 1.2 GeV/c. In particular, the relationship applies to the pure \( I \)-spin states \( \rho^0 \rho^0/\omega \rho^0 \), \( \rho^0 \omega/\rho^0 \omega \).

Figure 1 shows the elastic angular distribution. The data cover the range \(+0.97\) to \(-0.98\) corresponding to minimum lab. momenta of 210 MeV/c and 170 MeV/c for proton and anti-proton respectively. In addition to the statistical errors shown there is a 4% normalisation uncertainty which applies to all channels. The forward peak shows some evidence for curvature. Extrapolating from various regions of the forward peak (as shown) we get:

\[
\frac{d\sigma}{d\Omega} = 98 \pm 15 \text{ mb/sr},
\]

and

\[
\sigma_{el} = 30 (\pm 2) \text{ mb}
\]

At this point it is normal practice to ignore the difference between singlet and triplet scattering and attribute the difference

\[
\frac{d\sigma}{d\Omega} = \frac{k^2 \sigma_{tot}}{16\pi^2}
\]

to the real part of the forward scattering amplitude. One must remember however that the non-linearity of the optical theorem will destroy this argument if singlet and triplet scattering differ in either total cross section or ratio of real to imaginary parts.

With this reservation we have

\[
\text{modulus} \frac{\text{Ref}}{\text{Im}} = 0.28 (\pm 0.15) \text{ i.e. consistent with zero.}
\]

In the backward region we have no peak at all - the distribution as a whole certainly does not suggest the presence of \( s \)-channel resonances, although, as has been pointed out before at this meeting, quantitative models of diffraction which can be reliably extended into the backward region do not exist. As a result no firm conclusion may be drawn.

---

* Work supported in part by the US Atomic Energy Commission
Fig. 1 pp elastic scattering at 2.32 GeV/c (79,255 events).
We have searched our data very carefully for exotic mesons. None has been found. Other aspects of the multibody data will be published in a series of papers in the near future.

I would like to spend the rest of the time discussing the 2 body channels shown 4):

<table>
<thead>
<tr>
<th>Channel</th>
<th>Events</th>
<th>Cross Section (µb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>π⁺π⁻</td>
<td>223</td>
<td>45±3</td>
</tr>
<tr>
<td>K⁺K⁻</td>
<td>91</td>
<td>18±2</td>
</tr>
<tr>
<td>π⁰ρ⁰</td>
<td>72±19</td>
<td>Background subtracted; ( f⁰ → π⁺π⁻ )</td>
</tr>
<tr>
<td>π⁻ρ⁺</td>
<td>51±15</td>
<td>&quot;</td>
</tr>
<tr>
<td>ρ⁻π⁺</td>
<td>119±40</td>
<td>&quot;</td>
</tr>
<tr>
<td>K⁺⁺⁺⁺</td>
<td>0</td>
<td>&lt; 0.3</td>
</tr>
<tr>
<td>K⁺⁺⁺⁺(K⁺⁺⁺⁺)</td>
<td>2</td>
<td>( \sim 0.3 )</td>
</tr>
</tbody>
</table>

Figure 2 shows the relevant angular distributions. The \( K⁺⁺⁺⁺ \) data is consistent with hyperon exchange and indeed agrees quite well with backward KN scattering under line reversal (dotted curve) 4). There is no backward peak due to \( Z⁺⁺⁺⁺ \) exotic exchange. The small ratio \( K⁺⁺⁺⁺/K⁺⁺⁻⁻ \) may be interpreted as the result of either

a) \( I = 0 \) exchange or

b) equal in phase \( I = 0 \) and \( I = 1 \) s-channel amplitudes.

While these two descriptions are "dual" to one another, it is particularly interesting to notice that the same relative amplitude and phase of the \( I = 0 \) and \( I = 1 \) channels was found in the p/u interference 3). Furthermore both effects appear more or less independent of incident momentum above \( \sim 1 \) GeV/c suggesting either an absence of s-channel resonances or a strong degeneracy of resonances with respect to I-spin.

A very different situation appears to obtain in the \( 2\pi \) channel. The angular distribution shows a major peak corresponding to a 4-momentum transfer of 1.2 GeV² or a transverse momentum of 1 GeV/c! This suggests spatial structure of order 0.2 fermis in the interaction region. We have compared our data with those of other experiments and get moderate to good agreement 4,5,6). The line reversal predictions of the models of Barger & Cline 7) (dashed curve) and Berger & Fox 8) (solid curve) do not fit the data at all. We have also considered a simple diffractive model 9) involving 2 body production in the surface of the annihilation region. While this can give a peak at \( \sim 1 \) GeV/c in transverse momentum the primary peak near zero should be twenty times higher. We are forced to the conclusion that an s-channel resonance interpretation of data in this region is not only possible (as shown by Nicholson et al. 6 but necessary 10).

Why do these resonances only couple to \( \pi⁺\pi⁻ \)? Is the observation of this extraordinary resonance phenomenon to be associated with the equally unique combination of selection rules and absorption which constrains the s-channel amplitudes for \( \pi⁺\pi⁻ \)? It is only reasonable to conclude that other states also couple to s-channel resonances but that the large angle interference structure are washed out by the many different helicity and I-spin states.
Center of mass angular distributions. For the charged states the right hand side represents the 'no-charge-exchange' direction; neutral final states are folded about 90°. For $\rho$ and $f^0$ channels each point represents an independent resonance & background fit to the mass spectrum for that angular bin. The errors shown are statistical.
From these results we find positive support for a dual picture of towers of resonances with strong degeneracies such that s and t channel descriptions are equivalent. However only when there are a small number of amplitudes allowed in the s-channel does the resonance structure manifest itself.

REFERENCES

(1) W.W.M. Allison et al., Nucl. Inst. and Methods, 79, 841 (1970)
(4) T. Fields et al., to be published. Also Oxford preprint "The reaction $pp \rightarrow \pi^+ \pi^-$ and $K^+ K^-$ at 2.3 GeV/c" (October 1971)
(9) The word diffraction means different things to different people. Here we mean that the phases of the different partial waves are all relatively real and positive. This is discussed further in ref. 4.
(10) We do not imply that the details of the Nicholson analysis are necessarily unique, only that some such resonance interpretation is required.

DISCUSSION AND COMMENTS

Mr. Armenteros: How much time did you need for the measurements?

Mr. Allison: One year, 100 hour a week.

Mr. Butterworth: What is your scanning efficiency for backward elastic scattering and for $V^0$?

Mr. Allison: At 2.3 GeV/c, the angular range for backward elastic scattering with an unvisible antiproton is small. We introduce a cut at $\cos \theta^* = -0.97$. Our results on channels with strange particle are preliminary. In this experiment, emphasis was put on channels with large cross-sections.

Mr. Butterworth: Underlines that the strange behaviour of the $\pi^+ \pi^-$ angular distribution is not specific of the 2.3 GeV/c but is also absurd at other energies (1.2 to 1.5 GeV/c).
PROBLEMS WITH DEUTERIUM

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Istituto Nazionale di Fisica Nucleare, Sezione di Trieste

INTRODUCTION

The complete knowledge of the antinucleon-nucleon system ($\bar{N}N$) requests the study of both the $I = 0$ and $I = 1$ isospin channels.

An increasing number of experiments have been performed in the last years on the anti-proton-proton system ($pp$) which is a mixture of the $I = 0$ and $I = 1$ states.

The antiproton-neutron system ($\bar{p}n$) is a pure $I = 1$ isospin state but the absence of a free-neutron target makes much more complicated the study of this channel.

By charge conjugation the $\bar{p}n$ system is equivalent to the antineutron-proton system ($np$) but in this case the experimental difficulties in building up an $\bar{n}$ beam have not allowed up to now a real improvement. In fact the $\bar{n}p$ interaction has been studied with the bubble chamber technique in a two-steps process $^1$ ($pp \rightarrow np\pi^-, np \rightarrow$ elastic and total) with very poor statistics.

Only very recently some results on the $\bar{N}N$ interactions have been published using an antideuterium beam at Serpukov $^2$), but, of course, the problem of the free-antineutron interaction is not resolved in this way.

Therefore the main source of information on the $\bar{N}N$ system in the pure $I = 1$ state is at the moment the study of the $\bar{p}$-deuterium interaction ($\bar{p}d$), since the deuterium is the simplest and weakest bound proton-neutron system. However the difficulties encountered when working with a deuterium target have been in most of the cases underestimated and not all the effects due to the bound nucleon have been studied extensively $^3$). In fact while the shadow or Glauber effect $^4$) has been deeply analysed the three following complications have received much less attention:

a) the Fermi motion
b) the dependence of the cross section on the nucleon mass
c) the interaction between the incident particle and the entire deuteron.

Practically all the existing $\bar{p}d$ data have been analysed in the spirit of the Impulse Approximation (IA) and a suitable factor for the Glauber screening correction has been applied for the partial and total cross section computations.

In this paper the validity of the IA for the $\bar{p}d$ case will be briefly discussed in the first part. In the two following sections recent counter data on the $\bar{p}d$ total cross section will be analyzed and the criteria for studying particular final states will be reported, when the experiments with bubble chambers will be discussed. At the end the difficulties in extracting the free-neutron cross sections will be considered in some details taking as a reference a recent analysis of the $K^+$-nucleon system, where the situation is easier because of the spin zero of the incident $K^+$ and the possibility of making a phase shift analysis.

1. THE IMPULSE APPROXIMATION

The criteria for the applicability of this method for the $\bar{p}d$ case has been studied by P. E. Nemirovskii et al. $^5$).

It is well known that three main requirements must be fulfilled, $^6$) i.e.

I) The incident particle never interacts strongly with two constituents of the system at the same time. And this is verified because of the large deuteron radius ($R \simeq 4$ fm) compared to the range of the $\bar{N}N$ forces ($r \simeq 1$ fm).

II) The amplitude of the incident wave falling on each constituent (nucleon) is nearly the same as if that constituent were alone (the transparency assumption). If the wavelength $\lambda$ of the scattered particle is small compared with $R$, this condition may be written in the following way

\[
\frac{\lambda}{R} \left( \frac{\sigma}{4\pi} \right)^{1/2} \ll 1
\]
If $R = \frac{4}{9} \text{ fm}$ and the formula $\sigma = \frac{39.8}{6} \text{ mb}^2$ ($\beta$ is the $p$ velocity in the laboratory system) gives the magnitude of the $\bar{p}p$ and $\bar{p}n$ total cross sections, this limit may be expressed as $\gamma = \sqrt{\frac{39.8}{\beta}} \approx 0.052 \ll 1$ (in Fermi). As an example for an $p$ incident in the laboratory system of $0.5 \text{ GeV}/c \gamma = 0.063$ and for $0.2 \text{ GeV}/c \gamma = 0.22$.

III The binding forces between the constituents of the system are negligible during the collision, when the incident particle interacts strongly with the system, and this is true if $\lambda \ll \eta$.

Therefore the IA may be applied to the $\bar{p}d$ interaction even at low energies; in II it has been shown that for a momentum of $0.5 \text{ GeV}/c$ ($\approx 0.13 \text{ GeV}$ of kinetic energy) the correction is of the order of $0\%$.

But the effect of the Fermi motion at low energy may be very important; this effect in two extreme situations has been made evident in the plot of $\lambda$ (the relative $p$-particle target wavelength) of Fig. 1; here the nucleon target momentum $p_N$ is $0.5 \text{ GeV}/c$ has the beam direction and the same ($p_N = +0.300$) or the opposite ($p_N = -0.300$) versus of the beam; since the deuteron mass is considered to be conserved, the nucleon target is taken off-mass shell.

Nevertheless the diffraction theory $^4$ cannot be applied to the complete range where the IA is trustworthy. In fact it has been shown in a convincing fashion that the standard Glauber formula

$$\sigma_{ad} = \sigma_{an} \sigma_{ap} - 2 \sigma_{ap} \sigma_{an} C$$

where $\sigma_{ad}$, $\sigma_{an}$ and $\sigma_{ap}$ are the $\bar{p}d$, $\bar{p}p$ and $\bar{p}n$ absorption cross sections and $C$ is a positive constant does not work because of their large values also at relatively high energy $^8$.

Moreover at very low energy ($0.050-0.100 \text{ GeV of the incident } \bar{p}$) some theoretical calculations give the impossible result that $C$ be negative $^5$.

In these cases the correct prescription, which saves the unitarity, is the following: compute the elastic amplitude from the Glauber theory and from it both the elastic and total cross sections (through the optical theorem); the absorption cross section is then the difference between them.

A comment about the formula

$$\sigma_{td} = \sigma_{tp} + \sigma_{tn} - \xi \sigma$$

(where $\sigma_{td}$, $\sigma_{tp}$ and $\sigma_{tn}$ are now the $\bar{p}d$, $\bar{p}p$ and $\bar{p}n$ total cross sections, and $\xi \sigma$ is the Glauber cross section defect) may be useful at this point; assuming the Glauber theory all the effects due to the bound two-nucleon system are considered to be summarized in $\xi \sigma$. This term is zero if we take into account only the contribution from the single scattering, but in this case the unitarity is violated because the imaginary part of the amplitude at zero degrees cannot be represented in the form of a sum of amplitudes for the free particles, each of them satisfying separately the unitarity condition.

2. THE TOTAL $\bar{p}d$ CROSS SECTION

In Fig. 2 the $\bar{p}d$ total cross section is shown $^9-14$). If explicitly given by the authors, also the $\bar{p}n$ total cross section is plotted.

In a high-precision $^{15}$) transmission experiment the $\bar{p}p$ and $\bar{p}d$ total cross sections have been measured between $1.00$ and $3.30 \text{ GeV}/c$ and the authors give the pure isospin total cross sections $\sigma_o$ and $\sigma_i$.

The computation of the these cross sections has been carried out in the following way:

The total cross section formula has been written as

$$\sigma_{pd} = \sigma_{pp}^m + \sigma_{pn}^m - \delta \sigma$$

Here

$$\delta = \frac{cp}{4\pi} \left[ 2 \sigma_{pp}^m \sigma_{pn}^m (1-\rho_p \rho_n) - \frac{1}{2} \left[ \sigma_{pp}^m (1-\rho_p^2) \sigma_{pn}^m (1-\rho_n^2) \right] \right]$$

is the Glauber-Wilkin cross section defect where $\rho_p$ is the ratio of the real to imaginary part of the forward scattering amplitude for the $\bar{p}p(\bar{p}n)$ scattering. The $\sigma_{pp}$ and $\sigma_{pn}$ are the $\bar{p}p$ and $\bar{p}n$ total cross section smeared out over the Fermi motion.

The procedure for the extraction of the free-neutron cross section is rather compli-
<table>
<thead>
<tr>
<th>Reaction</th>
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<th>p spectrum</th>
<th>Comments</th>
<th>Authors</th>
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</thead>
</table>
| $\bar{p}d \rightarrow \text{all}$ | assumed | Hulthén 
large tail | 20% simultaneous $ar{p}n$ interactions | Chinowsky 1966 |
| $\bar{p}d \rightarrow p_s \pi^- \phi, \phi \rightarrow \bar{K}K$ | assumed $p_s < 0.250$ | Hulthén, normalization 0.12 - 0.18, large tail | 28% proton visible and stopping in the chamber | Bettini 1967 |
| $\bar{p}d \rightarrow p_s \pi^- \pi^+$ | assumed $p_s < 0.150$ | Hamada - Johnson 20 | If the proton has large momentum, it cannot be treated as a spectator | Anninos 1969 |
| $\bar{p}d \rightarrow \bar{p}N N N [\pi^+ p] [\Delta N^+(n\pi)]$ | no 3-body reaction | 84% okey Hulthén 16% no Hulthén Rescattering not possible or not sufficient $\rightarrow \Delta$ body | Bizzarri 1969 |
| $\bar{p}d \rightarrow p_s K^+ (n\pi)$ | assumed $p_s < 0.250$ | | Bizzarri 1969 |
| $\bar{p}d \rightarrow p_s \omega^- \pi^+$ | assumed $p_s < 0.150$ | | Bizzarri 1970 |
| $\bar{p}d \rightarrow p + N$ $\rightarrow n \pi$ | $\bar{p}$ $\rightarrow$ $M$ $\rightarrow$ $n \pi$ | Hulthén okey .150-.200 | Efficiency for identifying protons by ionization unity up to 0.800 | Gray 1971 |

Cated and, as yet, a little arbitrary. Moreover for the $\bar{p}n$ case $\rho_n$ is taken equal to $\rho_p$, because of lack of data on this quantity, and $<\pi^2>$ is not well known.

Nevertheless the result seems reliable if the $\bar{p}N$ cross-section is reasonably smooth or better if the $\bar{p}N$ cross-sections do not vary rapidly because of the presence of sharp resonances.

If the conditions for the extraction of the free-neutron cross section from deuterium and hydrogen data is supposed to be fulfilled, the pure isospin total cross section are then

$$\sigma_t = \sigma_{\bar{p}n}$$

$$\sigma_t = 2 \cdot \sigma_{pp} - \sigma_{\bar{p}n}$$

where $\sigma_{pp}$ and $\sigma_{\bar{p}n}$ are the total free-target $pp$ and $\bar{p}n$ cross sections.

In Fig. 3 a compilation of all the existing data on $\sigma_0$ and $\sigma_t$ is shown; if not given by the authors they have been computed from formula (1) propagating the errors.

3. BUBBLE CHAMBER EXPERIMENTS

In Table I and II the present situation is summarized.

Table I deals in the $\bar{p}d$ experiments at rest while in Table II the $\bar{p}$ in flight are
considered. The following general comments can be drawn:

a) The \( \bar{p}d \) annihilation at rest with a proton (\( p_b \)) in the final state can be explained as an \( \bar{p}n \) annihilation provided that the \( p_b \) momentum be less than 0.150-0.200 GeV/c.

In about 20% of the cases the \( \bar{p}d \) annihilation is better explained as a 3-body interaction.

b) Almost all the \( \bar{p}d \) annihilation in flight experiments have assumed the simple IA. There is some evidence that the old prescription for cooking the deuterium (momentum spectator less than momentum not spectator, momentum spectator less than 0.25 GeV/c) seems not to work properly even at high energies (see for example the 5.5 GeV/c experiment \( ^{37,38,39} \)).

4. THE EXTRACTION OF FREE-NEUTRON CROSS SECTIONS

A) From the experimental point of view:

I) In a recent \( K^+p \) and \( K^+d \) experiment \(^{41} \) the same reaction has been measured in hydrogen and in deuterium:

\[
\begin{align*}
K^+p & \rightarrow K^0 p + \gamma \\
K^+p(n) & \rightarrow K^0 p + \gamma(n) 
\end{align*}
\]

The parentheses denote the spectator nucleon defined with the two conditions

\[
\begin{align*}
P_n & < 0.250 \\
P_n & < P_p 
\end{align*}
\]

These two cross sections are shown in Fig. 4 together with previously measured data. The cross section for reaction (2) is systematically larger than that of reaction (3) by an average factor \( R = 1.26 \pm 0.06 \).

Therefore in this case the standard Glauber correction factor \( \frac{1}{2} = 1.12 \), often used in the literature, does not work.

II) In Fig. 5 the \( \bar{p}p \) and \( \bar{p}n \) annihilation cross sections measured in the 0.050-0.200 GeV interval of the incident \( \bar{p} \) kinetic energy are shown (full-point) from a \( \bar{p}d \) experiment \(^{42} \).

The data shown that these two cross section are not equal.

In the same figure the \( \bar{p}p \) annihilation cross section from an \( \bar{p}p \) experiment is shown \(^7 \) (open points); the lack of the shadow in this case is apparent.

Therefore it is evident that some caution is necessary in the extraction of free-neutron cross sections, since the effects of the bound nucleon seem to be strongly dependent on the particular channel and on the particular energy range.

B) From the theoretical point of view:

If the main point of interest is the extraction of the free-neutron cross section (and not the coherent reaction, in which there is a deuteron in the final state) then only the "elastic incoherent" break-up reaction has been studied in some details. A process is here called "elastic incoherent" if it is of the type

\[
xd \rightarrow y NN
\]

where \( x \) is a hadron, \( NN \) a nucleon pair, and \( y \) is \( x \) or \( \bar{x} \) or the charge-exchanged particle of \( x \) (for example \( \pi d \rightarrow \pi n, K^0 d \rightarrow K^0 pp, \bar{p}d \rightarrow \bar{p}p \) etc.).

The results of a particular analysis on the \( K^0 d \rightarrow K^0 pp \) reaction \(^{43} \) are reported here, just to understand all the difficulties.

The deuteron break-up in the \( K^0 d \rightarrow K^0 pp \) case has been described by means of the impulse approximation, assuming that the two following Feynman graphs, obtained one from the other by interchanging the two nucleon lines, describe the process:

\[
\begin{align*}
\begin{array}{cccc}
2 & 3 & K^0 & \\
\bar{d} & \alpha & P & \\
1 & & & \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{cccc}
2 & 3 & K^0 & \\
\bar{d} & \alpha & P & \\
1 & & & \\
\end{array}
\end{align*}
\]
The charge exchange differential cross section in the lab. frame, where the deuteron is at rest, can be expressed by means of the K-nucleon free amplitudes in the following way:

\[
\frac{d\sigma}{d\Omega_{\text{lab}}} = \frac{1}{m_1 p_1} \int d^3 p_2 \left\{ \left| \psi_s - \psi_d \right|^2 \left[ \frac{1}{4} \left| f^1 - f^0 \right|^2 + \frac{1}{2} \left| g^1 - g^0 \right|^2 \right] + \left| \psi_s + \psi_d \right|^2 \left[ \frac{1}{T^2} \left| g^1 - g^0 \right|^2 \right] \right\} \frac{p_3^2}{E_2 \left[ p_3 \left( E_2 - E_4 \right) - E_3 \left( E_3 - E_4 \right) \right]} \]

(4)

where we label the particles following, for example, the first of the two diagrams.

The kinematical variables i.e. \( p_1 \) and \( E_4 \), are all defined in the lab. frame while the standard no spin-flip and spin-flip amplitudes in \( I=1 \) and 0 state, \( f^- (s) \) and \( g^- (s) \), are defined in the c.m. of the \( K^- N \) system. \( \psi_0 (s) \) is equal to \( S_{34}^{J3} (s) \) \( \phi (-p_3) \) where \( \phi (-p_3) \) is the deuteron wave function in the momentum space and \( S_{34}^{J3} (s) \) is the total energy of the K-nucleon system in their c.m. frame.

The amplitudes \( f^- (s) \) and \( g^- (s) \) depend on \( p_3 \) through \( S_{34}, S_{35} \) and \( \cos \phi \). If we suppose that the dependence of \( f^- (s) \) and \( g^- (s) \) on \( \cos \phi \), \( S_{34}, S_{35} \) is slow we can evaluate them for

\[ S_{34} = S_{35} = S' \]

\[ \cos \phi = \cos \phi' \]

where the primed quantities are obtained considering the struck nucleon at rest in the target. Under this assumption it is also possible to give the cross section in the approximative \( K^- N \) c.m. frame just by multiplying \( \frac{d\sigma}{d\Omega_{\text{lab}}} \) by the jacobian of the transformation from the \( K^- N \) lab. frame to the \( K^- N \) c.m. frame (if we suppose the nucleon at rest in the target this jacobian does not depend on \( p_3 \)).

With these hypothesis equation (4) becomes

\[
\frac{d\sigma}{d\Omega \ast} \ast \text{c.o.} = \left[ \frac{1}{4} \left| f^1 - f^0 \right|^2 + \frac{1}{2} \left| g^1 - g^0 \right|^2 \right] I_{\text{ppt}} + \left[ \frac{1}{T^2} \left| g^1 - g^0 \right|^2 \right] I_{\text{pps}}
\]

(5)

Where \( I_{\text{ppt}} \) and \( I_{\text{pps}} \) are the so called deuteron weight factors for the proton-proton final charge state and respectively triplet and singlet spin state \( \left( \right. \). Now \( I_{\text{ppt}} = I_\ast - J_\ast \) and \( I_{\text{pps}} = J_\ast + J_\ast \), and

\[
J_\ast = \int \frac{d^3 p_2}{2m_2 p_2} \int d^3 p_3 \left[ \left| \psi (-p_3) \right|^2 \left| \psi (-p_3) \right|^2 \left[ \frac{p_3 \left( E_3 - E_4 \right) - p_3 \left( E_3 - E_4 \right)}{E_2 \left[ p_3 \left( E_2 - E_4 \right) - E_3 \left( E_4 - E_4 \right) \right]} \right] p_3^2 \right]
\]

The behaviour of \( I_{\text{pps}} \) for two different moments are given in Fig. 6 as can be seen, in the high energy limit, this result is coherent with the closure approximation.

In Fig. 7.a and 7.b the behaviour of the function that must be integrated for the \( I_\ast - J_\ast \) computation is shown for two different values of \( \cos \phi \). The spectator shoulder and the quasi-elastic peak are apparent. In the second case the interference between the two single scatterings is visible.

Finally the charge exchange differential cross section in deuterium can be expressed as

\[
\frac{d\sigma}{d\Omega} = \left[ \frac{1}{2} \left| f^1 - f^0 \right|^2 \left( I_\ast - J_\ast \right) + \left| g^1 - g^0 \right|^2 \left( I_\ast - \frac{J_\ast}{2} \right) \right]
\]

but even when the \( K^- N \to K^- p \) differential cross section is derived from the \( K^- d \to K^- p \), nevertheless the \( K^- N \to K^- p \) cross section on the free-neutron is not obtainable because of the lack of knowledge of the \( f^- \), \( g^- \) and \( g^- 6 \). Therefore this free-neutron cross section will be a by-product for example of a complete phase shift analysis in which the amplitudes will be fitted directly to the deuteron data rather than to free-nucleon data \( \left( \right. \).
5. CONCLUSION

The era in which the deuterium was considered as an easy device for the extraction of free neutron cross sections seems definitively ended.

From a naive experimental point of view it is possible to say that there is no fundamental difficulty in the particle-deuterium analysis.

From a phenomenological point of view the extraction of particle free-neutron cross-section largely depends upon the type of observations that are made, but in general becomes more and more complicated.

If we are interested in angular distributions or Dalitz plots form the \( \bar{p}n \) annihilations the situation seems reasonably good if the momentum of nucleon spectator is less than \( \sim 0.2 \text{ GeV}/c \). On the contrary the absolute values depend from the proton spectator distribution. And in this distribution there are two discontinuity points, at about 0.1 \( \text{GeV}/c \) and at about 0.3 \( \text{GeV}/c \). Between 0 and \( \sim 0.1 \text{ GeV}/c \) the proton is not visible and the events are practically with 2 constraints. Between \( \sim 0.1 \) and \( \sim 0.3 \text{ GeV}/c \) the proton stops in the chamber and the events are really with 4 constraints. Above \( \sim 0.3 \text{ GeV}/c \) the proton does not stop in the chamber and therefore its identification becomes difficult.

At the end from a theoretical point of view the situation is very far from the satisfaction. The large tail of high momentum spectators has not been explained up now. And this may be due either to insufficiencies in the deuteron wave function or to the failure of the simple impulse approximation.

FIGURE CAPTIONS

Fig. 1 - The relative \( \bar{p} \)-particle target wavelength \( \lambda \) versus \( P_{\text{lab}} \) (laboratory incident momentum), \( T_{\text{lab}} \) (laboratory incident kinetic energy) and \( \mu \) (\( \bar{p} \)-nucleon at rest invariant mass). The effect of the Fermi motion is shown (see text). \( R \) and \( r \) are approximately the deuteron radius and the range of the \( \bar{p} \) forces.

Fig. 2 - \( \bar{p}d \) and \( \bar{p}n \) total cross sections versus laboratory incident momentum \( P_{\text{lab}} \).

Fig. 3 - \( I = 0 \) (\( \sigma_0 \)) and \( I = 1 \) (\( \sigma_1 \)) cross sections for the \( \bar{p}n \) system versus laboratory incident momentum \( P_{\text{lab}} \).

Fig. 4 - The cross sections \( K^+p \to K^0\pi^+ \), \( K^+d \to K^0\pi^+(n) \) and \( K^+d \to K^0\pi^+(p) \) versus laboratory incident momentum. The cross sections for the last two reactions are the raw data, after the cuts on the spectator nucleon in parenthesis \( { }^{(1)} \).

Fig. 5 - \( pp \) and \( \bar{p}n \) annihilation + charge exchange cross sections as measured in hydrogen \( { }^7 \) (open points) and in deuterium (full point) \( { }^{(2)} \).

Fig. 6 - \( I_0(\theta^\#) \) and \( J_0(\theta^\#) \) for two different \( K^+ \) momenta (see text).

Fig. 7 - Behaviour of the function to be integrated for the \( I_0 \pm i\lambda \) computation for:

a) \( \cos \phi = 0 \) and b) \( \cos \phi = 0.9 \). In the b) case the saddle of the interference between the quasi-elastic peak and the spectator shoulder is apparent. (see text).
\[ \begin{array}{|c|c|c|c|c|}
\hline
\text{Reaction} & \text{IA} & \text{p spectrum} & \text{Comments} & \text{Authors} \\
\hline
\bar{p}d \to p_p \bar{p} \bar{p} \pi^- \pi^- & \text{assumed} & P_{\bar{p}} < P_n & \text{Hulthên okey up to 0.2} & \text{Bacon}^{25}\text{) 1965} \\
\hline
\bar{p}d \to p_p \bar{p} \bar{p} \pi^- \pi^- & \text{"} & \text{"} & \text{"} & \text{Bacon}^{26}\text{) 1967} \\
\hline
\bar{p}d \to \bar{p} \bar{p} n & \text{assumed} & \text{Hulthên okey (up to?)} & N_a < 0.150 & \text{Berryhill}^{27}\text{) 1968} \\
\hline
\bar{p}d \to n_p p p \pi^- & \text{assumed} & P_n < 0.250 & \text{"} & \text{Bacon}^{28}\text{) 1969} \\
\hline
\bar{p}d \to p_p \bar{p} \bar{p} \pi^- \pi^- & \text{assumed} & P_p < P_n & \text{Hulthên okey even in the high momenta} & \text{Bacon}^{28}\text{) 1970} \\
\bar{p}d \to p_p \bar{p} \bar{p} \pi^- \pi^- & \text{assumed} & P_p < P_n & \text{\(\sigma = \sigma_{\text{measured}} \times 1.12\)} & \text{Bacon}^{28}\text{) 1970} \\
\bar{p}d \to \bar{p}d \pi^+ \pi^- & \text{assumed} & P_p < P_n & \text{plot of \(\cos \theta_{\text{f.i.}}\) isotropy; no IA valid sity (1) but compensation between various effects; Glauber correction <r^2> = 0.027 mb^-1} & \text{Braun}^{30}\text{) 1970} \\
\hline
\end{array} \]
TABLE II (cont.)

<table>
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<tr>
<td>$\bar{p}d + p \rightarrow p^+ n^-$</td>
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<td>$p_\pi &lt; 0.150$</td>
<td>McGehee $^3b)$ okey up to 0.2</td>
<td>Bettini $^35)$ 1971</td>
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<td>$(\bar{p}n \rightarrow p^+ n^-)$</td>
<td></td>
<td></td>
<td>unfolding Fermi motion</td>
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<tr>
<td>$\bar{p}d + p \rightarrow p^+ n^-$</td>
<td>assumed</td>
<td></td>
<td>Hulthén okey 0.10-0.28</td>
<td>Braun $^37)$ 1971</td>
</tr>
<tr>
<td>$(\bar{p}n \rightarrow p^+ n^-)$</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\bar{p}d + p \rightarrow p^+ n^- + \Lambda$</td>
<td>$p_\pi &lt; 0.29$; accumulation for $\cos\theta_p &gt; 0$; $p_n &lt; p_p$, removal too large; $1 &lt; p_n/p_p &lt; 2$</td>
<td>No Hulthén</td>
<td>Braun $^38)$ 1971</td>
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</tr>
<tr>
<td>$(\bar{p}n \rightarrow p^+ n^- + \Lambda)$</td>
<td></td>
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</tr>
<tr>
<td>$\bar{p}d + p \rightarrow p^+ n^- + \Lambda + MM$</td>
<td></td>
<td></td>
<td>No $\bar{p}$ annihilation on the entire deuteron</td>
<td>Camerini $^39)$ 1971</td>
</tr>
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<td>$(\bar{p}n \rightarrow p^+ n^- + \Lambda + MM)$</td>
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<tr>
<td>$\bar{p}d + p \rightarrow p^+ n^- + \Lambda + MM$</td>
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</table>
REFERENCES

    Rev. 128, 869 (1962).
12) R.J. Abrams, R.L. Cool, G. Giacomelli, T.F. Kycia, B.A. Leontic, K.K. Li and D.N. Michael,
17) L. Gray, P. Hagerty, T. Kalogeropoulos, G. Nicodemi, S. Zenone, R. Bizzarri, G. Ciapetti,
19) A. Aminosu, L. Gray, P. Hagerty, T. Kalogeropoulos, S. Zenone, R. Bizzarri, G. Ciapetti,


43) G. Alberi, E. Castelli, P. Poropat and M. Sessa, On the extraction of the particle neutron amplitude from particle deuteron scattering with break up, INPN internal report of the University of Trieste (in preparation).


46) N.W. Dean, Inelastic scattering from deuterium in the impulse approximation, Iowa State University preprint.
Fig. 1
Fig. 2
Fig. 7a

$K^*d \rightarrow K\Lambda N$

$p_3 = 0.6$ GeV/c

$\cos\theta_3 = 0.0$

Fig. 7b

$K^*d \rightarrow K\Lambda N$

$p_3 = 0.6$ GeV/c

$\cos\theta_3 = 0.9$
Mr. Bizzarri: For small enough momenta of the spectator proton (say below 150 MeV) impulse approximation for $\bar{p}n$ annihilations should work quite well. There is then a tail of energetic protons (15 to 20%) which is not accounted for by impulse approximation but it could be due for instance to 3-body ($\bar{NNN}$) annihilations.

After all, from the deuteron wave function, we have a probability of 10 to 15% of finding both nucleons inside the interaction volume.

There are however two difficulties:

1) The direction of the low momentum spectator proton is unknown and this could cause troubles in fitting;

2) The absolute normalization is unknown since we do not know how to estimate the loss of energetic spectators: by means of the deuteron wave function or using the experimental proton spectrum.

These difficulties in normalization are clear in two examples shown by Castelli: $\bar{p}d$ annihilations for 0.4 to 0.6 GeV/c and $K^+d \rightarrow K^0, \bar{p} (n_s)$.

I should like to point out however that these cross-section excess or defects are not rapidly varying with momentum and therefore this uncertainty in the absolute cross-sections should not prevent detection of resonances by formation experiments.
METHODS OF ANALYSIS - BROAD RESONANCES (CUTS AND RATES)

R.A. Donald

The University of Liverpool

In giving a survey of analysis methods for broad resonances I face a difficulty. Several aspects have already been dealt with - I refer you to the articles by Laloum and Lillestol in this report - and hence there is an overlap to be avoided. I shall make no reference to specific models used to analyse data. This does not mean that I regard the use of such models as of no value. Although my personal prejudice is that no model of strong interactions is at the same time sufficiently well grounded theoretically, and corresponds to reality in sufficient detail, to regard it as being a satisfactory description of experiment, it is necessary to compare experiment with models in order to provide a test of the theorist's ideas and I do not dispute this. Equally I regard it as important to provide accurate reliable data as a base for future theoretical ideas so I intend to use this brief survey to concentrate on this aspect of analysis, with all due acknowledgement to the necessity and value of the comparison between models and experiment.

This being so, I cannot avoid some discussion of the standard methods used to date. I refer you to Phys. Rev. Lett. 25, 783, 1970. This is a paper by George L. Trigg, an editor of Phys. Rev. Lett., in which he comments on the characteristics of some papers claiming new resonances which are submitted to the journal and in which he lays down minimum standards* to be met by such papers. I detail these below

1) Is the data compatible with no peak?
2) Is it really the peak that causes the incompatibility?
3) A renormalization of the phase space
4) Establishment that the peak is significant
5) Determination of the parameters by a fitting procedure.

I do not intend to dwell on such elementary considerations, but there are other problems to be touched on.

Surveying the literature one finds various ways of studying a suspected enhancement.

1) A smooth freehand background is drawn, a Breit-Wigner is superimposed, and a

* Criteria provided by Maglic
cross-section and significance is computed

2) A fit to a phase space background plus a Breit-Wigner is made by minimising $\chi^2$ on the appropriate distribution

3) A fit is made to several distributions simultaneously by minimising the $\chi^2$ computed by summing the $\chi^2$ from the separate distributions

4) A maximum likelihood fit is made to the distribution of events in the appropriate multidimensional space

Methods 1) and 2) are based on the idea that the background under a peak is a smooth continuation of the shape on either side of it. This idea has a certain simple appeal but it is dangerous. Figure 1 is taken from Nuclear Physics B1, 551, 1969, and shows two things. Firstly the dashed curve (fig. 1a) is the background underneath the distribution of $\pi^+\pi^-$ combinations associated with a $\rho$ meson (defined as a dipion falling in a given mass range) produced by the contribution of final states containing one resonance only. The bump in the background is a kinetic effect and the use of a smooth background would overestimate the cross-section for the final state $\rho^0\rho^{-}\pi^+\pi^-$. Figure 1b shows a more remarkable effect. In the experimental distribution of $\pi^+\pi^-$ combinations associated with a $\rho^0$ there is a prominent enhancement which might be taken as evidence for the existence of a final state $\rho^0\rho^0\pi^0$. The full line is the prediction of the overall fit which does not contain any such contribution. Figure 2 shows an experimental distribution with a very prominent peak in the $\Lambda_2$ region which again is totally misleading. The data is unpublished data from the final state

$$\bar{p} + p \rightarrow \pi^+ + \pi^- + \pi^+ + \pi^- + \pi^0$$

and the recipe to create the peak is as follows

a) remove all events with a combination in the $\rho^0$ region (suitably defined)

b) plot all combinations $\pi^+\pi^-\pi^0$ where the $\pi^+\pi^-$ combination has a mass in the $\rho^0$ region

The mechanism is quite clear; it is well known that $\pi^+\pi^-$ from $\omega^0$ mesons form a broad enhancement centred at about 450 MeV/$c^2$. If one replaces such a combination by a dipion external to the $\omega^0$ which has a larger mass $M$, restricted to some range $M_1 \leq M \leq M_2$, one, so to speak, moves the $\omega^0$ meson up by approximately the difference between $(M_1 + M_2)/2$ and 450 MeV/$c^2$, thus creating the observed peak.

Now, of course, this peak will not deceive the experimenter (it will not, for example,
produce an $\Lambda^+_2$). However, this peak does lie buried in the background and reinforces my prejudice against any analysis based on the idea of backgrounds varying smoothly underneath a broad enhancement. In this context it is interesting to note that a few months ago a paper was published which contained elaborate methods of dealing with the background under peaks found in missing mass experiments. Hidden away in this paper is the crucial phrase "since the background has presumably a smooth behaviour we can safely extrapolate the expression derived for $M > 2$ GeV/c$^2$ into the resonance region."

![Fig. 1a](image1.png) ![Fig. 1b](image2.png) ![Fig. 2](image3.png)

Figure 2 is also perhaps an illustration of the dangers of enhancements which require cuts to make them visible.

Going back to the enumeration of some of the methods of analysis, the third method (evaluation of $\chi^2$ on several distributions) suffers from obvious theoretical objections in that the distributions are not independent of each other. This leaves the Maximum Likelihood method of fitting, which is theoretically adequate, although subject to technical problems of computer usage and there may also be other problems.

I must now return to the main theme. The symposium programme specifies "Cuts and Rates" as the title. The literature is full of examples where groups have attempted to make a structure more obvious or more convincing. These cuts have broadly speaking two subdivisions. They are (without implying any value judgments)

a) Simple cuts, which exploit a knowledge of, or a hypothesis regarding, the decay mode of the resonance e.g. the study of $\rho$ combinations in studying the $\Lambda^+_2$. Also in this category are cuts which seek to eliminate some known major component of the background.
b) *Intelligent* cuts which seek to exploit a supposed understanding of the production mechanism. These include momentum transfer cuts and cuts based on exchange diagrams (see Nuclear Physics B35, 237, 1971 for some examples of these). These cuts often work but one finds that even in the same experiment they are successful in one final state, but not in another.

Also in this category comes selections based on well founded theoretical ideas. An example of this type is the use of the properties of Zeeman spin coefficients to select resonances. This was very successful in the case of the CERN - College de France analysis of antiprotons at rest (Nuclear Physics B16, 239, 1970) and indeed led on to the impressive analysis explained by Liljestol in an earlier session. The drawback here is that this was a special case where the initial states are few and well known. When the same ideas are used in a case where we do not know the initial states of the $\bar{p}p$ system (see Nuclear Physics B35, 237, 1971 again) they are not nearly so successful.

There does not seem to be any recipe for determining the best method of selecting data in order to isolate particular resonances. In this context the idea of "best" includes one knowledge of how the application of the cut distorts the distributions. However, recently there has been some interesting work which may help us. This is the work of Pless et al. (Phys. Rev. Lett. 27, 1431, 1971). If we have an unpolarised beam and target, an N-body final state requires $3N-5$ independent parameters to describe that state completely. A 3 body final state thus requires 4 parameters, a 4 body final state, 7 parameters and so on. Thus if one analyses a 3 body state in terms of the Dalitz plot, not all the available information is being used. The suggestion of Pless et al. consists of a convenient choice of the parameters necessary to describe the state. For the 3 body state they use a 3 dimensional plot, produced by the kinetic energies of two particles and the Van Hove angle of the event. Fig. 3 taken from Pless et al. shows the definition of the Van Hove angle, and also that of the fourth parameter required $R/R_{max}$ ($R_{max}$ is the maximum value of the radius vector in Figure 3 at any given Van Hove angle $\Theta$. In figure 3 the axes labelled $p, p^+, \pi^0$ represent the longitudinal momenta of the 3 particles in the final state (here a proton and two pions). Different regions of $\Theta$ correspond to different ordering of the magnitudes of these longitudinal momenta and it is this extra information which helps resolve overlapping resonances.
The two kinetic energies and the Van Hove angle define a prism shown in fig. 4 where each event is represented by a point. The base of the prism is the well known Dalitz-Fabri plot. The data shown in fig. 4 are from

\[ \bar{p} + p \rightarrow \pi^+ \pi^- \pi^0 \]

in the momentum range 1.5 - 2.0 GeV/c. There is a complicated structure as can be seen by viewing the prism from different points (fig. 5a and 5b). In the original work of Pless, they found very marked structures and selecting various regions of the prism plot gave remarkably total separation of the various resonances produced. The same was true of the corresponding 4 body final state.

Using the data of fig. 4 as an example, it is found that events with a \(\pi^+ \pi^-\) mass in the region of the \(\rho^0\) show strongly localised structures (fig. 6). Selection of events lying along the centre of the spiral like structure in this region yields fig. 7 with fig. 8 showing the unselected sample for comparison. Examining the events of fig. 7 as a function of \(R/\rho_{\text{max}}\) would then provide the most complete separation possible for these events. A more spectacular example is provided by the events of the type

\[ \bar{p} + p \rightarrow \bar{p} + p + \pi^0 \]

\[ \rightarrow \bar{p} + n + \pi^+ \]  

\[ \rightarrow p + \bar{n} + \pi^- \]  

over the same momentum range
These final states are dominated by the production of $\Delta (1236)$. The prism plot is shown in fig. 9. Selection along the main structure of the spiral in the densely populated region at the top gives almost total separation of the $\Delta (1236)$ events (fig. 10a, 10b).

For a four body final state one uses the kinetic energy of 3 final state particles to form a tetrahedron which is a generalisation of the Dalitz-Fabbri plots, two generalised Van Hove coordinates, $R/R_{\text{max}}$, and one mass combination chosen to suit the experiment. Once again structures can be seen. Fig. 11a, 11b show examples of this, the data being from the 8.25 GeV/c $K^-$ - p experiment of the Athens - Democritos - Liverpool - Vienna collaboration.
I suggest that, in any given experimental situation, it will be well worth while to examine the distributions in all of the $3N - 5$ independent parameters. The structures found (if any) will suggest the most effective cuts possible. There is clearly scope for the ingenious physicist to devise alternative combinations of plotting 3 out of the $(3N-5)$ parameters to suit particular experimental circumstances.
DISCUSSION AND COMMENTS

**Mr. Bizzari** : In the study of $\bar{p}p$ annihilations at rest, there is a clear trend to obtain an increasing percentage of resonance production as the statistical accuracy increases. This is presumably due to the fact that, as statistical accuracy increases, less events can be accounted for by phase space, but result from interference effects of the tail of the resonances.

In the in flight annihilations, such detailed studies cannot be made because of the lack of knowledge of the initial angular momentum state. In this situation I wonder which is the meaning of the rates for broad resonances production and if one should not rather try to invent new different parameters to characterize the annihilation properties?
ANALYSIS OF NARROW EFFECTS IN \( \bar{p}p \) ANNIHILATIONS

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1. INTRODUCTION

Putting aside the question of more "classic" resonances such as \( \eta^0 \) and \( \chi^0 \), the narrow effects of interest are observed generally through rather high-multiplicity meson final states. According to the number of like particles forming these final states, one can distinguish two circumstances.

1) If this number is rather low, the number of particle combinations of a given type is limited, for instance from 1 to 4. Such is the case of the non-strange narrow effects which are observed in \( \bar{p}p \to K\bar{K}n\pi \) (n=3 to 5) at 5.7 GeV/c \(^1\). These effects are detected in the \((K\bar{K})^0\) and \((K\bar{K}n\pi)^0\) mass distributions (Fig.1) at respectively 2.36 and 2.62 GeV/c\(^2\) near the \(U^-\) and \(X^-\) positions. Neither cuts nor special selection methods are used.

2) The situation becomes different from the first if the number of like particles is such that the number of indistinguishable combinations is great. Such is the case if one has to analyze the \((7\pi)\) final state, \( \bar{p}p \to \pi^+\pi^+\pi^\pi^-\pi^-\pi^-\pi^- \). (nine \((\pi^+\pi^-\pi^-)\) combinations, 18 \((\pi^+\pi^-\pi^-\pi^-)\) combinations etc... per event).

Now, at low antiproton energy, this reaction has been analyzed to look for the possibility of cascading decays of the \(D^0\) and \(E^0\) mesons, through narrow \( \eta^0\pi \) effects near the \(K\bar{K}\)-threshold mass, for instance \(^2\). Furthermore, observation of effects associated with such channels are complicated by reflections coming from another channel.

In the following, we shall describe briefly some methods of analysis that final states of this type require. A first method consists of separating two competing channels to minimize the reflections due to the undesirable one. Later techniques of analysis lead to the isolation of the only channel of interest and circumvent the problems of background and reflections due to irrelevant final states. Generally, all these processes are based on the presence of a narrow and identified resonance, for example the \( \eta^0 \) or \( \omega^0(\pi^\pi^-\pi^-) \). To be efficient, it is necessary that the observed width of such a basic resonance not be increased too much by experimental errors.
2. PROBLEMS MET IN THE DETECTION OF THE POSSIBLE $\delta^0$ DECAY OF THE $D^0$

The first evidence for a cascade decay of the $D^0$ was observed in $\bar{p}p$ annihilations into $7\pi$ at 1.2 GeV/c via the process $^3$:

$$\bar{p}p \rightarrow D^0\pi^+\pi^- \xrightarrow{\delta^0} \eta^0\pi^+\pi^- \xrightarrow{\eta^0} \pi^+\pi^-\pi^0$$

One can see in Fig.2 the $\eta^0\pi^\pm$ and $\eta^0\pi^+\pi^-$ mass distributions obtained from a $(\pi^+\pi^-\pi^0)$ selection at the $\eta^0$ mass. A narrow signal is observed at 975 MeV in $\eta^0\pi^-$ ($\Gamma_{\eta^0} \approx 20$ MeV) and a bump at the $D^0$ mass in $\eta\pi\pi^+$ (M=1310 MeV; $\Gamma = 40$ MeV). These two signals are strongly correlated: The $\eta\pi$ signal is found again in the $\eta^0\pi^+\pi^-$ combinations selected in the $D^0$ region (1310 ± 30 MeV); furthermore the mass distribution of $\eta\pi\pi$ having at least one $\eta\pi$ combination in the $\delta$ region (975 ± 20 MeV) gives back the $D^0$ signal. However the $(7\pi)$ final state is dominated by $\omega^0$ production (about 70% - Fig.3). Unfortunately, because of this strong $\omega^0$ production, an $\eta^0$ selection results in an $\omega^0$ reflection in the $\eta\pi$ distribution in the neighborhood of the $\delta$-mass. This reflection can be faked by a Monte-Carlo method (the result of such a simulation is shown in Fig.2) and does not explain either the narrow width of the signals nor the very localized correlation between the $D^0$, $\delta^\pm$ and $\eta^0$ regions. Nevertheless, one can never be sure that such a Monte-Carlo description is sufficiently realistic, given the complexity of the $(7\pi)$ final-state.

On the other hand, two other similar experiments simply add to the confusion: - at 1.1 GeV/c, in the 2-meter H.B.C. of CERN, where no distinction can be made between an $\omega^0$ reflection and a genuine $\delta\pi$ effect$^4$;
- at 0.7 GeV/c in the 81-cm H.B.C.$^5$ for which the distributions are shown in Fig. 4: the $\eta^0\pi^+\pi^-$ mass distribution shows a possible bump in the $D^0$ region, but this is right on the top of the phase space. The $\eta^0\pi^\pm$ distribution is no more enlightening, showing a shoulder at the $\delta$-mass, which is the location of the $\omega^0$ reflection.

3. METHOD OF SEPARATING TWO COMPETING CHANNELS

To reduce the non-$\eta^0$ background and attempt to overcome the ambiguity, $\omega^0$ reflection-$\delta\pi$ effect, a first possibility consists of separating the $\eta^04\pi$ channel from the dominating $\omega^04\pi$. Ideally, one would calculate the probability for each event to come from one or the other reaction. Actually, we know neither the structure of the complete matrix elements $\bar{p}p \rightarrow \omega^04\pi$ and $\eta^04\pi$, nor what are the other channels going into the $(7\pi)$ final state. The available information comes from the decay properties of the two basic resonances, $\eta^0$ and $\omega^0$ which characterize
each channel. Therefore, one possibility is to construct estimators for taking into account the Breit-Wigner form, the experimental resolution (for example supposing a gaussian spread due to the errors) and the decay matrix element \( \lambda^a \) of both resonances \( \alpha = \eta^8 \) or \( \omega^8 \). By using these elements, the probability \( p^a \) for the \( n \)th \( (\pi^+ \pi^- \pi^0) \) combination (with an effective mass \( m_1 \)) in one event to be an example of the resonance \( \alpha \) can be deduced from the couple of measured variables \( (m_1, \lambda^a_1) \). The estimator which has been chosen is:

\[
P_\alpha = \sum_{i=1,9} p^a_1 (m_1, \lambda^a_1)
\]

Each event corresponds to a point in the \( (P_{\eta^8}, P_{\omega^8}) \) plane (Fig.5). By using as criteria the attenuation of the \( \omega^8 \) signal and the enhancement of the \( \eta^8 \) channel we find an optimal selection. The efficiency of the method is checked by a Monte-Carlo generation of the probability distribution of various \( (\pi^8) \) final-states in the \( (P_{\eta^8}, P_{\omega^8}) \) plane. The \( \eta^8 \) channel is found to be enhanced by a factor 4 as compared with the other channels, and only 3% of all the \( \omega^8 \) remain in the sample. The Fig.6 shows the results of the selection. A \( D^0 - \delta^+ \) correlation is found again. \( D^0 \) is found narrow (28 MeV for an experimental resolution equal to 12 MeV). In addition, there is some indication of an \( \eta^8 \pi^+ \pi^- \) and \( \delta^+ \) decay of another object near 1400 MeV (G-parity equal to 1), which may be the \( E^0 \).

4. METHOD TO LOCALIZE CHAINS OF DECAY THROUGH SUCCESSIVE NARROW OBJECTS

If the chain involves a well-identified narrow resonance or leads to such an object, one can proceed by displaying all the possible combinations of cascades as follows. For instance, if we are interested by cascading decays leading to the \( \eta^8 \), such as the \( D^0 \) or \( E^0 \), we can consider all the cascades included in the different \( \pi^+ \pi^- \pi^0 \) combinations by drawing a plane whose axes denote the mass of the \( (5\pi)^8 \) combinations and the mass of each \( \pi^+ \pi^- \pi^0 \)(and c.c.) combination contained in the \( (5\pi)^8 \); the two \( \pi^+ \pi^- \pi^0 \) combinations included in this \( (4\pi)^\pm \) group are associated with each \( (5\pi)^0 - (4\pi)^\pm \) pair \( (3\pi)^0 \subset (4\pi)^\pm \subset (5\pi)^8 \).

The \( (4\pi)^\pm - (5\pi)^8 \) mass-plane correlated kinematically with the \( \eta^8 \) region is divided into sections (100 in the present case), each corresponding to a range of \( (5\pi)^0 \) and \( (4\pi)^\pm \) masses, and the mass distributions of the two \( (3\pi)^0 \) combinations for all points in each section are histogrammed. The most significant sections of such a display are shown in Fig.7 for the \( pp \to 7\pi \) analysis at 0.7 GeV/c. The \( \eta^8 \) signal is spread out over all the squares, usually with a significance of less
than 3 s.d. in each one - except in one square shown in the center of the figure (square k - 6 s.d.), at the (D° - δ±) coordinates. So, a clear and narrow correlation is observed between the (5π)° combinations in the D° region, the included (4π)± combinations near the δ mass and the η° production. A less significant correlation can be seen in the square c between the E° - δ± - η° regions as compared with the nearby squares.

One notices that this procedure circumvents the possibility of reflection coming from other channels, particularly the reflection of the ω° which is not contained in the field of observation.

5. **ISOLATION OF THE CHANNEL OF INTEREST FROM A SAMPLE WITH REDUCTION OF THE NUMBER OF BODIES IN THE FINAL STATE.**

Furthermore, one can project from the squares onto the (5π)° axes all the η°-signals previously estimated above their respective backgrounds. One thus obtains the "true η°π±π-" mass distribution. Similarly, the projection onto the (4π)° axis gives the "true η° π±" mass distribution. If we limit ourselves to the evolution of the η° signal in the D° band, we get the "true η°π±" distribution in D° etc...(Fig. 8). Obviously it is not necessary to pass by such a projection from the display to construct the ηπ or ηππ mass distributions. These last and all others - (for instance for π+π-, π+π-π-, ρ°π±π- etc. recoiling against the η°) can be deduced in a straightforward way by measuring the η°-signal evolution versus the successive ranges of their mass values.

This method seems relevant to the analysis of the ω±4π channel in the (7π) sample. In fact, Figure 3 shows that there are about 4 indistinguishable (3π)° combinations for 1 "true ω°" in the ω° mass range. An ordinary selection of these combinations to construct the "ω°π±π", "ω°πππ" etc... mass distributions results in a strong non-ω° background which overwhelms the effects associated with the ω° production. Therefore the analysis of this channel by measuring the evolution of the ω° signal versus the different multipion effective masses (or angular emission), circumvents the problem of selection of the "true ω°" combination. At the same time, the (7π) analysis is reduced to a five-body problem, and, except for the statistical fluctuations, such a manner of selecting the ω°4π channel from the sample does not bring with it events of the non-ω° channel. Finally the channel of interest is selected in its entirety.

The eventual bias of the method can be controlled by adding to the study of variation of the ω° signal in the (3π)° mass distribution, that of the decay matrix element λω° in the ω° mass range 6).
The analysis of this channel at 0.7 and 1.2 GeV/c by means of this method leads to evidence for a narrow effect (6.s.d.) in the $\omega^0\pi^\pm$ mass distribution near 1050 MeV, shown in Fig. 9b for $\bar{p}p$ annihilations at 700 MeV/c ($\Gamma = 30$ MeV for an experimental resolution equal to about 15 MeV). This last enhancement was also observed by using several simpler methods of background subtraction 6). However, given the loss of statistics these methods involve, this observation was less significant (Fig. 9a). This study is not ended, but this effect seems to be correlated with an $\omega^0\pi^+\pi^-$ enhancement centered at 1.33 GeV ($\Gamma \approx 50$ MeV).

Since all that corresponds to the G-parity and usual mass and width of the $A_2$ observed in the $\bar{p}p$ annihilation through its $\rho\pi$ and $\eta\pi$ decay modes, maybe an interpretation of this effect is possible in terms of a new $A_2$ decay.
REFERENCES

1) H.W. Atherton et al., Evidence for narrow non-strange neutral bosons of masses 2.37 and 2.61 GeV produced in \( pp \) annihilations at 5.7 GeV/c, CERN/D.Ph II/Phys 71-18 (1971).

2) A. Astier et al., Further study of the \( I=1 \) \( K\bar{K} \) structure near threshold, Physics Letters, vol.25B, n° 4 (1967).

Ch. d'Andlau et al., Analysis of the \( I=0 \) (\( K\bar{K}\gamma \)) resonances produced in \( pp \) annihilations at 0.7 GeV/c : the \( D^\ast \), \( E \) and \( f^\prime \) mesons; Nuclear Physics, B14, 63 (1969).

3) C. Defoix et al., Evidence for the existence of a narrow \( \eta^\prime \pi^\pm \) resonance at 975 MeV, interpreted as a decay of the \( \delta^\ast \) meson, and evidence for a \( \delta^\ast \pi^- \) decay of the \( D^\ast \) meson, Physics Letters 28B, p. 353 (1968).

4) R.A. Donald et al., Search for \( D^\ast \) and \( \delta^\ast \) (962) mesons in \( \bar{p}p+3\pi^\pm 3\pi^-\pi^0 \) XVth International Conference on High Energy Physics – KIEV (1970).

5) C. Defoix et al., Evidence for decays of the \( D \) and \( E \) mesons into \( \delta^\pi \) in \( pp \) annihilations at 700 MeV/c. Submitted to Nuclear Physics B.

6) C. Defoix et al., Resonance production in the \( \bar{p}p+\omega 2\pi^-2\pi^- \) annihilations at 0.7 and 1.2 GeV/c. Proceedings of the XVth International Conference on Elementary Particles – KIEV (1970).
FIGURE CAPTIONS

Fig. 1) Distribution of neutral $\bar{K}\pi\pi$ and $K\bar{K}\pi$ masses from the following $\bar{p}p$ final states at 5.7 GeV/c:  
   a) $\bar{p}p \rightarrow K^0(K^0)2\pi^+2\pi^-$  
      $\rightarrow K_L^{\pm}K^\mp\pi^\mp\pi^\pm$  
   b) $\bar{p}p \rightarrow K^\pm K^\mp\pi^\mp\pi^\pm\pi^0$  
      $\rightarrow K^0_L K^0_1 2\pi^+2\pi^-$

Fig. 2) Distributions of $\eta^0\pi^\mp$ and $\eta^0\pi^+\pi^-$ masses from $\bar{p}p \rightarrow 3\pi^+3\pi^-\pi^0$ at 1.2 GeV/c.

Fig. 3) Distributions of $\pi^+\pi^-\pi^0$ mass and $\lambda\omega$ from $\bar{p}p \rightarrow 3\pi^+3\pi^-\pi^0$ at (a) 0.7 GeV/c and (b) 1.2 GeV/c.

Fig. 4) Distribution of $\eta^0\pi^\pm$ and $\eta^0\pi^+\pi^-$ masses from $\bar{p}p \rightarrow 3\pi^+3\pi^-\pi^0$ at 0.7 GeV/c.

Fig. 5) Distribution of $(\eta^0, \omega)$ pairs from $\bar{p}p \rightarrow 7\pi$ at 0.7 GeV/c.

Fig. 6) From the reaction $\bar{p}p \rightarrow 7\pi$ at 0.7 GeV/c, distributions of the following mass combinations using $(\eta^0, \omega)$ selection method:  
   (a) $\eta^0\pi\pi^\pm$  
   (b) $\xi^\pm\pi$  
   (c) $\eta^0\pi^\pm$  
   (d) $\eta^0\pi^\pm$ from $\eta^0\pi^+\pi^-$ in $D^0$ region,  
   (e) $\eta^0\pi^\pm$ from $\eta^0\pi^+\pi^-$ in $E^0$ region.

Fig. 7) Display of $\pi^+\pi^-\pi^0$ mass combinations, in $\bar{p}p \rightarrow 7\pi$ at 0.7 GeV/c, associated with $(4\pi)^\pm$ and $(5\pi)^0$ masses such that $(3\pi)^0 \subset (4\pi)^\pm \subset (5\pi)^0$, $(5\pi)^0$ and $(4\pi)^\pm$ mass ranges are indicated, respectively, at left and above the display; $(3\pi)^0$ mass and number of combinations, below and at right.

Fig. 8) Distributions of $\eta^0\pi(\pi)$ masses from $\bar{p}p \rightarrow 7\pi$ at 0.7 GeV/c made by counting the number of $\eta^0$ above background for each $\eta^0\pi(\pi)$ mass division  
   (a) Total $\eta^0\pi^\pm$  
   (b) $\eta^0\pi^\pm$ contained in $(5\pi)^0$ in the $D^0$ band.  
   (c) $\eta^0\pi^\pm$ contained in $(5\pi)^0$ in the $E^0$ band.  
   (d) Total $\eta^0\pi^+\pi^-$.  

Fig. 9) Distributions of $\omega^0\pi^\pm$ masses from $\bar{p}p \rightarrow \omega^0\pi^+\pi^-\pi^0$ at 0.7 GeV/c.  
   (a) Various background-selection methods used.  
   (b) Distribution generated by counting events in $\omega^0$ signal above background.
$$\bar{p} p \rightarrow K K n \pi \ (n=3,4,5) \ \text{at} \ 5.7 \ \text{GeV/c}$$

(a) 2798 combinations

(b) 2690 combinations

Fig. 1

$$\bar{p} p \rightarrow \pi^+ \pi^- \pi^+ \pi^- \pi^0 \ \text{at} \ 12 \ \text{GeV/c}$$

Fig. 2
Fig. 3 $(TTTT)^*$ EFFECTIVE MASS DISTRIBUTIONS AND $\lambda$ DISTRIBUTIONS

- $a$ = 0.7 GeV/c ($M_{WW} = 783 \pm 0.015$ MeV)
- $b$ = 1.2 GeV/c ($M_{WW} = 783 \pm 0.030$ MeV)

FOR $\lambda$
Fig. 4
Fig. 6
Fig. 7
Fig. 8
Figure 9a

1045 GeV

Number of combinations
0.015 GeV

Figure 9b

(\ell^+ \tau^-) distribution
in pp \rightarrow (\ell^+ \tau^- \tau^- \tau^-)
at 0.7 GeV/c
Mr. Donald: Since we learned about Defoix's method of displaying the data, we have looked at the same distributions. We tend to find $\eta$ for every selection of $5\pi$ mass and $4\pi$ mass, we have examined so far. The interpretation is not clear, but the effect $D \rightarrow \delta \pi \rightarrow \eta \pi \pi$ is not nearly so strong as Defoix apparently sees at 700 MeV/c.
WHAT HAVE WE LEARNED ON MESONS?

J. Moebes
CERN - Geneva

Since this is not a conference, where especially new results are presented, all the facts concerning the title-question are more or less known to you. With the space permitted for this review I have to restrict the discussion to only a few problems and I will not try to make up a complete meson-list with all the pros and cons coming from $p$-results$^1$.

The annihilation channels are a very rich source of mesons and nearly all mesons in the Particle-Data-Table (PDT)$^2$ have a $p$-reference. The attractive features of these channels are well known:

- No baryon in the final state,
- No deck-type-mechanism,
- At low $p$-momenta (< 2 GeV/c) no difficulty in identifying the events,
- At rest only few initial states allowed.

However we have heard in the preceding talks how these attractions are balanced by serious problems. In most of the channels the number of contributing amplitudes is so large, that it is very difficult to disentangle them. So the interpretation of the data depends to a high degree on the kind of method used in the analysis, and it is in several channels no longer only meager statistics that limits the conclusions, but also the missing appropriate method. Keeping this in mind, I will underline my doubts with respect to some of the results rather heretically, even at the risk of being accused as being too pessimistic.

The following items will be discussed:

1. $\omega^0$
2. $K\bar{K}$-threshold-effects: $\pi^*_N$ (1016), $\pi^*$ (1070)
3. "Diffractive" mesons: $A_1$, $B$, $C$
4. $K\bar{K}^*$-effects: $D$, $E$, $F$

1. $\omega^0$

I will start with the $\omega$-meson as the earliest successful example of a $p\bar{p}$-annihilation study. Fig. 1 shows two ($\pi^+\pi^-\pi^0$)-mass-spectra from the reaction $p\bar{p} \rightarrow 2\pi^+2\pi^-\pi^0$. Fig. 1a comes from the historical Berkeley-experiment$^{3a}$, where the $\omega$ was seen for the first time and where its quantum-numbers have been determined. Fig. 1b shows the latest result for the same channel at rest with more than 7000 $\omega\pi\pi$-events. For the detailed analysis of this reaction see$^{3c}$.

There is a slight problem concerning the $\omega$-mass-determination. In principle well suited for this measurement are the annihilation-channels at rest, where the $\omega$ recoils from a pair of kaons ($K^O_1 K^O_1 \omega$ or $K^*K^-\omega$). But the results obtained differ by more than 2 s.d.$^4$. The PDT$^2$ quotes without further comment two mass values:
a) \( M_w = 780.60 \pm 0.52 \) MeV from \( \bar{p}p \to K_1^0 K_1^0 \) at rest (3 experiments with 730 events)
b) \( M_w = 783.86 \pm 0.30 \) MeV from all other experiments
[c] \( M_w = 783.4 \pm 1.0 \) MeV from \( \bar{p}p \to K^+ K^- \), ref. 4.)

Up to now no obvious systematical reason is found, to explain this 5 s.d. difference between the values a) and b).

Fig. 1: a) Ref. 3a, \( \bar{p}p \to 2\pi^+ 2\pi^- \pi^0 \) (1.61 GeV/c) (\( \sim 100 \) \( \omega^0 \)-events)
b) Ref. 3c, \( \bar{p}p \to 2\pi^+ 2\pi^- \pi^0 \) (at rest), (\( \sim 7400 \) \( \omega^0 \)-events)

2. \( K\bar{K} \)-Threshold-Effects

2.1 \( \pi_N(1016) \to K^0\pi^\pm \ (|q^0| = 1^0^+) \)

This item falls nearly fully in the domain of \( \bar{p} \)-studies. With positive evidence the \( \pi_N(1016) \) is seen only in \( \bar{p} \)-reactions, where nearly every channel of the type \( pp \to K^+ K^- \pi^0(n\pi) \) shows a threshold enhancement in the \( K^0 \pi^\pm \)-mass-spectrum. Fig. 2 shows 2 examples. The nature of this effect is not yet clear. It can be described with comparable probabilities either with a Breit-Wigner- or with a scattering-length (S.L.-) formula. From fig. 2a the Breit-Wigner parameters are quoted to be \( M = 1016 \pm 10 \) MeV and \( |\gamma| < 25 \) MeV. For the S.L., values between \( |s| = 1.5 \sim 2.5 \) fm have been found (ref. 5a, d, e, lla, b).

The physical interpretation of the S.L.-description depends on whether the S.L. is real or complex. The most attractive solution would be a complex S.L. with a negative real part, because this could be connected to a virtual bound state below the \( K\bar{K} \)-threshold\(^6\). The obvious candidate for such a state is the \( \delta \)-meson, as found in \( K^0 N \)-experiments\(^7\), with
a mass $M \lesssim 980$ MeV and decay-mode $\delta \to \eta\pi$. But it seems hard to test this hypothesis directly, as the information on the complexity of the S.L. and the sign of its real part can only be obtained via its interference with other amplitudes. (From the $K\bar{K}$-spectrum one gets only the absolute value of the S.L.).

The most detailed analysis concerning this problem can be found in ref. 5a,b, where the reaction $\bar{p}p \to K^O K^\pm \pi^\mp$ at rest was studied. The results show, that the description of the data is not very sensitive on the type of S.L. chosen. Three equally probable solutions are given:

$$a_1 = (2.0 \pm 1.0)_{\text{fm}}; \quad a_2 = (2.3 \pm 0.3)_{\text{fm}}; \quad a_3 = (2.3 \pm 0.3)_{\text{fm}}.$$

The last complex solution corresponds to a bound state with a mass of $975 \pm 15$ MeV.

Though the data do not allow an experimental proof of the connection between the $\eta^\prime_{N}(1016)$ and the $\delta$-meson, the speculation is supported by the quantum-numbers of the $K\bar{K}$-system. They are unambiguously determined to be $1^{G}P = 1^{0+}$, which leads to the preferred pionic decay-mode into $\eta\pi$, as was indeed indicated for the $\delta$-meson. The confirmation of the equivalence between $\eta^\prime_{N}(1016)$ and $\delta$ would need very accurate studies of corresponding $K\bar{K}$- and $\eta\pi$-channels with the determination of the mass-, width- and cross-section-values.

What is done up to now? In the reaction $\bar{p}p \to K^O K^\pm \pi^\mp$ at rest (fig. 2a, ref. 5c) a rate $R(\eta^\prime_{N} \to K^O K^\pm) \sim 10^{-4}$ is quoted, when the $\eta^\prime_{N}(1016)$ is interpreted as a S.L. effect. The corresponding pionic channel $\bar{p}p \to \eta\pi\pi$ at rest (ref. 5f with 460 $\eta\pi\pi$, $\eta = \pi^+\pi^-\pi^0$-events from a total of $\sim 80$ 000 4-prongs) show no $\delta$-signal in the $\eta\pi$-spectrum and an upper limit on the rate $R(\delta \to \eta\pi) < 1.6 \times 10^{-5}$ is given. So at least a factor of 10 in statistics would be needed for a conclusive test. This - corresponding to the analysis of $\sim 10^6$ events - seems not to be an inviting task with the conventional bubble chamber technique.

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**Fig. 2:**

a) Ref 5a, $\bar{p}p \to K^O K^\pm \pi^\mp$ (at rest)

b) Ref 5d, $\bar{p}p \to K^O K^\pm \pi^\mp \pi^0$ (1.2 GeV/c)
One way to reduce the necessary statistics would be to pick up the \( \eta \) in its neutral decay-modes, in the channel \( \bar{p}p \rightarrow \eta \pi \) with \( \eta \rightarrow \) neutrals. Information may also come from the D-meson-decay, which is thought to go mainly via \( D \rightarrow \eta \Xi(1016) \tau \) or \( D \rightarrow \delta \pi \) (ref. 12b,d). But it is not sure that these possibilities will provide sufficient accuracy to solve the problem.

How is the situation in experiments with incoming \( \eta \)'s and \( K \)'s? In these cases one would expect the production of \( \Xi(1016) \) or \( \delta \) to take place via graphs as shown in fig. 3.

Fig. 5

In \( \bar{p}p \rightarrow K^0 \bar{K}^0 \) no \( K \bar{K} \)-threshold-effect is found (see fig. 5) and the claims for \( \delta \)-production are very weak. The negative result is usually explained by the small \( \eta \)-nucleon-coupling. In \( K \bar{K} \)-experiments the situation is somehow opposite to that in \( pp \)-studies. Only upper limits are quoted for a \( K \bar{K} \)-effect, but three claims are given for a \( \delta \rightarrow \eta \pi \)-meson \(^7\). The branching-ratios (resp. the upper limits) for \( (\eta \Xi + \bar{K} \bar{K})/(\delta \rightarrow \eta \pi) \) as determined from \( pp \)- (ref. 5f, 11d) and \( K \bar{K} \)- (ref. 7a,b) experiments are so far not in contradiction, but they are much too gross to draw conclusions. So one has to wait for higher statistics experiments.

2.2 \( S^* \rightarrow K_{11}^0 K_{11}^0 \) \((\Gamma_{P}^0 = 0^+0^+)\)

As for the charged \( K \bar{K} \)-system one finds also for the \( \bar{K}_{11}^0 \)-system in several annihilation channels of the type \( \bar{p}p \rightarrow K_{11}^0 K_{11}^0 \eta \pi \) \((n=1)\) a sharp enhancement near threshold. Fig. 4 shows some \( (K_{11}^0 K_{11}^0) \)-mass-spectra at various \( \bar{p} \)-momenta. Contrary to the \( \eta \Xi(1016) \) the \( (K_{11}^0 K_{11}^0) \)-effect is better described by a Breit-Wigner- than by a Scattering-Length-formula (ref. 5d, 8c), with mass-values ranging from \( M = 1020-1070 \) MeV and width-values between \( 20-70 \) MeV.

A similar enhancement is found in the reaction \( \pi^{-}p \rightarrow K_{11}^0 K_{11}^0 \eta \) at several \( \eta \)-momenta (ref. 8a,b; fig. 5b), but with a much broader width than seen in \( pp \)-reactions. In ref. 8a, the following estimate for mass and width is given: \( M(S^*) = 104C-1100 \) MeV; \( \Gamma(S^*) = 150-300 \) MeV. Comparing in fig. 5 the charged and neutral \( K \bar{K} \)-mass spectra from \( \pi^{-}p \rightarrow K_{11}^0 K_{11}^0 \) and \( \pi^{-}p \rightarrow K_{11}^0 K_{11}^0 \eta \), the absence of any threshold effect in the \( (K_{11}^0 K_{11}^0) \)-spectrum clearly favours \( I = 0 \) for the \( S^* \) in \( \pi^{-}p \)-reactions. This is not necessarily true for the \( K_{11}^0 K_{11}^0 \)-effect found in \( pp \)-reactions, where also the \( \eta \Xi(1016) \) with \( I = 1 \) is present. It might well be possible that this difference explains the variation of the mass- and width-values, and that interference between the \( I = 0 \ S^* \) and the \( I = 0 \) part of the \( \eta \Xi(1016) \) produces the observed sharpening in the \( pp \)-reactions.
Fig. 4  

a) Ref 8c: $\bar{p}p \rightarrow K^0_1 \bar{K}^0_1 \pi^+ \pi^- (7 + 1.2$ GeV/c) $K^* \rightarrow \pi^0$ out

b) Ref 5d: $\bar{p}p \rightarrow K^0_1 K^0_1 \pi^+ \pi^- (1.2$ GeV/c)

c) Priv. Comm. Fast $\bar{p}$-group CERN: $\bar{p}p \rightarrow K^0_1 K^0_1 \pi^+ \pi^- (3.6$ GeV/c)

d) Priv. Comm. Fast $\bar{p}$-group CERN: $pp \rightarrow K^0_1 K^0_1 \pi^+ \pi^- \pi^0 (n \geq 1) (5.7$ GeV/c)

Fig. 5  

a) Ref 8a: $\pi^- p \rightarrow K^0_1 \bar{K}^0_1 \pi^- (17.2$ GeV/c) from G. Gayer et al. PL 34B (1971) 333

b) Ref 8a: $\pi^- p \rightarrow K^0_1 K^0_1 \pi^- (6$ GeV/c)
Another difference seems to be present in the $S^*$ decay angular distributions. The data coming from $\pi^-p$ show the flat behaviour expected for a $S$-wave-state, but $\bar{p}p$-results at 0.7 and 1.2 GeV/c favour more a $D$-wave-behaviour in the $S^*$-region. Final state interactions may be responsible for this fact, but on the other hand a new and different phenomenon cannot be excluded. In conclusion I think, that for the $S^*$ - as the resonant state with the quantum-numbers $I^GJ^P = 0^+0^+$ - one has to rely more on the information coming from the $\pi^-p$-data.

3. "DIFFRACTIVE" MESONS

The question about the interpretation of enhancements like the $A_1(\rho\pi)$, $A_3(\pi\pi)$, $Q(K^{*}\pi)$ or $L(K^{*}\pi)$, as found in $ar{p}p$- and $K\pi$-experiments is still not fully answered. Deck-type-mechanisms or equivalently Double-Regge-exchange-graphs may at least partly explain these effects. The advantageous absence of such diffractive mechanisms in the annihilation channels makes them in principle a good place, to clarify the nature of these objects.

3.1 $A_1(1070) \rightarrow \rho\pi$

The claims for an $A_1$-meson in $ar{p}p$-reactions are so far rather weak and contradictory. Fig. 6a-d shows the results published up to now. The difficulties coming from the large combinatorical background, from simple mass-cuts or from adding up different channels may partly explain the strongly differing results and it is too early to draw any definite conclusion.

3.2 $B(1235) \rightarrow \omega\pi$

When the $B$-meson was first found in $\pi\pi$-experiments, it was interpreted as being possibly due to a kinematical effect. But this explanation was ruled out after the $B$ was found in the channel $\bar{p}p \rightarrow \omega\pi\pi$ at rest. Fig. 7 shows the $\omega\pi$-mass-spectra from the Columbia and the CERN-HERA-experiments, where a strong accumulation of events is seen around $M = 1220$ MeV. For the questions raised in the $J^P$-analysis of the $B$ see ref. 3c,d,e.

It is a bit mysterious, that the $B$-meson is only seen in the reaction $\bar{p}p \rightarrow \omega\pi\pi$ at rest, although the $\omega$ itself is strongly produced also in other annihilation channels and at higher $\bar{p}$-momenta.

3.3 $C(1240) \rightarrow K^\pi, K\pi$

The last candidate in this "diffractive" group is the $C$-meson. It has in common with the $B$, that it is claimed only in one channel and only at rest. The evidence comes mainly from the reaction $\bar{p}p \rightarrow K^0\pi^0, K^\pm\pi^\mp$, where a 4 s.d. peak is observed in the $(K_1^{0}\pi^\pm\pi^\mp)$ mass-spectrum at 1240 MeV (fig. 8a). The results of ref. 11a can be summarized as follows: A maximum likelihood fit to the channel $\bar{p}p \rightarrow K^0\pi^0, K^\pm\pi^\mp$, including contributions from $K^0\pi^\pm, K^0\pi^\mp, K^\pm\pi^\mp$ and $(K\pi)_{S}$-wave, demands a $J^P = 1^+$ $C$-meson with $M = 1242 \pm 10$ MeV and $\Gamma = 127^{+7}_{-25}$ MeV with a production rate of more than 50%. A branching ratio $(C\rightarrow K\pi)/(C\rightarrow K\pi) = 2.5 \pm 1.2$ is given. In the channel $\bar{p}p \rightarrow K^0\pi^0, K^\pm\pi^\mp$, a similar fit is prevented by the large number of contributing amplitudes, but the authors quote, that the data are compatible with the
Fig. 6  
a) Ref. 9a : \( \bar{p}p \rightarrow \rho^0 2\pi^+ 2\pi^- \) \( (\text{From a total of 538 } \bar{p}p + 7\pi \text{ evts, 5.7 GeV/c}) \)

b) Ref. 9b : \( \bar{p}p \rightarrow \rho^0 \omega \pi^+ \pi^- \) \( (\text{From a total of 733 } \bar{p}p + 7\pi \text{ evts, 3.0 GeV/c}) \)

c) Ref. 9c : \( \bar{p}p \rightarrow \rho^0 \pi^+ \pi^- \) \( (\text{From a total of 1503 } \bar{p}p + 4\pi \text{ evts, 1.2 GeV/c}) \)

d) Ref. 9d : \( \bar{p}p \rightarrow K\bar{K} \, m\pi, \, m = 4,5 \) \( (5.7 \text{ GeV/c, 1948 evts}) \)
Fig. 7  
a) Ref 10b: $\bar{p}p \rightarrow 2\pi^+ 2\pi^- \pi^0$ (at rest)  
total 16,934 evts.

b) Ref 3c: $\bar{p}p \rightarrow 2\pi^+ 2\pi^- \pi^0$ (at rest)  
total 20,615 evts.

Fig. 8  Ref. 11a
   a) $\bar{p}p \rightarrow K_1^0 K_1^0 \pi^+ \pi^-$ (at rest) 1143 evts
   b + c) $\bar{p}p \rightarrow K_1^0 K \pi^+ \pi^0$ (at rest) 3780 evts
C-meson-hypothesis used to fit the $K^{-}_{1/2} \pi^{-} \pi^{-}$-channel (see fig. 8b,c for the $(K\pi)^{0}$-spectra of this reaction).

The effect seems doubtless to be real. A Columbia group\textsuperscript{11b}) analysed the same channels with about 40% of the statistics of ref. 11a and they see the same enhancements. But in contrast to ref. 11a, they consider these reactions to be too complex to establish a new resonance.

With regard to the fact that the C-meson is not seen at other $\bar{p}$-momenta or in other annihilation channels, caution is advisable concerning the conclusions drawn in ref. 11a. On the other hand, it is clearly of great interest to study the behaviour of this effect in connection with the lower part of the $Q$-bump.

4. $KK\pi$-effects

In this last section I will comment on the $D^{0}$, $E^{0}$ and $F_{0}$-meson, which have been seen originally as $KK\pi$-enhancements. For the latest evidence of the decay-mode $D_{0}E_{0} \to \delta \pi, \delta \to \pi \pi$ see Defoix's talk and ref. 12d.

4.1 $D^{0}(1285) \to (KK\pi)^{0}$ ($I^{G} = 0^{+}; J^{P} = 1^{+} ?$)

Most of the information concerning the $D$-meson comes from $\bar{p}p$-studies, but it is seen also in $\bar{p}p$-reactions and there exists no serious doubt on its existence\textsuperscript{21}).

What has been found in $\bar{p}p$ can be summarized as follows:

a) The $D$ is seen at 0.7 GeV/c\textsuperscript{12b)} and 1.2 GeV/c\textsuperscript{12a,c,5d)}.

b) The $D$ is found preferentially in quasi-two-body-processes $\bar{p}p \to D^{0} X^{0}$, with $X = \pi, \eta, \rho, \omega$. The best example is shown in fig. 9, where in the reaction $\bar{p}p \to K^{0}_{1/2} \pi^{+} \pi^{-} \pi^{-}$ at 1.1-1.2 GeV/c $\sim 60\%$ of the events go through the intermediate state $\bar{p}p \to D^{0} \omega^{0}$.

c) The decay-mode is said to be mainly $D^{0} \to \pi_{N}^{0}(1016)\pi \to (KK\pi)^{0}$. From this the search for the $D \to \delta \pi, \delta \to \pi \pi$-decay is motivated.

The established quantum-numbers are:

\begin{itemize}
  \item $I = 0$ (I $\neq 1$ : no $D^{+}$ found; I $\neq 2$ : $\bar{p}p \to D^{0} \omega^{0}$ seen)
  \item $G = +1$ (from $G_{\pi_{N}^{0}(1016)} = -1$, or $D \to \pi \pi, \rho \pi \pi$ decay)
  \item $C = +1$ (C = G ($\simeq 1$) and $D \to (KK\pi)^{0}$ seen).
\end{itemize}

The determination of the $J^{P}$ value from the decay $D \to KK\pi$ is not unique. It depends strongly on the assumptions about the nature of the $(KK)$-subsystem\textsuperscript{12b}). The unnatural series $J^{P} = 0^{-}, 1^{+}, 2^{-} \ldots$ is favoured over the natural one, with $J^{P} = 1^{+}$ preferred. Additional and possibly more conclusive information can be obtained by the study of the decay correlations in quasi-two-body-processes. The only channel with sufficient statistics, namely $\bar{p}p \to D^{0} \omega^{0}$ at 1.1-1.2 GeV/c was analysed with this respect in ref. 12c. The conclusions are, that $J^{P}(D) = 0^{-}$ is definitely excluded; under the assumption, that $D$ and $\omega$ are in an $S$-state - the reaction takes place $\sim 150$ MeV above threshold - $J^{P}(D) = 1^{+}$ is the only possible solution.
Fig. 9 Ref. 12c: $\bar{p} p \rightarrow K_1^{0} K^\mp \pi^+ \pi^- \pi^0$ (1.1-1.2 GeV/c) 325 evts

a) $M(\pi^+ \pi^- \pi^0)$,
$1.26 \leq M(K_1^{0} K^\mp \pi^0) \leq 1.34 \text{ GeV}$

b) $M(K_1^{0} K^\mp \pi^0)$,
$0.75 \leq M(\pi^+ \pi^- \pi^0) \leq 0.81 \text{ GeV}$

4.2 $E(1422) \rightarrow (K\bar{K}\pi)^0 (I^G = 0^+, J^P = ?)$

Here I have to announce serious doubts on the interpretation of the results published up to now\textsuperscript{13a,b, 12b, 5d}. My question marks concern the production rates, the decay-mode-fraction and connected to that the $J^P$-assignment.

The $E$-meson is quoted to be seen mainly in the channel $\bar{p} p \rightarrow K\bar{K} 3\pi$ and all the detailed information comes essentially from the annihilation at rest. Other positive claims are given at 0.7 GeV/c and 1.2 GeV/c.

Let me first recall the results\textsuperscript{13a)} from the study of the reactions (1) $\bar{p} p \rightarrow K_1^{0} K^\mp \pi^+ \pi^-$ and (2) $\bar{p} p \rightarrow K_1^{0} K^\mp \pi^+ \pi^- \pi^0$ at rest. The $E$ is seen in the neutral $K\bar{K}\pi$-spectrum with $M = 1424 \pm 8 \text{ MeV}$ and $\Gamma = 80 \pm 15 \text{ MeV}$. Fig. 10a shows the $(K\bar{K}\pi)^0$-spectrum from reaction (1) (2 comb./evt) with a strong enhancement around 1420 MeV. To eliminate the "false" combination the double-charged $(K\bar{K}\pi)^{\pm\pm}$-spectrum from the same channel was subtracted and the resulting histogram can be seen in fig. 10b. A Breit Wigner-fit led to the conclusion, that reaction (1) can be described by ~ 100% $E$-production. For reaction (2), where only one $(K\bar{K}\pi)^0$-combination is present, a similar fit resulted in an $E$-production-rate of 76% (fig. 10d). The $(K\pi \bar{K}\pi) -$Dalitz-plot for the events in the $E$-region for react. (1) is shown in fig. 10c. Most of the events are situated in the $K^*-K^*$-overlap-region. These events have simultaneously a low-mass $(K\bar{K})$-subsystem and produce a strong threshold-effect in the $(K\bar{K})$-spectrum. This is displayed in fig. 11, where the $(K\bar{K}\pi)^0$- and $(K\bar{K})$-spectrum for react. (1) are shown in a linear mass-scale\textsuperscript{13c)}. The curves superimposed in fig. 10a and fig. 11 are trials to explain these spectra and especially the E-peak with the assumption of either 100% constructively or destructively interfering $K^*-K^*$-amplitudes (curve A in fig. 10a and dashed curve in fig. 11 respectively) or of a mixture of 50% $K^*$ and 50% $(K\bar{K})$-threshold effect-production (full curve...
in fig. 11). One sees how, in the case of only $K^*$-production, the reflection peaks dangerously near the $E$-position, and I could well imagine with some refinements, that the $K\bar{K}$-spectrum is reproducible with this effect. But clearly the description of the $(K\bar{K})$-distribution is bad. Vice versa, the introduction of a $K\bar{K}$-threshold effect allows an acceptable description of the $K\bar{K}$-spectrum, but then the $E$-enhancement is not accounted for. So from the impossibility to describe the data only with $K^*$- and $(K\bar{K})_{\text{thresh.}}$-amplitudes one is led to accept the existence of the $E$-meson.

![Graphs showing $M^2(K\bar{K})$ vs $M^2(K\pi)$ and $M^2(K\pi)\times M^2(K\bar{K})$](image)

**Fig. 10**

a) Ref. 13a: $\bar{p}p \to K_1^{0}K_1^{\pm}\pi^+\pi^- \ (\text{at rest}) \ Q = 0 : M^2(K_1^{0}K_1^{\pm})^O; \ Q = 2 : M^2(K_1^{0}K_1^{\pm})^{\pm\pm}$. The curve corresponds to 100% $K^*$ production with constructive interference.

b) Ref. 13a: From fig. 10a: $M^2(K_1^{0}K_1^{\pm})^O - M^2(K_1^{0}K_1^{\pm})^{\pm\pm}$

c) Ref. 13b: $\bar{p}p \to K_1^{0}K_1^{\pm}\pi^+\pi^- \ (\text{at rest})$ Dalitz plot $M^2(K\pi)$ vs $M^2(K\bar{K})$ for events with $1.84 \text{ GeV}^2 < M^2(K\bar{K})^O < 2.14 \text{ GeV}^2$ (469 evts)

d) Ref. 13a: $\bar{p}p \to K_1^{0}K_1^{-}\pi^+\pi^- \ (\text{at rest})$
My doubts are now connected to the question: How can one disentangle an E-independent $K^*$-production, which reflects in the E-region, from a genuine E-meson effect? Every kind of $K^*$-amplitude, coherent or incoherent, which produces an enhanced Dalitz-plot-density in the $K^*\bar{K}^*$-overlap region, will reflect a peak in the $(K\bar{K}\pi)^0$-spectrum around 1400-1500 MeV, and that independent of the incoming $\bar{p}$-momentum. So the interpretation of the data for this channel will depend crucially on the choice and strength of the $K^*$-amplitudes.

Since in ref. 13a all conclusions concerning the E-branching-ratio and the $J^P$-assignment are based on the assumption, that react. (1) goes nearly completely via $\bar{p}p \rightarrow E^0\pi^+\pi^-$, to my mind the results must be looked at with caution.

A last obvious remark to fig. 10b: Both $(K\pi)^-\pi^0$-combinations from the double-charged $(K\bar{K}\pi)^\pm\pi^-$-system have $I = 3/2$. So by subtraction of the double-charged from the neutral $K\bar{K}\pi$-spectrum one does not eliminate any $K^*$-effect.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig11.png}
\caption{Ref. 13c: $\,\bar{p}p \rightarrow K^0L\pi^+\pi^+$ (at rest) 535 evts. The dashed curve corresponds to 100% $K^*$ production with destructive interference, and the full curve to a sum of 50% $K^*$ production (destr. interference) and 50% $K\bar{K}$-threshold effect (Breit-Wigner curve with $M = 1.00$ GeV and $\Gamma = 0.05$ GeV).}
\end{figure}
Coming now to the results at higher $\bar{p}$-momenta, fig. 12 shows the $(K\bar{K}\pi)^O$-spectra from data at 0.7 GeV/c for the channels $\bar{p}p \rightarrow K^0_{1,2}K^\mp \pi^\pm \pi^\mp$, $pp \rightarrow K^0_{1,2}K^\mp \pi^\pm \pi^\mp$ and $pp \rightarrow K^0_{1,2}K^\mp \pi^\pm \pi^\mp$, where E-production-rates of 5%, 60% and 40% are quoted respectively. At 1.2 GeV/c the strongest E-production with $\sim 23\%$ is again quoted in the reaction $\bar{p}p \rightarrow K^0_{1}K^\mp \pi^\pm \pi^\mp$ (see fig. 13). Since the kinematical situation between E-meson and possible $K^*$-reflection is the same as at rest, the same questions are raised on the interpretation of the E-enhancements in fig. 12 and 13. It is not sure that higher statistics in this channel will solve the problem, as long as the behaviour of the $K$-amplitudes is not understood.

To circumvent these difficulties one would have to look for the E in other channels or other decay modes.

A promising reaction could be $\bar{p}p \rightarrow E^0 \omega \rightarrow K^0_{1,2}K^\mp \pi^\pm \pi^\mp$ (threshold at 1.35 GeV/c $\bar{p}$-momentum), allowing the type of analysis mentioned for the D-meson$^{13d}$. The search for the pionic mode $E \rightarrow \delta \eta, \delta^* \pi \pi$ in the channel $\bar{p}p \rightarrow 7\pi$ was presented by Defoix$^{12d}$. The analysis is complicated by the strong $\omega^0$-production and the large number of possible combinations. Both difficulties may be cut down a bit in the 4-prong channel $\bar{p}p \rightarrow 4\pi \eta^0$, where the $\eta$ decays into neutrals. A large number of 4-prong multineutral-events is collected at different $\bar{p}$-momenta$^{13e}$, but so far no analysis concerning the E-meson is reported.
Along the same line of arguments as for the \( \pi_N(1016) \), one could search for E-production via its proposed decay channels, e.g. in \( K^-p \)-reactions with \( K^* \)-exchange or in \( \pi^-p \)-reactions with \( \pi_N(1016) \)-exchange (fig. 14).

![Diagram](image)

No convincing evidence for the E-meson is reported so far in these channels. In \( K^-p \), in reactions like \( K^-p \rightarrow \Lambda K^+ \pi^- \) the difficulties are, that two Vees should be seen, leaving only 2/9 of the events for a useful analysis, and also, that other resonances like \( Y^* \), \( N^* \) or \( K^* \)'s are strongly produced\(^{15f} \). The only indication for an E-meson in a non-\( \bar{p}p \)-reaction was found in the channel \( \pi^-p \rightarrow n K^{0}_L K^- \pi^- \) in 1965\(^{15b} \)), but no new positive results are published, though many experiments in the same energy range have been performed since that time.

4.3 \( F_{1}(1540) \rightarrow K\bar{K} \) \( I = 1; J^{PG} = ? \)

The last meson in my sketchcy review is the \( F_1(1540) \). It was first seen at 0.7 GeV/c as charged \( (K^* \bar{K}) \)-enhancement with \( M = 1540 \pm 5 \) MeV and \( \Gamma = 40 \pm 15 \) MeV in the channel \( pp \rightarrow K^{0}K^{0}\pi^{+}\pi^{-} \) (fig. 15a). A statistical significance of 5-6 s.d. is quoted. It was proven with a Maximum Likelihood fit, that the peak is not produced by \( K^* \)-reflections. \( J^P(F_1) = 1^+ \) or \( 2^- \) is favoured from the study of the \( K^* \)-decay angular distribution.

The search for the \( F_1 \) at 1.2 GeV/c in the same channel \( K^{0}K^{0}\pi^{+}\pi^{-} \) was inconclusive, only a \( < 3 \) s.d. effect was seen\(^{14b} \)). But in the reaction \( pp \rightarrow K^{0}K^{0}\pi^{+}\pi^{-} \) \( (MM) \) an enhancement in the neutral \( (K^* \bar{K})^{0} \)-system was observed with mass-, width- and possible \( J^P \)-values similar to those of the charged \( F_1 \) found at 0.7 GeV/c. So it was identified as the neutral \( F_1 \) (fig. 15b). A study of the neutral system recoiling from the \( F_1 \) showed evidence for the process \( pp \rightarrow F_{1}^{0} \eta^0 \). This - taking place only \( \sim 40 \) MeV above threshold - favours a S-wave final state and allows such a determination of the C-parity, being -1 or +1 for \( J^P(F_1) = 1^+ \) or \( 2^- \) respectively. Though the analysis looks promising, one has to keep in mind, that the \( F_1 \) in fig. 15b has a statistical significance of \( \sim 3 \) s.d., and it is clearly not very satisfactory to see the \( F_1 \) at different momenta only in different channels. So again the standard demand for more data has to be placed at the end.
Fig. 15  

a) Ref. 14a \( \bar{p}p \rightarrow K^0\bar{K}^{*0} \pi^+ \pi^- \) (0.7 GeV/c, total 1789 evts)  
The channel \( \bar{p}p \rightarrow K^{*+} \) being removed and a \((K\pi)\) combination  
being plotted, if it contains at least one \(K^*\).

b) Ref. 14b \( \bar{p}p \rightarrow K^{*+} \pi^0 \) \((1.1 - 1.2 \text{ GeV/c})\)  
\((KK\pi)^0\) contains at least one \((K\pi)\) combination in the \(K^*\)-region  
(329 evts).

If you ask me for some final conclusions, I feel rather ambivalent. Looking back in  
time - to answer the title-question - there are doubtlessly important contributions from  
the \(\bar{p}\)-annihilation studies to the field of meson spectroscopy. On the other hand numerous  
and difficult problems are still waiting to be attacked and one has to ask, if, without the  
development or more powerful methods in the analysis and without more elaborate experimental  
techniques, progress will be possible in this area of high energy physics.

I would like to express my thanks to Drs. B.R. French and L. Montanet for their valuable  
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REFERENCES

1) For more extended reviews see e.g.:
   - L. Montanet "p-reactions" Rapporteur's talk, Lund-Conference 1969

2) Review of Particle Properties,

    b) R. Bizzarri et al, Nuclear Physics B14 (1969) 169
    c) P. Frenkel et al, Preprint CERN/D.Ph.II/PHYS 72-9
    d) Talks of E. Lillestøl and C. Ghersièrè, these proceedings.
    e) S.U. Chung et al, Nuclear Physics B30 (1971) 525

4) R. Bizzarri et al, Nuclear Physics B27 (1971) 140

5) a) A. Astier et al, Physics Letters 25B (1967) 294
    b) B. Conforto et al, Nuclear Physics B5 (1967) 469
    c) A. Bettini et al, Nuovo Cimento 63A (1969) 1199
    d) J. Duboc et al, Preprint Paris VI - Liverpool 1972
    e) J. Bariow et al, Nuovo Cimento 50 (1967) 710
    f) P. Espigat et al, Nuclear Physics B36 (1972) 93
    M. Foster et al, Nuclear Physics B8 (1968) 174


8) a) W. Beusch in Exp. Meson Spectr. 1970, p. 185 (Editor: C. Baltay and A.H. Rosenfeld)
    b) W. Beusch - Introductory talk, Bologna-Conference, April 1971
    c) M. Aguilar-Benitez et al, Physics Letters 29B (1969) 241

9) a) A. Fridman et al, Phys. Review 167 (1966) 1268
    c) R.A. Donald et al, Nuclear Physics B6 (1968) 174
    d) H.W. Atherton et al, Preprint CERN/D.Ph.II/PHYS 71-20


11) a) A. Astier et al, Nuclear Physics B10 (1969) 65
    b) N. Barash et al, Phys. Review 145 (1966) 1095
    c) B. Marschel - Thèse d'Etat Paris 1969

12) a) C. D'Andlau et al, Nuclear Physics B5 (1968) 693
    b) B. Lörstad et al, Nuclear Physics B14 (1969) 63
    c) M. Goldberg et al, Lettere al Nuovo Cimento 1 (1971) 627
    d) C. Defoix et al, Preprint CERN/D.Ph.II/PHYS 72-2 and talk, these proceedings

13) a) P. Baillon et al, Nuovo Cimento 50 (1967) 393
    b) M. Baubillier - Thèse d'Etat, Paris 1967

14) a) A. Ferrando, CERN (priv. comm.) The normalisation of the FOWL-curves is done for the
     (KKr)0-spectrum roughly on the non-E-region and for the (K0)0-spectrum on the total
     histogram
    d) J. Duboc, D.N. Edwards, private communication
    e) For the channel pp → 2π+ 2π− X0 (X > 2π) the following number of events are analysed:
       ~17 000 at rest (P. Espigat, Thèse 1972, Paris)
       ~10 000 at 0.7 GeV/c (M. Baubillier, priv. comm.)
       ~30 000 at 1.2 GeV/c (D.N. Edwards, priv. comm.)
       ~27 000 at 3.6 GeV/c (J. Debray, Thèse d'Etat Paris 1971)
    f) The highest statistics (~550 evts) for the reaction K*+ → K0*0K− is to my know-
       ledge collected by the Samios-Group in Brookhaven at 3.9 and 4.6 GeV/c. No evidence
       for the E-meson can be shown (M. Aguilar-Benitez, priv. comm.)
    g) D.H. Miller et al, Phys. Rev. Letters 14 (1965) 1074

Mr. Duboc: Mentions that the analysis of $\bar{p}p$ annihilations around 1.5 GeV/c indicates some $E^0$ production of the type $\bar{p}p \rightarrow E^0 \pi^0$.

Mr. Montanet: The fact that the C-meson is only observed in $\bar{p}p$ annihilations at rest is of course puzzling. But this is not an isolated case. Indeed, $\bar{p}p$ annihilations at rest appear to be often quite different from annihilations in flight as far as the production of resonance is concerned: if the C, the E and the B are mainly observed at rest, the $S^*$ and the D, on the contrary are mainly observed in flight... If what is observed at rest for the $K\pi\pi$ system can be attributed to a resonance, then its quantum numbers are: $J^P = 1^+$, $I = \frac{1}{2}$, and its decay properties suggest that the C belongs to the $^3P$, nonet, with $C = +1$. There are also some indications that the same $K\pi\pi$ enhancement (with $M = 1250$ MeV rather than 1320) is observed in in-diffractive processes like

$$\bar{p}p \rightarrow \Lambda K^*\pi^*\pi^-$$

About the E-meson, we believe that the direct production of $K^*$ may be more abundant than quoted in the original paper, therefore changing the branching ratio. But for the spin-parity determination, if we keep only the information coming from the decay, $J^P = 0^+$ is still favoured over $1^+$.

Mr. Baubillier: gives an upper limit for direct production of $K^*$ in $\bar{p}p \rightarrow K^0 \bar{K}^+\pi^-\pi^+$ of 10%. Note also that the number of $E^0$ decaying into neutral particles (80±20) is in agreement with expected number (90±4) if the all five body annihilation $K^0 \bar{K}^+\pi^-\pi^+\pi^-$ go through $E^0\pi^+\pi^-$.

Mr. Chaloupka: Can you comment on the width of the D?

Mr. Makowsky: The width of the D: 56 MeV comes from a first analysis. A more careful analysis gives 46 MeV. If we introduce a cut on the $K\bar{K}$ mass, the apparent width is reduced to 35 MeV.

Mr. Chaloupka: A comment on properties of mesons, as quoted from $\bar{p}p$ experiments:

Troubles with the $A_2$ in $\bar{p}p$ at rest are not limited to the mass discrepancy between $K\bar{K}$ and $\rho\pi$ mode. The branching ratio $A_2 \rightarrow \frac{K\bar{K}}{\rho\pi}$ does not agree with $np$ experiments, e.g. the difference from the 7 GeV $\pi^+p$ experiment is more than 4$\sigma$. I can think of two possibilities:

a) there is indeed some strong interference effect present;
b) there is some additional complication in at least one of the channels.

Both alternatives represent serious difficulties for measuring the mass, width and branching ratios.
Mr. Bizzarri: Be careful with what we call A_2; a Veneziano analysis of \( \bar{p}p \) at rest into \( \eta\pi\pi \) suggests that the \( \eta\pi \) enhancement observed at 1300 MeV is mainly \( 0^+ \), not \( 2^+ \). This could explain the rather large differences observed for the mass of the "A_2" according to its decay mode.

Mr. Donald: Preliminary results on \( \omega\pi \) system for \( \bar{p}p \rightarrow 2\pi^+ 2\pi^- \pi^0 \) between 1.5 and 2.0 GeV/c shows no evidence for the \( B \).

Mr. Bizzarri: Recalls that the mass of the \( \omega^0 \), as observed in \( \bar{p}p \rightarrow K^0\bar{K}^0 \omega^0 \) is discussed in a publication (ref. 4).

Mr. French: In view of all the difficulties met in the analysis of these meson resonances, it would be nice to have a clear cut example, as for instance \( \bar{p}p \rightarrow E^{0}\pi^0 \).
ANTIPROTON-NUCLEON ANNIHILATIONS AT LOW ENERGIES
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1. INTRODUCTION
I shall first discuss some recent experimental results which strongly suggest that
interesting physics still "resides" at low $\bar{N}\bar{N}$ energies. Then I shall briefly discuss how
this energy region may be studied experimentally and finally, I shall make some general
remarks based on my experience with antiproton physics and my feelings from this symposium.

2. THE LOW ENERGY REGION
2.1 Definition
The energy region corresponding to $s_{\bar{N}\bar{N}} < (2m_p + m_N)^2$ (i.e. threshold for single pion
production) shall arbitrarily be defined as the low energy region. This definition
includes all experiments with $\bar{p}(\leq 800 \text{ MeV/c}) + p(\text{rest})$ and the annihilations with bound
nucleons which lead to $s < (2m)^2$ as well. In this energy region the $\bar{N}\bar{N}$ interactions lead
to annihilations only if elastic and charge exchange processes are excluded. Experimentally
only annihilations at rest in hydrogen and deuterium have been studied to some
extent and very little has been done for $\bar{p}(\leq 300 \text{ MeV/c}) + p(\text{rest})$ collisions. The studies
on $\bar{p}(\text{rest}) + d$ have been exploring the region with $s \leq M_d^2$. I shall now summarize some of
the available data bearing on my main thesis, namely, that interesting physics does still
remain unexplored in this energy region.

2.2 On the $\bar{p}p$ S-capture hypothesis
The observation$^1$ of the $\bar{p}p \rightarrow 2\pi^0$ at rest has brought the most direct challenge to
this hypothesis. The observation of the $2\pi^0$ and $\pi^+\pi^-$ branching ratios imply the following:

$$\frac{\bar{p}p}{L=0,1} \rightarrow 2\pi/\bar{p}p + 2\pi = 0.39 (\pm 20\%)$$

(1)

to be constrained to analogous ratio

$$\frac{\bar{p}K^0}{L=0,1} \rightarrow K^0\bar{p} + K^0\bar{p} \leq 1.5 \times 10^{-2}$$

(2)

which is based on the observation of the $K^0\bar{K}^0$ ratio. Observation (2) has been taken
as evidence in favor of the S-capture at rest but recent in flight data are accumulating
which show that (2) is essentially unchanged in spite of the several high waves present in
the annihilation. It has also been observed$^1$) that the ratio (1) does not change in flight
($\approx 200 - 400 \text{ MeV/c}$). Moreover, the annihilation at rest into $\pi^0\eta^0$ has been observed$^2$) with
a "typical" two-body annihilation rate in spite of the fact that it is not allowed from
$L(\bar{N}\bar{N})$ = even states. Table I summarizes the various experimental facts and important
comments are made which directly or indirectly can be looked upon in terms of the S-state
hypothesis. From the information in this table we find:
i) no support of the S-dominance hypothesis and
ii) that important dynamical effects are clearly present which are likely due to initial
$(\bar{N}\bar{N})$ state interactions.
2.3 \( \bar{p}d \) Annihilations at rest and the nature of the dynamical effects:

\[ N^0 \] narrow bound states?

We have been studying for some years annihilations at rest in deuterium in collaboration with Rome. Recently our attention has been focusing on the behavior of annihilations as a function of the "spectator" momentum. Namely, we are examining

\[ \bar{p}(\text{rest}) + d \rightarrow N^- + p. \]  

The energy of the \((\bar{p}n) \rightarrow N^-\) system is defined by the proton momentum:

\[ s = (m_d + m_p - \omega_p)^2 - p_p^2 \]  

and consequently if a \((\bar{p}n)\) s-channel effect is present it will show up in the spectator momentum distribution. Gray et al.\(^3\)) observed such a peak at 320 MeV/c associated with even number of pions \((G = +1)\) and absent for annihilations into an odd number of pions. This peak corresponds to a \((\bar{p}n)\) mass of \((2m - 83)\) MeV and a width of \(< 8\) MeV. Another observation comes from the study of \(\bar{p}d \rightarrow \pi^-\pi^0 p\) (in progress: Democritos, Rome, Syracuse). In Fig. 1a the missing-mass squared spectra are presented for one (invisible spectator) - and two (visible stopping) - pronged events of the recoiling neutrons against the \(\pi^-\) and proton (setting its momentum equal to zero for one-pronged events). The momentum spectra for the visible proton (stopping and non-stopping) is shown in Fig. 1b and the majority of the stoppings \((\sim 80\%)\) are part of the Hulthen distributions and on this basis are assumed.
to be as good spectators as the invisible protons. A clear π^0 (Fig. 1a) is associated with the one pronged events corresponding to $\bar{p}d = \pi^-\pi^0p$ but no evidence of it is present among the two prong with stopping proton! Clearly this result demonstrates that the ($\bar{p}n$) coupling to $\pi^-\pi^0$ changes within ± 5 MeV of the ($\bar{p}n$) energy. This can be interpreted that a narrow ($\pm 10$ MeV) $\bar{p}n$ state exists just below threshold. If this interpretation is accepted then such bound states can completely change the predictions on the P/S ratio based on smooth-like extrapolations of scattering lengths and also make the data presented in Table I less puzzling. If this is the case then most of the channels should be going through such states. No dramatic variation like the $\pi^-\pi^0$ have yet been seen but we should not be surprised. Most of the initial states do not lead to $\pi^-\pi^0$ - due to selection rules - but the selection rules are not restrictive to multi-pion annihilations. There are however, the following surprising phenomena for annihilations into many pions ($N_\pi = 3, 4, 5, 6, 7$):

i) 20% of all annihilations have a proton "tail" beyond the δ-hadron independent of the pion multiplicity (Fig. 2). This is a significant effect. (It may be worth to consider it when searching for s-channel effects suggested by total cross-sections).

ii) No differences in $\pi^+p$, $\pi^-p$ mass spectra are observed. Particularly for the events in the tail (Fig. 3). If these events were due to rescattering of the pions with the spectator then these distributions are predictable and as shown in Fig. 3.

The absence of $p_S$ final state interactions has been a puzzling one to us for some time. Moreover, if the annihilating system goes via narrow bound states its special separation with the spectator will alter by $\bar{p}_S T_{\text{ann}}$ where $\bar{p}_S$ is the velocity of the spectator proton ($\leq 0.1$ c) and $T_{\text{ann}}$ the annihilation time. We reconcile the absence of final state interactions with a $T_{\text{ann}} \leq 5 \times 10^{-22}$ sec which corresponds to $\Gamma_{\text{ann}} \leq 1$ MeV.

There are a number of other phenomena which may be relevant to the supposition of narrow bound states. There are dramatic variations:

i) in $\pi^-\pi^+$ angular distributions\(^4\) close to $\sqrt{s} - 2m$ and

ii) in the 2$\pi^-\pi^+$ Dalitz plots at rest\(^5\) (very strong structures) and $\gtrsim 200$ MeV/c (no significant structures) (Fig. 4).\(^6\)

There are also the following variations\(^5\) for $\bar{p}d$ annihilations at rest with spectator momenta $\lesssim 200$ MeV/c and $\gtrsim 200$ MeV/c:

i) The 2$\pi^-\pi^+$ Dalitz plot loses its structure for $p_S \gtrsim 200$ MeV/c.

ii) The production of resonances ($\rho, \omega$) is suppressed for the events with $p_S > 200$ MeV/c in comparison to those with $p_S \lesssim 200$ MeV/c.

Is this conjecture of bound $NN$ states against predictions based on $NN$ potentials suitably adjusted to $NN^*$? Dalakov et al.\(^7\) have been doing such calculations and are predicting many bound states.

It is clear from this very brief review that much more work is necessary. This may lead to a nucleon-antinucleon nuclear physics spectroscopy which may or may not be related to meson spectroscopy.

### 3. VERY LOW ENERGY EXPERIMENTS: WHIPLASH

Up to now we do not even know cross-sections below $\sim 20$ MeV in the cms due to known experimental difficulties. If narrow bound states exist close to threshold then experiments below this energy will be of great interest. Even independently of this the
Coulomb field will become important in this energy region and its influence on cross-sections will be significant and therefore extrapolations of cross-sections for very low energies which are of interest to cosmological problems are highly unreliable.

We have suggested that studies at very low energies can be done in intersecting storage rings with the $\bar{p}p$ beams running in the same direction (whiplash). We estimate that for the next generation of $\bar{p}p$ ISR's high statistics ($10^6$ events/10 hrs) experiments can be done at $\bar{p}p$ center-of-mass energies of $1/4$ MeV.

4. THE FUTURE OF ANTINUCLEON PHYSICS

I shall give here a brief summary of the questions which I consider important for experimental exploration.

4.1 Symmetries

i) We have been taking for granted that the $\bar{N}\bar{N}$ relative parity is negative. To my knowledge this is a consequence of field theory applying to particles of spin 1/2 but has never been tested experimentally. If whiplash becomes reality then a study of $\bar{p}p \to 2\pi^0$ at very low energies may reveal the $\bar{p}p$ parity, if we assume that at some low energies the S-waves should be dominant in which case $\sigma(2\pi^0)$ should be zero if $\bar{p}p$ has negative intrinsic parity. The study of angular distributions would be another constraint.

ii) Another important assumption in the study of annihilations is the assumption of I-spin conservation, an assumption never tested in the annihilations. A possible investigation of this may be carried out by studying the branching ratios $\bar{p}d \to 2\pi\bar{N}$, namely the ratio $\pi^+\pi^-/\pi^0\pi^0/2\pi^0$ in deuterium. Charge independence implies that $N(\pi^+\pi^-) = 2N(\pi^0\pi^0) + 2N(\pi^+\pi^-)$.

4.2 Matrix elements

We know very little about the matrix elements

$$\langle \bar{N}N | F_{\text{ann}} | \text{Mesons} \rangle.$$ 

The various attempts (experimental and theoretical) are often concentrated on either initial (s-channel resonances) or final (production of resonances) state interactions. In the final state we see copious production of resonances but all attempts to see s-channel effects have not produced as yet convincing evidences. The phenomena, however, observed at low energies (see section 2 above) clearly indicate the importance of the initial state.

In the search for the substructures in the matrix elements factorization approximations are very useful but we should not push them too far. Be aware that even "strong" correlations rapidly "disappear" by the effects of averages and only by projecting them into the "proper" space may "strike" the eye.

4.3 Annihilation cross-sections

The following cross-sections are badly needed:

i) $\sigma_{\text{ann}}$ as $\nu \to 0$. How does this cross-section rise? Does it rise as $1/\nu$ or $1/\nu^2$ or in other ways? At these energies the Coulomb, nuclear ($\pi$-range), vector ($\rho$, $\omega$ etc.)
Annihilation potentials are all important and interesting interference effects will show up.

i) $\sigma_{\text{ann}}$ as a function of energy. We know well $\sigma_{\text{tot}}$, but not $\sigma_{\text{ann}}$. It will be of great interest to see its behavior versus energy. How fast does it go to zero or to a constant and whether $\sigma_{\text{ann}} = \sigma_{\text{tot}} - \sigma_{\text{pp}}$.

4.4 Annihilation multiplicity

Good experiments are needed for studying the annihilation multiplicity versus energy. This information together with the $\sigma_{\text{ann}}$ will be very interesting to know.

4.5 Phase relations

I am puzzled with the fact that in spite of the complexities of final states there are fixed phase relations as for example in $\pi^+\pi^-\rho$, $\pi^+\pi^-\omega$ and possibly in K$^0\bar{K}^0$. This means that the assumption of "randomness" is a poor one.

4.6 Polarization

The existence of "phase relations" will make polarization experiments very attractive and informative. Very little work has been done so far along this line.

4.7 Deuteron

The deuteron may be a useful tool for studying initial state interactions (annihilation radii and times) or it may be a trap - Pandora's box - which I doubt.

4.8 Conclusions

I hardly see the end of interesting problems with antinucleons, we just started! I foresee that important and surprising discoveries are ahead of us and we will get an increasing membership into "our club" from the $e^+e^- + \text{hadrons}$, nuclear physicists and cosmologists. I hope that ultimately the annihilation phenomena may give a greater perspective to hadron physics in general.

***

REFERENCES

2) J. Skelly, Ph.D. dissertation, Syracuse University, November 1971.
6) L. Gray, Ph.D. dissertation, Syracuse University, June 1969.
Fig. 2
$\bar{p}d \rightarrow 2\pi^-\pi^+p_s$

302 events at $\sim 200-300$ MeV/c

Fig. 4
DISCUSSION AND COMMENTS

Mr. Moneti: You called the phenomena in the high momentum tail of the proton: "deuteron annihilation"; they are those which take place when proton and neutron are very close to each other and both partake in the annihilation. Now, how can you reconcile the fact that \( \bar{p}, p \) and \( n \) interact together, with the following:

1) formation of a \( \bar{p}n \) bound states;
2) no production of \( \Lambda \);
3) no production of meson resonances.

Mr. Kalogeropoulos: You have 3 bodies close together: \( \bar{N}N \), which occur 20% of the time. Then you form a state \( \bar{N}N \) plus a nucleon \( N \). If this object lives long enough, you do not form any \( \Lambda \). All the tail of the recoil momentum distribution is of this type, with no \( \Lambda \). But why there are no resonances produced in the decay of this object is just as dramatically mysterious as when the \( \bar{N}N \) goes into \( \Lambda K \) + pions which also shows a dramatic absence of resonances!
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