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QUANTIZATION OF THE RIGID STRING USING THE BACKGROUND-FIELD METHOD

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ABSTRACT

The string theory containing extrinsic curvatures is quantized by the background-field approach. Renormalization is carried out keeping two-dimensional general covariance, where we adopt "the background-field gauge". The ghost contribution is evaluated explicitly. As an application, the rigid string within the space-time background fields (graviton, dilaton, etc.) is discussed.

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1. - INTRODUCTION

String theories originate from the Nambu-Goto action

$$\begin{aligned} S_{NG}[x^\mu] &= \mu_r \int d^2\xi \sqrt{g} \quad , \quad g = \det g_{ab} \quad , \\ g_{ab} &= \partial_a x^\mu \partial_b x^\mu \quad , \\ a &= 1, 2 \quad , \quad \mu = 1, 2, \dots, D \end{aligned} \quad (1.1)$$

where $x^\mu = x^\mu(\xi^1, \xi^2)$ describes the two-dimensional surface in the D-dimensional space-time. μ_r is the (renormalized) coupling constant (string tension). As an extension of this model, the Polyakov string [1] is one direction where the two-dimensional metric $g_{ab}(\xi)$ and the fields of surface $x^\mu(\xi)$ are introduced independently. The other direction was also given by Polyakov [2]. He suggested a possibility of adding another term to (1.1):

$$\begin{aligned} S_{EXT}[x^\mu] &= \frac{1}{2\alpha_r} \int K_{ab}^i K^{iab} \sqrt{g} d^2\xi \quad , \\ \partial_a \partial_b x^\mu &= \Gamma_{ab}^c \partial_c x^\mu + K_{ab}^i n_i^\mu \quad , \\ \Gamma_{ab}^c &= \frac{1}{2} g^{cd} (\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab}) \quad , \end{aligned} \quad (1.2a)$$

where n_i^μ are normals on the surface of a string configuration. They are defined as

$$\begin{aligned} \partial_a x^\mu \cdot n_i^\mu &= 0 \quad , \quad n_i^\mu n_j^\mu = \delta_{ij} \quad , \\ i, j &= 1, 2, \dots, D-2 \quad , \end{aligned} \quad (1.2b)$$

K_{ab}^i is the extrinsic curvature. We note that S_{EXT} is written as

$$S_{EXT}[x] = \frac{1}{2\alpha_r} \int \sqrt{g} x_{a;b}^\mu x^{\mu a;b} d^2\xi \quad (1.2c)$$

Both terms are invariant under the two-dimensional general co-ordinate transformation (reparametrization)

$$\begin{aligned} \delta \xi^a &= \epsilon^a(\xi) , \\ \delta x_a^\mu &= -\partial_a x_b^\mu \cdot \epsilon^b - x_b^\mu \partial_a \epsilon^b , \\ x_a^\mu &\equiv \partial_a x^\mu , \end{aligned} \quad (1.3a)$$

and under the D-dimensional space-time Poincaré transformation

$$\begin{aligned} x'^\mu &= a^\mu_\nu x^\nu + b^\mu , \\ a^\mu_\nu , b^\mu &: \text{constants} , \\ a^\mu_\nu a^\nu_\lambda &= \delta^\mu_\lambda . \end{aligned} \quad (1.3b)$$

The term (1.2a) is distinct from other possible interaction terms of x^μ , because it is the unique one that satisfies all the following conditions:

- 1) it is invariant under (1.3a,b);
- 2) it is dynamically meaningful (not a total derivative);
- 3) it is invariant under the global scale transformation in the D-dimensional space-time.

$$\begin{aligned} x'^\mu &= \rho x^\mu , \quad \rho: \text{constant} . \\ (g'_{ab} &= \rho^2 g_{ab}) \end{aligned} \quad (1.4)$$

The dimensionless coupling α_r reflects this symmetry. One candidate of the following form

$$\int K_a^{i a} K_b^{i b} \sqrt{g} d^2 \xi \quad (1.5)$$

is excluded by condition 2). There is a relation between extrinsic curvatures and intrinsic ones:

$$K_a^{i a} K_b^{i b} - K_{ab}^i K^{i ab} = R . \quad (1.6)$$

This model is called the "rigid string" because the term S_{EXT} , in the classical theory, tends to prevent curving of the surface. The inverse of α_r is called the "rigidity" or "stiffness".

The importance of S_{EXT} is due to the following facts.

- 1) The coupling α_r , which is the expansion parameter, has no dimension in the space-time world. Furthermore, this coupling is asymptotically free [2].
- 2) When S_{EXT} is written in terms of tangents (to the surface) or normals, their interactions are similar to those of the $SO(D)/SO(2) \times SO(D-2)$ Grassmannian σ -model [2]. Some non-trivial non-perturbative effects can be expected.
- 3) There is a strong possibility that S_{EXT} changes the critical dimension (in Ref. [3], $D_c = 14$ is reported for a Weyl-invariant generalization of the model).

Classical aspects of the rigid string theory were extensively examined in Ref. [4]. The Regge trajectories for closed strings, especially, deviate substantially from straight lines. In the present paper, we are interested in quantum effects. The quantization of the rigid string theory is not so easy, however. The difficulty is twofold: i) higher derivativeness; ii) highly non-linear couplings. Canonical quantization has been examined in Refs. [5] and [6], but the situation is not yet so satisfactory that we can deduce some physical conclusions. At present, the functional integral approach gives interesting physical results [2,5]. We adopt this approach.

In order to quantize the theory with the above difficulties, it is best to make use of all (global and local) symmetries of the Lagrangian (1.2a). The choice of gauge is the most important step for that purpose. We quantize the rigid string theory by the use of the background-field method and introduce the "background-field gauge" [7]. One of the characteristic points of this approach is that renormalization can be done only in terms of invariants (under the two-dimensional co-ordinate transformation).

Compared with previous works [2,5], the following points become clear in this paper:

- 1) we give a ghost Lagrangian corresponding to our gauge and evaluate its contribution clearly;
- 2) the present formalism gives a consistent perturbation theory with respect to α_r (or the number of loops);
- 3) the correct β -function (one-loop) for the coupling α_r is obtained.

This paper is organized as follows. In Section 2, we explain the background-field formulation of the rigid string. The one-loop renormalization is explicitly carried out, keeping the symmetries, in Section 3. Finally, in Section 4, we

comment on two points: the higher-order structure of this formalism and the rigid string within the space-time background fields. Some detailed analysis is done in two Appendices. The symmetry properties of coefficient functions, which appear in the background-field method, are explained in Appendix A. For a model of the rigid string within the space-time background fields, the classical energy-momentum tensor θ_a^b is calculated and is shown to vanish in Appendix B.

2. - BACKGROUND FIELD APPROACH

In general, when quantizing theories with local gauge symmetries, the most natural way is to use the background-field method [8,9]. It keeps all symmetries of the starting invariant Lagrangian explicitly. We begin with the combined action of (1.1) and (1.2):

$$S[\bar{x}] = S_{NG}[\bar{x}] + S_{EXT}[\bar{x}] \quad , \quad (2.1)$$

where we put bars on full fields to distinguish them from the background fields. We split the full field $\bar{x}^\mu$ into the background (classical) field x^μ and the quantum field y^μ ,

$$\bar{x}^\mu = x^\mu + \sqrt{\alpha_r} y^\mu \quad , \quad (2.2)$$

where $\sqrt{\alpha_r}$ appears in order to make the y^μ propagator independent of α_r . The field x^μ satisfies the classical field equation

$$\left[\mu_r x_c^\mu + \frac{1}{2\alpha_r} \left\{ x_{a;b}^\nu x^{\nu a;b} x_c^\mu - 2 x^{\nu a;b} x_{c;b}^\nu x_a^\mu + 2 (h^{\mu\nu} - \delta^{\mu\nu}) x_c^{\nu;b} \right\} \right]^{;c} = 0 \quad , \quad (2.3)$$

where we use the following notations and definitions:

$$\begin{aligned} x_a^\mu &\equiv \partial_a x^\mu \quad , \quad g_{ab} = \partial_a x^\mu \partial_b x_\mu \quad , \\ h^{\mu\nu} &= x_a^\mu x_b^\nu g^{ab} \quad , \\ x_{a;b}^\mu &= \partial_b x_a^\mu - \Gamma_{ab}^c x_c^\mu = x_{b;a}^\mu \quad . \end{aligned} \quad (2.4)$$

g^{ab} is the inverse of g_{ab} . We raise and lower two-dimensional suffixes (a,b,...) by the classical metrics g_{ab} and g^{ab} :

$$x^{\mu a} = g^{ab} x_b^{\mu} \quad , \quad x_a^{\mu ; b} = g^{bc} x_{a;c}^{\mu} \quad .$$

Quantities without bars are those constructed from background fields only. For example,

$$\Gamma_{ab}^c = \Gamma_{ab}^c(g) = \frac{1}{2} g^{cd} (\partial_a g_{db} + \partial_b g_{da} - \partial_d g_{ab}) \quad .$$

$h^{\mu\nu}$ and $\delta^{\mu\nu} - h^{\mu\nu}$ are projection operators for the tangent components and for the normal components respectively:

$$\begin{aligned} h^{\mu\nu} h^{\nu\lambda} &= h^{\mu\lambda} \quad , \quad (\delta^{\mu\nu} - h^{\mu\nu}) h^{\nu\lambda} = 0 \quad , \\ (\delta^{\lambda\mu} - h^{\lambda\mu}) (\delta^{\mu\sigma} - h^{\mu\sigma}) &= \delta^{\lambda\sigma} - h^{\lambda\sigma} \quad , \\ h^{\mu\nu} x^{\nu a} &= x^{\mu a} \quad , \quad (\delta^{\mu\nu} - h^{\mu\nu}) n_i^{\nu} = n_i^{\mu} \quad , \\ h^{\mu\nu} n_i^{\nu} &= 0 \quad , \quad (\delta^{\mu\nu} - h^{\mu\nu}) x_a^{\nu} = 0 \quad , \end{aligned} \quad (2.5a)$$

and have the following properties:

$$h^{\mu\nu} h^{\mu\nu} = g_a^a = n \quad , \quad h^{\mu\nu} x_{a;b}^{\mu} = 0 \quad , \quad (2.5b)$$

where n is the number which is used in the n -dimensional regularization of the two-dimensional parameter space (ξ^1, ξ^2) . n may be put equal to 2 in one-loop divergence calculations. This note can be said also about the following in the text. The second equation of (2.5b) is due to the following:

$$x_a^{\mu} x_{b;c}^{\mu} = 0 \quad , \quad x_{b;c}^{\mu} = K_{bc}^i n_i^{\mu} \quad , \quad (2.5c)$$

The expanded Lagrangian $S[x + \sqrt{\alpha} y]$ has two typical kinds of local symmetries, which come from the symmetry (1.3a) for the full field $\bar{x}^{\mu}$.

(i) Q-type transformation:

$$\begin{aligned} \delta x_a^{\mu} &= 0 \quad , \\ \sqrt{\alpha_r} \delta y_a^{\mu} &= -\partial_a (x_b^{\mu} + \sqrt{\alpha_r} y_b^{\mu}) \cdot \epsilon^b - (x_b^{\mu} + \sqrt{\alpha_r} y_b^{\mu}) \partial_a \epsilon^b \quad , \end{aligned} \quad (2.6a)$$

(ii) C-type transformation:

$$\begin{aligned} \delta x_a^{\mu} &= -\partial_a x_b^{\mu} \cdot \epsilon^b - x_b^{\mu} \partial_a \epsilon^b \quad , \\ \delta y_a^{\mu} &= -\partial_a y_b^{\mu} \cdot \epsilon^b - y_b^{\mu} \partial_a \epsilon^b \quad , \end{aligned} \quad (2.6b)$$

where we used the notation $y_a^\mu = \partial_a y^\mu$. We will soon make use of these symmetries which are characteristic of the background field method.

Here we give useful expansion expressions.

(i) metric

$$\begin{aligned}\bar{g}_{ab} &= \partial_a \bar{x}^\mu \partial_b \bar{x}^\mu = g_{ab} + g'_{ab} + g^2_{ab} , \\ g'_{ab} &= x_a^\mu y_b^\mu + y_a^\mu x_b^\mu , \quad g^2_{ab} = y_a^\mu y_b^\mu , \\ \bar{g}^{ab} &= g^{ab} + g_1^{ab} + g_2^{ab} + O(y^3) , \\ g_1^{ab} &= -g'^{ab} = -(x^{\mu a} y^{\mu b} + y^{\mu a} x^{\mu b}) , \\ g_2^{ab} &= g'^{ac} g'^b_c - g^2^{ab} = (x^{\mu a} y^{\mu c} + y^{\mu a} x^{\mu c})(x_c^\nu y^{\nu b} + y_c^\nu x^{\nu b}) \\ &\quad - y^{\mu a} y^{\mu b} ,\end{aligned}\tag{2.7a}$$

(ii) Christofel symbol (connection)

$$\begin{aligned}\bar{\Gamma}_{ab}^c &= \Gamma_{ab}^c + \Gamma'_{ab}^c + \Gamma^2_{ab}^c + O(y^3) , \\ \Gamma'_{ab}^c &= x^{\mu c} y_{a;b}^\mu + y^{\mu c} x_{a;b}^\mu , \\ \Gamma^2_{ab}^c &= y^{\mu c} y_{a;b}^\mu - (x^{\nu c} y^{\nu d} + y^{\nu c} x^{\nu d})(x_d^\mu x_{a;b}^\mu + y_d^\mu x_{a;b}^\mu) , \\ y_{a;b}^\mu &= \partial_b y_a^\mu - \Gamma_{ab}^c y_c^\mu = y_{b;a}^\mu ,\end{aligned}\tag{2.7b}$$

(iii) determinant

$$\begin{aligned}\bar{g} &= \det \bar{g}_{ab} = g \exp \{ E_1 + E_2 + O(y^3) \} , \\ \sqrt{\bar{g}} &= \sqrt{g} \left\{ 1 + \frac{1}{2} E_1 + \left(\frac{1}{2} E_2 + \frac{1}{8} E_1^2 \right) + O(y^3) \right\} , \\ E_1 &= g'^a_a = 2 y^{\mu a} x_a^\mu , \\ E_2 &= g^2^a_a - \frac{1}{2} g'^a_b g'^b_a = y^{\mu a} y_a^\mu - y^{\mu a} y^{\nu b} x_b^\mu x_a^\nu - y^{\mu a} y_a^\nu h^{\mu\nu} .\end{aligned}\tag{2.7c}$$

Using the above expressions, we find the expanded action explicitly as follows:

$$S[x + \sqrt{\alpha} y] = S[x] + S_1[x, y] (=0) + S_2[x, y] + O(y^3),$$

$$S_2[x, y] = \int d^2x \mathcal{L}_2^{inv}[x, y],$$

$$\mathcal{L}_2^{inv} = \sqrt{g} \left\{ \frac{1}{2} (y_{a;b}^\mu y_{c;d}^\mu - y_{a;b}^\mu y_{c;d}^\nu h^{\mu\nu}) g^{ab} g^{cd} + y_{a;b}^\mu y_c^\nu N_{\mu\nu}^{abc} + \frac{1}{2} y_a^\mu y_b^\nu M_{\mu\nu}^{ab} \right\}, \quad (2.8)$$

where $N_{\mu\nu}^{abc}$ and $M_{\mu\nu}^{ab}$ are functionals only of background fields x^μ . Due to the total derivative arbitrariness, we can always symmetrize N and M as

$$\begin{aligned} N_{\mu\nu}^{abc} &= N_{\nu\mu}^{bac}, \\ N_{\mu\nu}^{abc} - N_{\nu\mu}^{abc} &= \text{totally symmetric w.r.t. } (a, b, c), \\ M_{\mu\nu}^{ab} &= M_{\nu\mu}^{ba}. \end{aligned} \quad (2.9)$$

This technical point is relegated to Appendix A. The explicit forms of N and M are given by

$$\begin{aligned} N_{\mu\nu}^{abc} &= \frac{1}{3} (-g^{ac} x^{\mu b}_{;d} x^{\nu d} + g^{ac} x^{\nu b}_{;d} x^{\mu d} + g^{ab} x^{\nu c}_{;d} x^{\mu d} + a \leftrightarrow b), \\ N_{\mu\nu}^{abc} - N_{\nu\mu}^{abc} &= \frac{2}{3} g^{ab} (-x^{\mu c}_{;d} x^{\nu d} + x^{\nu c}_{;d} x^{\mu d}) \\ &\quad + \text{cycl}(a, b, c), \\ M_{\mu\nu}^{ab} &= \frac{1}{2} \mu_t \left\{ (\delta^{\mu\nu} - h^{\mu\nu}) g^{ab} + (x^{\mu a} x^{\nu b} - x^{\mu b} x^{\nu a}) \right\} \\ &\quad + g^{ab} \left\{ -\frac{1}{2} x^{\nu c;d} x^{\mu}_{c;d} + x^{\lambda}_{c;e} x^{\lambda e}_{;d} x^{\mu c} x^{\nu d} + \frac{1}{4} (\delta^{\mu\nu} - h^{\mu\nu}) x^{\lambda}_{c;d} x^{\lambda c;d} \right\} \\ &\quad - (\delta^{\mu\nu} - h^{\mu\nu}) x^{\lambda a}_{;c} x^{\lambda b;c} + \end{aligned}$$

$$\begin{aligned}
& + 2 x_{c;d}^{\lambda} x^{\lambda a;c} (x^{\nu d} x^{\mu b} - x^{\nu b} x^{\mu d}) + x_{c;d}^{\lambda} x^{\lambda a;b} x^{\mu c} x^{\nu d} \\
& + \frac{1}{4} x_{c;d}^{\lambda} x^{\lambda c;d} (x^{\mu a} x^{\nu b} - x^{\mu b} x^{\nu a}) + x_d^{\lambda;a} x_c^{\lambda;b} x^{\mu c} x^{\nu d} \\
& + \frac{1}{2} (\delta^{\mu\nu} - h^{\mu\nu}) R^{ab} \\
& - 2 \nabla_c \left\{ -\frac{1}{4} h^{\mu\nu}_{;d} (g^{ac} g^{bd} - g^{ab} g^{cd}) - g^{ab} x^{\mu c}_{;d} x^{\nu d} \right. \\
& \quad \left. - g^{bc} x^{\mu a}_{;d} x^{\nu d} - 2 x^{\nu a;c} x^{\mu b} + x^{\mu a;c} x^{\nu b} - x^{\mu a;b} x^{\nu c} \right\} \\
& + \mu a \leftrightarrow \nu b
\end{aligned}
\tag{2.10}^{*})$$

Note that N and M are covariant under the C-type transformation (2.6b).

Now we fix the Q-type local symmetry (2.6a) to quantize the theory. The most natural way, in the present approach, is to take the following gauge:

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} &= \frac{1}{2} \sqrt{g} g_{ab} C^a C^b, \\
C^a &\equiv y^{\mu}_{;c} x^{\mu a}
\end{aligned}
\tag{2.11}^{**})$$

It keeps the C-type local symmetry (2.6b). The higher-derivative gauge is admitted for the present theory with the propagator $1/p^4$, not with $1/p^2$. We call the gauge the "background-field gauge", because it is a natural extension (to the string theory) of 't Hooft's gauge [8].

*) We frequently use the properties (2.5c).

**) Note that $-\mathcal{L}_{\text{gauge}}$ is just the second of the "kinetic terms" of $\mathcal{L}_2^{\text{inv}}$, i.e., $\frac{1}{2} \sqrt{g} (y^{\mu}_{a;b} y^{\mu}_{c;d} - y^{\mu}_{a;b} y^{\mu}_{c;d} h^{\mu\nu}) \times g^{ab} g^{cd}$. Here we find a similarity to the case of the pure Yang-Mills theory. The kinetic part of the theory, after the expansion $(A_{\mu} = A_{\mu}^{\text{cl}} + a_{\mu})$ takes the form $\frac{1}{2} (-D_{\mu}^{\text{cl}} a_{\nu}^a D_{\mu}^{\text{cl}} a_{\nu}^a + D_{\mu}^{\text{cl}} a_{\nu}^a D_{\mu}^{\text{cl}} a_{\nu}^a)$, and we take the background field gauge $\mathcal{L}_{\text{gauge}} = -\frac{1}{2} D_{\mu}^{\text{cl}} a_{\nu}^a D_{\mu}^{\text{cl}} a_{\nu}^a$ to subtract the second term of the kinetic part.

The ghost Lagrangian is obtained by the usual procedure, from (2.6a) and (2.11), as

$$\begin{aligned}\mathcal{L}_{ghost} &= \sqrt{g} (\eta_b^* x^{\mu b})^{;a} \{ (\eta_c x^{\mu c})_{;a} + \sqrt{\alpha_r} (\eta_c y^{\mu c})_{;a} \} \\ &= \mathcal{L}_{ghost}^2 + C(\eta^* \eta y), \\ \mathcal{L}_{ghost}^2 &= \sqrt{g} (\eta_b^* x^{\mu b})^{;a} (\eta_c x^{\mu c})_{;a},\end{aligned}\quad (2.12)$$

where η_a and η_a^* are ghost fields which are complex vectors. We treat the ghost fields as quantum fields. In the next section, we explicitly renormalize the theory at the one-loop order, where we may consider the $\mathcal{L}_{ghost}^2$ only.

3. - ONE-LOOP RENORMALIZATION

Now we investigate the ultra-violet behaviour of the theory (2.1). At the one-loop order, we may consider the quadratic part of the quantum fields (y^μ , η_a , η_a^*):

$$\mathcal{L}_{1-loop} = \mathcal{L}_2^{inv} + \mathcal{L}_{gauge} + \mathcal{L}_{ghost}^2. \quad (3.1a)$$

It can be written as

$$\begin{aligned}\mathcal{L}_{1-loop} &= \sqrt{g} \left\{ \frac{1}{2} y_a^\mu{}^{;a} y_b^\mu{}^{;b} + y_{a;b}^\mu y_c^\nu N_{\mu\nu}^{abc} + \frac{1}{2} y_a^\mu y_b^\nu M_{\mu\nu}^{ab} \right\} \\ &+ \sqrt{g} \left\{ \eta_{a;b}^* \eta^{a;b} + (\eta^a \eta^{*b;c} + \eta^{*a} \eta^b{}_{;c}) \tilde{N}_{abc} + \eta_a^* \eta_b \tilde{M}^{ab} \right\}, \\ \tilde{N}_{abc} &= C, \\ \tilde{M}^{ab} &= x^{\mu a}{}_{;c} x^{\mu b}{}_{;c},\end{aligned}\quad (3.1b)$$

where $N_{\mu\nu}^{abc}$ and $M_{\mu\nu}^{ab}$ are given by (2.10). $\tilde{M}^{ab}$ is C-type gauge invariant, like N and M.

We proceed in several steps. First we obtain all counterterms for general coefficient functions (N, M, $\tilde{N}$, $\tilde{M}$), i.e., the one-loop counterterm formula. For the moment, we forget the contents of those functions. The formula is obtained in two steps:

(i) flat case ($g_{ab} = \delta_{ab}$)

In the flat case, the Lagrangian (3.1b) becomes

$$\begin{aligned} \mathcal{L}_{\text{flat}} = & \frac{1}{2} \partial_a \partial_a y^\mu \partial_b \partial_b y^\mu + \partial_a \partial_b y^\mu \partial_c y^\nu N_{\mu\nu}^{abc} + \frac{1}{2} \partial_a y^\mu \partial_b y^\nu M_{\mu\nu}^{ab} \\ & + \partial_b \eta_a^* \partial_b \eta_a + (\eta_a \partial_c \eta_b^* + \eta_a^* \partial_c \eta_b) \tilde{N}_{abc} + \eta_a^* \eta_b \tilde{M}^{ab} . \end{aligned} \quad (3.2)$$

Feynman rules for the flat Lagrangian $\mathcal{L}_{\text{flat}}$ are given in Fig. 1. All one-loop divergent diagrams are shown in Fig. 2. In the calculations, we put an infra-red regularization mass m into the propagators to separate the infra-red divergences from ultra-violet ones. We take the dimensional regularization as the ultra-violet one, i.e., we consider n -dimensional field theory instead of two-dimensional field theory. Diagrams (a) and (d) vanish. The tadpole diagrams (c) and (f), besides (b) and (e), contribute to the counterterms. (This is a distinct contrast to four-dimensional theories, where we have no serious infra-red singularity for those diagrams.) The one-loop counterterm formula for $\mathcal{L}_{\text{flat}}$ is obtained as

$$\begin{aligned} \Delta \mathcal{L}_{\text{flat}} = & \frac{1}{2^8 \cdot 3 \pi \epsilon} \{abcdef\} (N_{\mu\nu}^{abc} - N_{\nu\mu}^{abc}) (N_{\mu\nu}^{def} - N_{\nu\mu}^{def}) \\ & - \frac{1}{2^3 \pi \epsilon} M_{\mu\nu}^{ab} \delta_{ab} + \frac{1}{2 \pi \epsilon} (\tilde{M}^{ab} \delta_{ab} + \tilde{N}_{abc} \tilde{N}_{abc}) , \\ \epsilon = 2 - n , \quad \{abcdef\} = & \delta_{ab} \delta_{cd} \delta_{ef} + \text{total sym.}(a,b,c,d,e,f) . \end{aligned} \quad (3.3)$$

(ii) general case

The one-loop counterterm formula for $\mathcal{L}_{\text{one-loop}}$ (3.1b) is obtained as the most general invariant form that reduces to $\Delta \mathcal{L}_{\text{flat}}$ at the flat limit $g_{ab} = \delta_{ab}$:

$$\Delta \mathcal{L} = \sqrt{g} \left\{ \frac{1}{2^3 \cdot 3 \pi \epsilon} [abcdef] (N_{\mu\nu}^{abc} - N_{\nu\mu}^{abc}) (N_{\mu\nu}^{def} - N_{\nu\mu}^{def}) \right. \\ \left. - \frac{1}{2^3 \pi \epsilon} M_{\mu\mu}^{ab} g_{ab} + \frac{1}{2 \pi \epsilon} (\tilde{M}^{ab} g_{ab} + \tilde{N}_{abc} \tilde{N}^{abc}) \right. \\ \left. + \frac{r}{\epsilon} R(g) \right\}, \\ [abcdef] = g_{ab} g_{cd} g_{ef} + \text{total sym.}(a, b, c, d, e, f), \quad (3.4)$$

where r is some constant. However, we know that the term

$$\frac{r}{\epsilon} \int \sqrt{g} R(g) d^2 \xi \quad (3.5)$$

is a topological invariant and does not contribute to renormalization. The result $\Delta \mathcal{L}$ (3.4) is the one-loop counterterm formula for $\mathcal{L}_{\text{one-loop}}$ (3.1b).

Now we apply the formula $\Delta \mathcal{L}$ to our model of the rigid string. We insert the concrete contents of $(N, M, \tilde{N}, \tilde{M})$ into the formula, using the following results:

$$[abcdef] (N_{\mu\nu}^{abc} - N_{\nu\mu}^{abc}) (N_{\mu\nu}^{def} - N_{\nu\mu}^{def}) \\ = 2^3 (n+2)(n+4) x_{a;b}^{\mu} x^{\mu a;b}, \\ M_{\mu\mu}^{ab} g_{ab} = \left\{ \left(\frac{n}{2} - 2 \right) (D-n) + n+2 \right\} x_{a;b}^{\mu} x^{\mu a;b} \\ + 2 x_a^{\mu ; a} x_b^{\mu ; b} + (D-n) R + (D-n) n \mu_r, \\ \tilde{M}^a_a = - x^{\mu a} x_{a;b}^{\mu ; b}, \quad \tilde{N}_{abc} \tilde{N}^{abc} = 0, \quad (3.6)$$

All the one-loop counterterms for our model are given by

$$\Delta \mathcal{L} = + \frac{D-2}{8 \pi \epsilon} \sqrt{g} x_{a;b}^{\mu} x^{\mu a;b} - \frac{D-2}{4 \pi \epsilon} \alpha_r \mu_r \sqrt{g} \quad (3.7)$$

They are absorbed by the coupling constant and mass renormalizations:

$$\mu_0 = \mu_r \left\{ 1 - \frac{D-2}{4\pi\epsilon} \alpha_r + O(\alpha_r^2) \right\},$$

$$\alpha_0 = \alpha_r - \frac{D-2}{4\pi\epsilon} \alpha_r^2 + O(\alpha_r^3) \quad (3.8)$$

The β -function for the coupling α_r is given by

$$\beta = \left(1 - \alpha_r \frac{\partial}{\partial \alpha_r} \right) \left(\frac{D-2}{4\pi} \alpha_r^2 + O(\alpha_r^3) \right)$$

$$= - \frac{D-2}{4\pi} \alpha_r^2 + O(\alpha_r^3) \quad (3.9)$$

The result is proportional to the physical degrees of freedom (number of components of the string field x^μ - number of gauge freedom) and is reasonable^{*}). It is just the half of the β -function for the $SO(D)/SO(2) \times SO(D-2)$ Grassmannian σ -model (at the one-loop level) [10]. This fact is again reasonable because the physical degree of freedom of the present string theory, $D-2$, is just the half of that of the σ -model, $2(D-2)$. The sign of β (3.9) shows that the theory is asymptotically free.

4. - DISCUSSIONS

We first comment on the higher-loop structure of this theory. The expanded Lagrangian takes the following form:

$$\mathcal{L} = \sqrt{g} \left\{ \frac{1}{2} y_a^\mu y_b^\mu + y_{a;b}^\mu y_c^\nu N_{\mu\nu}^{abc} + \frac{1}{2} y_a^\mu y_b^\nu M_{\mu\nu}^{ab} \right.$$

$$+ y_{a;b}^\mu y_{c;d}^\nu y_e^\lambda \Xi_{\mu\nu\lambda}^{abcde} + y_{a;b}^\mu y_c^\nu y_d^\lambda \Omega_{\mu\nu\lambda}^{abcd} + y_a^\mu y_b^\nu y_c^\lambda \Lambda_{\mu\nu\lambda}^{abc}$$

$$+ y_{a;b}^\mu y_{c;d}^\nu y_e^\lambda y_f^\alpha \Gamma_{\mu\nu\lambda\alpha}^{abcdef} + y_{a;b}^\mu y_c^\nu y_d^\lambda y_e^\alpha \Sigma_{\mu\nu\lambda\alpha}^{abcde}$$

$$\left. + y_a^\mu y_b^\nu y_c^\lambda y_d^\alpha \Theta_{\mu\nu\lambda\alpha}^{abcd} \right\} + O(y^5) \quad (4.1)$$

where we stop the expansion at the relevant terms for two-loop calculation. The new coefficient functions Ξ , Ω , Λ , Γ , Σ and Θ are, like N and M , C-type gauge-

^{*}) Our result is different from that of Polyakov [2]: $\beta = -(D/8\pi)\alpha_r^2 + O(\alpha_r^3)$.

invariant, and are made of background (classical) fields. As is easily obtained from the dimensional analysis, or explicit calculations, they have the following contents:

$$\begin{aligned}
 \Xi_{\mu\nu\lambda}^{abcde} &\sim \sqrt{\alpha_r} \left\{ g^{ac} g^{bd} \delta_{\mu\nu} x^{\lambda e} + \dots \right\}, \\
 \Omega_{\mu\nu\lambda}^{abcde} &\sim \sqrt{\alpha_r} \left\{ g^{ad} g^{bc} x_{e;f}^{\mu} x^{\nu f} x^{\lambda e} + \dots \right\}, \\
 \Lambda_{\mu\nu\lambda}^{abc} &\sim \sqrt{\alpha_r} \left\{ g^{ab} x_{;d}^{\mu c} x^{\nu d}_{;e} x^{\lambda e} + \mu_r x^{\mu a} x^{\nu b} x^{\lambda c} + \dots \right\}, \\
 \Gamma_{\mu\nu\lambda\alpha}^{abcdef} &\sim \alpha_r \left\{ g^{ac} g^{bd} g^{ef} \delta_{\mu\nu} \delta_{\lambda\alpha} + \dots \right\}, \\
 \Sigma_{\mu\nu\lambda\alpha}^{abcde} &\sim \alpha_r \left\{ g^{ab} g^{cd} \delta_{\mu\nu} x^{\lambda e}_{;f} x^{\alpha f} + \dots \right\}, \\
 \textcircled{H}_{\mu\nu\lambda\alpha}^{abcd} &\sim \alpha_r \left\{ g^{ab} \delta_{\mu\nu} x^{\lambda c}_{;e} x^{\alpha d}_{;e} + \mu_r x^{\mu a} x^{\nu b} x^{\lambda c} x^{\alpha d} + \dots \right\}.
 \end{aligned}
 \tag{4.2}$$

The structure of (4.1) is similar to that appearing in the case of four-dimensional gravitational theories [9]. Our splitting of fields is linear, i.e., $\bar{x}^\mu = x^\mu + y^\mu$, and the subdivergence problem is treated similarly to that given in Ref. [9]. From the expressions (4.2), and simple power-counting of divergent diagrams, it is explicitly seen that all higher-loop contributions sum up to the following form of counterterms:

$$\begin{aligned}
 \Delta \mathcal{L}_{full} &= \sqrt{g} \left\{ f(\alpha_r) x_{a;b}^{\mu} x^{\mu a;b} + g(\alpha_r) \mu_r \right\}, \\
 f(\alpha_r) &= \frac{D-2}{8\pi\epsilon} + O(\alpha_r), \quad g(\alpha_r) = -\frac{D-2}{4\pi\epsilon} \alpha_r + O(\alpha_r^2).
 \end{aligned}
 \tag{4.3}$$

The exact β -function for the coupling α_r is derived from the full form of $f(\alpha_r)$. It is very important to know the non-perturbative result of $f(\alpha_r)$ in order to see the phase transition phenomena in this model. For this purpose, we must do some non-perturbative expansion, unlike the one given in the present paper. Reference [5] treats it in $1/D$ expansion and finds no non-trivial fixed point in the β -function in the leading order.

Secondly, we see what happens when we apply the rigid string theory to the unified theories. One of the popular approaches is the effective action represented by Refs. [11]. Usually, the equations of motion of the space-time fields (gravitons, dilatons,...) are derived from the vanishing condition of the generalized β -functions, which guarantees the non-existence of Weyl anomalies. Let us discuss our case. For simplicity, we consider only graviton fields $G_{\mu\nu}(x)$. As the most general model, which leads to the model (2.1) for the flat space-time $G_{\mu\nu} = \delta_{\mu\nu}$, we take:

$$S[x] = \int d^2\xi \sqrt{g} \left[\frac{\mu_r}{2} + G_{\mu\nu}(x) \left\{ \frac{1}{2\alpha_r} x_{a;b}^\mu x^{\nu a;b} + \frac{1}{2\beta_r} x_a^{\mu;a} x_b^{\nu;b} \right\} \right. \\ \left. + \frac{1}{2\gamma_r} \tilde{R}_{\mu\nu} x_a^\mu x^{\nu a} + \frac{1}{\delta_r} \tilde{R} \right. \\ \left. + \frac{1}{\omega_r} \tilde{R}_{\mu\nu\lambda\sigma} x_a^\mu x^{\lambda a} x_b^\nu x^{\sigma b} \right],$$

$$g_{ab}(\xi) \equiv \partial_a x^\mu \partial_b x^\nu G_{\mu\nu}(x),$$

$$\tilde{R}_{\mu\nu}(x) = \tilde{R}^\lambda_{\mu,\nu\lambda}(x), \quad \tilde{R}^\lambda_{\sigma,\mu\nu} = \partial_\mu \tilde{\Gamma}^\lambda_{\sigma\nu} + \tilde{\Gamma}^\lambda_{\mu\tau} \tilde{\Gamma}^\tau_{\sigma\nu} - \mu \leftrightarrow \nu, \quad (4.4)$$

where $\tilde{\Gamma}^\lambda_{\mu\nu}$ is the Christoffel symbol constructed from the space-time metric $G_{\mu\nu}$. The dimension of μ_r is L^{-2} , and those of α_r , β_r , γ_r , δ_r and ω_r are L^0 under the following assignment:

$$[x^\mu] = L, \quad [\xi^a] = L, \quad [G_{\mu\nu}] = L^0, \quad [S] = L^0. \quad (4.5)$$

As in the case of the flat space-time [4], the theory (4.4) has the vanishing energy-momentum tensor ($\theta_b^a = 0$). It is shown by explicit calculations in Appendix B.

In order to quantize the theory keeping not only two-dimensional general covariance, but also the symmetry of the space-time geometry, we must take the normal co-ordinate expansions

$$\bar{x}^\mu = x^\mu + \xi^\mu - \frac{1}{2} \tilde{\Gamma}^\mu_{\nu\lambda} \xi^\nu \xi^\lambda + O(\xi^3), \\ \bar{G}_{\mu\nu} = G_{\mu\nu}(x) - \frac{1}{2} \tilde{R}_{\mu\lambda\nu\sigma} \xi^\lambda \xi^\sigma + O(\xi^3), \quad (4.6)$$

and, as a natural extension of (2.11), take the following gauge:

$$\begin{aligned} \mathcal{L}_{\text{gauge}} &= \frac{1}{2} \sqrt{g} g_{ab} C^a C^b, \\ C^a &\equiv \int d^2x \, \epsilon^{ac} x^{\nu a} G_{\mu\nu}(x). \end{aligned} \quad (4.7)$$

Because the present approach keeps all symmetries in quantization and the perturbation parameter is dimensionless, we easily see that all ultra-violet divergences of the theory (4.4) are absorbed only by the coupling constants ($\mu_r, \alpha_r, \beta_r, \gamma_r, \delta_r, \omega_r$). We need no redefinition of the space-time metric $G_{\mu\nu}(x)$. Therefore we can conclude that the generalized β -function for the metric $G_{\mu\nu}$ vanishes. In other words, the space-time geometry does not change under the two-dimensional (surface) quantum effects.

How is this model (4.4) related to the popular one, i.e., the Polyakov string within external space-time fields? The renormalizability situation above reminds us of the analogous one in (3+1)-dimensional gravitational theories. The (3+1)-dimensional conformal gravity is renormalizable in the usual sense (we need no redefinition of fields). On the other hand, the (3+1)-dimensional Einstein gravity is not (even in the generalized sense). There have been many attempts to deduce the Einstein gravity from the conformal one by the spontaneous breakdown mechanism. Based on this analogy, we might be right to suppose that the popular model [which is not invariant under the scale transformation of the space-time co-ordinates (1.4)] is deduced from the rigid string model (4.4), or from its certain generalization (see below), by the spontaneous breakdown of the global scale invariance.

After the work of Polyakov [2], there was progress in generalizing the model. Weyl-invariant models are proposed in Refs. [3] and [12], and supersymmetric models in Refs. [13] and [12]. The present approach is also important for those models, especially for the examination of local properties such as renormalization effects.

Unitarity problems are serious in higher-derivative theories, as in the present model. In order to see the unitarity as the two-dimensional theory, one approach is to look at the two-dimensional full propagator and examine a ghost pole. Some non-perturbative analysis (say, $1/D$ expansion for the present case) is needed. It is far more difficult to show the unitarity as the D -dimensional space-time theory. That is beyond the present scope of our research.

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A P P E N D I X A

SYMMETRIES OF N AND M IN EQUATIONS (2.8) and (3.2)

The case of (2.8) is treated in the same way as that of (3.2) because we use only a common property of the usual derivative ∂_a and the covariant one ∇_a . When we write the one-loop Lagrangian as (3.2),

$$\mathcal{L} = \frac{1}{2} \partial_a \partial_a y^\mu \cdot \partial_b \partial_b y^\mu + \partial_a \partial_b y^\mu \cdot \partial_c y^\nu N_{\mu\nu}^{abc} + \frac{1}{2} \partial_a y^\mu \partial_b y^\nu M_{\mu\nu}^{ab}, \quad (\text{A.1})$$

there are, in general, some symmetries in N and M under the exchange of suffixes because of the total derivative arbitrariness. First, by definition, we may assume

$$N_{\mu\nu}^{abc} = N_{\mu\nu}^{bac}, \quad M_{\mu\nu}^{ab} = M_{\nu\mu}^{ba}. \quad (\text{A.2})$$

The N-term is rewritten as

$$\begin{aligned} \partial_a \partial_b y^\mu \cdot \partial_c y^\nu \cdot N_{\mu\nu}^{abc} &= \partial_a \partial_b y^\mu \cdot \partial_c y^\nu \cdot \tilde{N}_{\mu\nu}^{abc}(\alpha) \\ &\quad - \frac{\alpha}{2} \partial_a y^\mu \partial_b y^\nu \cdot (\partial_c N_{\mu\nu}^{cab} + \partial_c N_{\nu\mu}^{cba}) + \text{total derivative}, \\ \tilde{N}_{\mu\nu}^{abc}(\alpha) &= (1-\alpha) N_{\mu\nu}^{abc} - \frac{\alpha}{2} (N_{\nu\mu}^{acb} + N_{\nu\mu}^{bca}), \end{aligned} \quad (\text{A.3})$$

where α is some number to be specified. The most general linear symmetry is written as

$$C_1 \tilde{N}_{\mu\nu}^{abc} + C_2 \tilde{N}_{\mu\nu}^{cab} + C_3 \tilde{N}_{\mu\nu}^{bca} + C_4 \tilde{N}_{\nu\mu}^{abc} + C_5 \tilde{N}_{\nu\mu}^{cab} + C_6 \tilde{N}_{\nu\mu}^{bca} = 0, \quad (\text{A.4})$$

We seek solutions for α where non-trivial (i.e., $\sum_{i=1}^6 C_i^2 \neq 0$) C_i 's exist and find the following three:

(i) $\alpha = \frac{1}{2}$

$$\tilde{N}_{\mu\nu}^{abc} + \tilde{N}_{\mu\nu}^{cab} + \tilde{N}_{\mu\nu}^{bca} + \tilde{N}_{\nu\mu}^{abc} + \tilde{N}_{\nu\mu}^{cab} + \tilde{N}_{\nu\mu}^{bca} = 0 \quad (A.5)$$

(ii) $\alpha = 2/3$

$$\begin{aligned} \tilde{N}_{\mu\nu}^{abc} - \tilde{N}_{\nu\mu}^{abc} &= \tilde{N}_{\mu\nu}^{cab} - \tilde{N}_{\nu\mu}^{cab} = \tilde{N}_{\mu\nu}^{bca} - \tilde{N}_{\nu\mu}^{bca} \\ &= \text{totally symmetric w.r.t. } (a, b, c) \end{aligned} \quad (A.6)$$

(iii) $\alpha = 2$

$$\begin{aligned} \tilde{N}_{\mu\nu}^{abc} + \tilde{N}_{\nu\mu}^{abc} &= \tilde{N}_{\mu\nu}^{cab} + \tilde{N}_{\nu\mu}^{cab} = \tilde{N}_{\mu\nu}^{bca} + \tilde{N}_{\nu\mu}^{bca} \\ &= \text{totally symmetric w.r.t. } (a, b, c) \end{aligned} \quad (A.7)$$

We do not take (i), because the symmetry is not useful. We take (ii) in the text. We may take (iii) instead. The final conclusion (3.7) does not change (we have checked).

A P P E N D I X B

ENERGY-MOMENTUM TENSOR OF THE RIGID STRING WITHIN EXTERNAL GRAVITY

In this Appendix we explicitly calculate the two-dimensional energy-momentum tensor of the rigid string within the space-time gravity:

$$\begin{aligned} \mathcal{L}[x] &= \frac{1}{2\alpha'} \sqrt{g} G_{\mu\nu}(x) x_{a;b}^{\mu} x^{\nu a;b}, \\ x_{a;b}^{\mu} &= \partial_b x_a^{\mu} - \Gamma_{ab}^c x_c^{\mu} = x_{b;a}^{\mu}, \\ g_{ab}(\xi) &= \partial_a x^{\mu} \partial_b x^{\nu} G_{\mu\nu}(x). \end{aligned} \quad (B.1)$$

g^{ab} is defined to be the inverse of g_{ab} . For the flat case $G_{\mu\nu} = \delta_{\mu\nu}$, explicit calculations were carried out in Ref. [4]. We treat the general case. The more general case (4.4) is treated similarly.

The energy-momentum tensor is generally given as the conserved current associated with the translation invariance of the theory. For the higher-derivative case $\mathcal{L} = \mathcal{L}[x, \partial_a x, \partial_a \partial_b x]$, it is given by

$$\begin{aligned} \theta^a_b &= -\delta^a_b \mathcal{L} + x_b^{\mu} \frac{\delta \mathcal{L}}{\delta x_a^{\mu}} - x_b^{\mu} \partial_c \frac{\delta \mathcal{L}}{\delta x_{a,c}^{\mu}} + x_{b,c}^{\mu} \frac{\delta \mathcal{L}}{\delta x_{a,c}^{\mu}}, \\ \partial_a \theta^a_b &= 0, \quad x_a^{\mu} \equiv \partial_a x^{\mu}, \quad x_{a,b}^{\mu} \equiv \partial_b \partial_a x^{\mu}, \end{aligned} \quad (B.2)$$

where we have used the equation of motion.

As in the flat case treated in the text, we introduce normals n_i^{μ} defined, in terms of tangents x_a^{μ} , by

$$\begin{aligned} n_i^{\mu} G_{\mu\nu} x_a^{\nu} &= 0, \quad n_i^{\mu} n_j^{\nu} G_{\mu\nu} = \delta_{ij}, \\ i &= 1, 2, \dots, D-2; \quad a = 1, 2; \quad \mu = 1, 2, \dots, D. \end{aligned} \quad (B.3)$$

Before calculation, we note two things. First, $G_{\mu\lambda} h^{\lambda\nu}$ ($h^{\mu\nu} \equiv x_a^{\mu} x_b^{\nu} g^{ab}$) and $\delta_{\mu}^{\nu} - G_{\mu\lambda} h^{\lambda\nu}$ are projection operators. This is a natural extension of the flat case (2.5a):

$$\begin{aligned}
 G_{\mu\lambda} h^{\lambda\nu} G_{\nu\alpha} h^{\alpha\tau} &= G_{\mu\nu} h^{\nu\tau}, \quad (\delta_\mu^\nu - G_{\mu\lambda} h^{\lambda\nu}) G_{\nu\alpha} h^{\alpha\tau} = 0, \\
 (\delta_\mu^\nu - G_{\mu\lambda} h^{\lambda\nu}) (\delta_\nu^\alpha - G_{\nu\tau} h^{\tau\alpha}) &= \delta_\mu^\alpha - G_{\mu\lambda} h^{\lambda\alpha}, \\
 G_{\mu\lambda} h^{\lambda\nu} x_a^\mu &= x_a^\nu, \quad (\delta_\mu^\nu - G_{\mu\lambda} h^{\lambda\nu}) n_i^\mu = n_i^\nu, \\
 G_{\mu\lambda} h^{\lambda\nu} n_i^\mu &= 0, \quad (\delta_\mu^\nu - G_{\mu\lambda} h^{\lambda\nu}) x_a^\mu = 0. \quad (B.4a)
 \end{aligned}$$

Furthermore, we have a relation

$$h^{\mu\lambda} G_{\lambda\alpha} h^{\alpha\nu} = h^{\mu\nu}. \quad (B.4b)$$

Secondly, $x_{a;b}^\mu$ has tangent components in the external gravity

$$G_{\mu\nu} x_c^\mu x_{a;b}^\nu = -\frac{1}{2} \{ x_c^\mu (x_b^\nu x_a^\lambda + x_a^\nu x_b^\lambda) - x_a^\mu x_b^\nu x_c^\lambda \} \partial_\lambda G_{\mu\nu}. \quad (B.5)$$

Compare this with the flat case (2.5c).

We calculate θ_b^a in three steps:

(i) $\delta\mathcal{L}/\delta x_{ab}^\mu$ calculation

Using the following equations

$$\begin{aligned}
 \Gamma_{ab}^c &= G_{\mu\nu} x^{\mu c} x_{a;b}^\nu + \frac{1}{2} (x^{\mu c} x_a^\nu x_b^\lambda + x^{\mu c} x_b^\nu x_a^\lambda \\
 &\quad - x_a^\mu x_b^\nu x^{\lambda c}) \partial_\lambda G_{\mu\nu}, \\
 \frac{\delta \Gamma_{ab}^c}{\delta x_{e,f}^\mu} &= \frac{1}{2} G_{\mu\nu} x^{\nu c} (\delta_a^e \delta_b^f + \delta_a^f \delta_b^e), \quad (B.6)
 \end{aligned}$$

we obtain

$$\begin{aligned}
 \frac{\delta \mathcal{L}}{\delta x_{a,b}^\mu} &= \frac{1}{2\alpha_r} \times 2 \times \sqrt{g} \left\{ x^{\nu a;b} G_{\mu\nu} - \frac{\delta \Gamma_{de}^c}{\delta x_{a,b}^\mu} x_c^\lambda x^{\sigma d;e} G_{\lambda\sigma} \right\} \\
 &= \frac{1}{\alpha_r} \sqrt{g} (\delta_\mu^\nu - G_{\mu\lambda} h^{\lambda\nu}) G_{\nu\sigma} x^{\sigma a;b}. \quad (B.7)
 \end{aligned}$$

(ii) $x_b^\mu (\delta \mathcal{L} / \delta x_a^\mu)$ calculation

Useful equations about x_a^μ -differentiation are given as follows:

$$\frac{\delta g_{ab}}{\delta x_c^\mu} = G_{\mu\nu} (\delta_a^c x_b^\nu + \delta_b^c x_a^\nu) ,$$

$$\frac{\delta g^{ab}}{\delta x_c^\mu} = -g^{ad} \frac{\delta g_{de}}{\delta x_c^\mu} g^{eb} = -G_{\mu\nu} (g^{ac} x^{\nu b} + g^{bc} x^{\nu a}) ,$$

$$\frac{\delta \sqrt{g}}{\delta x_a^\mu} = \frac{1}{2} \sqrt{g} \frac{\delta g_{bc}}{\delta x_a^\mu} g^{bc} = \sqrt{g} G_{\mu\nu} x^{\nu a} ,$$

$$\frac{\delta x_b^\nu}{\delta x_a^\mu} = \delta_\mu^\nu \delta_b^a ,$$

$$\frac{\delta x^{\nu b}}{\delta x_a^\mu} = \delta_\mu^\nu g^{ab} - G_{\mu\lambda} (g^{ab} h^{\lambda\nu} + x^{\nu a} x^{\lambda b}) ,$$

$$\frac{\delta h^{\nu\lambda}}{\delta x_a^\mu} = (\delta_\mu^\nu - G_{\mu\sigma} h^{\sigma\nu}) x^{\lambda a} + (\delta_\mu^\lambda - G_{\mu\sigma} h^{\sigma\lambda}) x^{\nu a} .$$

(B.8)

Using these equations, we can evaluate every term of the following expression:

$$x_b^\mu \frac{\delta \mathcal{L}}{\delta x_a^\mu} = \frac{1}{2\alpha_r} x_b^\mu \left\{ \frac{\delta \sqrt{g}}{\delta x_a^\mu} x_{c;d}^\nu x^{\lambda c;d} G_{\nu\lambda} - 2\sqrt{g} \frac{\delta(\Gamma_{cd}^e x_e^\nu)}{\delta x_a^\mu} x^{\lambda c;d} G_{\nu\lambda} + 2\sqrt{g} x_{c;e}^\nu x_d^{\lambda;e} \frac{\delta g^{cd}}{\delta x_a^\mu} G_{\nu\lambda} \right\} .$$

(B.9)

Especially, we obtain

$$x_b^\mu \frac{\delta(\Gamma_{cd}^e x_c^\nu)}{\delta x_a^\mu} = \frac{1}{2} h^{\nu\lambda} x_b^\sigma \left\{ \delta_c^a x_d^\tau (\partial_\tau G_{\lambda\sigma} - \partial_\lambda G_{\tau\sigma}) + \delta_d^a x_c^\tau \partial_\sigma G_{\lambda\tau} \right\} + c \leftrightarrow d \quad (B.10)$$

Then (B.9) is evaluated as

$$x_b^\mu \frac{\delta \mathcal{L}}{\delta x_a^\mu} = \frac{1}{2\alpha_r} \sqrt{g} G_{\mu\nu} \left[\delta_b^a x_{c;d}^\mu x^{\nu c;d} - 4 x^{\mu a}_{;c} x_b^{\nu;c} - 2 x^{\mu c;d} h^{\nu\lambda} x_b^\sigma \left\{ \delta_c^a x_d^\tau (\partial_\tau G_{\lambda\sigma} - \partial_\lambda G_{\tau\sigma}) + \delta_d^a x_c^\tau \partial_\sigma G_{\lambda\tau} \right\} \right] \quad (B.11)$$

(iii) θ_b^a calculation

The results (B.7) and (B.11) help us to calculate θ_b^a of (B.2). Noting the following equations,

$$\begin{aligned} x_{a,b}^\mu (\delta_\mu^\nu - G_{\mu\lambda} h^{\lambda\nu}) &= x_{a;b}^\mu (\delta_\mu^\nu - G_{\mu\lambda} h^{\lambda\nu}) , \\ \partial_a h^{\mu\nu} &= (\delta_\lambda^\mu - G_{\lambda\sigma} h^{\sigma\mu}) x^{\nu b} x_a^\lambda{}_{;b} \\ &\quad - \frac{1}{2} h^{\mu\lambda} h^{\nu\sigma} x_a^\tau \partial_\tau G_{\lambda\sigma} + \mu \leftrightarrow \nu , \end{aligned} \quad (B.12)$$

we find

$$\theta_b^a = 0 \quad (B.13)$$

We have confirmed that the string theory (B.1) has the vanishing two-dimensional energy-momentum tensor, not only for the flat space-time, but also for the curved space-time. This reflects the reparametrization (two-dimensional general co-ordinate) invariance of the theory.

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FIGURE CAPTIONS

Fig. 1 : Feynman rules of the Lagrangian (3.2). m is the infra-red regularization mass. The metric is Euclidean.

Fig. 2 : All one-loop divergent diagrams.

$$\frac{i\delta_{\mu\nu}}{(p^2+m^2)^2}$$

$$\frac{1}{2}(\delta_{\mu\alpha}\delta_{\nu\beta}q_aq_br_c + \delta_{\mu\beta}\delta_{\nu\alpha}r_ar_bq_c)$$

$$\frac{1}{4i}(\delta_{\mu\alpha}\delta_{\nu\beta}q_ar_b + \delta_{\mu\beta}\delta_{\nu\alpha}r_aq_b)$$

$$\frac{i\delta_{ab}}{p^2+m^2}$$

$$\frac{1}{2}\{\delta_{ad}\delta_{be}q_c - \delta_{ae}\delta_{bd}r_c - (d \leftrightarrow e)\}$$

$$\frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$$

Fig. 1

(a)

(d)

(b)

(e)

(c)

(f)

Fig. 2