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NEW THRESHOLD AND THE UA4 EXPERIMENT

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ABSTRACT

We derive a general bound of the total cross-sections for both pp and $p\bar{p}$ at a given high energy for a large class of the threshold models constrained only by the recent UA4 measurement of $p\bar{p}$ scattering. In particular, we find
 $66.61 \text{ mb} < \sigma_{\text{tot}}(\sqrt{s}=900 \text{ GeV}) < 79.41 \text{ mb}$
and
 $75.51 \text{ mb} < \sigma_{\text{tot}}(\sqrt{s}=1.8 \text{ TeV}) < 127.08 \text{ mb}$
independently of the values of the threshold parameters.

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The recent publication [1] of the UA4 collaboration on the proton-antiproton elastic scattering at the CERN SPS-collider at CM energy $\sqrt{s} = 546$ GeV in the Coulomb interference region has renewed the interest in the problem of elastic hadron-hadron scattering. This is because the real part of the forward $p\bar{p}$ scattering amplitude is surprisingly large, i.e.,

$$\rho_{p\bar{p}} = \frac{\text{Re } F_{p\bar{p}}(s, t=0)}{\text{Im } F_{p\bar{p}}(s, t=0)} = 0.24 \pm 0.04 \text{ at } \sqrt{s} = 546 \text{ GeV} \quad (1)$$

if the earlier measurement [2]

$$\sigma_{p\bar{p}} (1 + \rho_{p\bar{p}}^2) = 63.3 \pm 1.5 \text{ mb} \quad (2)$$

is used to fix the absolute normalization of the differential cross-section. This is contrary to the expectations based on the conventional "wisdom" that $F_{-} \rightarrow 0$ at high energies and $\sigma_{p\bar{p}}$ rises compatibly with the Froissart-Martin bound. In this case, one expects

$$\rho_{p\bar{p}}(\sqrt{s} = 546 \text{ GeV}, t=0) = 0.1386 \pm 0.02 \quad (3)$$

In 1975, one of us (K.K.), in collaboration with B. Nicolescu, predicted [3], based on derivative dispersion relations that saturate the rigorous asymptotic bounds and the experimental data existing before 1975, that:

$$\rho_{p\bar{p}} = 0.27 \text{ and } \sigma_{p\bar{p}} = 63 \text{ mb at } \sqrt{s} = 546 \text{ GeV} \quad (4)$$

Such a high value of $\rho_{p\bar{p}}$ was possible because of the presence of a new singularity in the crossing-odd amplitude in the complex J -plane at $J = 1$ in the forward direction. This line of approach has since been known as the odderon model. Apart from the odderon explanation, a rapid energy variation [4] of the σ_{+} in some energy interval can also explain a high $\rho_{p\bar{p}}$.

In this paper we speculate yet another possibility, i.e., the existence of a new threshold [5] in the crossing-even forward amplitude above a certain energy. This will cause an observable consequence already at Tevatron energies and therefore can be tested even at the Fermilab collider. In particular, we derive a general bound of the total cross-section at a given high energy for a large class of threshold models constrained only by the UA4 experiments. We parametrize the new effect by adding

$$\text{Im } F_+^{(1)}(s) = C s^\beta \left(\frac{s-s_1}{s} \right)^{\alpha'} \theta(s-s_1) \quad (5)$$

to the imaginary part of the crossing-even amplitude. Then the real part is found from the dispersion relation for the forward amplitude:

$$F_+^{(1)}(E) = F_+^{(1)}(0) + \frac{2E^2}{\pi} \int_m^\infty dx \frac{\text{Im } F_+^{(1)}(x)}{x(x^2 - E^2)} \quad (6)$$

where $E = (s-u)/2m = s/2m - m$ is the lab-energy and m is the mass of the proton or the antiproton. The result from Eq. (6) is added to the well-known analytical parametrization [3,6]

$$F_+^{(0)}(s) = i s \left[A + B \left(\ln \frac{s}{s_0} - i \frac{\pi}{2} \right)^2 \right] + C_R \left[i - \cot \left(\frac{\pi}{2} \alpha'_R(0) \right) \right] s^{\alpha'_R(0)} \quad (7)$$

so that

$$F_+(s) = F_+^{(0)}(s) + F_+^{(1)}(s) \quad (8)$$

To preserve the property that $F_+^{(0)} = 0$ at $s = 0$ we set the subtraction constant $F_+^{(1)}(0)$ so that $F_+^{(1)}(s=0) = 0$. The exponent β is chosen to be one so that C is to carry the same dimensions as A . At high energies the contribution from the new threshold reduces to a mere redefinition of A .

The total cross-section is normalized through the optical theorem:

$$\sigma_{ph} = \frac{1}{s} \text{Im } F_{ph}(s, t=0) \quad , (h=p, \bar{p}) \quad (9)$$

where

$$F_\pm = \frac{1}{2} (F_{pp} \pm F_{p\bar{p}}) \quad (10)$$

and

$$F_-(s) = -C_R \left[i + \tan \left(\frac{\pi}{2} \alpha'_R(0) \right) \right] s^{\alpha'_R(0)} \quad (11)$$

Amaldi's fit [7] for the low energy data up to ISR energies suggests that $\sigma_- = \frac{1}{2}(\sigma_{pp} - \sigma_{p\bar{p}}) = -24E^{-0.55}$ mb, E being the lab energy in GeV. This may imply that at

high energies the crossing-even amplitude becomes dominant. However, rigorous asymptotic theorems allow up to a logarithmic growth of σ_- with s . The original odderon model [3] saturates maximally the asymptotic bound of this type. Here, since F_- is assumed solely due to an exchange-degenerate Regge term R , σ_- as well as $\rho_{pp} - \rho_{p\bar{p}}$ approach zero with increasing s , and σ_T 's and ρ 's take the following forms:

$$\sigma_{p\bar{p}} = A + B \left(\ln^2 \frac{s}{s_0} - \frac{\pi^2}{4} \right) + 2C_R s^{\alpha_R(0)-1} + C \left(1 - \frac{s_1}{s} \right)^\alpha \theta(s-s_1)$$

$$\sigma_{pp} = A + B \left(\ln^2 \frac{s}{s_0} - \frac{\pi^2}{4} \right) + C \left(1 - \frac{s_1}{s} \right)^\alpha \theta(s-s_1)$$

$$\rho_{p\bar{p}} \sigma_{p\bar{p}} = \pi B \ln \frac{s}{s_0} - 2C_R \cot(\pi\alpha_R(0)) s^{\alpha_R(0)-1} + \pi C K(s, s_1, \alpha) \quad (12)$$

$$\rho_{pp} \sigma_{pp} = \pi B \ln \frac{s}{s_0} - 2C_R \csc(\pi\alpha_R(0)) s^{\alpha_R(0)-1} + \pi C K(s, s_1, \alpha)$$

where

$$K(s, s_1, \alpha) = \frac{1}{\pi^2} \mathcal{P} \int_0^1 dz (1-z)^\alpha \left(\frac{1}{\frac{s_1}{s} + z} + \frac{1}{\frac{s_1}{s} - z} \right) \quad (13)$$

is the principal valued dispersion integral.

Since σ_- is independent of the new effect the low-energy fit can be used to determine $C_R = 33.92$ and $\alpha_R(0) = 0.45$. A good description ($\chi^2 \approx 1.1/\text{point}$) of σ_{pp} , $\sigma_{p\bar{p}}$, ρ_{pp} and $\rho_{p\bar{p}}$ between $\sqrt{s} = 10$ GeV and the ISR energies can already be accomplished with the following parameters:

$$A = 38.593 \text{ mb}, \quad B = 0.313 \text{ mb}, \quad s_0 = 55.46 (\text{GeV})^2 \quad (14)$$

Then $F_+^{(0)}$ alone predicts Eq. (3) and

$$\sigma_{p\bar{p}}(\sqrt{s} = 546 \text{ GeV}) = 60.96 \text{ mb}, \quad \sigma_{pp}(\sqrt{s} = 900 \text{ GeV}) = 66.61 \text{ mb},$$

$$\sigma_{p\bar{p}}(\sqrt{s} = 1.8 \text{ TeV}) = 75.51 \text{ mb} \quad (15)$$

which are to be compared to the UA4 measurement 60 ± 2 mb, the UA5 suggestion [8] 65.3 ± 0.7 mb and the preliminary indication from Tevatron ~ 80 mb respectively. This suggests that if a new threshold of the type Eq. (5) exists, the value of the threshold $\sqrt{s_1}$ must be greater than 546 GeV. On the other hand, if $\sqrt{s_1} > 546$ GeV, the typical corrections to ρ_{pp}^- (ISR), due to the threshold, are at most of the order of $O(10^{-3})$. This is because we have

$$\Delta \rho_{p\bar{p}}(\sqrt{s}=546 \text{ GeV}) = \frac{\pi C K(s=(546 \text{ GeV})^2, s_1, \alpha)}{\sigma_{p\bar{p}}(\sqrt{s}=546 \text{ GeV})} = 0.1 \pm 0.04 \quad (16)$$

and the principal value integral $K(s, s_1, \alpha)$ can well be approximated below the threshold, i.e., for $s < s_1$ by

$$K(s < s_1, s_1, \alpha) \approx \frac{2/\pi^2}{\alpha+1} \left(\frac{s}{s_1}\right) \left[1 + \frac{2}{(\alpha+2)(\alpha+3)} \left(\frac{s}{s_1}\right)^2\right] \quad (17)$$

as illustrated in Fig. 1. This approximation becomes better as α and s_1 increase from 1 and s respectively. Thus while C is given by

$$C \approx 0.1571(\alpha+1) \sigma_{p\bar{p}}(\sqrt{s}=546 \text{ GeV}) \left(\frac{\sqrt{s_1}}{546}\right)^2 \text{ mb} \quad (18)$$

the correction to $\rho_{p\bar{p}}$ at the ISR energy $\sqrt{s} = 62 \text{ GeV}$ for example is tiny,

$$\Delta \rho_{p\bar{p}}(\sqrt{s}=62 \text{ GeV}) \approx 0.1 \frac{\sigma_{p\bar{p}}(\sqrt{s}=546 \text{ GeV})}{\sigma_{p\bar{p}}(\sqrt{s}=62 \text{ GeV})} \left(\frac{62}{546}\right)^2 = 0.002 \quad (19)$$

In view of the experimental uncertainties in ρ , the constraint Eq. (16) warrants that the contribution from the new threshold at ISR energies to ρ is totally negligible. In fact, Eq. (16) is the only constraint on the threshold parameters C , s_1 and α because the low-energy fitting of the experimental data on both $\sigma_{p\bar{p}}$ and $\rho_{p\bar{p}}$ is more or less independent of the new threshold terms in Eq. (12).

Other than $\sqrt{s_1} > 546 \text{ GeV}$ then, what can we say about these parameters? Indeed, there are infinitely many sets of the threshold parameters possible, as can be seen from Fig. 2, that can satisfy the constraint Eq. (16), even if $\sqrt{s_1}$ is restricted to be no less than 546 GeV. Nevertheless, we will argue in the following that there is a rather narrow range of C values for the total cross-section to receive maximum contributions from the threshold term. This allows us to make a definitive prediction on the upper bound of the total cross-section at any given energy that can easily be tested.

As noted above, the asymptotic behaviour of the total cross-sections is independent of the threshold power α , i.e., the new threshold terms in the total cross-sections merely redefine the parameter A in Eq. (12) by $A+C$. Thus the power α has to be decided from the properties near the threshold region. But the experimental as well as theoretical uncertainties of $\rho_{p\bar{p}}$ are such that the parameter range for

the case $\alpha = 1$ cannot be distinguished from those of α 's between 0 and 3, as one can see from Fig. 3. Although we can consider all α 's as in Figs. 1 and 2, $\alpha > 1$ may be preferred because the total cross-section then has a smooth derivative and the principal value integral $K(s, s_1, \alpha)$ behaves smoothly at the threshold. On the other hand, $K(s, s_1, \alpha)$'s all approach a similar value for $\alpha > 5$ so that we may say that the range of α between 1 and 5 is interesting. For this reason, we present and compare the results of $\alpha = 1$ and $\alpha = 4$ to each other in Figs. 4 and 5.

Figure 2 shows several trends of the solutions to Eq. (16). First of all, for a given α the strength parameter C is nearly proportional to s_1 , i.e., the square of the threshold value. Thus in the case of $\alpha = 2$, for example, one can have not only $C = 27$ mb with $\sqrt{s_1} = 546$ GeV, but also $C = 104$ mb with $\sqrt{s_1} = 1044$ mb [see also Eq. (18)]. Secondly, for a fixed C the higher the power α is the lower the threshold $\sqrt{s_1}$ is. For example, $C = 100$ mb can combine with ($\alpha = 1$, $\sqrt{s_1} = 1270$ GeV) and also with ($\alpha = 4$, $\sqrt{s_1} = 800$ GeV). Thirdly, for a fixed threshold value $\sqrt{s_1}$, a higher α requires a larger value of C . For example, at $\sqrt{s_1} = 546$ GeV, $C = 15$ mb for $\alpha = 1$ while $C = 47$ mb for $\alpha = 4$. These properties are all expected from the fact that the product CK ($\sqrt{s} = 546$ GeV, s_1, α) has to adjust to remain at the same value, as the three parameters vary independently. In fact, because of the first property, one can easily show that the correction due to the new threshold term to the total cross-section becomes maximal at a given s if the threshold is such that

$$\frac{s_1}{s} \cong \frac{1}{\alpha+1} + \frac{4\alpha}{(\alpha+2)(\alpha+3)} \left(\frac{546}{\sqrt{s} \text{ GeV}} \right)^4 \quad (20)$$

The maximal contribution that the new threshold term can make to the total cross-section at a given $s > s_1 > (546 \text{ GeV})^2$ is then

$$\sigma_{\max}^{(1)}(s) \cong C_{\max} \left(\frac{\alpha}{1+\alpha} \right)^\alpha \theta(\alpha s_1) \quad (21)$$

where

$$C_{\max} \cong 9.4248 \left(\frac{\sqrt{s} \text{ GeV}}{546} \right)^2 \text{ mb} \quad (22)$$

Note that $C_{\max}$ is independent of α and all $\sigma_{\max}^{(1)}$ approaches to the value $C_{\max} e^{-1}$ for $\alpha > 5$. Thus we can say that both total cross-sections of Eq. (12) are bounded at a given $s > s_1 > (546 \text{ GeV})^2$ from both below and above by

$$\sigma^{(0)}(s) \leq \sigma_{\text{tot}}(s) \leq \sigma^{(0)}(s) + \sigma_{\max}^{(1)}(s) \quad (23)$$

where $\sigma^{(0)}(s)$ is the respective conventional contribution in Eq. (12). The situation is summarized in Fig. 6, in which the lower curve is the extrapolation of the conventional contributions $\sigma^{(0)}$ that fits excellently the low-energy data, and the

upper curve branched out at $\sqrt{s} = 546$ GeV is $\sigma^{(0)}(s) + \sigma_{\max}^{(1)}(s)$. For example, we get

$$66.61 \text{ mb} \leq \sigma_{\text{tot}}(\sqrt{s} = 900 \text{ GeV}) \leq 79.41 \text{ mb} \quad (24)$$

and

$$75.51 \text{ mb} \leq \sigma_{\text{tot}}(\sqrt{s} = 1.8 \text{ TeV}) \leq 129.08 \text{ mb} \quad (25)$$

These ranges can be compared to some typical predictions based on the solutions of Eq. (16) (see Fig. 2) as given in Tables 1 and 2. These examples are chosen to check the cases when the upper bounds of Eqs. (24) and (25) are nearly saturated, but it should be emphasized that the threshold model is also capable of explaining any value of the total cross-sections within those bounds. If the experiments show the total cross-section to be near the upper bound of Eq. (24), it will imply the threshold to be in the interval of 546 GeV and 666 GeV, while if the upper bound of Eq. (25) is confirmed at $\sqrt{s} = 1.8$ TeV, the threshold can be anywhere between 546 GeV and 1 278 GeV with $C = 104$. On the other hand, if the experiments turn out to be near the lower bound of Eq. (24), we may conclude that the threshold is above 900 GeV. We show in Fig.4, Fig. 5 and Table 3 what one can expect from the threshold model for σ_{tot} and ρ for one such case. Similarly if the total cross-section at 1.8 TeV is found to be closer to the lower bound of Eq. (25), it would entail the threshold to be around 1.8 TeV as shown in Table 4.

In this way, for the general class of the threshold models constrained by Eq. (16) based on the UA4 result, we can predict the possible range of the total cross-sections at higher energies, which can readily be tested at the Tevatrons. Depending on the actual values of the cross-section at those energies, we can deduce the likely value of the threshold, its strength and form. The threshold model appears to be one of the two possible explanations, the other being the odderon model, for the UA4 experiment. In fact, the updated odderon model [9], which is essentially the same as that of the 1975 model (Ref. [3]) except for removing the exchange degeneracy of the Regge terms and relaxing one more freedom of parameter in $\text{Re}F_{\pm}$, predicts the cross-sections numerically closer to the lower bounds of Eqs. (24) and (25). But there are two important aspects of difference: the threshold models as formulated in this paper make σ_{pp}^{-} and σ_{pp}^{+} to approach each other without ever having to cross, and secondly the ρ -parameters are generally larger at Tevatron energies than those of the odderon model. At energies much higher than the threshold region, both ρ_{pp}^{+} and ρ_{pp}^{-} approach each other and decrease somewhat like the ρ_{pp}^{+} predicted from the odderon model. The measurements of σ_{tot} and ρ for both pp and $p\bar{p}$ scattering at Tevatron energies will be important to test the two possibilities.

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α	$\sqrt{s_1}$ (GeV)	C(mb)	σ_{tot} (mb)
1	666	26	78.38
2	563	27	76.62
3	552	36	75.36
4	551	46	73.64
5	551	56	71.96

Table 1

Total cross-section at $\sqrt{s} = 900$ GeV for some typical cases that are close to the upper bound of Eq. (24).

α	$\sqrt{s_1}$ (GeV)	C(mb)	σ_{tot} (mb)
1	1278	104	127.08
2	1044	104	121.30
3	905	104	118.89
4	802	102	117.60
5	743	105	116.77

Table 2

Total cross-section at $\sqrt{s} = 1.8$ TeV for some typical cases that are close to the upper bound of Eq. (25).

	$\alpha=1, C=58\text{mb}, \sqrt{s_1}=960\text{GeV}$		$\alpha=4, C=147\text{mb}, \sqrt{s_1}=960\text{GeV}$	
$\sqrt{s}$ (TeV)	σ_{pp}^- (mb)	ρ_{pp}^-	σ_{pp}^- (mb)	ρ_{pp}^-
0.9	66.61	0.44	66.61	0.40
1.8	117	0.3	114	0.65
4	142	0.15	203	0.28
10	160	0.1	244	0.11
40	188	0.09	277	0.06

Table 3

Typical predictions from the threshold model when σ_{pp}^- is close to the lower bound of Eq. (2).

α	$\sqrt{s_1}$ (GeV)	C(mb)	σ_{tot} (mb)
1	1779	202.61	80.21
2	1680	271.13	80.01
3	1566	314.15	80.02
4	1464	343.22	80.01
5	1376	363.85	80.02

Table 4

Typical cases when σ_{tot} (1.8 TeV) is close to the lower bound of Eq. (25).

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FIGURE CAPTIONS

- Fig. 1 : The principal value integrals $K(s, s_1, \alpha)$ of Eq. (13). The dashed line is the leading order approximation for $s < s_1$. Note that the approximation is better for smaller s/s_1 and larger α .
- Fig. 2 : Solutions of Eq. (16) for $\sqrt{s_1} > 546$ GeV when the central value $\Delta\rho_{pp}^-$ ($\sqrt{s} = 546$ GeV) = 0.1 is used.
- Fig. 3 : Due to the uncertainties in $\Delta\rho_{pp}^-$ ($\sqrt{s} = 546$ GeV), the threshold power $\alpha = 1$ may encompass the parameter regions of C and s_1 from $\alpha = 0$ to $\alpha = 3$. Dashed curves correspond to $\Delta\rho_{pp}^-$ ($\sqrt{s} = 546$ GeV) = 0.14 and 0.06 in Eq. (16).
- Fig. 4a : Total cross-sections for a typical case $\alpha = 1$, $C = 58$ mb and $\sqrt{s_1} = 960$ GeV.
- Fig. 4b : ρ_{pp}^- for the case corresponding to Fig. 4a.
- Fig. 5a : Total cross-sections when $\alpha = 4$, $C = 147$ mb and $\sqrt{s_1} = 960$ GeV.
- Fig. 5b : ρ_{pp}^- for the case considered in Fig. 5a.
- Fig. 6 : The upper and lower bounds of the total cross-sections. The lower curve is the extrapolation of the low-energy fitted form $\sigma^{(0)}$ without the threshold term in Eq. (12) and the upper curve branched out at 546 GeV is the exact maximal contribution $\sigma_{\max}^{(1)}$ which reduces to Eq. (21) to the leading order from the threshold terms added to $\sigma^{(0)}$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4a

Fig. 4b

Fig. 5a

Fig. 5b

Fig. 6